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Color Superconductivity in Ultra-Dense Quark Matter

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Color superconductivity in ultra-dense quark matter

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Outline

I High density QCD

Color-superconducting quark matter, Color-flavor locking (CFL)

II Quark matter in compact stars

weak interactions, neutrality, M_s;

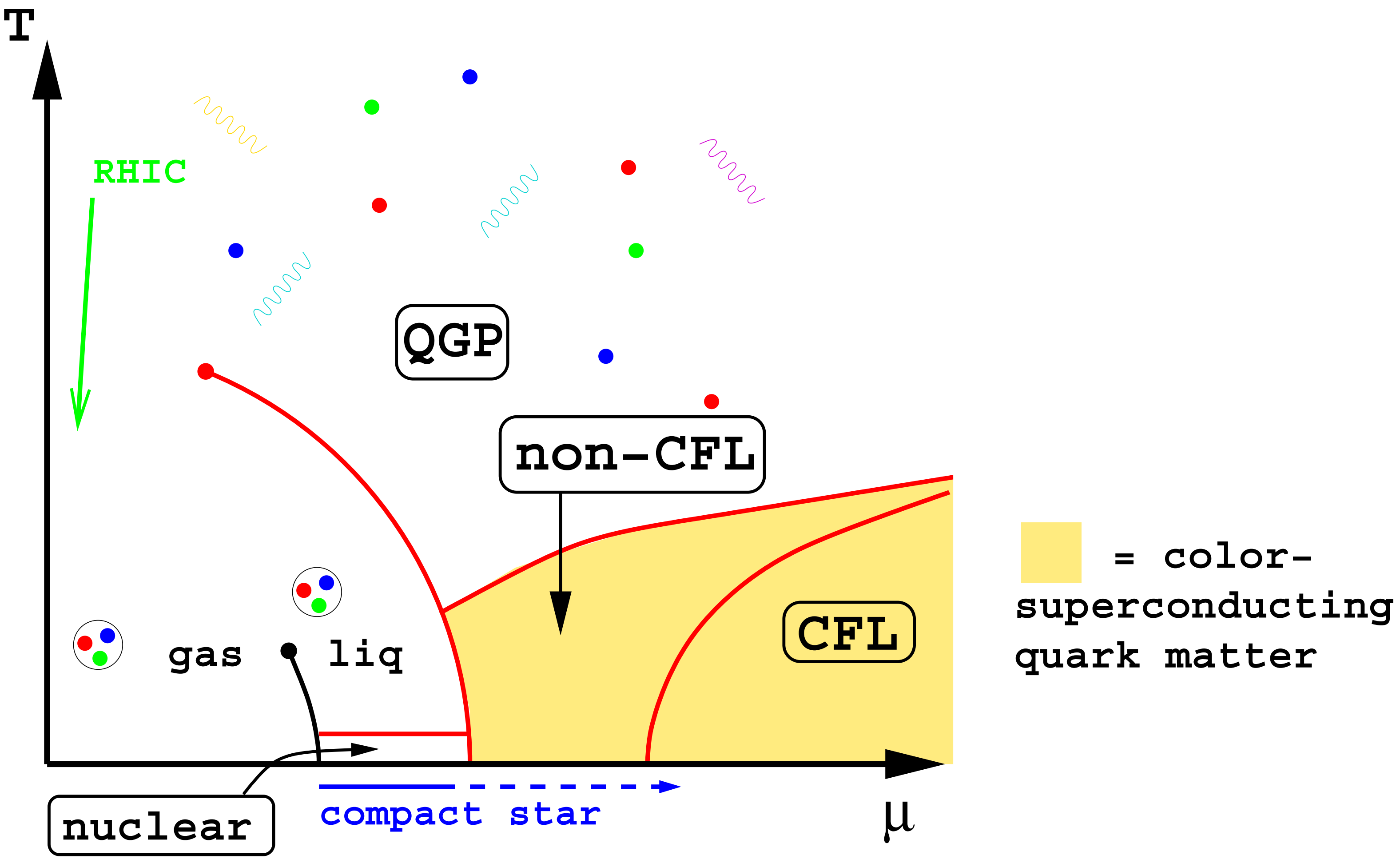
III Signatures of color superconductivity in compact stars

