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Particle-Nucleus and Nucleus-Nucleus Scattering at Relativistic Energies

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Topological Structure of Dense Hadronic Matter

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May 2006
Trieste

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1. Introduction
2. A theory for Nuclear Structure
3. Dense Skyrmion Matter
4. Pions in Dense Skyrmion Matter
5. Sliding Vaccua
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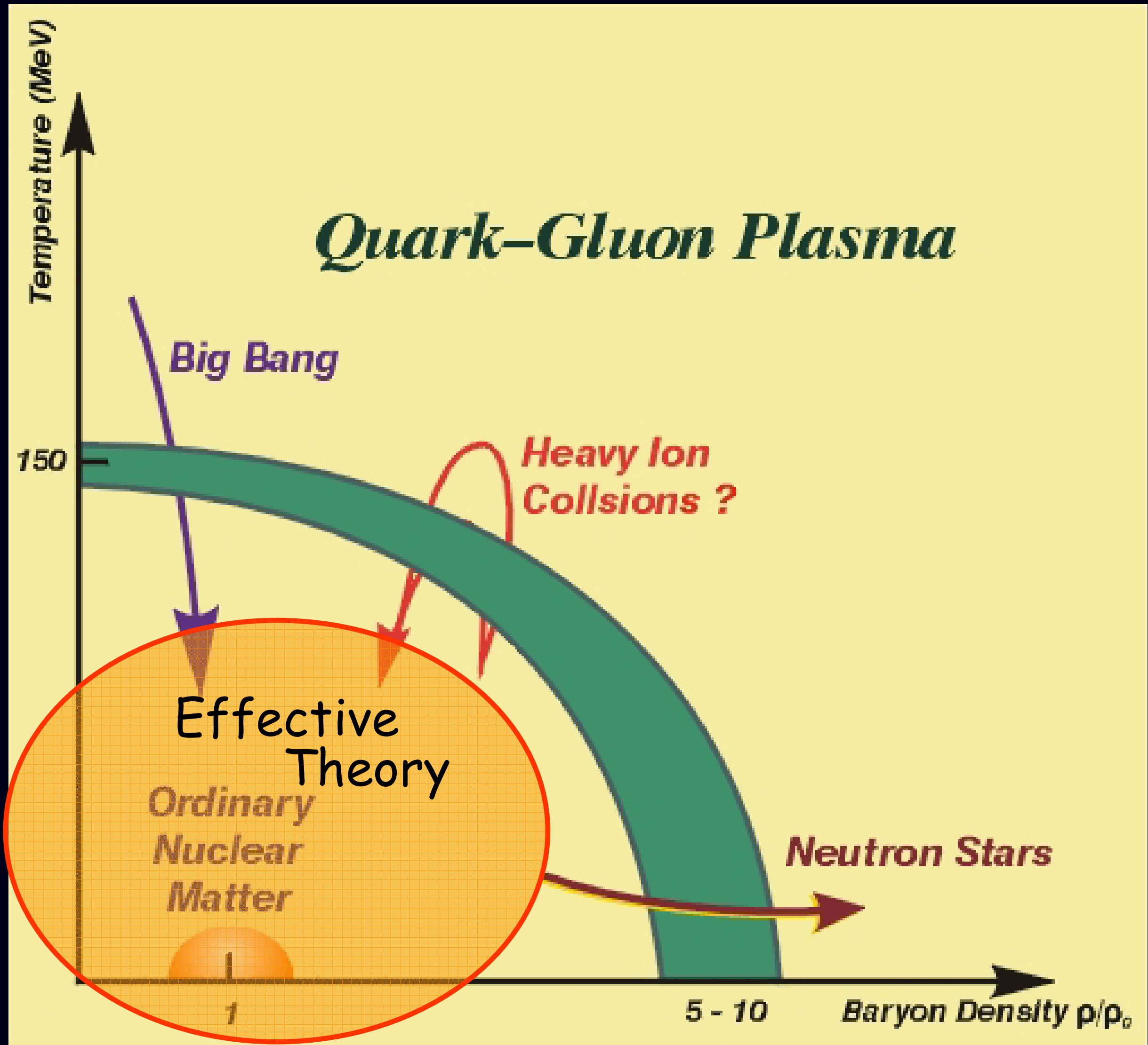
7. Ongoing work
8. Concluding remarks

1. Introduction



Relevant degrees of freedom?

Dynamics?

QCD

Effective
Theory

Observation

Quantum Chromo-Dynamics

↓

Effective Theory
at zero
Temp./Density

↓

Effective Theory
at finite
Temp./Density

2. A theory for Nuclear Structure

Skyrme's Old Idea

1960, T. H. R. Skyrme

$$\mathcal{L} = \frac{f_\pi^2}{4}\mathrm{Tr}(\partial_\mu U^\dagger \partial^\mu U)$$
$$= \frac{1}{2}\partial_\mu \varphi^a \partial^\mu \varphi^a + \cdots$$

← $U_\pi = \exp(i\vec{\tau}\cdot\vec{\varphi}/f_\pi)$

Skyrme's Old Idea

1960, T. H. R. Skyrme

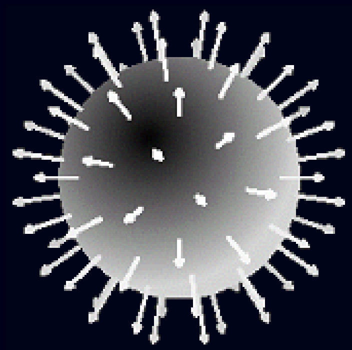
$$\mathcal{L} = \frac{f_\pi^2}{4}\mathrm{Tr}(\partial_\mu U^\dagger \partial^\mu U) + \frac{1}{32e^2}\mathrm{Tr}[U^\dagger \partial_\mu U, U^\dagger \partial_\nu U]^2$$

$U(\vec{x})$: mapping from R^3-{∞}=S^3 to $SU(2)=S^3$

⟶ topological soliton

R ~ 1 f m

M ~ 1.5 GeV



BARYON

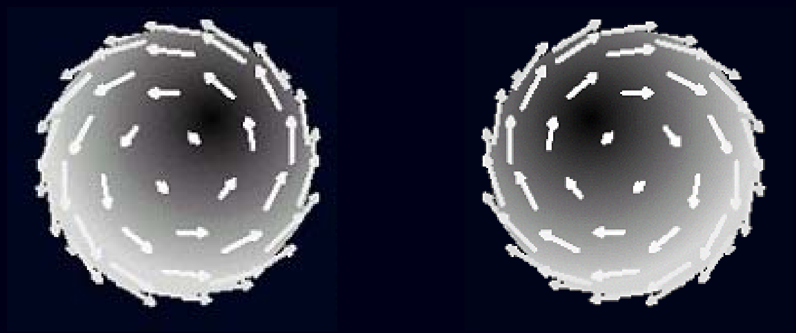
1983 Adkins,Nappi,Witten,….

Two Skyrmions

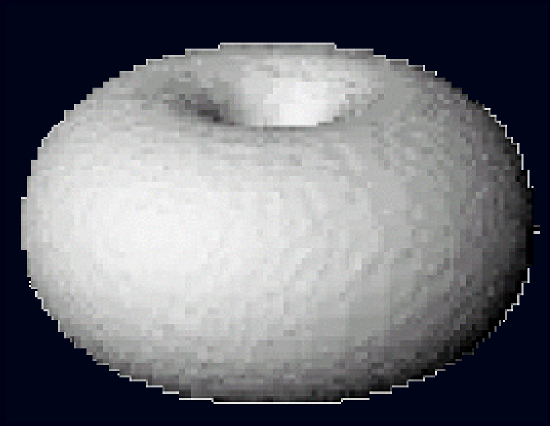
Product Ansatz

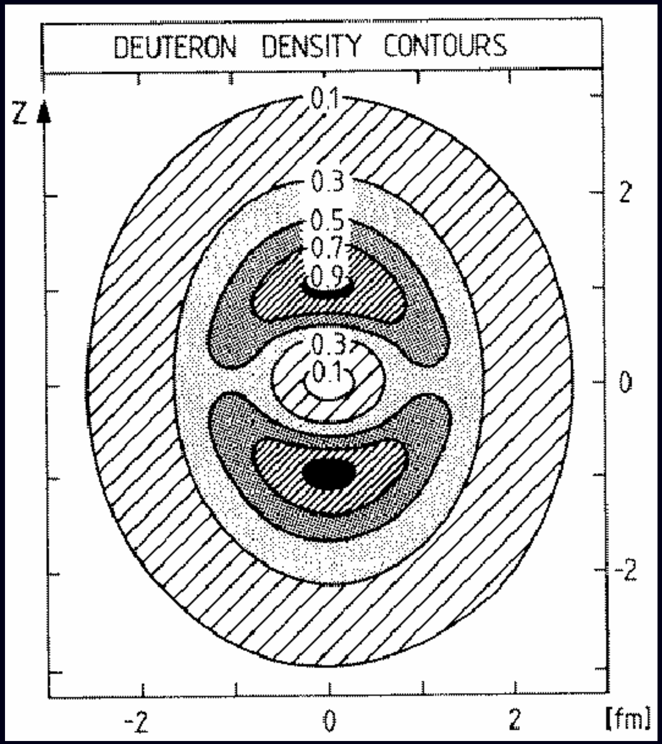
1984 Orsay group, VV,..

$$U(\vec{x}) = AU(\vec{x}-\vec{x}_1)A^\dagger BU(\vec{x}-\vec{x}_2)B^\dagger$$



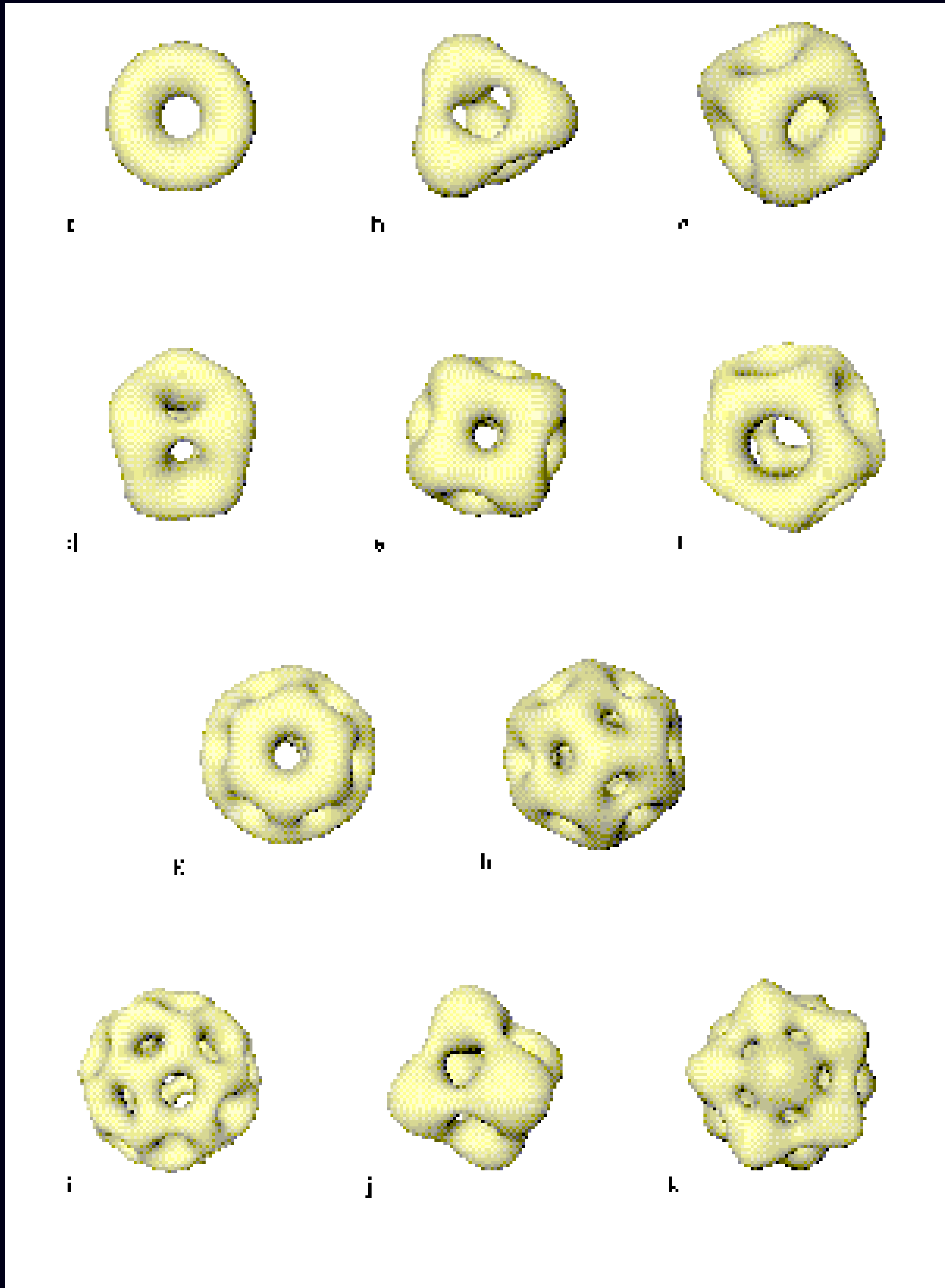
Toroidal B=2 Skyrmion





1988, Braaten,Carson,….

Manton and collaborators



Multi-Skyrmion Systems

(surface of constant baryon number density)

Quantization is progressing

Manton (B=2), Carson (B=3), Walhout (B=4), Irwin (B=4 to 9), Krusch (general formalism!)

