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Color Strings, Pomerons and CGC

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INTRODUCTION:
Parton saturation at small x

• Recent years: Growing activity around systems and experiments DIS at HERA, heavy-ion experiments at RHIC and LHC involving **large number of partons due to high energy and high number of participants**

• **High parton densities:**

$$\frac{dN^{ch}_{AA}}{d\eta}(b) = a(\eta, b)N_{part}(b) + c(\eta, b)N_{coll}(b).$$

A) $N_{part}(b) \propto A$: number of participant nucleons, valence-like contribution.
B) $N_{coll}(b) \propto A^{4/3}$: number of inelastic nucleon-nucleon collisions

• **To get the right multiplicities at RHIC and LHC it is necessary to lower the contribution of B**

• Two different approaches for the initial state:

Geometrical approach: Interaction/percolation of strings
Saturation in high-density QCD: CGC

STRING MODELS: PERCOLATION

(Armesto, Braun, Ferreiro, Pajares)

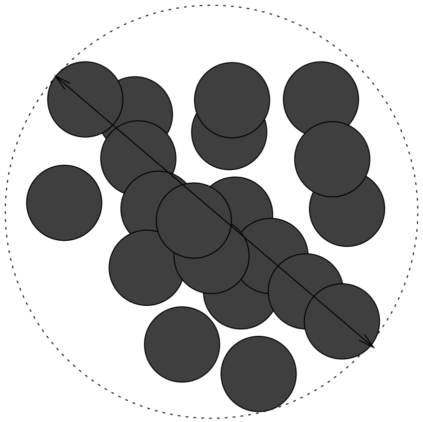
• **Phenomenological model for the soft region**

• In a collision **color strings** are streched between the colliding partons

• **Strings = color sources** of particles which are successively broken by creation of $q\bar{q}$ pairs from the sea

• **Color strings = small areas** in the transverse space filled with color field created by the colliding partons

• If the density of strings increases ⇒ Ovelapping in the transverse space
Phenomenon of string fusion and percolation



$\eta = N_{st}\frac{S_1}{S_A}$
$S_1 = \pi r_0^2$
$r_0 = 0.2 \div 0.3$ fm
$\eta_c = 1.1 \div 1.5.$

Percolation theory

• Consider N small circular discs (color sources: strings, partons) of **radius r** onto a large circular manifold (the transverse nuclear plane) of **radius R**:

$$\textbf{Density: } \eta = \frac{N\pi r^2}{\pi R^2}$$

• **Percolation:** at a critical value η_c of the density the cluster size diverges:

the size of the cluster reaches the size of the system

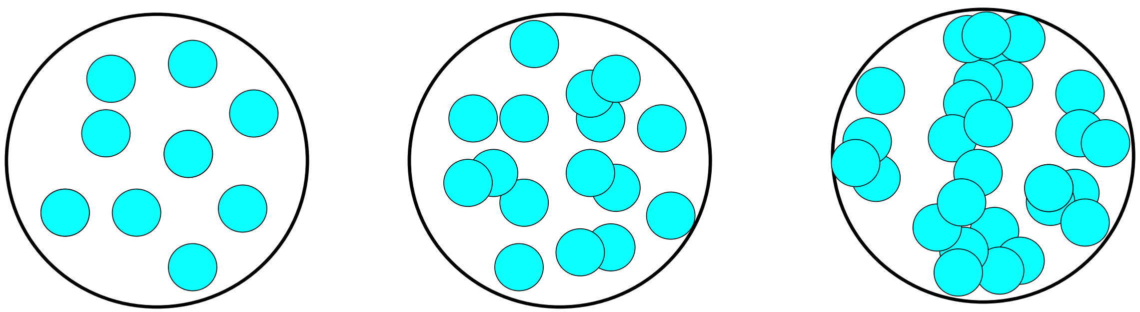
• $\eta_c = 1.12 \div 1.175$ (from MC, direct connectedness expansion and other methods)

• In our model: **fixed radius** for the independent color sources **r=0.2 - 0.3 fm** which corresponds to a momentum around **1 GeV**

• To estimate the density η, one needs to know the number of sources N
N depends on the energy $\sqrt{s}$ and on the number of participant nucleons A
The condition to achieve percolation depends on A and s, $\eta = \eta(A, x)$

• **Critical threshold for percolation:** central Pb-Pb collisions at SPS, Au-Au and lighter central collisions at RHIC, p-p collisions at LHC energies

From left to righ: Density of strings in the transverse space, from low energy and/or low number of participants to high energies and/or high number of participants. In the last circle we show percolation.



The size of the color sources

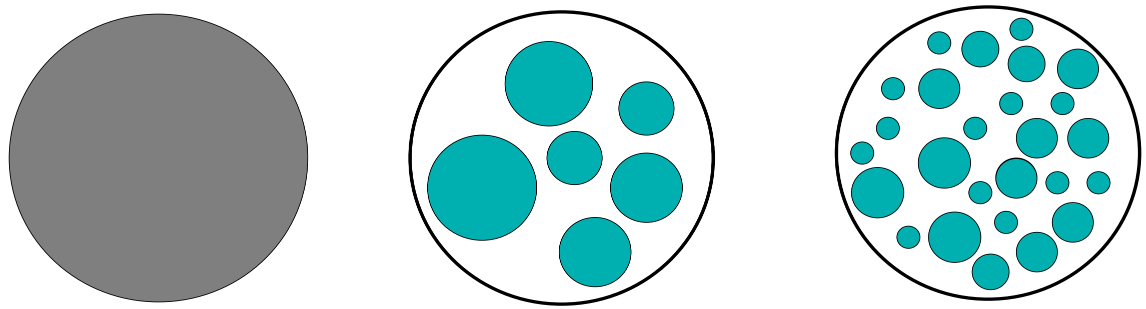
If you look at a fast nucleon coming at you, what do you see?

It depends on who is looking

- Another nucleon sees a disc of radius $r \simeq 1$ fm and a certain greyness.

- A hard photon with resolution scale $Q^{-1} << 1$ fm sees a swarm of partons.

How many there are depends on the resolution scale: given a finer scale, you can see smaller partons, and there are more the harder you look.



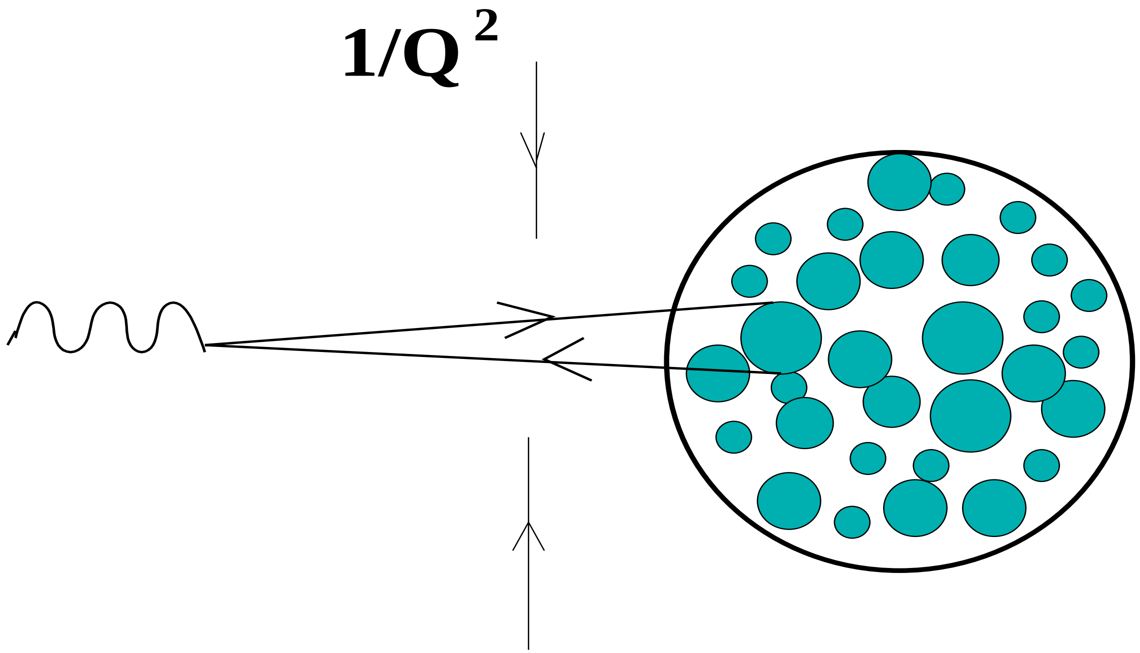
The structure of an incoming nucleon seen for increasing resolution

Partons in a nucleon: transverse size r_T determined by their k_T: $r_T \sim 1/k_T$

The resolution scale Q^{-1} specifies the minimum k_T^{-1} resolved

The probing photon sees all partons in the range $0 \leq k_T \leq Q$

It sees all the partons with a radius $r_T \geq Q^{-1}$



The partonic size is through the uncertainty relation determined by its average transverse momentum, $r^2 \sim 1/ < k_T^2 >$, for a given resolution scale, $< k_T^2 > \sim Q^2$

The percolation density depends on the number of sources N and their size

- **Two models:**

- **Critical density (Armesto, Braun, Ferreiro, Pajares)**

N calculated in the SFM, $N = N(A, s)$

Initial sources of a fixed size of r=0.2-0.3 fm $\equiv$ momentum of 1 GeV

- **Critical momentum (Nardi, Sazt, Digal)**

N calculated through the WNM using P.D.F.'s for the nucleonic density

Instead of fixing the radius of the initially created sources, they estimate the momentum of those sources that will lead to percolation

$$\eta_c = \frac{N\pi r_c^2}{\pi R^2} = \frac{N}{Q_c^2 R^2}$$

Condition for percolation: $\eta_c = 1.12 \Rightarrow$ **Critical momentum** $Q_c^2 = \frac{N}{1.12R^2}$

For central Pb-Pb collisions at SPS energies: $Q_c^2 \approx 1$ GeV^2

For central Au-Au collisions at RHIC: $Q_c^2 \approx 2.5$ GeV^2

The condition for percolation depends on A and s: N=N(A,s), $Q_c=Q_c$(A,x)

Some remarks

- What we are trying is to determine **under what conditions the initial state configurations can lead to color connection**, and more specifically, **if variations of the initial state can lead to a transition from disconnected to connected color clusters**.

- **This is not a final state interaction phenomenon**: the results of such a study of the pre-equilibrium state in nuclear collisions do not depend on the subsequent evolution and thus **not** require any kind of **thermalization**.

- When the density of strings becomes high the string color fields overlap and individual strings fuse, forming **a cluster which has a higher color charge**, corresponding to the sum of the color charges of the original strings.

