Nongeometric Flux Compactifications

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Strings see spacetime very differently than point particles do!

- **Singularity Resolution**
- **T-duality**
- **Mirror Symmetry**

In particular, the last two on this list describe an equivalence between naively **unrelated** geometries.

Is it possible to extend this to relate geometries to something else entirely, but which are still perfectly good string compactifications?

In this talk, I will do exactly that:
Use known symmetries of string theory to motivate the existence of good string compactifications which are not describable as geometry, i.e. "**nongeometric**."

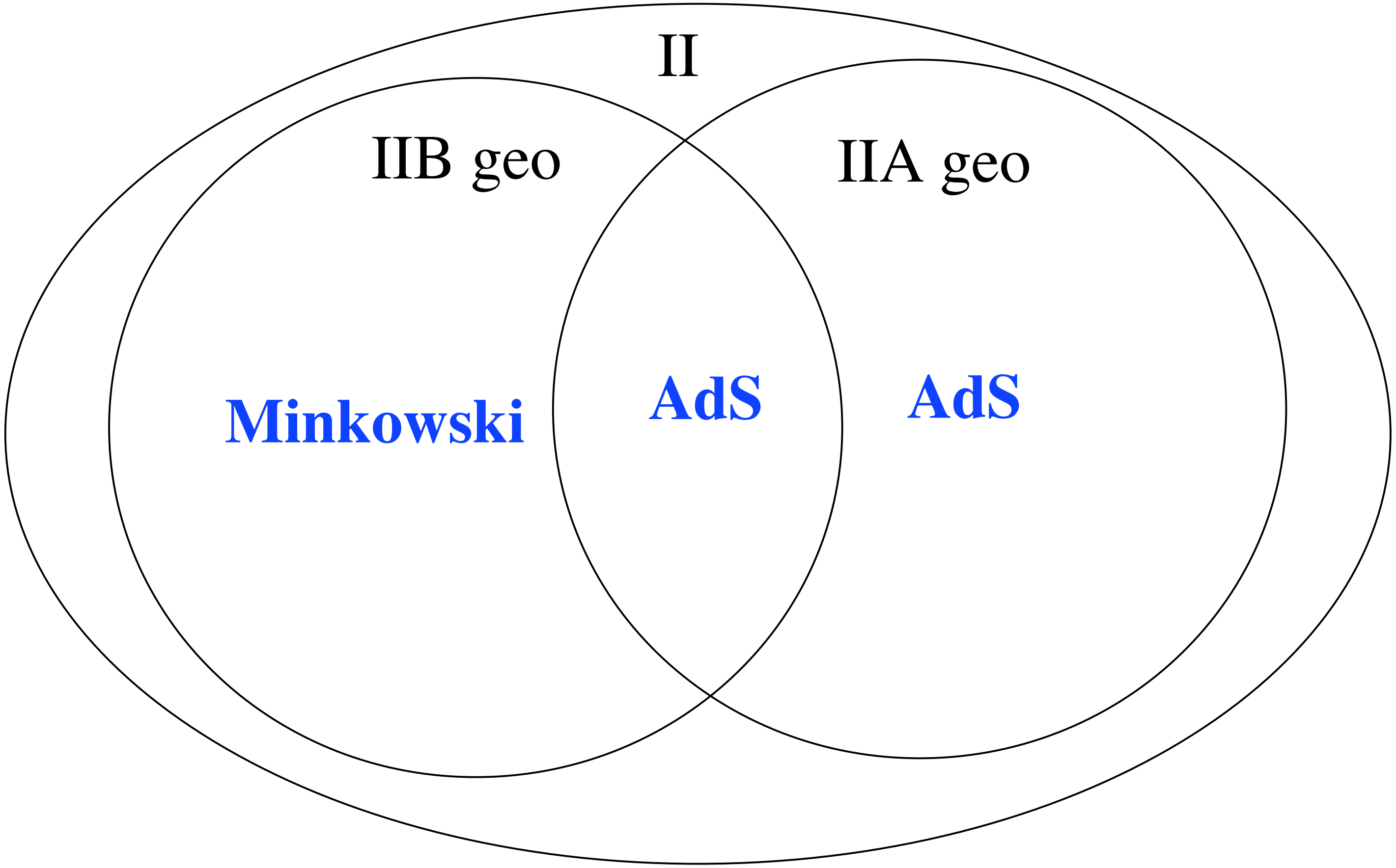
Nongeometric backgrounds have been studied for a while now.