PHYSICAL REVIEW D, VOLUME 61, 114024

Zero mode quantization of multi-Skyrmions

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(Received 14 September 1999; published 10 May 2000)

A zero mode quantization of the minimal energy SU(2) Skyrmions for nucleon numbers four to nine and seventeen is described. This involves quantizing the rotational and isorotational modes of the configurations. For nucleon numbers four, six and eight the ground states obtained are in agreement with the observed nuclear states of helium, lithium and beryllium. However, for nucleon numbers five, seven, nine and seventeen the spins obtained conflict with the observed isodoublet nuclear states.

PACS number(s): 12.39.Dc, 02.20.Rt, 14.80.Hv, 21.60.−n

Homotopy of rational maps and the quantization of Skyrmions

Steffen Krusch

Department of Pure Mathematics, School of Mathematics, University of Leeds, Leeds LS2 9JT, UK

Received 5 November 2002

Abstract

The Skyrme model is a classical field theory which models the strong interaction between atomic nuclei. It has to be quantized in order to compare it to nuclear physics. When the Skyrme model is semi-classically quantized it is important to take the Finkelstein–Rubinstein constraints into account. The aim of this paper is to show how to calculate these FR constraints directly from the rational map ansatz using basic homotopy theory. We then apply this construction in order to quantize the Skyrme model in the simplest approximation, the zero mode quantization. This is carried out for up to 22 nucleons and the results are compared to experiment.

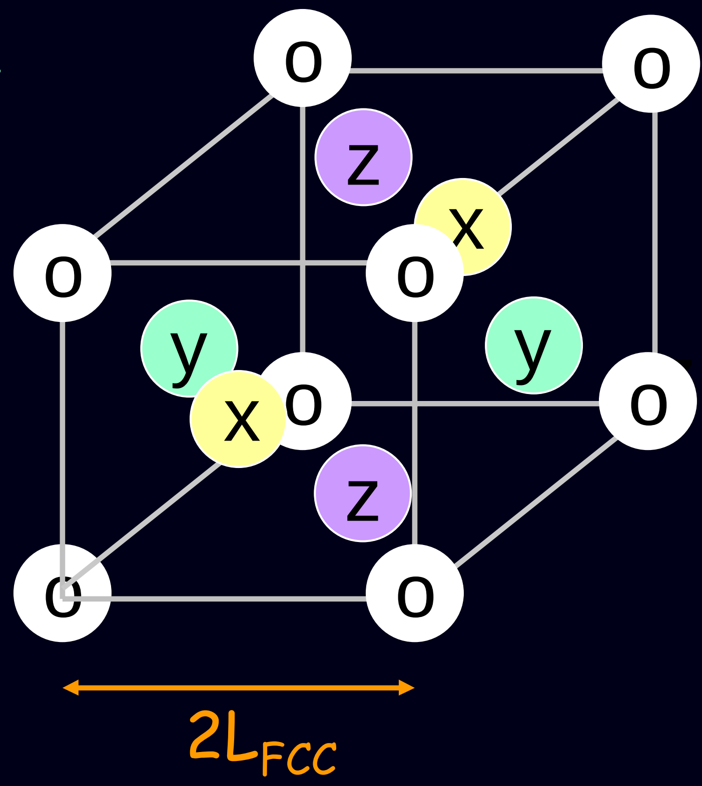
PACS: 12.39.Dc

3. Dense Skyrmion Matter

B.-Y. Park, D.-P. Min, M. Rho, V. Vento,

Skyrmion Crystal

1985, *CC*,
I. Klebanov,
1987, Half-Skyrmion BCC,
A. S. Goldhaber & N. S. Manton
1989, FCC & Half-Skyrmion CC,
L. Castillejo *et al.*,
M. Kugler *et al.*



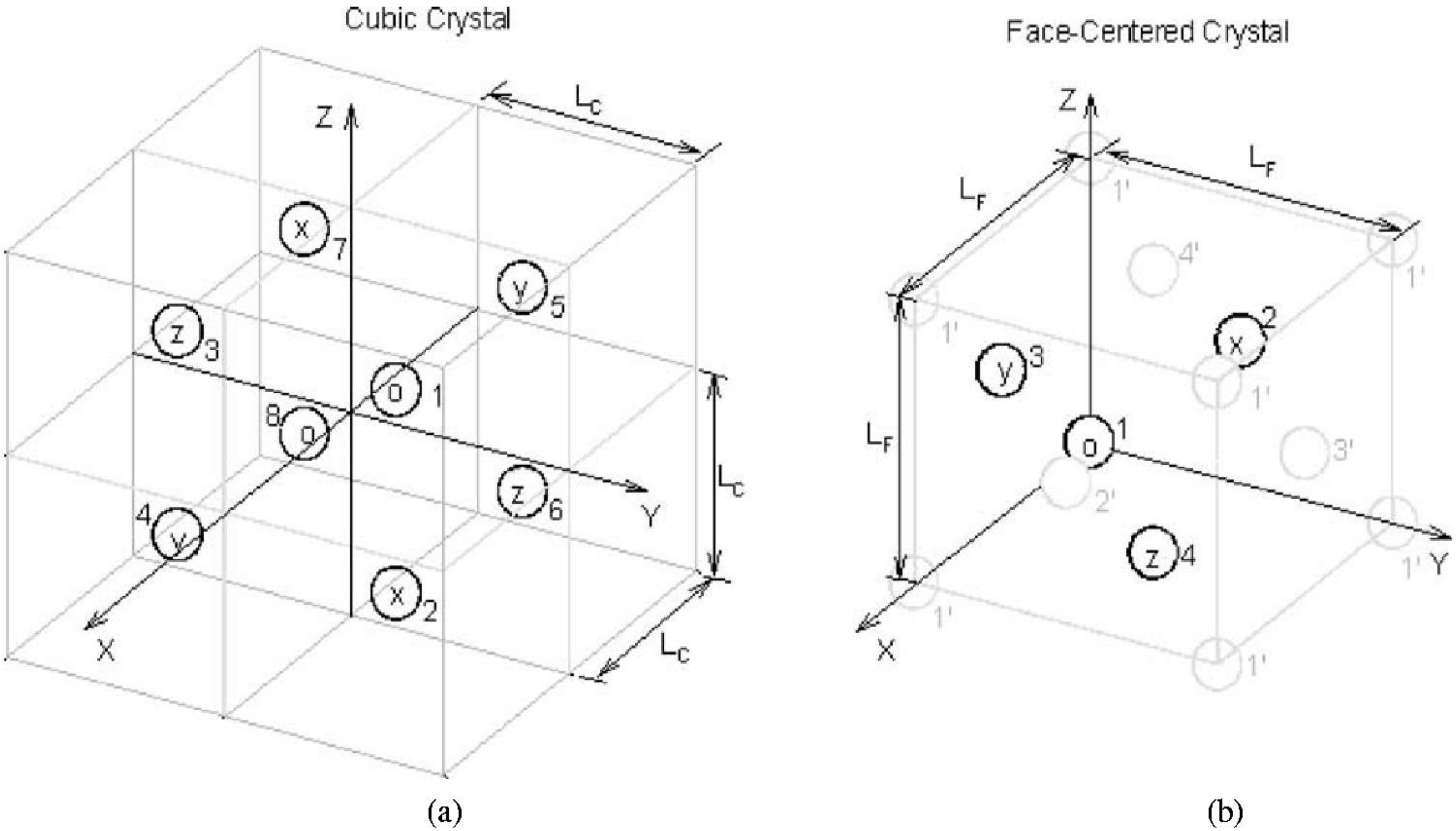
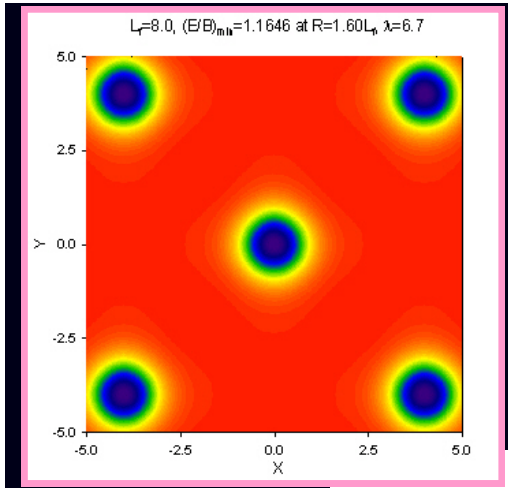
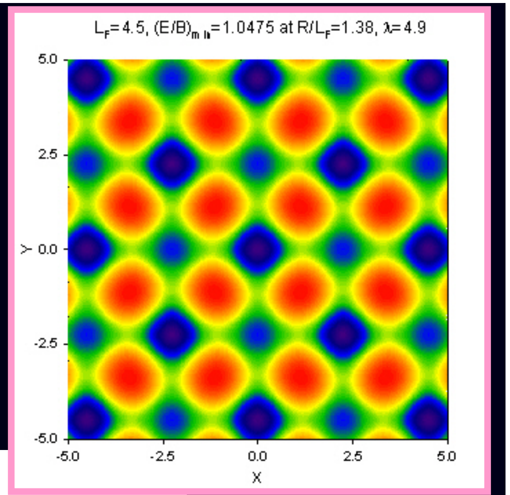


Fig. 1. Face centered instanton crystal. The location of each instanton is denoted by a circle. The letters “x”, “y” and “z” inside the circle mean that the instanton is rotated by π around this axis. “o” means unrotated instanton. The image particles belonging to the neighboring boxes are presented by gray circles and labelled as “i'”, $i = 1, 2, \ldots$.

