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Nonperturbative QCD Calculations of (quasi) elastic processes

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Nonperturbative QCD calculations of (quasi) elastic processes

$$\gamma^* \mathrm{p} \to \mathrm{Vp} \quad , \quad \mathrm{V} = \rho, \omega, \phi, \psi, \Upsilon$$

EXPLORING

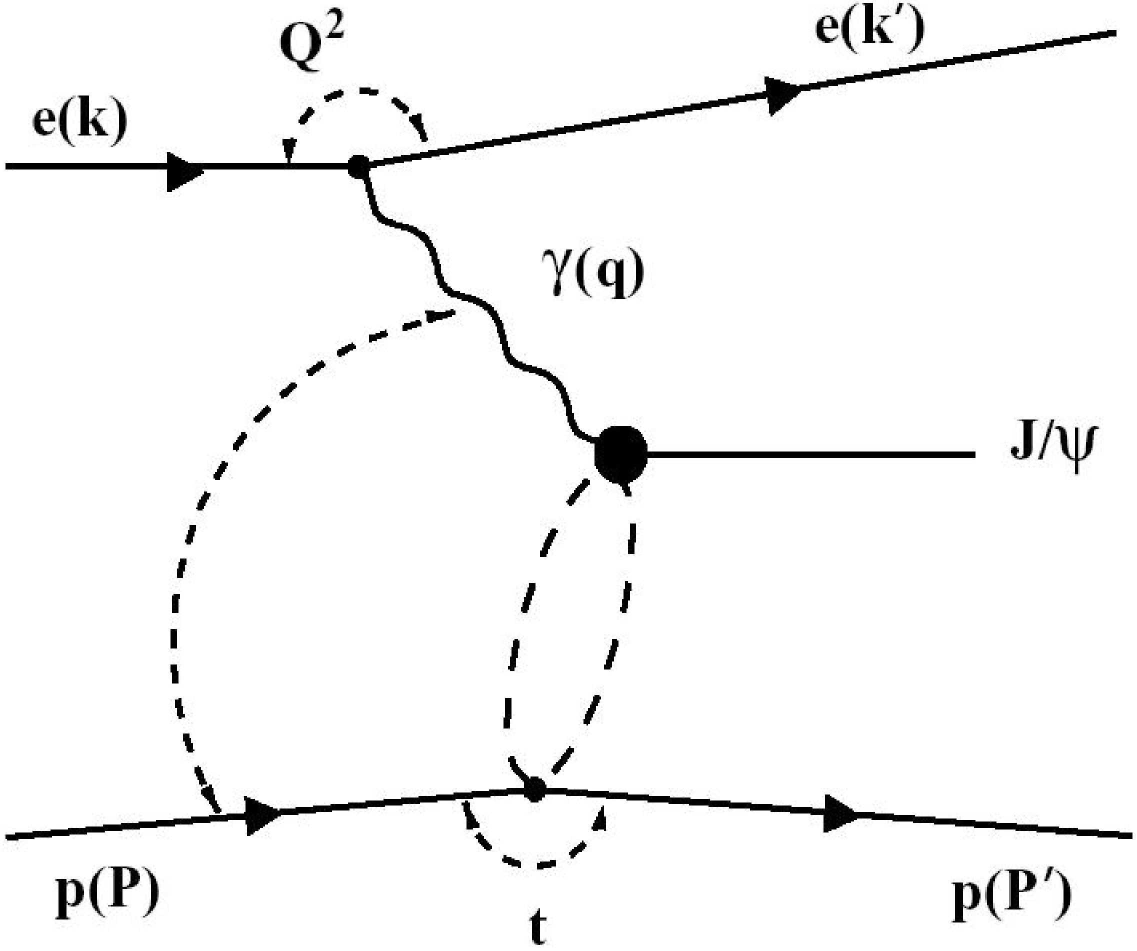
A

NON-PERTURBATIVE QCD APPROACH

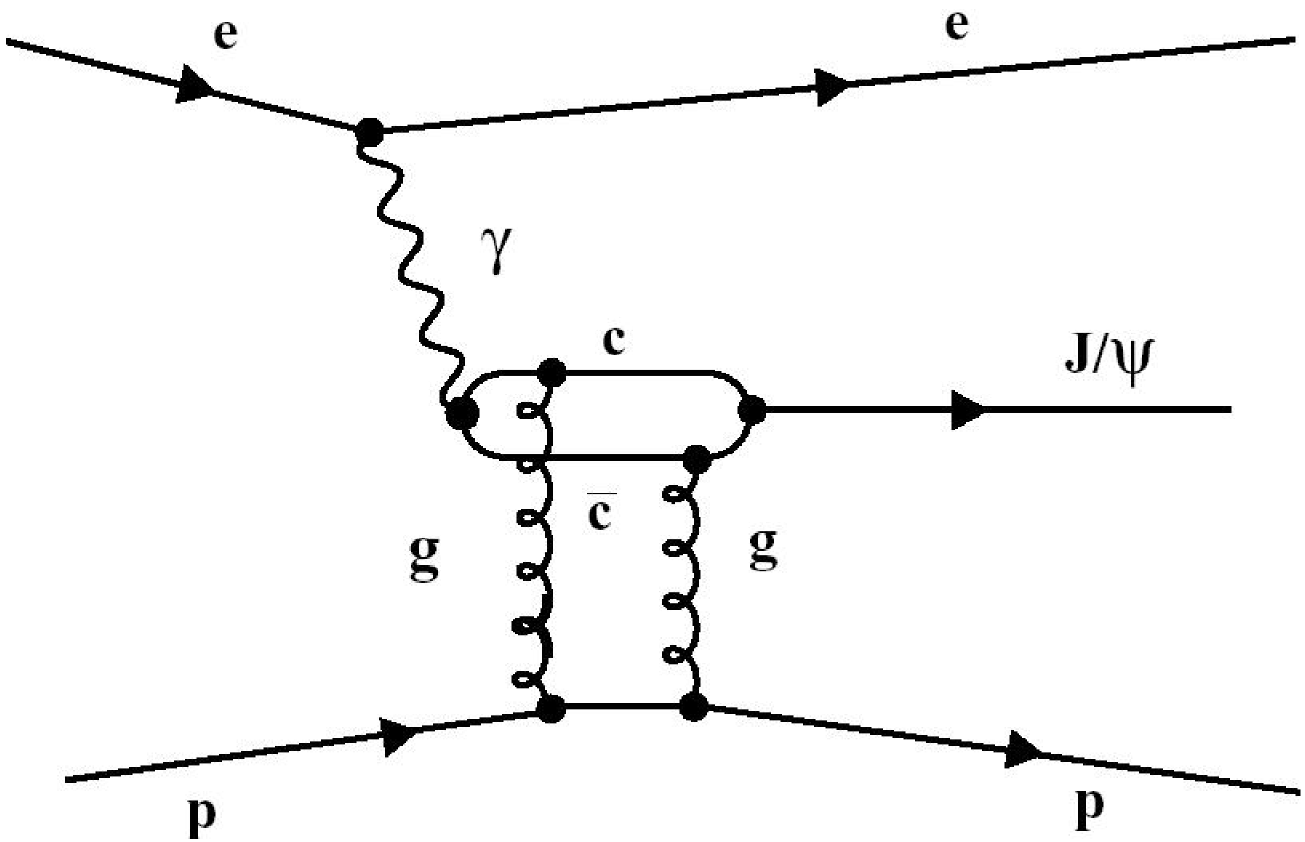
Initial reference work : H.G.Dosch, T. Gousset, G. Kulzinger and J.J. Pirner - Phys. Rev. D55 (1997) 2602

Further work by H.G. Dosch, E. Ferreira, V.Baltar.

Present situation : Data of increasing accuracy and covering more observables from HERA on photo and electroproduction of vector mesons require and allow extended theoretical and phenomenological analysis.



Q²
e(k′)
e(k)
γ(q)
J/ψ
p(P)
t
p(P′)


e
e
γ
c
J/ψ
$\bar{c}$
g
g
p
p

VECTOR MESONS

ρ , ω , ϕ , Ψ , Υ

VIRTUAL PHOTON

γ*(Q2)

PROTON

p

Polarizations : T , L

Wave functions

Photons and vector mesons: Quark-antiquark systems

Proton: quark-diquark

Vector mesons : BL and BSW wf

Overlaps
<photon*| dipole(r) |VM>

dipole-proton interaction

Integrate over all dipoles
that form the wave functions

Mechanisms for the dipole-dipole interaction

Pomerons,
QCD vacuum correlations,
two-gluon exchanges

Perturbative Vs Nonperturbative
Unique calculations? Free parameters? Unified description of all vector mesons

$$\gamma^* \mathrm{p} \longrightarrow \mathrm{Vp} \quad , \quad \mathrm{V} = \rho, \omega, \phi, \psi, \Upsilon$$

Amplitude for electroproduction with polarization λ

$$T_{\gamma^* p \to Vp,\lambda}(s,t;Q^2) = \int d^2\mathbf{R}_1 dz_1 \; \rho_{\gamma^* V,\lambda}(Q^2; z_1, \mathbf{R}_1) \; J(s, \mathbf{q}, z_1, \mathbf{R}_1)$$

$$J(s, \mathbf{q}, z_1, \mathbf{R}_1) = \int d^2\mathbf{R}_2 d^2\mathbf{b} \, e^{-i\mathbf{q}.\mathbf{b}} |\psi_p(\mathbf{R}_2)|^2 S(s, b, z_1, \mathbf{R}_1, z_2 = \frac{1}{2}, \mathbf{R}_2)$$

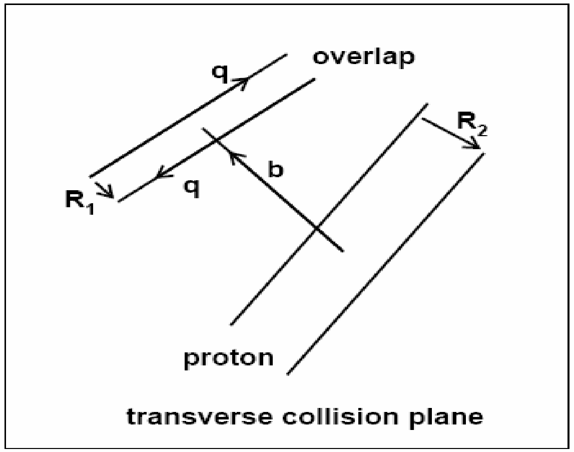
Overlap of virtual photon and VM wave functions

$$\rho_{\gamma^* V,\lambda}(Q^2; z_1, \mathbf{R}_1) = \psi_{V\lambda}(z_1, \mathbf{R}_1)^* \psi_{\gamma^*\lambda}(Q^2; z_1, \mathbf{R}_1)$$

Scattering of two dipoles $\mathbf{R}_1, \mathbf{R}_2$

$$S(s, b, z_1, \mathbf{R}_1, 1/2, \mathbf{R}_2)$$

Impact parameter $\vec{\mathbf{b}}$, **Momentum transfer** $t \approx -\mathbf{q}^2$



Wave functions

Light-cone coordinates : dipole pair : $q \to \bar{q}$ vector in the transverse plane $\mathbf{r} = (r\cos\theta, r\sin\theta)$, and momentum fractions z , $\bar{z} = (1-z)$ of quark and antiquark.

Photon wave functions **$q\bar{q}$ photon wave function: virtuality Q^2 polarization state λ, flavours $(f, \bar{f})$, helicities $h, \bar{h})$, $\hat{e}_f$ quark charge for flavour f**

$$\psi_{\gamma^*,+1}(Q^2; z, r, \theta) = \hat{e}_f \frac{\sqrt{6\alpha}}{2\pi}$$

$$[i\epsilon_f e^{i\theta}(z\delta_{h,+}\delta_{\bar{h},-} - \bar{z}\delta_{h,-}\delta_{\bar{h},+})K_1(\epsilon_f r) + m_f \delta_{h,+}\delta_{\bar{h},+}K_0(\epsilon_f r)]$$

$$\psi_{\gamma^*,-1}(Q^2; z, r, \theta) = \hat{e}_f \frac{\sqrt{6\alpha}}{2\pi}$$

$$[i\epsilon_f e^{-i\theta}(\bar{z}\delta_{h,+}\delta_{\bar{h},-} - z\delta_{h,-}\delta_{\bar{h},+})K_1(\epsilon_f r) + m_f \delta_{h,-}\delta_{\bar{h},-}K_0(\epsilon_f r)]$$

$$\psi_{\gamma^*,0}(Q^2; z, r) = \hat{e}_f \frac{\sqrt{3\alpha}}{2\pi}(-2z\bar{z})\ \delta_{h,-\bar{h}}\ Q\ K_0(\epsilon_f r)$$

$$\epsilon_f = \sqrt{z(1-z)Q^2 + m_f^2}$$

m_f quark masses

$$\mathrm{m_u} = \mathrm{m_d} = 0.1 - 0.2,\ \ \mathrm{m_s} = 0.2 - 0.3,\ \ \mathrm{m_c} = 1.25,\ \ \mathrm{m_b} = 4.2\ \mathrm{GeV}$$

Vector meson wave functions

Spin structure similar to photon

$$\psi_{V,+1}(z,r,\theta) = (-ie^{i\theta}\partial_r(z\delta_{h,+}\delta_{\bar{h},-} - \bar{z}\delta_{h,-}\delta_{\bar{h},+}) + m_f\delta_{h,+}\delta_{\bar{h},+})\phi_V(z,r)$$

$$\psi_{V,-1}(z,r,\theta) = (-ie^{-i\theta}\partial_r(\bar{z}\delta_{h,+}\delta_{\bar{h},-} - z\delta_{h,-}\delta_{\bar{h},+}) + m_f\delta_{h,-}\delta_{\bar{h},-})\phi_V(z,r)$$

$$\psi_{V,0}(z,r) = (\omega 4 z\bar{z}\delta_{h,-\bar{h}})\ \phi_V(z,r)$$

Scalar function $\phi_V(z,r)$ has two parameters, N and ω, fixed by normalization condition and leptonic decay width for each meson. ω controls the meson size.