	Asymmetric Orbifolds	**Narain, Sarmadi, Vafa**
1987!	**T-folds/Monodrofolds**	**Hull, Dabholkar, Hellerman, Walcher, Williams, Flournoy, Lawrence, Schulz, BW**
	Other Constructions	**Hellerman, McGreevy, Williams**

In this talk, I will discuss how nongeometric backgrounds fit **naturally** into the flux compactification story.

The talk is aimed at a non-string theory audience. String theorists may be bored, insulted, or ideally, both.

The goal of the work I'll describe was to resolve the following puzzle for a T^6/Z_2 :



Where are the IIA Minkowski vacua?

Outline

I. Review of T-duality

II. T^3 with NS Flux

III. The $T^6/\mathbf{Z}_2$ Effective Theory

IV. Equations of Motion and Solutions

V. Interpretation of Nongeometric Fluxes

VI. Open Questions

I. Review of T-duality

T-duality is an important symmetry of string theory.
Let's briefly review what it is.

The simplest form is that string theory on a circle of radius R is **equivalent** to string theory on a circle of radius 1/R.

The spectrum of a string compactified on a circle is

$$m^2 = \frac{n^2}{R^2} + w^2 R^2 + 2(N + \widetilde{N} - 2)$$

momentum

strings stretched around circle ("winding number")

excited oscillators

This is invariant under $n \iff w, \;\; R \to 1/R$

We can simply restate the inversion of the radius as

$$\boxed{G_{xx} \to \frac{1}{G_{xx}}}$$

(circle is in the *x* direction)

But a general string theory background has more stuff in it:

$$\textbf{NS-NS sector} \left\{ \begin{array}{ll} \text{metric} & G_{\mu\nu} \\ \text{two-form} & B_{\mu\nu} \\ \text{dilaton} & \Phi \end{array} \right.$$

Can we generalize T-duality to a more complicated background?

Yes!

(Buscher, 1987)

say, in the *x* direction

The "Buscher rules" say that anytime we have an isometry, we can get a sigma model equivalent to the original one by taking

$$G'_{xx} = \frac{1}{G_{xx}} \qquad G'_{ab} = G_{ab} - \frac{G_{xa}G_{xb} - B_{xa}B_{xb}}{G_{xx}}$$

$$G'_{xa} = -\frac{B_{xa}}{G_{xx}} \qquad B'_{ab} = B_{ab} - \frac{G_{xa}B_{xb} - B_{xa}G_{xb}}{G_{xx}}$$

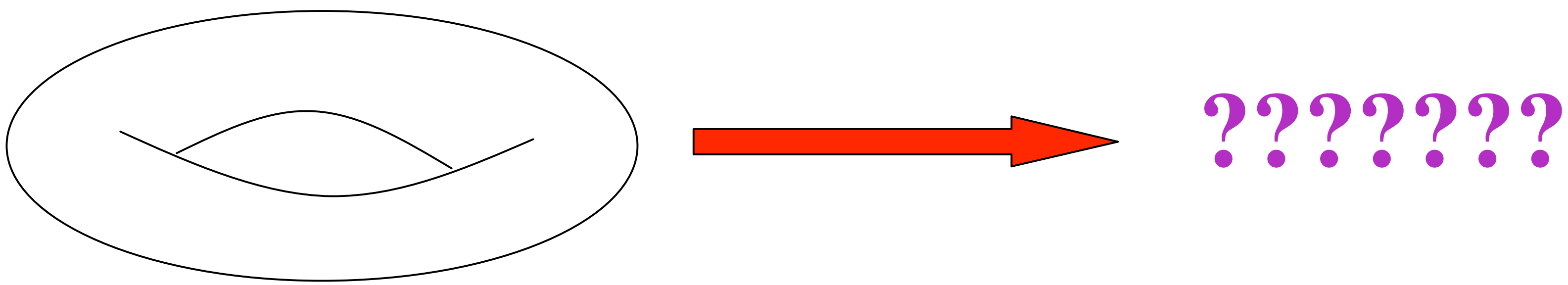
$$B'_{xa} = -\frac{G_{xa}}{G_{xx}} \qquad e^{2\Phi'} = \frac{e^{2\Phi}}{G_{xx}}$$

Notice that some components of *G* and *B* switch!

