The Abdus Salam
International Centre for Theoretical Physics

United Nations Educational, Scientific and Cultural Organization
International Atomic Energy Agency

SMR.1751 - 52

Fifth International Conference on
PERSPECTIVES IN HADRONIC PHYSICS
Particle-Nucleus and Nucleus-Nucleus Scattering at Relativistic Energies

22 - 26 May 2006

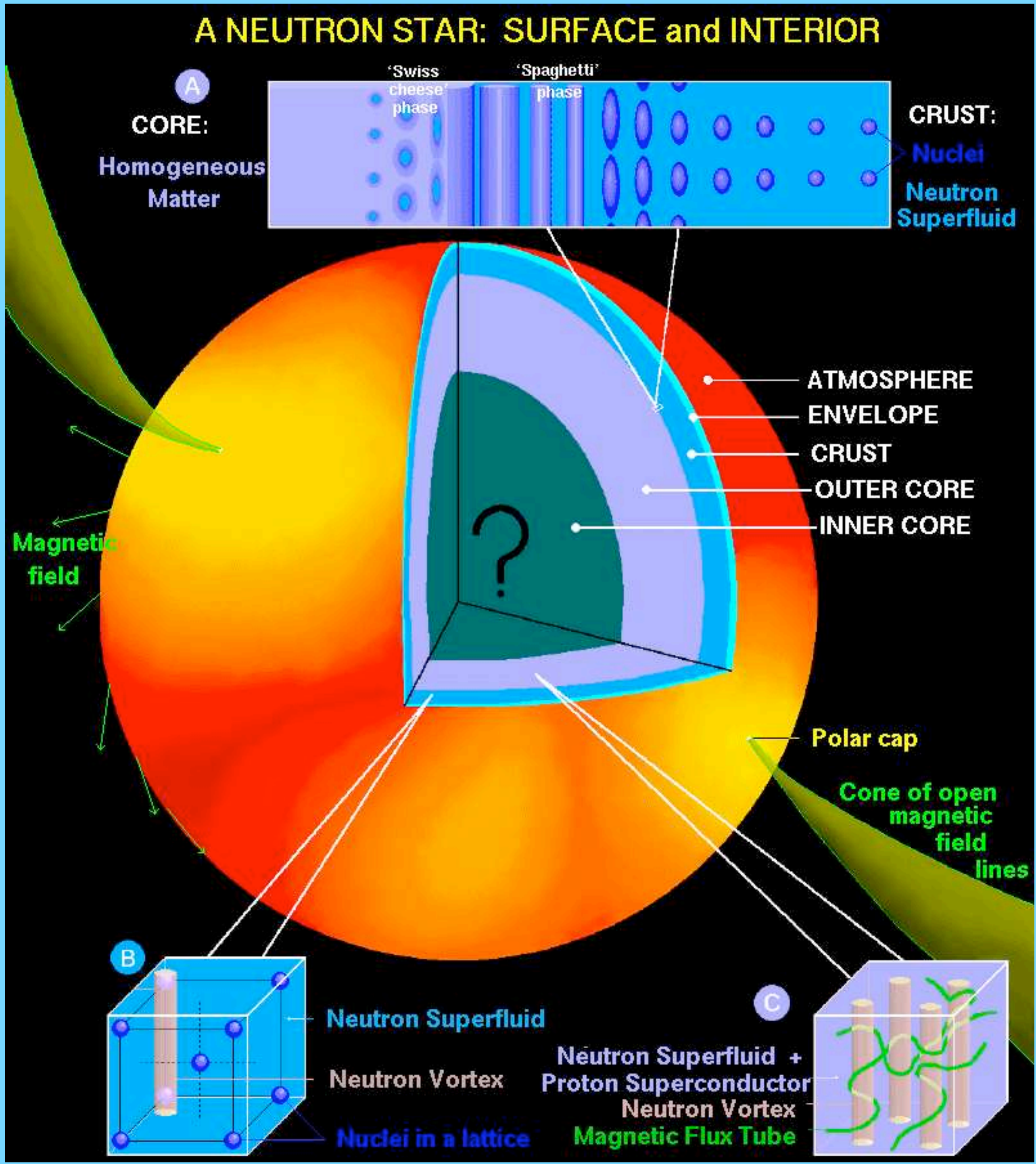
The Nuclear EoS at High Density

Marcello BALDO
Istituto Nazionale di Fisica Nucleare
INFN
Dipartimento di Fisica
Universita' di Catania
Via S. Sofia 64
I-95129 Catania
ITALY

The nuclear EoS at high density

A section (schematic)

of a neutron star



Motivations

1. Set the uncertainity in the many-body treatment comparing different methods

2. Try to fix constraints on the quark phase EoS comparing different simple model predictions with observations.

OUTLOOO
K

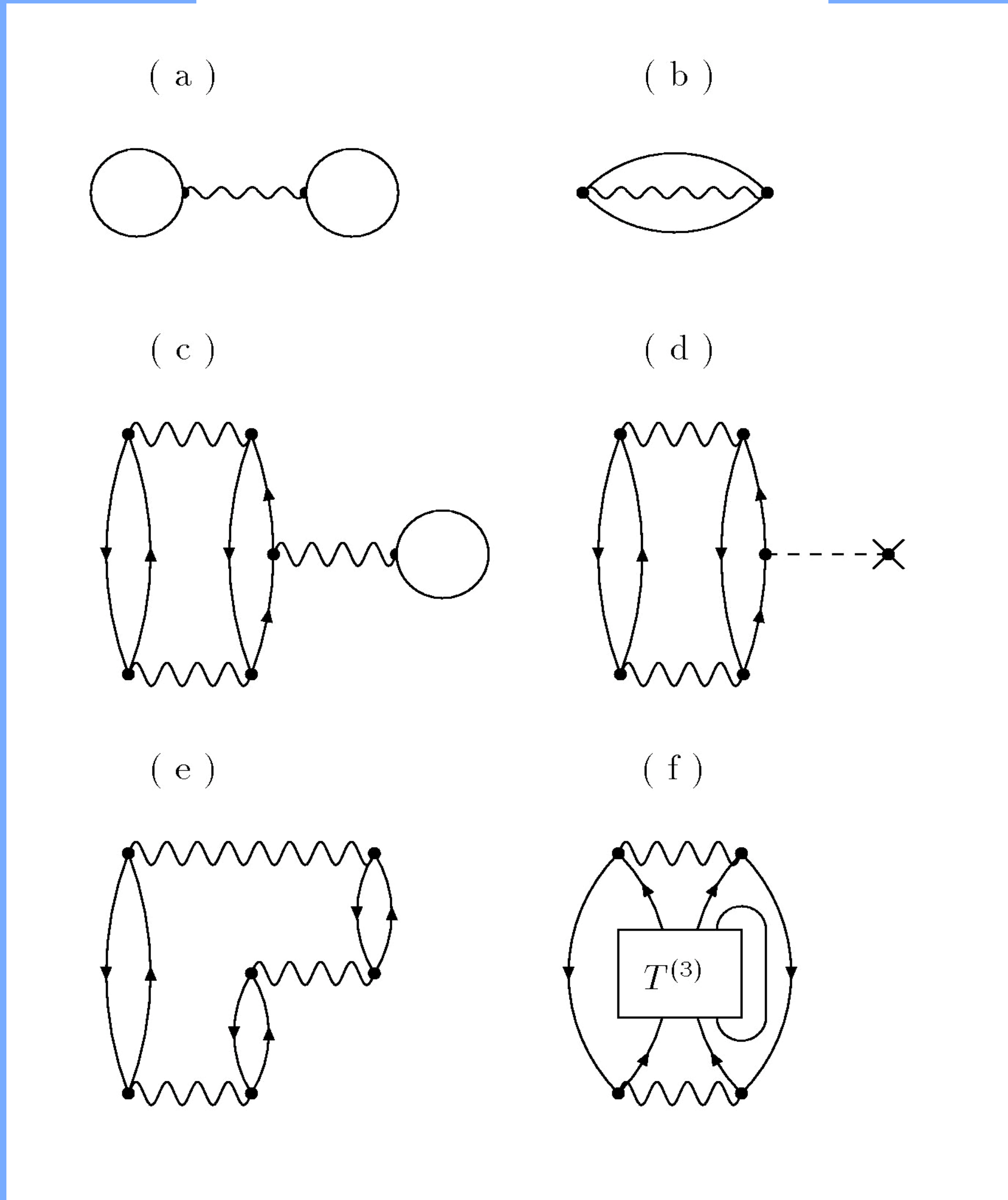
A. Formal comparison between different methods (sketch) in the hadronic sector
Comparing the computed EOS

B. Possible transition to quark matter
Comparing dfifferent simple models

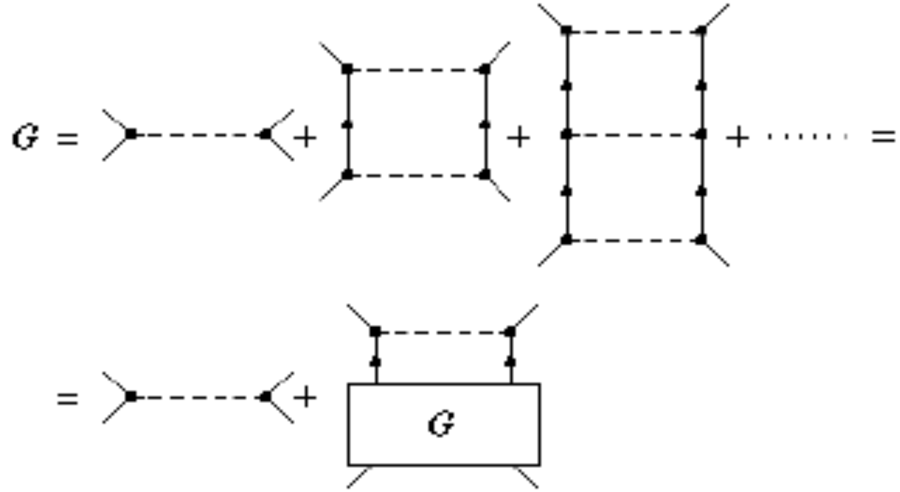
C. Neutron star structure

D. Summary and conclusions

The BBG expansion



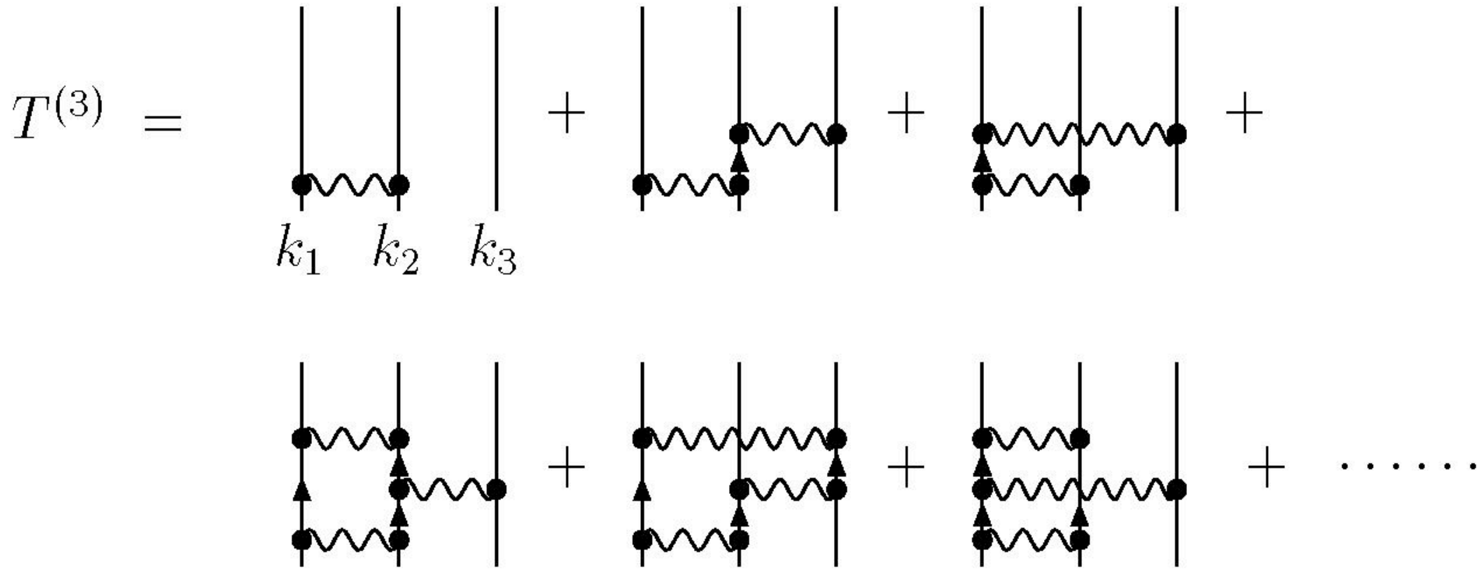
Two and three hole-line diagrams in terms of the Brueckner G-matrixs



Ladder diagrams for the scattering G-matrix

$$G = V + V\frac{Q}{e}G$$

Graphical representation of the Brueckner self-consisten potential



The ladder series for the three-particle scattering matrix

$$T_3 = G + GX \frac{Q_3}{e} T_3$$

$$E_{3h} =$$

$$\frac{1}{2} \sum_{k_1 k_2 k_3} \sum_{[k' k'']} < k_1 k_2 | G | k_1' k_2' >_A$$

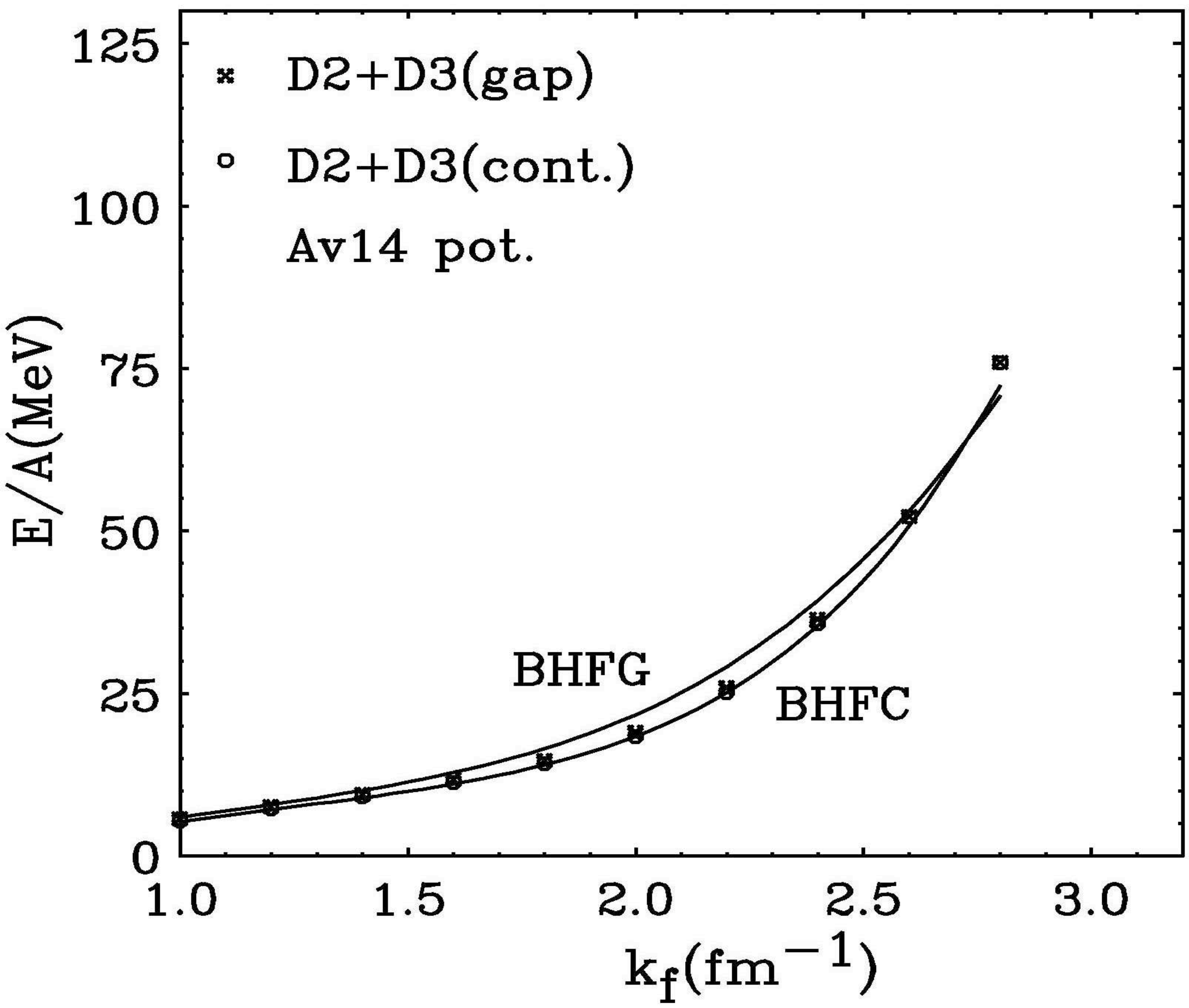
$$\frac{1}{e} < k_1' k_2' k_3 | X T_3 X | k_1'' k_2'' k_3 > \frac{1}{e'}$$

$$< k_1'' k_2'' | G | k_1 k_2 >_A$$

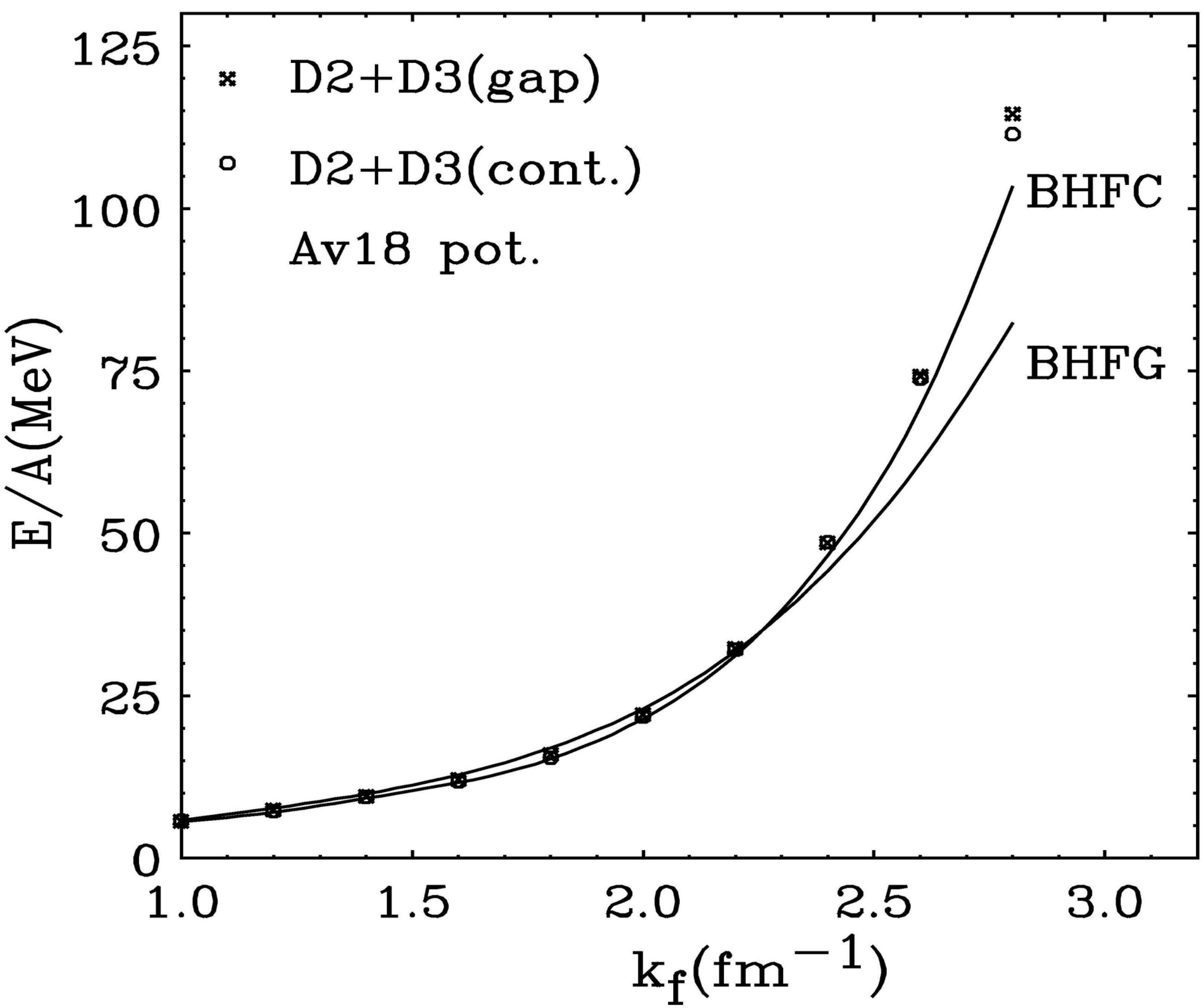
$$k_1, k_2, k_3 \leq k_F$$

$$k_1', k_2', k_1'', k_2'' \geq k_F$$

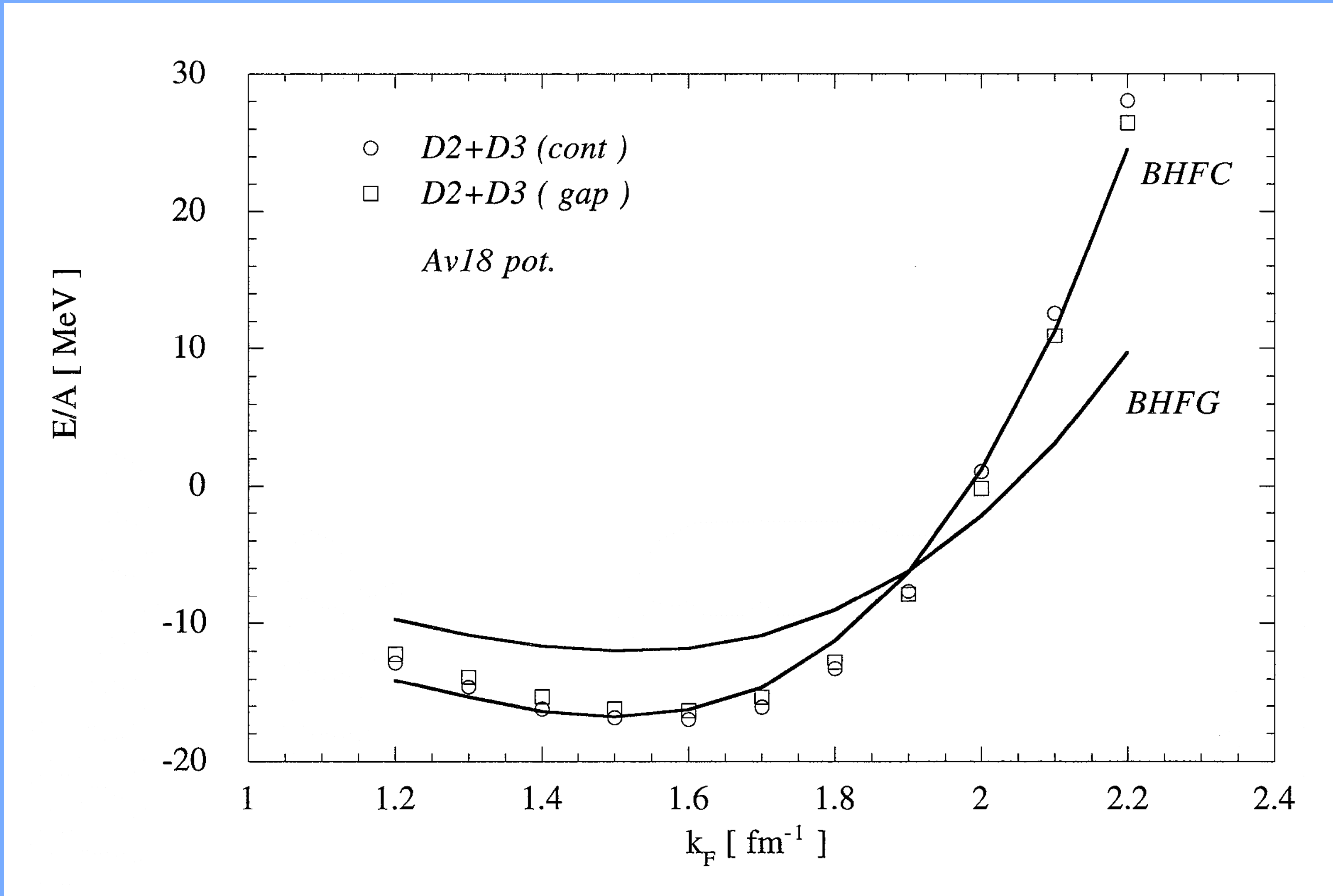
Three hole-line contribution



Neutron matter



Neutron matter



Evidence of convergence

. The final EOS is independent on the choice of the single particle potential

. The three hole-line contribution is small in the continuous choice

Alternative methods

The Coupled Cluster Method

Ansatz on the the exact ground state Ψ in terms of the unperturbed one Φ

$$|\Psi\rangle = e^{\hat{S}}|\Phi\rangle$$

$$\hat{S} = \sum_{k_1,k_2...k_n,k'_1,k'_2...k'_n} \frac{1}{n!^2}\langle k'_1, k'_2...k'_n|S_n|k_1,k_2...k_n\rangle a_{k'_1}{}^{\dagger} a_{k'_2}{}^{\dagger}...a_{k'_n}{}^{\dagger} a_{k_n}...a_{k_2}a_{k_1}$$

k ' s are hole momenta and all the k' ' s are particle momenta.

$$\langle\Phi|\Psi\rangle = 1.$$

The functions $\mathbf{S_n}$ are expected to describe the n-body correlations in the ground state.