- The string clusters break into hadrons according to their higher color. As a result, there is **a reduction of the total multiplicity**. Also, as the energy-momenta of the original strings are summed to obtain the energy-momentum of the cluster, **the mean transverse momentum is increased** compared to the one of the particles created from individual sources.

Numerical results

(Armesto, Braun, Ferreiro, Pajares)

- For a cluster of n overlapping strings covering an area S_n:

Color charge of the cluster=Vectorial sum of the strings charges

$$\vec{Q}_n = \sum_{i=1}^{n} \vec{Q}_{1i}, \quad \langle \vec{Q}_{1i} \cdot \vec{Q}_{1j} \rangle = 0, \quad \vec{Q}_n^2 = n\vec{Q}_1^2,$$

$$Q_n = \sqrt{\frac{nS_n}{S_1}} Q_1, \quad \mu_n = \sqrt{\frac{nS_n}{S_1}} \mu_1, \quad \langle p_T^2 \rangle_n = \sqrt{\frac{nS_1}{S_n}} \langle p_T^2 \rangle_1.$$

For strings without interaction:

$S_n = nS_1$, $Q_n = nQ_1 \rightarrow \mu_n = n\mu_1$, $\langle p_T^2 \rangle_n = \langle p_T^2 \rangle_1$

For strings with max overlapping:

$S_n = S_1$, $Q_n = \sqrt{n} Q_1 \rightarrow \mu_n = \sqrt{n}\mu_1$, $\langle p_T^2 \rangle_n = \sqrt{n} \langle p_T^2 \rangle_1$

- Moreover, one can obtain the analytic expression:

$$\eta = \frac{N_{strings} S_1}{S_A}, \qquad < \frac{nS_1}{S_n} > = \frac{\eta}{1 - e^{-\eta}} \equiv \frac{1}{F(\eta)^2} \quad \Rightarrow$$

$$\mu = N_{strings} \; F(\eta) \mu_1 \; , \qquad < p_T^2 > = \frac{1}{F(\eta)} < p_T^2 >_1$$

A-dependence

(Armesto, Braun, Ferreiro, Pajares)

- Taking

$$N_{strings} \propto A^{4/3},\ S_A \propto A^{2/3},\ \eta = \frac{N_{strings} S_1}{S_A} \propto A^{2/3},\ F(\eta) \propto \frac{1}{\sqrt{\eta}}$$

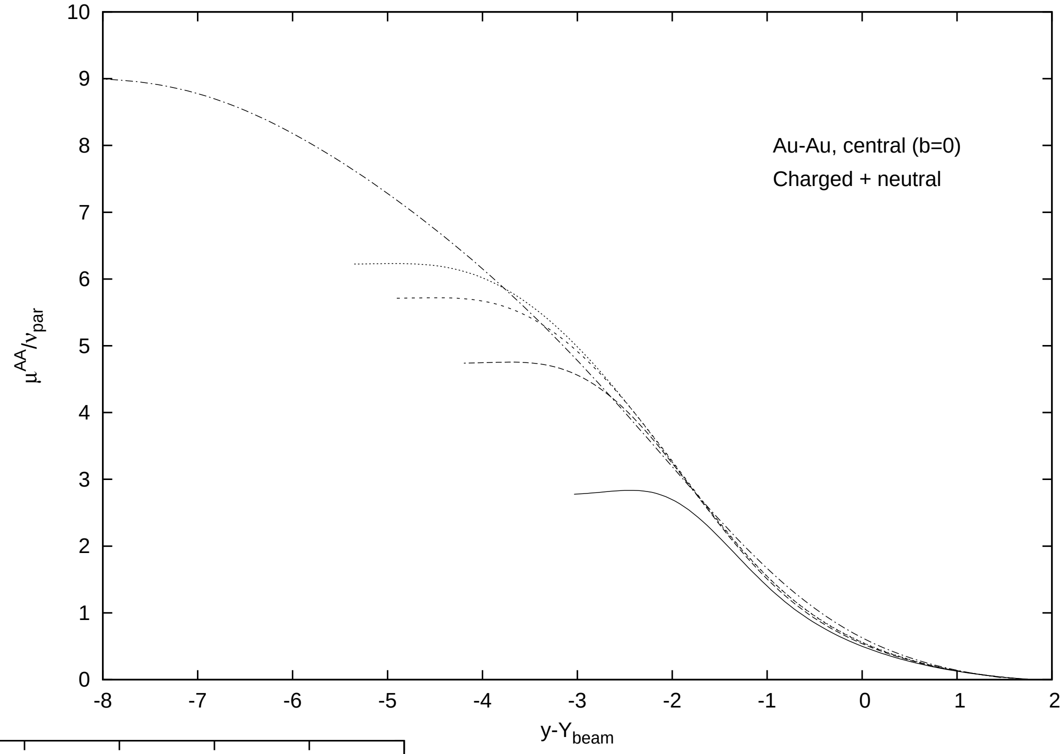
we obtain

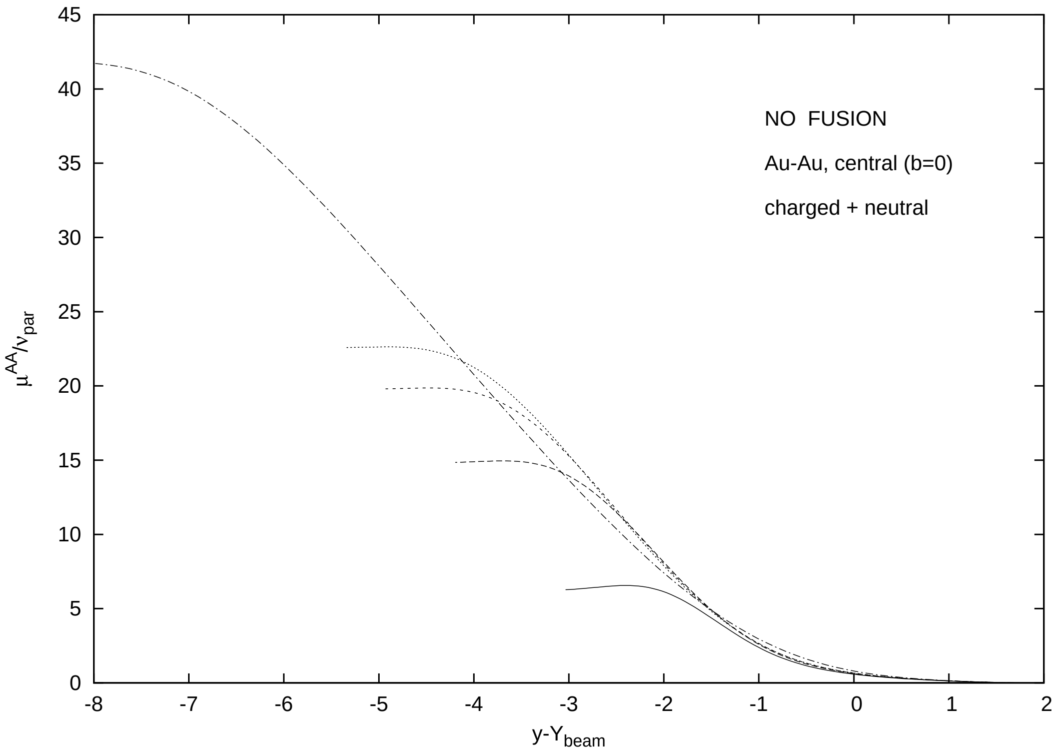
- **Without interaction:**

$$\mu_{AA} = N_{strings}\ \mu_1 \propto A^{4/3}, \qquad < p_T^2 >_{AA} = < p_T^2 >_1$$

- **With clustering:**

$$\mu_{AA} = N_{strings}\ F(\eta)\ \mu_1 \propto A, \qquad < p_T^2 >_{AA} = \frac{1}{F(\eta)} < p_T^2 >_1 \propto A^{1/3}$$


Au-Au, central (b=0)
Charged + neutral
y-Y_beam
μ^AA/ν_par


NO FUSION
Au-Au, central (b=0)
charged + neutral
y-Y_beam
μ^AA/ν_par

- **Beyond the percolation point, one has a condensate, containing interacting and hence color-connected sources of all scales $k_T \leq Q$**

- **The percolation point thus specifies the onset of color deconfinement**

- **It says nothing about any subsequent thermalization**

THE COLOR GLASS CONDENSATE

(Iancu, Leonidov, McLerran)

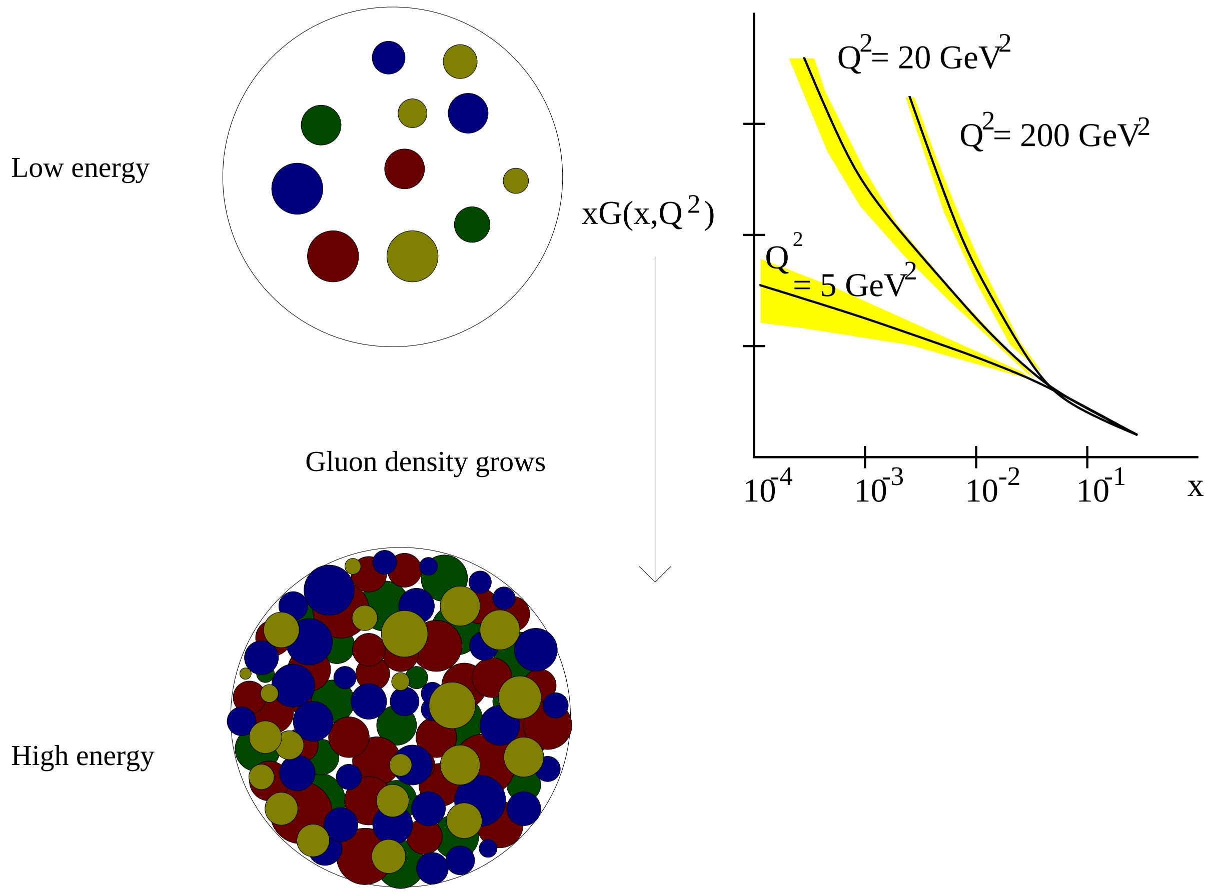
• At high energy the QCD cross sections are controlled by small x gluons in the hadron wavefunction, whose density grows rapidly with the energy (or with decreasing Bjorken's x) due to the enhancement of radiative process.