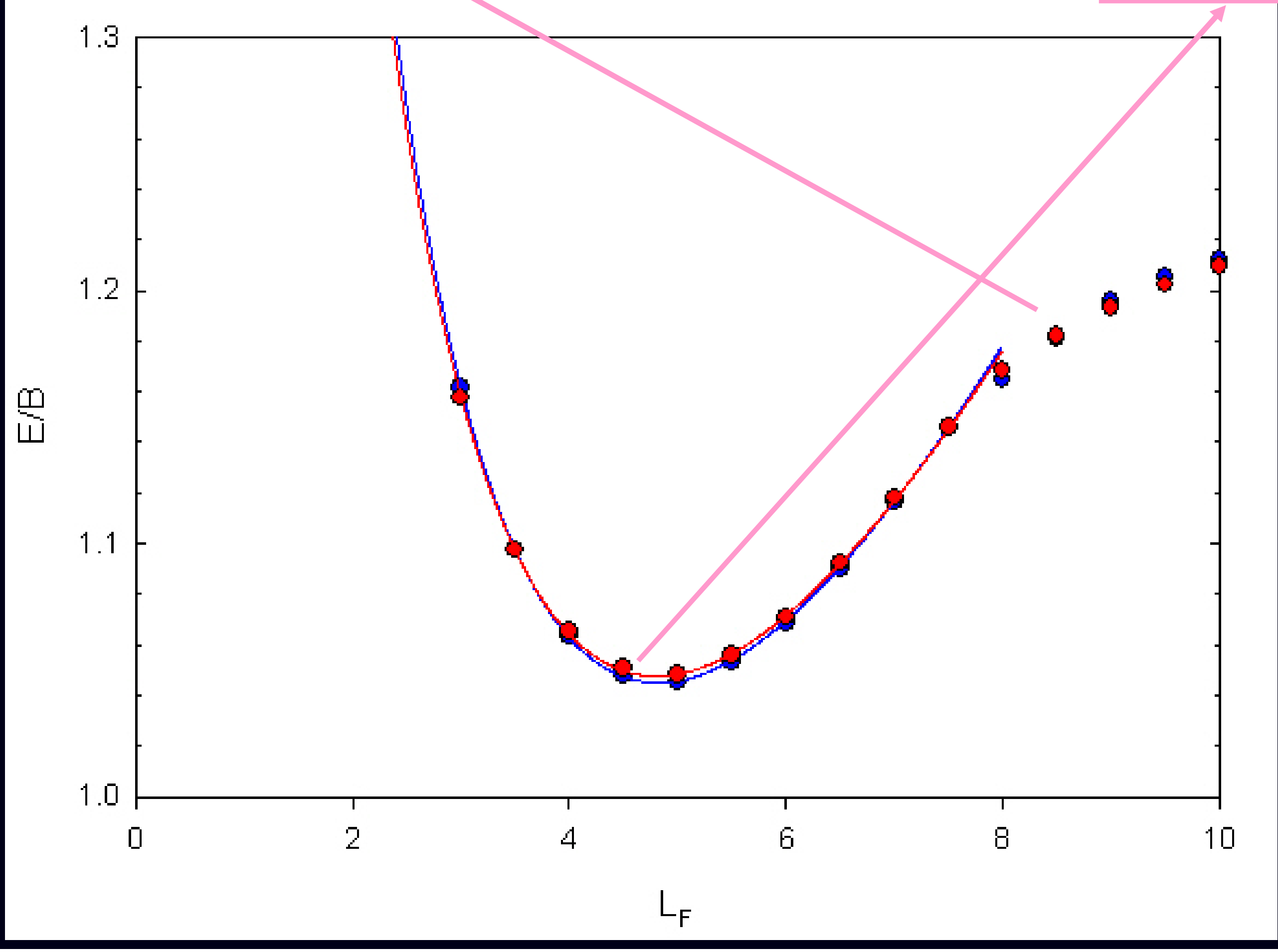
E/B vs. L_F



L_F=8.0, (E/B)_min=1.1646 at R=1.60L_F, λ=6.7
5.0
2.5
0.0
-2.5
-5.0
Y
X

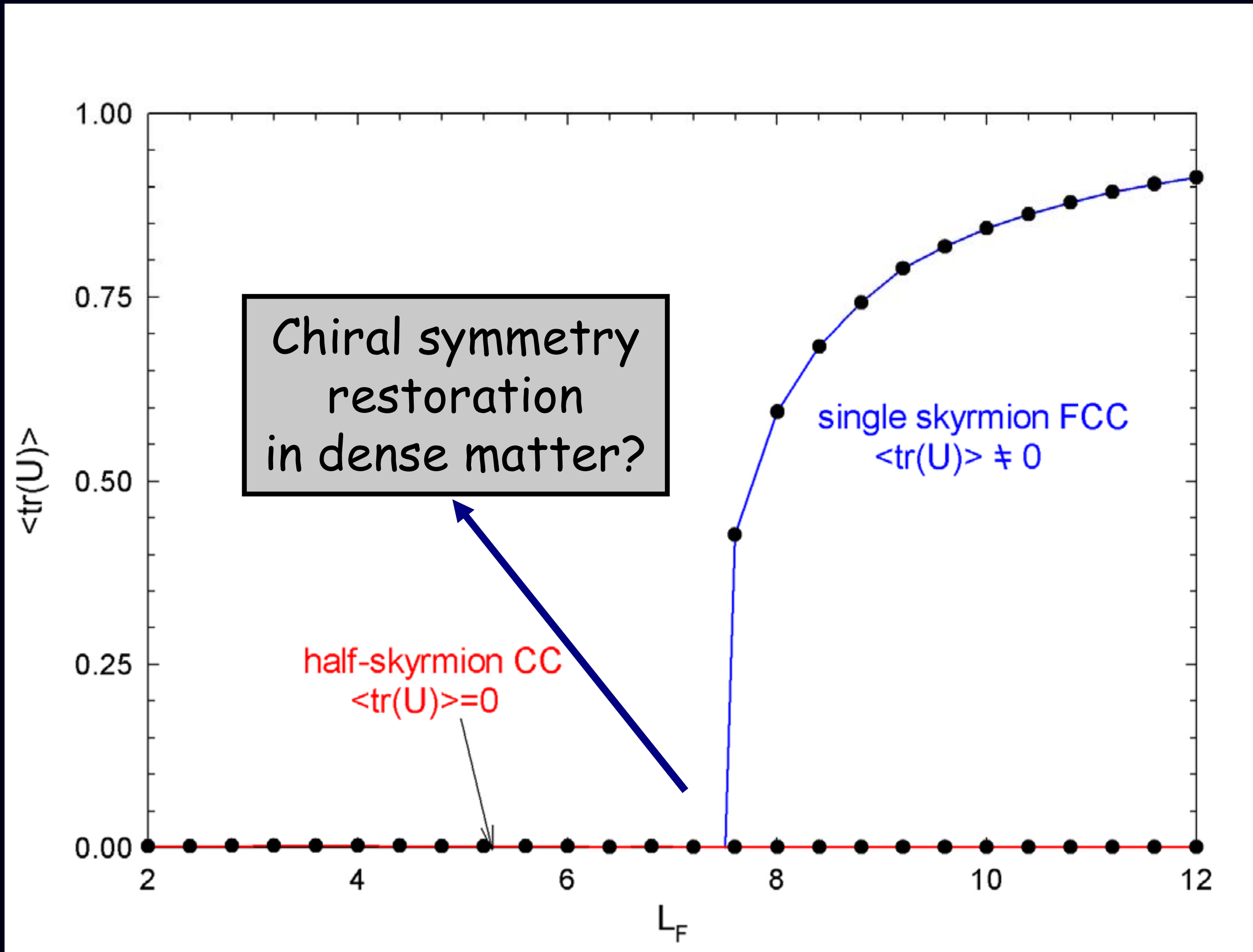


L_F=4.5, (E/B)_min=1.0475 at R/L_F=1.38, λ=4.9
5.0
2.5
0.0
-2.5
-5.0
Y
X



1.3
1.2
1.1
1.0
E/B
0
2
4
6
8
10
L_F

<tr(U)>



Chiral symmetry restoration in dense matter?
single skyrmion FCC
<tr(U)> ≠ 0
half-skyrmion CC
<tr(U)>=0
1.00
0.75
0.50
0.25
0.00
<tr(U)>
2
4
6
8
10
12
L_F

3. Pions in Dense Skyrmion Matter

B.-Y. Park, H.-J. Lee, M. Rho, V. Vento

Skyrme Model($m_\pi \neq 0$)

$$\mathcal{L} = \frac{f_\pi^2}{4}\mathrm{Tr}(\partial_\mu U^\dagger \partial^\mu U) + \frac{1}{32e^2}\mathrm{Tr}[U^\dagger\partial_\mu U, U^\dagger \partial_\nu U]^2$$
$$+ \frac{f_\pi^2 m_\pi^2}{4}\mathrm{Tr}(U + U^\dagger - 2)$$

$$U_\pi = \exp(i\vec{\tau}\cdot\vec{\varphi}/f_\pi)$$

$$\mathcal{L} = \tfrac{1}{2}\partial_\mu\varphi_a\partial^\mu\varphi_a + \tfrac{1}{2}m_\pi^2\varphi_a\varphi_a + \cdots$$

π dynamics($\rho\mp 0$)

$$\mathcal{L} = \frac{f_\pi^2}{4}\mathrm{Tr}(\partial_\mu U^\dagger \partial^\mu U) + \frac{1}{32e^2}\mathrm{Tr}[U^\dagger\partial_\mu U, U^\dagger \partial_\nu U]^2$$

$$+ \frac{f_\pi^2 m_\pi^2}{4}\mathrm{Tr}(U + U^\dagger - 2)$$

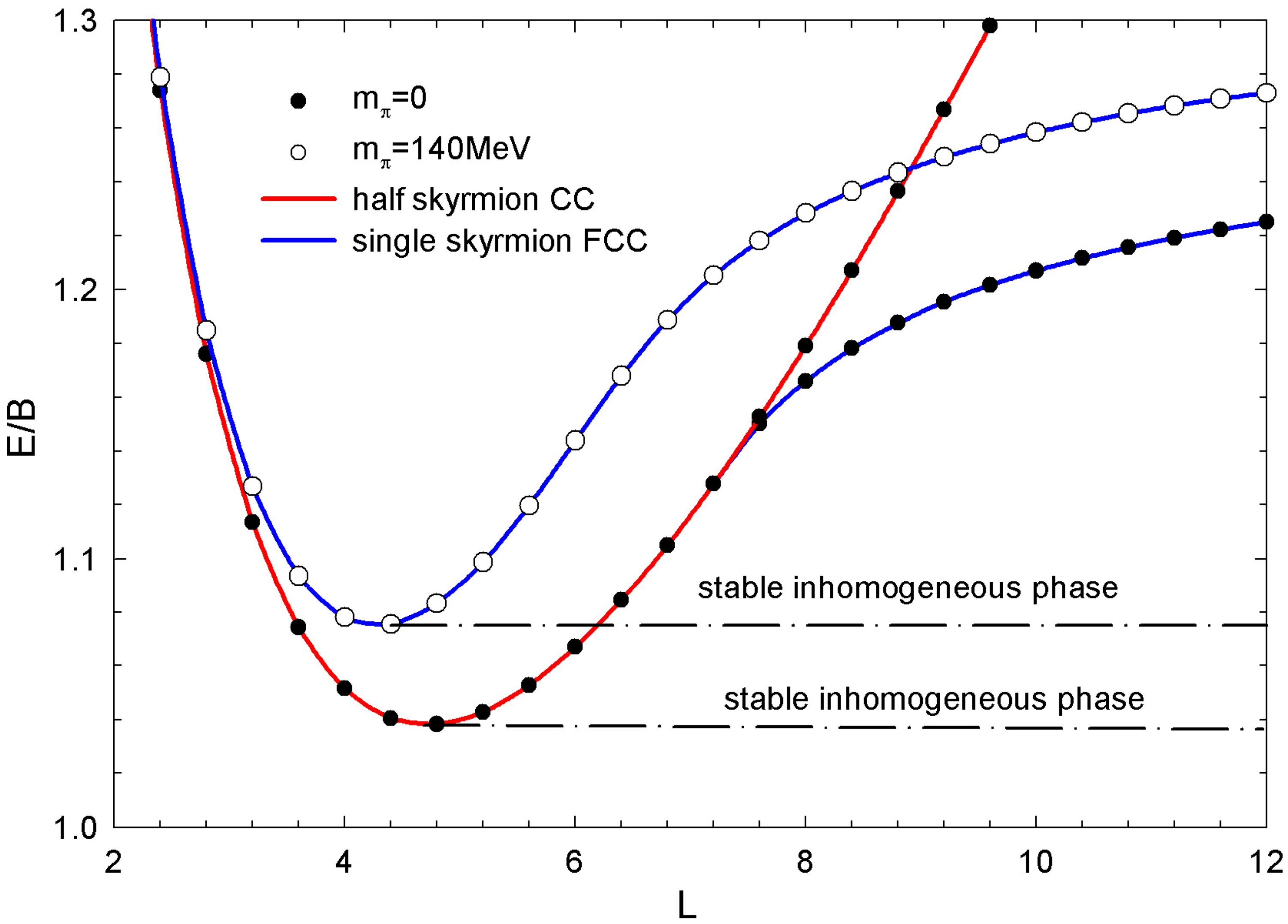
Skyrmion matter

$$U = \sqrt{U_\pi}\, U_0 \sqrt{U_\pi}$$

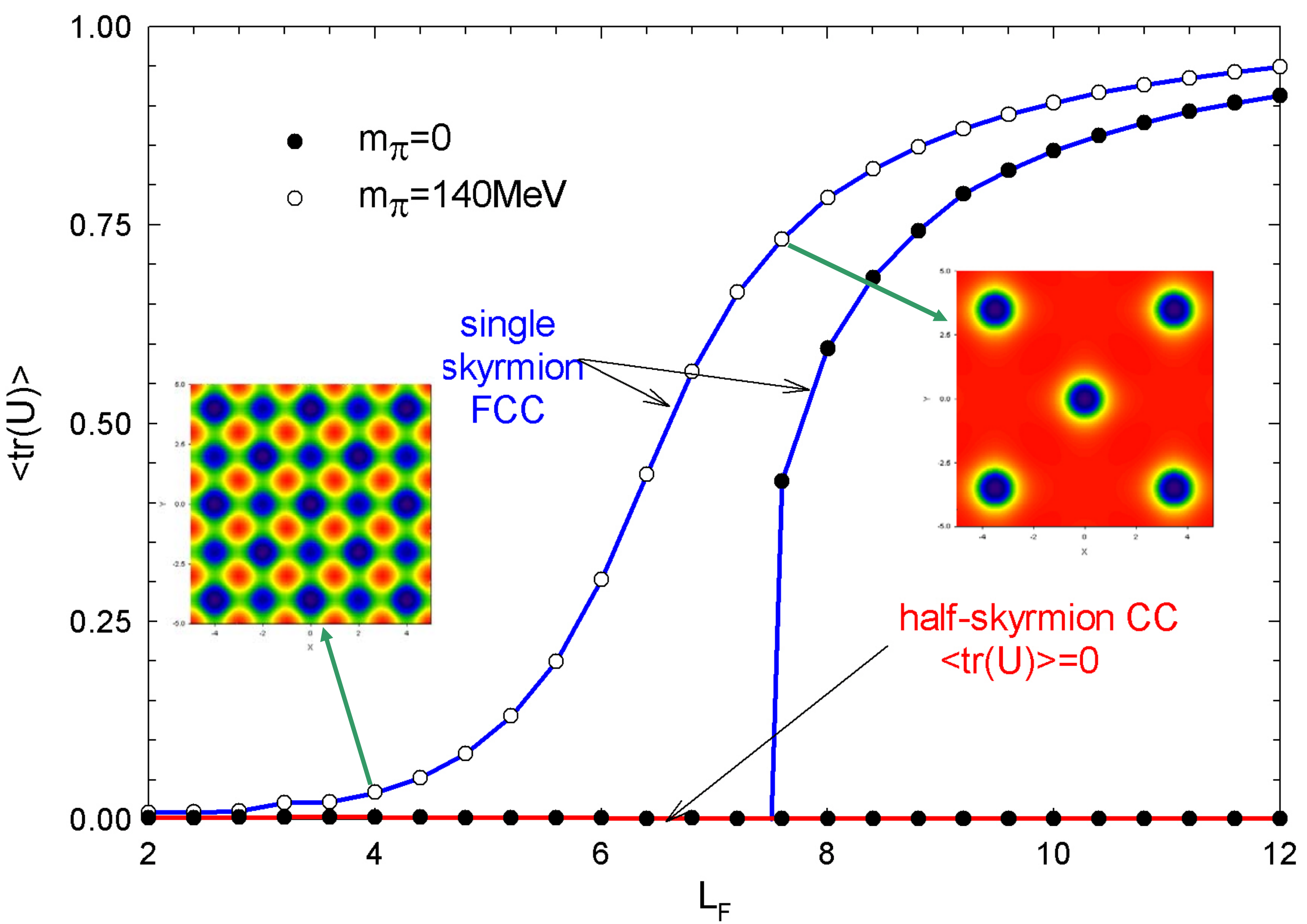
Pion fluctuations on top of the skyrmion matter

$$\mathcal{L} = \frac{1}{2}\partial_\mu \varphi_a \partial^\mu \varphi_a + \frac{1}{2} m_\pi^2 \varphi_a \varphi_a + \cdots$$

+ π-skyrmion matter interactions



1.3
1.2
1.1
1.0
E/B
m_π=0
m_π=140MeV
half skyrmion CC
single skyrmion FCC
stable inhomogeneous phase
stable inhomogeneous phase
2
4
6
8
10
12
L



1.00
0.75
0.50
0.25
0.00
2
4
6
8
10
12
<tr(U)>
L_F
m_π=0
m_π=140MeV
single
skyrmion
FCC
half-skyrmion CC
<tr(U)>=0

π dynamics($\rho \neq 0$)

$$\mathcal{L} = \frac{1}{2}G^{ab}(\vec{x})\partial_\mu\varphi_a\partial^\mu\varphi_b + \frac{1}{2}m_\pi^2\sigma(\vec{x})\varphi_a\varphi_a + \varepsilon_{abc}\partial_i\varphi_a\varphi_b V_i^c(\vec{x})$$

$$f_\pi^*(\vec{x}) \sim f_\pi \frac{1}{3}(1 + 2\langle\sigma(\vec{x})^2\rangle)^{\frac{1}{2}}$$

Wavefunction renormalization constant Z^{-1}

$$m_\pi^* = -m_\pi \left(\frac{\langle\sigma\rangle}{\frac{1}{3} + \frac{2}{3}\langle\sigma^2\rangle} \right)^{\frac{1}{2}}$$

Chiral Symmetry Restoration

π-N sigma term

$$\frac{\langle \bar{q}q \rangle_\rho}{\langle \bar{q}q \rangle_{\rho=0}} = 1 - \frac{\Sigma_{\pi N}}{m_\pi^2 f_\pi^2}\rho + \cdots$$

nuclear matter density

pion properties in dense medium?