The point: Given a string theory background, we can use the Buscher rules to produce new backgrounds!

And because *G* and *B* can mix in with each other, sometimes these backgrounds will be something weird.

Let's apply the Buscher rules to a simple example and see what happens!

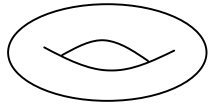


II. T^3 with NS Flux

Let's start with a warm-up example.

NB: I do not solve the string EOM.

Step 1: A flat three-torus with *N* units of NS-NS flux.



$$ds^2 = dx^2 + dy^2 + dz^2$$

$$(x, y, z) \sim (x+1, y, z) \quad \text{etc.}$$

$$\int_{T^3} H_3 = N$$

$$H = dB$$

We choose $B_{xy} = N z$ to accomplish this.

Notice that nothing depends on *x* or *y*, so we have directions on which we can perform **T-duality.**

Let's T-dualize in the *x* direction!

Kachru, Schulz, Tripathy, Trivedi

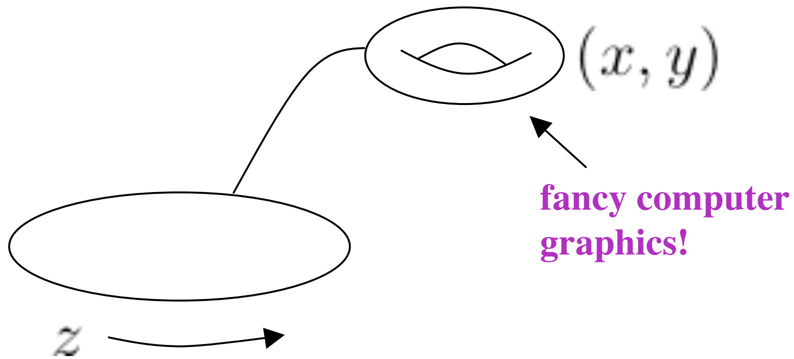
Step 2: The Buscher rules take our ***B*-field** into an off-diagonal component of the **metric**.

$$\boxed{\begin{gathered} ds^2 = (dx + Nz\,dy)^2 + dy^2 + dz^2 \\ B = 0 \end{gathered}}$$

We can make the metric globally well-defined by choosing

$$(x, y, z) \sim (x - Ny, y, z + 1) \sim (x, y + 1, z) \sim (x + 1, y, z)$$

We can usefully think of this as a torus fibered over a circle, where the coordinates of the fiber mix upon going around the base.



$$\tau \rightarrow \tau - N \text{ as } z \rightarrow z + 1$$

complex structure

This is an example of a **twisted torus.**

We can write this metric as

$$ds^2 = (dx + f^x_{\ yz} z dy)^2 + dy^2 + dz^2$$

This is just *N* in this case.

f is often referred to as **geometric flux** and characterizes the twisting in the twisted torus.

T-duality took NS **3-form flux** to **geometric flux**!

$$H_{xyz} \xrightarrow{T_x} f^x_{\ yz}$$

Can really think of this as a 2-form

And there's still another direction in which we can do T-duality.

Let's go for it.

Step 3: T-dualize in the y direction.

$$ds^2 = \frac{1}{1+N^2z^2}(dx^2+dy^2)+dz^2$$

$$B_{xy} = \frac{Nz}{1+N^2z^2}$$

What *is* this background?

It's not too hard to see what's going on here:

$$\rho = B + iV = \frac{Nz+i}{1+N^2z^2} \Rightarrow \frac{1}{\rho} = Nz - i$$

Kahler

So as $z \longrightarrow z+1, \; \frac{1}{\rho} \rightarrow \frac{1}{\rho} + N$

This is **nongeometric**, since $\dfrac{1}{\rho} \longrightarrow \dfrac{1}{\rho} + N$ mixes B and G.