• **Perturbative QCD**: By resumming dominant radiative corrections at high energy, the BFKL eq. leads to a gluon density that grows like a power of s $\Rightarrow \sigma$ also grows like a power of s and violates Froissart bound.

• **BFKL and DGLAP: Linear equations** that neglet the interaction among the small x gluons

With increasing energy, recombination effects favoured by the high density of partons should become more important and lead to eventual saturation of the parton densities.

Gluon density at small x



- Increase at fixed Q^2
- More rapid increase at larger Q^2

The saturation momentum

• Non-linear effects in the hadron wavefunction become important when the interaction probability for the gluons becomes of $\mathcal{O}(1)$ (**gluons overlap**):

$$\frac{\alpha_s N_c}{Q^2} \times \frac{xG(x, Q^2)}{\pi R^2} \sim 1$$

Transverse size of the gluon **Density of gluons**

• Equiv: For a given energy, saturation occurs for those gluons having a sufficiently large transverse size $r_\perp^2 \sim 1/Q^2$, larger than a critical value $1/Q_s(x) \Rightarrow$

$\Rightarrow$ Gluons with momenta $Q^2 \lesssim Q_s^2(x)$ where

$$Q_s^2(x) = \alpha_s N_c \frac{xG(x, Q_s^2)}{\pi R^2} \equiv \frac{(\text{color charge})^2}{\text{area}}$$

$$\sim A^{1/3} x^{-\lambda\alpha_s}$$

• For sufficiently large energy (x small enough):

$$Q_s^2(x) \gg \Lambda_{QCD}^2 \, \text{and} \, \alpha_s(Q_s) \ll 1$$

$\Rightarrow$ Weak coupling QCD

• But although the coupling is small the effects of the interactions are amplified by the large gluon density:
At saturation: $xG(x, Q_s^2) \sim 1/\alpha_s \gg 1 \Longrightarrow$

large ocupation numbers
semi-classical regime **[McLerran, Venugopalan (94)]**
ordinary perturbation theory breaks down

• **The strategy:** To construct an effective theory in which the **small-x gluons** are describe by **classical color fields** radiated by a **random color source**, that of the **fast partons** with larger x

• **The advantage:** Non-linear effects in a classical context $\Rightarrow$ Exact calculations are possible

Effective theory for the CGC

(Iancu, Leonidov, McLerran, Ferreiro)

• General idea: **Fast partons** (valence quarks with large longitudinal momentum) are considered as a **classical souce** ρ that emits **soft gluons** (with smaller longitudinal momenta) with are treated as **classical color fields** $\mathcal{A}[\rho]$

• Yang Mills eqs. describing soft gluon dynamics:

$$D_\nu F^{\nu\mu} = \delta^{\mu +}\rho(x^-, \boldsymbol{x})$$

• Physical quantities, as the unintegrated gluon distribution, are obtained as an average over ρ:

$$< A^i(X)\, A^i(Y) >_x = \int D[\rho]\; W_x[\rho]\; \mathcal{A}^i[\rho](X)\, \mathcal{A}^i[\rho](Y)$$

$A^i(X)$= classical solution for given ρ, $F^{+i} = \delta(x^-)\frac{i}{g}V(x_\perp)(\partial^i V(x_\perp)^\dagger) = \partial^+ A^i$
$W_x[\rho]$= gauge-invariant weight function for ρ

The Renormalization Group Equation

- ρ and its correlations change with increasing $\tau \equiv \ln 1/x$

$$\frac{\partial W_\tau[\rho]}{\partial \tau} = \frac{1}{2} \int_{x_\perp, y_\perp} \frac{\delta}{\delta \rho_x^a} \, \chi_{xy}^{ab}[\rho] \, \frac{\delta}{\delta \rho_y^b} \, W_\tau[\rho]$$

Jalilian-Marian, Kovner, Leonidov, Weigert, 97;

Iancu, Leonidov, McLerran, 2000

Functional diffusion equation. It encompasses previous evolution equations by : Balitsky (96), Kovchegov (99), Weigert (2000)

χ depends upon ρ via Wilson lines:

$$V^\dagger(x_\perp) \equiv \mathrm{P}\exp\left\{ ig \int dx^- A^+(x^-, x_\perp) \right\}; \; -\nabla_\perp^2 A^+ = \rho$$

Solutions

Physical quantities, as the gluon density, are obtained as an average over ρ

$$n(x, k_\perp) \equiv \frac{1}{\pi R^2} \frac{\mathrm{d}N}{\mathrm{d}\tau\, \mathrm{d}^2 k_\perp} \propto \langle F^{+i}(k_\perp) F^{+i}(-k_\perp) \rangle_x$$

- **Low density:** high Q^2 or low energy $\quad k_\perp \gg Q_s(x)$

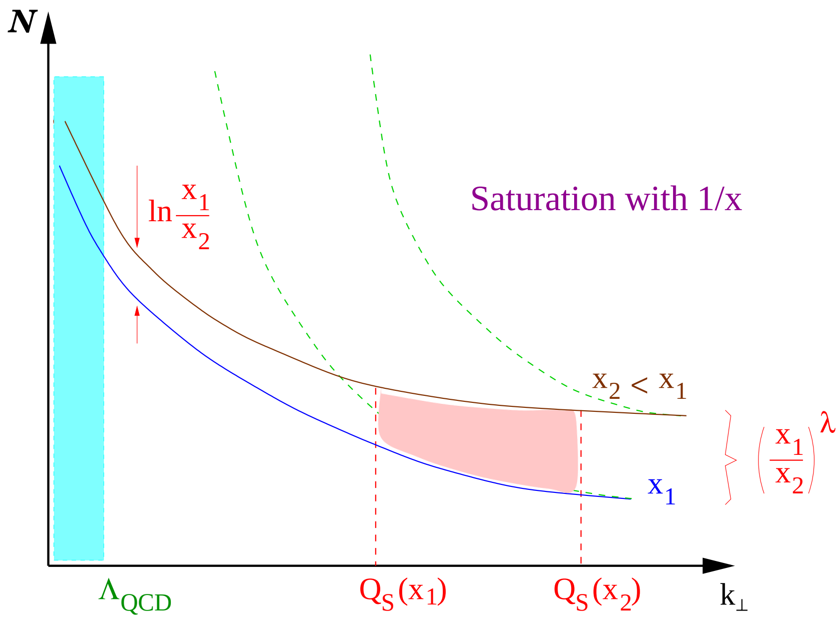
$$n(x, k_\perp) \sim \frac{1}{k_\perp^2} \frac{1}{x^{\omega \alpha_s}} \qquad \text{(Bremsstrahlung)}$$

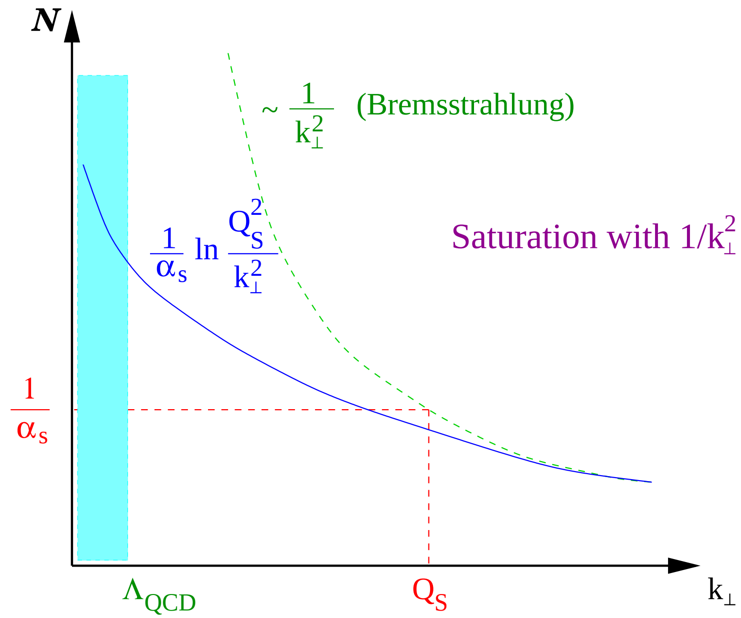
$\Rightarrow$ **Gluon density grows both with** $1/k_\perp^2$ **and** $\tau = \ln \frac{1}{x}$

- **High density:** low Q^2 or high energy $\quad k_\perp \ll Q_s(x)$

$$n(x, k_\perp) \sim \frac{1}{\alpha_s} \ln \frac{Q_s^2(x)}{k_\perp^2} \propto \ln \frac{1}{x}$$

$\Rightarrow$ **SATURATION: Gluon density increases linearly with** τ **and logaritmically with the energy: Unitarity is restored**


N
$\ln \frac{x_1}{x_2}$
Saturation with 1/x
$x_2 < x_1$
$\left(\frac{x_1}{x_2}\right)^\lambda$
x_1
Λ_{QCD}
$Q_S(x_1)$
$Q_S(x_2)$
$k_\perp$


N
$\sim \frac{1}{k_\perp^2}$ (Bremsstrahlung)
$\frac{1}{\alpha_s} \ln \frac{Q_S^2}{k_\perp^2}$
Saturation with $1/k_\perp^2$
$\frac{1}{\alpha_s}$
Λ_{QCD}
Q_S
$k_\perp$