GellMann-Oakes-Renner Relation

$$f_\pi^2 m_\pi^2 = -m_q \langle \bar{q}q \rangle_{\rho=0}$$

$$f_\pi^{*2} m_\pi^{*2} \approx -m_q \langle \bar{q}q \rangle_\rho$$

Chiral symmetry restoration

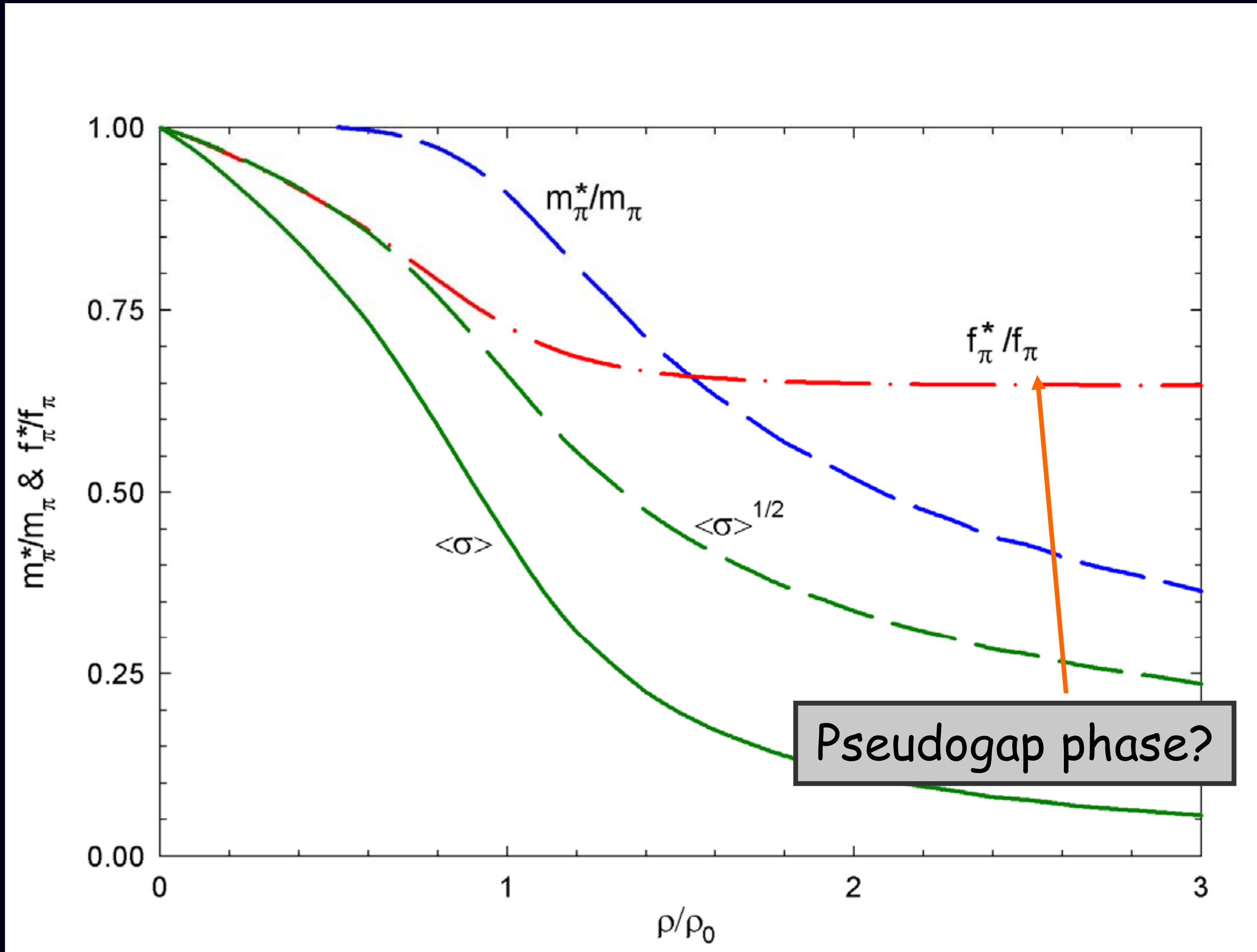
GellMann-Oakes-Renner Relation

$$f_\pi^2 m_\pi^2 = -m_q \langle \bar{q} q \rangle_{\rho=0}$$

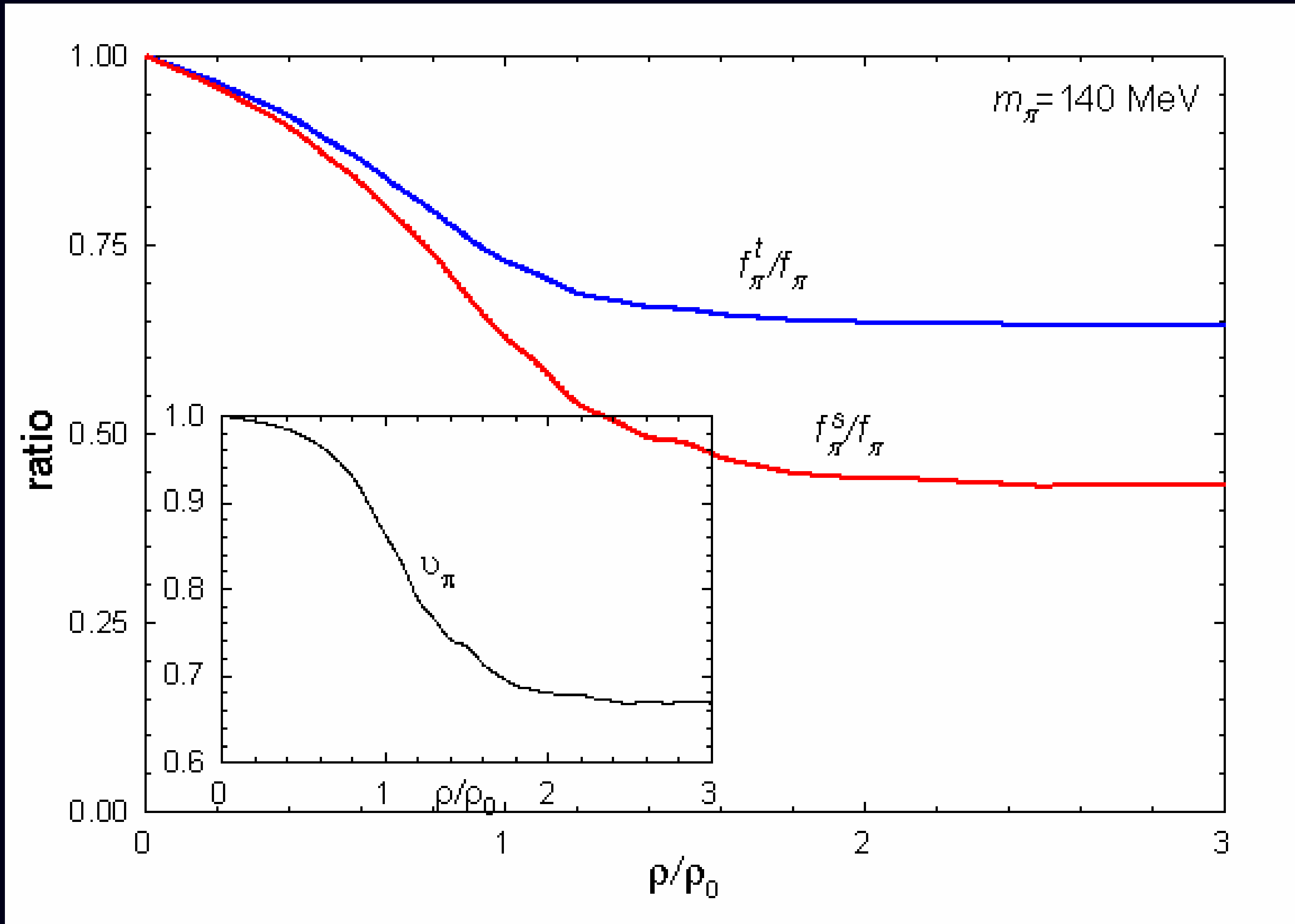
$$f_\pi^{*2} m_\pi^{*2} \approx -m_q \langle \bar{q} q \rangle_\rho$$

pion condensation?

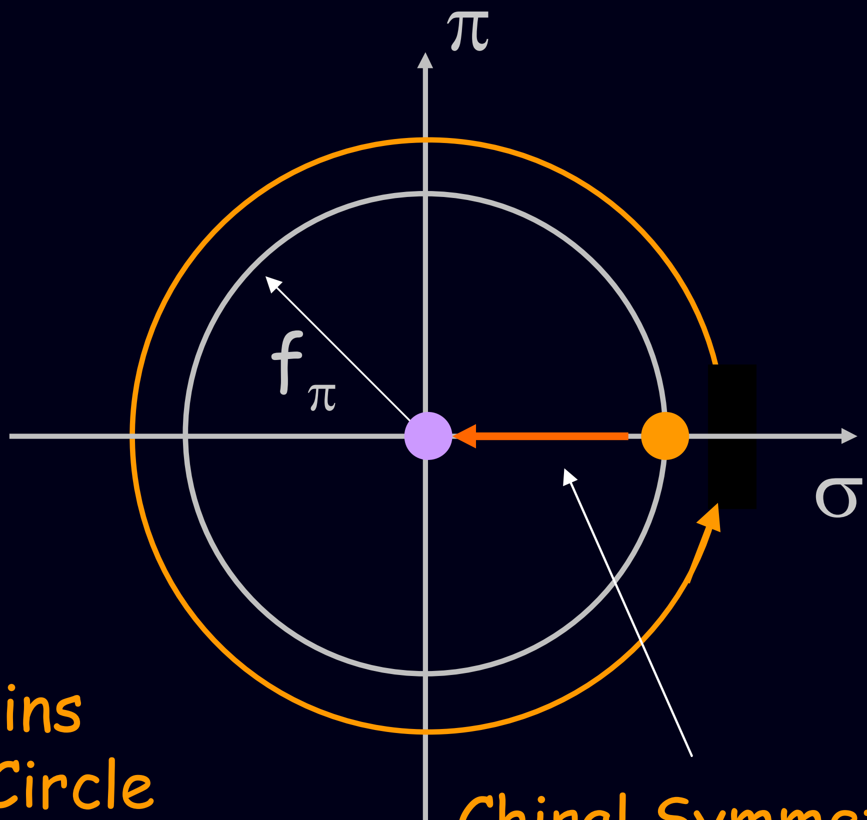
Pion effective mass



Pion velocity in medium



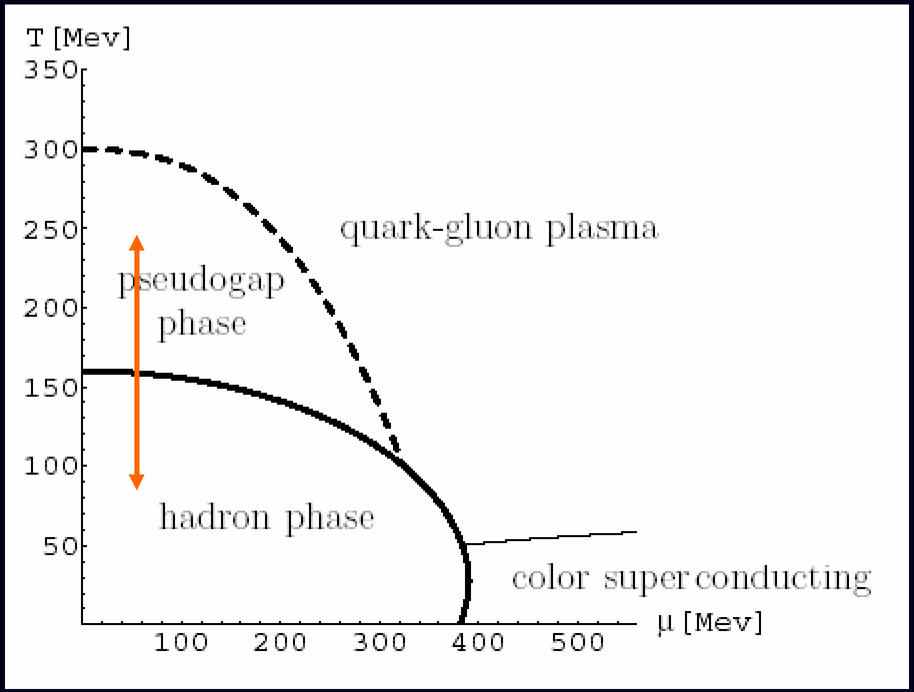
Pseudogap?