Transport properties, mass-radius relationship

IV Looking to the future

I. High density QCD

Conjectured QCD phase diagram

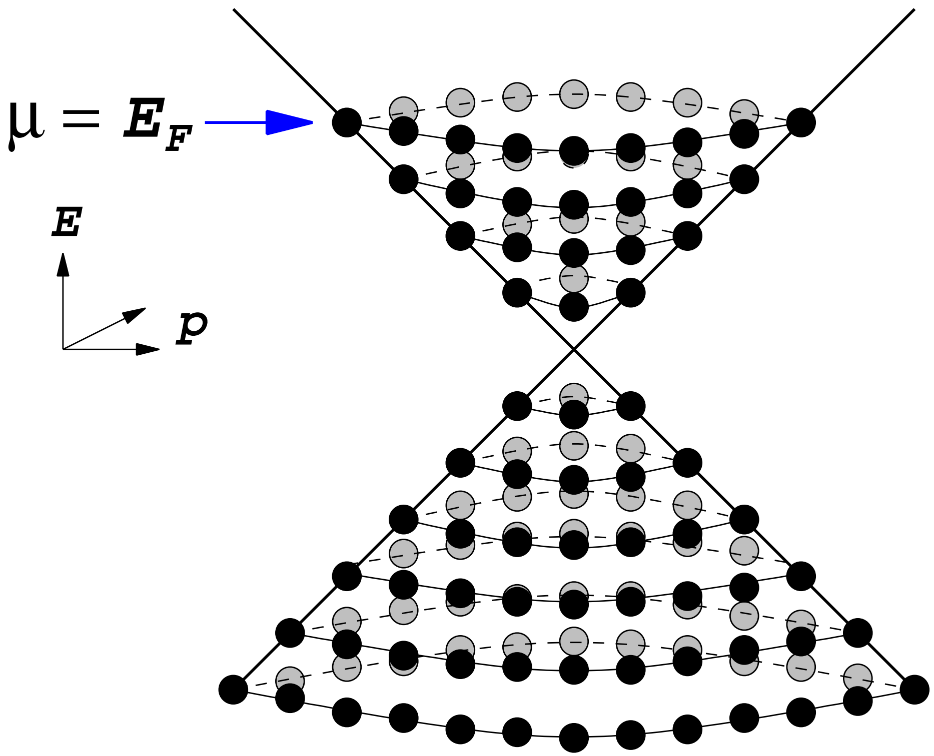


heavy ion collisions: chiral critical point and first-order line

compact stars: color superconducting quark matter core

Cooper pairing of quarks at high density

At sufficiently high density and low temperature, there is a Fermi sea of almost free quarks.



$$F = E - \mu N$$

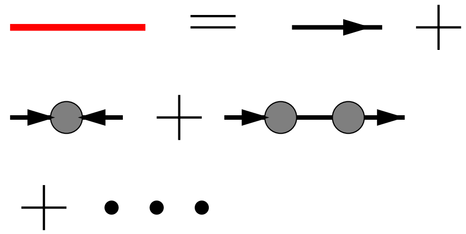
But quarks have attractive QCD interactions.

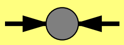
Any attractive quark-quark interaction causes pairing instability of the Fermi surface. This is the Bardeen-Cooper-Schrieffer (BCS) mechanism of superconductivity.

High-density QCD calculations

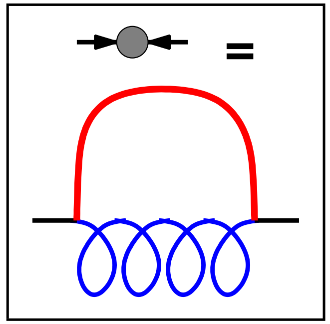
Guess a color-flavor-spin pairing pattern P

minimize free energy wrt Δ: gap equation for Δ.

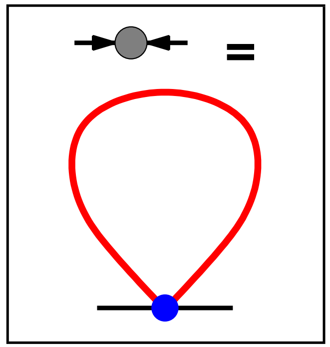


 $= \langle q^{\alpha}_{ia} q^{\beta}_{jb} \rangle_{1PI} = P^{\alpha\beta}_{ij\,ab} \Delta$

1. Weak-coupling methods. First-principles calculations direct from QCD Lagrangian, valid in the asymptotic regime, currently $\mu \gtrsim 10^6$ MeV.



2. Nambu–Jona-Lasinio models, ie quarks with four-fermion coupling based on instanton vertex, single gluon exchange, etc. This is a semi-quantitative guide to physics in the compact star regime $\mu \sim 400$ MeV, not a systematic approximation to QCD.



NJL gives $\Delta \sim 10-100$ MeV at $\mu \sim 400$ MeV.

Both methods agree on the favored pairing pattern.