Phenomenology at RHIC

(Kharzeev, Levin, Nardi; McLerran, Schaffner-Bielich, Venugopalan)

Density of partons at saturation:

$$xG(x, Q_s^2) = \frac{\pi R_A^2 \, Q_s^2(x, A)}{\alpha_s(Q_s^2)} \sim N_{part} \ln N_{part}$$

- $\pi R_A^2 \propto N_{part}^{2/3}$ = the nuclear overlap area
- $Q_s^2(x, A) \propto N_{part}^{1/3}$ = the saturation momentum
- $1/\alpha_s(Q_s^2) \approx \ln(Q_s^2/\Lambda_{QCD}^2) \sim \ln N_{part}$(evolution)

Transverse momentum spectra at saturation:

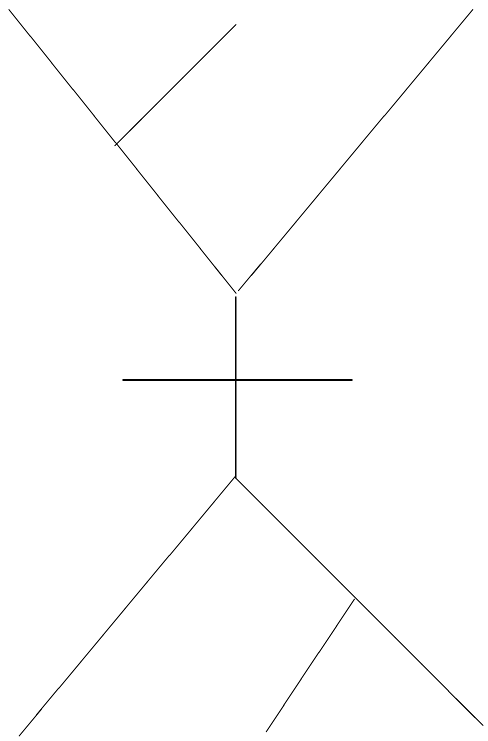
Intrinsic p_T broadening in the partonic phase: **Scaling relations:**

$$< p_T >^2 \sim \frac{1}{\pi R_A^2}\frac{dN}{dy}, \quad \frac{1}{\sigma}\frac{dN}{d\eta d^2 p_T} = \frac{1}{\alpha_s} f(\frac{p_T^2}{Q_s^2}), \quad \frac{1}{\sigma}\frac{dN}{dy d^2 m_T} = \frac{1}{\alpha_s} f(\frac{m_T}{Q_s^2})$$

PERTURBATIVE QCD POMERON

(Braun, Pajares)

Consider now the A-B interaction as governed by the exchange of pomerons:



A typical diagram for the inclusive cross-section in nucleus-nucleus collisions

Its interaction is realized by the triple pomeron vertex

Inclusive cross section in pQCD, taking $A = B$ and constant nuclear density for $|b| < R_A$: convolution of two sums of fan diagrams (AGK rules satisfied for BFKL pomerons interacting via the triple pomeron vertex Bartels and Wuesthoff, Ciafaloni)

$$I_A(y,k) = A^{2/3}\pi R_0^2 \frac{8N_c\alpha_s}{k^2}\int d^2r e^{ikr}[\Delta\Phi_A(Y-y,r)][\Delta\Phi_A(y,r)],$$

where $\Phi(y,r)$ is the sum of all fan diagrams connecting the pomeron at rapidity y and of the transverse dimension r with the colliding nuclei, one at rest and the other at rapidity Y.

In the momentum space, function $\phi_A(y,r) = \Phi(y,r)/(2\pi r^2)$ satisfies the BK equation

$$\frac{\partial\phi(y,q)}{\partial\bar{y}} = -H\phi(y,q) - \phi^2(y,q),$$

where $\bar{y} = \bar{\alpha}y$, $\bar{\alpha} = \alpha_s N_c/\pi$, α_s and N_c are the strong coupling constant and the number of colors, respectively, and H is the BFKL Hamiltonian

This equation has to be solved with the initial condition at $y = 0$ determined by the color dipole distribution in the nucleon smeared by the profile function of the nucleus.

We take the initial condition in accordance with the Golec-Biernat distribution saturation in CGC:

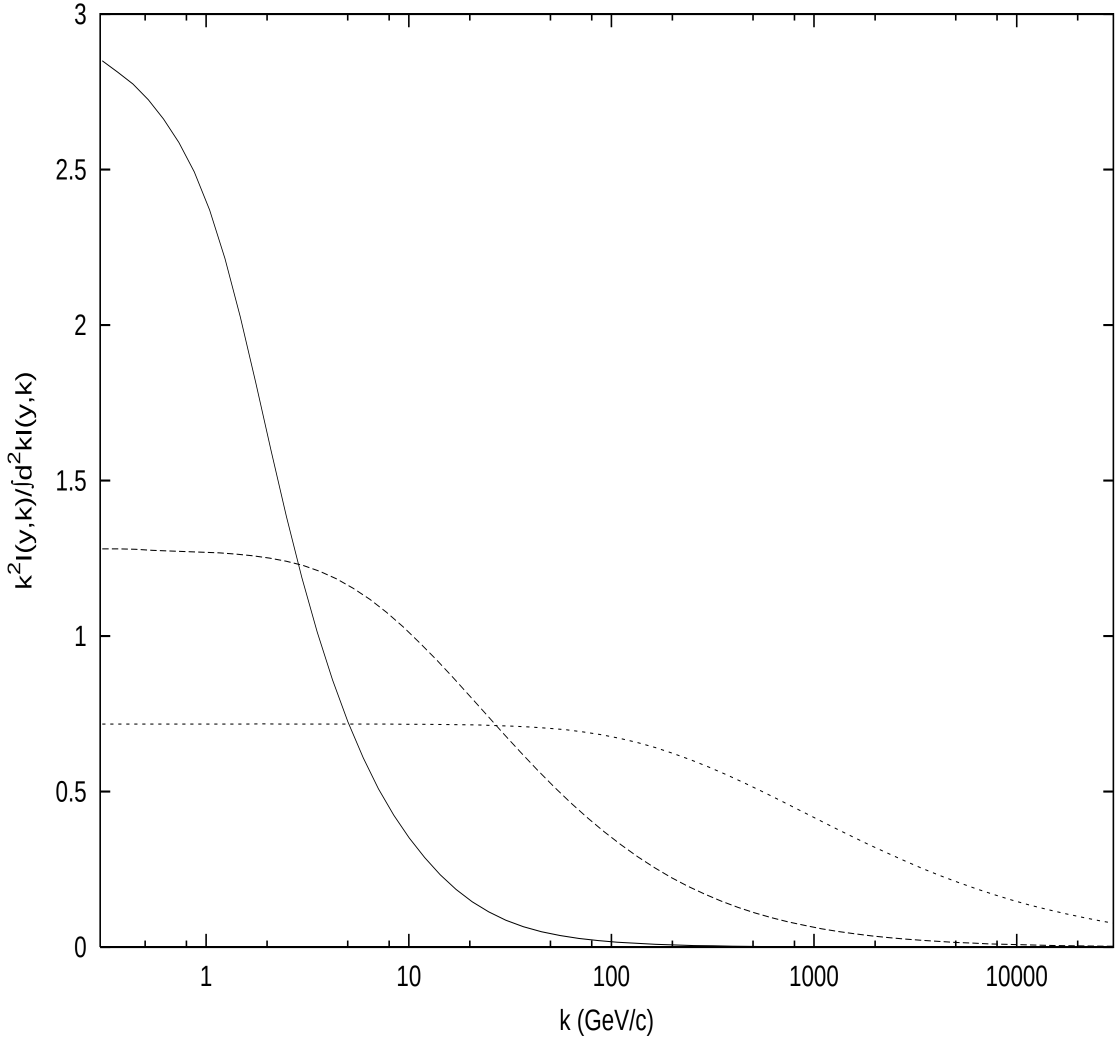
$$\phi(0, q) = -\frac{1}{2} a \, \mathrm{Ei}\left(-\frac{q^2}{0.3567 \, \mathrm{GeV}^2}\right),$$

with

$$a = A^{1/3} \frac{20.8 \, \mathrm{mb}}{\pi R_0^2}.$$

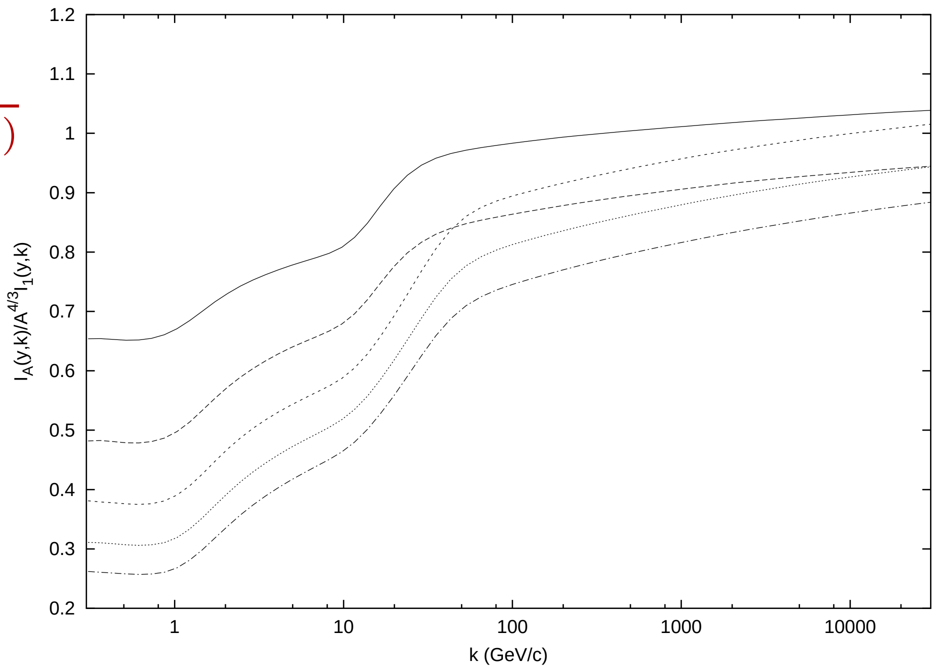
Evolving $\phi(y, q)$ up to values $\bar{y} = 3$ we found the inclusive cross-section at center rapidity for energies corresponding to the overall rapidity $Y = \bar{Y}/\bar{\alpha}$. With $\bar{Y} = 6$ and $\alpha_s = 0.2$ this gives $Y \sim 31$.