Structure of the wave function

Let us consider only $\mathbf{S_2}$ for simplicity and let us assume that it can be considered local in coordinate space,

$$\mathbf{S_2(r_i - r_j) = \chi_{ij}}.$$

Then the correlated ground state can be written

$$\mathbf{\Psi(r_1, r_2, ...) = \Pi_{i<j} f_{ij} \Phi(r_1, r_2, ...)}$$

where the product runs over all possible distinct pairs of particles and

$$\mathbf{f_{ij} = \exp(2\chi_{ij})}.$$

In general, however, the functions S_n are highly non-local in coordinate space and the expression for the ground state wave function cannot be written in such a simple form. The eigenvalue equation for the exact ground state Ψ can be re-written as a (non-hermitean) eigenvalue equation for the unperturbed ground state Φ with a modified hamiltonian, transformed according to a similarity transformation generated by $\hat{S}$

$$\mathbf{e^{-\hat{S}} H e^{\hat{S}} |\Phi\rangle = E |\Phi\rangle}$$

The energy in the CCM scheme

The equations for the total energy E and for the S_n can be obtained by multiplying systematically by n particle-n hole states. The multiplication by $\langle\Phi|$ gives a simple expression for the total energy. If only two-body interaction is present, one gets

$$\mathbf{E} = \langle\Phi|\mathrm{e}^{-\hat{\mathbf{S}}}\,\mathbf{H}\,\mathrm{e}^{\hat{\mathbf{S}}}\,|\Phi\rangle = \mathbf{E}_0 + \langle\Phi|\{\mathbf{V} + [\mathbf{V}, \hat{\mathbf{S}}_2]_-\}|\Phi\rangle$$

all the other terms in the expansion vanish. Therefore, in principle the exact total energy can be obtained from the knowledge only of the exact two particle - two hole amplitude S_2.

$$\mathbf{E} = \mathbf{E}_0 + \frac{1}{2}\sum_{\mathbf{k}_1,\mathbf{k}_2<\mathbf{k}_F}\langle\mathbf{k}_1\mathbf{k}_2|\mathbf{W}_2|\mathbf{k}_1\mathbf{k}_2\rangle$$

$$\langle\mathbf{k}_1\mathbf{k}_2|\mathbf{W}_2|\mathbf{k}_1\mathbf{k}_2\rangle = \langle\mathbf{k}_1\mathbf{k}_2|\{\mathbf{V}+\mathbf{V}\mathbf{S}_2\}|\mathbf{k}_1\mathbf{k}_2\rangle = \langle\mathbf{k}_1\mathbf{k}_2|\mathbf{V}|\mathbf{k}_1\mathbf{k}_2\rangle + \sum_{\mathbf{k}'_1,\mathbf{k}'_2>\mathbf{k}_F}\langle\mathbf{k}_1\mathbf{k}_2|\mathbf{V}|\mathbf{k}'_1\mathbf{k}'_2\rangle\langle\mathbf{k}'_1\mathbf{k}'_2|\mathbf{S}_2|\mathbf{k}_1\mathbf{k}_2\rangle$$

Of course, the amplitude S_2 is connected with the higher order amplitudes $S_3....S_n.....$ These equations are the constitutive "Coupled Cluster" equations, which are equivalent to the eigenvalue equation for the ground state. Approximations can be obtained by truncating these chain of equations to a certain order m, i.e. neglecting S_n for $n > m$. The meaning of the truncation can be read from the ansatz for Ψ, it amounts to consider correlated n particle n hole components in the ground state up to $n = m$, while higher order components with $n > m$ are just antisymmetrized products of the lower ones (note the exponential forms, which produces components of arbitrary higher orders).

The CCM scheme from the variational principle

This form of the CCM equations can be also obtained from the variational principle, i.e. by demanding that the mean value of the hamiltonian in the ground state Ψ is stationary under an arbitrary variation of the state vector orthogonal to Ψ.

$$\delta|\Psi\rangle = e^{-\hat{S}^\dagger}\delta\hat{S}e^{-\hat{S}}|\Psi\rangle$$

where $\delta\hat{S}$ corresponds to an arbitrary variation of the function S_n. Such a variation is orthogonal to Ψ.

However, the CCM equations, as they stand, *cannot be applied to calculations in nuclear matter*. The main correlations in nuclear systems come from the strong short range repulsive core, and this part of the NN interaction requires special treatment. This must be incorporated systematically in the correlation functions S_n, otherwise no truncation of the expansion would be feasible.

Problem of the hard core

The simplest way to proceed is to *renormalize* the original NN interaction and introduce an effective interaction which takes into account the two-body short range correlations from the start. In the BBG expansion this is done by introducing the G-matrix, and a similar procedure can be followed within the CCM scheme. In the modified CCM equations, one introduces the effective interaction

$$\hat{\mathbf{W}} = \frac{1}{2}\sum_{\{\mathbf{k_i}\}}\langle \mathbf{k_1 k_2}|\mathbf{v}|\mathbf{k_3 k_4}\rangle \mathbf{a_{k_1}^\dagger a_{k_2}^\dagger}\left(\mathbf{e}^{-\hat{\mathbf{S}}}\mathbf{a_{k_4} a_{k_3}}\mathbf{e}^{\hat{\mathbf{S}}}\right)_{\mathbf{c}}$$

The subscript c indicates ordered product, i.e. no $a_k^\dagger$ with $k < k_F$ or a_k with $k > k_F$ are retained.

Incorporating the "G-matrix" in the CCM scheme

The operator W can be also expanded in n particle - n hole operators

$$\hat{\mathbf{W}} = \sum_{\mathbf{n}} \sum_{\mathbf{k_1,k_2...k_n,k'_1,k'_2...k'_n}} \frac{1}{\mathbf{n!^2}} \langle \mathbf{k'_1, k'_2...k'_n | W_n | k_1, k_2...k_n} \rangle \mathbf{a_(k'_1)^\dagger a_(k_2)^\dagger....a_(k'_n)^\dagger a(k_n)...a(k_2)a(k_1)}$$

vskip 0.3 cm The functions W_n are related with the functions S_n. Schematically this relation can be written

$$\mathbf{W_n} = \mathbf{v}\delta_{\mathbf{n,2}} + \mathbf{vS_{n-1}} + \mathbf{vS_n} + \sum_{\mathbf{k \le n-2}} \mathbf{vS_k S_{n-k}}$$

Together with the previous relationship, a closed set of equations is then obtained, which is again equivalent to the original eigenvalue problem for the ground state. The ground state energy is still given by the same equation since the relation between W_2 and S_2 still holds. The truncation at order m corresponds now to neglecting the functions W_n and S_n for $n > m$. If one truncates the expansion at $m = 2$, only W_2 and S_2 are retained, the quantity W_2 can be readily identified with the on-shell G-matrix of the BBG expansion and the function S_2 with the corresponding defect function. If the self-consistent single particle potential is introduced, one then gets at this level exactly the Brueckner approximation.

As in thed BBG expansion, the G-matrix can be introduced in all the terms of the Coupled-Cluster expansion. In this case each term of the expansion coincides with one diagram in the BBG method.

Since the CCM is based on the ansatz for the ground state wave function, it is likely that the same structure of the ground state is underlying the BBG expansion. At Brueckner level the ground state is then given by

$$|\mathbf{\Psi_{Bru}}\rangle = \mathbf{e}^{\hat{\mathbf{S}}_2} |\mathbf{\Phi}\rangle$$

with S_2 the Brueckner defect function.

The variational method in its practical form

The variational method

The variational acquires a particular form in nuclear physics because of the peculiarities of the NN interaction. The strong repulsion at short distance has been treated by introducing a Jastrow-like trial wave function. In the simple case of a central interaction the trial ground state wave function is written as

$$\Psi(\mathbf{r_1}, \mathbf{r_2}, ...) = \Pi_{\mathbf{i<j}} \mathbf{f}(\mathbf{r_{ij}}) \Phi(\mathbf{r_1}, \mathbf{r_2}, ...)$$

where Φ is the unperturbed ground state wave function, properly antisymmetrized, and the product runs over all possible distinct pairs of particles. The similarity with the wave function of the CCM method is apparent and indicates a definite link with BBG and CCM methods. The correlation function $\mathbf{f}(\mathbf{r_{ij}})$ is here determined by the variational principle, i.e. by imposing that the mean value of the hamiltonian gets a minimum (or in general stationary)

$$\frac{\delta}{\delta \mathbf{f}} \frac{\langle \Psi | \mathbf{H} | \Psi \rangle}{\langle \Psi | \Psi \rangle} = 0$$

The problem of non-central correlations

Channel dependent correlation factors

In principle this is a functional equation for the correlation function f, which however can be written explicitly in a closed form only if additional suitable approximations are introduced. A practical and much used method is to assume a parametrized form for f and to minimize the energy with respect to the set of parameters which constrain its form. The function $\mathbf{f(r_{ij})}$ is assumed to converge to $\mathbf{1}$ at large distance and to go rapidly to zero as $\mathbf{r_{ij} \to 0}$, with a shape similar to the the defect function. For nuclear matter it is necessary to introduce a channel dependent correlation factor, which is equivalent to assume that f is actually a two-body operator $\hat{\mathbf{F}}_{\mathbf{ij}}$. In principle, the condition of energy minimum (or extremal) should produce a set of **Euler-Lagrange** equations which determine the correlation factors. In practice, a viable explicit form can be used only for the two-body cluster terms. If the two-body NN interaction is local and central, its mean value is directly related to the pair distribution function $\mathbf{g(r)}$

$$< \mathbf{V} > = \frac{\mathbf{1}}{\mathbf{2}}\rho \int \mathbf{d^3r \;\; v(r)\, g(r)}$$

where

$$\mathbf{g(r_1 - r_2)} \; = \; \frac{\int \mathbf{\Pi_{i>2} d^3r_i |\Psi(r_1, r_2 ....)|^2}}{\int \mathbf{\Pi_i d^3r_i |\Psi(r_1, r_2 ....)|^2}}$$

The main job in the variational method is to relate the pair distribution function to the correlation factors F. In general this cannot be done exactly, and one has to rely on some suitable expansion. The expansion is in the quantity

$$\mathbf{h(r) = 1 - F(r)^2}$$

which is directly related to the defect function.

The pair distribution function

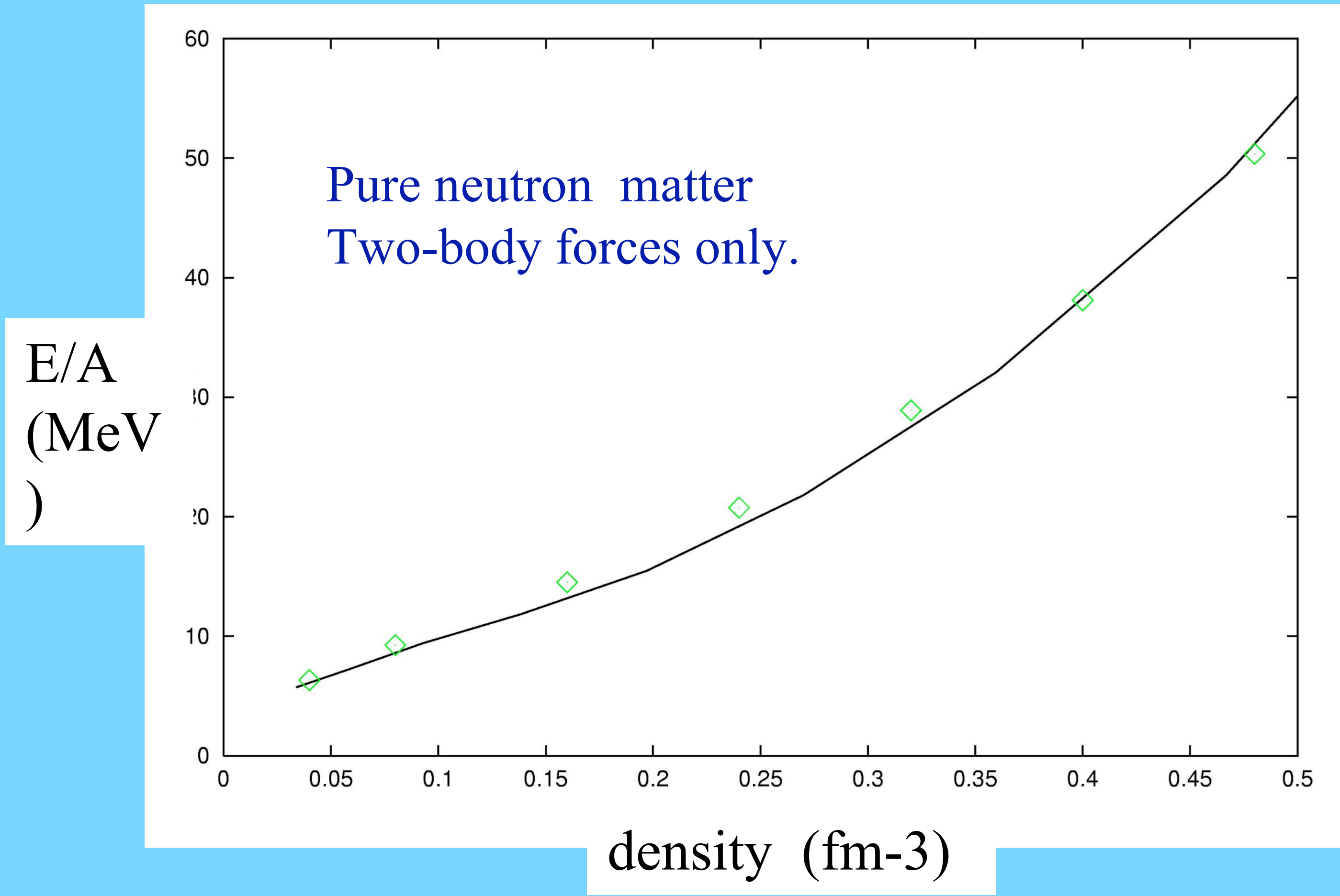
Summary of the formal comparison

1. The CCM and BBG are essentially equivalent, which indicates that the
   w.f. is of the type
   $\Psi = e^S \Phi$, if $S = S_2$ one gets the Brueckner approximation

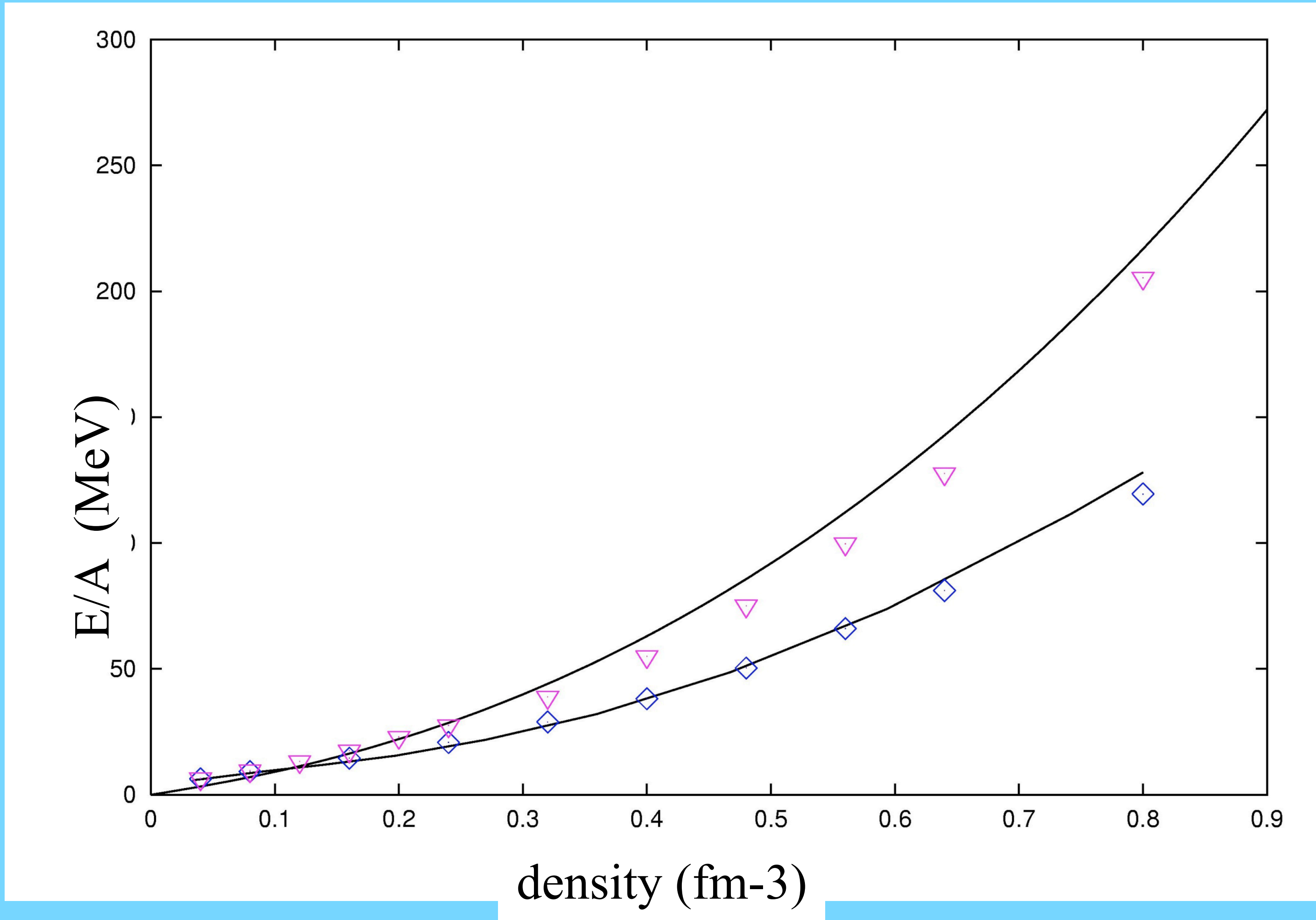
   Once the single particle potential is introduced, the methods are not variational at a given truncation.

2. The main differences in the variational method

   a) The correlation factors are local and momentum independent (eventually gradient terms).
   b) No single particle mean field is introduced, so that the meaning of "clusters" is quite different
   c) Chain summations include long range correlations
      Short range 3-body cluster calculated in PRC 66 (2002) 0543308



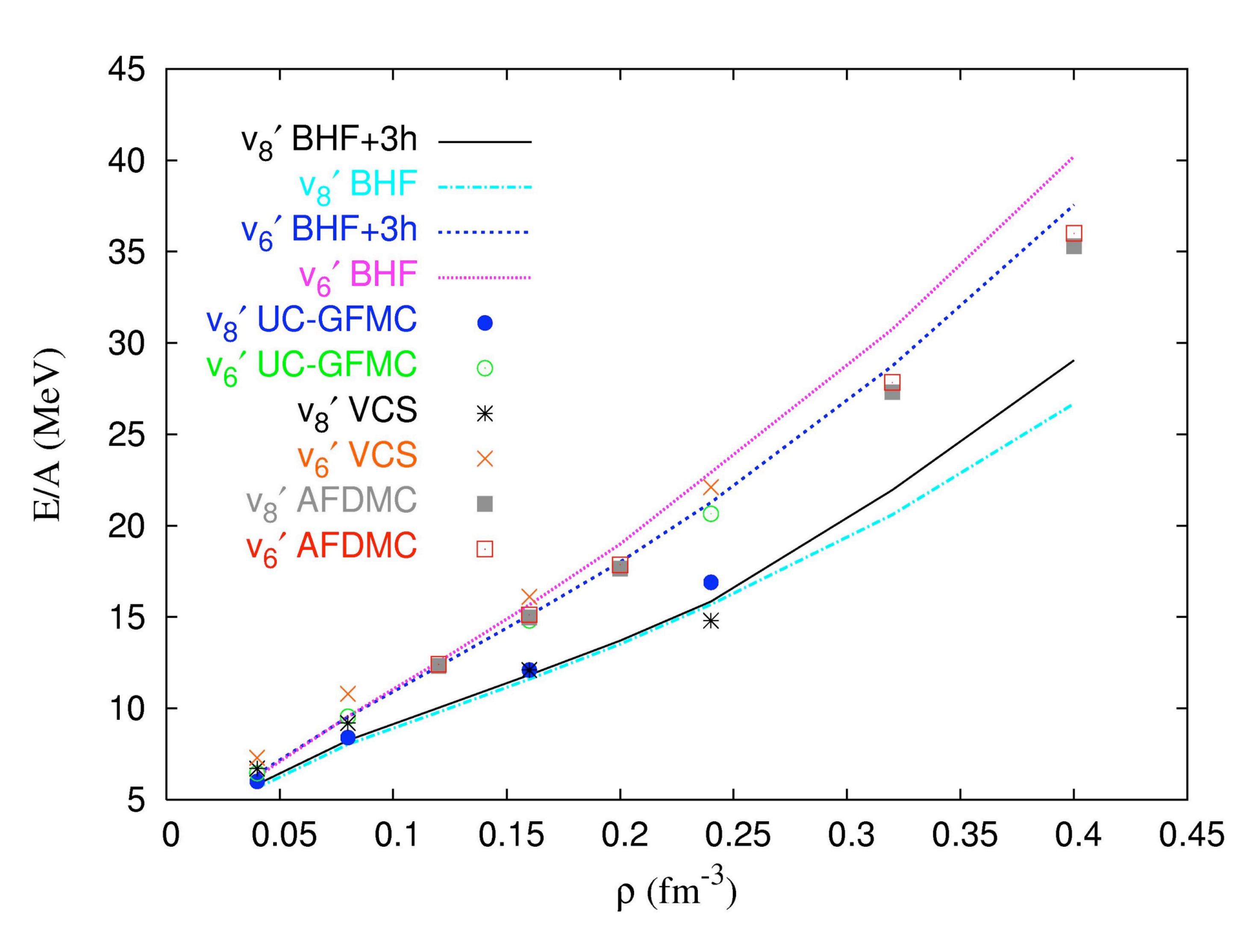
Comparison between BBG (solid line)
Phys. Lett. B 473,1(2000)
and variational calculations (diamonds)
Phys. Rev. C58,1804(1998)



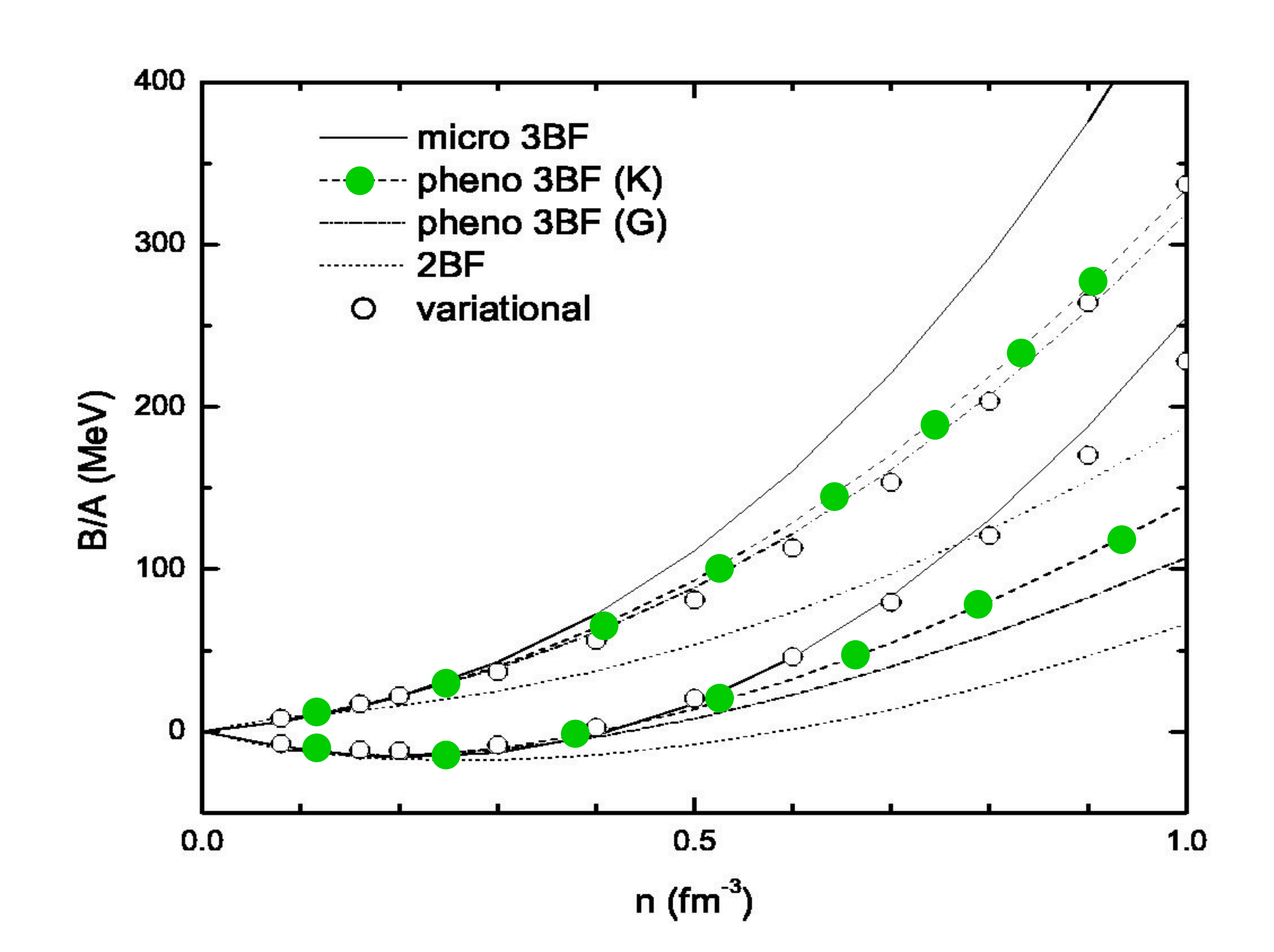
Including TBF and extending the comparison to "very high" density.

