



**Fourth Stig Lundqvist Conference on  
Advancing Frontiers of Condensed Matter Physics**

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**Superfluid disorder in He-4:  
from edges and interfaces to superglass**

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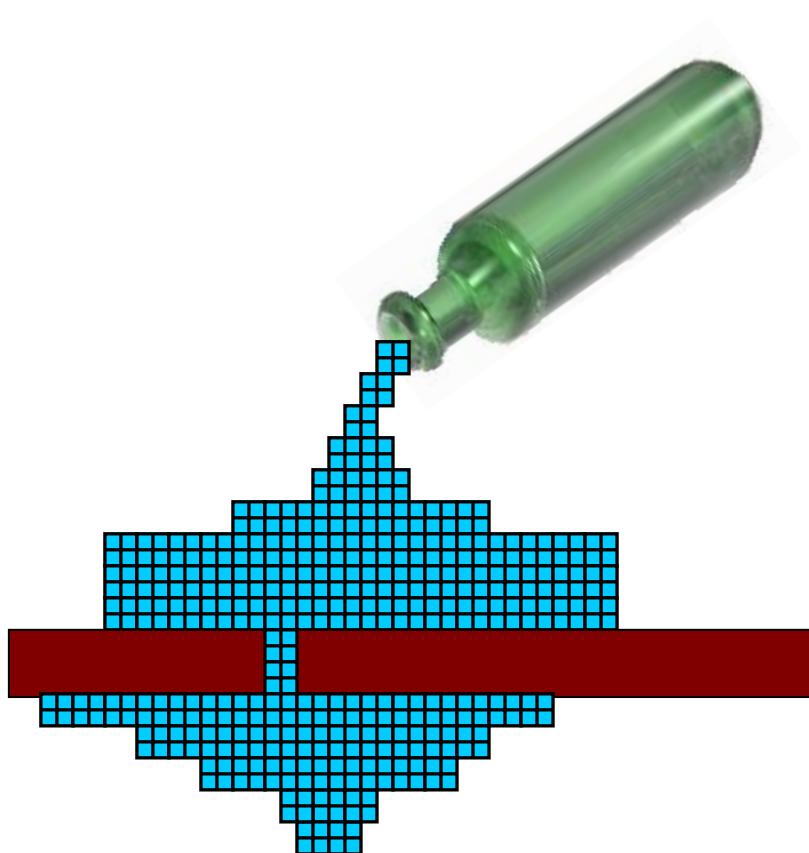
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These are preliminary lecture notes, intended only for distribution to participants

# Superfluid disorder in He-4. From edges and interfaces to superglass



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Evgeni Kozik      UMass, Amherst  
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Anatoly Kuklov      CSI CUNY

NASA

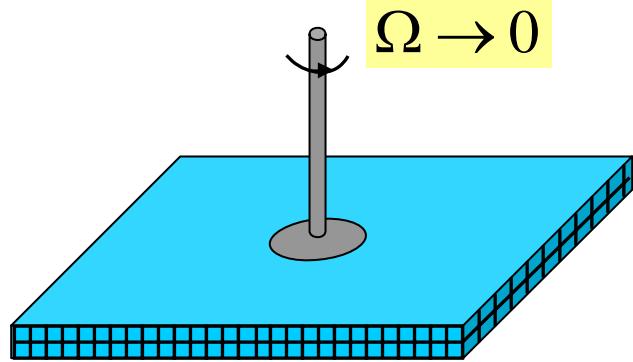
Trieste  
July 3-7 (2006)

 National Science Foundation  
WHERE DISCOVERIES BEGIN

# Outline

- Andreev-Lifshitz & Chester scenario for supersolids
- Experimental facts.
  - Theorem: Continuous-space supersolids are always incommensurate
  - He-4 *hcp* crystals are commensurate and insulating.
  - Vacancies and interstitials are activated; vacancy gas is unstable
- Superfluid interfaces in lattice models (proof of principle)
- He-4 superglass
- Superfluid grain boundaries and ridges in polycrystalline samples

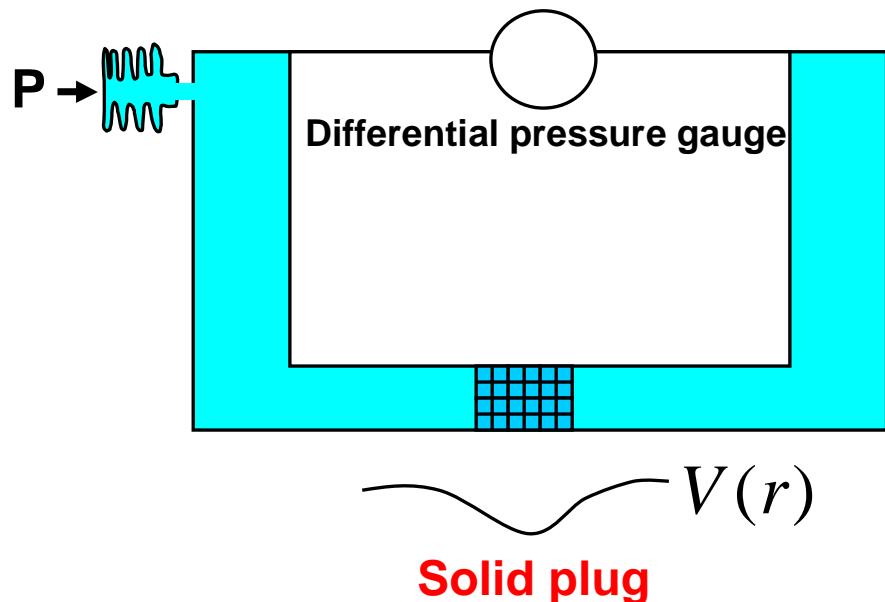
## What is supersolid?