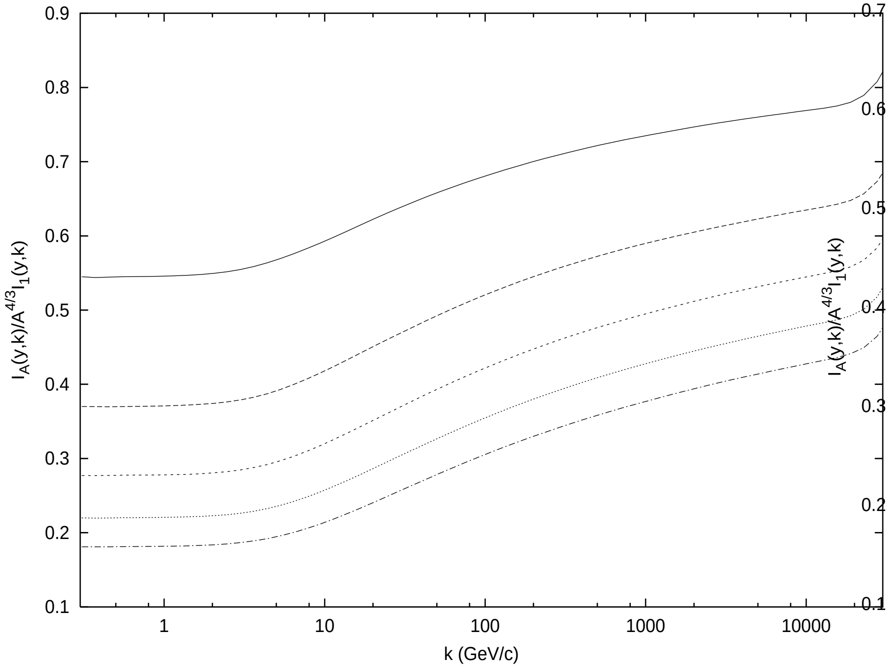
Energy dependence

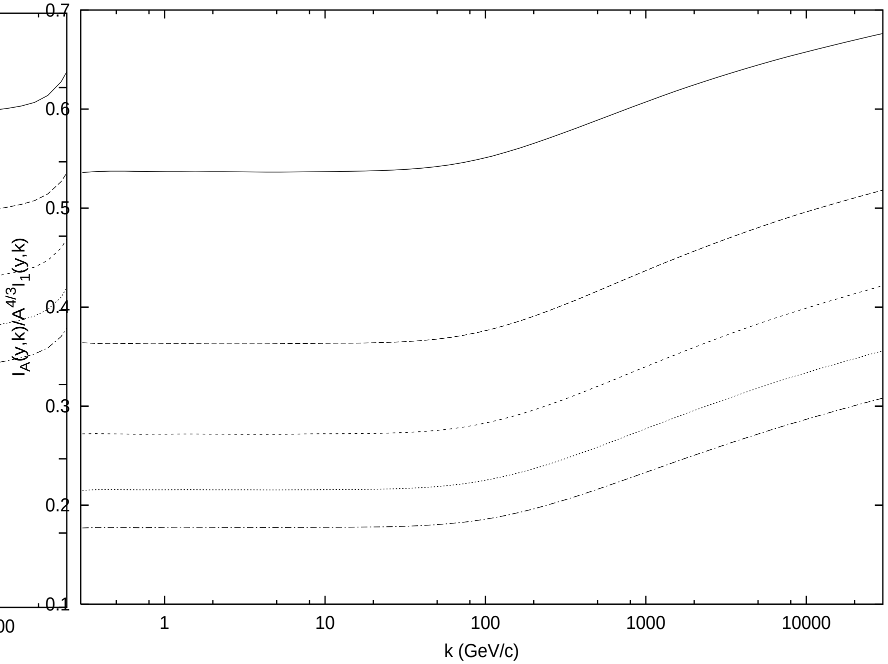


Normalized distributions for $A = 1$ and $y = \frac{Y}{2}$. Curves from top to bottom at small k correspond to scaled overall rapidities $\bar{Y} = 1, 3, 6$. With the growth of energy, the distributions are shifted towards higher values of k.

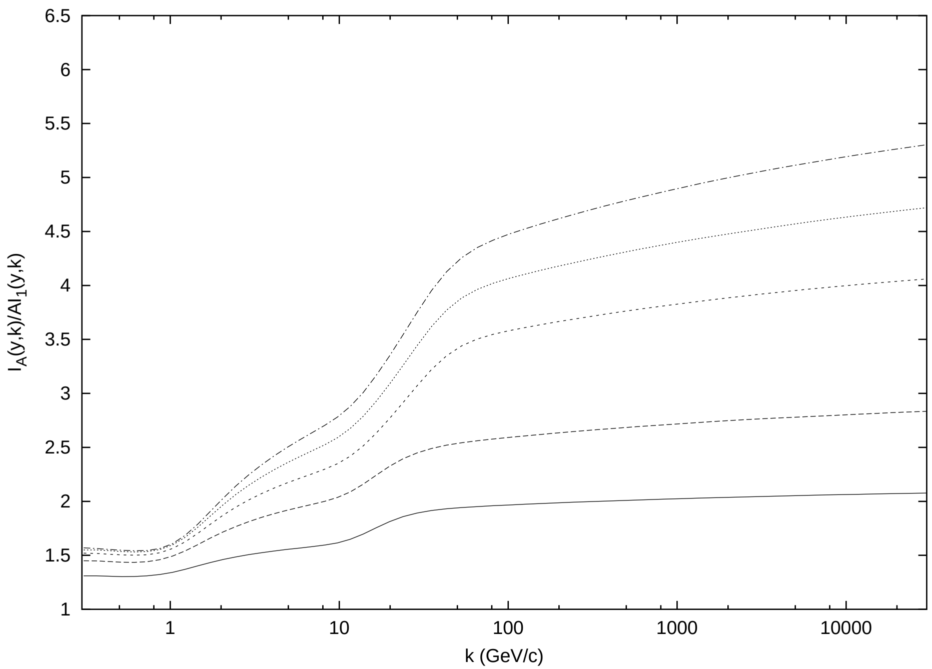
$$R_A^{col} = \frac{I_A(y,k)}{A^{4/3} I_1(y,k)}$$

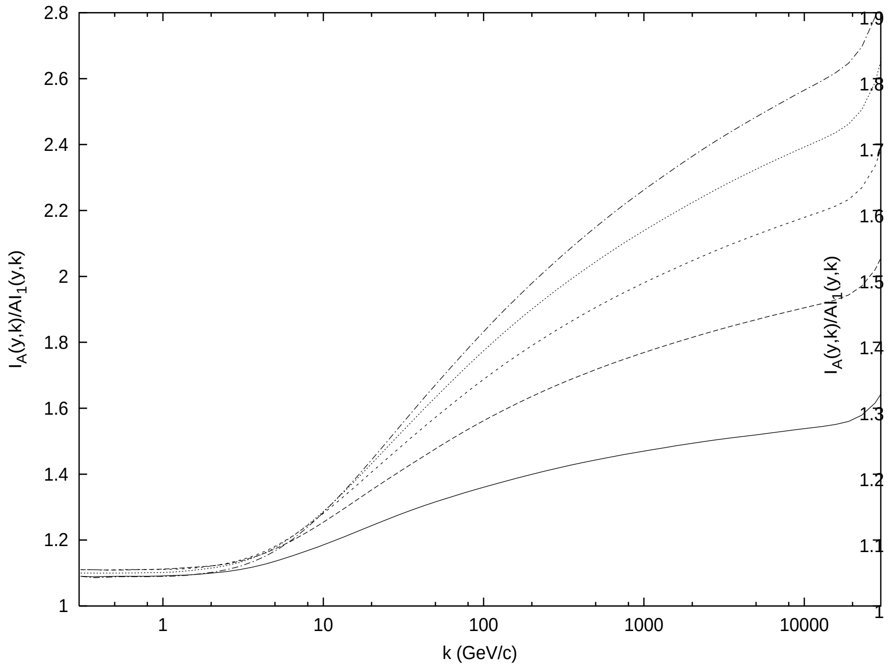


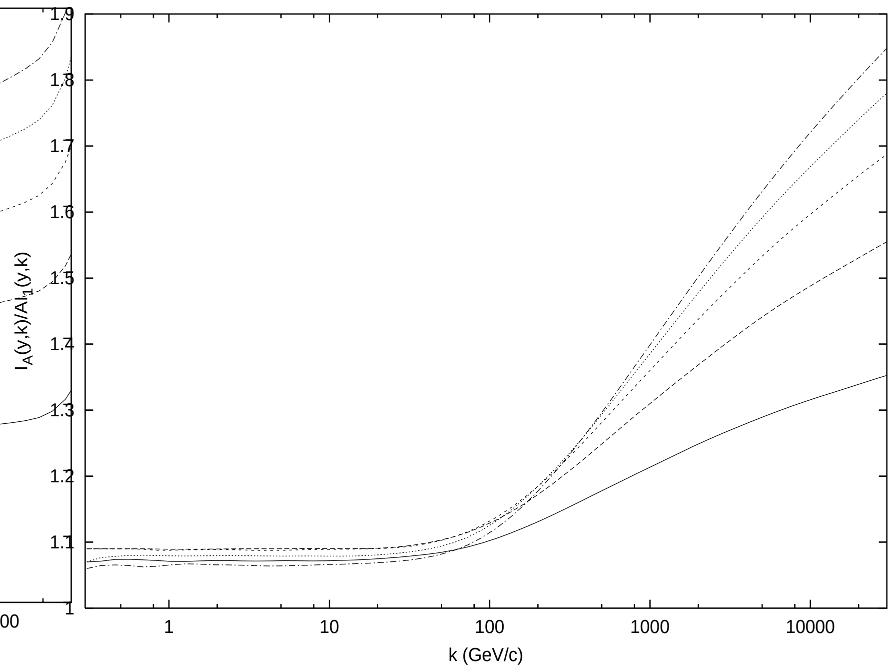




$$R_A^{part} = \frac{I_A(y,k)}{AI_1(y,k)}$$





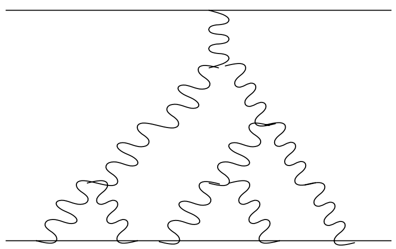


At relatively small momenta the inclusive cross-sections are proportional to A, that is to *the number of participants*

At larger momenta they grow with A faster, however noticeably slowlier than the number of collisions, approximately as $A^{1.1}$

The interval of momenta for which $I_A \propto A$ is growing with energy, so that one may conjecture that at infinite energies all the spectrum will be proportional to A

SHADOWING



(Capella, Kaidalov)