Two forms for $\phi_V(z,r)$

1) Bauer-Stech-Wirbel (BSW) - based on relativistic harmonic oscillator

$$\phi_{BSW}(z,r) = \frac{N\sqrt{z\bar{z}}}{\sqrt{4\pi}}\exp\left[-\frac{M_V^2}{2\omega^2}(z-\frac{1}{2})^2\right]\exp\left[-\frac{1}{2\omega^2 r^2}\right]$$

2) Brodsky-Lepage (BL) construction of a light-cone wave function from a non-relativistic one

$$\phi_{BL}(z,r) = \frac{N}{\sqrt{4\pi}}\exp\left[-\frac{m_f^2(z-\frac{1}{2})^2}{2z\bar{z}\omega^2}\right]\exp[-2z\bar{z}\omega^2 r^2]$$

M_V **- vector meson mass**

Overlap functions

After summation over helicity indices ===>

Transverse BSW and BL:

$$\rho_{\gamma^* V,\pm 1;BSW}(Q^2; z, r) =$$

$$\hat{e}_V \frac{\sqrt{6\alpha}}{2\pi}(\epsilon_f \, \omega^2 r[z^2+\bar{z}^2]K_1(\epsilon_f \, r)+m_f^2 K_0(\epsilon_f \, r)) \, \phi_{BSW}(z, r)$$

$$\rho_{\gamma^* V,\pm 1;BL}(Q^2; z, r) =$$

$$\hat{e}_V \frac{\sqrt{6\alpha}}{2\pi}(4\epsilon_f \, \omega^2 r z\bar{z}[z^2+\bar{z}^2]K_1(\epsilon_f \, r)+m_f^2 K_0(\epsilon_f \, r)) \, \phi_{BL}(z, r)$$

Longitudinal $X=$ BSW or BL:

$$\rho_{\gamma^* V,0;X}(Q^2; z, r) =$$

$$-16\hat{e}_V \frac{\sqrt{3\alpha}}{2\pi} \, \omega \; z^2\bar{z}^2 \; Q \; K_0(\epsilon_f \, r)\phi_X(z, r)$$

The ranges (in the r variable) of the photon-meson overlap functions are crucial in the factorization process that give the amplitude with a factor containing all Q^2 dependence of the amplitude.

Overlap strengths

Integration of the overlap functions over the internal variables of the quark-antiquark pairs, introducing a factor r^2

$$Y_{\gamma^* V,T;X}(Q^2) = \int_0^1 dz \int d^2\mathbf{r}\ r^2\ \rho_{\gamma^* V,\pm 1,X}(Q^2; z, r)$$
$$\equiv \hat{e}_V\ \widetilde{Y}_{\gamma^* V,T;X}(Q^2)$$

$$Y_{\gamma^* V,L;X}(Q^2) = \int_0^1 dz \int d^2\mathbf{r}\ r^2\ \rho_{\gamma^* V,0,X}(Q^2; z, r)$$
$$\equiv \hat{e}_V\ \widetilde{Y}_{\gamma^* V,L;X}(Q^2)$$

X = BSW or BL

Charge factors

$\hat{e}_V^2 = 1/2,\ 1/18,\ 1/9,\ 4/9,\ 1/9$ for $\rho, \omega, \phi, \psi, \Upsilon$

====> **approximate universalities**

Q^2 dependence of the squared strengths

$$\widetilde{Y}^2_{\gamma^* V,T}(Q^2) = \frac{A_T}{(1 + Q^2/M_V^2)^{n_T}}$$

$$\widetilde{Y}^2_{\gamma^* V,L}(Q^2) = \frac{A_L(Q^2/M_V^2)}{(1 + Q^2/M_V^2)^{n_L}}$$

Similar to behaviour found in experimental data on cross sections

Energy Dependence

Factorization of Q^2 dependence

$$T_{\gamma^* p \to Vp, T/L}(s,t;Q^2) \approx (-2is)\ G(t)\ Y_{\gamma^* V, T/L; X}(Q^2)$$

$G(t)$ depends on specific framework and model for the dipole-dipole interaction and proton structure. It may contain energy dependence due to the dipole-p interaction.

Model for energy dependence : the two-pomeron model of Donnachie and Landshoff

Split the integration over R_1 :

Small dipoles: $R_1 \leq r_c \approx 0.22$ fm hard pomeron (h) coupling - energy dependence $(s/s_0)^{0.42}$

Large dipoles: $R_1 \geq r_c \approx 0.22$ fm soft pomeron (s) coupling - energy dependence $(s/s_0)^{0.0808}$

Reference energy $s_0 = (20\ \text{GeV})^2$. Numerical values for r_c and s_0 taken from previous work

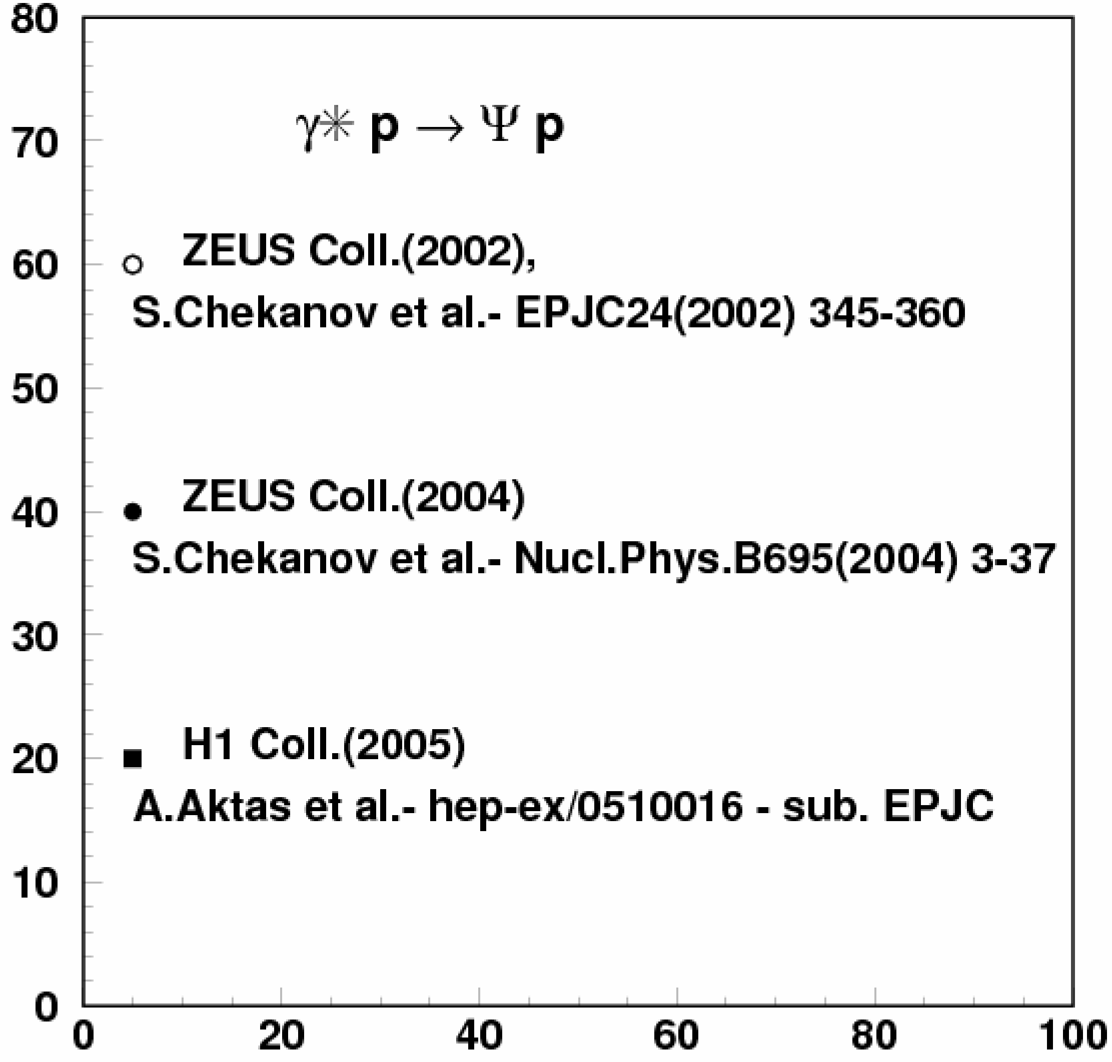
$$T_{\gamma^* p \to Vp, T/L}(s,t;Q^2)$$
$$\approx (-2is)\ G(t)\ \left(T_h^{T/L}(Q^2)\left(\frac{s}{s_0}\right)^{\epsilon_h} + T_s^{T/L}(Q^2)\left(\frac{s}{s_0}\right)^{\epsilon_s}\right)$$

From here ====> all observables

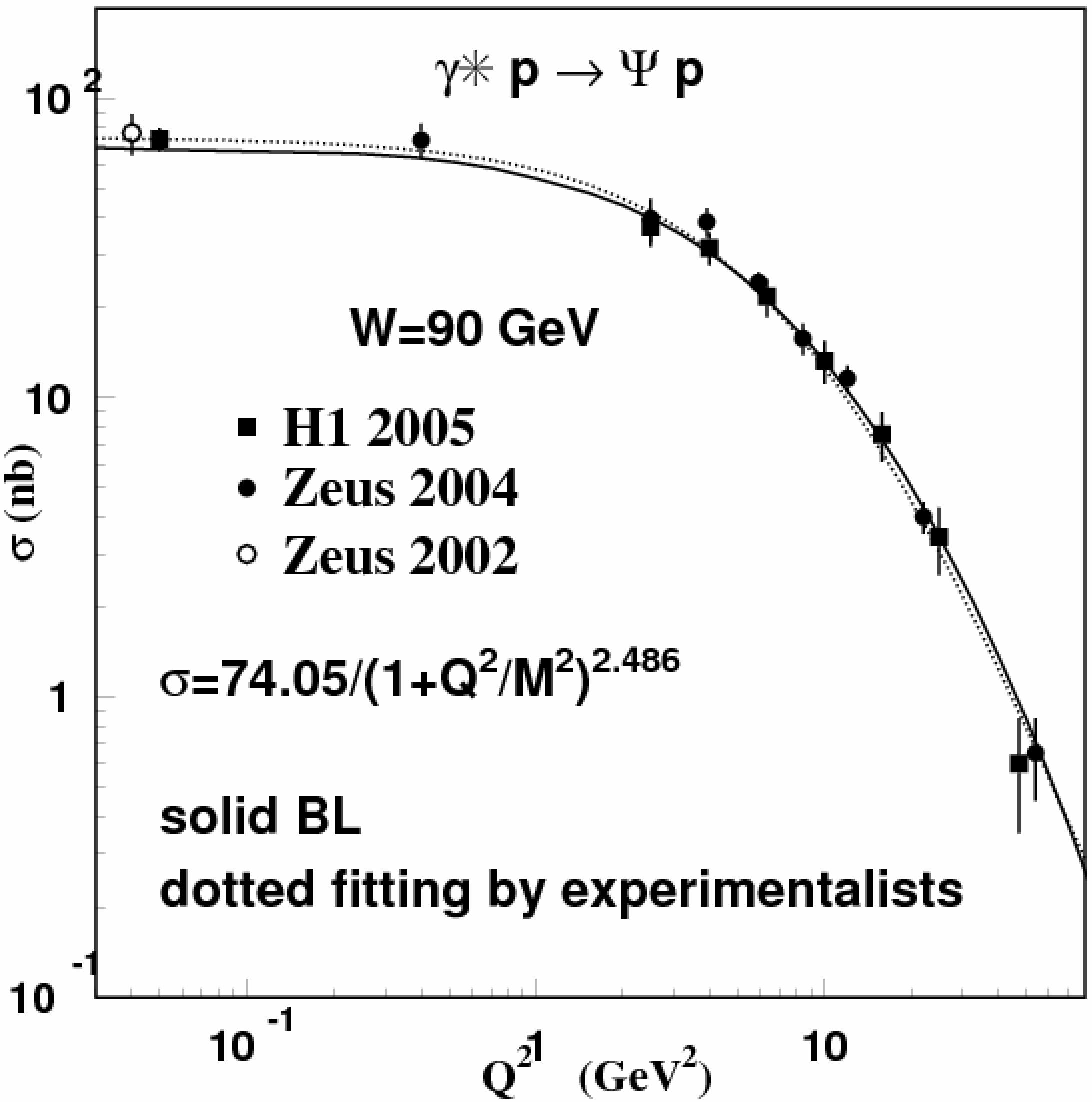
To form energy dependent overlap strengths

$$Y_h^{T/L}(Q^2) = 2\pi \int_0^{r_c} dR_1 \int_0^1 dz_1 \left(\frac{R_1^2}{r_c^2}\right)^{\epsilon_h} R_1^3 \rho_{\gamma^*,V,T/L}(Q^2, z_1, R_1)$$