CAVEAT : TBF are not exactly the same.

Confronting with “exact” GFMC for v6 and v8



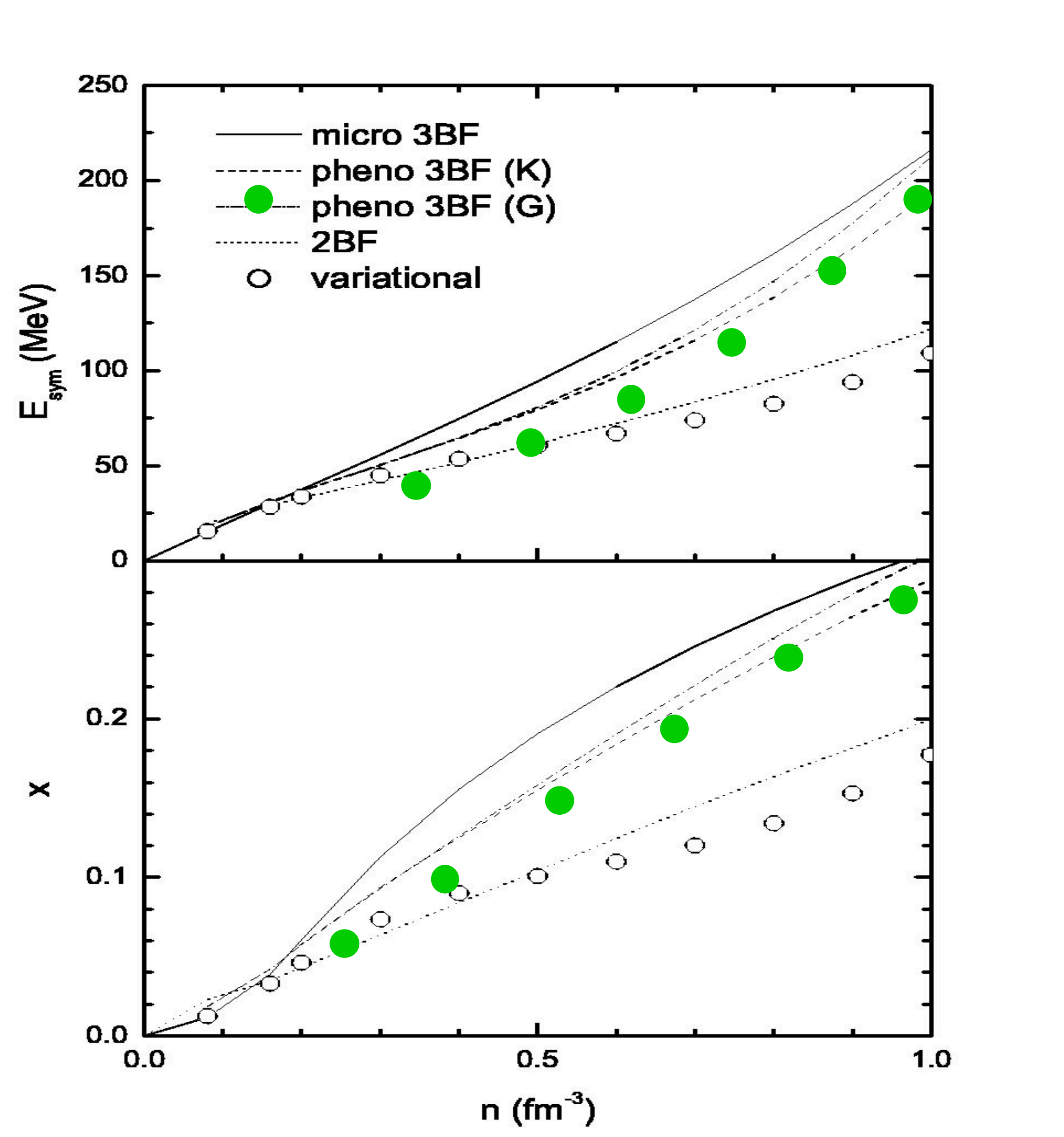
Variational and GMFC : Carlson et al. Phys. Rev. C68, 025802(2003)
BBG : M.B. and C. Maieron, Phys. Rev. C69,014301(2004)



Neutron and Nuclear matter EOS.
Comparison between BBG and variational method.

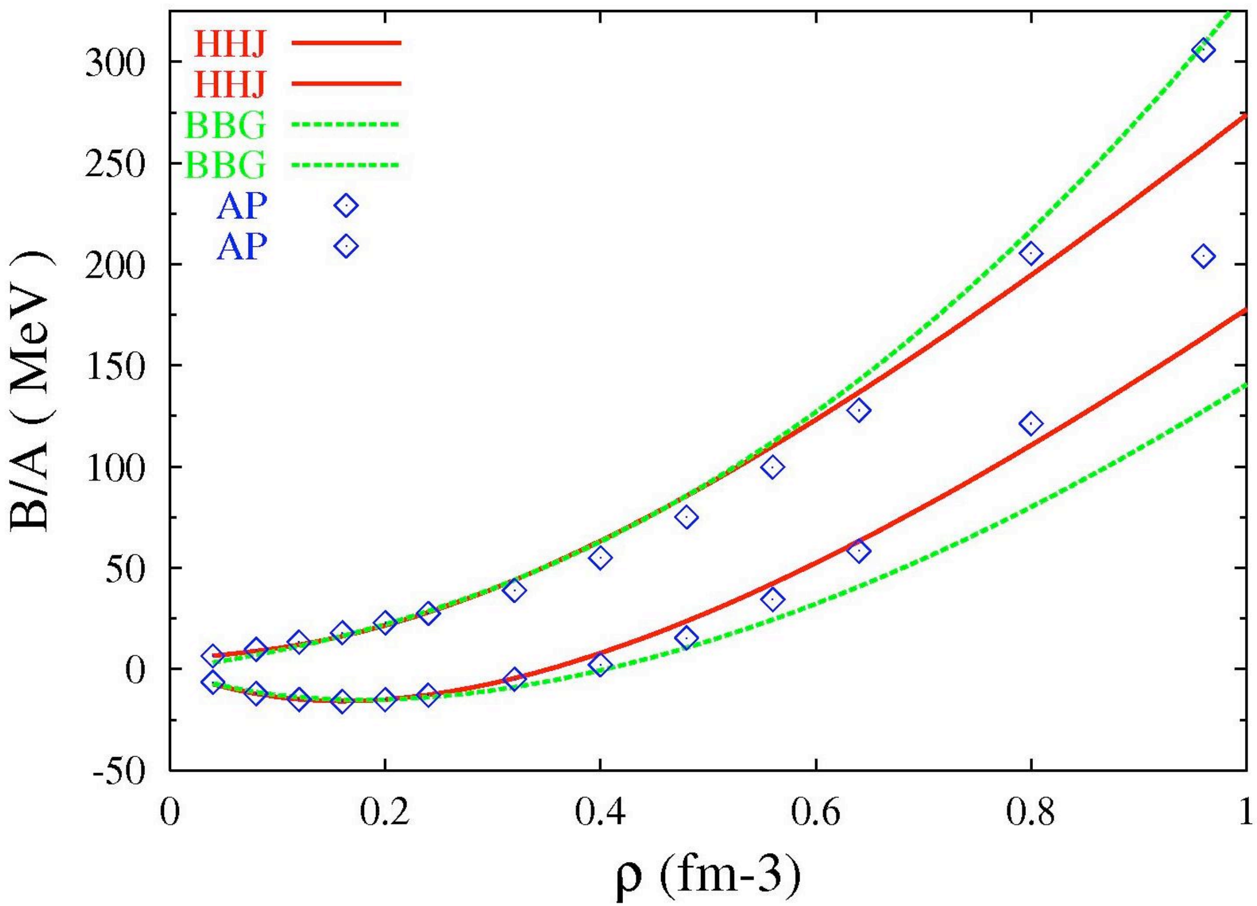
Symmetry energy as a function of density

Proton fraction as a function of density in neutron stars



AP becomes superluminal and DU process is at too high density

The baryonic Equations of State



HHJ : Astrophys. J. 525, L45 (1999

BBG : PRC 69 , 018801 (2004)

AP : PRC 58, 1804 (1998)

Summary for the nucleonic sector

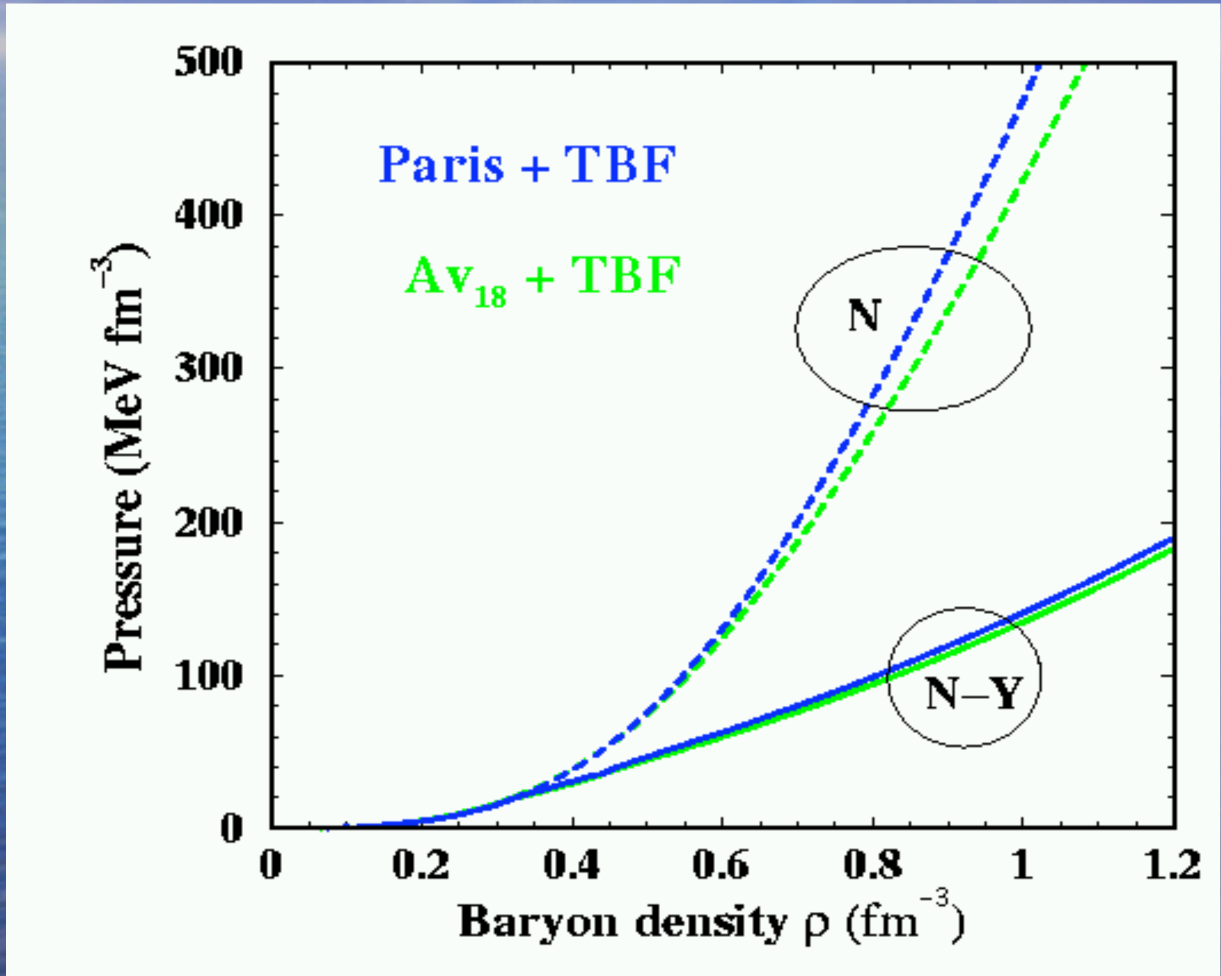
1. Similarities and differences between variational and BBG

2. At v6-v8 level excellent agreement between var. and BBG as well as with GFMC (at least up to 0.25 fm-3) for neutron matter.

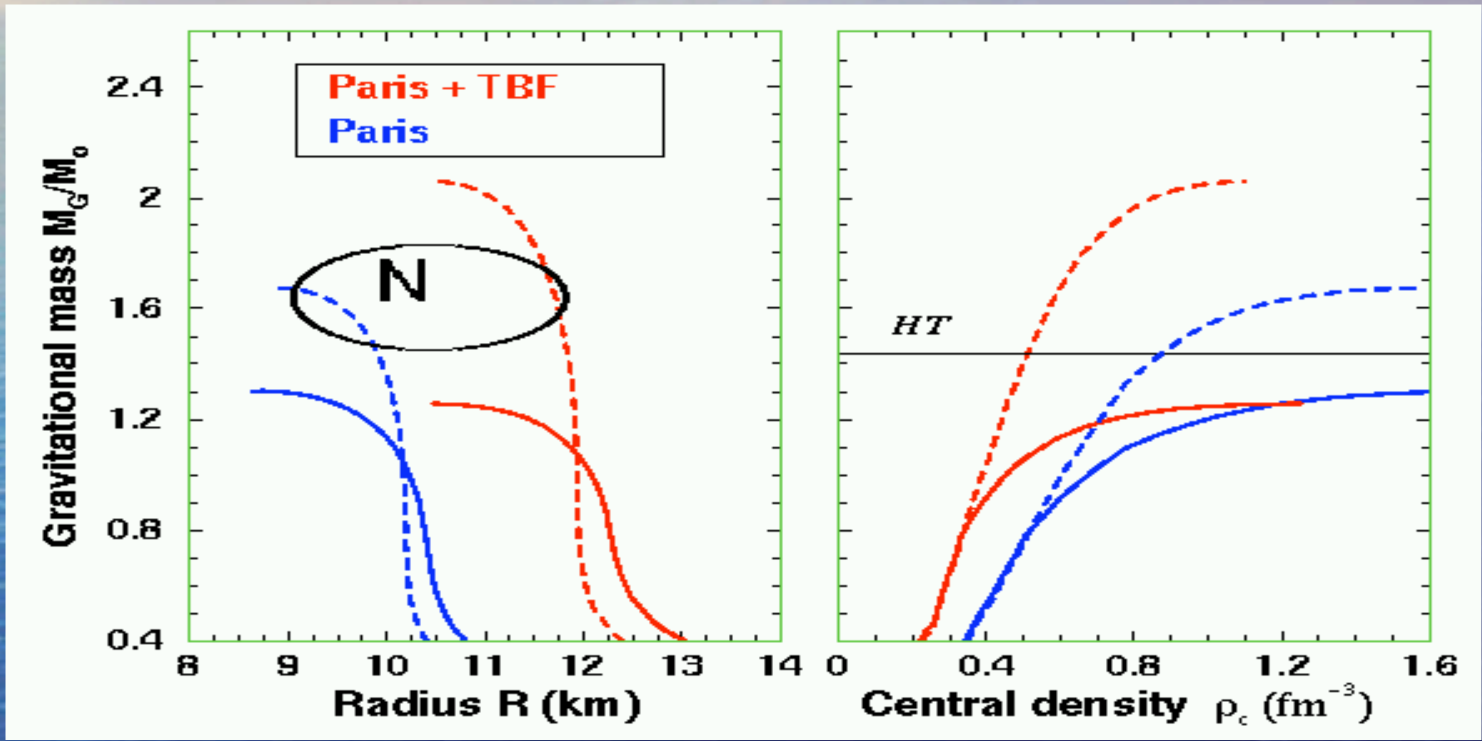
3. For the full interaction (Av18) good agreement between var. and BBG up to 0.6 fm-3 (symmetric and neutron matter).

4. The many-body treatment of nuclear matter EOS can be considered well understood. Main uncertainity is TBF at high density (above 0.6 fm-3).

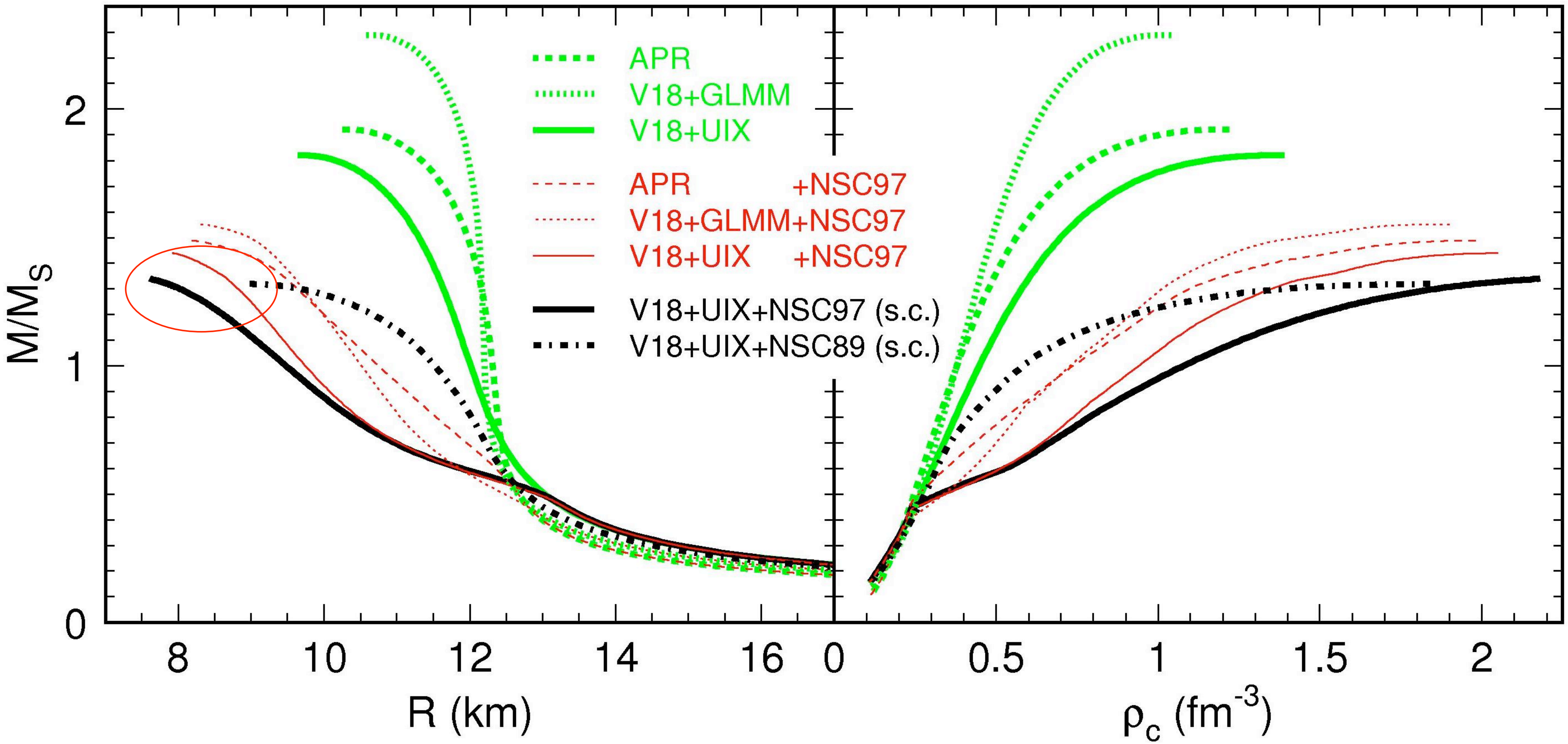
Hyperon influence on hadronic EOS



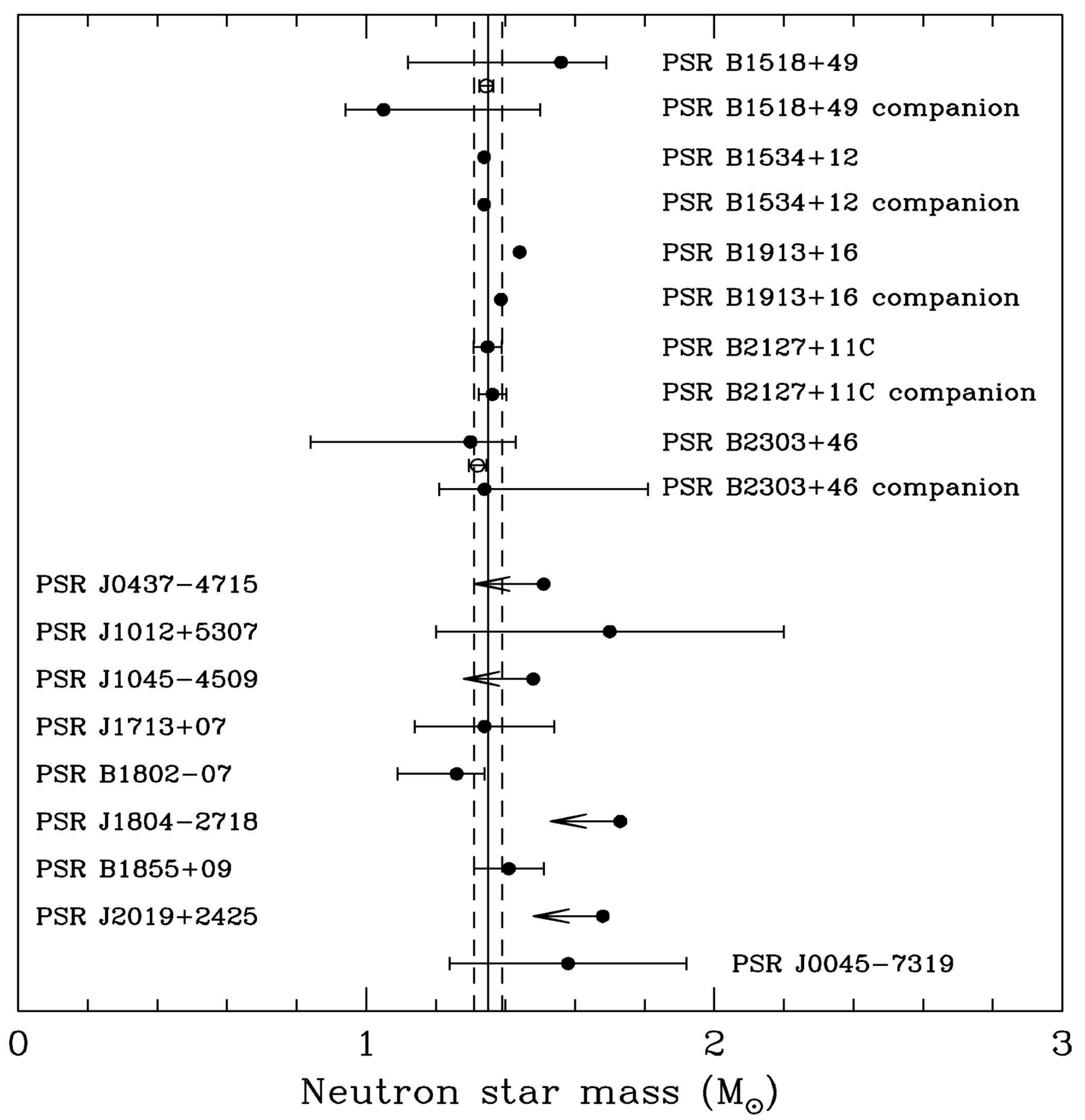
Mass-Radius relation

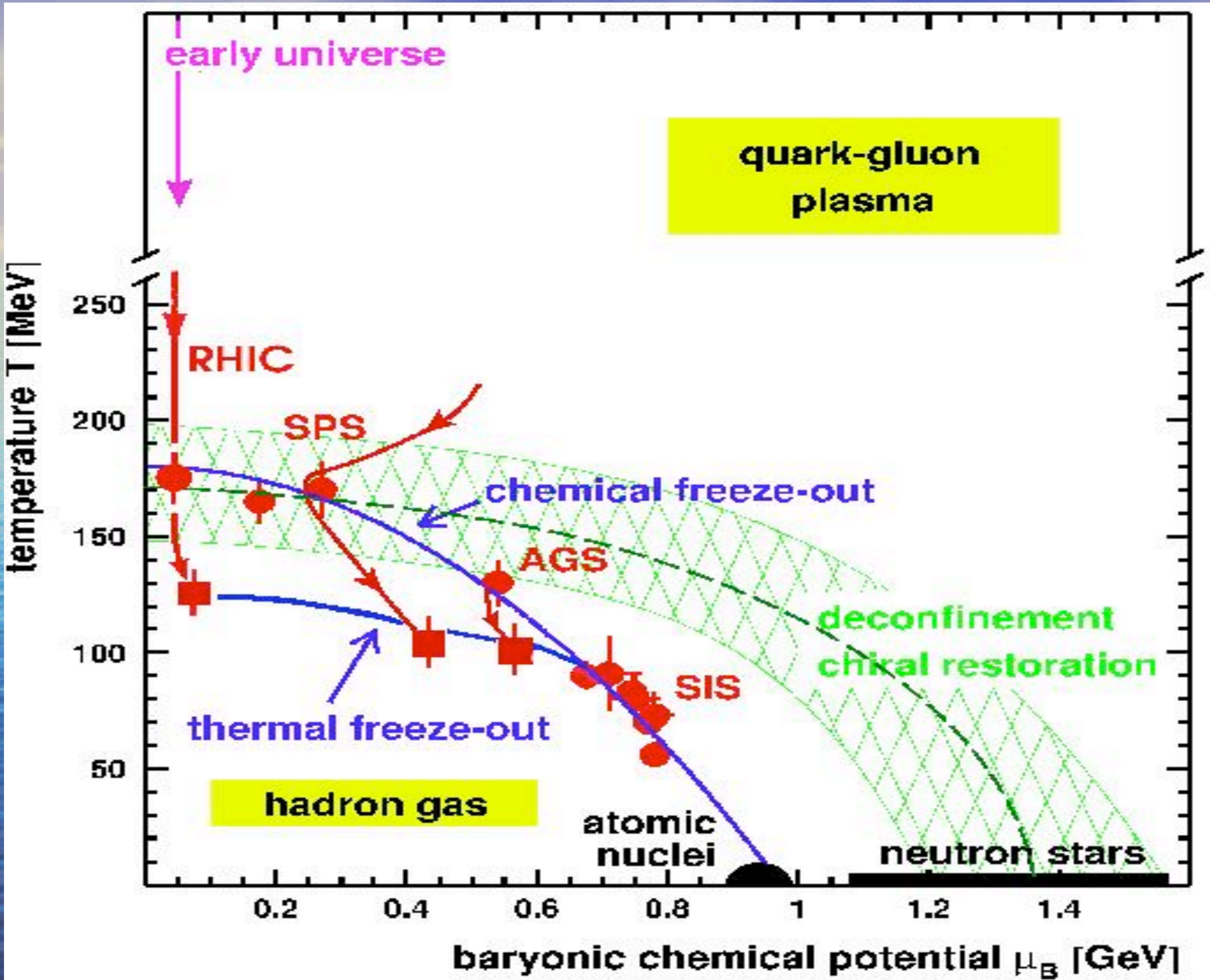


- *Inclusion of Y decreases the maximum mass value*



H.J. Schulze et al., PRC 73, 058801 (2006)