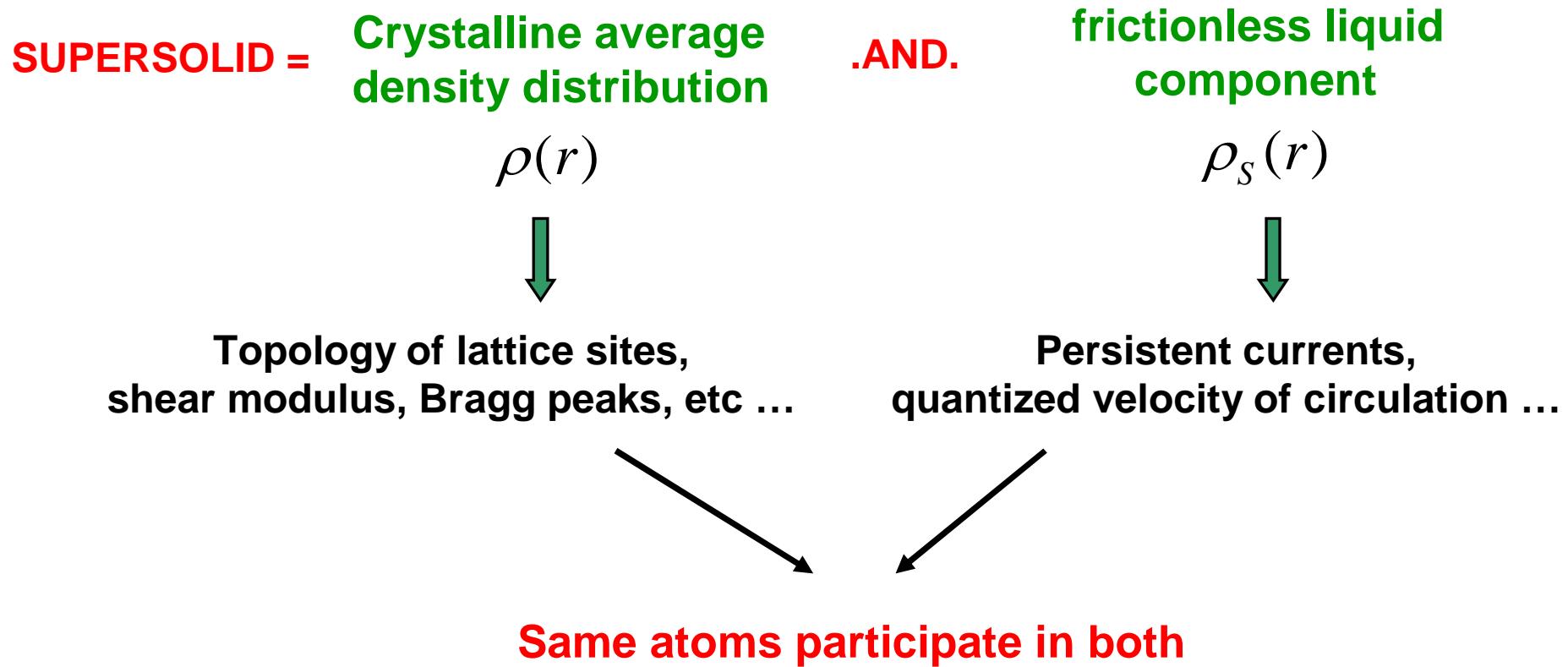
$$E = \frac{1}{2} I \Omega^2$$

$$I \neq I_0 = \int \rho(r) r^2 dV$$

**Non classical moment of inertia.  
Some of the crystal mass is not  
rotating with the lattice**



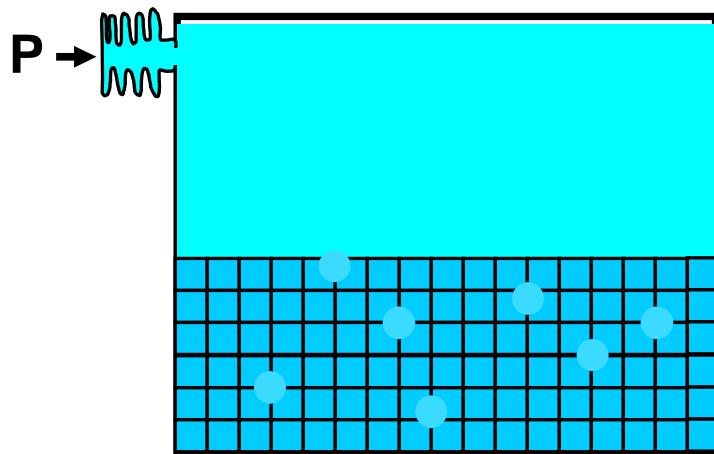
**Superflow or persistent current  
through the solid state; pressure  
Equilibration, etc.**



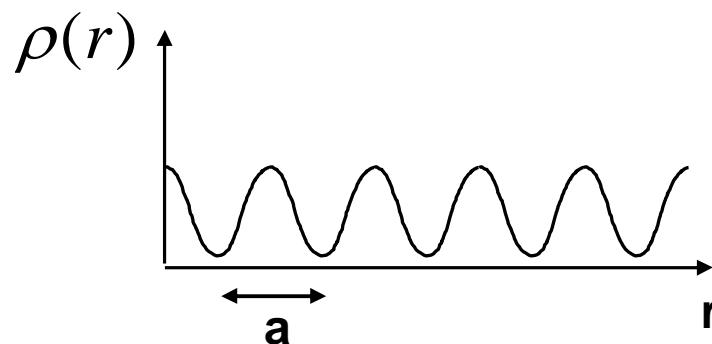
How can this happen?

Andreev & Lifshitz '69  
Chester '70

Zero-point vacancies = “incommensurate” solid



Number of lattice points  
 $\neq$   
Number of atoms in  
the ground state



Perfectly periodic, but  $\frac{\int_0^{am} \rho(r) dr}{m} \neq 1$  (integer)

$$\frac{\int_0^{am} \rho(r) dr}{m} \neq 1 \text{ (integer)}$$

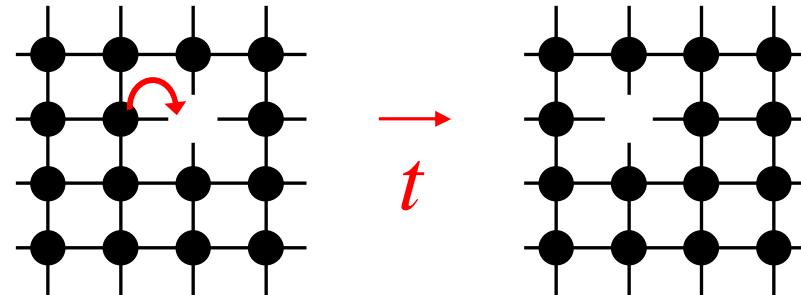
Start from a perfect structure

1. Create a static vacancy;  $\delta\epsilon = E_0 > 0$

2. Let the vacancy move around;  $\delta\epsilon = -zt < 0$

Reduction of kinetic energy of n.n. atoms

$$\delta p \sim h / \delta r$$



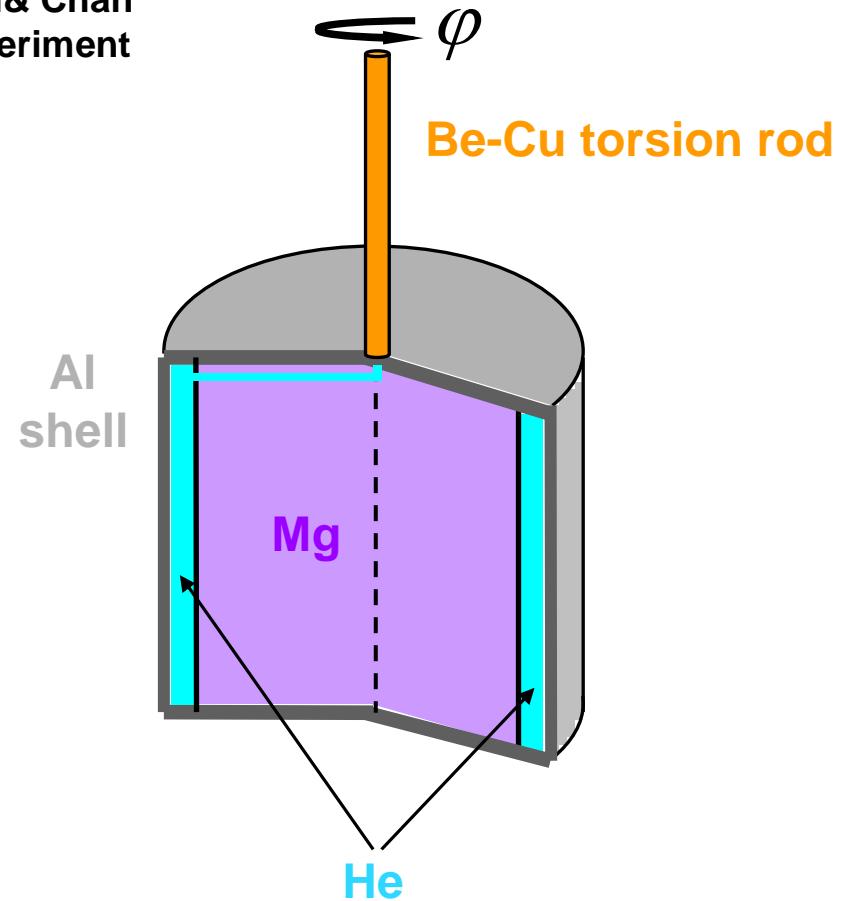
3. IF  $\epsilon_v = E_0 - zt < 0$ , making vacancies is favorable for the structure.