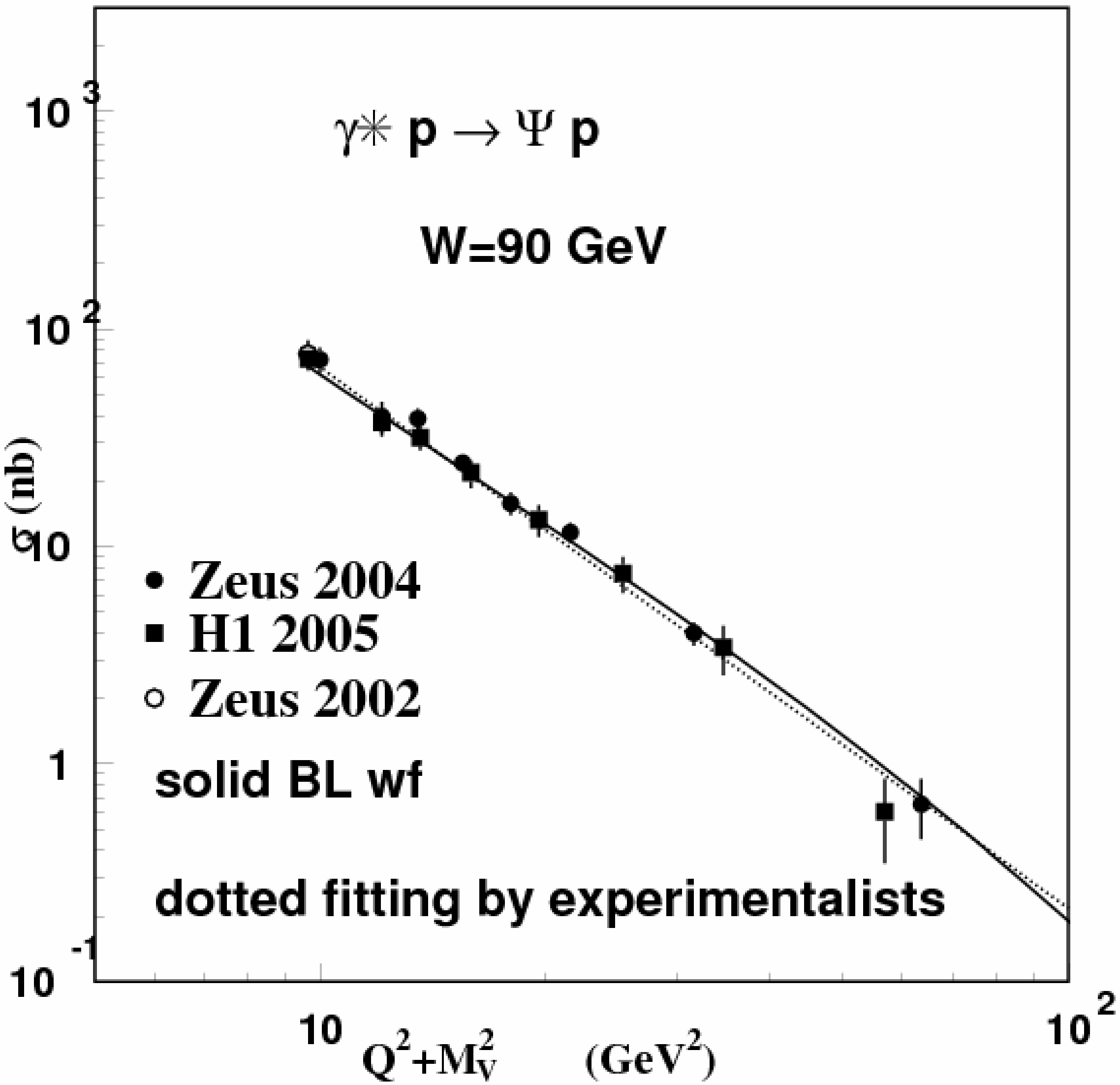
$$Y_s^{T/L}(Q^2) = 2\pi \int_{r_c}^{\infty} dR_1 \int_0^1 dz_1 \, R_1^3 \; \rho_{\gamma^*,V,T/L}(Q^2, z_1, R_1)$$



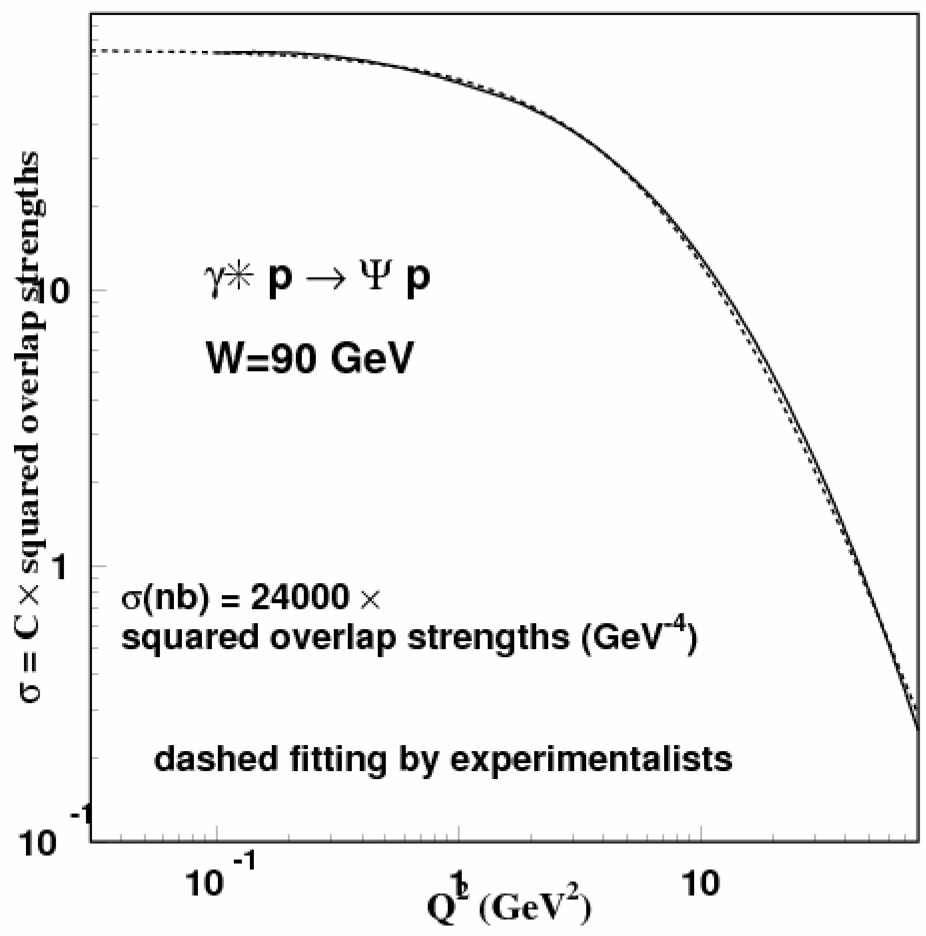
γ* p → Ψ p
ZEUS Coll.(2002),
S.Chekanov et al.- EPJC24(2002) 345-360
ZEUS Coll.(2004)
S.Chekanov et al.- Nucl.Phys.B695(2004) 3-37
H1 Coll.(2005)
A.Aktas et al.- hep-ex/0510016 - sub. EPJC
0
10
20
30
40
50
60
70
80
0
20
40
60
80
100

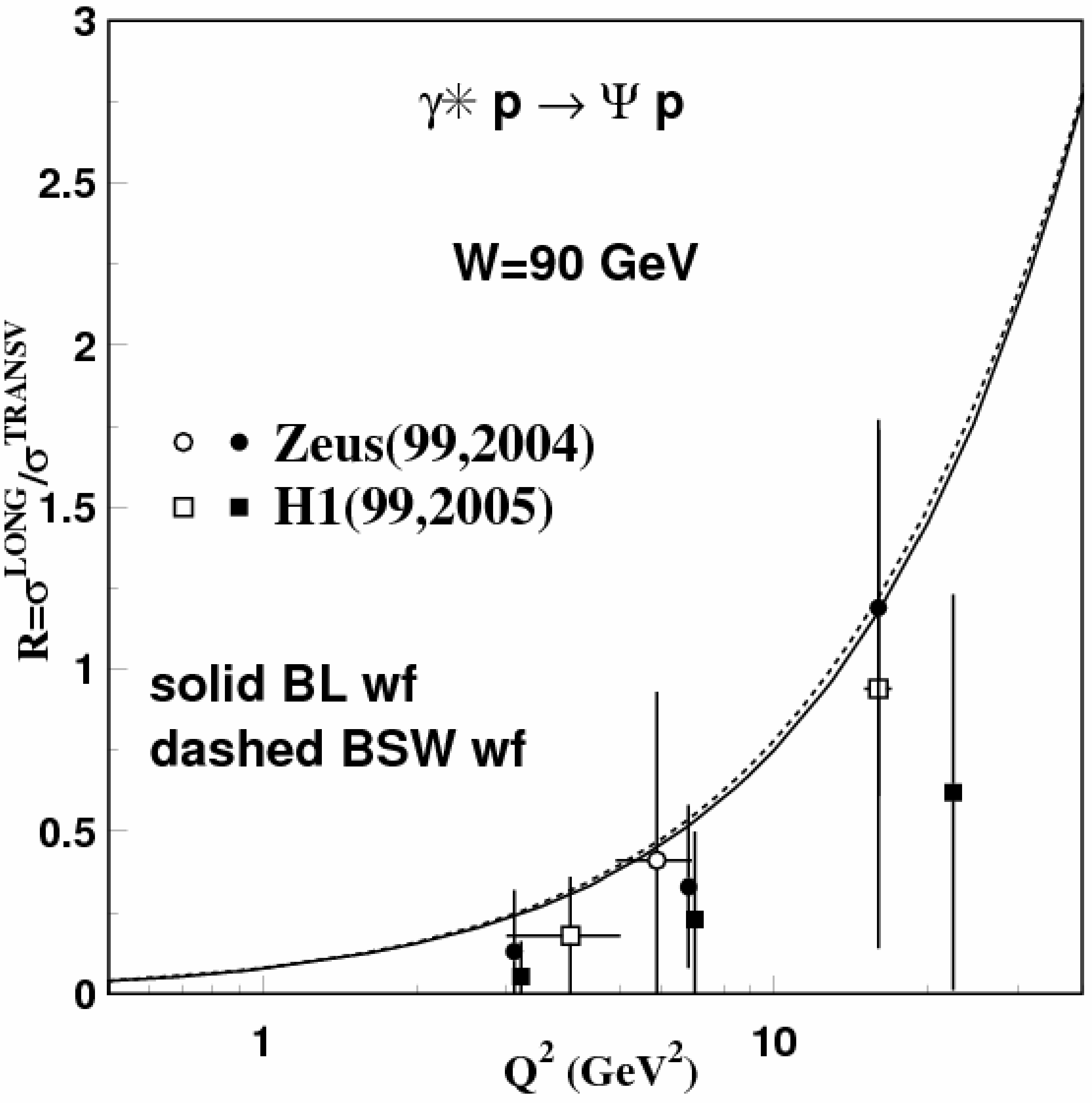


$\gamma^* p \to \Psi p$
W=90 GeV
H1 2005
Zeus 2004
Zeus 2002
$\sigma=74.05/(1+Q^2/M^2)^{2.486}$
solid BL
dotted fitting by experimentalists
σ (nb)
Q^2 (GeV^2)
10^2
10
1
10^{-1}
10^{-1}
1
10