PSR B1518+49
PSR B1518+49 companion
PSR B1534+12
PSR B1534+12 companion
PSR B1913+16
PSR B1913+16 companion
PSR B2127+11C
PSR B2127+11C companion
PSR B2303+46
PSR B2303+46 companion
PSR J0437−4715
PSR J1012+5307
PSR J1045−4509
PSR J1713+07
PSR B1802−07
PSR J1804−2718
PSR B1855+09
PSR J2019+2425
PSR J0045−7319
0
1
2
3
Neutron star mass ($M_\odot$)



early universe
quark-gluon plasma
temperature T [MeV]
250
200
150
100
50
RHIC
SPS
chemical freeze-out
AGS
deconfinement
chiral restoration
SIS
thermal freeze-out
hadron gas
atomic nuclei
neutron stars
0.2
0.4
0.6
0.8
1
1.2
1.4
baryonic chemical potential μ_B [GeV]

CAVEAT

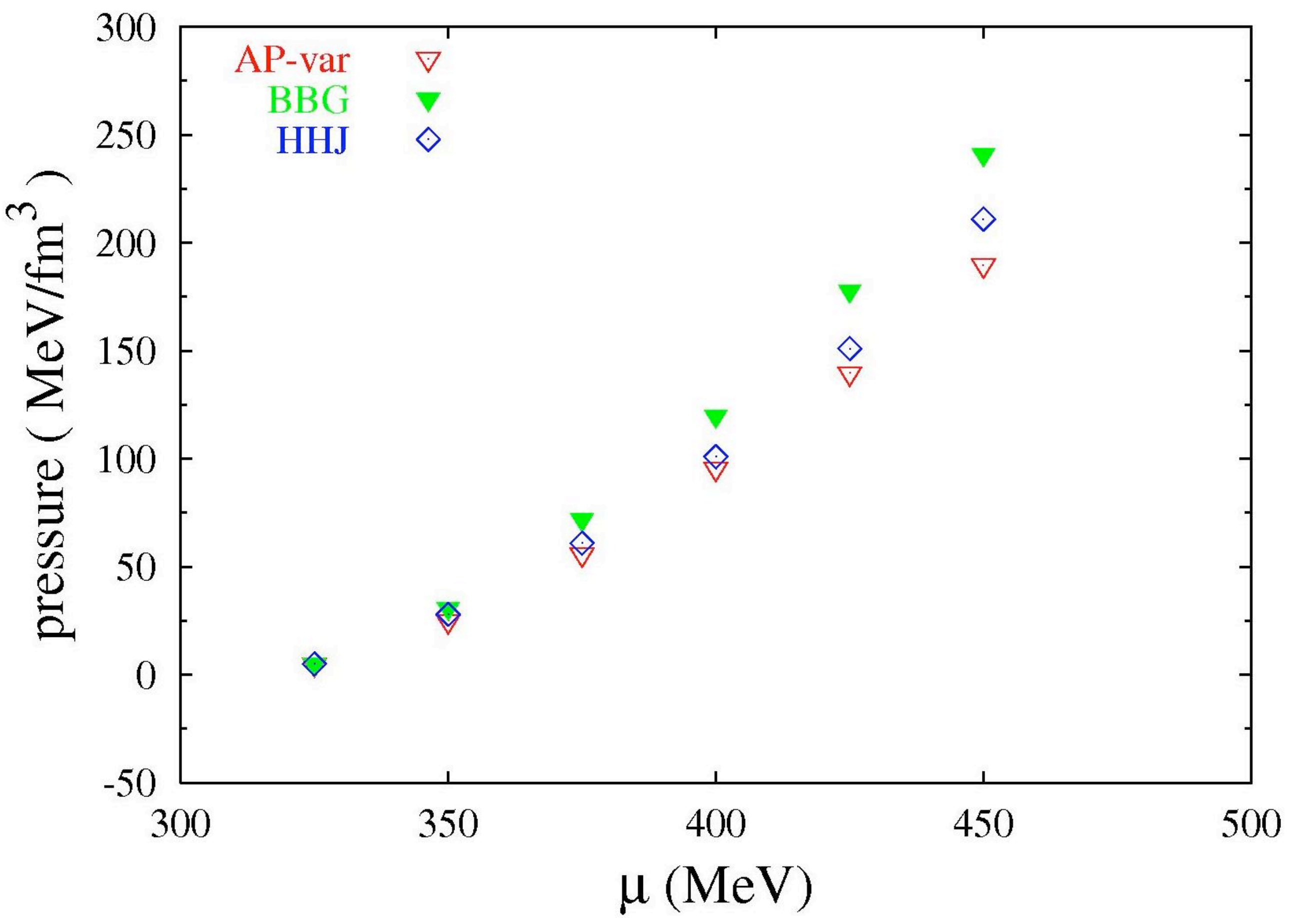
This picture is too simplified .
It neglects the isotopic effect. Nuclear matter inside neutron stars is highly asymmetric and the possible transition to quark matter is located at quite different densities than in symmetric matter.

Including Quark matter

Since we have no theory which describes both confined and deconfined phases, we uses two separate EOS for baryon and quark matter and assumes a first order phase transition.

a) Baryon EOS. BBG
AP
HHJ

b) Quark matter EOS. MIT bag model
Nambu-Jona Lasinio
Coloror dielectric model



The three baryon EOS for beta-stable neutron star matter in the pressure-chemical potential plane.

MIT bag model. “Naive version”

$$\Omega_q = -\frac{3m_q^4}{8\pi^2}\Big[\frac{\eta_q x_q}{3}(2x_q^2-3)+\ln(x_q+\eta_q)\Big]$$
$$+\frac{3m_q^4\alpha_s}{4\pi^3}\Big\{2\Big[\eta_q x_q-\ln(x_q+\eta_q)\Big]^2-\frac{4}{3}x_q^4+2\ln(\eta_q)$$
$$+4\ln\Big(\frac{\sigma_{\rm ren}}{m_q\eta_q}\Big)\Big[\eta_q x_q-\ln(x_q+\eta_q)\Big]\Big\}$$

m_q , μ_q : q quark mass and chemical potential.

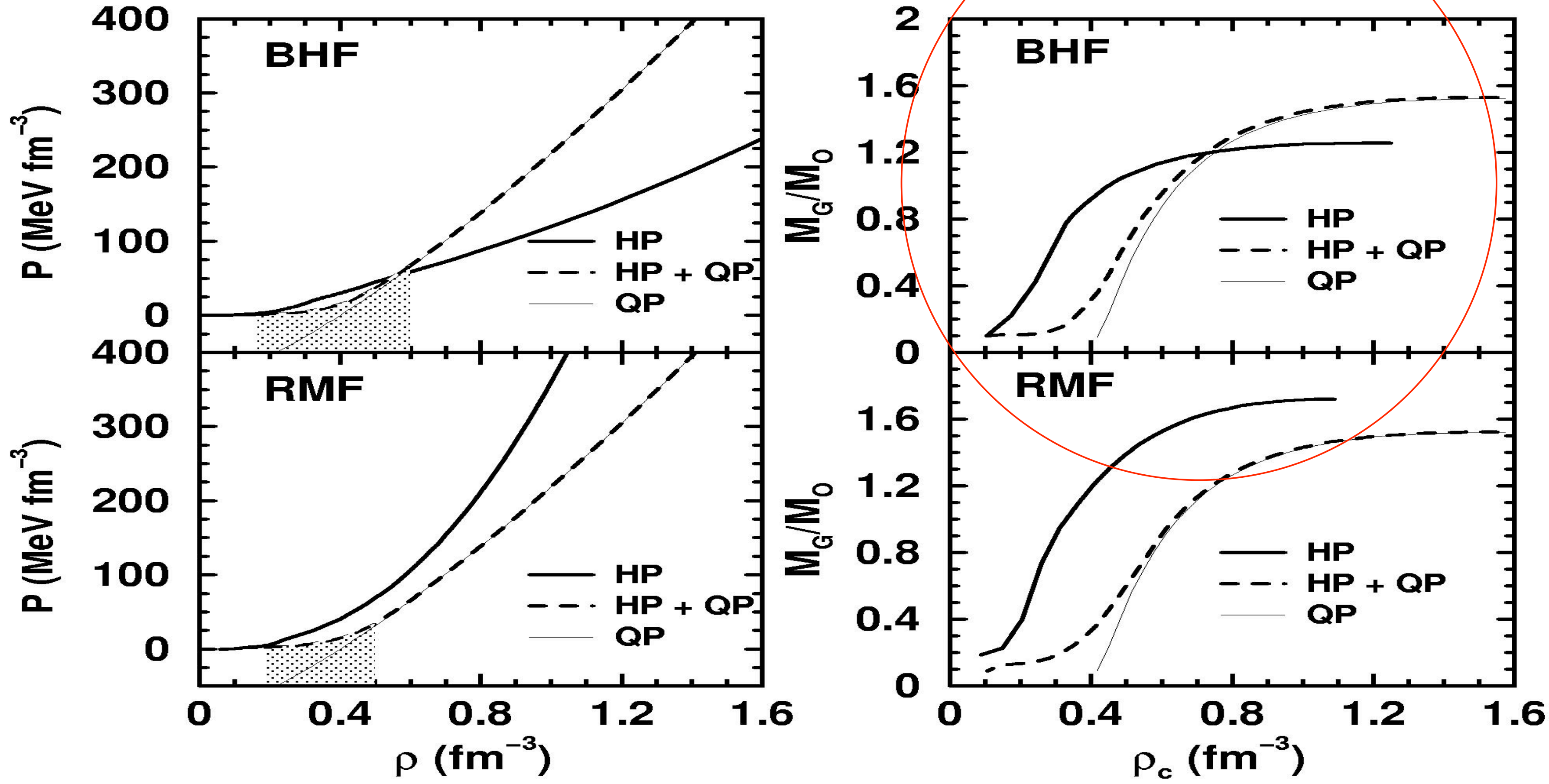
$x_q = \sqrt{\mu_q^2 - m_q^2}/m_q$

$\eta_q = \sqrt{1+x_q^2} = \mu_q/m_q$

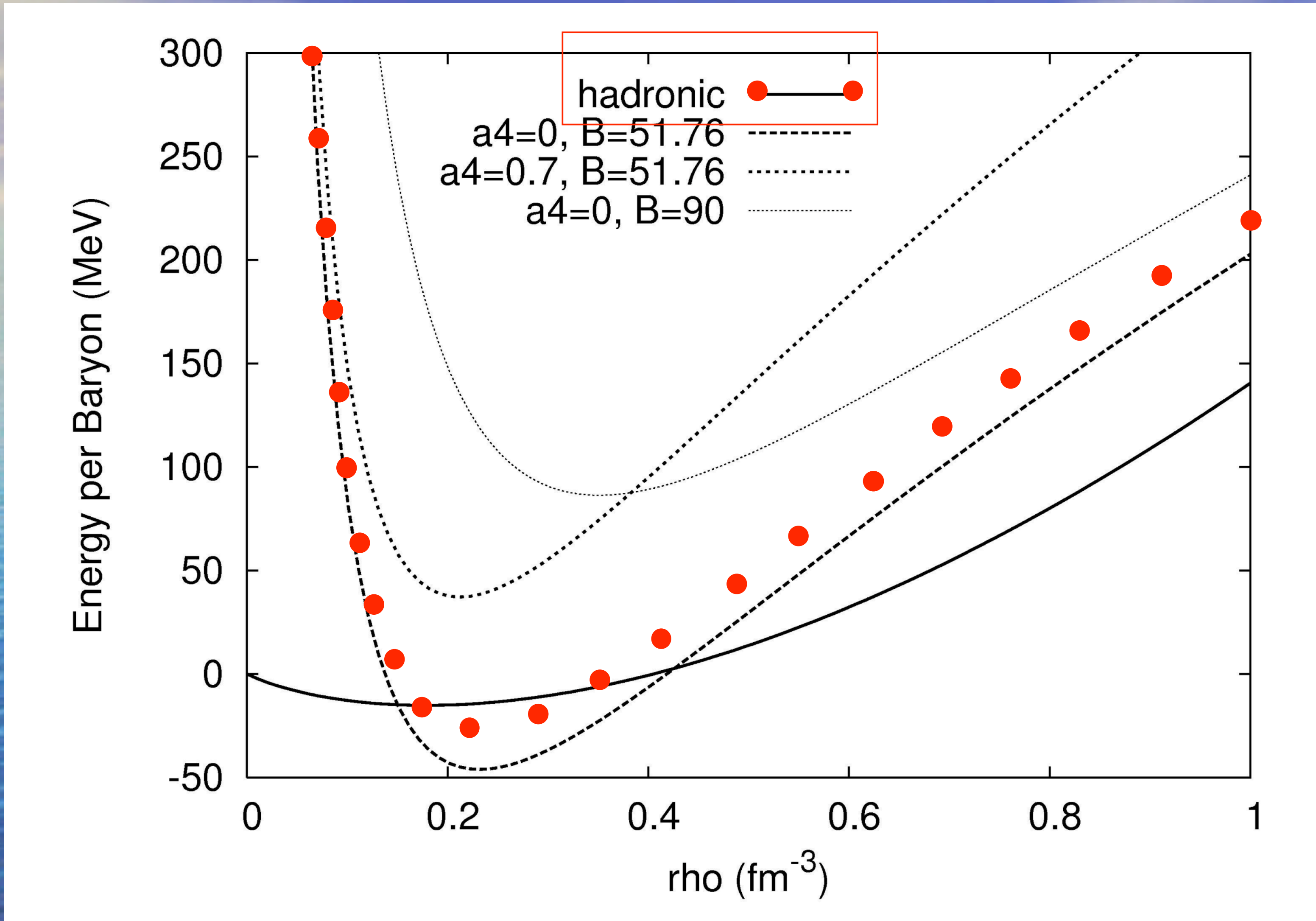
α_s : QCD fine structure constant

$$\rho_q = -\frac{\partial\Omega_q}{\partial\mu_q}$$

$$\epsilon_Q = \sum_q(\Omega_q+\mu_q\rho_q)+B$$
$$P_Q = -\sum_q\Omega_q - B$$

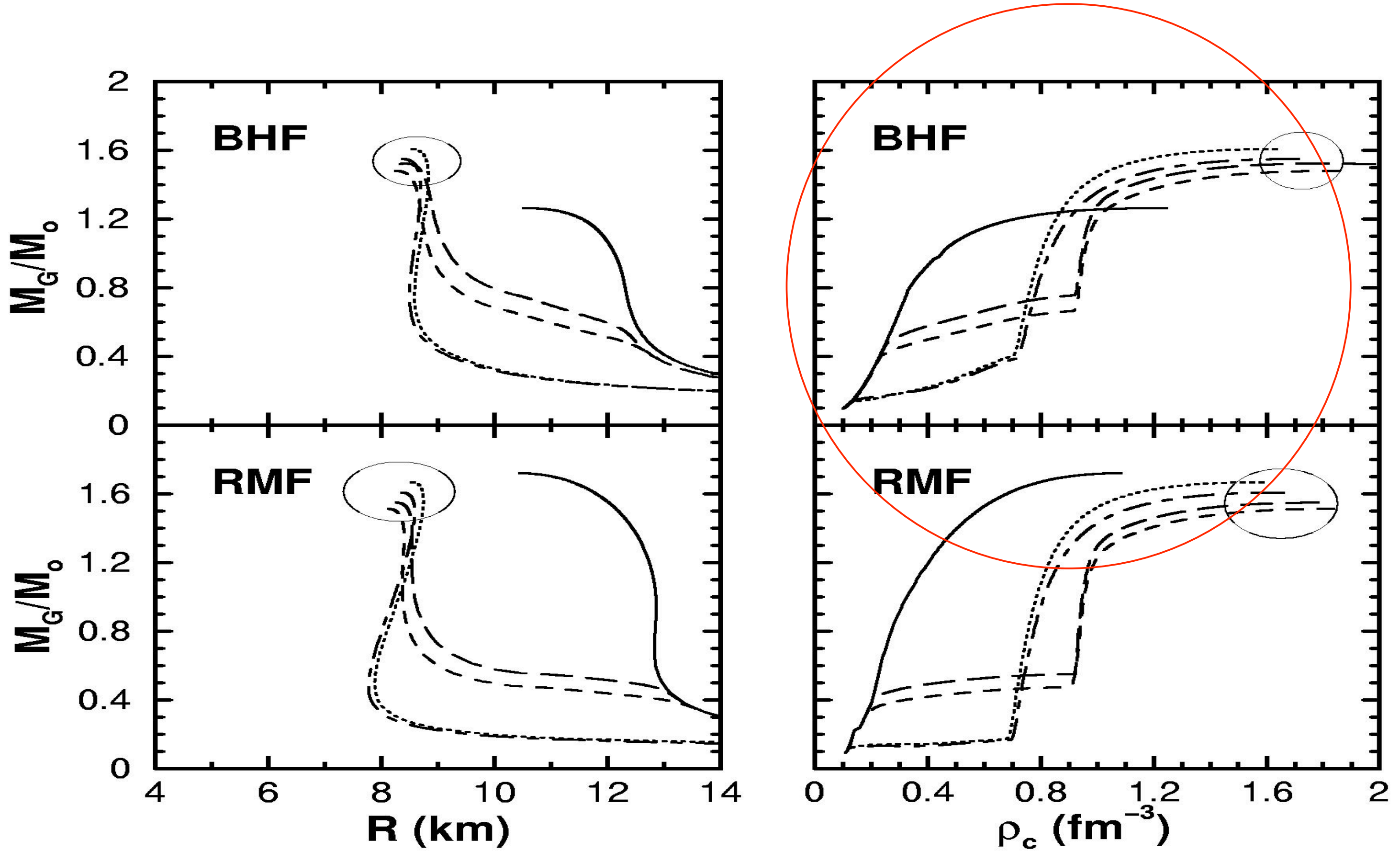


PRC , 025802 (2002)

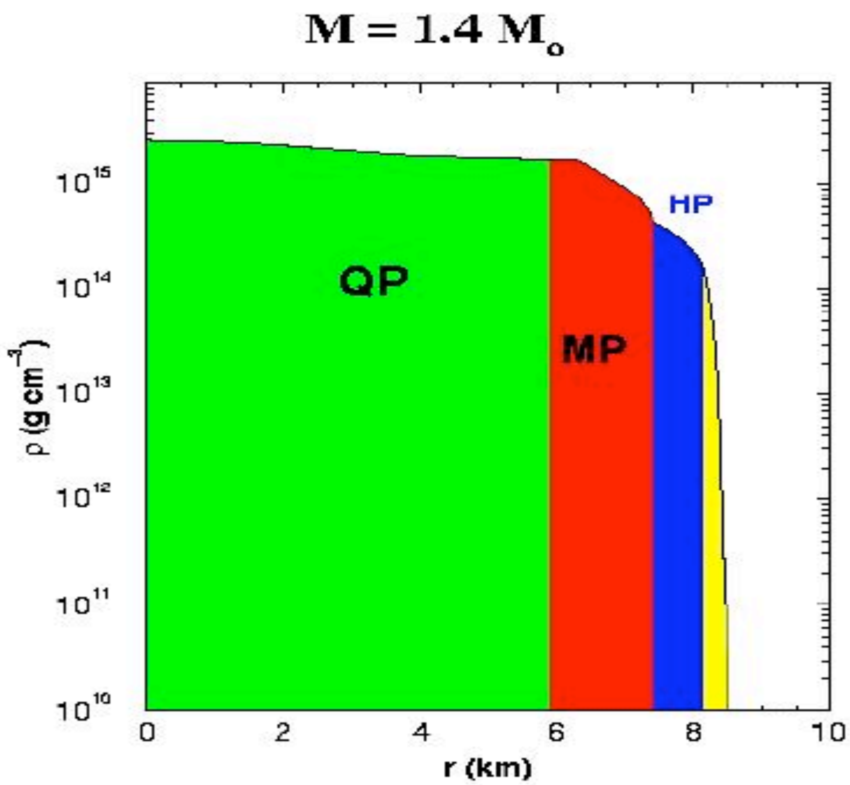


hadronic
a4=0, B=51.76
a4=0.7, B=51.76
a4=0, B=90
Energy per Baryon (MeV)
300
250
200
150
100
50
0
-50
0
0.2
0.4
0.6
0.8
1
rho (fm-3)

Density dependent bag "constant"



$$\varepsilon_Q = 1.1 \quad GeV\, fm^{-3}$$



Density profiles of different phases

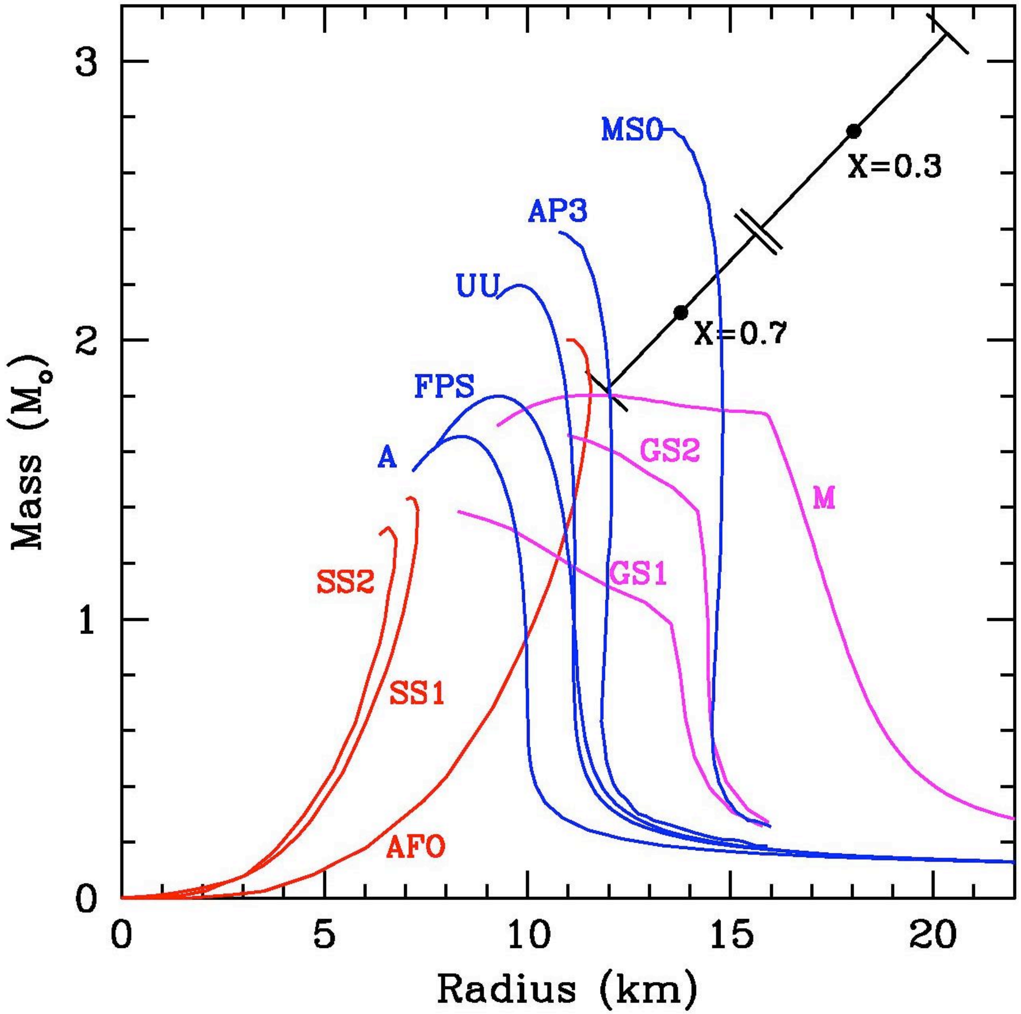
MIT bag model

Evidence for "large" mass ?