Color superconductivity in three flavor quark matter: Color-flavor locking (CFL)

Equal number of colors and flavors allows a special pairing pattern
(Alford, Rajagopal, Wilczek, hep-ph/9804403)

$$\langle q_i^\alpha q_j^\beta \rangle \sim (\kappa+1)\delta_i^\alpha \delta_j^\beta + (\kappa-1)\delta_j^\alpha \delta_i^\beta = \epsilon^{\alpha\beta N}\epsilon_{ijN} + \kappa(\cdots)$$

color α, β — This is invariant under equal and opposite
flavor i, j — rotations of color and (vector) flavor

$$SU(3)_{\text{color}} \times \underbrace{SU(3)_L \times SU(3)_R}_{\supset U(1)_Q} \times U(1)_B \to \underbrace{SU(3)_{C+L+R}}_{\supset U(1)_{\tilde{Q}}} \times \mathbb{Z}_2$$

- Breaks chiral symmetry, but *not* by a $\langle \bar{q} q \rangle$ condensate.
- There need be no phase transition between the low and high density phases: ("quark-hadron continuity")
- Unbroken "rotated" electromagnetism, $\tilde{Q}$, photon-gluon mixture.

Color-flavor-locked ("CFL") quark pairing

$\tilde{Q}$	0	0	0	-1	$+1$	-1	$+1$	0	0
	u	d	s	d	u	s	u	s	d
u		Δ	Δ						
d	Δ		Δ						
s	Δ	Δ							
d					$-\Delta$				
u				$-\Delta$					
s							$-\Delta$		
u						$-\Delta$			
s									$-\Delta$
d								$-\Delta$	

II. Quark matter in compact stars

Where in the universe is color-superconducting quark matter most likely to exist? In compact stars.

A quick history of a compact star.

A star of mass $M \gtrsim 10M_\odot$ burns Hydrogen by fusion, ending up with an Iron core. Core grows to Chandrasekhar mass, collapses $\Rightarrow$ supernova. Remnant is a compact star:

mass	radius	density	initial temp
$\sim 1.4M_\odot$	$\mathcal{O}(10\text{ km})$	$\geqslant \rho_{\text{nuclear}}$	~ 30 MeV

The star cools by neutrino emission for the first million years.

The real world: M_s and neutrality

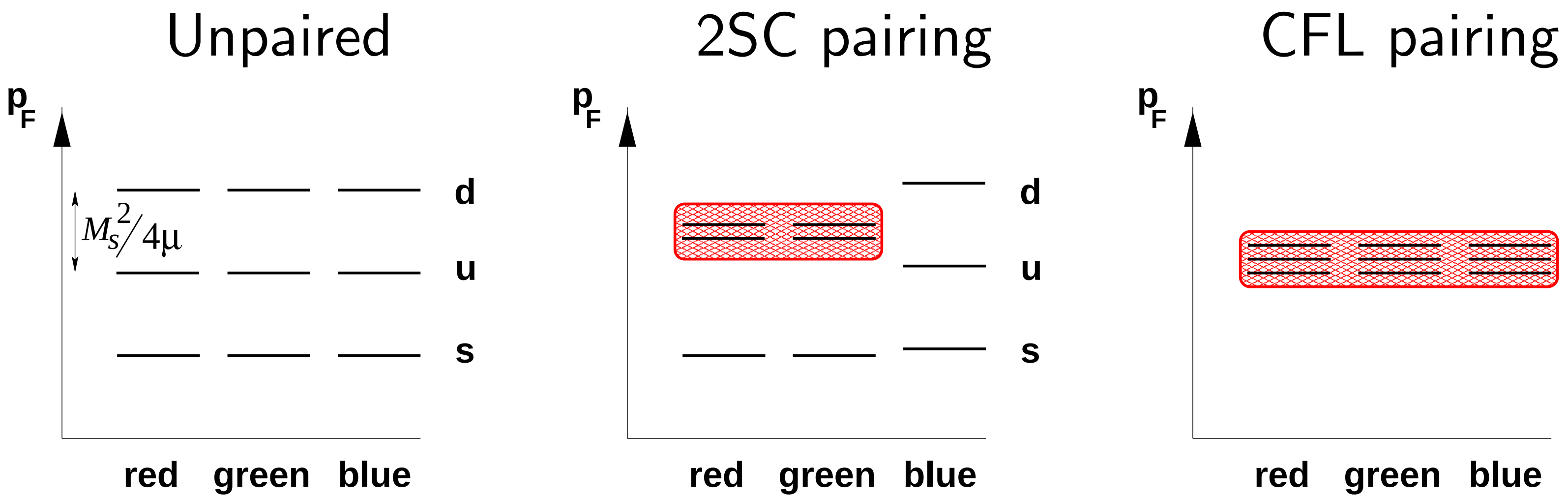
In the real world there are three complications that disfavor the CFL phase at realistic compact-star densities ($\mu \sim 400$ MeV).

1. Strange quark mass is not infinite nor zero, but intermediate. It depends on density, and ranges between about 500 MeV in the vacuum and about 100 MeV at high density.
2. Neutrality requirement. Bulk quark matter must be neutral with respect to all gauge charges: color and electromagnetism.
3. Weak interaction equilibration. In a compact star there is time for weak interactions to proceed: neutrinos escape and flavor is not conserved.