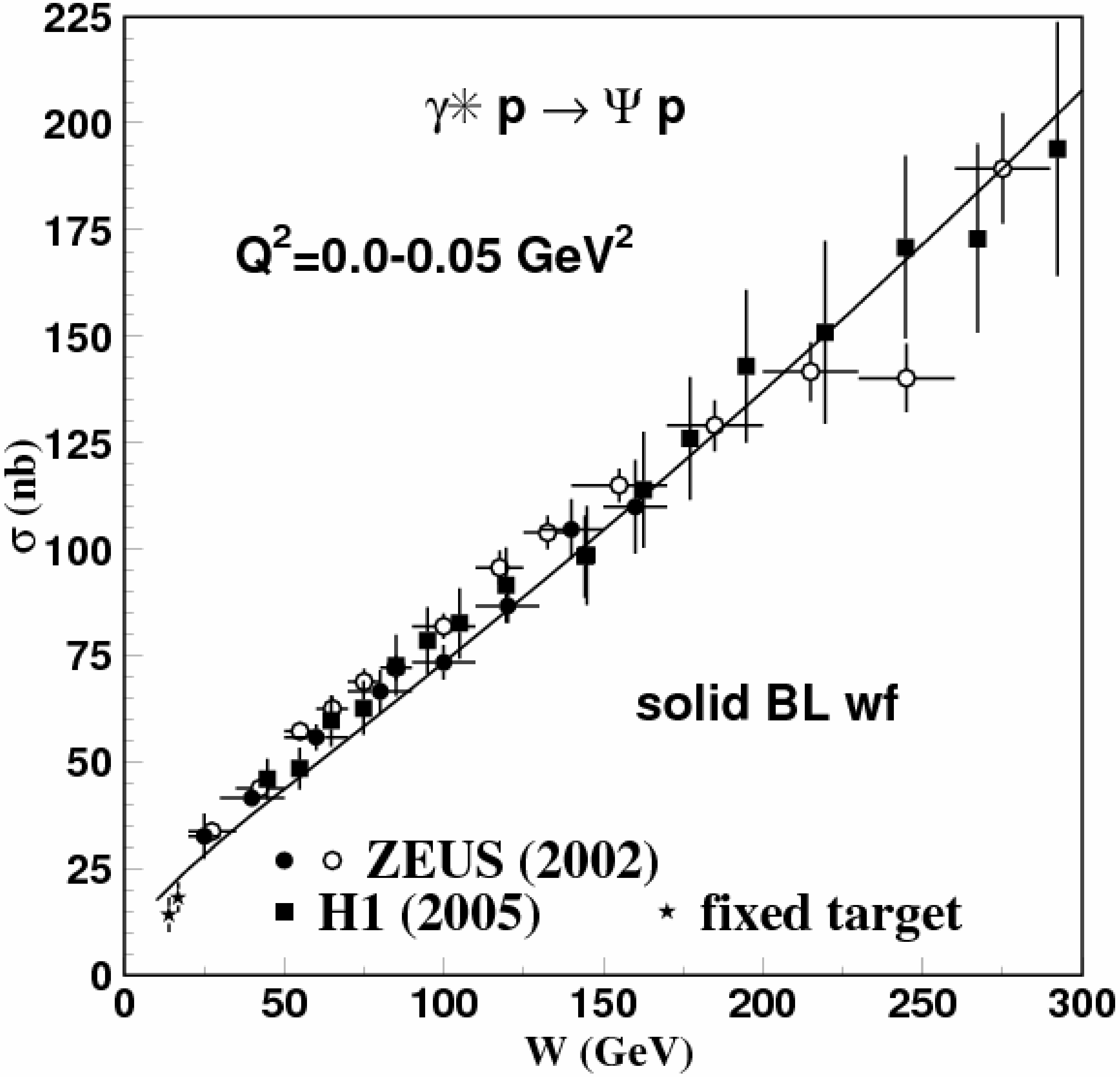


$\gamma^* p \to \Psi p$
W=90 GeV
σ (nb)
Zeus 2004
H1 2005
Zeus 2002
solid BL wf
dotted fitting by experimentalists
$Q^2+M_V^2$ (GeV^2)
10^3
10^2
10
1
10^{-1}
10
10^2

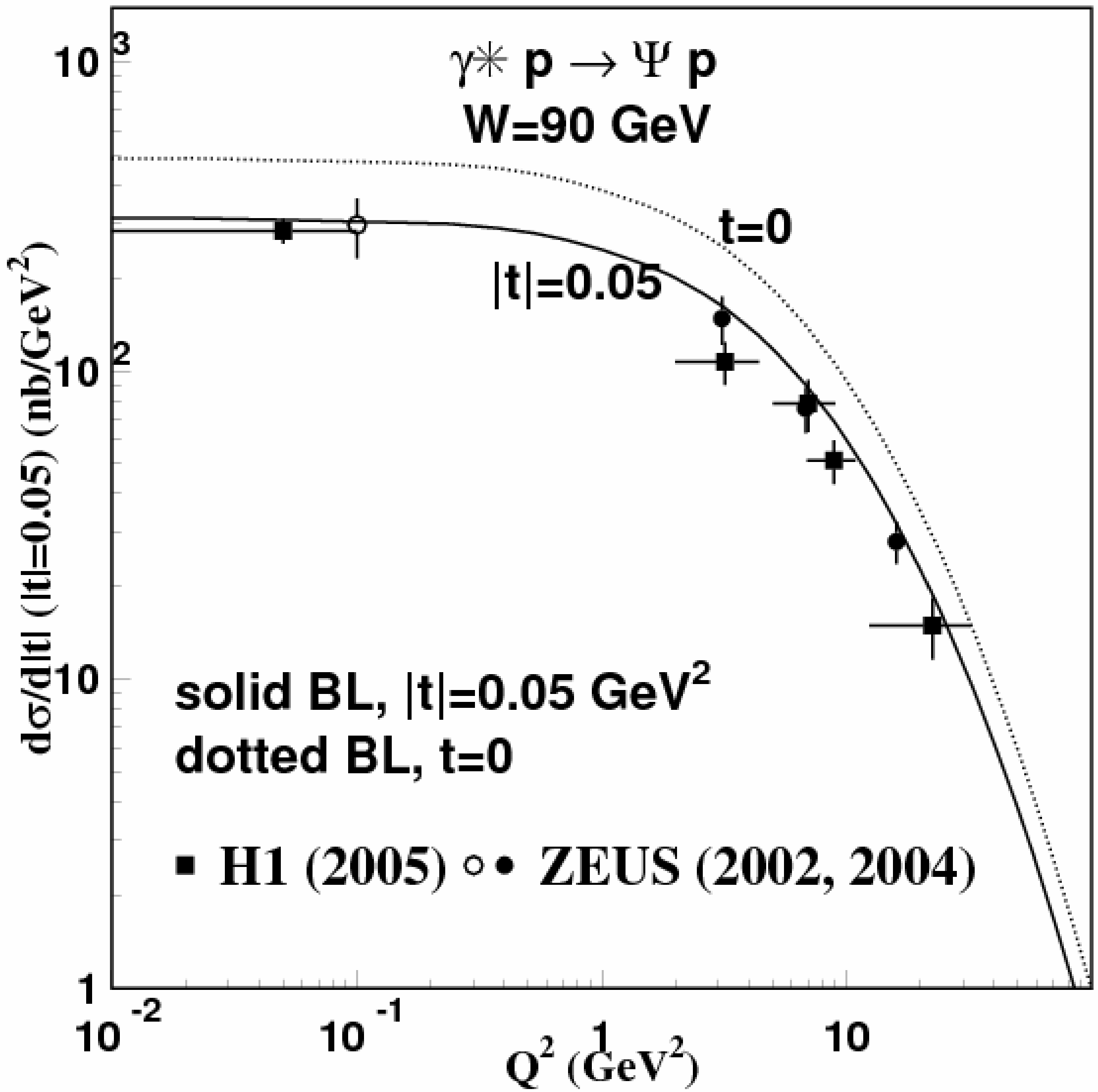

γ* p → Ψ p
W=90 GeV
σ(nb) = 24000 ×
squared overlap strengths (GeV-4)
dashed fitting by experimentalists
σ = C × squared overlap strengths
Q2 (GeV2)
10-1
1
10
10



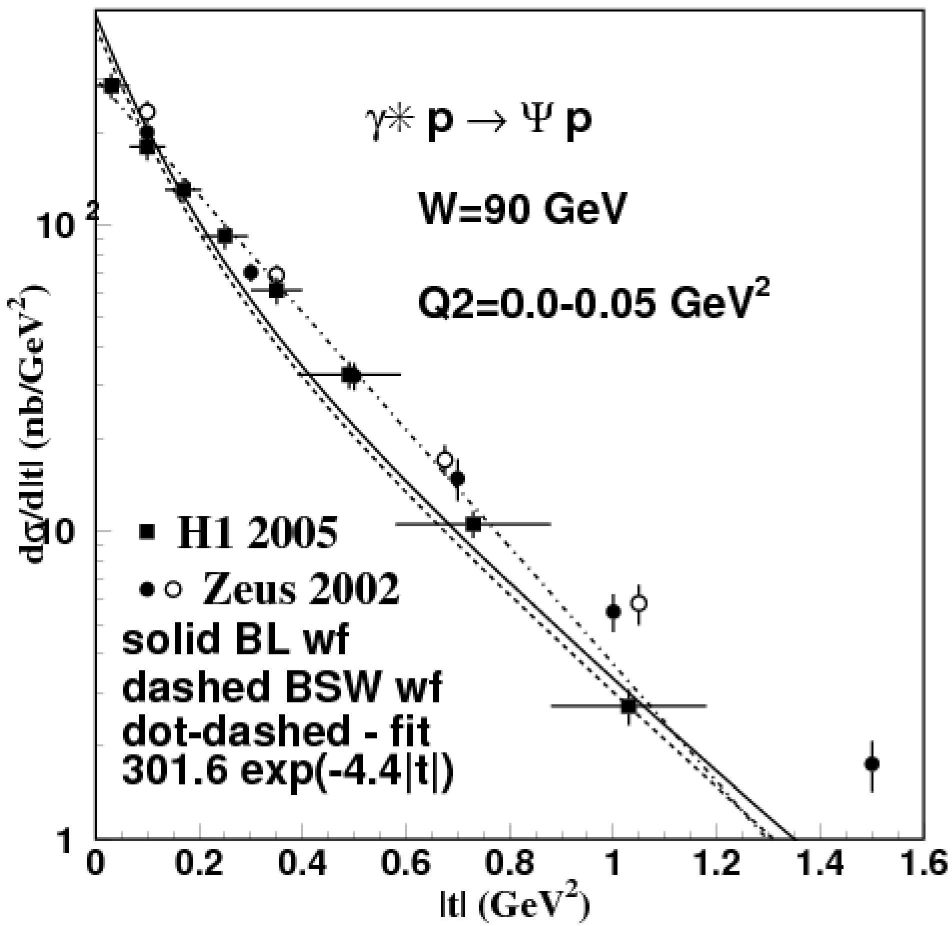
γ* p → Ψ p
W=90 GeV
○ ● Zeus(99,2004)
□ ■ H1(99,2005)
solid BL wf
dashed BSW wf
R=σ^LONG/σ^TRANSV
Q^2 (GeV^2)
0
0.5
1
1.5
2
2.5
3
1
10



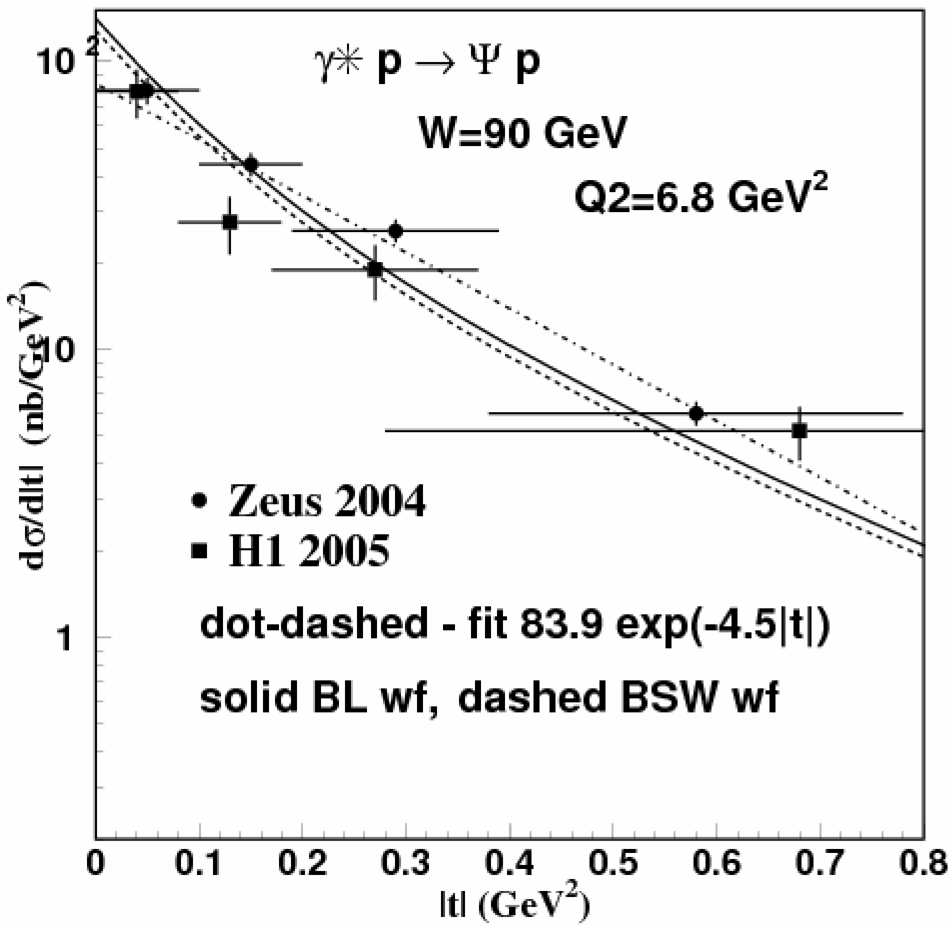
γ* p → Ψ p
Q²=0.0-0.05 GeV²
solid BL wf
ZEUS (2002)
H1 (2005)
fixed target
σ (nb)
W (GeV)
0
25
50
75
100
125
150
175
200
225
0
50
100
150
200
250
300