$$E \approx \epsilon_v n_v + \frac{U}{2} n_v^2 \rightarrow n_v = \frac{|\epsilon_v|}{U}$$

Weakly-interacting gas goes superfluid in 3D below

$$T_C \sim n_v^{2/3}$$

Kim& Chan  
experiment



$$H = \frac{\kappa\varphi^2}{2} + \frac{I\dot{\varphi}^2}{2}$$

$$\Omega = \sqrt{\kappa / I}$$

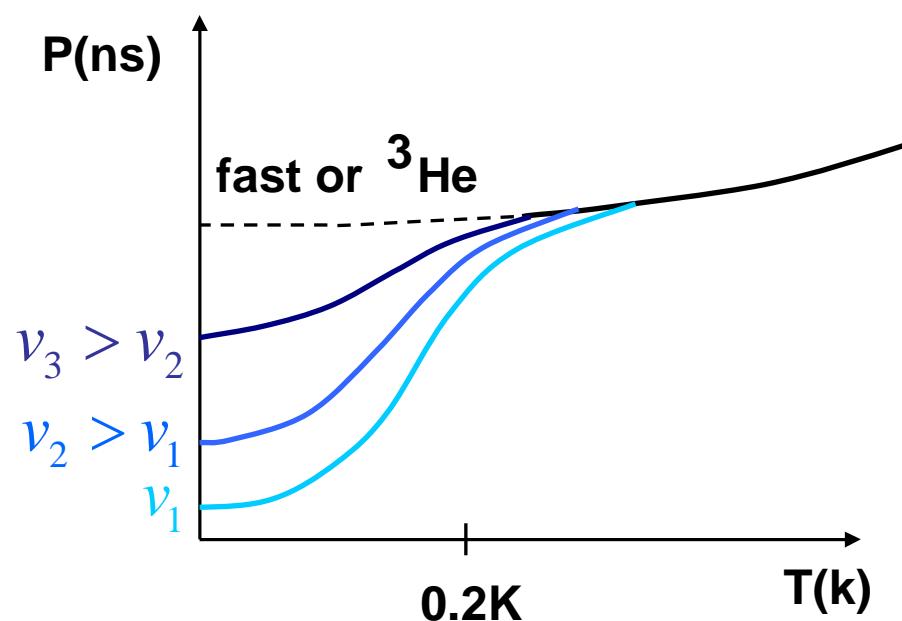


$$I = I_{classical} \quad \text{if} \quad \rho_s = 0$$

$$I < I_{classical} \quad \text{if} \quad \rho_s > 0$$

Oscillation period drop = onset of superfluidity

$$v = \Omega R \text{ - linear velocity}$$

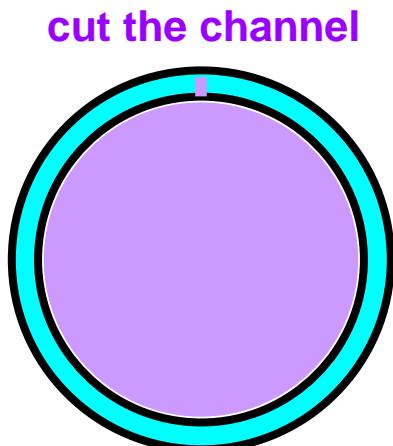


Systems

- ${}^4\text{He}$  in Vycor
- ${}^4\text{He}$  in porous gold
- ${}^4\text{He}$  frozen in bulk



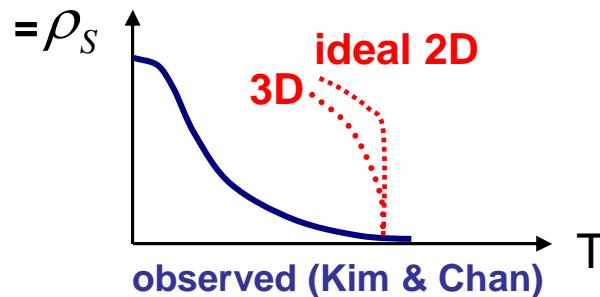
1-2 % of  ${}^4\text{He}$   
did not follow  
the rotation



$\rho_S \rightarrow$  reduced by nearly two orders of magnitude

## Problems with homogeneous 3D interpretation

- Decoupled mass



Strongly disordered system  
with a broad distribution of  
 $T_c(\vec{r})$

- He-3 impurities:  $\rho_3 / \rho_s \sim 10^{-3}$  !

impossible to understand in  
the homogeneous bulk state

physics of Josephson-junctions  
or weak-links cutting the flow is  
involved

- “Elimination of the supersolid state  
through crystal annealing”

Rittner & Reppy ‘06. Supersolid  
samples are far more dissipative !

- $C(T) \propto T$  at low temperatures < 0.1 K

Kim & Chan measurements. Typical  
for a glass (or 1D superfluid).