But it's not *that* bad – the background is **locally** geometric, but not **globally** geometric.

In this background, the integer N still shows up, but has a different interpretation.

$$H_{xyz} \xrightarrow{T_x} f^{x}_{yz} \xrightarrow{T_y} Q^{xy}_{z}$$

geo **geo** **nongeo**

Can we go further?

$$H_{xyz} \xrightarrow{T_x} f^{x}_{yz} \xrightarrow{T_y} Q^{xy}_{z} \xrightarrow{T_z} R^{xyz}$$

Seemingly **not** – we have used up all the isometries!

$$H_{xyz} \xrightarrow{T_x} f^{x}_{yz} \xrightarrow{T_y} Q^{xy}_{z} \xrightarrow{T_z} \cancel{R^{xyz}}$$

this is very sad

However, we will soon argue that the *R* flux **must** exist by using commonplace symmetries of string theory.

III. The T^6/Z_2 Effective Theory

(J. Shelton, W. Taylor, BW '05)

As our flagship example, we consider a symmetric $T^6/\mathbf{Z}_2$ orientifold.

$$T^6 = \underset{(\alpha,i)}{T^2} \times \underset{(\beta,j)}{T^2} \times \underset{(\gamma,k)}{T^2}$$ with all 2-tori same

There are **three** moduli here:

S	**axio-dilaton**
τ	**complex structure**
U	**Kahler**

in IIB

This theory has been studied by many people.
Let's just review some relevant facts.

When we turn on fluxes, we'll get an $\mathcal{N} = 1$ theory in 4d, with

$$K = -3\log(-i(U - \bar{U})) - 3\log(-i(\tau - \bar{\tau})) - \log(-i(S - \bar{S}))$$

and scalar potential

$$V = e^K \left(\sum_{i,j,=\{\tau,U,S\}} K^{ij} D_i W \overline{D_j W} - 3|W|^2 \right)$$

where $D_i = \partial_i W + W \partial_i K$

The superpotential has been worked out for IIA and IIB with only geometric fluxes. Let's write down what's known.

IIB: $$W = \int (F_3 - SH_3) \wedge \Omega_3$$

Gukov, Vafa, Taylor, Witten

$dz^1 \wedge dz^2 \wedge dz^3$

but our complex coordinates are $z^1 = x^\alpha + \tau y^i$ etc.

$$\Rightarrow W = P_1(\tau) + SP_2(\tau)$$

where both polynomials are **cubic**.

IIA: $$W = P_1(\tau) + SP_2(\tau) + UP_3(\tau)$$

where the first polynomial is **cubic** and the others are **linear**!

Villadoro, Zwirner; Camara et. al. ; Derendinger et. al. ; Dall'Agata, Ferrara; Hull, Reid-Edwards

There is a mismatch between the two superpotentials under duality!

$$\xleftrightarrow{T_{\alpha\beta\gamma}}$$

Term	IIA flux	IIB flux
1	$\bar{F}_{\alpha i\beta j\gamma k}$	$\bar{F}_{ijk}$
τ	$\bar{F}_{\alpha i\beta j}$	$\bar{F}_{ij\gamma}$
τ^2	$\bar{F}_{\alpha i}$	$\bar{F}_{i\beta\gamma}$
τ^3	$F^{(0)}$	$\bar{F}_{\alpha\beta\gamma}$
S	$\bar{H}_{ijk}$	$\bar{H}_{ijk}$
U	$\bar{H}_{\alpha\beta k}$	$Q_k^{\alpha\beta}$
$S\tau$	f^{α}_{jk}	$\bar{H}_{\alpha jk}$
$U\tau$	$f^j_{k\alpha}, f^i_{\beta k}, f^{\alpha}_{\beta\gamma}$	$Q_k^{\alpha j}, Q_k^{i\beta}, Q_{\alpha}^{\beta\gamma}$
$S\tau^2$	$Q_i^{\beta\gamma}$	$\bar{H}_{i\beta\gamma}$
$U\tau^2$	$Q_{\beta}^{\gamma i}, Q_{\alpha}^{\gamma j}, Q_k^{ij}$	$Q_{\gamma}^{i\beta}, Q_{\gamma}^{\alpha j}, Q_k^{ij}$
$S\tau^3$	$R^{\alpha\beta\gamma}$	$\bar{H}_{\alpha\beta\gamma}$
$U\tau^3$	$R^{ij\gamma}$	Q_{γ}^{ij}

$$H_{abc} \xleftrightarrow{T_a} f^a_{bc} \xleftrightarrow{T_b} Q^{ab}_c \xleftrightarrow{T_c} R^{abc}$$

To get from IIA to IIB (or the other way), T-dualize on the Greek indices.