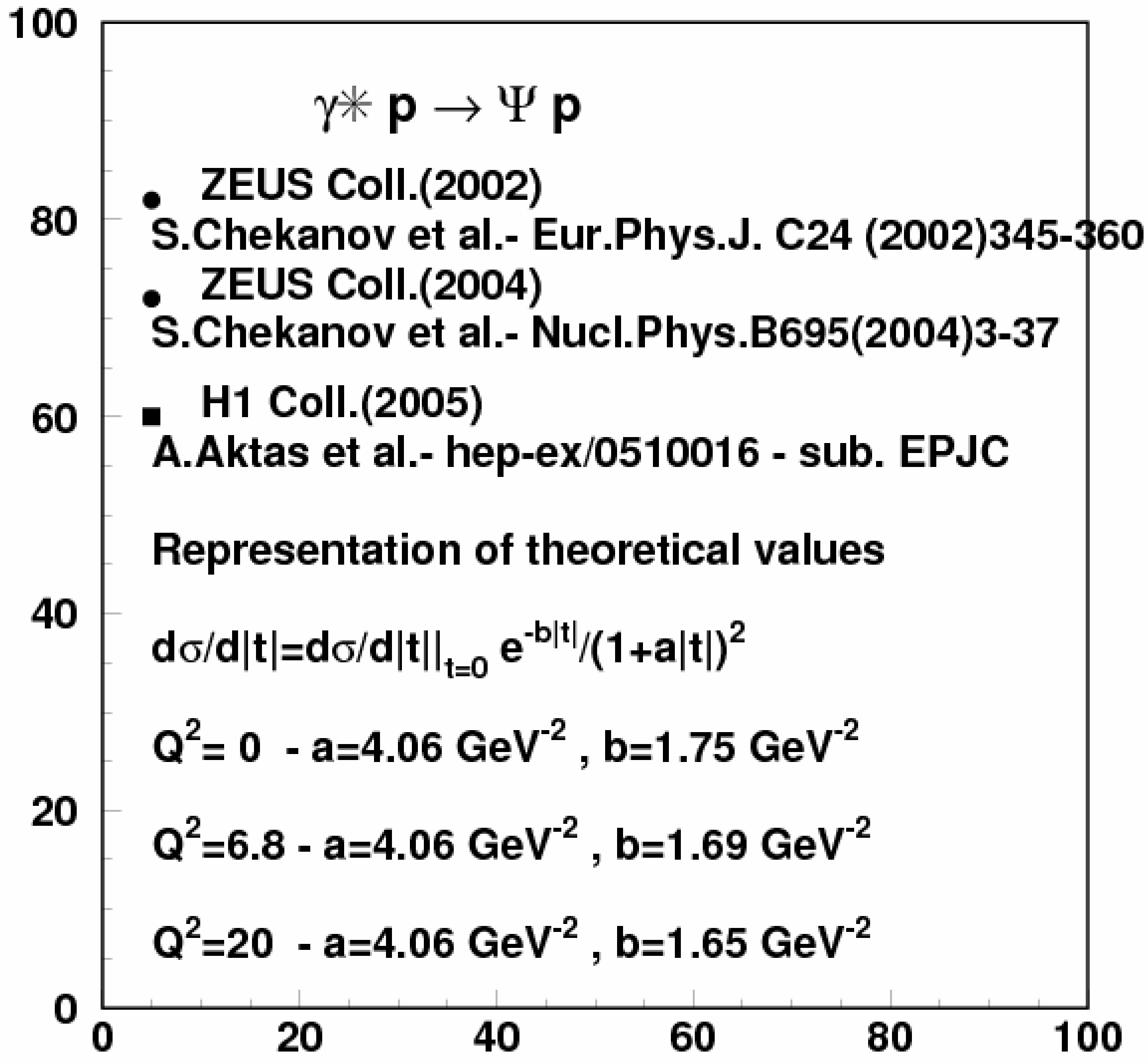


γ* p → Ψ p
W=90 GeV
t=0
|t|=0.05
solid BL, |t|=0.05 GeV²
dotted BL, t=0
■ H1 (2005) ○ ● ZEUS (2002, 2004)
dσ/d|t| (|t|=0.05) (nb/GeV²)
Q² (GeV²)
10³
10²
10
1
10⁻²
10⁻¹
1
10



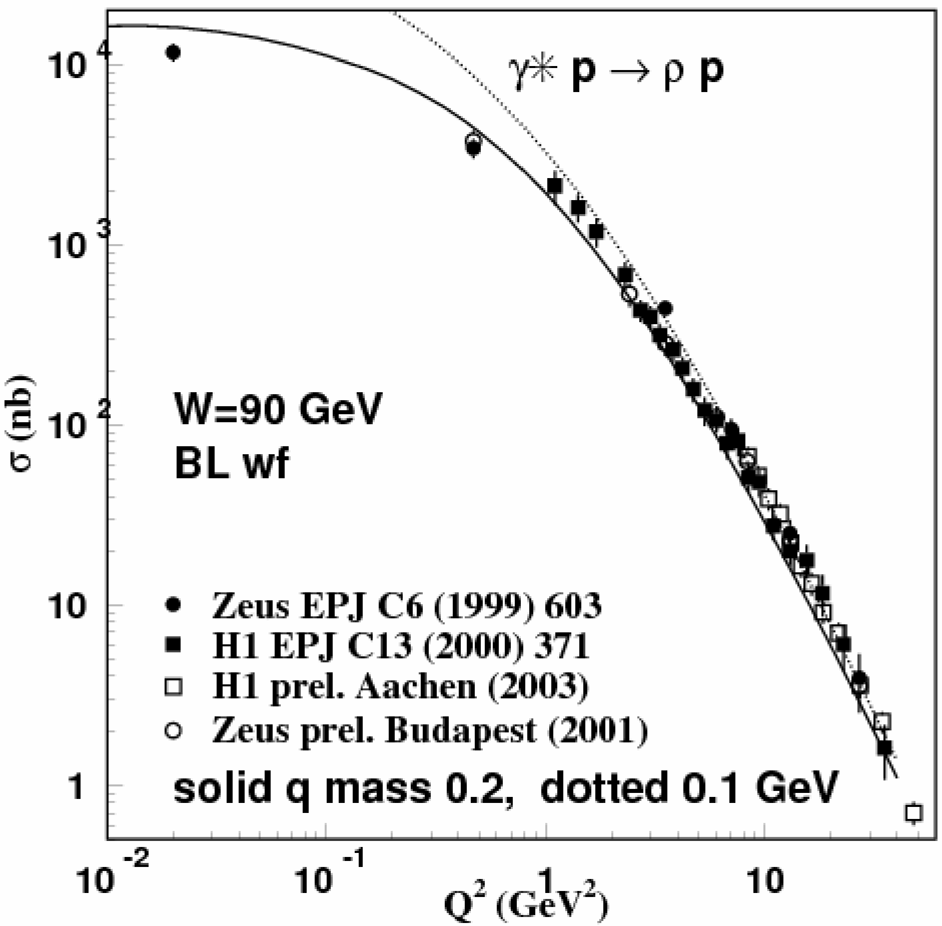
γ* p → Ψ p
W=90 GeV
Q2=0.0-0.05 GeV²
H1 2005
Zeus 2002
solid BL wf
dashed BSW wf
dot-dashed - fit
301.6 exp(-4.4|t|)
dσ/d|t| (nb/GeV²)
|t| (GeV²)
1
10
10²
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6


γ* p → Ψ p
W=90 GeV
Q2=6.8 GeV²
dσ/d|t| (nb/GeV²)
|t| (GeV²)
10²
10
1
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
Zeus 2004
H1 2005
dot-dashed - fit 83.9 exp(-4.5|t|)
solid BL wf, dashed BSW wf

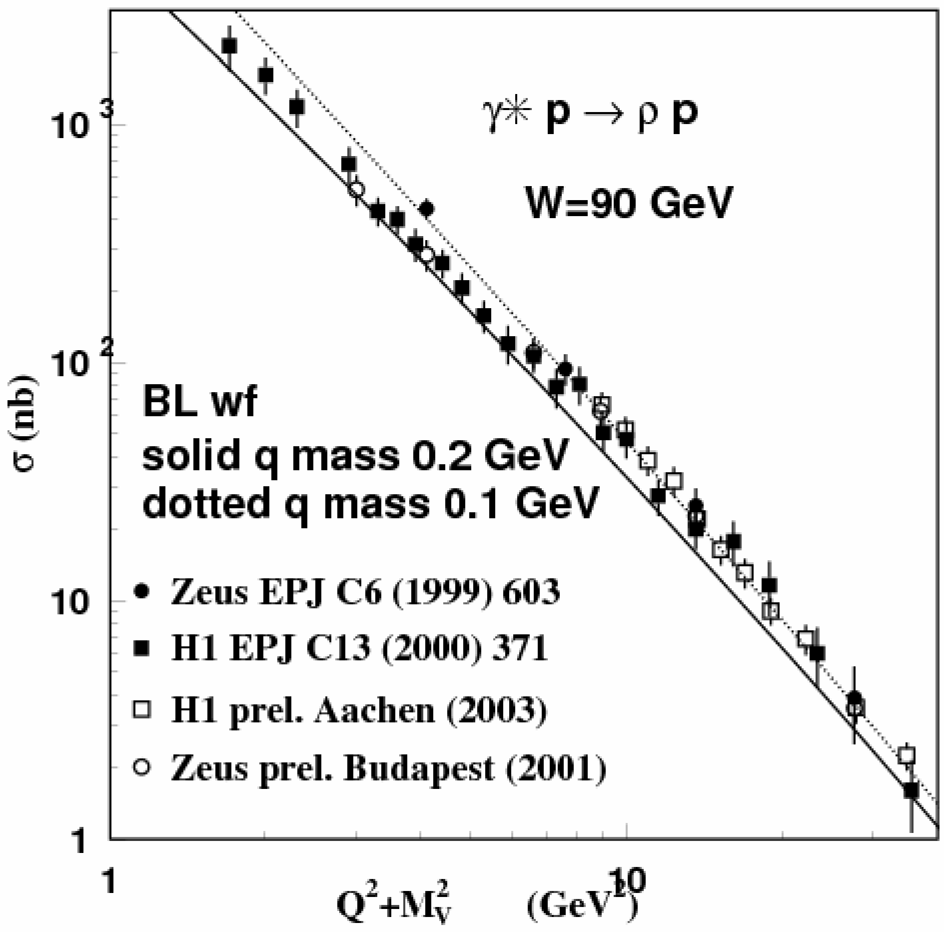

γ* p → Ψ p
ZEUS Coll.(2002)
S.Chekanov et al.- Eur.Phys.J. C24 (2002)345-360
ZEUS Coll.(2004)
S.Chekanov et al.- Nucl.Phys.B695(2004)3-37
H1 Coll.(2005)
A.Aktas et al.- hep-ex/0510016 - sub. EPJC
Representation of theoretical values
$d\sigma/d|t| = d\sigma/d|t||_{t=0}\, e^{-b|t|}/(1+a|t|)^2$
$Q^2 = 0$ - a=4.06 GeV^{-2} , b=1.75 GeV^{-2}
$Q^2 = 6.8$ - a=4.06 GeV^{-2} , b=1.69 GeV^{-2}
$Q^2 = 20$ - a=4.06 GeV^{-2} , b=1.65 GeV^{-2}
0
20
40
60
80
100