Chiral Symmetry Restoration

U still remains
on the Chiral Circle
But <U>=0

Zarembo, hep-ph/0104305



Zarembo, hep-ph/0104305

4. Sliding Vacuua

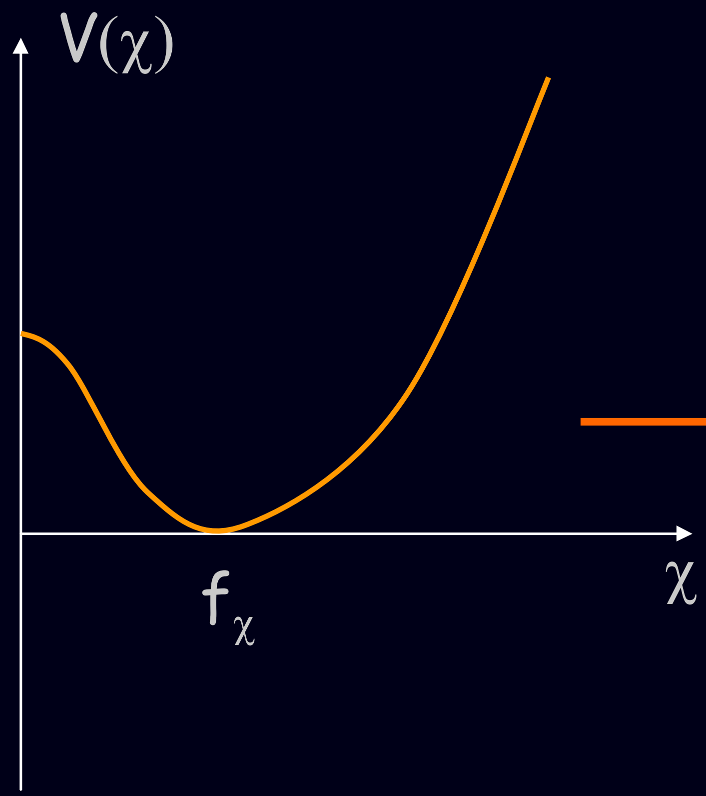
B.-Y. Park, H.-J. Lee, M. Rho, V. Vento

Skyrme Lagrangian

Ellis & Lanik, PLB(1985)

$$\mathcal{L} = \frac{f_\pi^2}{4}\left(\frac{\chi}{f_\chi}\right)^2 \mathrm{Tr}(\partial_\mu U \partial^\mu U) + \frac{1}{32e^2}\mathrm{Tr}[U^\dagger\partial_\mu U, U^\dagger\partial_\nu U]^2$$
$$+\frac{f_\pi^2 m_\pi^2}{4}\left(\frac{\chi}{f_\chi}\right)^3 \mathrm{Tr}(U^\dagger + U - 2)$$
$$+\tfrac{1}{2}\partial_\mu\chi\partial^\mu\chi - \tfrac{1}{4}m_\chi^2 f_\chi^2\left((\chi^4/f_\chi)^4(\ln(\chi/f_\chi) - \tfrac{1}{4}) + \tfrac{1}{4}\right)$$

m_χ ~ 720 MeV, f_χ~240 MeV

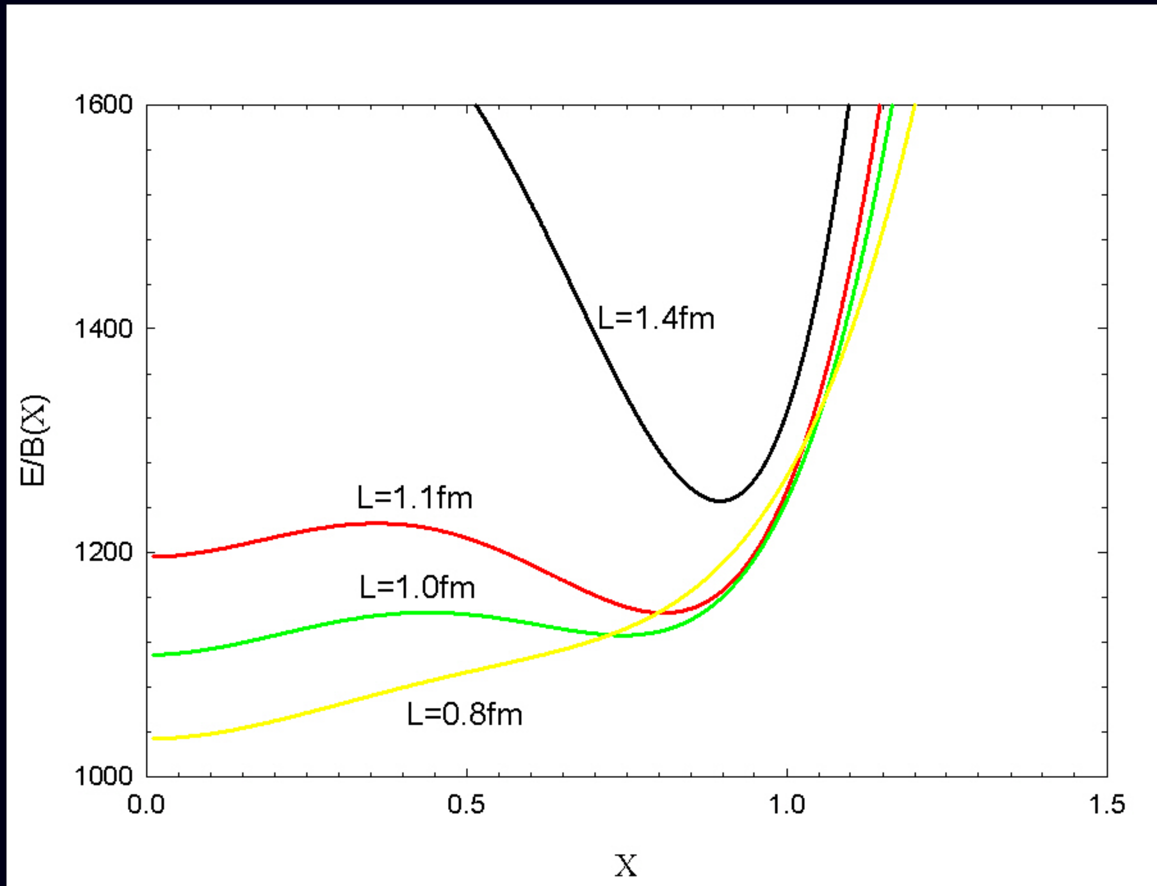


Vacuum ($\rho=0$)

$U=1$

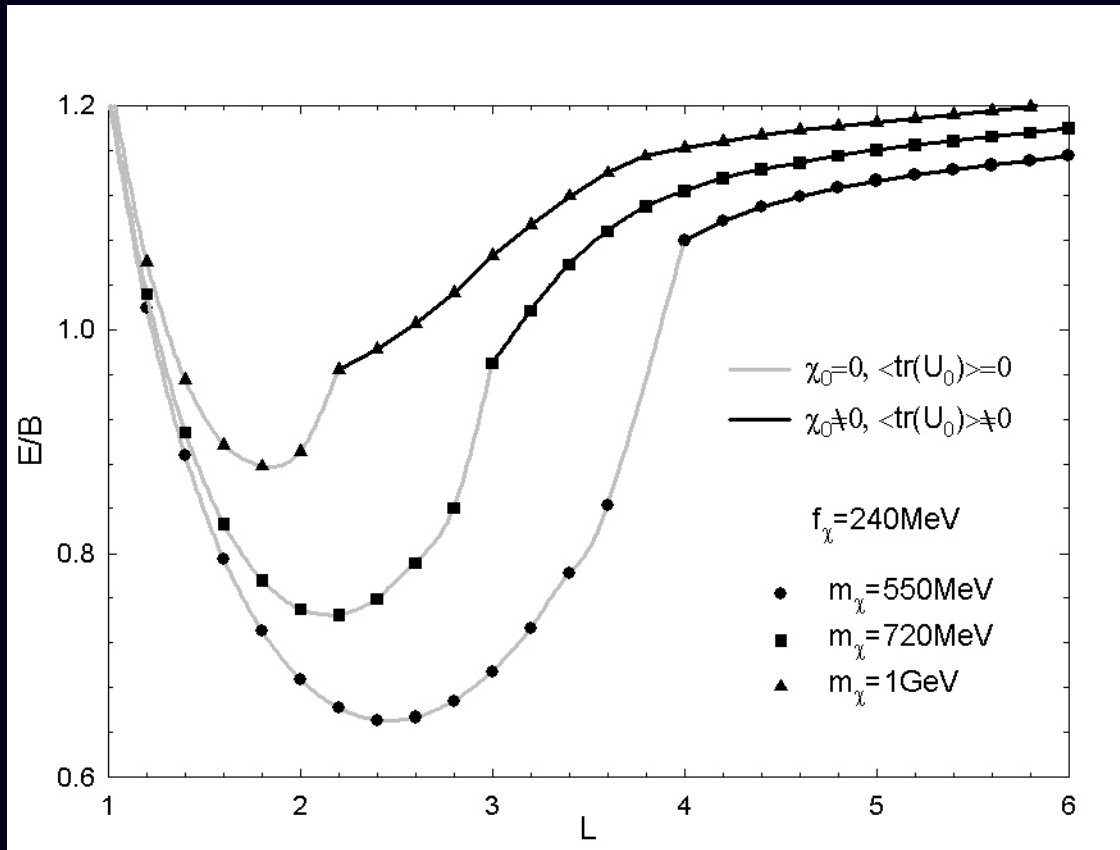
$\chi=f_\chi$

Naive Estimate

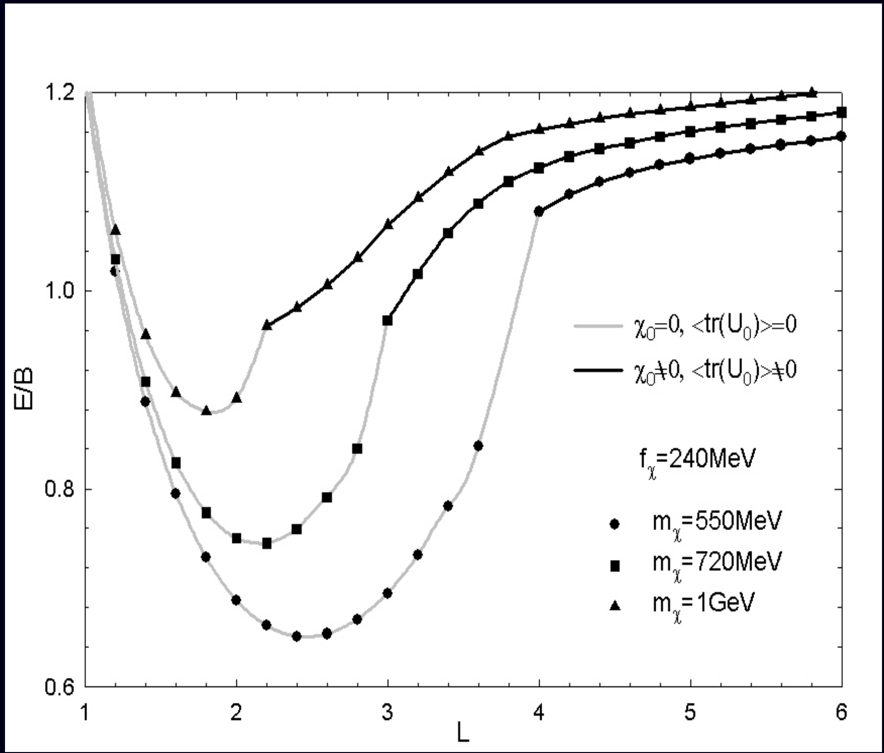


$$E/B = M_2(L)X^2 + M_4(L) + M_m(L)\,X^3 + V(X)$$

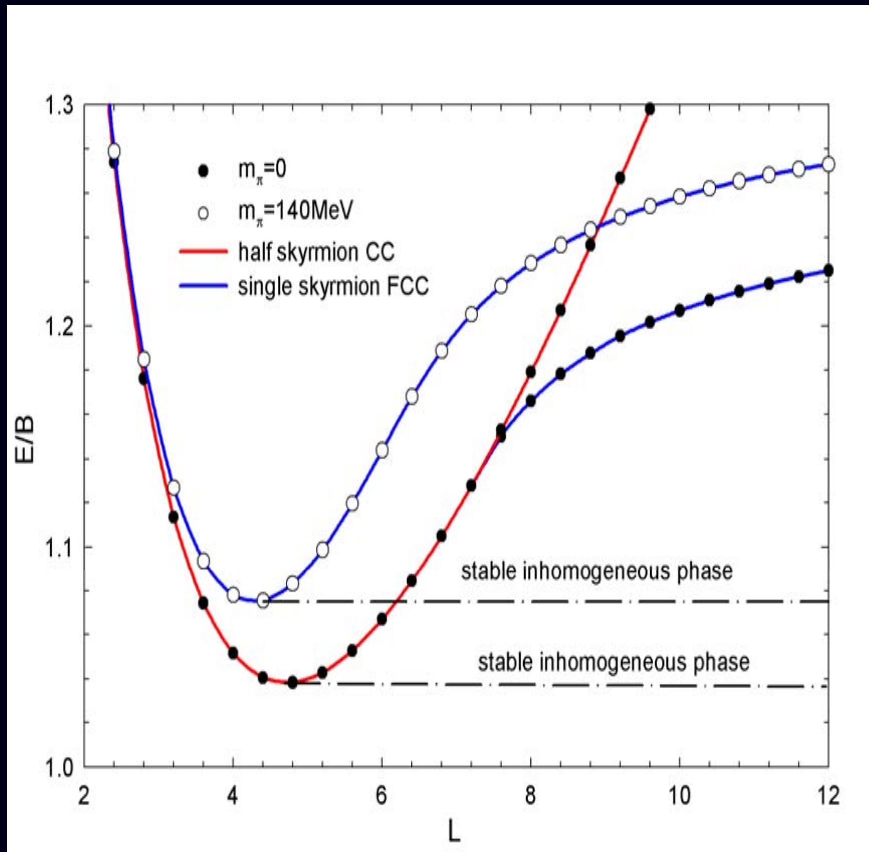
E/B



1.2
1.0
0.8
0.6
E/B
1
2
3
4
5
6
L
χ0=0, <tr(U0)>=0
χ0≠0, <tr(U0)>≠0
fχ=240MeV
mχ=550MeV
mχ=720MeV
mχ=1GeV