Nice et al. ApJ 634, 1242 (2005)
PSR J0751+1807 M = 2.1 +/- 0.2

Ozel, astro-ph /0605106
EXO 0748 – 676 M > 1.8

Quaintrell et al. A&A 401, 313 (2003)
NS in VelaX-1 1.8 < M < 2



Mass (M$_\odot$)
Radius (km)
0
1
2
3
0
5
10
15
20
MS0
AP3
UU
FPS
A
SS2
SS1
AFO
GS2
GS1
M
X=0.3
X=0.7

Ozel, 2006

Alford et al., ApJ 629 (2005) 969

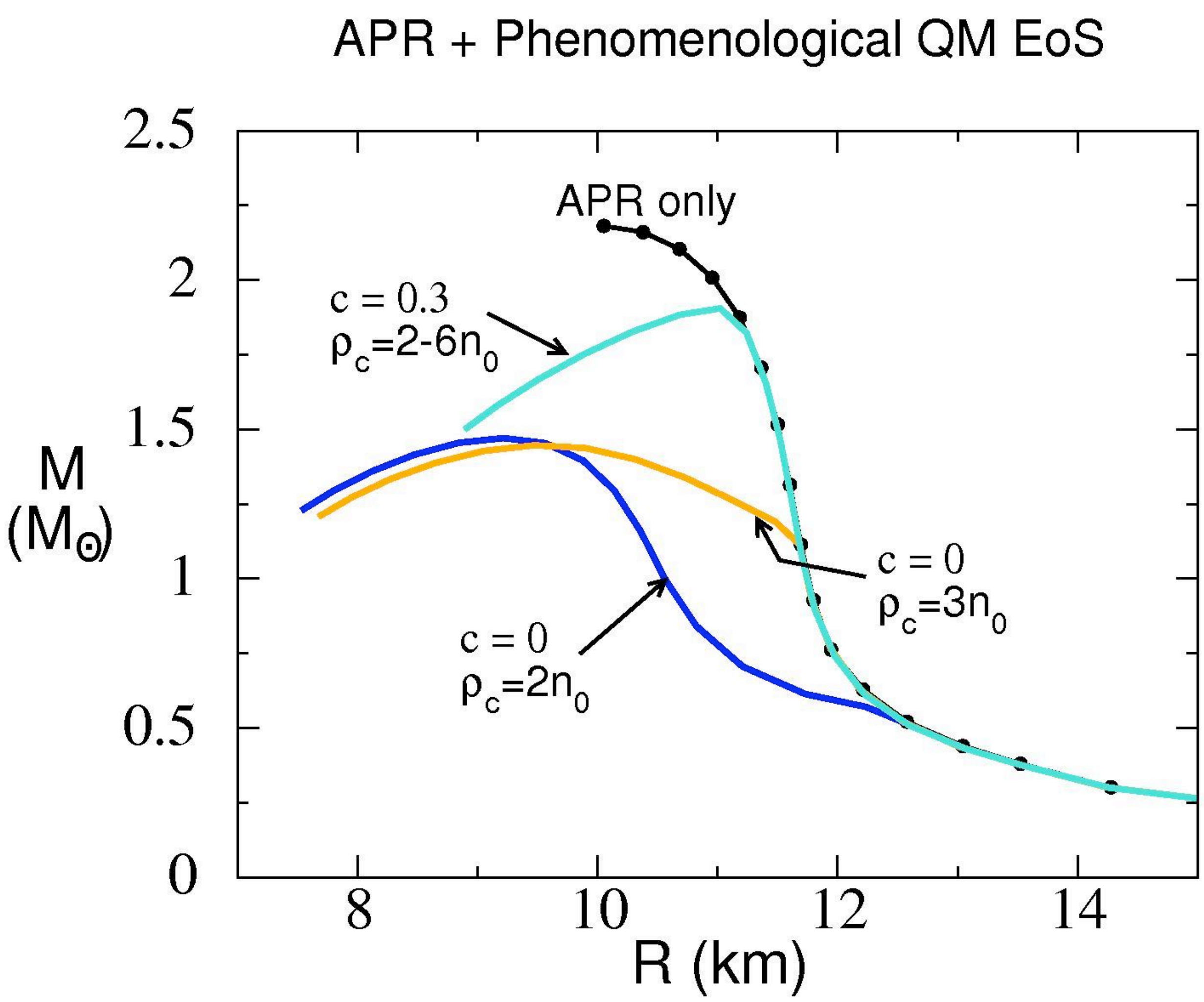
$$\Omega_{QM} = -\frac{3}{4\pi^2} a_4 \mu^4 + \frac{3}{4\pi^2} a_2 \mu^2 + B_{eff}$$

a_4 → Non-perturbative corrections ;

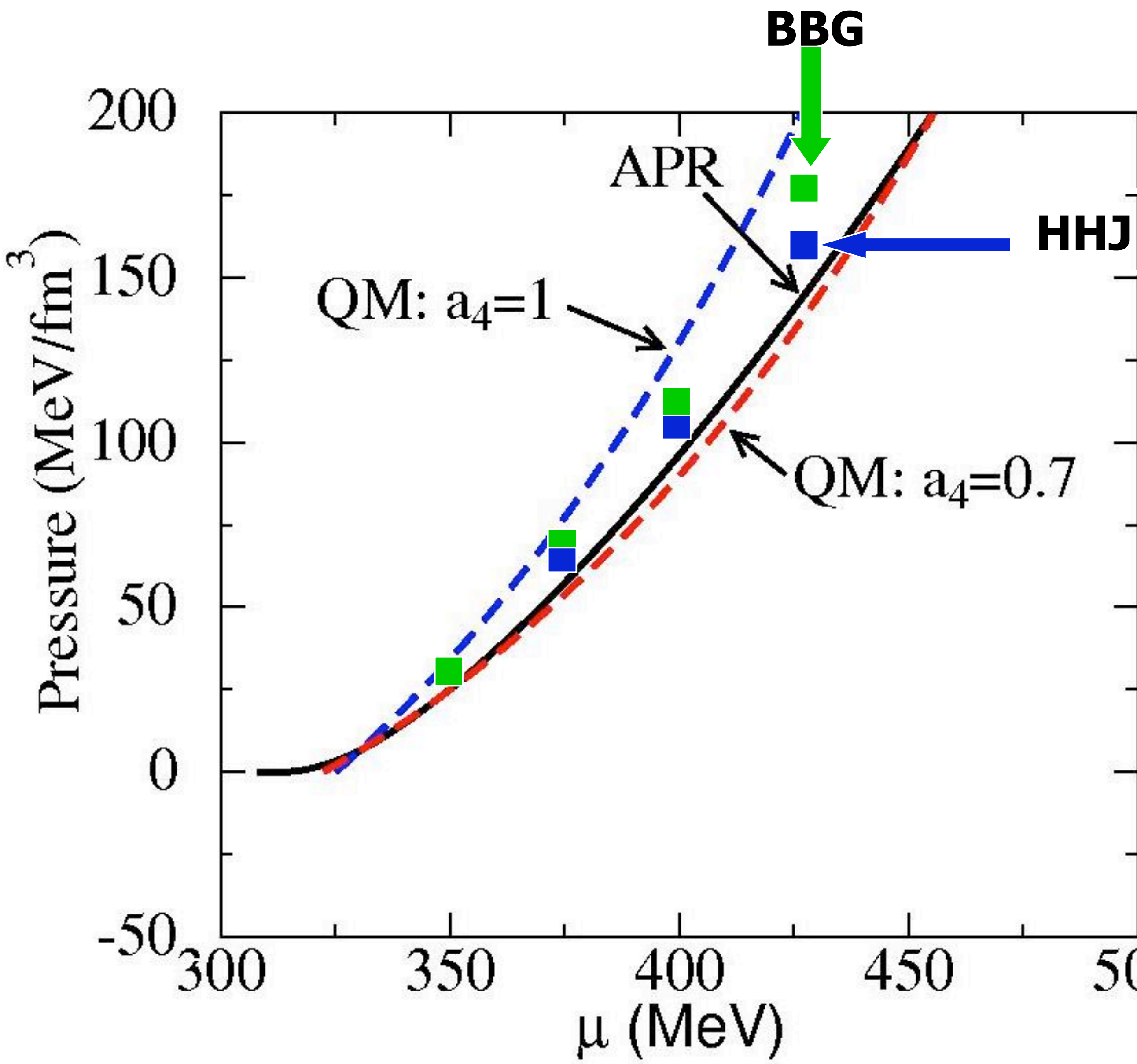
a_2 → Strange quark mass

$a_4 = 1$ corresponds to the usual MIT bag model

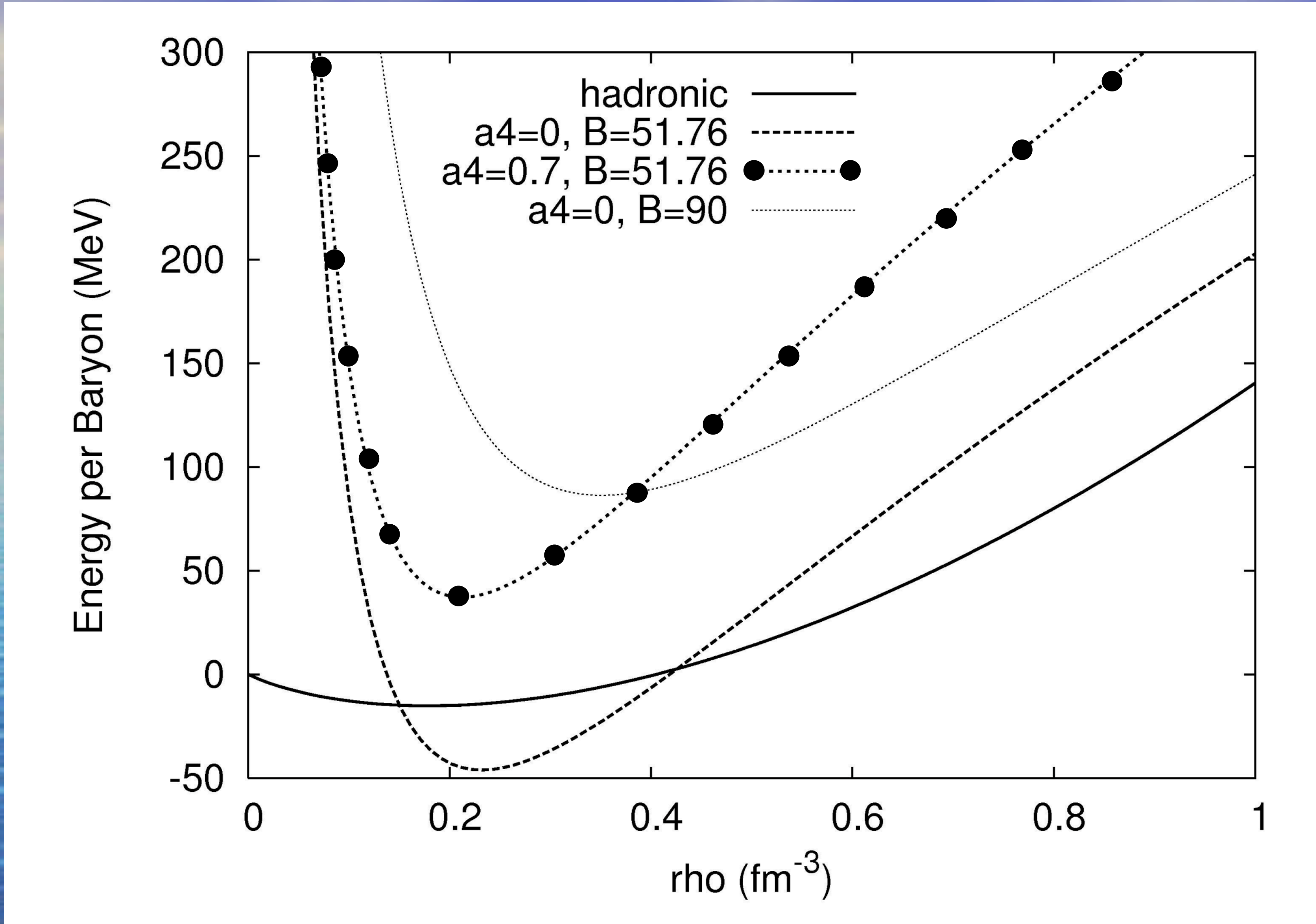
Freedman & McLerran 1978



Maximum mass depends mainly on the parametrization
and not on the transition point



BBG
HHJ
APR
QM: a4=1
QM: a4=0.7
Pressure (MeV/fm3)
μ (MeV)
200
150
100
50
0
-50
300
350
400
450
500



In any case one needs an additional repulsion in Quark matter at high density

NJL Model

Leptonic contribution from electrons and muons and a quark contribution.

$$\mathcal{L}_{eff} = \bar{\psi}(i\partial\!\!\!/ - \hat{m})\psi + \mathcal{L}_{q\bar{q}} + \mathcal{L}_{qq},$$

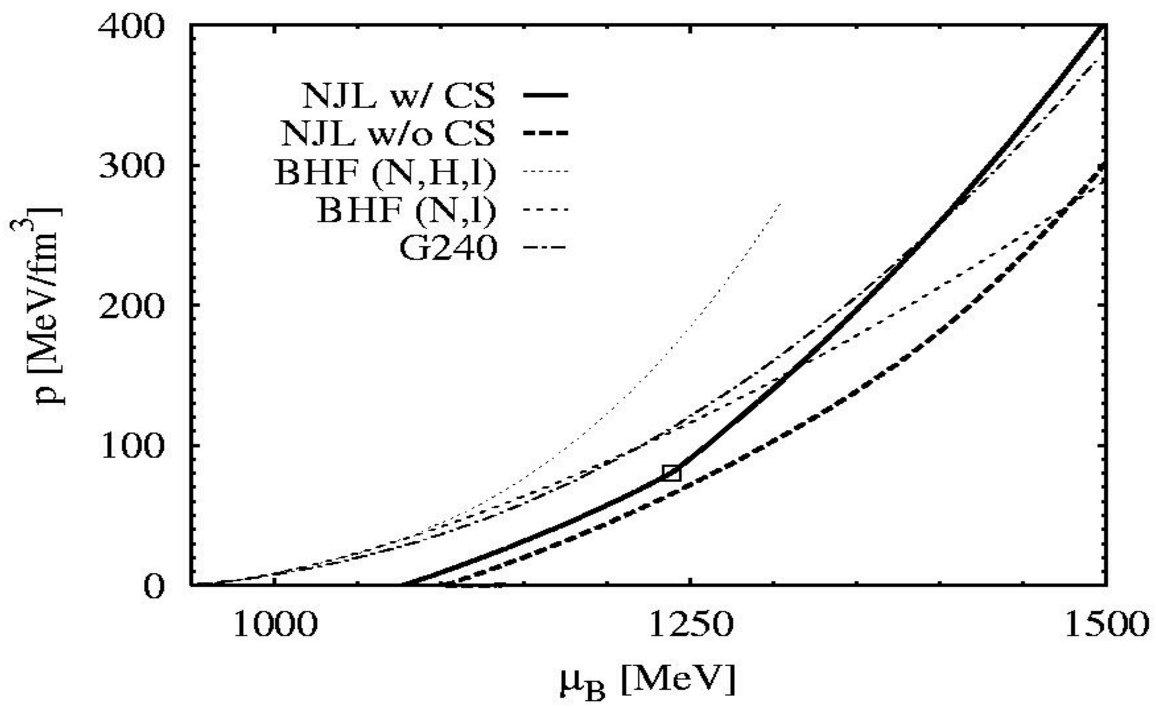
$$\mathcal{L}_{qq} = H \sum_{A=2,5,7} \sum_{A'=2,5,7} (\bar{\psi}\, i\gamma_5 \tau_A \lambda_{A'}\, C\bar{\psi}^T)(\psi^T C\, i\gamma_5 \tau_A \lambda_{A'}\, \psi)\,.$$

$$\mathcal{L}_{q\bar{q}} = G \sum_{a=0}^{8} \left[(\bar{\psi}\tau_a\psi)^2 + (\bar{\psi} i\gamma_5\tau_a\psi)^2 \right] - K \left[\det{}_f\left(\bar{\psi}(1+\gamma_5)\psi\right) + \det{}_f\left(\bar{\psi}(1-\gamma_5)\psi\right) \right].$$

Parameter adjusted to reproduce masses and decay constants of the pseudoscalar meson nonet.

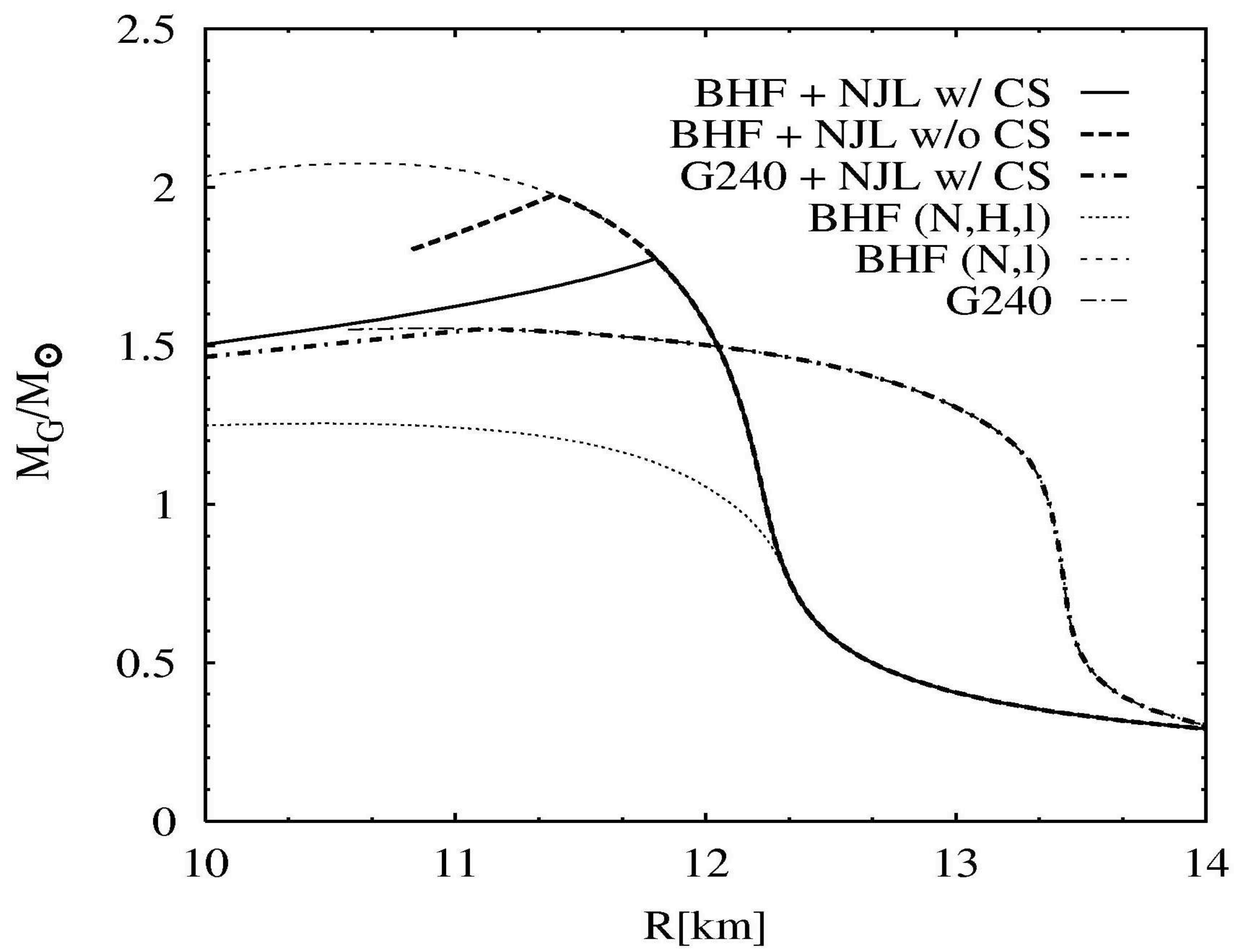
$H = G$

The model is questionable at high density where the cutoff can be comparable with the Fermi momentum



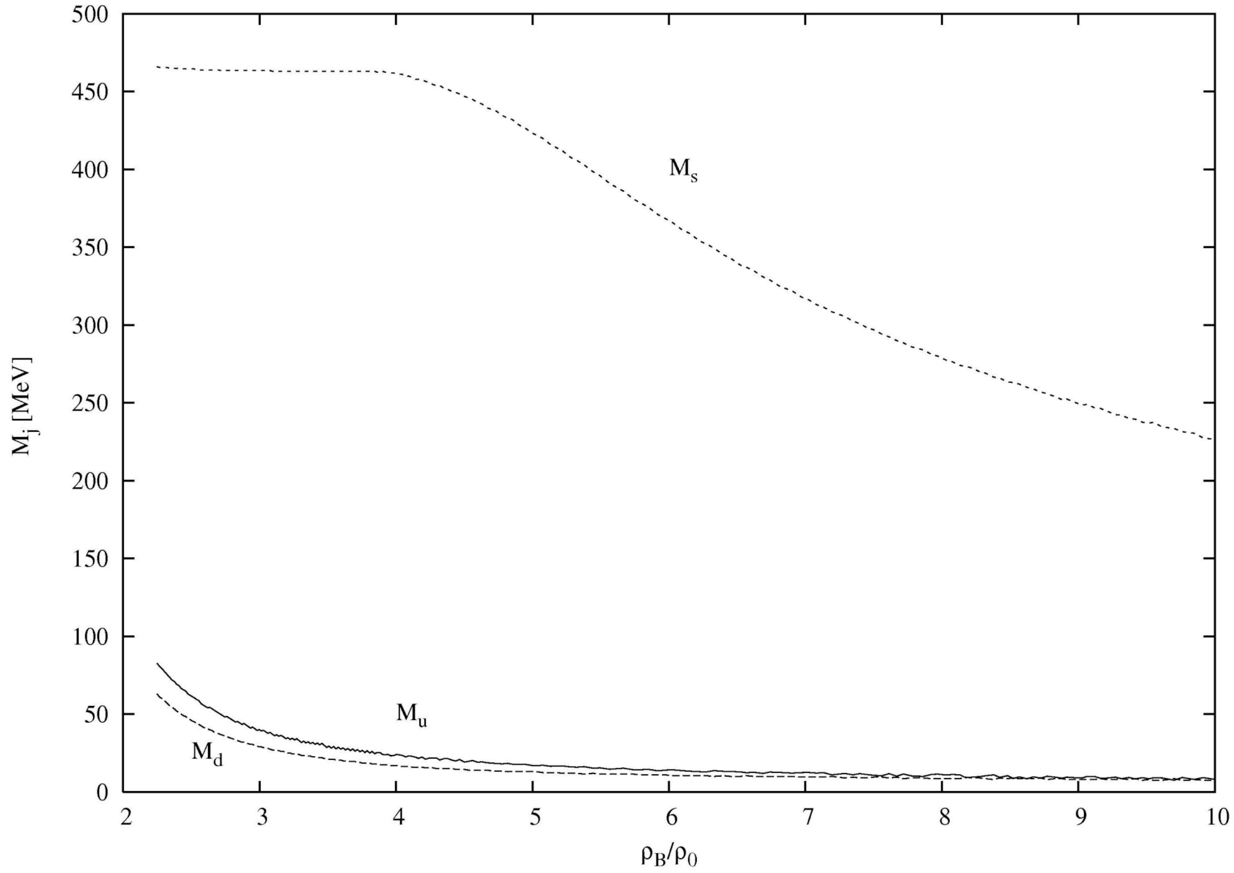
Including Color Superconductivity in NJL
Steiner,Reddy and Prakash 2002
Buballa & Oertel 2002.