$\gamma^* p \longrightarrow \psi p$

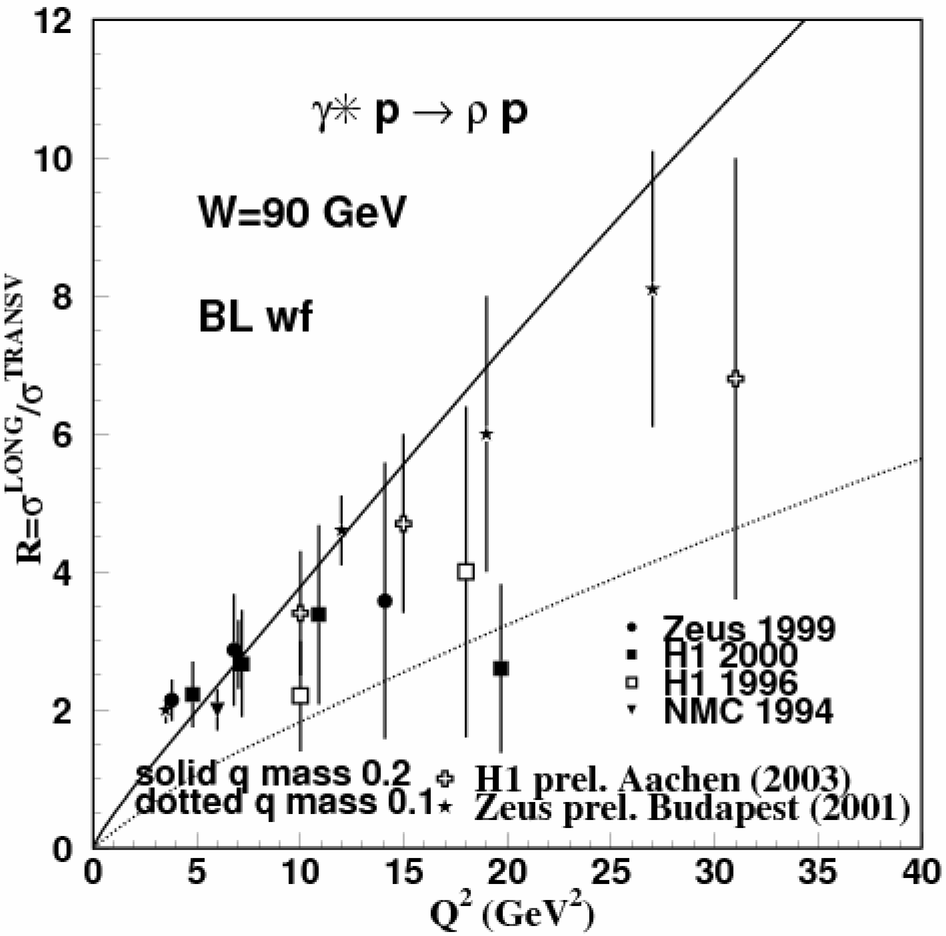
- **Factorization: the overlap strengths contain all Q^2 dependence.**
- **The $q\bar{q}$ pair of the $\gamma^* - \mathrm{VM}$ overlap enters in the amplitude only through the size r of the dipole.**
- **The Stochastic Vacuum Model, without any new free parameters, give perfect value for the constant C relating the overlap strengths to the cross section for Ψ electroproduction.**
- **Only specific quantities in the wf : quark mass m_f and electromagnetic decay rate Γ. BL wf case : M_V not mentioned at all in the construction of the overlap function. Miracle (of nonperturbative QCD) that it appears so neatly in the results for the cross sections.**



γ* p → ρ p
W=90 GeV
BL wf
σ (nb)
Q² (GeV²)
Zeus EPJ C6 (1999) 603
H1 EPJ C13 (2000) 371
H1 prel. Aachen (2003)
Zeus prel. Budapest (2001)
solid q mass 0.2, dotted 0.1 GeV



γ* p → ρ p
W=90 GeV
BL wf
solid q mass 0.2 GeV
dotted q mass 0.1 GeV
Zeus EPJ C6 (1999) 603
H1 EPJ C13 (2000) 371
H1 prel. Aachen (2003)
Zeus prel. Budapest (2001)
σ (nb)
10^3
10^2
10
1
1
10
Q^2+M_V^2 (GeV^2)



γ* p → ρ p
W=90 GeV
BL wf
R=σ^LONG/σ^TRANSV
Q^2 (GeV^2)
Zeus 1999
H1 2000
H1 1996
NMC 1994
solid q mass 0.2
dotted q mass 0.1
H1 prel. Aachen (2003)
Zeus prel. Budapest (2001)
0
2
4
6
8
10
12
0
5
10
15
20
25
30
35
40

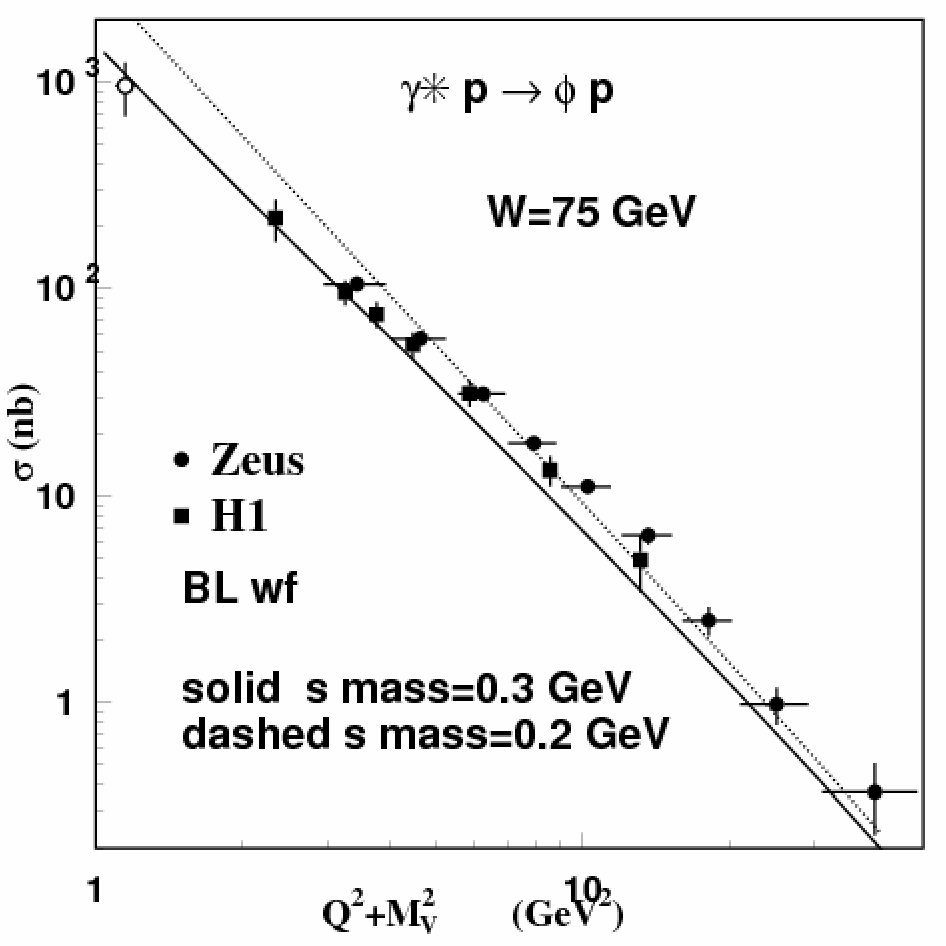
$\gamma^* p \rightarrow \rho\, p$

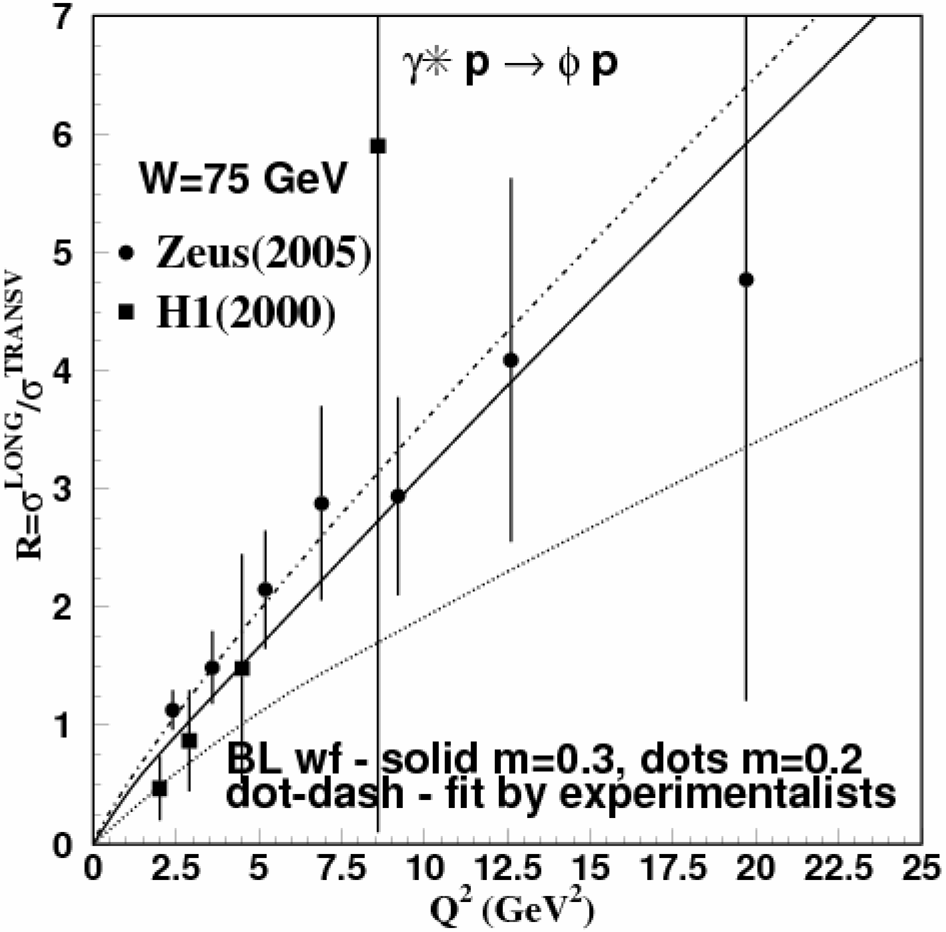
Q^2 dependence of cross section

$\sigma = Const/(1+Q^2/M^2)^n$

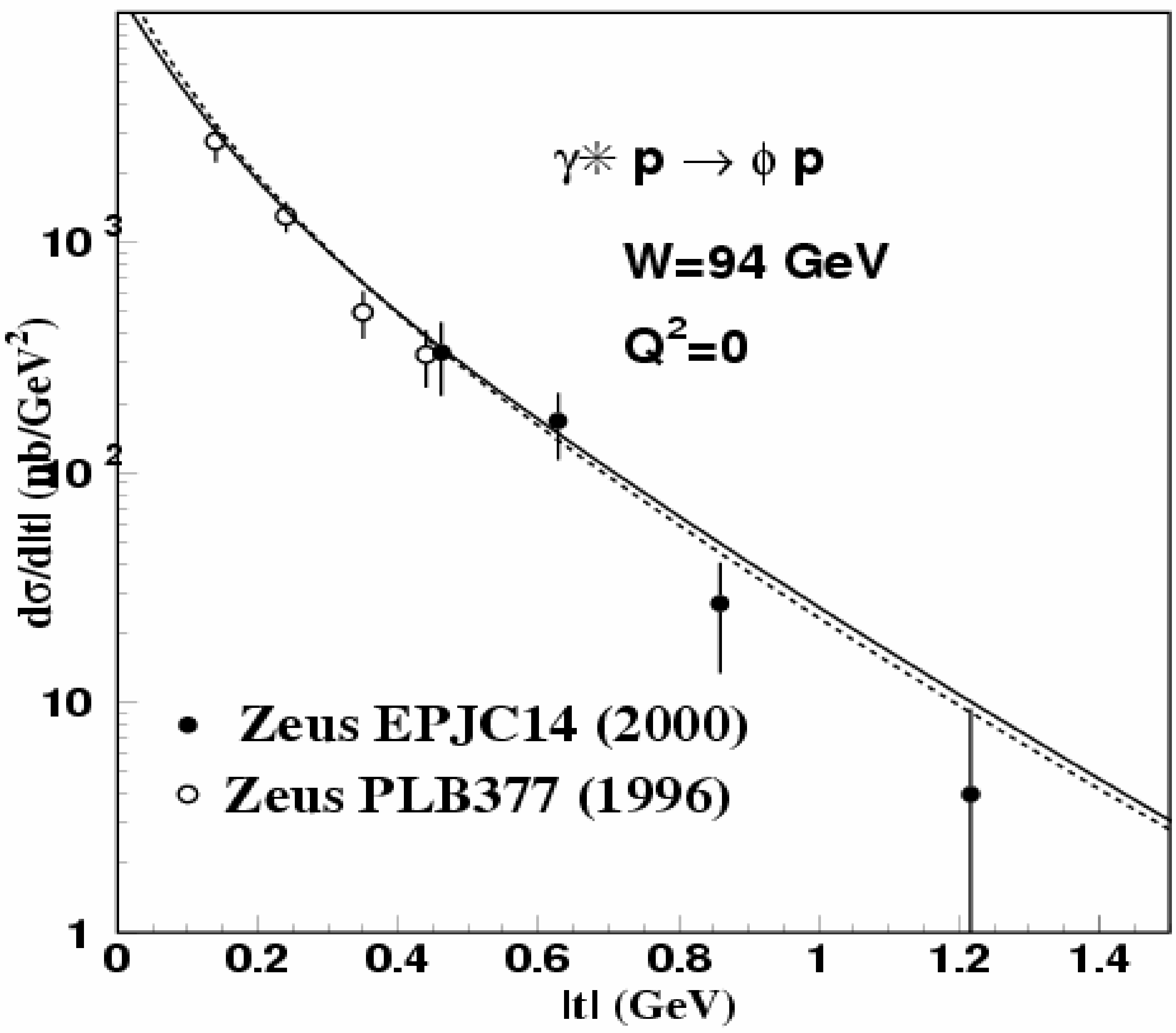
Oversimplification: cannot be a single n

- Fast change of n
- σ^T is small at large Q^2 :
  strongly influences R, but not σ
- With factorization at large Q^2+M^2
  R depends only on overlap strength
- Needed σ^T and σ^L separately
  to see where structure is
- Study ϕ and ω simultaneously