1. Start with geo IIA.

2. Dualize to get to IIB.

3. IIB has O3-planes, so we can rotate Greek into Latin, which takes $\tau^n \leftrightarrow \tau^{3-n}$

4. Dualize back to IIA.

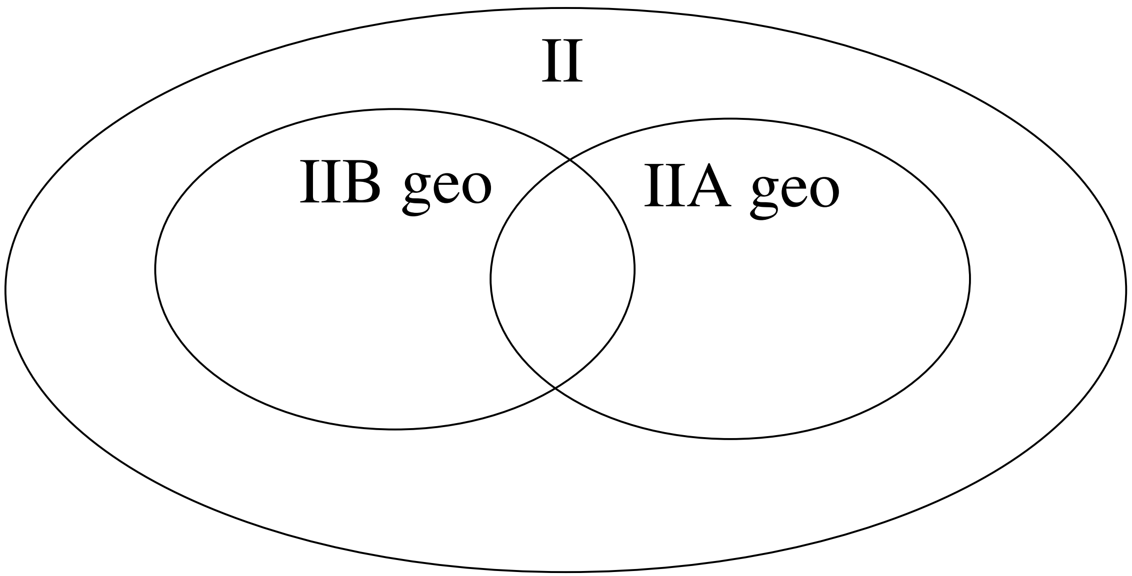
And now we've got a duality-invariant superpotential (with 16 parameters)!

So the superpotential

$$W = P_1(\tau) + SP_2(\tau) + UP_3(\tau)$$

with all three polynomials cubic
is what we need to use to get
things to match up on both sides.

This necessarily has nongeometric degrees of freedom,
even the threefold T-dual of *H*-flux. Weird!



But you can't just put in any old configuration of these new fluxes!

There are **several** constraints (e.g. tadpole) one must satisfy.

Let's just give **one** example, from the RR sector.

$$\bar{F}_{[abc}\bar{H}_{def]} = 0 \qquad \textbf{(no sources)}$$

under three T-dualities becomes

$$\bar{F}_{xyz[abc]}R^{xyz} - 9\bar{F}_{xy[ab}Q^{xy}_{c]} - 18\bar{F}_{x[a}f^{x}_{bc]} + 6F^{(0)}\bar{H}_{[abc]} = 0$$

There are similar constraints just from the NS-NS sector,
which can be interpreted as Jacobi identities for
a non-Abelian algebra whose structure constants are
the fluxes. **Kaloper, Myers**

IV. Equations of Motion And Solutions

So we have a superpotential $W = P_1(\tau) + SP_2(\tau) + UP_3(\tau)$

and equations of motion $\partial_i W + W \partial_i K = 0$ for SUSY vacua.

aka

$$P_1(\tau) + \bar{S}P_2(\tau) + UP_3(\tau) = 0$$

$$P_1(\tau) + SP_2(\tau) + \frac{1}{3}(2U + \bar{U})P_3(\tau) = 0$$

$$(\tau - \bar{\tau})\partial_\tau W - W = 0$$

In general, these must be solved numerically. **BORING!**

Let's give one interesting thing you can see **without** crunching numbers.