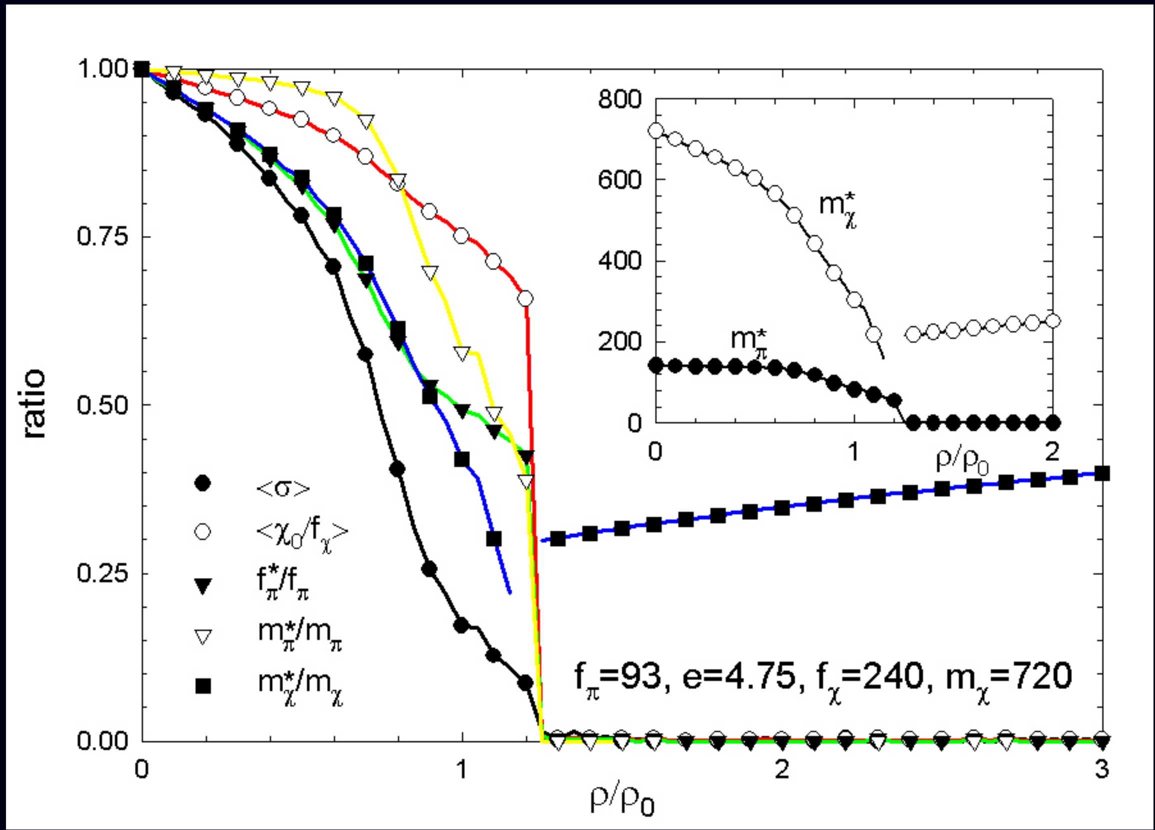


E/B
L
χ0=0, <tr(U0)>=0
χ0≠0, <tr(U0)>≠0
fχ=240MeV
mχ=550MeV
mχ=720MeV
mχ=1GeV



E/B
L
mπ=0
mπ=140MeV
half skyrmion CC
single skyrmion FCC
stable inhomogeneous phase
stable inhomogeneous phase

In-medium quantities



5. Vector Mesons

Hidden Local Gauge Symmetry

$$\mathcal{L} = \frac{f_\pi^2}{4}\mathrm{Tr}(\partial_\mu U \partial^\mu U) + \frac{f_\pi^2 m_\pi^2}{4}\mathrm{Tr}(U^\dagger + U - 2)$$

Trace Anomaly

dilaton

HLG

rho & omega vector mesons

+ Vector Meson Dominance

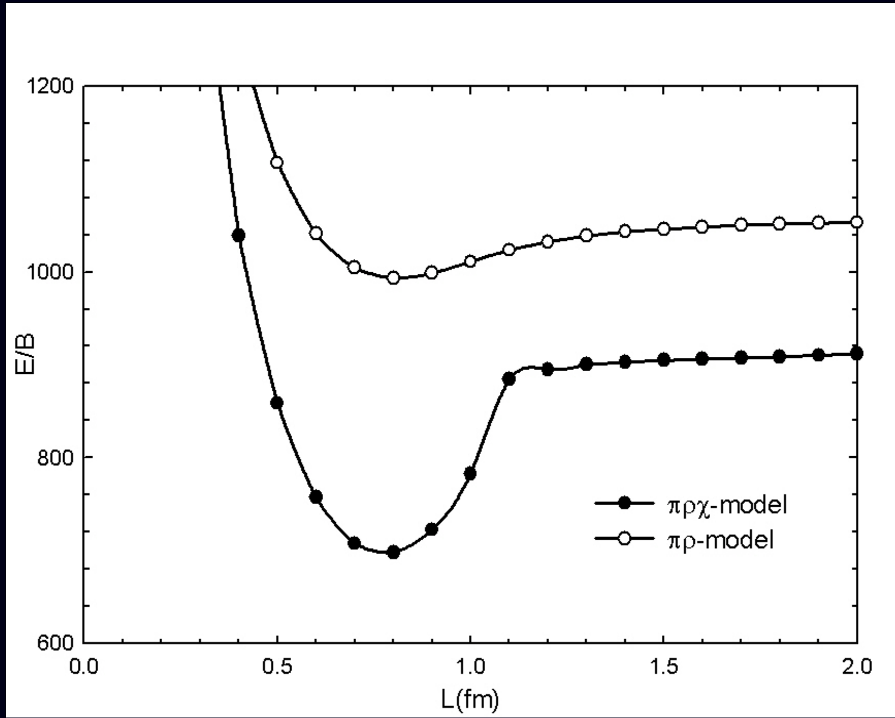
pions, chi, rho and omega

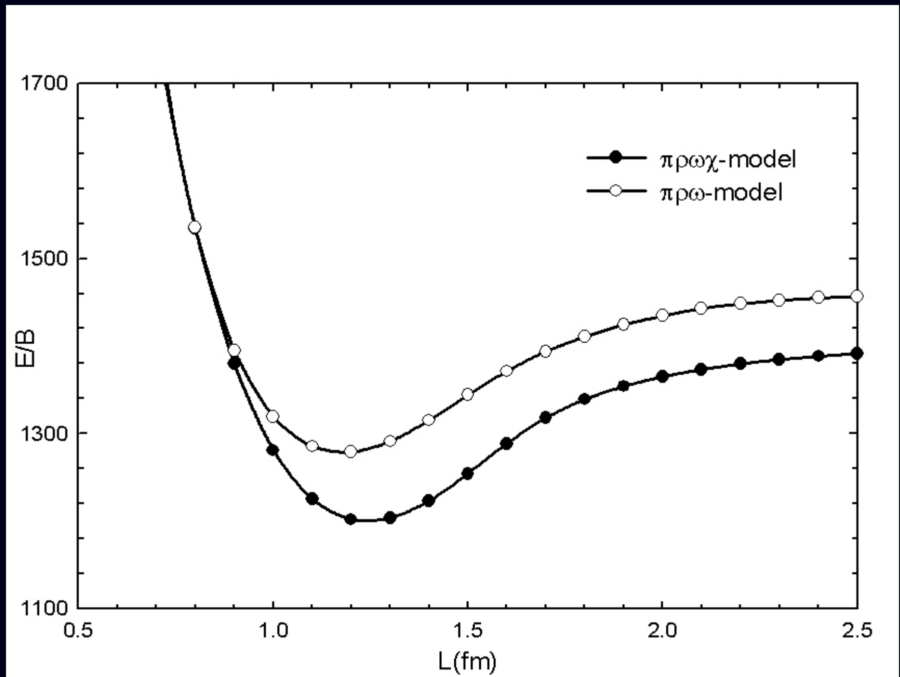
$$
\begin{aligned}
\mathcal{L} \;=\;& \frac{f_\pi^2}{4}\left(\frac{\chi}{f_\chi}\right)^2 \mathrm{Tr}(\partial_\mu U^\dagger \partial^\mu U) + \frac{f_\pi^2 m_\pi^2}{4}\left(\frac{\chi}{f_\chi}\right)^3 \mathrm{Tr}(U + U^\dagger - 2) \\
& - \frac{f_\pi^2}{4} a \left(\frac{\chi}{f_\chi}\right)^2 \mathrm{Tr}[\ell_\mu + r_\mu + i(g/2)(\vec{\tau}\cdot\vec{\rho}_\mu + \omega_\mu)]^2 \\
& - \tfrac{1}{4}\vec{\rho}_{\mu\nu}\cdot\vec{\rho}^{\mu\nu} - \tfrac{1}{4}\omega_{\mu\nu}\omega^{\mu\nu} + \tfrac{3}{2} g \omega_\mu B^\mu \\
& + \tfrac{1}{2}\partial_\mu\chi\partial^\mu\chi - \frac{m_\chi^2 f_\chi^2}{4}\left[(\chi/f_\chi)^4(\ln(\chi/f_\chi) - \tfrac{1}{4}) + \tfrac{1}{4}\right]
\end{aligned}
$$

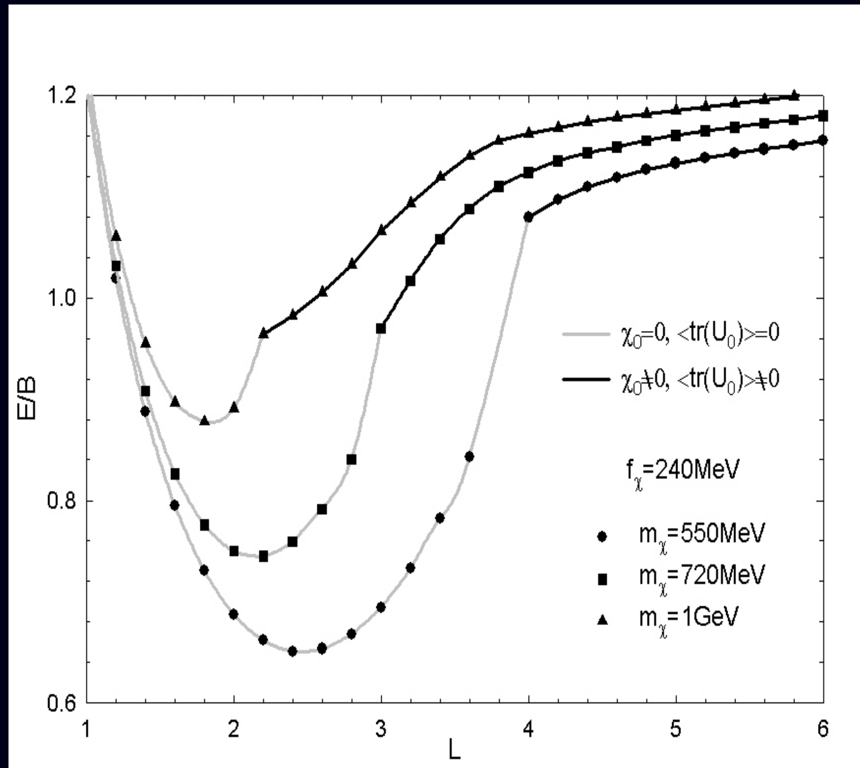
KSRF relation : $m_V^2 = a f_\pi^2 g^2$

Bando, Kugo, Yamawaki, Phys. Rep. (1988)