Dynamical, non linear shadowing
It is determined in terms of diffractive cross sections
It would lead to saturation at $s \longrightarrow \infty$
Controled by triple pomeron diagrams
Contribution to diffraction: positive
Contribution to the total cross-section: negative

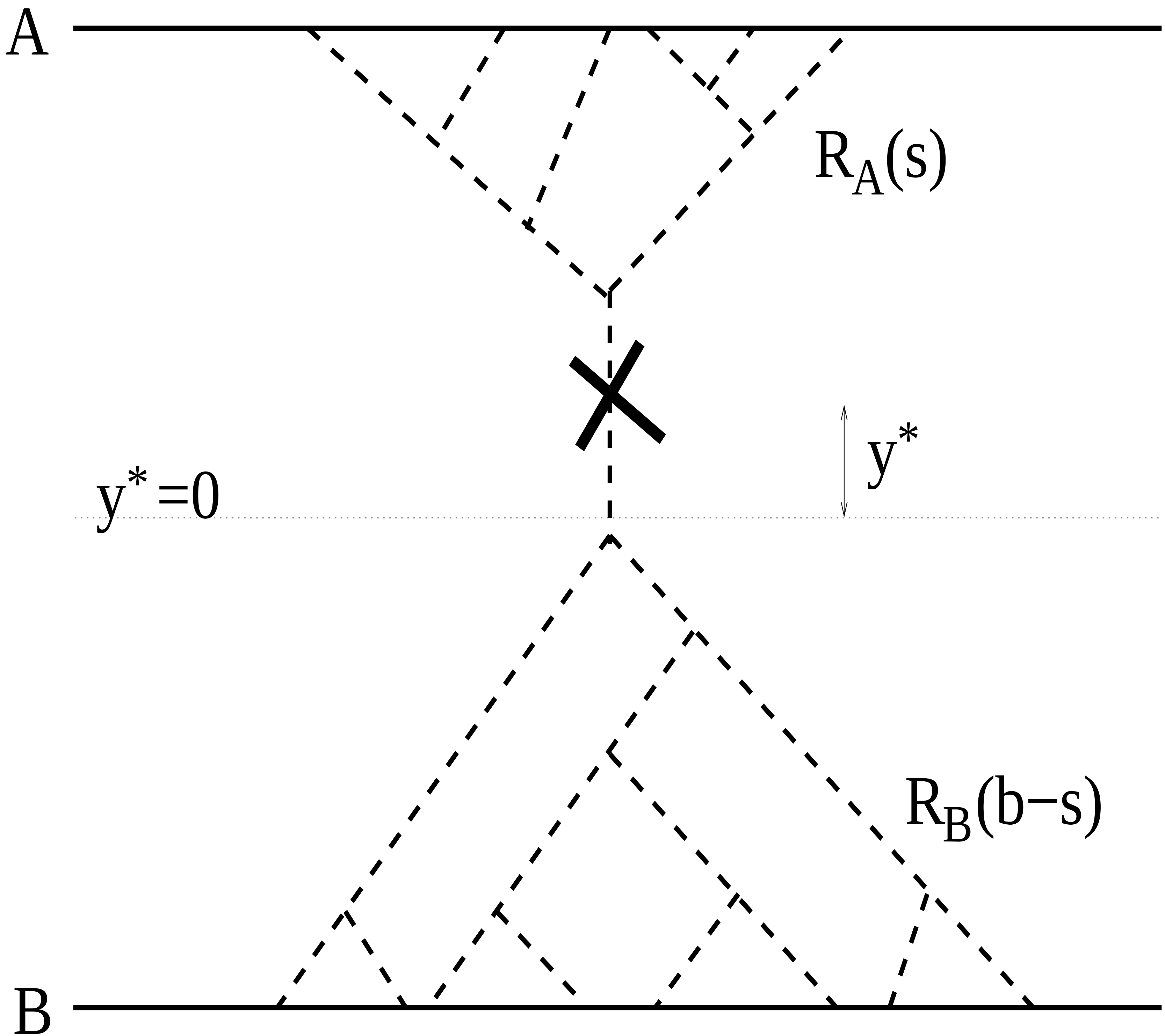
Reduction of multiplicity from shadowing corrections in AB collisions:

$$S_{sh} = \frac{\int d^2 s f_A(s) f_B(b-s)}{T_{AB}(s)} \, , \, f_A(b) = \frac{T_A(b)}{1 + AF(s) T_A(b)}$$

Function F: Integral of the triple P cross section over the single P one:

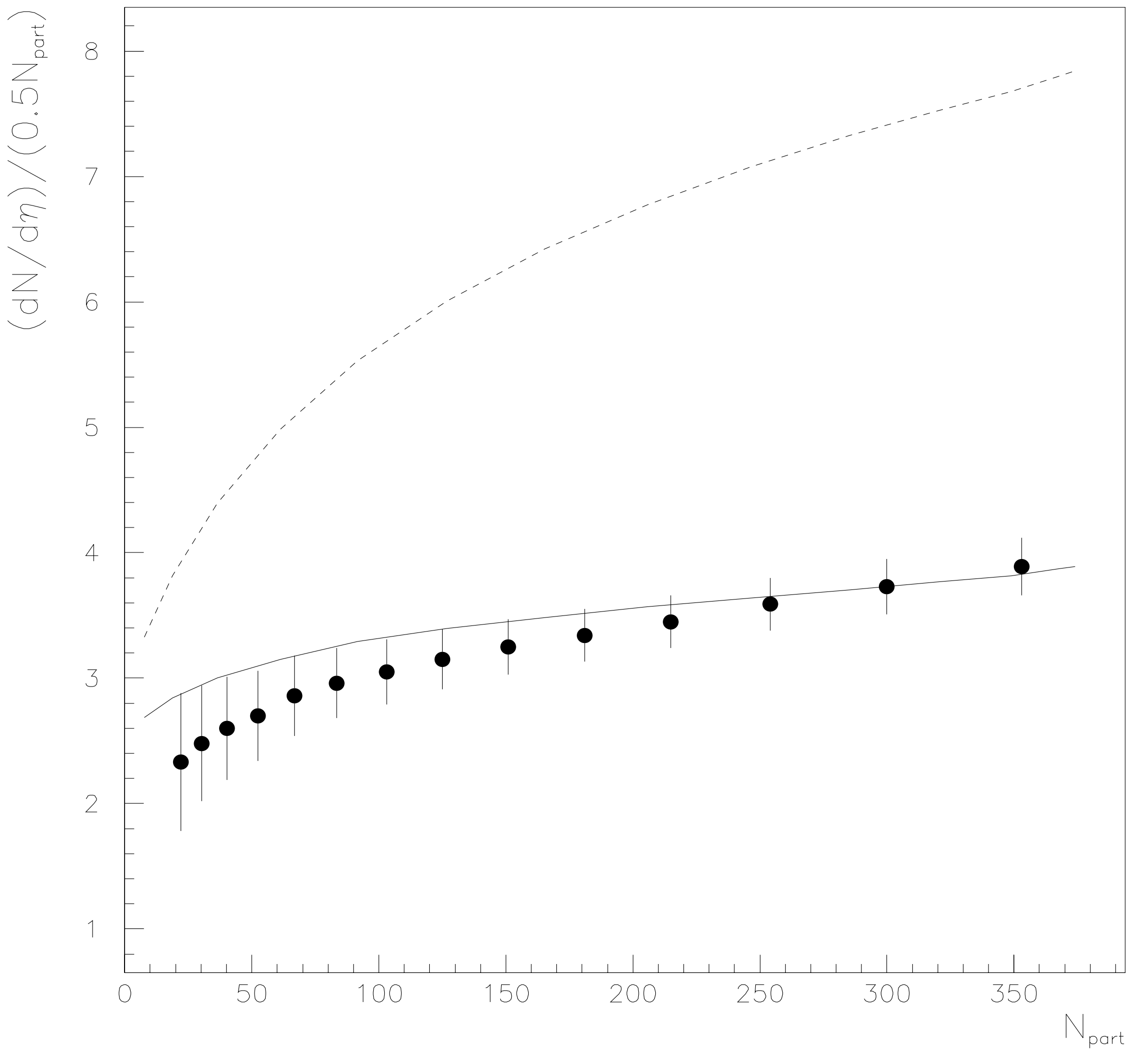
$$F(s) = 4\pi \int_{y_{min}}^{y_{max}} dy \frac{1}{\sigma_P(s)} \left. \frac{d^2\sigma^{PPP}}{dydt} \right|_{t=0} = C \left[\exp\left(y_{max}\right) - \exp\left(y_{min}\right) \right]$$

$y = \ln(s/M^2)$, $M^2 =$ squared mass of the diffractive system
$y_{max} = \frac{1}{2} \ln(s/m_T^2)$, $y_{min} = \ln(R_A m_N/\sqrt{3})$, $C =$ triple pomeron coupling


A
$R_A(s)$
y^*
$y^*=0$
$R_B(b-s)$
B

b (fm)	$Shadow(ch)$	$Shadow(J/\psi)$
0.	0.4959	0.7482
1.	0.4962	0.7485
2.	0.4973	0.7493
3.	0.5003	0.7513
4.	0.5058	0.7550
5.	0.5145	0.7607
6.	0.5268	0.7687
7.	0.5423	0.7792
8.	0.5649	0.7928
9.	0.5954	0.8109
10.	0.6318	0.8321
11.	0.6830	0.8599
12.	0.7447	0.8909
13.	0.8072	0.9200

Shadowing corrections for Au+Au collisions at RHIC



- **Maximal multiplicity in absence of shadowing:**
  $dN_{AA}/dy = A^{4/3}$
  AGK cancellation
  A dependence:
  hard $\equiv$ soft

- **Multiplicity with shadow corrections:**
  $dN_{AA}/dy = A^{\alpha}$
  $\alpha = 1.13$ at RHIC
  $\alpha = 1.1$ at LHC
  AGK violated by triple-P

Some similarities: CGC, strings and pomerons

• In the percolation approach when taking the saturation limit – all the strings overlap into a single cluster that occupies the whole nuclear overlap area–

$$\mu_{AA} = \mu_n = \sqrt{\frac{nS_n}{S_1}}\mu_1 = \sqrt{\frac{N_s S_{AA}}{S_1}}\mu_1 \propto A \ , < p_T^2 >_{AA} = \frac{S_1}{S_{AA}} \frac{< p_T^2 >_1}{\mu_1} \mu_{AA}$$

$N_s \propto N_{coll} \propto A^{4/3}$ = number of strings, $S_{AA} \propto A^{2/3}$ = nuclear overlap area