Application to NS
CT + GSI , PLB 562,,153 (2003)

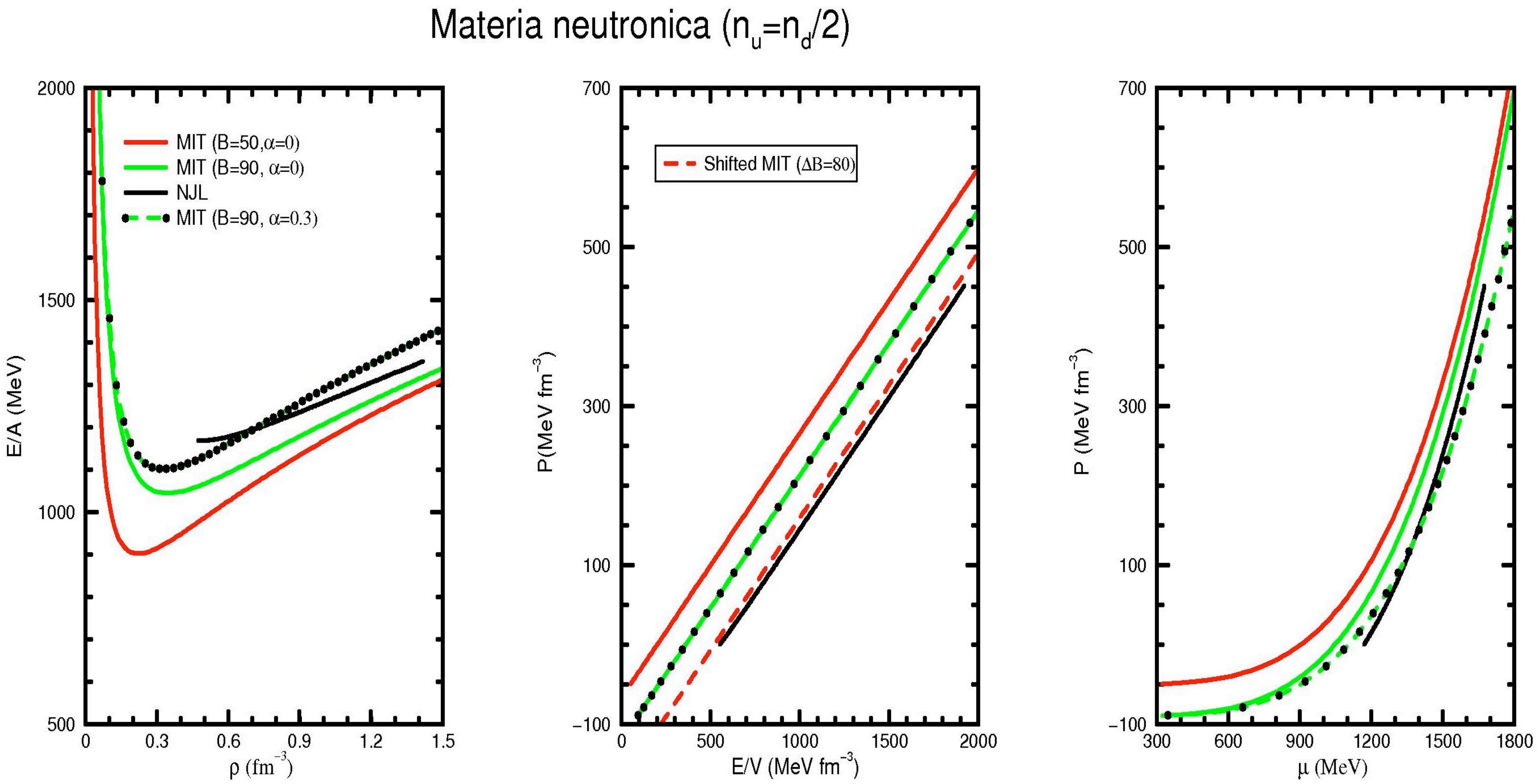


Mass radius relationship

Maximum mass

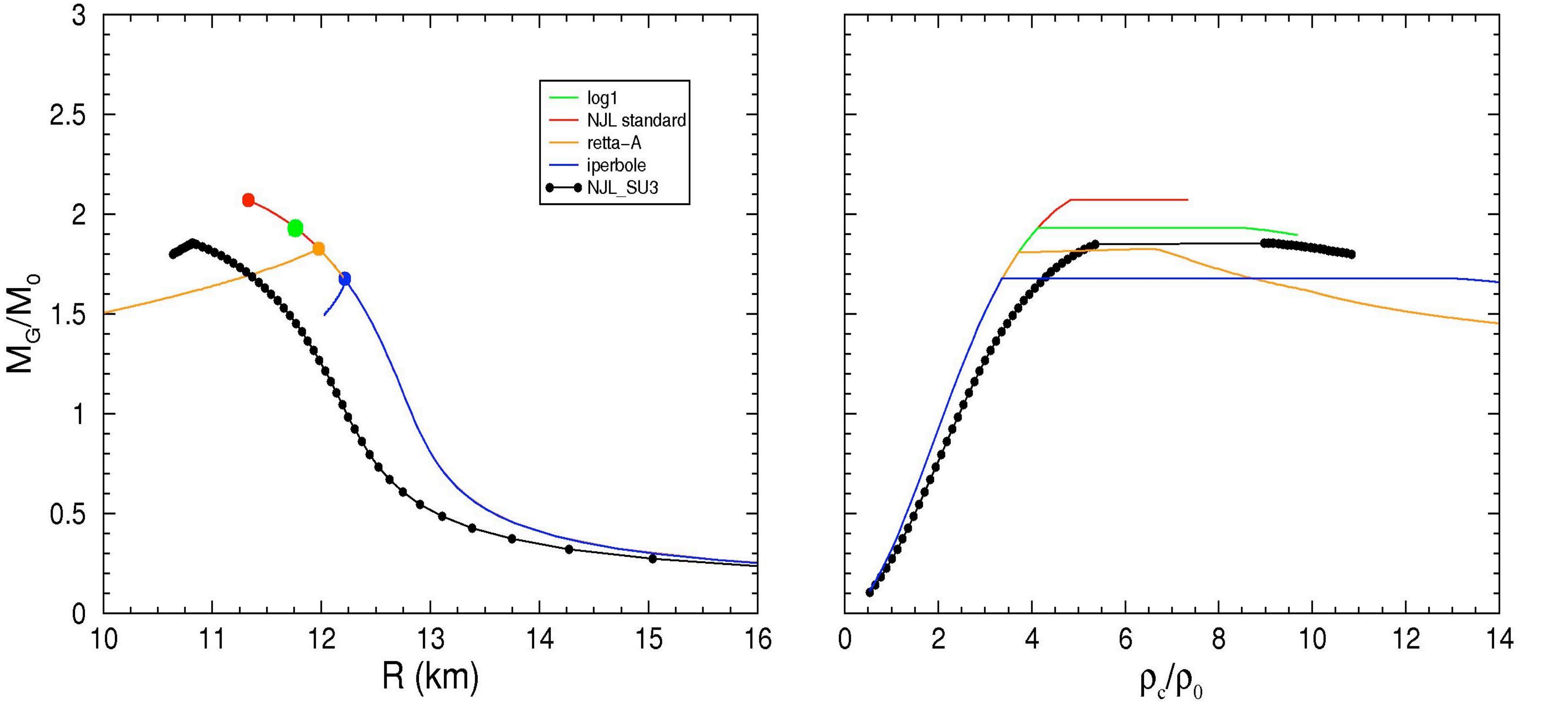


NJL , the quark current masses as a function of density



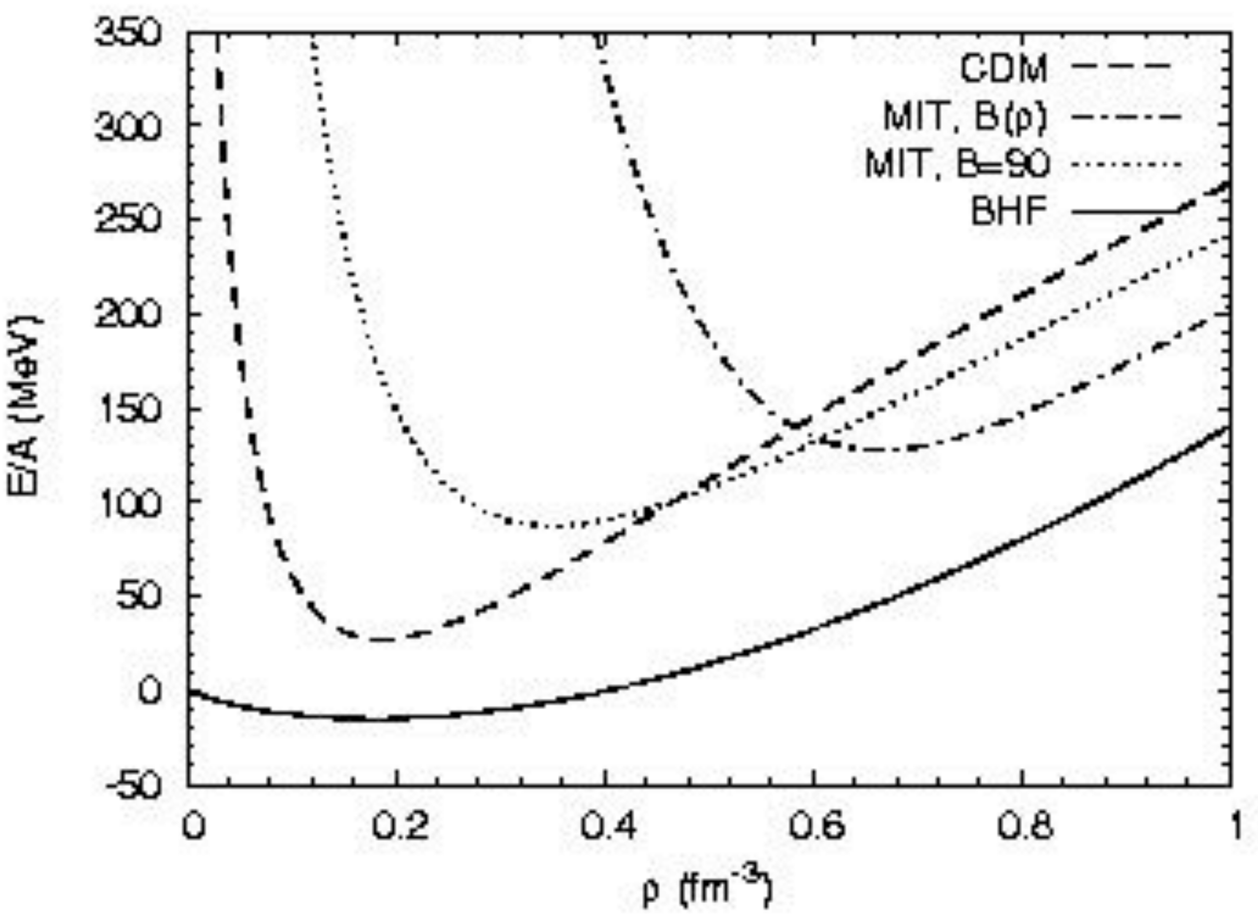
Equivalence between NJL and MIT bag model above chiral transition (two flavours). For NJL B = 170 MeV

The pressure is zero at zero density ! (no confinement)



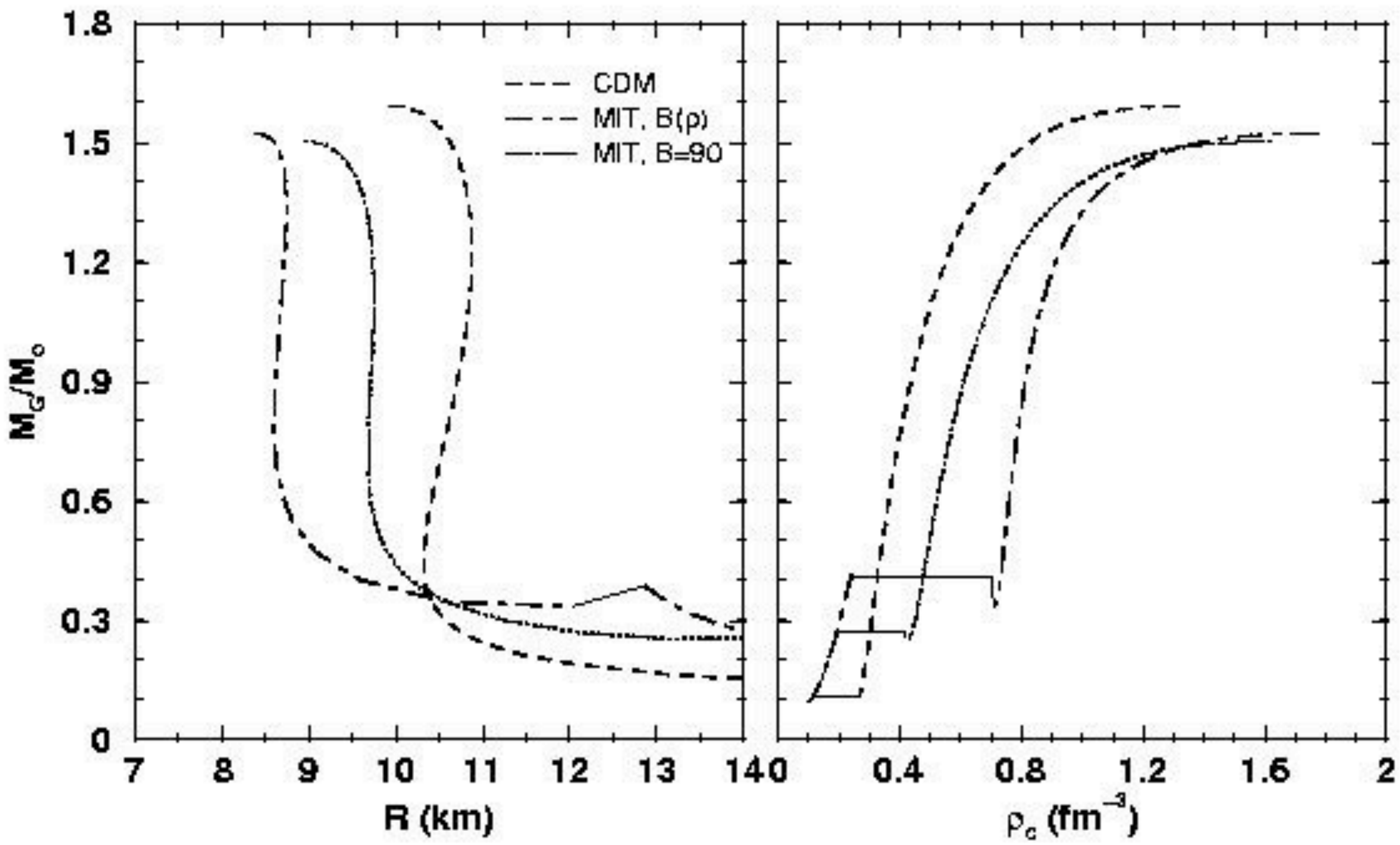
Trying a density dependent cutoff

(Work in progress)

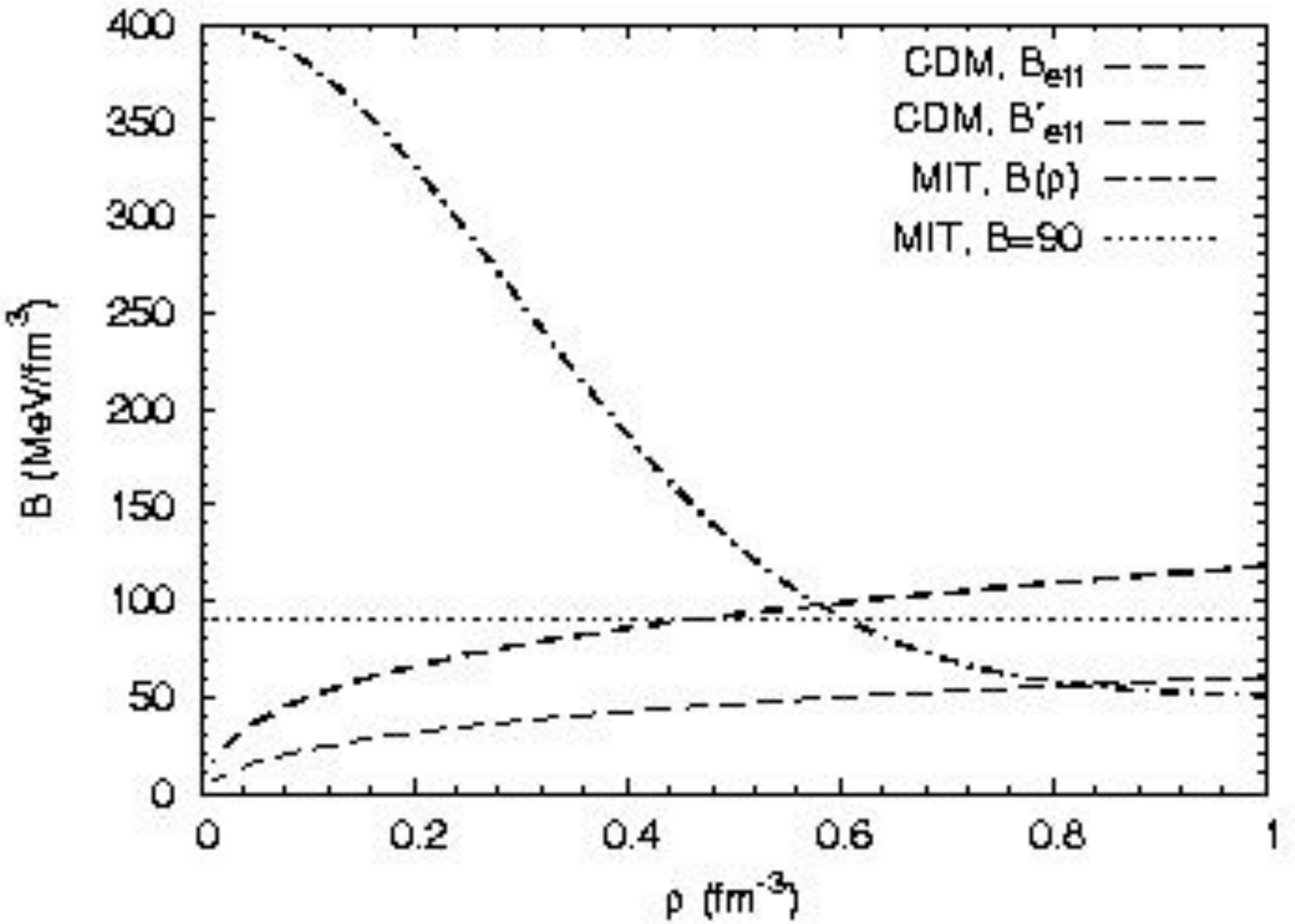


The CDM model : the equation of state for symmetric matter
C. Maieron et al., PRD 70, 043010 (2004)

The model is confining



The CDM model : maximum mass of neutron star



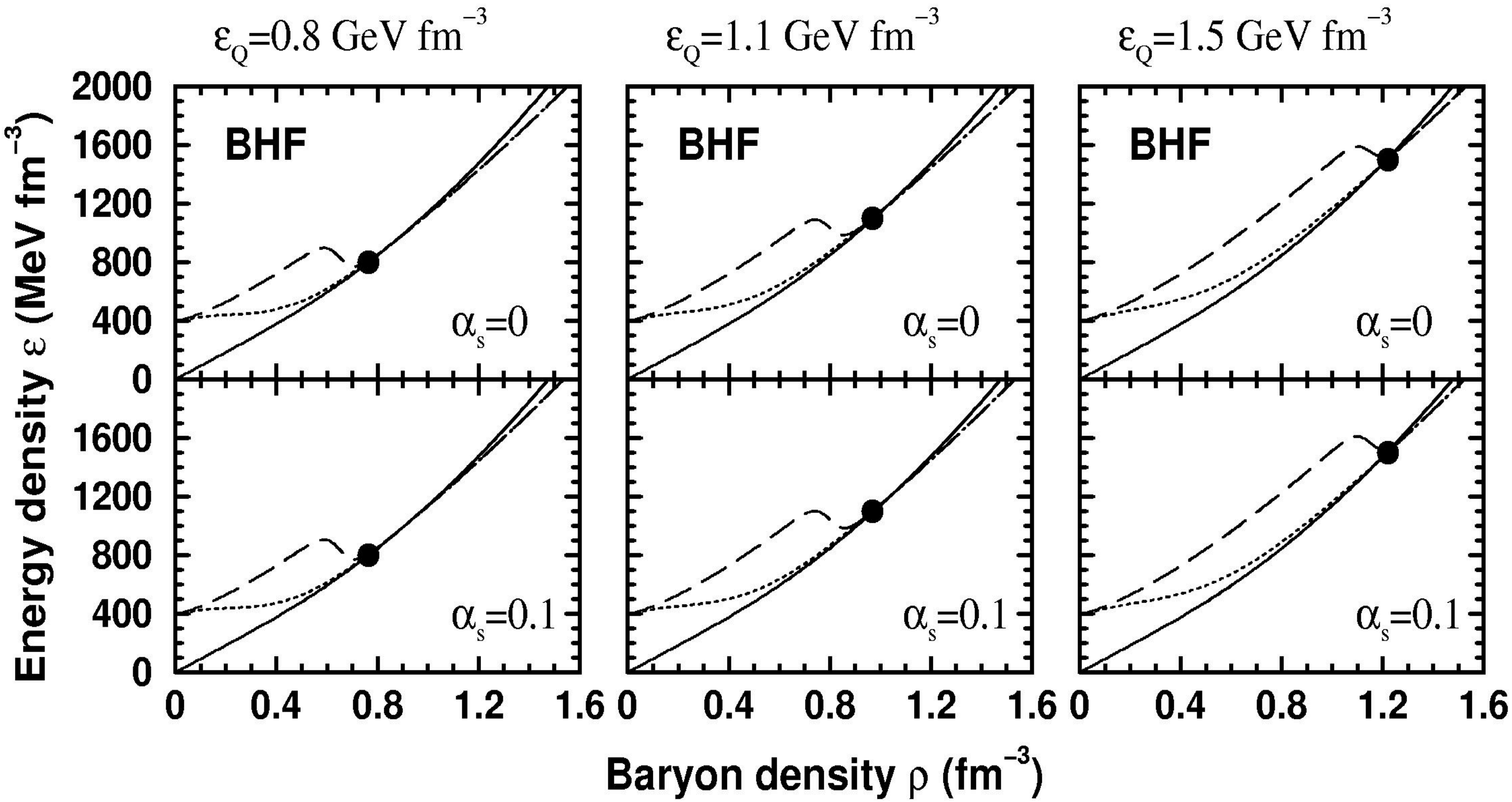
The effective Bag constsnt in the CDM model

Some (tentative) conclusions

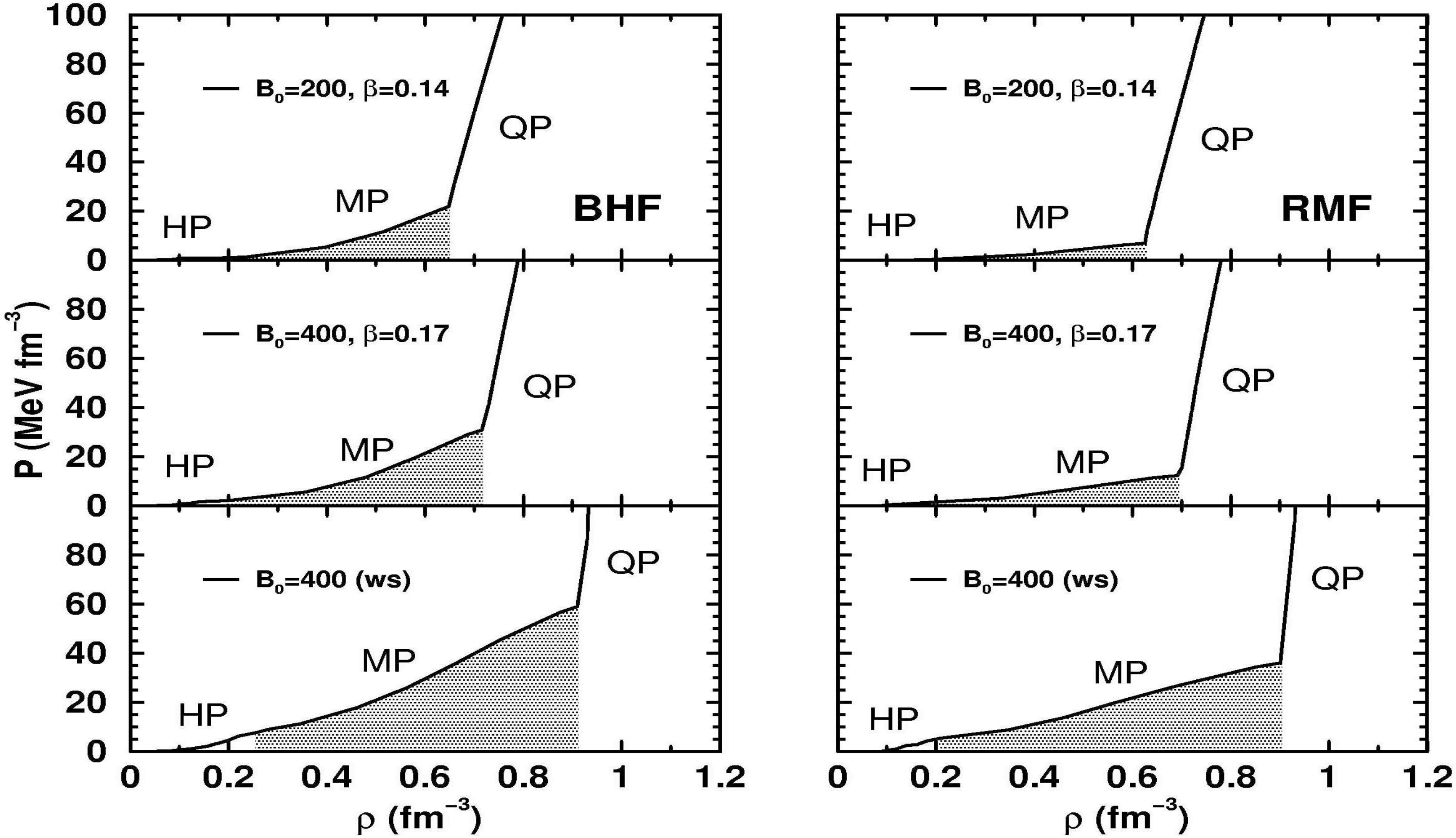
1. **The transition to quark matter in NS looks likely, but the amount of quark matter depends on the quak matter model.**