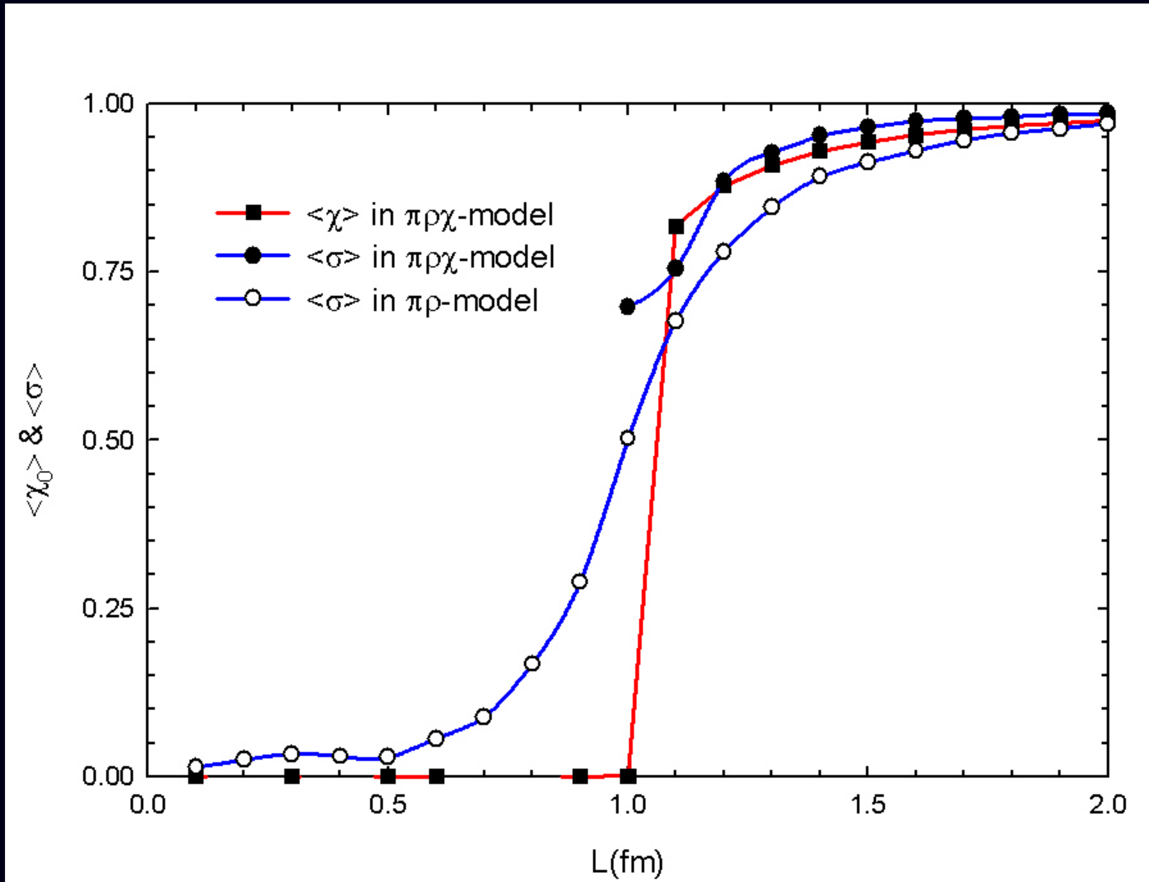
E/B


E/B
L(fm)
πρχ-model
πρ-model

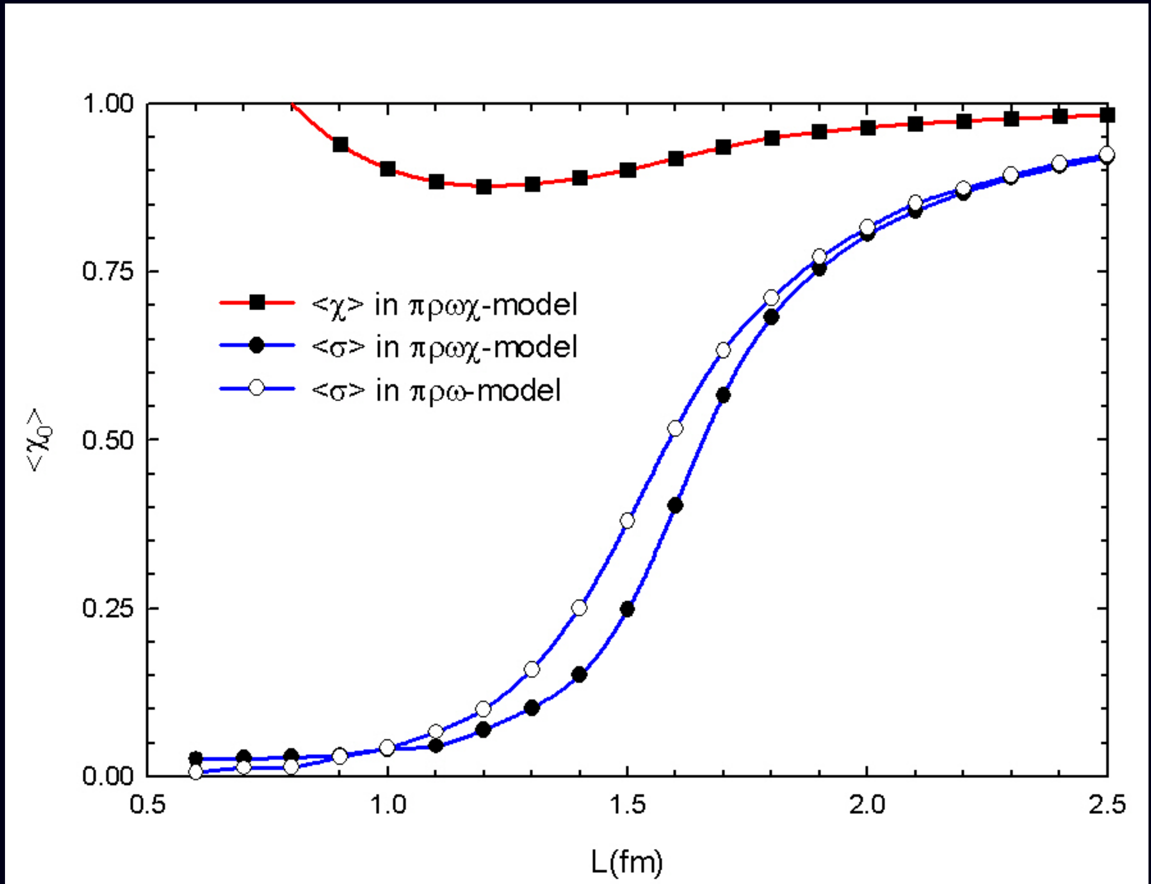

E/B
L(fm)
πρωχ-model
πρω-model


E/B
L
χ0=0, <tr(U0)>=0
χ0≠0, <tr(U0)>≠0
fχ=240MeV
mχ=550MeV
mχ=720MeV
mχ=1GeV

<σ> & <χ> without Omega



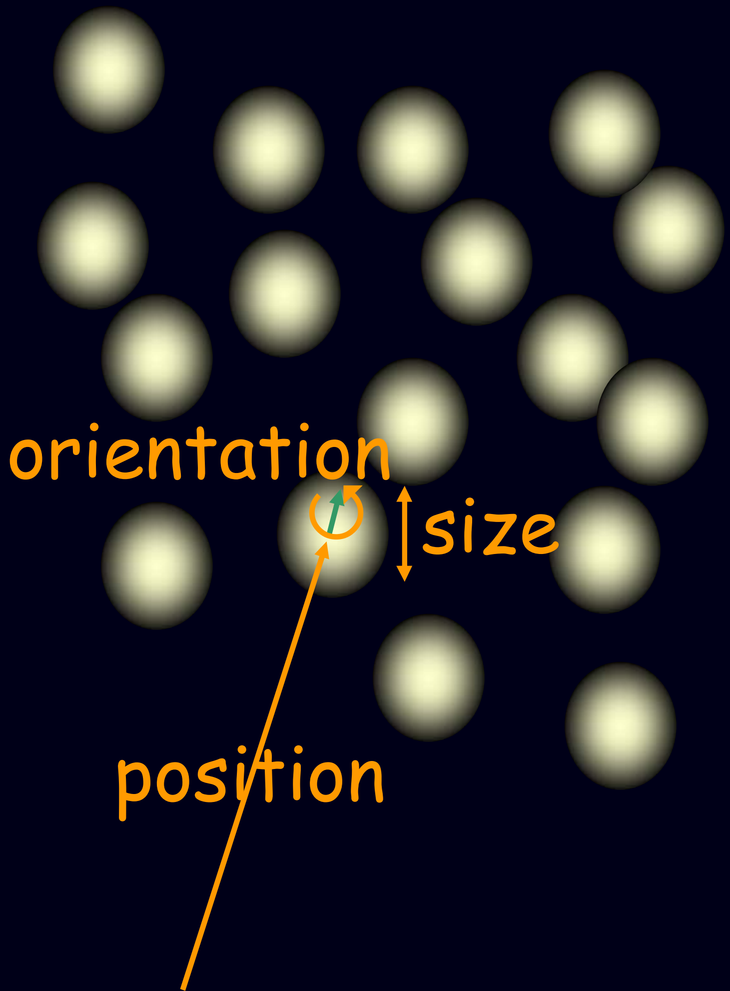
<σ> & <χ> with Omega



6. Ongoing work

Skyrmion from Instanton

1989, M. F. Atiyah & N. S.Manton



Introduce variables describing the single skyrmion dynamics

↓

Classical mechanics
Statistical mechanics
Quantum mechanics

$$U(\vec{x}) = C\mathcal{S}\left\{\mathcal{P}\exp\left[\int_{-\infty}^{\infty} -A_4(\vec{x},t)dt\right]\right\}C^{\dagger}$$

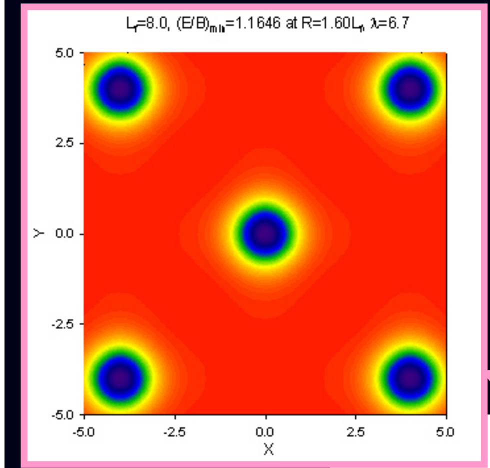
time component of SU(2) gauge potential for the instanton field of charge N

time-ordering

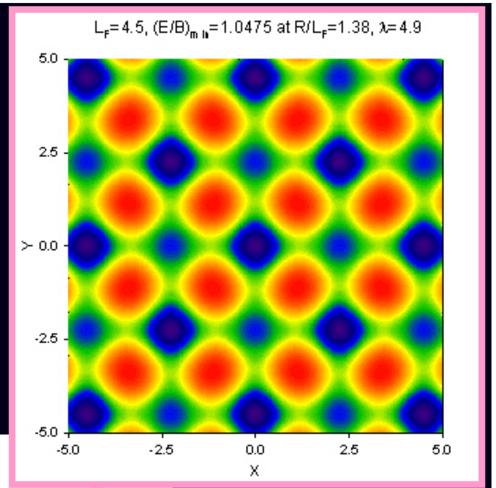
constant matrix to make U approach 1 at infinity