So quark matter in a compact star might be CFL, or something else: kaon-condensed CFL, 2SC, 1SC, crystalline (LOFF), diquark BEC . . .

Effect of M_s and electrical neutrality

Fermi momenta of flavors tend to split apart. If pairing is strong enough ($\Delta > M_s^2/(2\mu)$) it can hold them together.

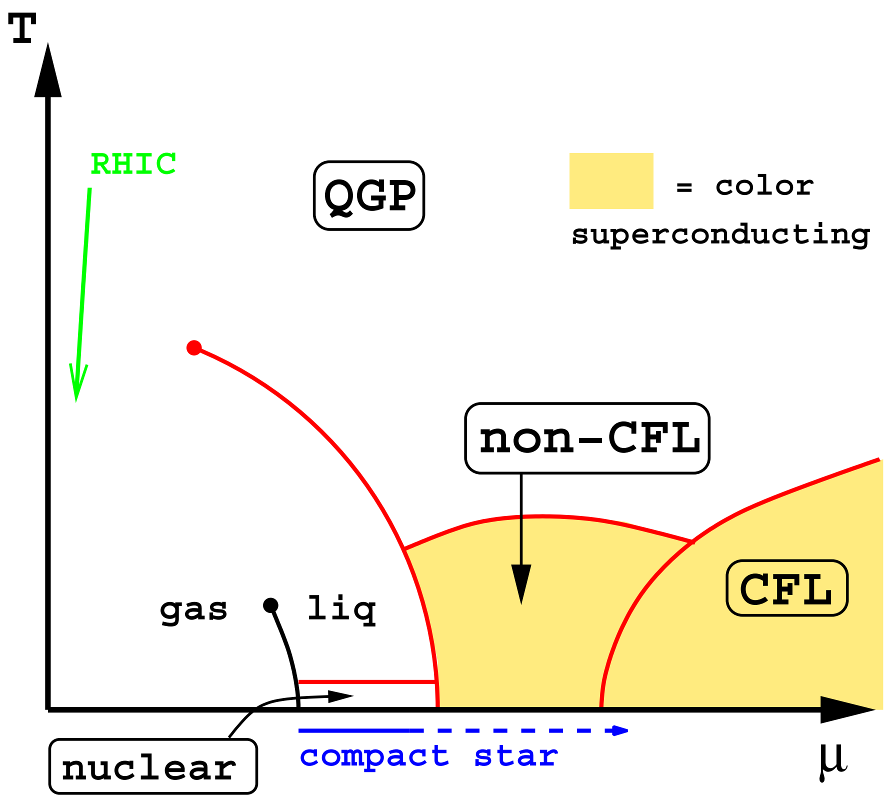


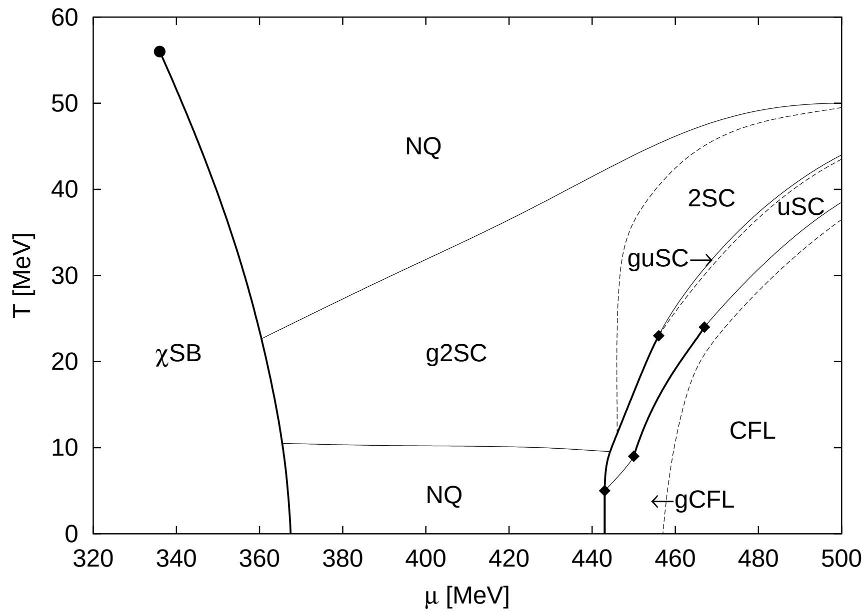
No electrons!

So do we get unpaired→2SC→CFL as density increases?

Not that simple.