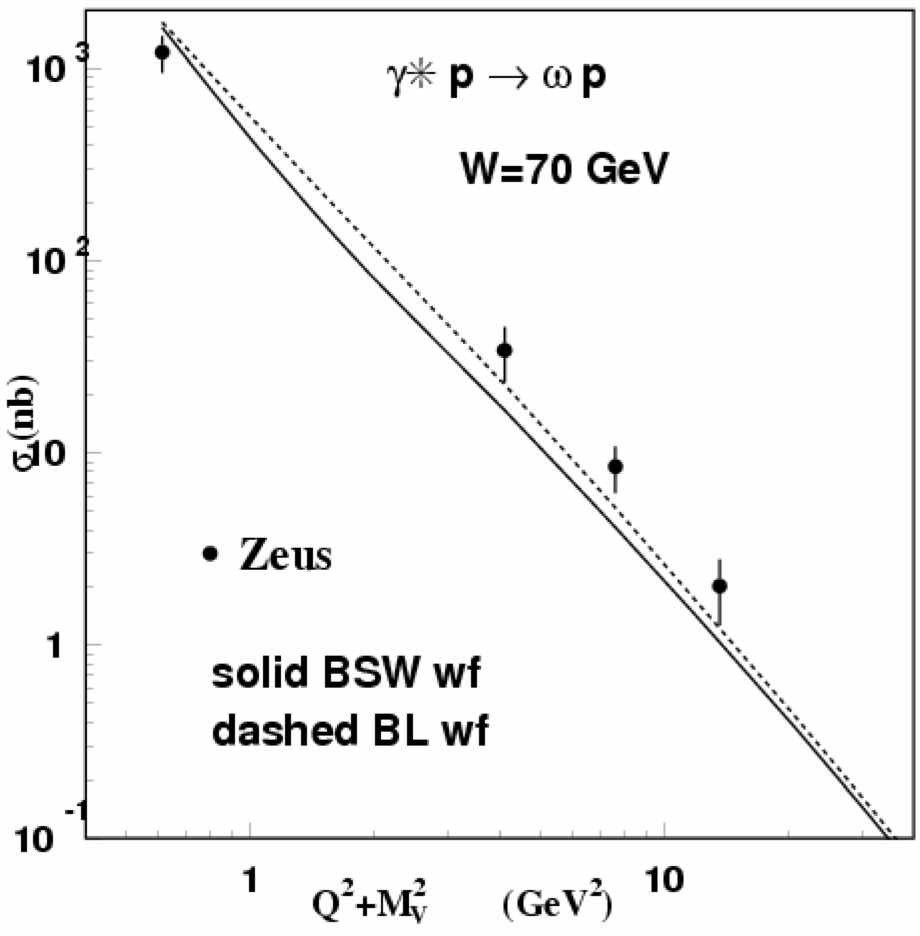

γ* p → φ p
W=75 GeV
σ (nb)
Zeus
H1
BL wf
solid s mass=0.3 GeV
dashed s mass=0.2 GeV
Q²+M²_V (GeV²)
10³
10²
10
1
1
10²



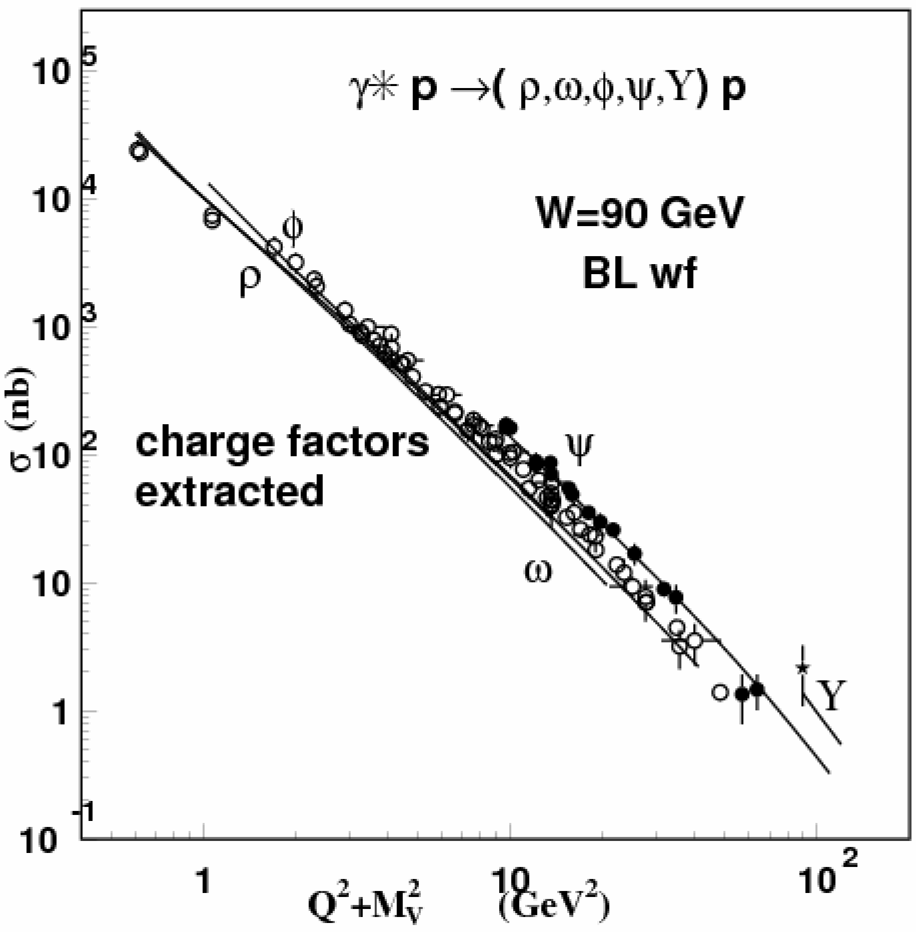
γ* p → φ p
W=75 GeV
Zeus(2005)
H1(2000)
BL wf - solid m=0.3, dots m=0.2
dot-dash - fit by experimentalists
R=σ^LONG/σ^TRANSV
Q^2 (GeV^2)
0
2.5
5
7.5
10
12.5
15
17.5
20
22.5
25
0
1
2
3
4
5
6
7



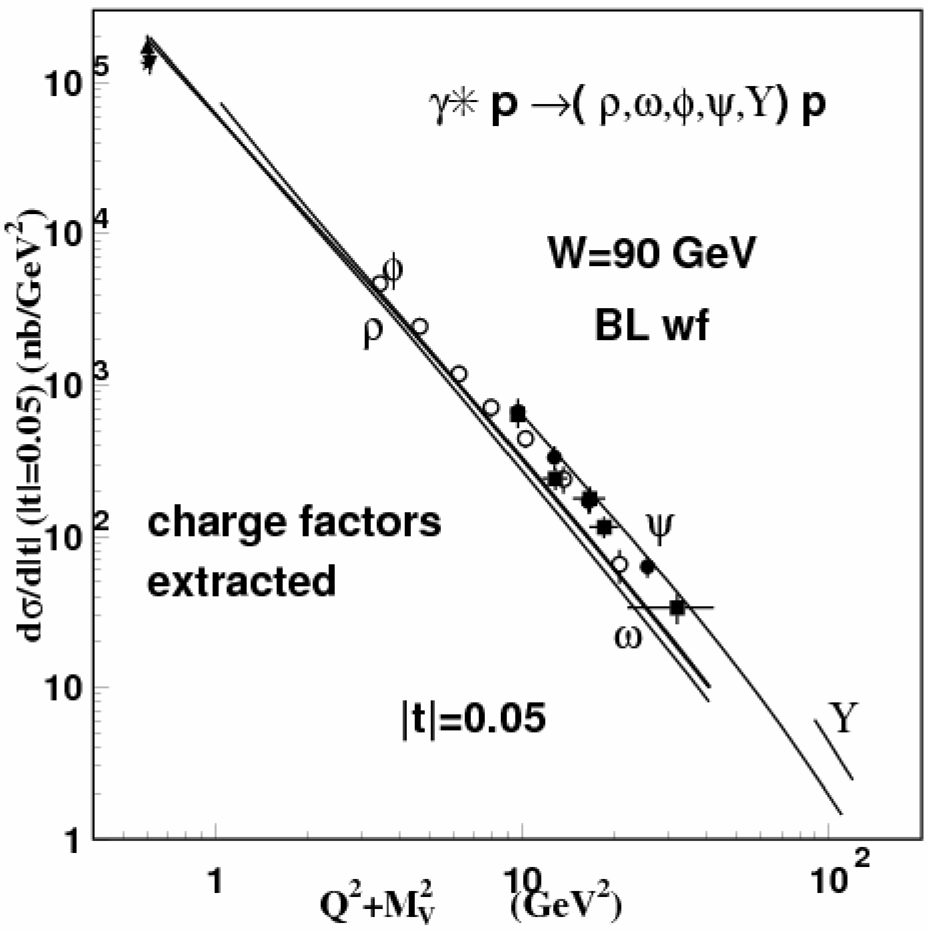
γ* p → φ p
W=94 GeV
Q²=0
Zeus EPJC14 (2000)
Zeus PLB377 (1996)
dσ/d|t| (μb/GeV²)
|t| (GeV)
10³
10²
10
1
0
0.2
0.4
0.6
0.8
1
1.2
1.4

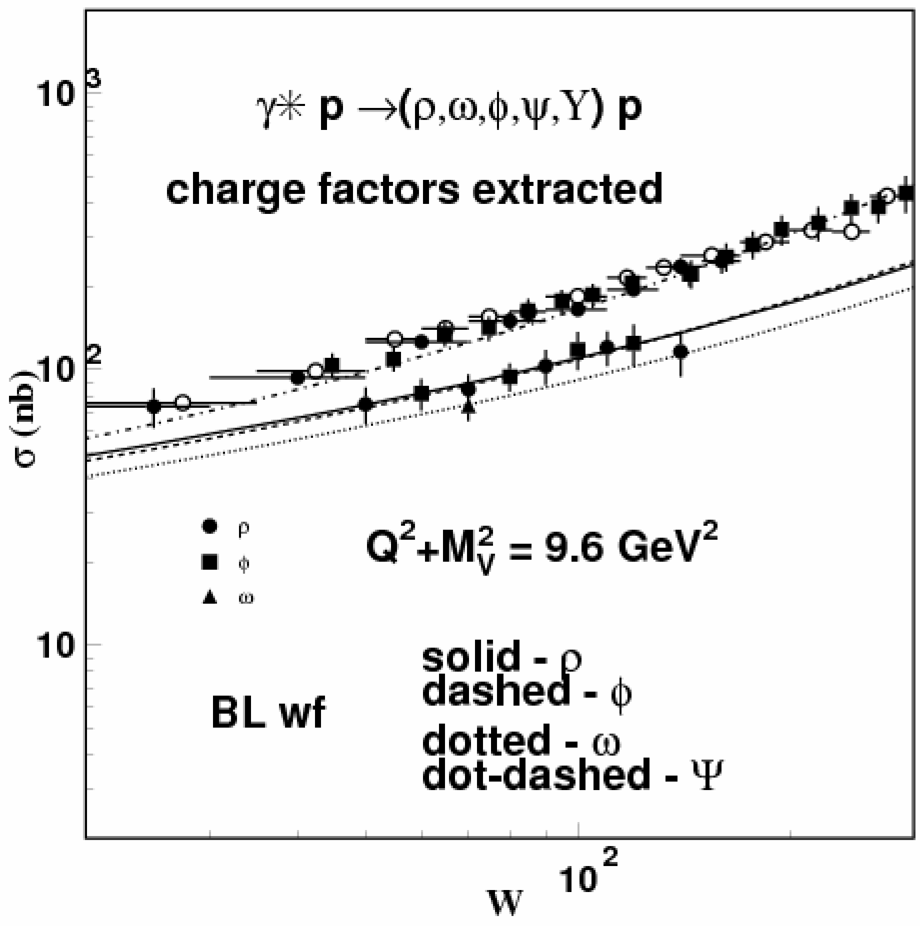


γ* p → ω p
W=70 GeV
σ_L(nb)
10³
10²
10
1
10⁻¹
• Zeus
solid BSW wf
dashed BL wf
1
10
Q²+M²_V (GeV²)