2. **If the "observed" high NS masses (about 2 solar mass) have to be reproduced, additional repulsion is needed with respect to "naive" quark models .**
   The situation resembles the one at the beginning of NS physics with the TOV solution for the free neutron gas
   The confirmation of a mass definitely larger than 2 would be a major breakthrough

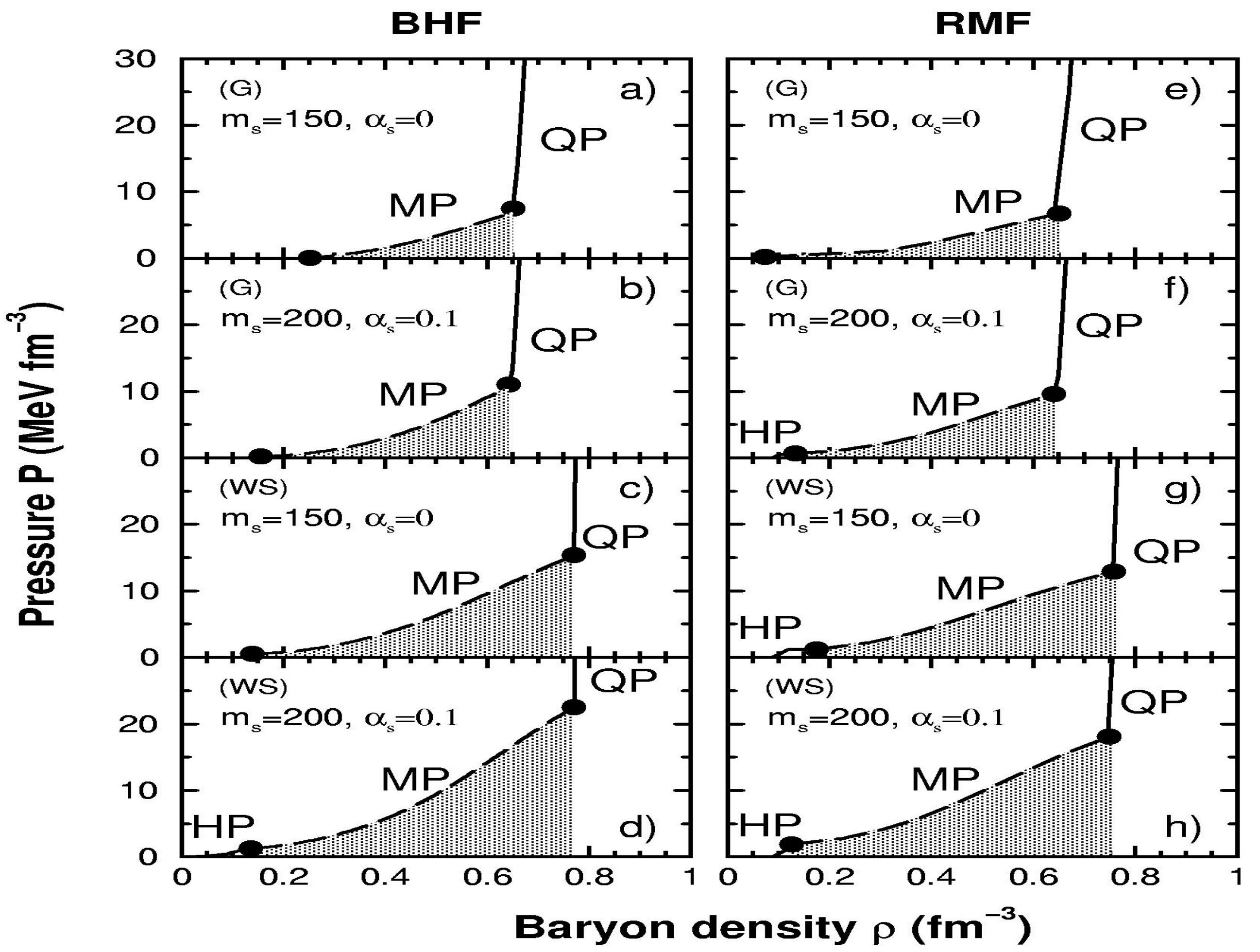
3. **Further constraints can come from other observational data (cooling, glitches ....)**



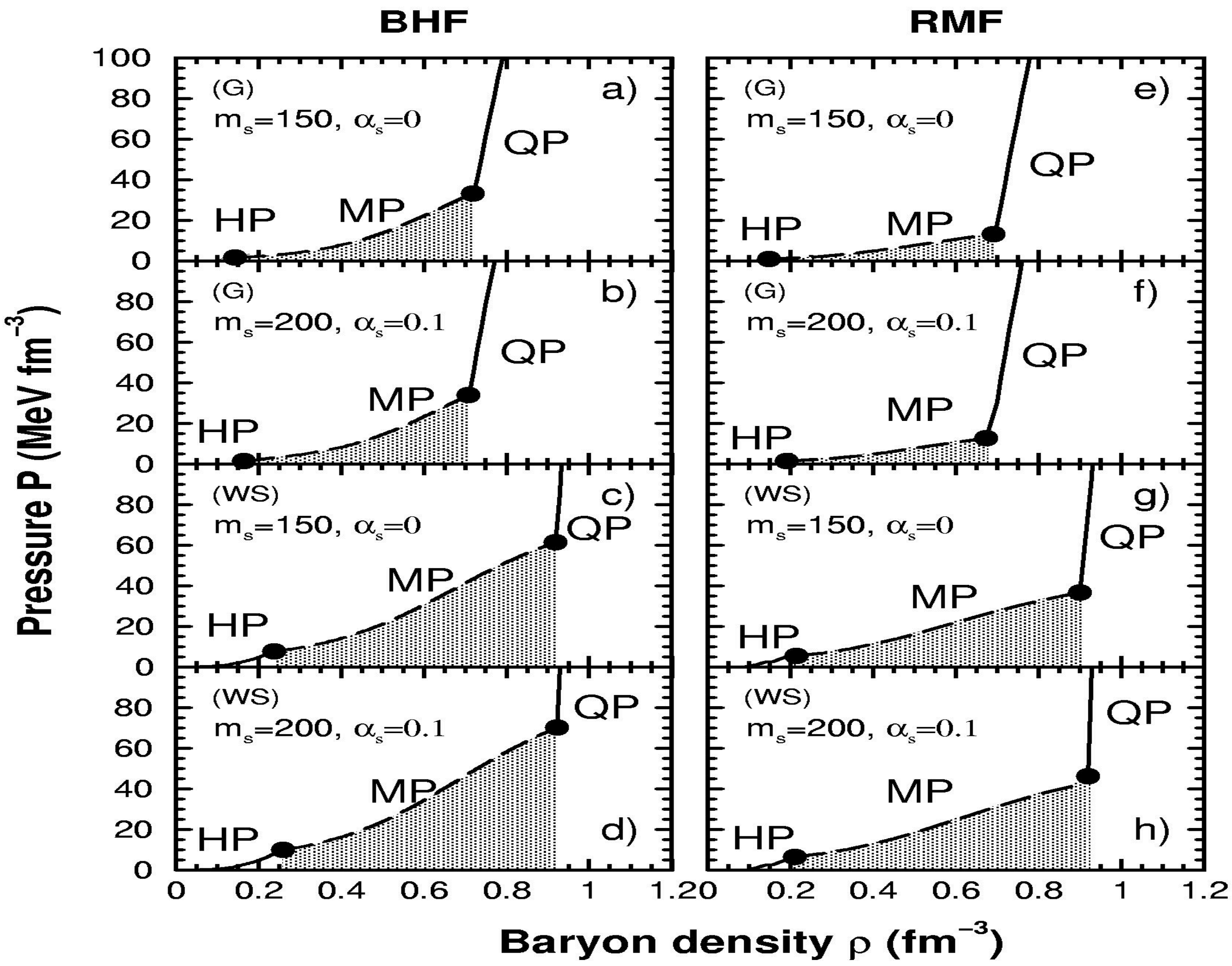
PRC 66, 025802 (2002)



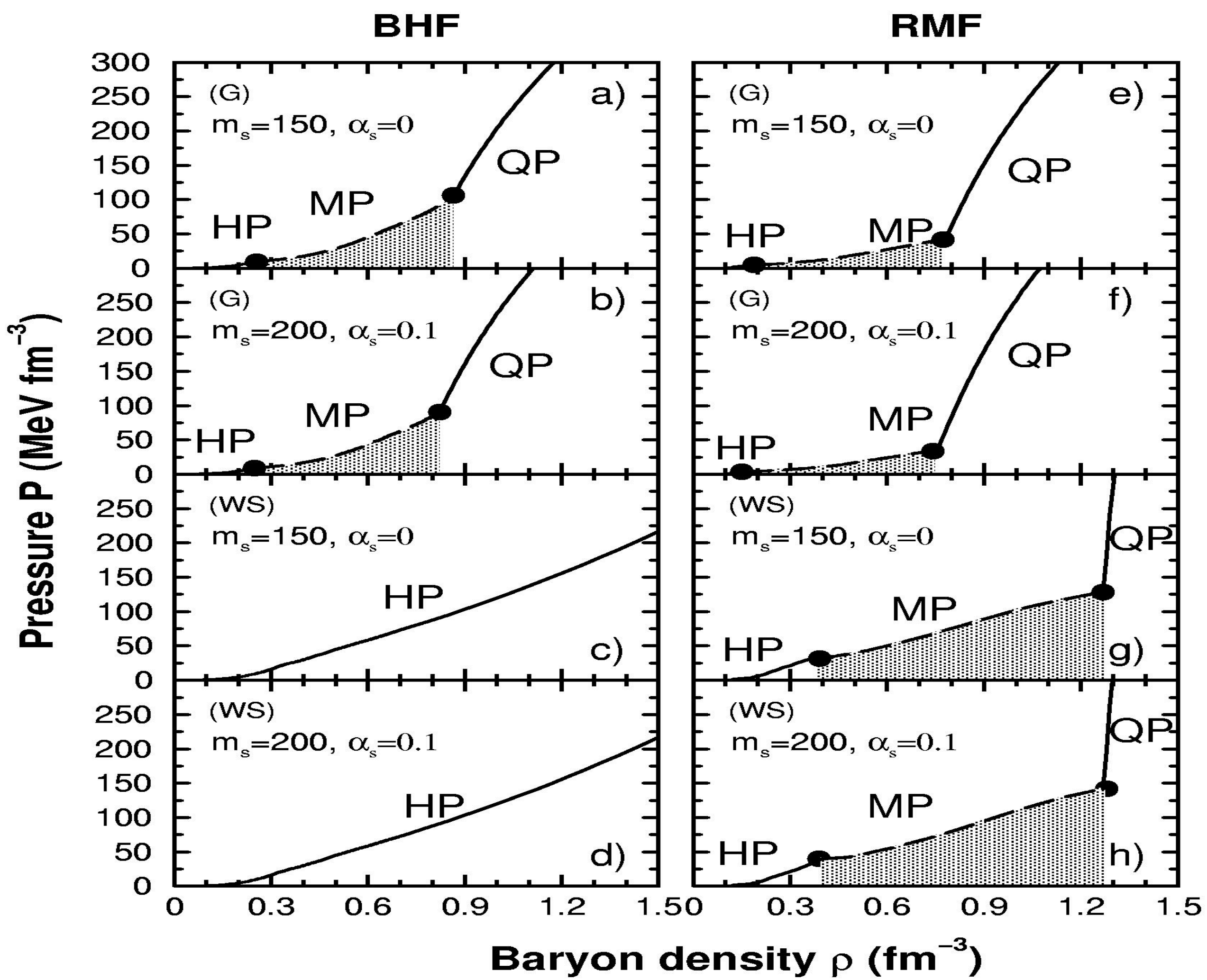
The Equation of State including the mixed phase (Glendenning construction)



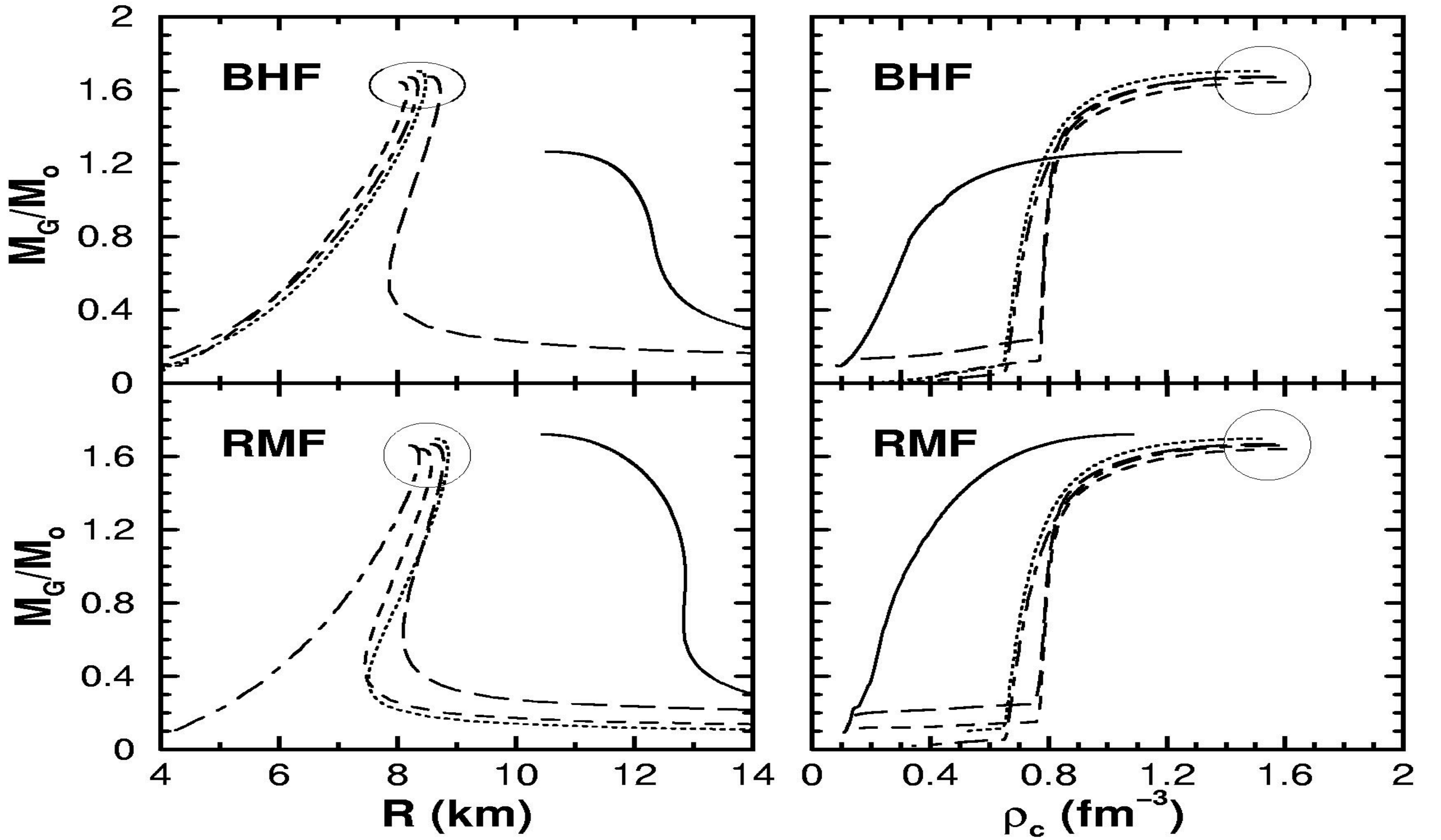
$$\varepsilon_Q = 0.8 \quad GeV\ fm^{-3}$$



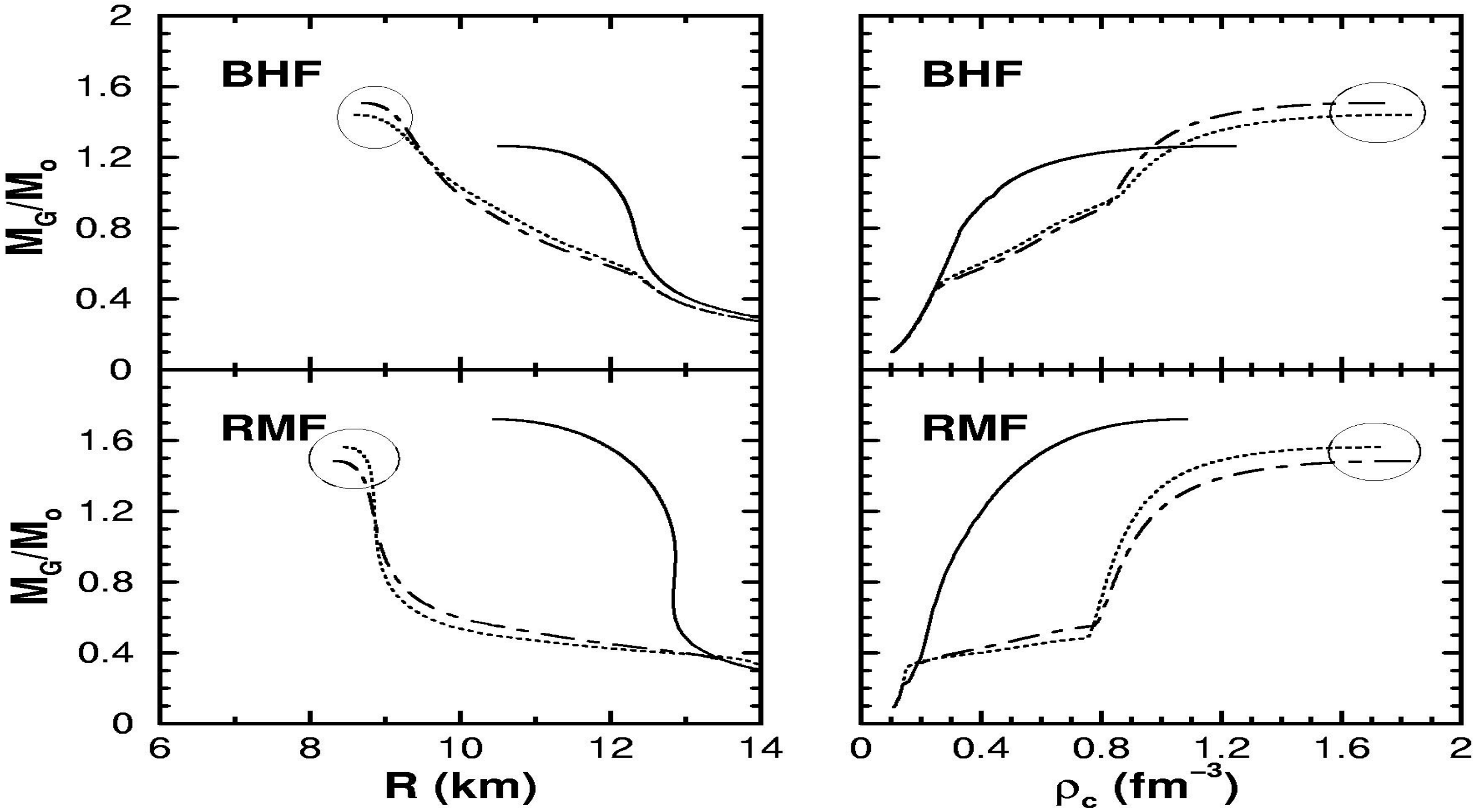
$$\varepsilon_Q = 1.1 \quad GeV\ fm^{-3}$$



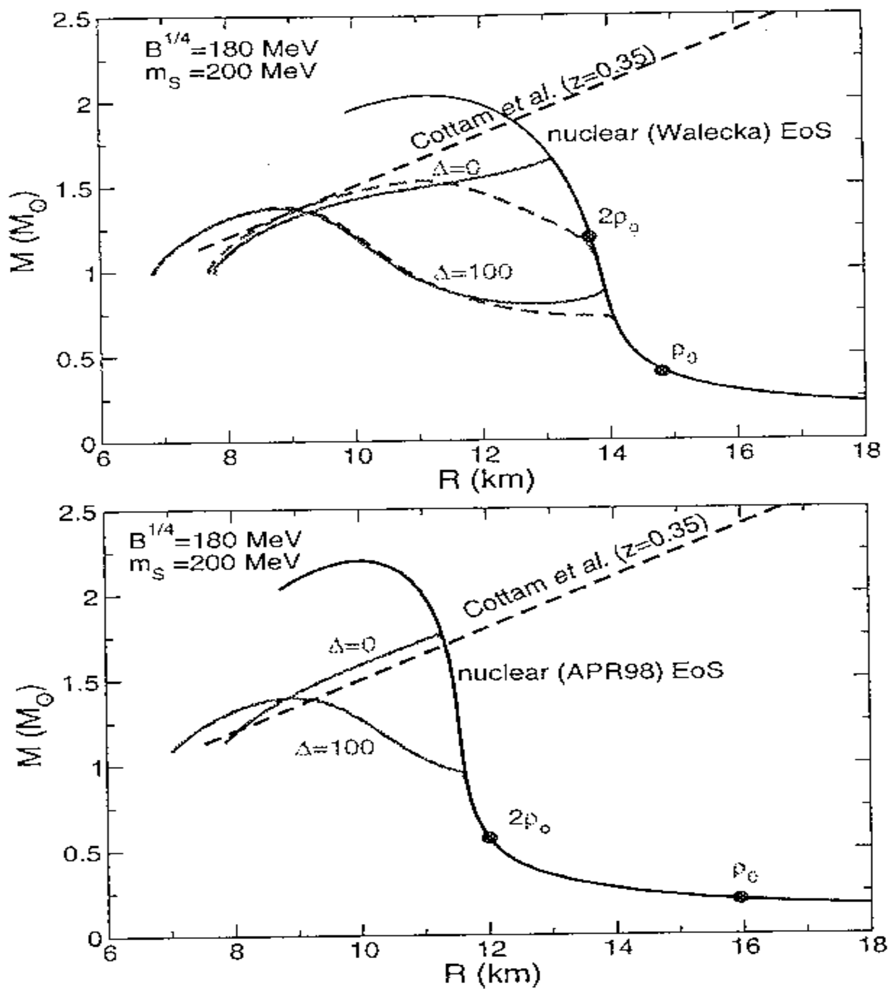
$$\varepsilon_Q = 1.5\ GeV\ fm^{-3}$$



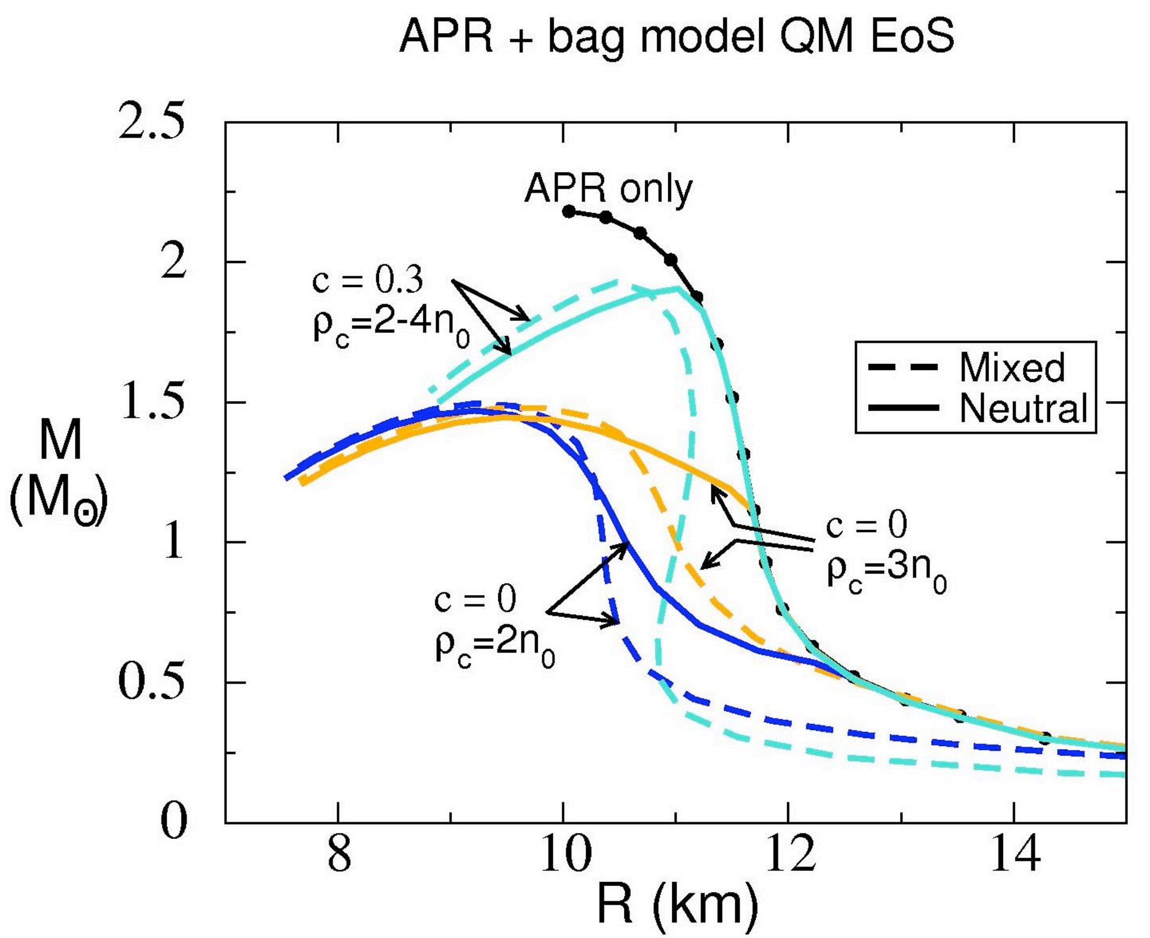
$$\varepsilon_Q = 0.8 \;\; GeV\ fm^{-3}$$