Phases of quark matter, again





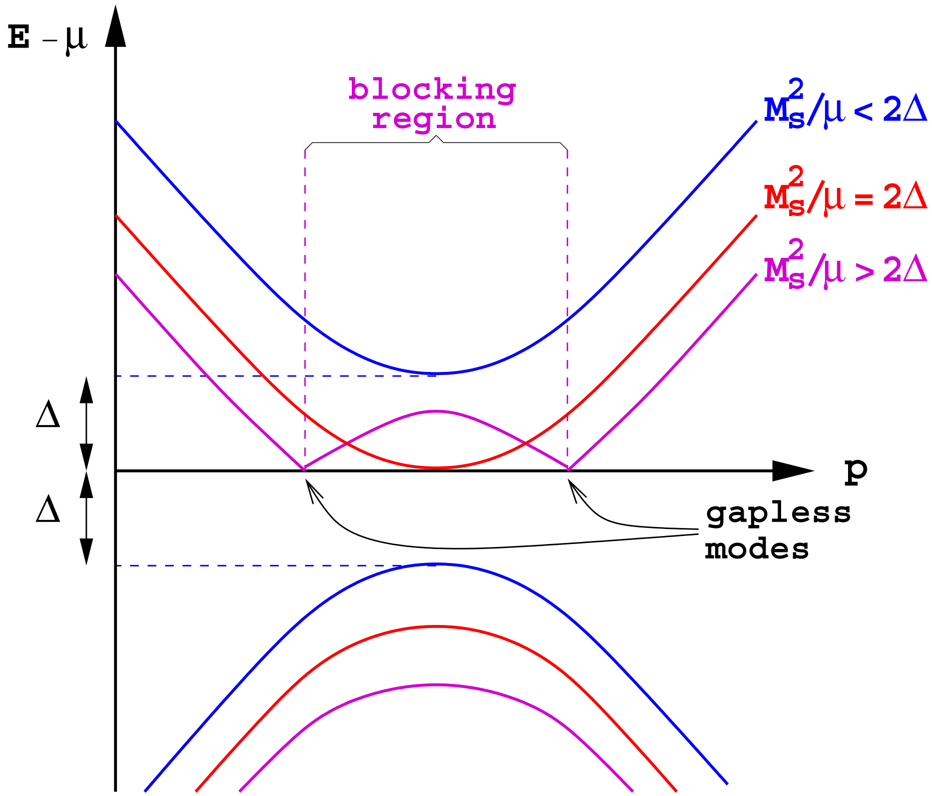
(Rüster, Werth, Buballa, Shovkovy, Rischke, hep-ph/0503184)

Rüster et. al. use an NJL model with coupled chiral and color-superconducting condensates.

The "g" phases are *gapless*, and are unstable: we don't know what replaces them.

Gapless phases

Quasiparticle dispersion relations for the ds sector:



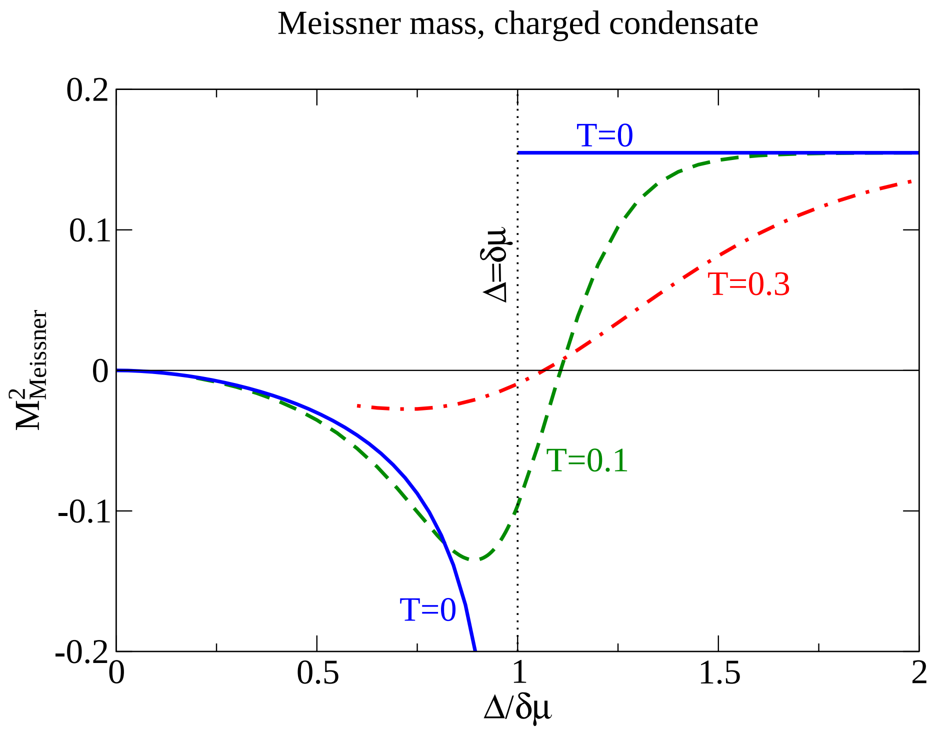
When M_s^2/μ reaches $2\Delta_{ds}$, unpairing begins: a "blocking" or "breached pairing" region opens up, bordered by gapless modes.