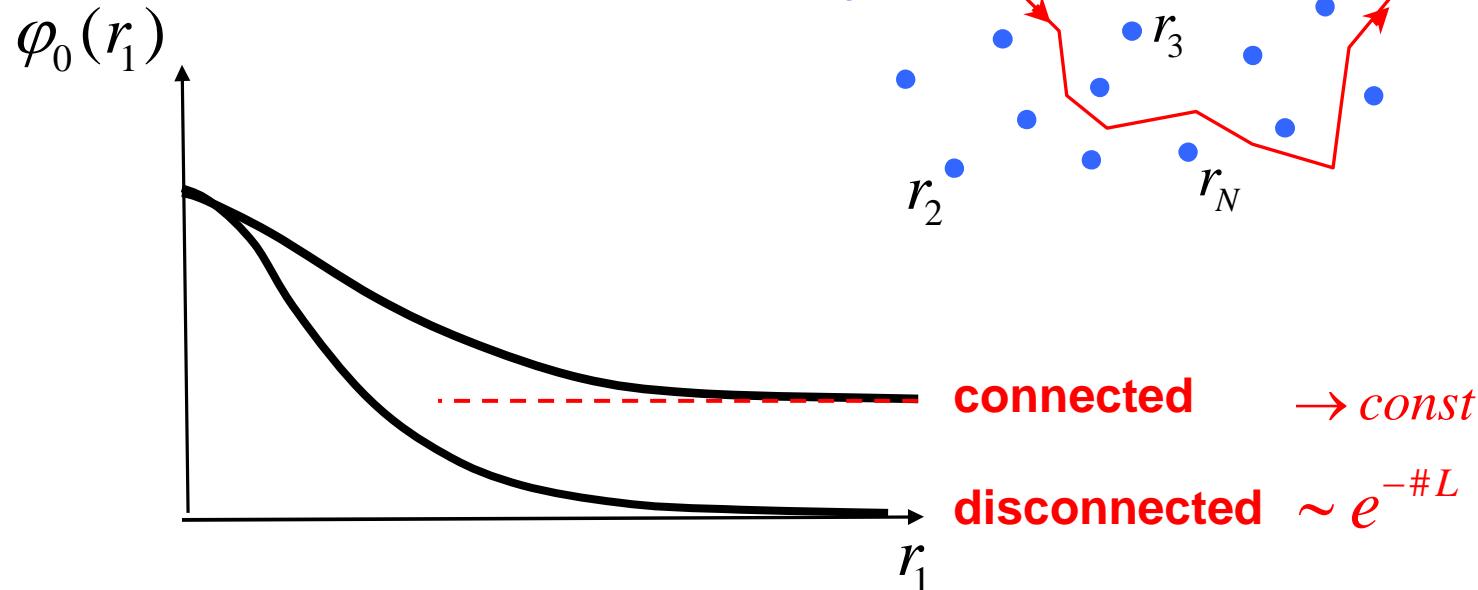
**Theorem: The state without vacancies and interstitials is normal.**  
 (Groundstate wave function; Path-integrals )

NP, Svistunov '04

Only “connected”ground states  
are superfluid !

$$\varphi_0(r_1, \underbrace{r_2, \dots, r_N}_{\text{fixed}})$$

Leggett '1970



## Insulating *hcp* crystals of He-4

No interstitial-vacancy symmetry in the supersolid  $\rightarrow n_v \neq n_i$ .  
Thus, supersolids are always incommensurate

### Experiments:

He-4 is a commensurate solid, X-ray (accuracy 0.1%)

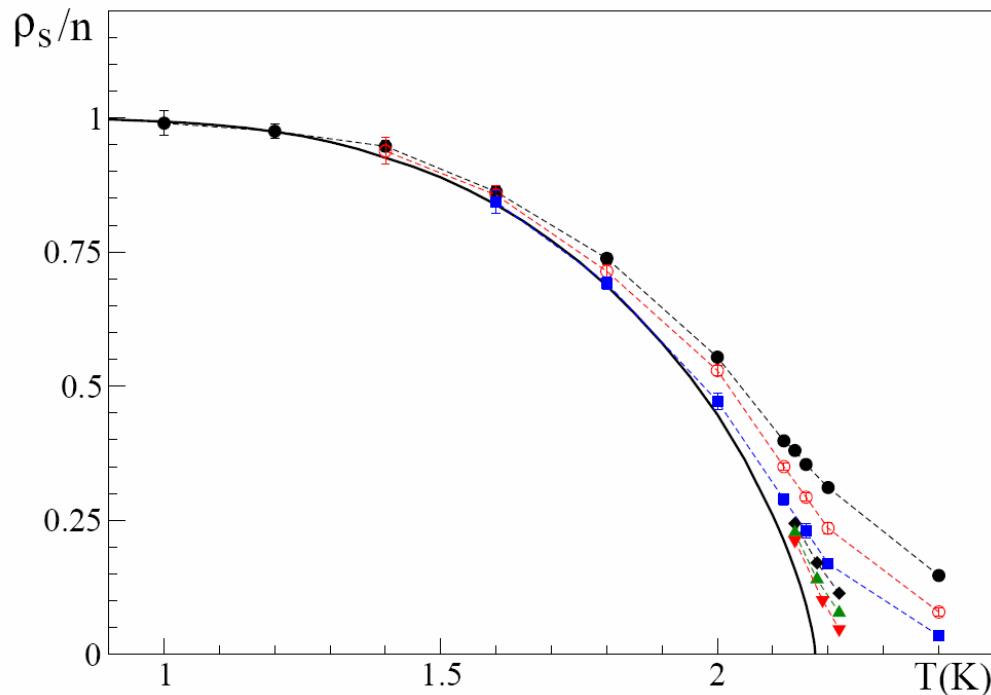
Vacancies and interstitials are activated, X-ray, ion and He-3 mobilities

Simulations: Density matrix (or ODLRO)  $n(r) = \langle \psi^+(r, +0) \psi(0, 0) \rangle$

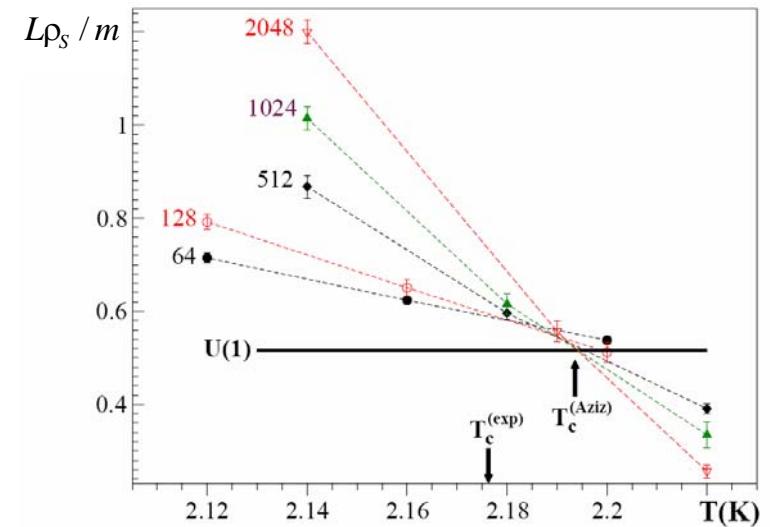
Green function  $G(p=0, \tau) = \int dr \langle \psi^+(r, \tau) \psi(0, 0) \rangle$

## Why should we trust simulations?

No free parameters; helium mass +  $V(r)$  and go ...