constant rotation matrix

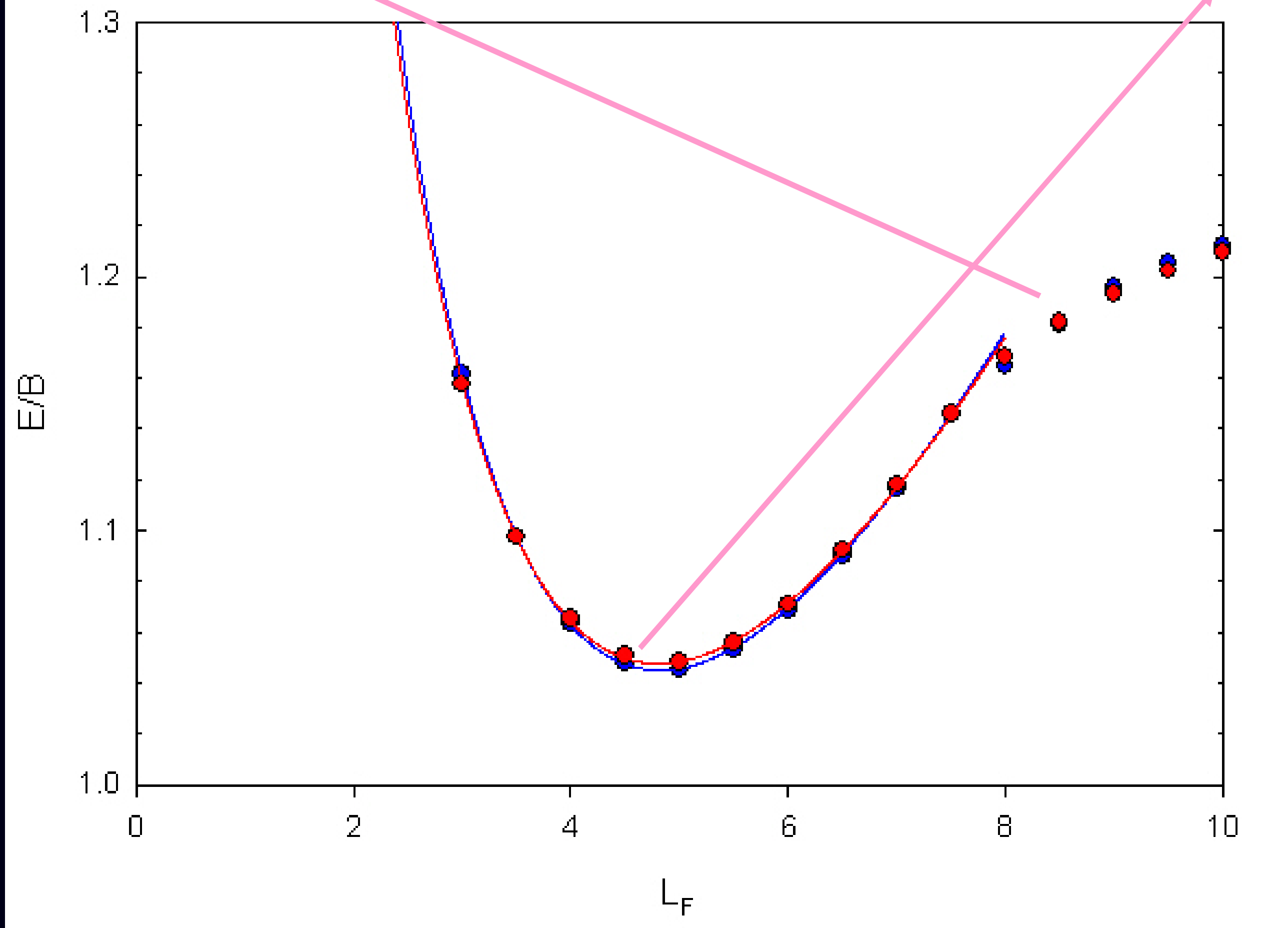
E/B vs. L_F



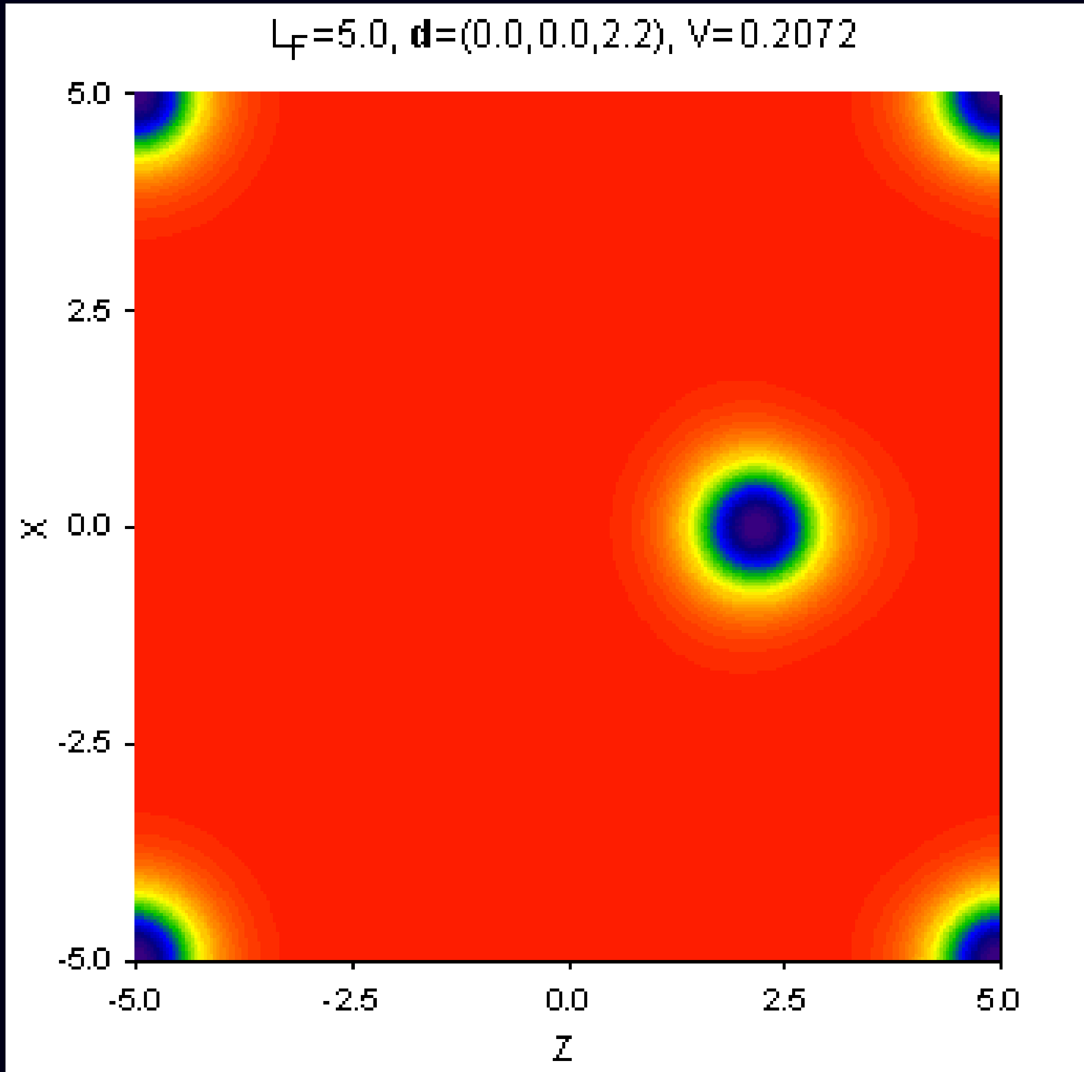
L_F=8.0, (E/B)_min=1.1646 at R=1.60L_F, λ=6.7
5.0
2.5
0.0
-2.5
-5.0
Y
-5.0
-2.5
0.0
2.5
5.0
X



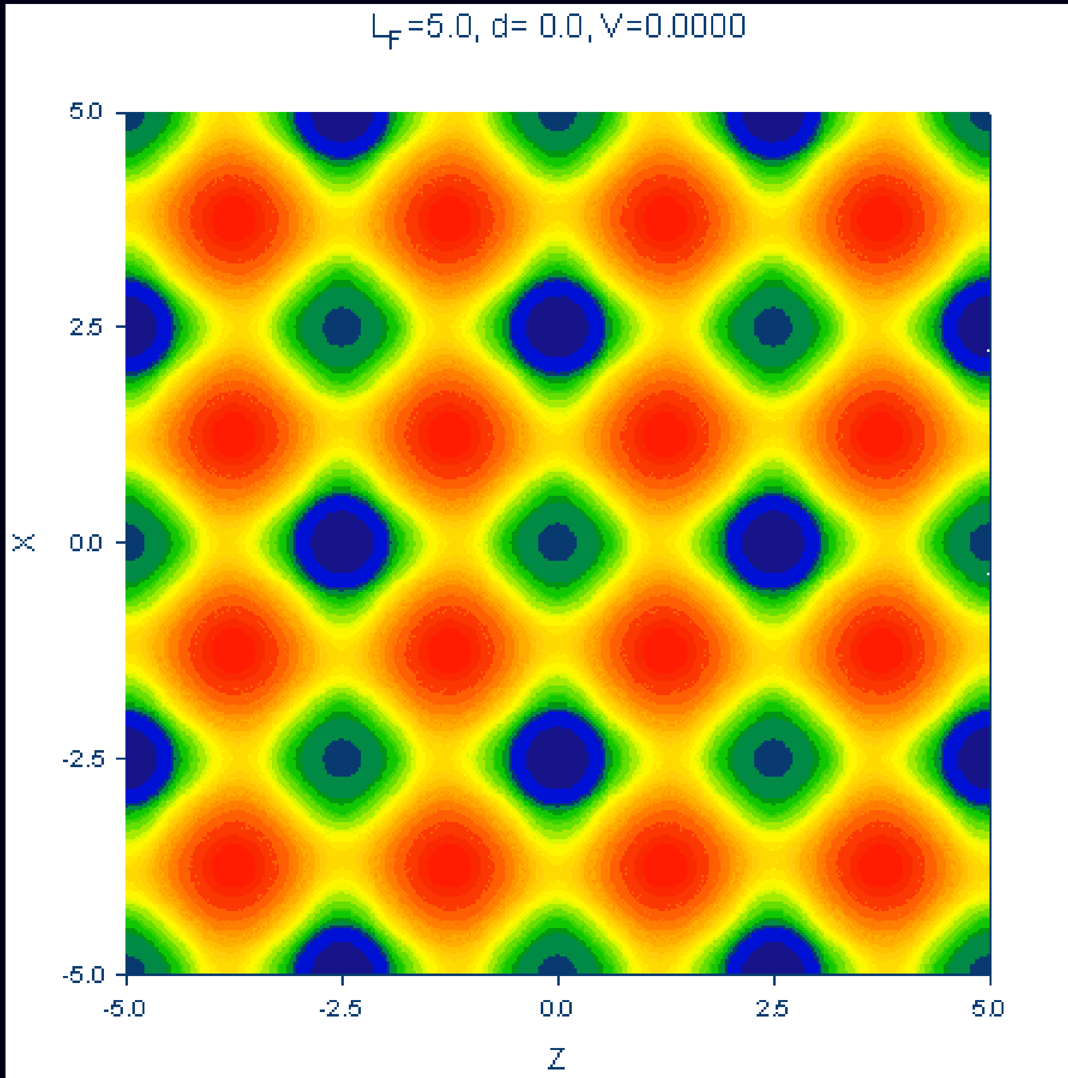
L_F=4.5, (E/B)_min=1.0475 at R/L_F=1.38, λ=4.9
5.0
2.5
0.0
-2.5
-5.0
Y
-5.0
-2.5
0.0
2.5
5.0
X



1.3
1.2
1.1
1.0
E/B
0
2
4
6
8
10
L_F

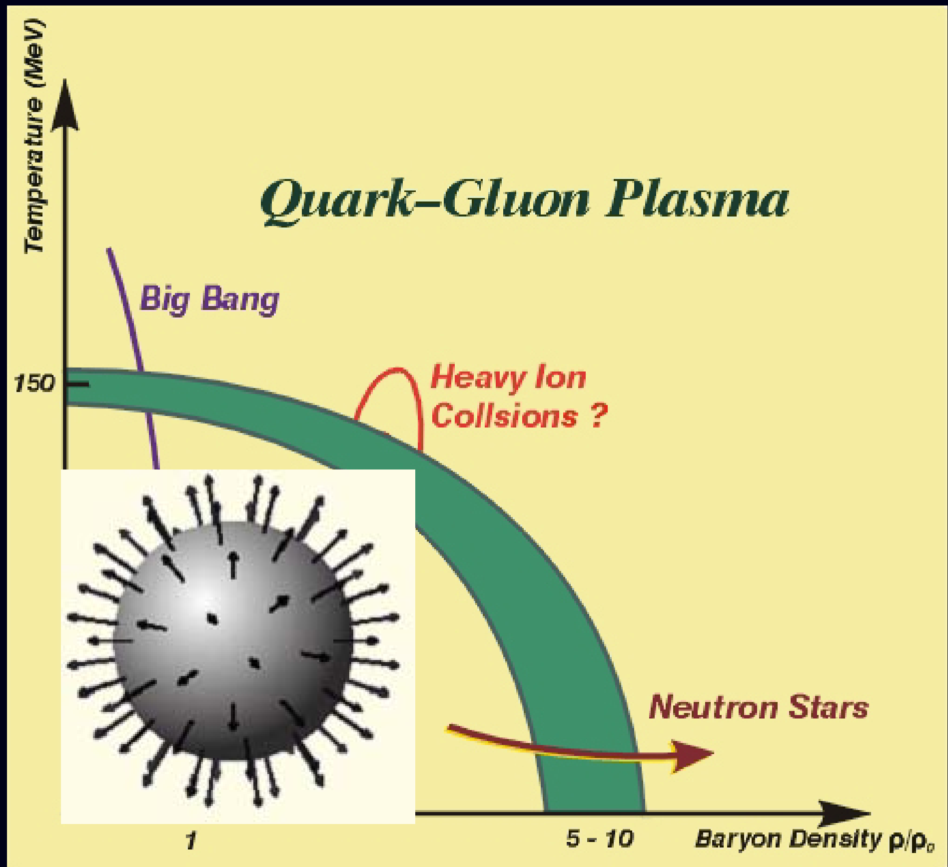


L_F=5.0, **d**=(0.0,0.0,2.2), V=0.2072
5.0
2.5
0.0
-2.5
-5.0
x
-5.0
-2.5
0.0
2.5
5.0
Z

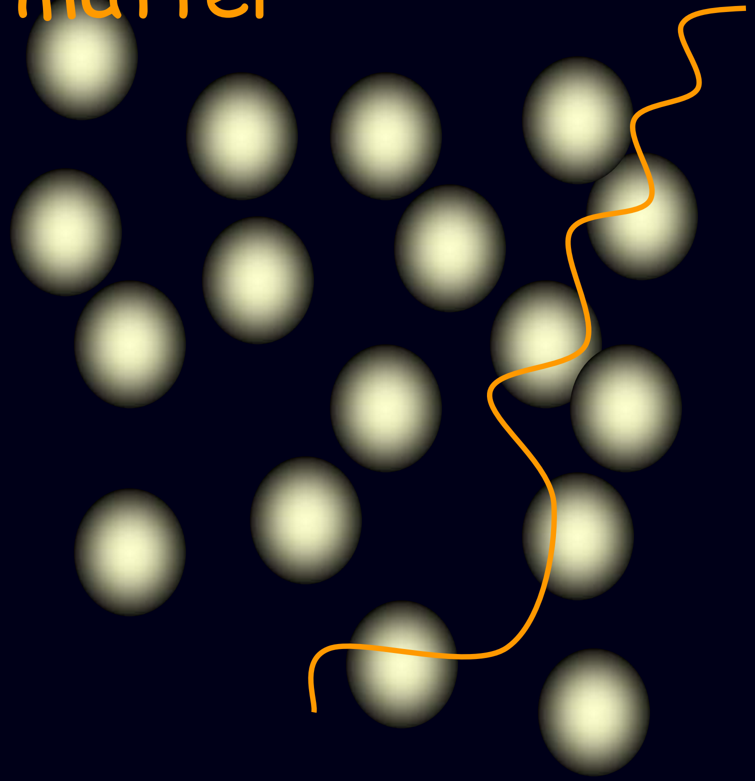

L_F=5.0, d= 0.0, V=0.0000
5.0
2.5
0.0
-2.5
-5.0
X
-5.0
-2.5
0.0
2.5
5.0
Z

7. Concluding remarks

The skyrmion role in dense matter



1. Construct dense skyrmion matter



2. Fluctuations on top of this skyrmion matter.

↓

Properties and Dynamics of hadrons in dense medium.

Skyrme model

- Universal theory of baryons and mesons
- Nuclei and meson fluctuations
- Nuclear matter and mesons in the medium

……... Nuclear physicists dream!

Caveats:

- Still very primitive! Crystal structures which should be Fermi liquids→ Quantum effects!
- Approach to QCD