Problem: magnetic instability of gapless phases

In the gCFL phase, the $r\bar{g}$ gluons have an imaginary Meissner mass

(Huang and Shovkovy, hep-ph/0408268; Casalbuoni, Gatto, Mannarelli, Nardulli, Ruggieri, hep-ph/0410401; K. Fukushima hep-ph/0506080).

This is a generic consequence of gapless quasiquark dispersion relations:



2 charged quark species, chem pots $\bar{\mu} \pm \delta\mu$, form Cooper pairs with gap parameter Δ.

The Meissner mass goes imaginary when the gap in the dispersion relation $\Delta - \delta\mu$ reaches zero.

(Alford and Wang, hep-ph/0501078)

What replaces the gapless phases?

Effective Hamiltonian for diquark condensate ϕ is

$$\mathcal{L}_{\text{eff}} = \kappa D_i \phi^* D_i \phi + \cdots = \kappa \partial_i \phi^* \partial_i \phi + \kappa |\phi|^2 A_i A_i + \cdots$$

$M^2_{\text{Meissner}} < 0 \;\Rightarrow \kappa < 0$: instability towards spatial variation $\phi(\vec{x})$.

Suggestions:

1. Crystalline “LOFF” phase: Cooper pairs have non-zero momentum.

   (Alford, Bowers, Rajagopal, hep-ph/0008208, Casalbuoni, Nardulli hep-ph/0305069)

2. p-wave K^0 meson condensate

   (Schäfer hep-ph/0508190, Kryjevski hep-ph/0508180)

3. Mixed phases, secondary pairing, gluon condensate, ...

IV. Signatures of color superconductivity in compact stars

Gaps in spectra affect Transport properties.

Pairing energy affects Equation of state.

Transport properties, mean free paths, conductivities, viscosities, etc.

1. Cooling by neutrino emission, neutrino pulse at birth

   (Page, Prakash, Lattimer, Steiner, hep-ph/0005094; Carter and Reddy, hep-ph/0005228

   Reddy, Sadzikowski, Tachibana, nucl-th/0306015)

2. Gravitational waves: r-mode instability

   (Madsen, astro-ph/9912418)

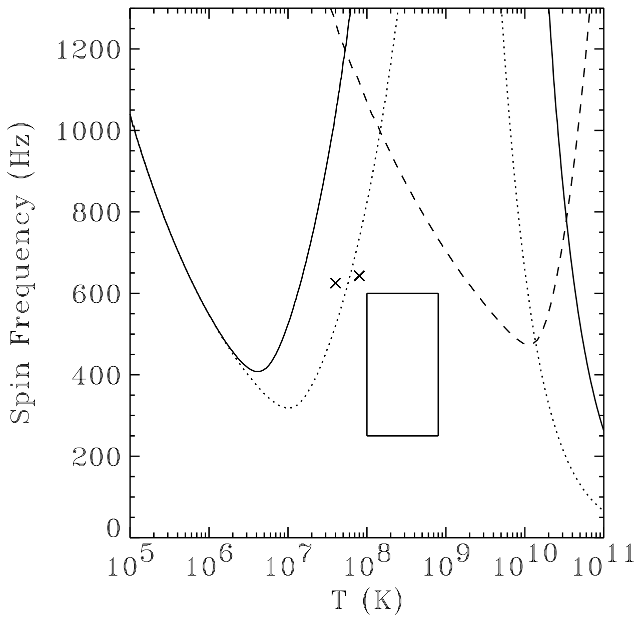
3. Glitches and crystalline ("LOFF") pairing

   (Alford, Bowers, Rajagopal, hep-ph/0008208)

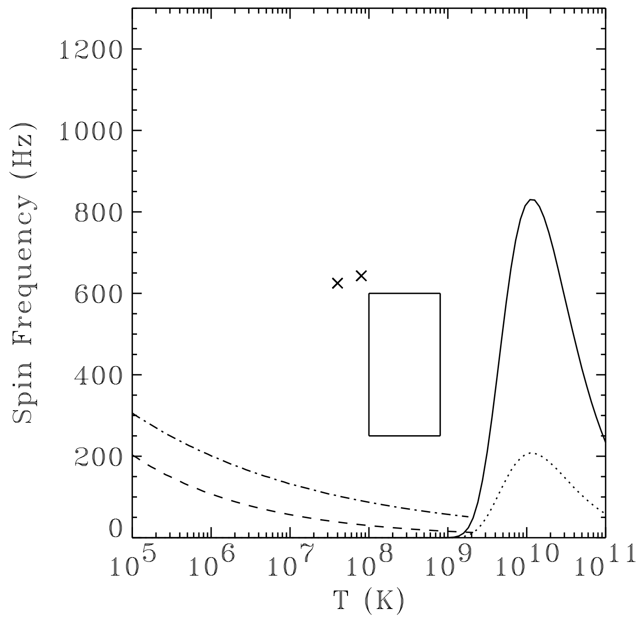
Constraints from r-modes (Madsen, astro-ph/9912418)

Rotation frequencies above curves are unstable: viscosity is too low to hold back the r-modes.