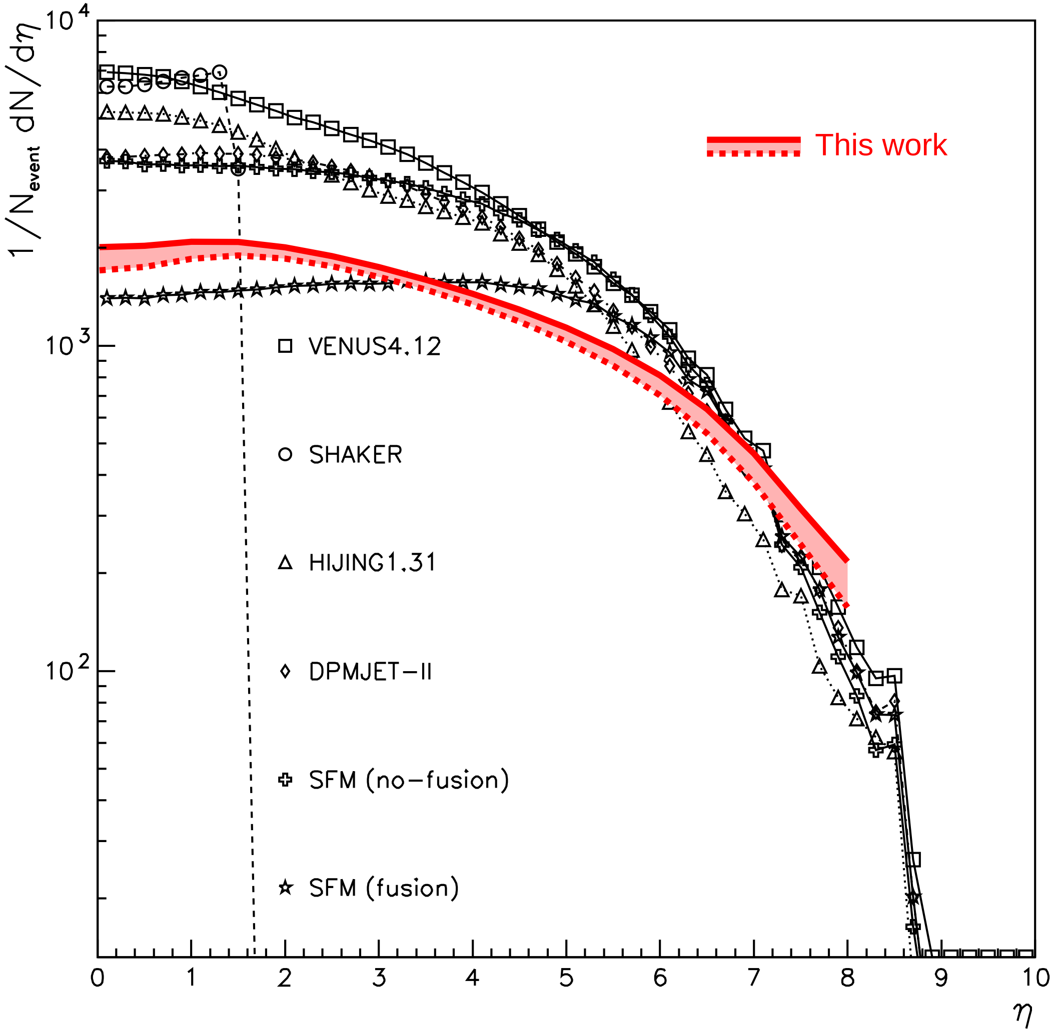
• In the CGC at saturation

$$\frac{dN}{dy} \sim xG(x, Q_s^2) \sim \frac{\pi R_A^2 \, Q_s^2(x, A)}{\alpha_s(Q_s^2)} \propto A \ , < p_T >^2 \sim \frac{1}{\pi R_A^2} \frac{dN}{dy}$$

$\pi R_A^2 \propto A^{2/3}$ = nuclear overlap area $Q_s^2(x, A) \propto A^{1/3}$ = saturation momentum

• In the RFT with maximal shadowing and pomeron approach: $I_A \propto A^{1.1}$

Predictions for LHC



Comparison of different predictions for charged hadron multiplicities in central ($b \leq 3$ fm) Pb-Pb collisions at LHC energies

—————— From: Kharzeev, Levin, Nardi, hep-ph/0408050

The original fig. is from: Armesto, Pajares, hep-ph/0002183

Prediction of SFM and CGC coincides

CONCLUSIONS

We have compared different models that takes into account saturation:

- **semi-phenomenological fusing color sources including percolation**
- **QCD saturation through the CGC**
- **pQCD pomeron with saturation in the initial conditions**.

Exchanged of elemental color sources –**strings**, **partons** or **pomerons**– lead to a saturation in the initial conditions when the densities are high enough.

In nuclear collisions there is a sudden onset of large-scale color connection
Above a critical density the elemental objects form one large cluster
They lose their independent existence and their relation to parent nucleons

Percolation = onset of color deconfinement

It may be a prerequisite for QGP formation, it does not imply thermalization

The idea of an ideal QGP –weakly coupled plasma of quarks and gluons– formed above a critical temperature $T_c \sim 160$ MeV is not longer valid.
For moderate temperatures $(1 \div 3)\,T_c$ the plasma is predicted by non-perturbative lattice to be strongly coupled (sQGP)
Its source could be a saturated initial state of the type described here.

Glasma

(Lappi, McLerran)

Glasma: highly coherent matter making in transition from Color Glass Condensate to Quark Gluon Plasma

It is characterized, in a manner reminiscent of the Lund string model, by the decay of a longitudinal electric and magnetic field into particles

This decay can happen both as classical radiation of the field and, quantum pair production from the classical background

Conclusions

The perturbative QCD hard pomeron approach leads to multiparticle production which qualitatively fully agrees with the color string approach with fusion.

Damping of the multiplicities turns out to be of the same strength as in the string picture, and also the average transverse momenta are found to rise nearly as predicted by the latter model.

This overall agreement may appear to be astonishing in view of very different dynamical pictures put in the basis of the two approaches and also quite different domains of their applicability: soft for the string picture and hard for the pomeron picture.

However, one may come to the conclusion that the dynamical difference between the two approaches is not so unbridgeable. Two phenomena are playing the leading role in both approaches:

One is fusion of exchanged elemental objects, strings in one picture and pomerons in the other. This explains damping of multiplicities per one initial elemental object.

Second phenomenon is the rise of average transverse momentum with this fusion.
It is generated by formation of strings of higher tension (color) in the string scenario.
In the pomeron model this rise occurs due the growth of the saturation momentum, which shifts the momentum distribution to higher momenta with A (and Y). Due to this shift non-linear effects in the pomerons in some sense reproduce formation of strings of higher color in the string model.

So we find an agreement between predictions of these two models, pertaining to completely different (in fact opposite) kinematical regions of secondaries, about certain basic features of multiparticle spectra.

These results are not fully unexpected. Indeed similar predictions were found previously in the framework of color glass condensate. Also in all considered approaches scaling in the transverse momentum distribution was observed. We consider this as a strong support for these predictions and thereby for the models.

NOTES

EFFECTIVE THEORY FOR THE CGC

• General idea: Fast partons (valence quarks with large longitudinal momentum) are considered as a classical souce ρ that emits soft gluons (with smaller longitudinal momenta) with are treated as classical color fields $\mathcal{A}[\rho]$

• Yang Mills eqs. describing soft gluon dynamics:

$$D_\nu F^{\nu\mu} \;=\; \delta^{\mu +}\rho(x^-, \boldsymbol{x})$$

• Physical quantities, as the unintegrated gluon distribution, are obtained as an average over ρ:

$$< A^i(X)\, A^i(Y) >_x = \int D[\rho]\; W_x[\rho]\; \mathcal{A}^i[\rho](X)\, \mathcal{A}^i[\rho](Y)$$

$A^i(X)$= classical solution for given ρ
$W_x[\rho]$= gauge-invariant weight function for ρ

Some characteristic of the source

The soft gluons (classical color fields) see the fast partons (random color source) as an effective color charge which is:

• Static: $E = q^- = \frac{q_\perp^2}{2q^+} = \frac{1}{\Delta x^+}$
The soft gluons have larger energies and shorter lifetimes

• Localized near the LC: $\lambda^- \sim \frac{1}{p^+}$, $q^+ \ll p^+$
For the soft gluons the fast partons appear sharply localized at the LC within $\lambda^- \sim \frac{1}{p^+}$, $q^+ \ll p^+$
For the soft gluons the fast partons appear sharply localized at the LC within a distance $\Delta x^- \sim \frac{1}{\Lambda^+}$

• With a random density $\rho(x^-, x_\perp)$ since this is the instantaneuos color charge of the fast partons $seen$ by the shortlived soft gluons at some arbitrary time

This color charge acts as a source of soft gluons that can be treated in the classical approximation

What we are doing is a kind of Born-Oppenhaimer approximation:
1. We study the dynamics of the classical fields (Weizsacher-William field) for a given configuration ρ of the color charges
2. We averages over all possible configurations

1. Classical solution:

$$F^{+i}(x^-, x_\perp) = \delta(x^-)\frac{i}{g}V(x_\perp)(\partial^i V(x_\perp)^\dagger) = \partial^+ A^i$$

$$V^\dagger(x_\perp) \equiv \mathrm{Pexp}\left\{ig\int dx^- A^+(x^-, \boldsymbol{x})\right\}; -\nabla_\perp^2 A^+ = \rho$$

2. The weight function:

$W_\tau[\rho]$ is obtained by integrating out the fast partons, so it depends upon the rapidity scale $\tau = \ln(1/x)$ at which one considers the effective theory

With increasing energy (or decreasing x) new quantum modes become relatively "fast" and must be included in the color source seen by the gluons. Thus, the classical description of the small-x gluons is to be seen as an effective theory valid at a given value of x, and whose "action" is evoluting with x

Non linear evolution and saturation

• When the rapidity τ is increase ny $\mathrm{d}\tau$ (i. e., the hadron is further accelerated) the quantum gluons with rapidity τ' in the interval $\tau < \tau' < \tau + \mathrm{d}\tau$ must be incorporated in the effective theory, they become part of the color glass

$\Rightarrow$ Change of the color source ρ that can be absorbed into an appropiated "renormalization" of the weight function $W_\tau \rightarrow W_{\tau+\mathrm{d}\tau}$

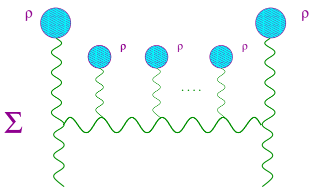
• We do that by integrating out the quantum modes in the background of the color fields generated at the previous step of the evolution

$\Rightarrow$ We obtain a non-linear evolution equation for $W_\tau[\rho]$

THE RENORMALIZATION GROUP EQUATION

- ρ and its correlations change with increasing $\tau \equiv \ln 1/x$

$$\frac{\partial W_\tau[\rho]}{\partial \tau} = \frac{1}{2} \int_{x_\perp, y_\perp} \frac{\delta}{\delta \rho_x^a} \chi_{xy}^{ab}[\rho] \frac{\delta}{\delta \rho_y^b} W_\tau[\rho]$$

$\chi = \Sigma$ 

Jalilian-Marian, Kovner, Leonidov, Weigert, 97;

Iancu, Leonidov, McLerran, 2000

- Functional diffusion equation

- It encompasses previous evolution equations by :

  Balitsky (96), Kovchegov (99), Weigert (2000)

- χ depends upon ρ via Wilson lines:

$$V^\dagger(x_\perp) \equiv \mathrm{Pexp}\left\{ig \int dx^- A^+(x^-, x_\perp)\right\}; \; -\nabla_\perp^2 A^+ = \rho$$

- Weak fields: high Q^2 or low energy

$V^\dagger(x_\perp) \approx 1 + igA^+(x_\perp) \implies$ BFKL equation.