Up to 2048 atoms  
(path-integral MC  
worm algorithm updates)

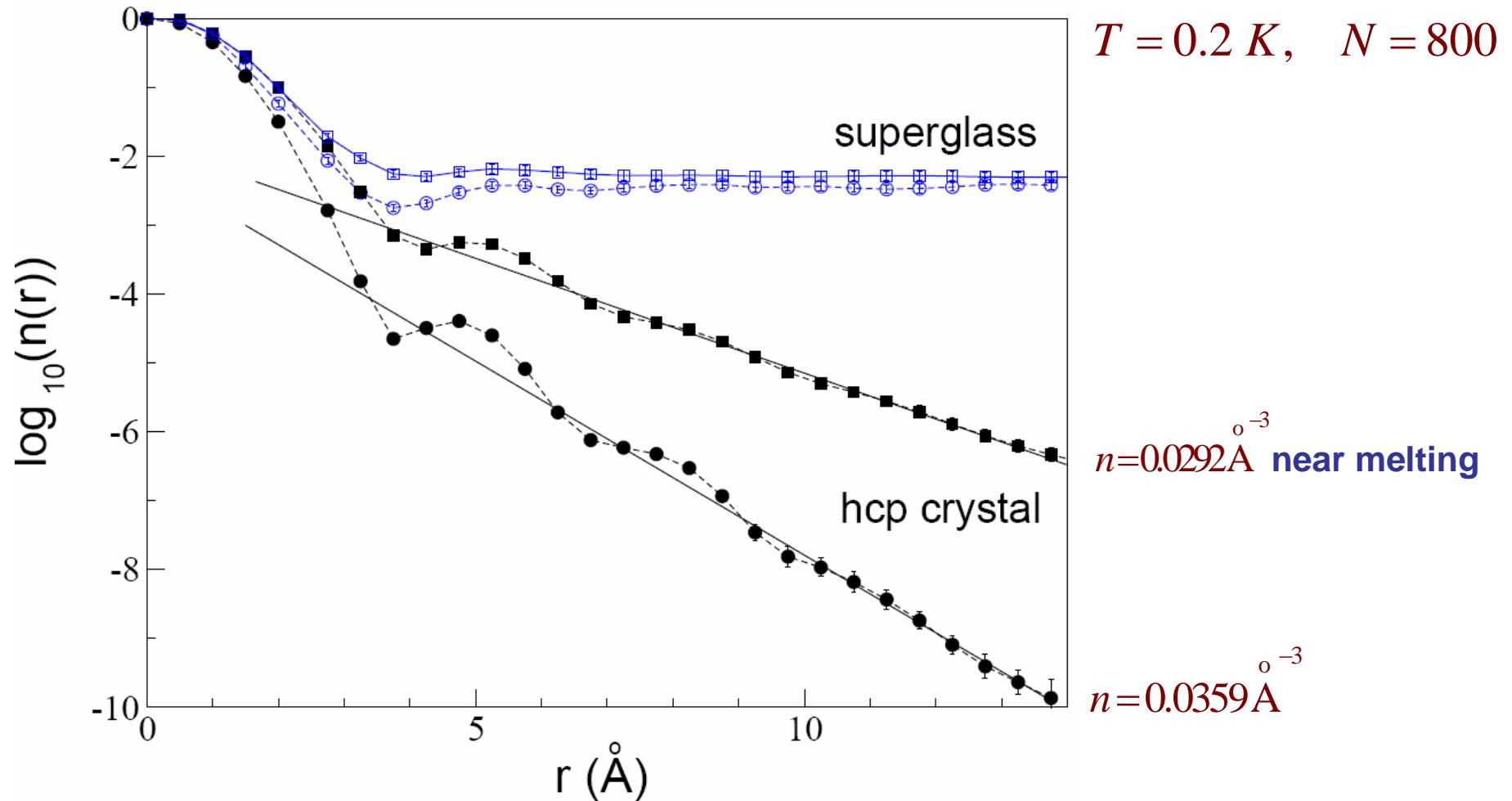


$T_c = 2.193$  calculated  
 $T_c = 2.177$  experiment

Better than 1%  
agreement at all  $T$   
after finite-size scaling

## Insulating *hcp* crystals of He-4

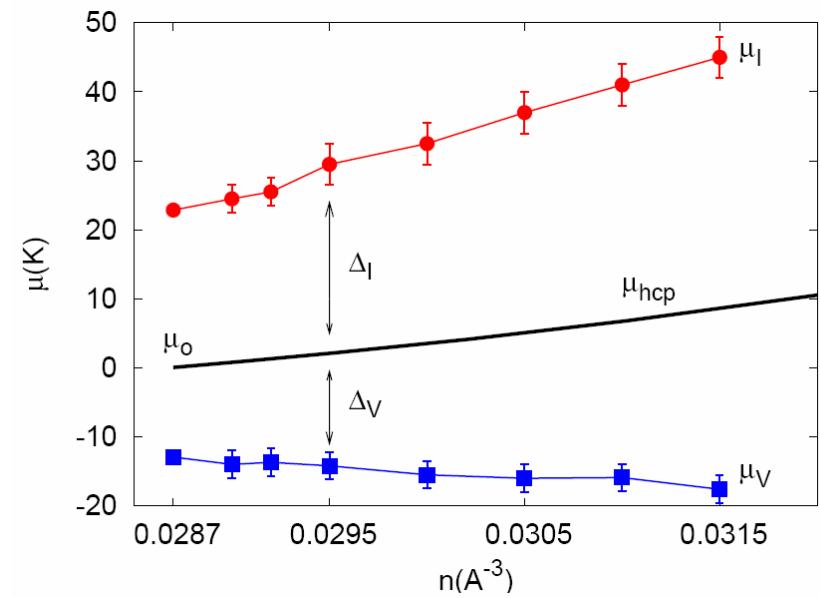
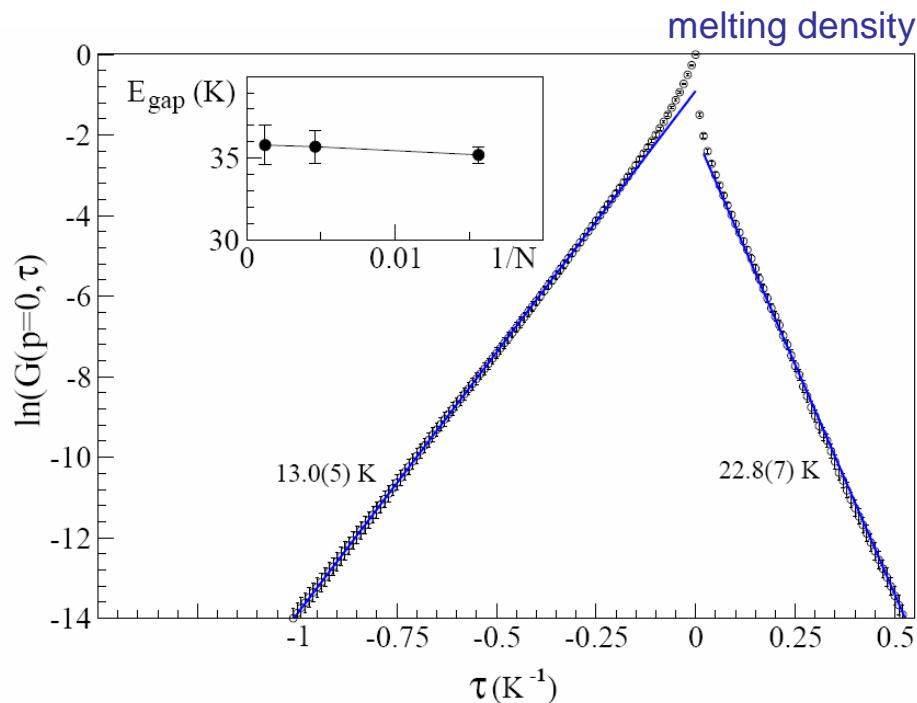
Exponential decay of the single-particle density matrix