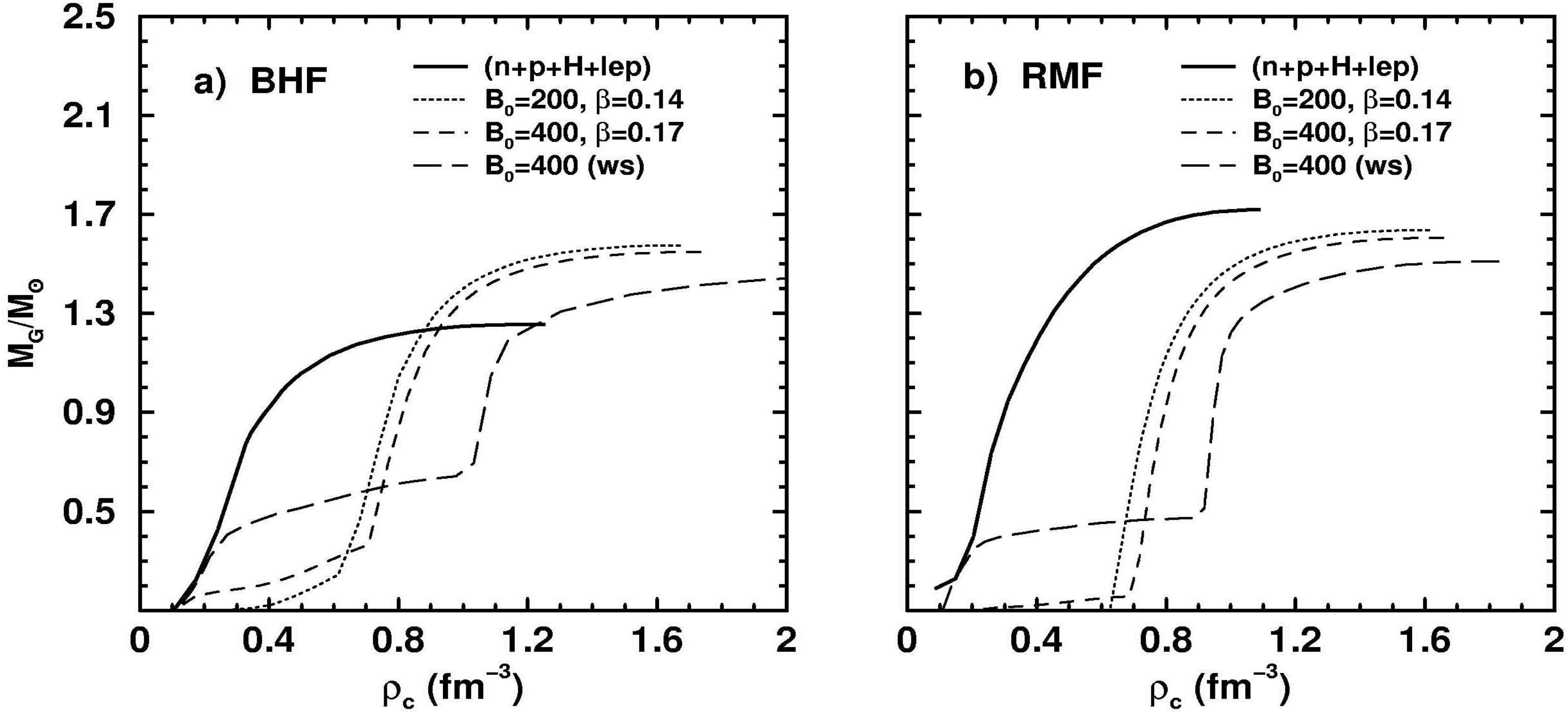


$$\varepsilon_Q = 1.5\ \ GeV\,fm^{-3}$$

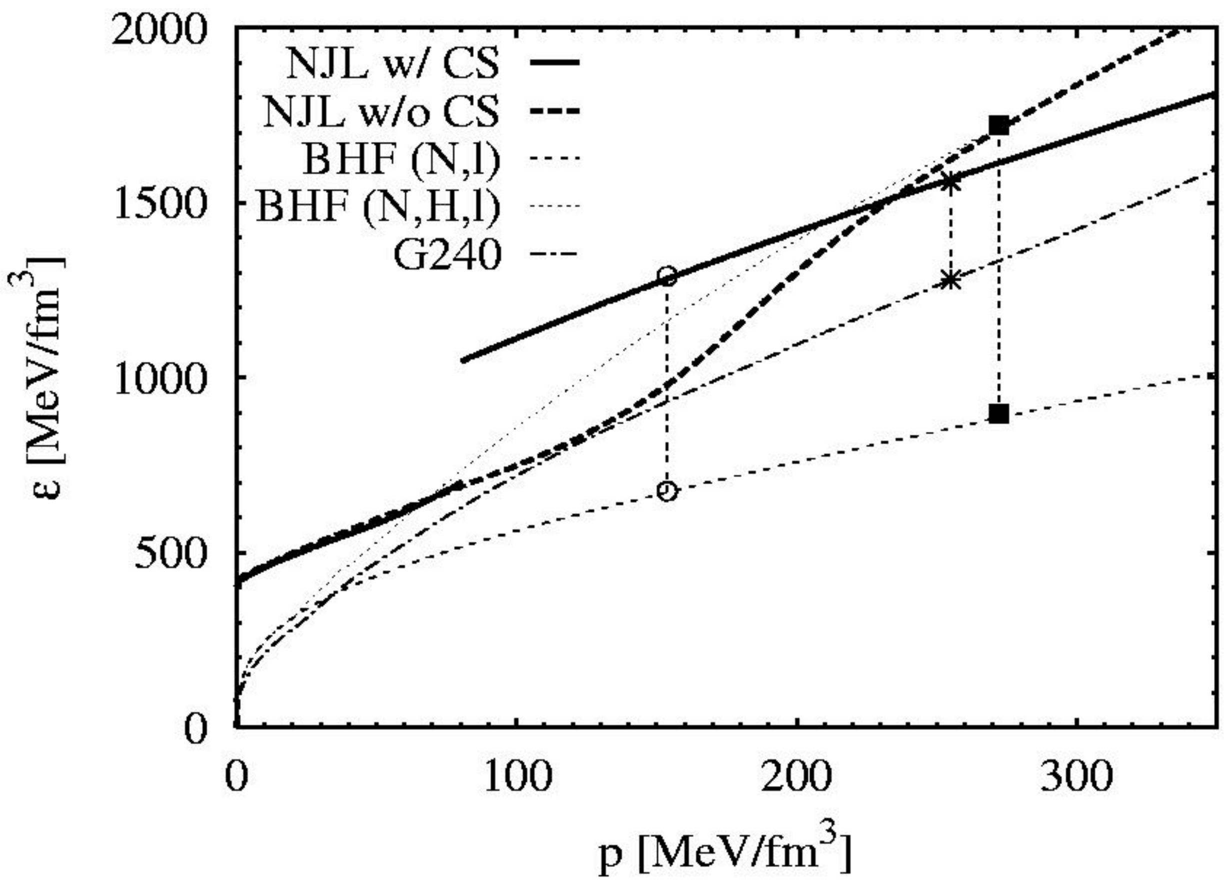


Mass radius relationship
Maximum mass

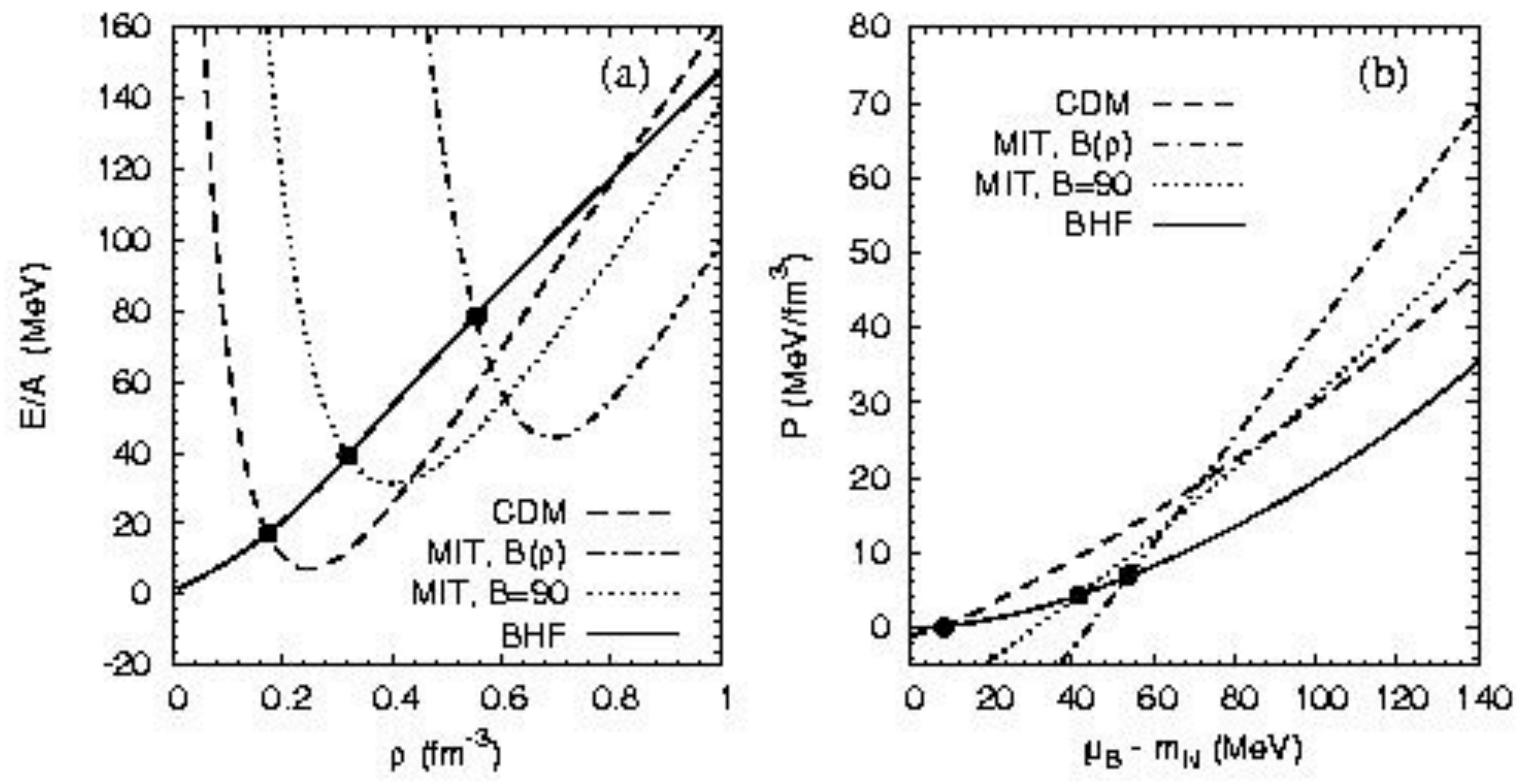

APR + bag model QM EoS
APR only
c = 0.3
ρc=2-4n0
Mixed
Neutral
M (M☉)
c = 0
ρc=3n0
c = 0
ρc=2n0
R (km)
2.5
2
1.5
1
0.5
0
8
10
12
14



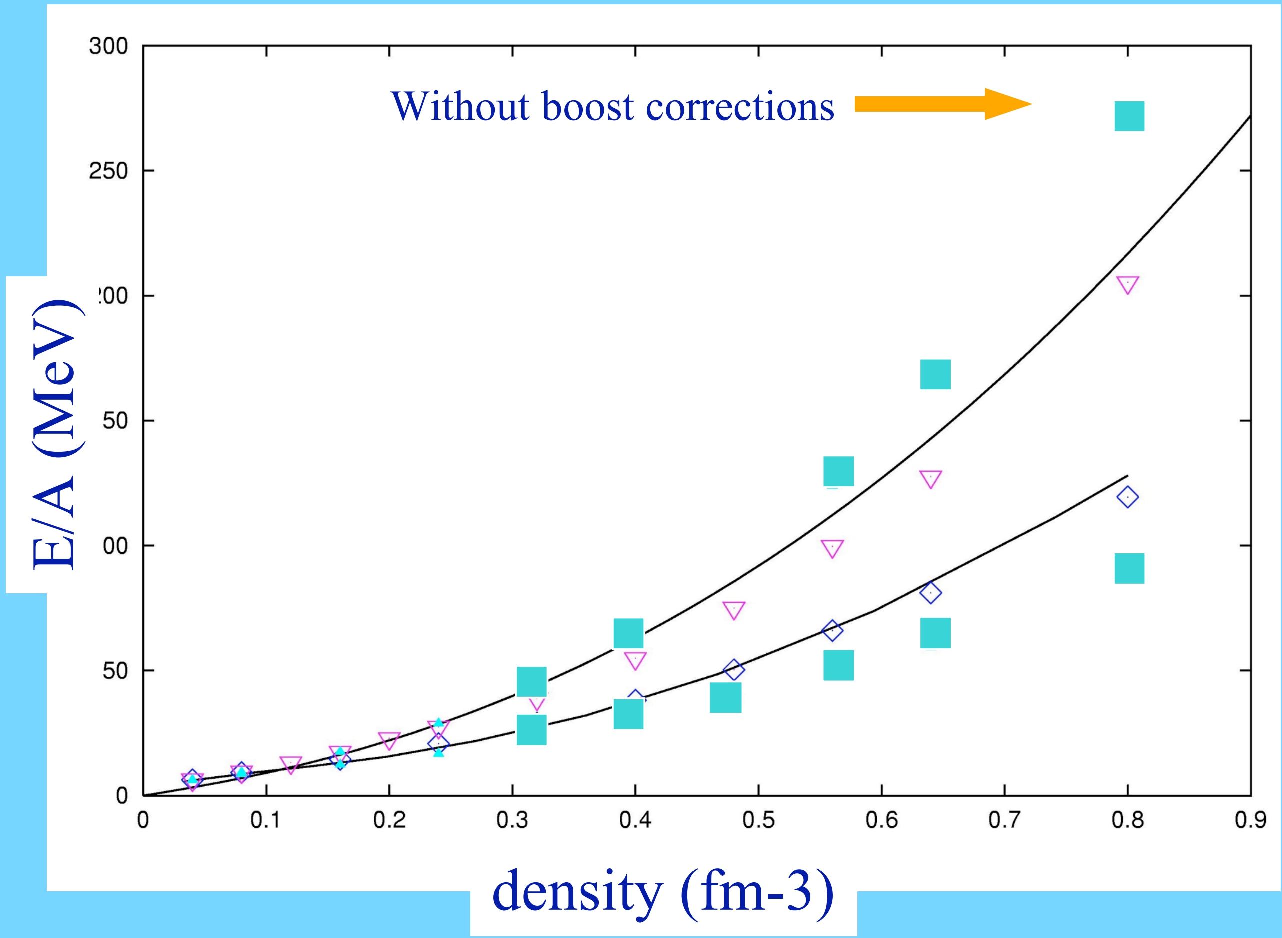
Using Glendenning construction



Transition to quark matter in neutron stars

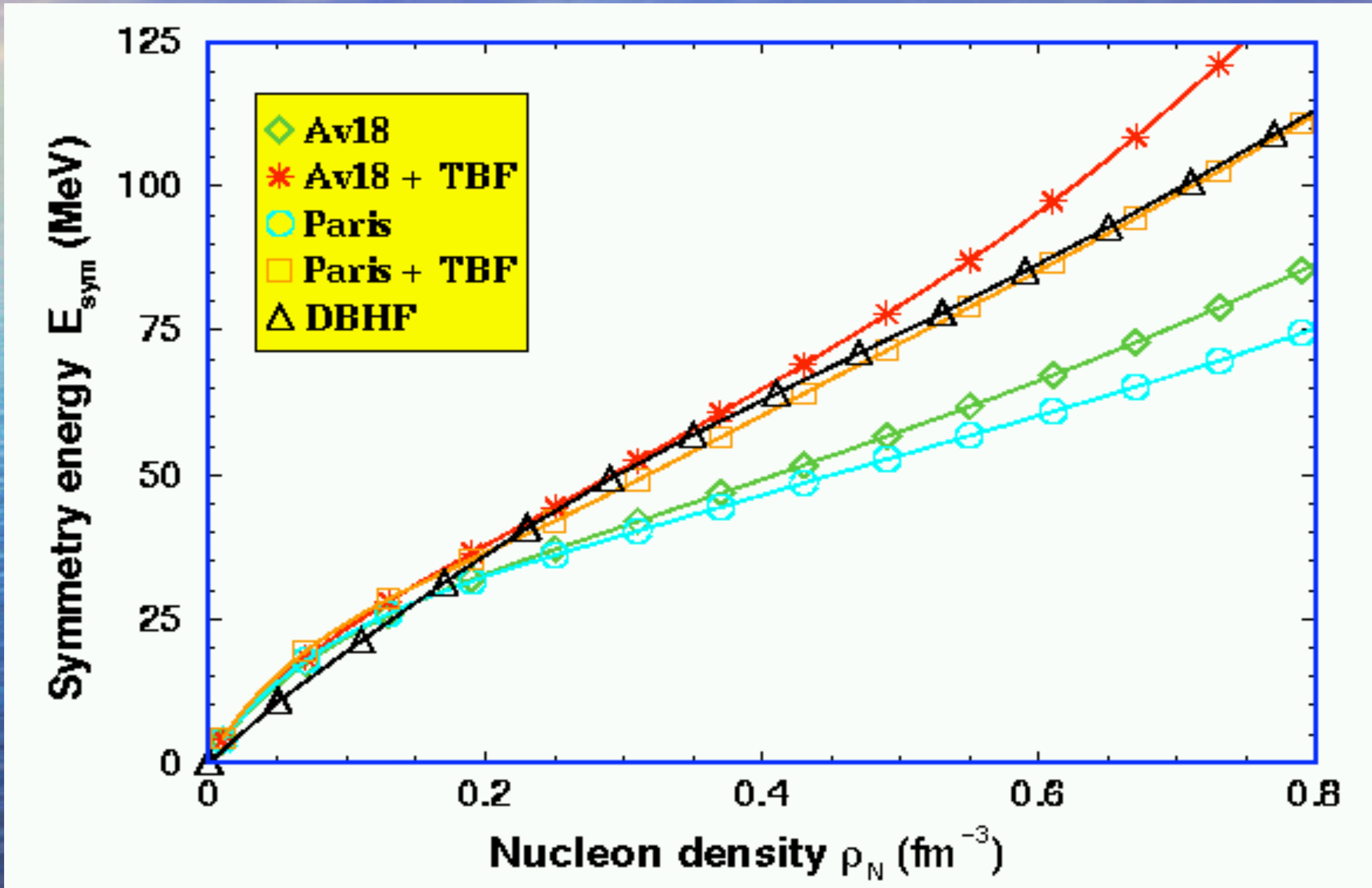


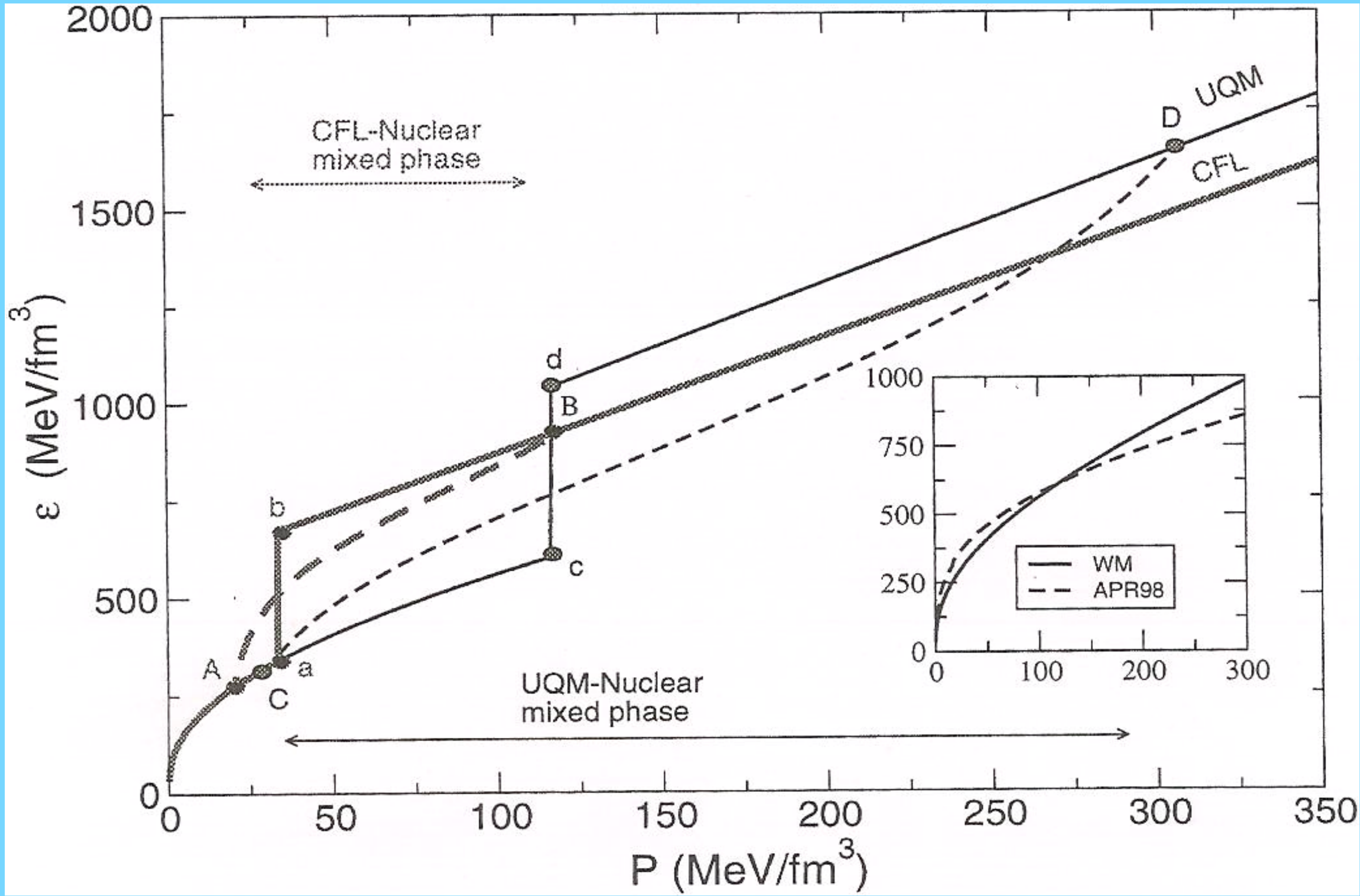
The CDM model : the equation of state in neutron star matter



It looks that if three-body forces produce the correct saturation point, then also neutron matter EoS is, to a large extent, fixed.   TBF can simulate boost corrections.

Symmetry energy





Alford & Reddy PRC 67, 074024 (2003)
Including Color Sup. in MIT bag model
No hyperons in hadronic EOS