Minkowsi IIA vacua

$$W = 0 \quad \text{and} \quad P_1(\tau) + \bar{S} P_2(\tau) + U P_3(\tau) = 0$$

$$\text{imply} \quad (S - \bar{S}) P_2(\tau) = 0$$

$$\text{so} \quad P_2(\tau) = 0$$

Now assume we only have **geometric** fluxes. Then $P_2 = a + b\tau$

$$\text{so} \quad \tau = -b/a$$

But this is a contradiction, since $\mathrm{Im}\,\tau > 0$

Minkowski vacua in IIA are necessarily nongeo (for T^6/Z_2)!

We are currently investigating general properties of the solutions to these EOM numerically.

Some things we notice:

1. It is difficult to find small string coupling and cc.
2. Roughly 20-30% of fluxes that satisfy the constraints actually give physical solutions for the moduli.
3. Difficult to find finite accumulation points for g and cc.

It would be **super great** to make these points more precise.

In general, it is **hard** to get control over these solutions!

However, we have had some nice solutions emerge…

We have found several infinte families of nongeometric vacua where we can tune string coupling and cosmological constant to be arbitrarily small.

$$g_s = \frac{n^3}{2m^3} \quad \text{and} \quad \Lambda = -\frac{3n^6}{16m^3}.$$

where n and m are integer fluxes.

These vacua all involve fluxes which are **nongeometric** in both IIA and IIB, so they naively appear to **not** have geometric duals

Details coming soon to a preprint server near you (I hope)!

V. Interpretation of Nongeometric Fluxes

OK, so nongeometric fluxes are around. But what's going on?

$$H_{abc} \xleftrightarrow{T_a} f^a_{bc} \xleftrightarrow{T_b} Q^{ab}_c \xleftrightarrow{T_c} R^{abc}$$

1. ***H* and *f* are geometric.**
_Q_ is locally but not globally geometric.
What about _R_?

We suspect that *R* is not even locally geometric: Consider a D3-brane on the 3-torus with *H*-flux. T-dualizing gives a D2-brane with inconsistent boundary conditions. Taking the chain to the end means **no D0-branes** on the *R*-space.

Lawrence
Schulz
BW

2. There is an interesting analogy between the NS-NS and RR-fluxes.

$$H_{xyz} \leftrightarrow F_3$$
$$f^x_{yz} \leftrightarrow F_2$$
$$Q^{xy}_z \leftrightarrow F_1$$
$$R^{xyz} \leftrightarrow F_0$$

Just like we can't use the Buscher rules to get the RR 0-form, we can't use them to get the NS-NS 0-form.

3. There is some math literature on such fluxes.

Q is related to non-commutative geometry.
R is related to non-associative geometry.
??

Bouwknegt
Mathai
Hannabuss
Rosenberg
Evslin

VI. Eight Open Questions

1. **How generic are nongeometric vacua?**

   My naïve guess is that a generic string compactification is nongeometric. But how do we show this, and make sure we're not overcounting?

2. **Is there an intrinsic definition or criterion for nongeometricity?**

   Can we define "nongeometric" more precisely?

3. **What is the relationship between these objects and G-structures and generalized geometry?**

   One is a manifold with extra structure, one isn't. Different?

4. **How controllable are α' corrections?**

   What does “large volume” mean, if anything?

5. **Can one extend these to more interesting Calabi-Yau’s?**

   We motivated our nongeo fluxes by dualizing on
   1-cycles. Can we still include them on a CY?
   How does SYZ give these?

6. **Non-SUSY vacua?**

7. **Can we get KK/winding modes under control?**

8. **What is the right description of the *R* fluxes?**

   Can they be lifted to 10 dimensions?
   Is there a worldsheet description?