# Insulating hcp crystals of He-4

**Green function (polaron effect + gaps)**

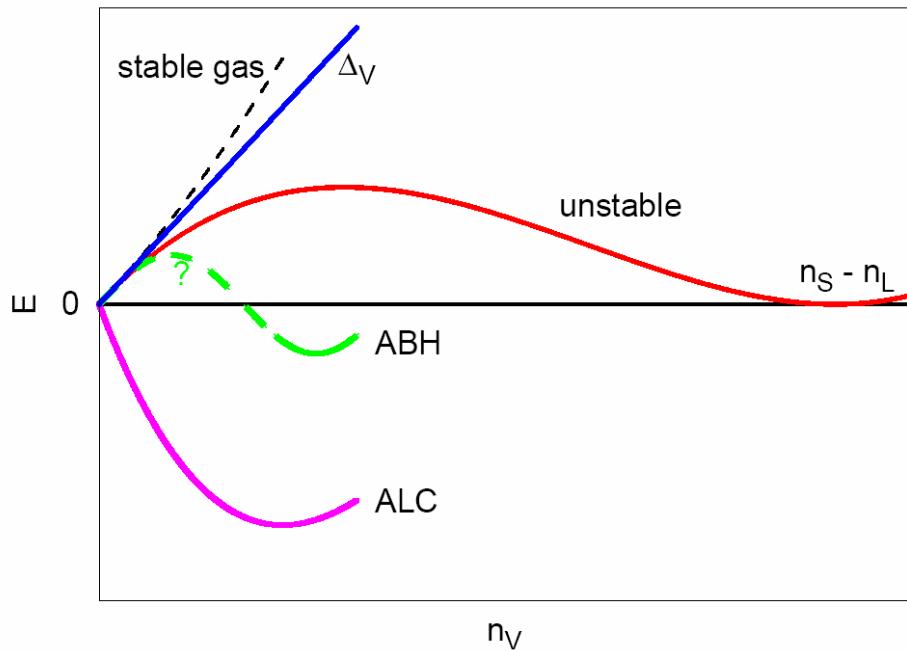
$$G(p, \tau \rightarrow \infty) \rightarrow Z e^{-E_{i,v} |\tau|}$$



**Large activation energies at all Pressures (thermodynamic limit)**

# Insulating hcp crystals of He-4

Various vacancy-induced supersolid scenarios



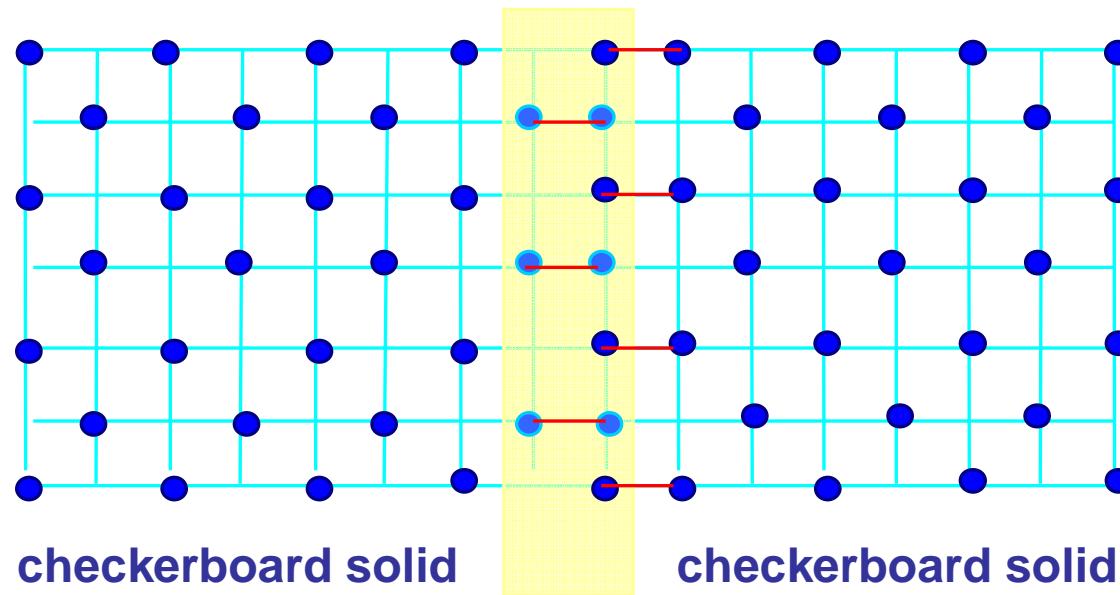
When introduced into the solid  
by hand vacancies form an unstable  
gas, cluster, and phase separate

In real experiments will anneal at the grain boundaries,  
dislocations, and other crystalline defects ...

# Superfluid disorder

Burovski ,Kozik,  
NP, Svistunov '05

## Proof of concept for grain boundaries



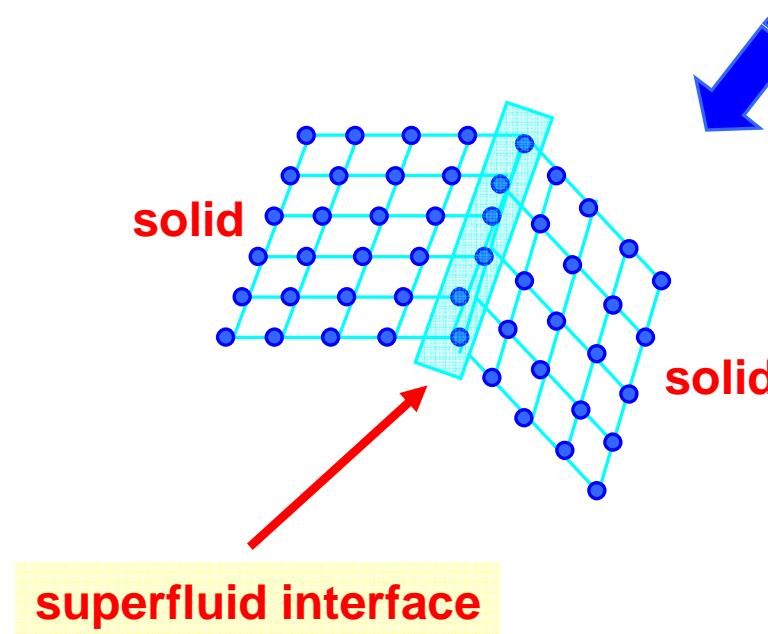
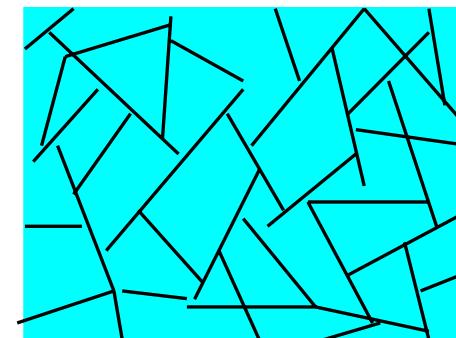
Superfluid domain walls  
both in 2D and 3D crystals