- Strong fields: low Q^2 or high energy

$gA^+ \sim 1 \implies V \ll 1 \implies$ Saturation

- Change of regime at $Q^2 \sim Q_s^2(x) \sim x^{-\lambda\alpha_s}$

SOLUTIONS

Physical quantities, as the gluon density, are obtained as an average over ρ

$$n(x,k_\perp) \equiv \frac{1}{\pi R^2}\frac{\mathrm{d}N}{\mathrm{d}\tau\,\mathrm{d}^2k_\perp} \propto \langle F^{+i}(k_\perp)F^{+i}(-k_\perp)\rangle_x$$

- Low density: high Q^2 or low energy $\quad k_\perp \gg Q_s(x)$

Weak fields: The Wilson lines can be expanded to the lowest order in A^+
$V^\dagger(x_\perp) \approx 1 + igA^+(x_\perp)$

RGE reduces to BFKL equation:

$$n(x,k_\perp) \sim \frac{1}{k_\perp^2}\frac{1}{x^{\omega\alpha_s}} \qquad \text{(Bremsstrahlung)}$$

$\Rightarrow$ Gluon density grows both with $1/k_\perp^2$ and τ

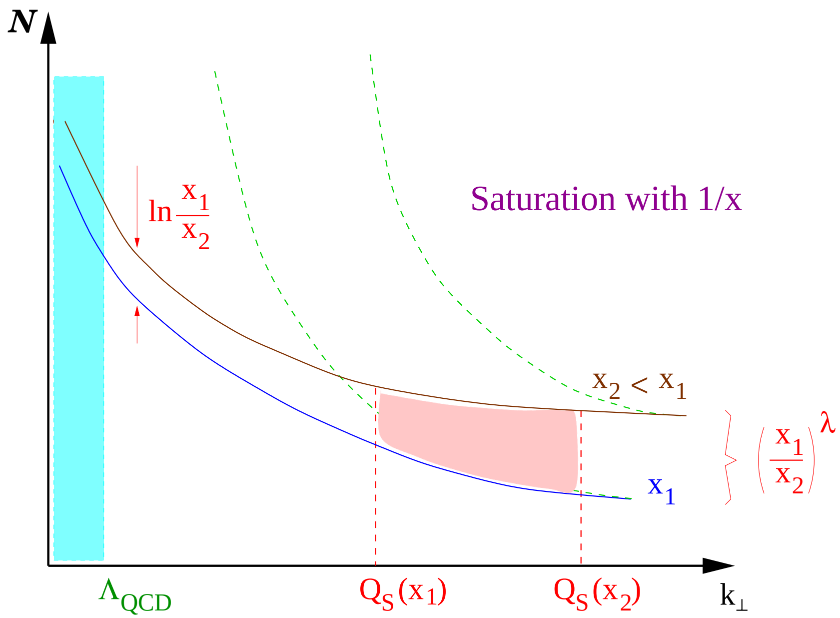
- High density: low Q^2 or high energy $\quad k_\perp \ll Q_s(x)$

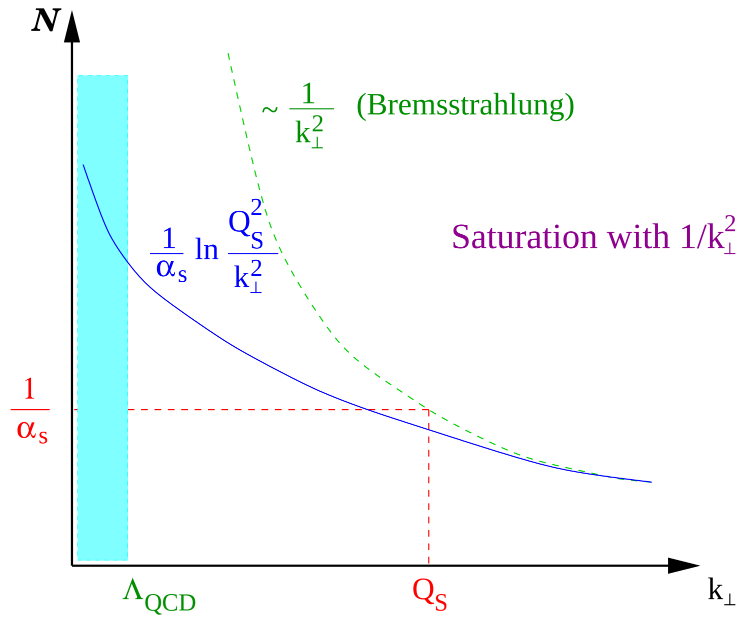
Strong fields: The Wilson lines oscillate and in average $V^\dagger(x_\perp) \approx 0$

RGE simplifies: χ becomes independent of ρ, $\chi = \frac{k_\perp^2}{\pi}$

$$n(x,k_\perp) \sim \frac{1}{\alpha_s}\ln\frac{Q_s^2(x)}{k_\perp^2} \propto \ln\frac{1}{x}$$

$\Rightarrow$ SATURATION: Gluon density increases linearly with τ and logaritmically with the energy: Unitarity is restored


N
ln x1/x2
Saturation with 1/x
x2 < x1
(x1/x2)^λ
x1
ΛQCD
QS(x1)
QS(x2)
k⊥


N
~ 1/k⊥^2 (Bremsstrahlung)
1/αs ln QS^2/k⊥^2
Saturation with 1/k⊥^2
1/αs
ΛQCD
QS
k⊥

Geometric Scaling at HERA

(Staśto, Golec-Biernat, and Kwieciński, 2000)

- At saturation: $n(x, k_\perp) \sim \frac{1}{\alpha_s} \ln \frac{Q_s^2(x)}{k_\perp^2}$

- For $x < 10^{-2}$ and $Q^2 \lesssim 400\text{GeV}^2$, data show scaling: $\sigma_{\gamma^* p}(x, Q^2) \approx \sigma_{\gamma^* p}(\mathcal{R})\,, \qquad \text{with } \mathcal{R} \equiv \frac{Q^2}{Q_s^2(x)}$

where: $Q_s^2(x) = Q_0^2 x^{-\lambda}$, $Q_0^2 = 1\text{GeV}^2$, $\lambda \simeq 0.3$

- This suggests : $\mathcal{N}(x, r_\perp) \approx \mathcal{N}\big(r_\perp^2 Q_s^2(x)\big)$

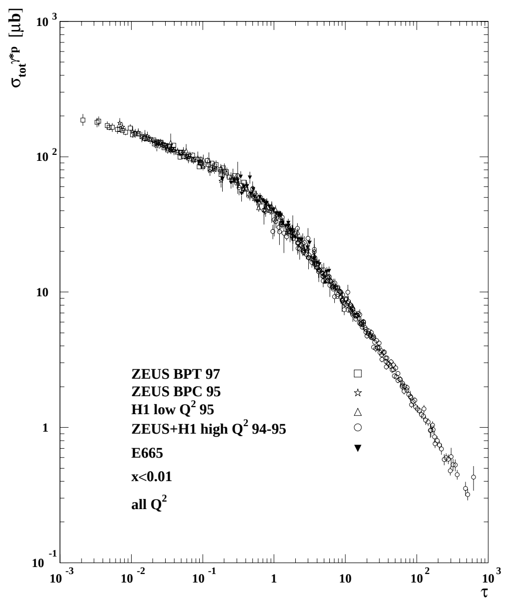
— Natural at saturation ($Q^2 < Q_s^2 \sim 1\text{GeV}^2$)

— Preserved by the BFKL evolution up to:

$Q^2 \leq \frac{Q_s^4}{\Lambda_{QCD}^2} \sim 100\text{GeV}^2$

Iancu, Itakura and McLerran, 2002;

Mueller and Triantafyllopoulos, 2002



• The strings form clusters, each of them with a constant color field $E_i = Q_i/S_i$, where Q_i and S_i correspond to the cluster color charge and the cluster area

• Schwinger formula for the production of $q - \bar{q}$ pairs of mass m_j in a uniform color field with charge g_j, per unit space-time volume:

$$\frac{dN_{q-\bar{q}}}{dy} = \frac{1}{8\pi^3} \int_0^\infty d\tau\tau \int d^2x_T |g_j E|^2 \sum_{n=1}^{\infty} \frac{1}{n^2} \exp\Big(- \frac{\pi n m_j^2}{|g_j E|} \Big)$$

• The charge and the field of each cluster before the decay

$$Q_{i0} = \sqrt{\frac{n_i S_i}{S_1}} Q_{10} \quad , \quad E_{i0} = \frac{Q_{i0}}{S_i} = \sqrt{\frac{n_i S_1}{S_i}} E_{10}$$

n_i= number of strings in the cluster, S_1= area of each individual string, S_i= total area of the cluster, Q_{10} and E_{10}= charge and field of the individual strings

• Taking into account the evolution of the field and the charge with the decay of the cluster

$$E_i = E_{i0} \frac{1}{(1 + \frac{\tau}{\tau_{i0}})^2} \quad , \quad Q_i = Q_{i0} \frac{1}{(1 + \frac{\tau}{\tau_{i0}})^2}$$

where $\tau \sim 1/\sqrt{E_{i0}}$

- We obtain

$$\frac{dN_{q-\bar{q}}}{dy} \propto \sum_{i=1}^{M} Q_{i0} \int_0^\infty dx x \frac{1}{(1+x)^4} \sum_{n=1}^{\infty} \frac{1}{n^2} \exp\left[- \frac{\pi n m_j^2}{|g_j E_{i0}|} (1+x)^2 \right]$$

where M is the total number of clusters