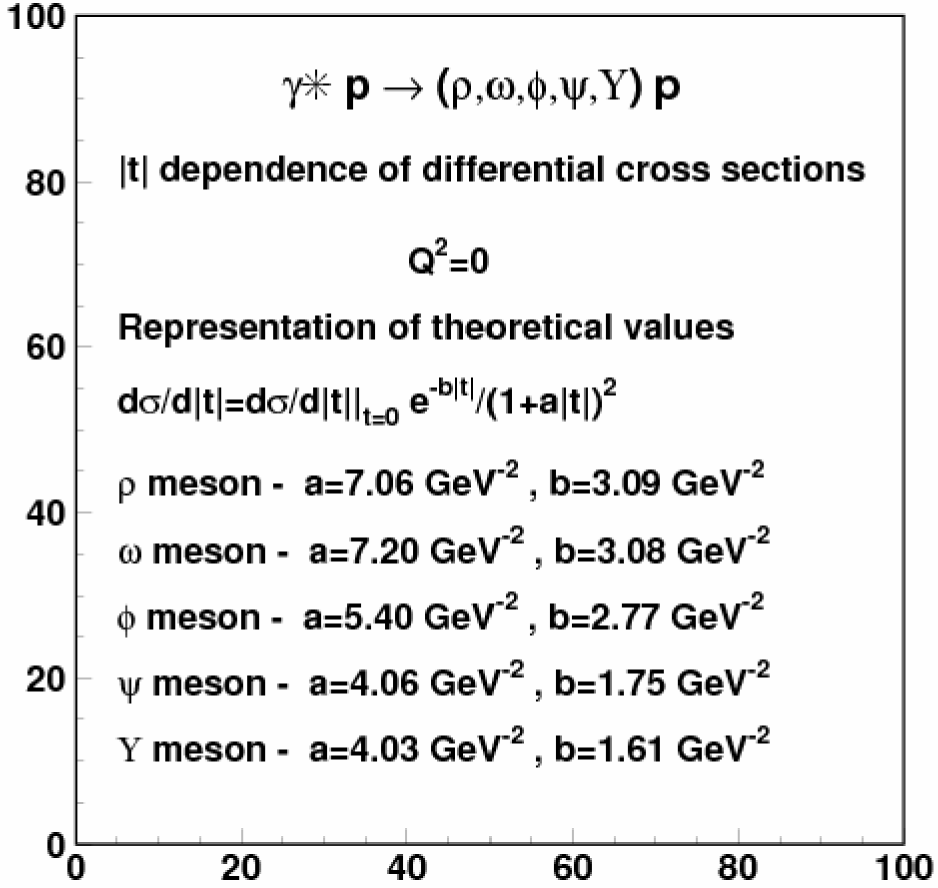


$\gamma^* p \to (\rho,\omega,\phi,\psi,\Upsilon)\, p$
W=90 GeV
BL wf
charge factors
extracted
ϕ
ρ
ψ
ω
Υ
σ (nb)
$Q^2+M_V^2$ (GeV^2)
10^5
10^4
10^3
10^2
10
1
10^{-1}
1
10
10^2

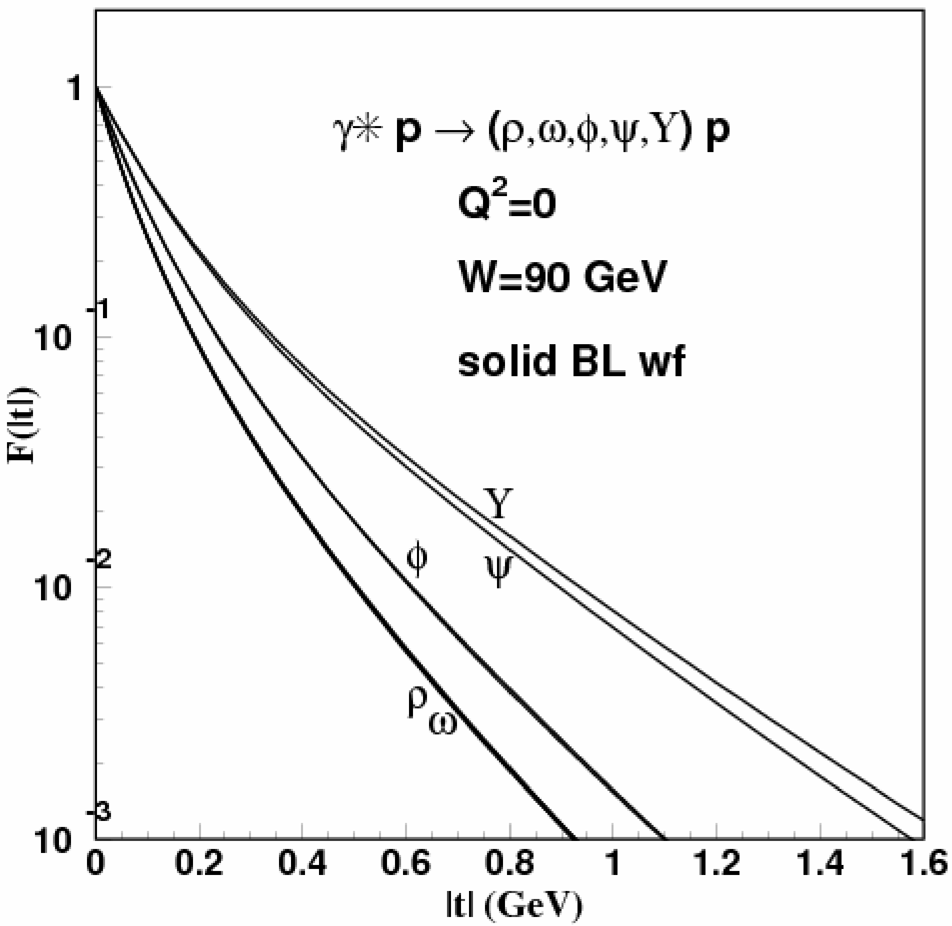

γ* p →(ρ,ω,φ,ψ,Υ) p
W=90 GeV
BL wf
charge factors
extracted
|t|=0.05
φ
ρ
ψ
ω
Υ
dσ/d|t| (|t|=0.05) (nb/GeV²)
Q²+M²V (GeV²)
10⁵
10⁴
10³
10²
10
1
1
10
10²

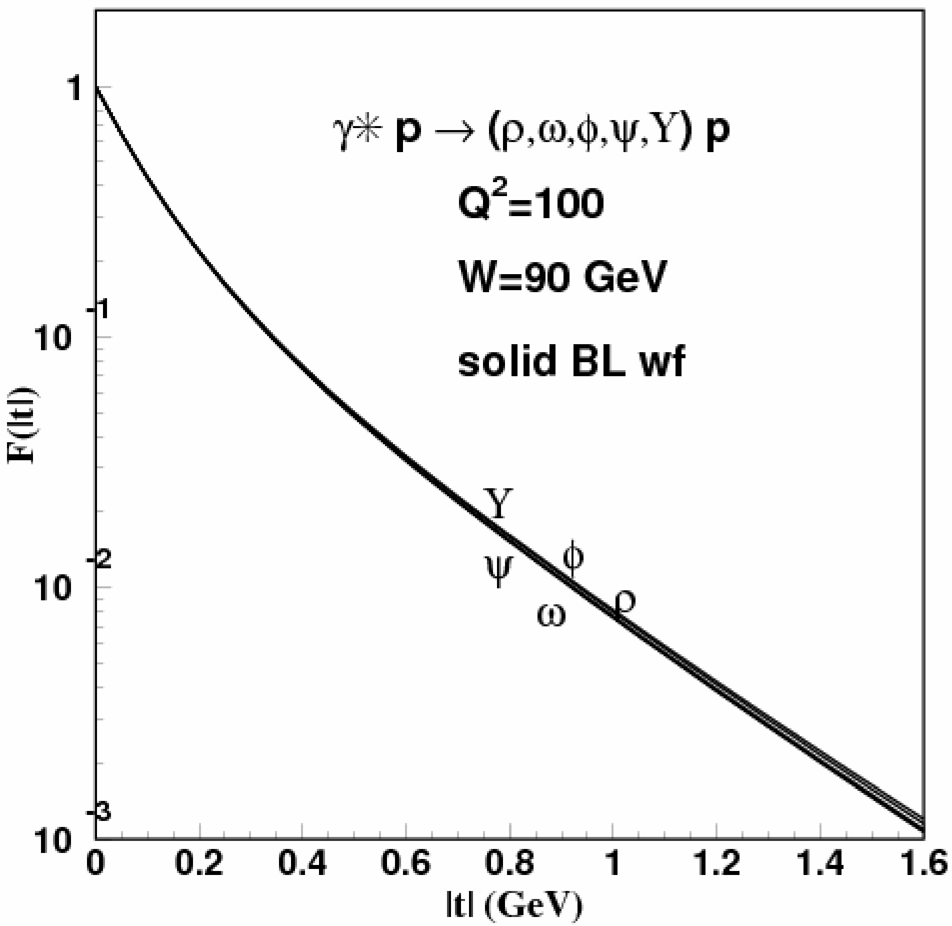


$\gamma^* p \to (\rho,\omega,\phi,\psi,\Upsilon)$ p
charge factors extracted
$Q^2+M_V^2 = 9.6$ GeV^2
ρ
φ
ω
BL wf
solid - ρ
dashed - φ
dotted - ω
dot-dashed - Ψ
σ (nb)
W
10^3
10^2
10

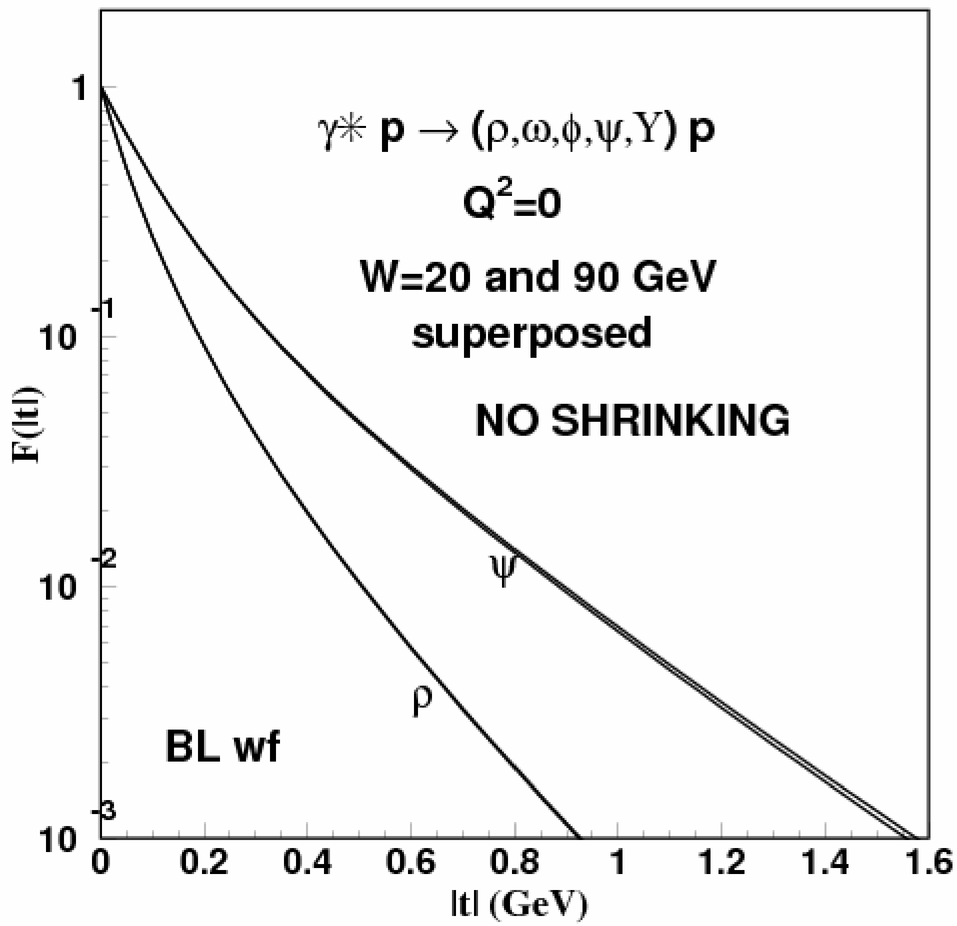


$\gamma^* p \rightarrow (\rho,\omega,\phi,\psi,\Upsilon)\, p$
|t| dependence of differential cross sections
$Q^2=0$
Representation of theoretical values
$d\sigma/d|t| = d\sigma/d|t||_{t=0}\, e^{-b|t|}/(1+a|t|)^2$
ρ meson - a=7.06 GeV^{-2} , b=3.09 GeV^{-2}
ω meson - a=7.20 GeV^{-2} , b=3.08 GeV^{-2}
ϕ meson - a=5.40 GeV^{-2} , b=2.77 GeV^{-2}
ψ meson - a=4.06 GeV^{-2} , b=1.75 GeV^{-2}
Υ meson - a=4.03 GeV^{-2} , b=1.61 GeV^{-2}
0
20
40
60
80
100


γ* p → (ρ,ω,φ,ψ,Υ) p
Q2=0
W=90 GeV
solid BL wf
Υ
φ
ψ
ρ
ω
F(|t|)
|t| (GeV)
1
10-1
10-2
10-3
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6



$\gamma^* p \to (\rho,\omega,\phi,\psi,\Upsilon) p$
Q^2=100
W=90 GeV
solid BL wf
Υ
ψ
φ
ω
ρ
F(|t|)
|t| (GeV)
1
10^{-1}
10^{-2}
10^{-3}
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6



$\gamma^* p \to (\rho,\omega,\phi,\psi,\Upsilon)\ p$
$Q^2=0$
W=20 and 90 GeV
superposed
NO SHRINKING
ψ
ρ
BL wf
F(|t|)
|t| (GeV)
1
10^{-1}
10^{-2}
10^{-3}
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6