(SF = rough wall = solid-solid recrystallization waves)

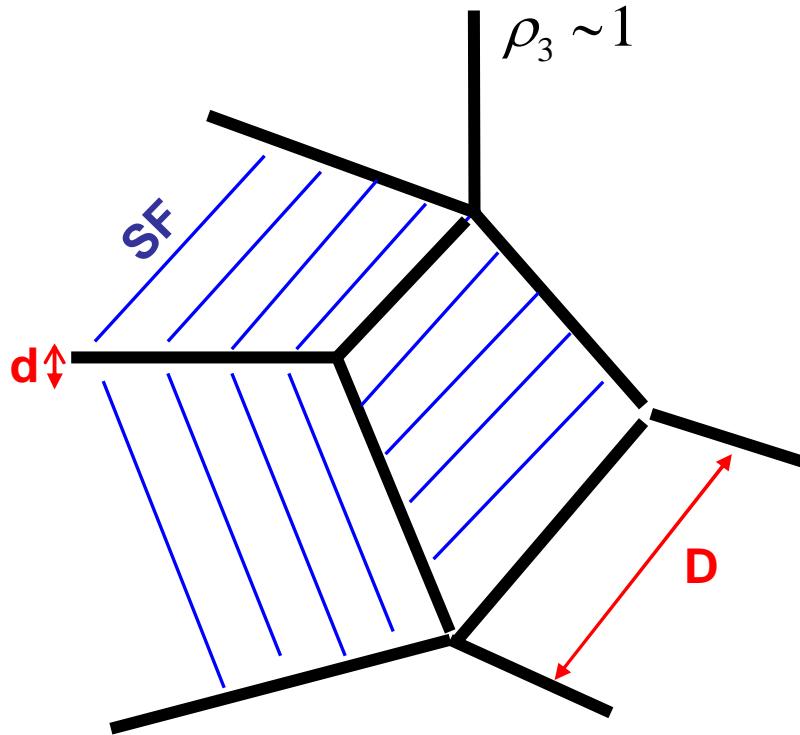
# Superfluid disorder

Quantum crystal, weak first-order  
liquid-solid transition

Frustrated crystalline interfaces



connected three-dimensional  
network of superfluid films  
(disordered ; ~ 2% volume)



3D network of superfluid interfaces and ridges. (disorder, different interfaces)

$$T_c(\vec{r})$$

He- 3 goes to interfaces and then to ridges connecting them.  
Ridges → Josephson junctions

He- 3 at grain boundaries, ridges and corners prevents crystals from growing (stabilizes disorder)

**Numbers:**  $\rho_s \sim d / D \longrightarrow D \sim \# \mu m$  “helium milk” (B. Hallock)

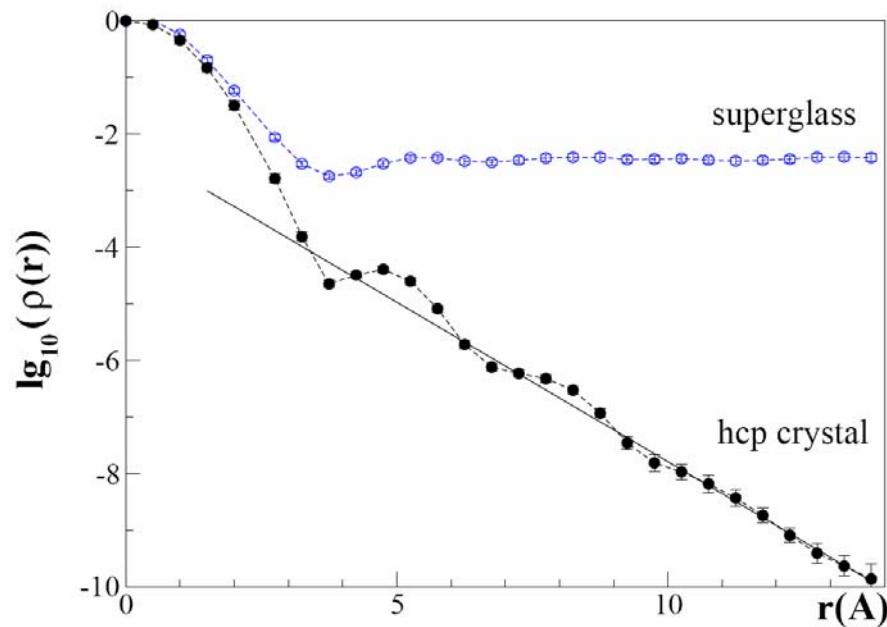
$$\rho_3 \sim (d / D)^2 \sim \rho_s^2 \longrightarrow \sim 10^{-4}$$

# Superglass state of He-4

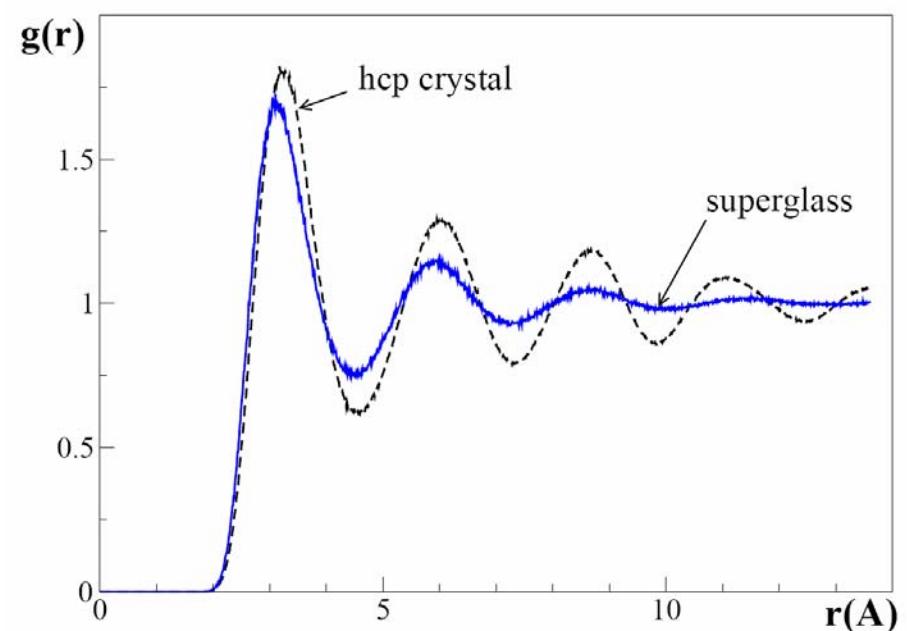
Monte Carlo temperature quench from normal liquid

$$n = 0.0359 \text{ \AA}^{-3}, \quad T = 100 \text{ K} \xrightarrow{\text{o}} 0.2 \text{ K}, \quad N = 800$$