No pairing



CFL pairing



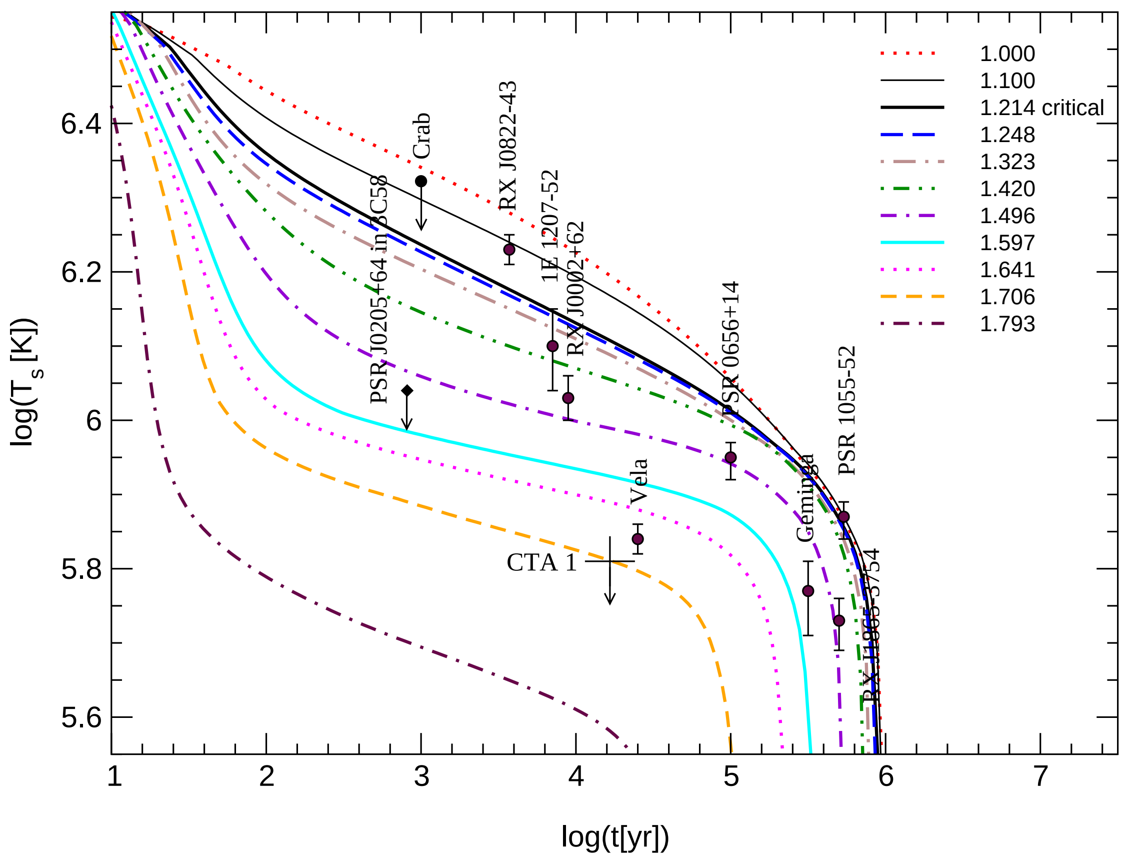
Solid and dotted lines: quark matter with $m_s = 200, 100$ MeV;

Dashed line: nuclear matter.

box: LMXBs; crosses: fastest pulsars.

Pure CFL quark matter stars are ruled out.

Cooling of a neutron star with quark matter core



(Grigorian, Blaschke, Voskresensky, `astro-ph/0411619`)

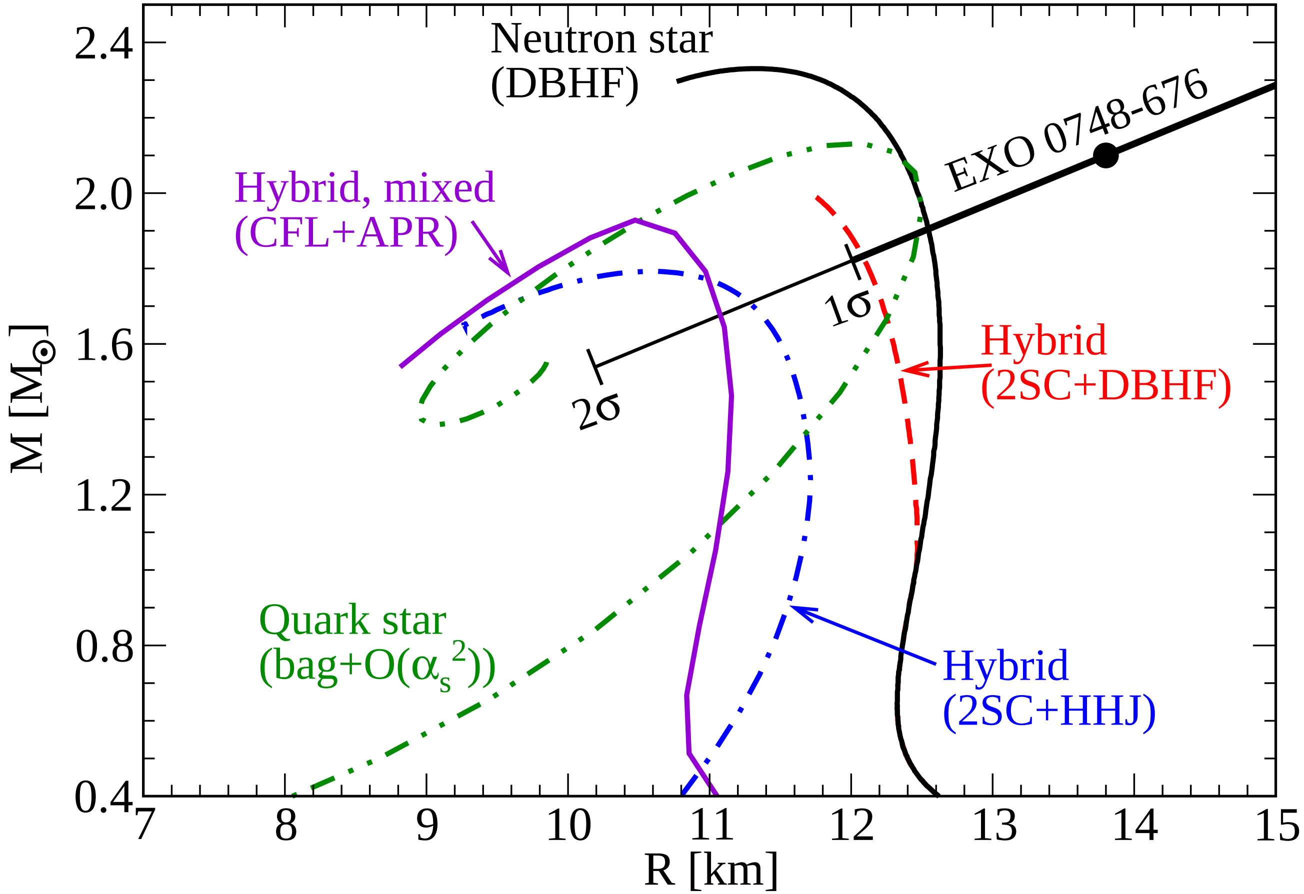
With 2-flavor color superconductivity, and additional weak pairing of the blue quarks. Can accomodate data with masses ranging from $1.1\ M_\odot$ to $1.7\ M_\odot$.

Equation of state (mass-radius relationship)

$$\text{pressure} \quad p = (1-c)\,(\cdot)\,\mu^4 - (\cdot)\,{M_s}^2\mu^2 + (\cdot)\,\Delta^2\mu^2 - B$$

Bag constant B and QCD correction parameter c have a strong effect on EoS and hence M vs R. But Δ is like a renormalization of M_s or even B.

Mass-radius relationship



(Alford, Blaschke, Drago, Pagliara, Schaffner-Bielich, in preparation)

It is hard to find values of M and R that would rule out quark matter in compact stars.

$M(R)$ measurements and quark matter

What would rule out quark matter?

Difficult. Most regions of M-R space that cannot be reached by any quark+nuclear matter hybrid also cannot be reached by nuclear matter alone. $M \gtrsim 2.1\ M_\odot$ seems hard to achieve with quark matter.

What would indicate the presence of quark matter?

Very Difficult. Regions of M-R space that cannot be reached by any nuclear matter EoS also cannot be reached by hybrid NM-QM EoS.

What would indicate the presence of color superconducting quark matter?

Impossibly difficult. Even if we found an $M(R)$ that was characteristic of quark matter, we would need an independent determination of the bag constant and M_s to claim that it was color-superconducting.

V. Looking to the future

- Neutron-star phenomenology of color superconducting quark matter:
  - Structure: nuclear-quark interface
  - Crystalline phase and glitches
  - Vortices but no flux tubes
  - Effects of gaps in quark spectrum
    - conductivity and emissivity (neutrino cooling)
    - shear and bulk viscosity (r-mode spin-down)
- Particle theoretic questions:
  - Response of CFL to M_s: gapless CFL, kaon condensation,...?
  - Magnetic instability of gapless phases
  - Better weak-coupling calculations, include vertex corrections
  - Go beyond mean-field, include fluctuations.