Single-particle density matrix



density-density correlator

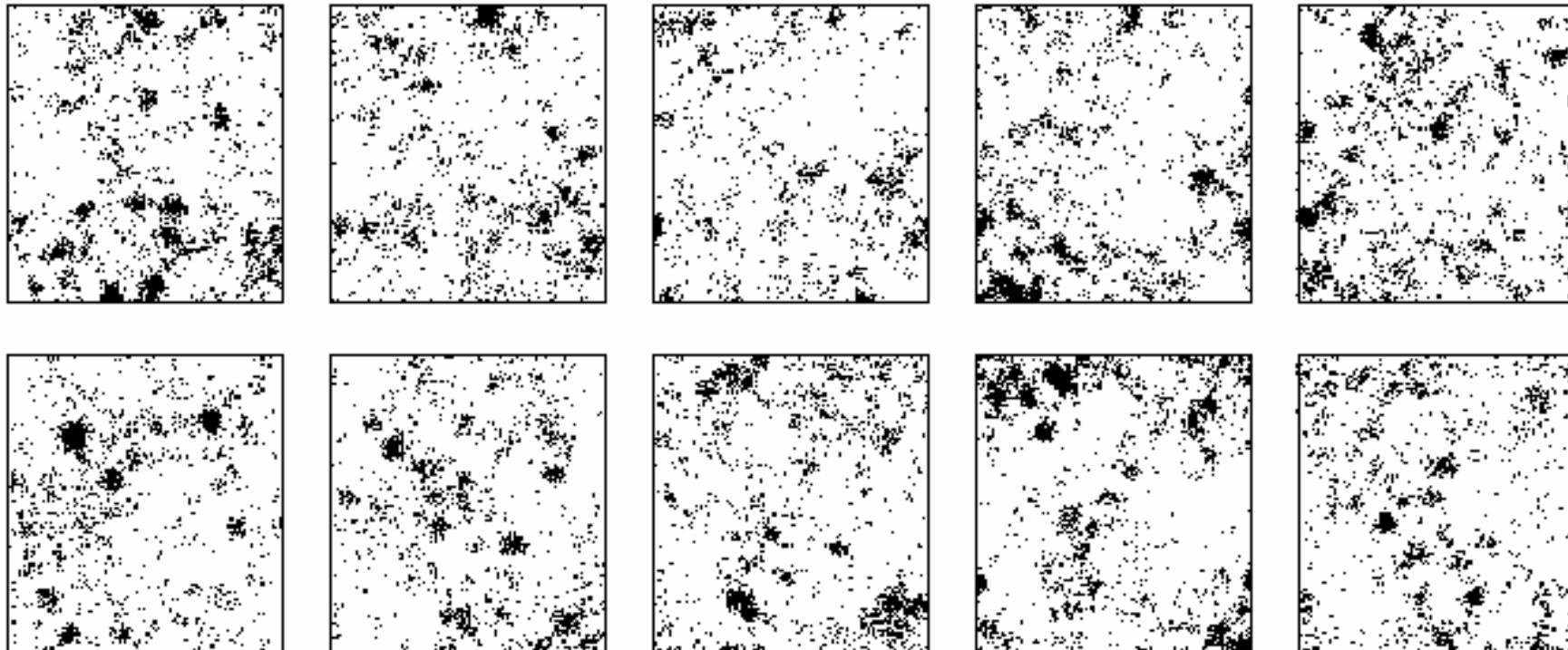


ODLRO,  $\rho_s = 0.07(2)$

# Superglass state of He-4

Condensate wave function maps  
reveal broken translation symmetry     $\phi_0(r) \sim \text{density of points}$

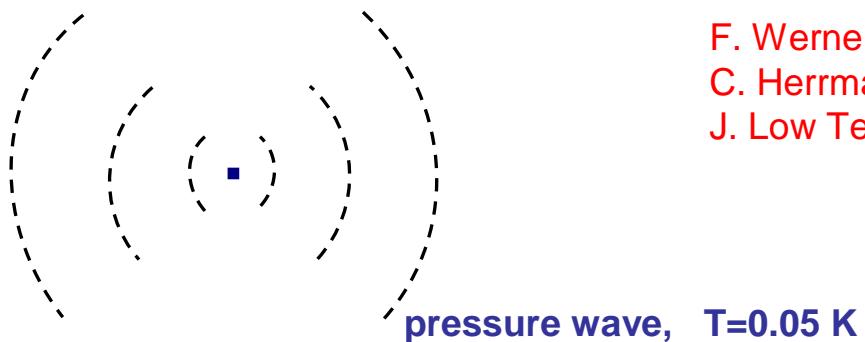
10 slices across the z-axis



A rough estimate of metastability:     $t_{\text{relax}} > 10^4 J_{4-4} \sim 10^{10} \omega_D^{-1} \sim 10^{-3} \text{ s}$

# Superglass state of He-4

“Strange” outcome of the quantum-nucleation experiment

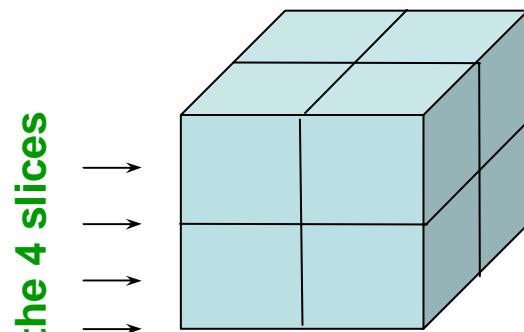


F. Werner, G. Beaume, A. Hobeika, S. Nascimbene,  
C. Herrmann, C. Caupin, and S. Balibar  
J. Low Temp. Phys., Vol. 136, Nos. 1/2, 2004

Bulk nucleation of a solid was predicted to occur  
at  $P \sim 65$  bar. Nothing was observed up to 160 bar !

Authors → very viscous (glassy), normal liquid.

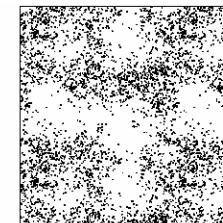
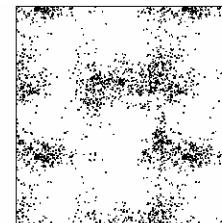
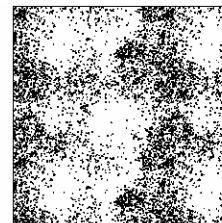
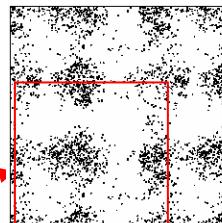
# Superfluid ridges and interfaces in He-4



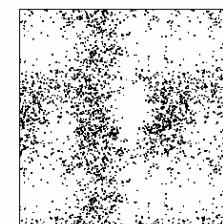
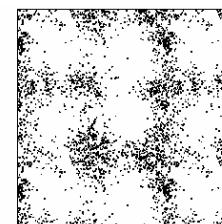
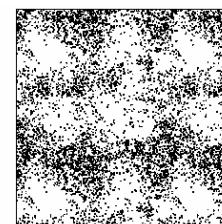
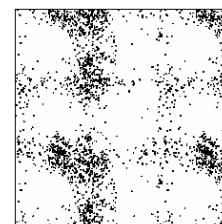
Each of the 8 cubes is a randomly oriented crystallite  
(24 interfaces)

Condensate maps

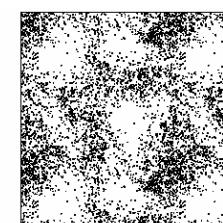
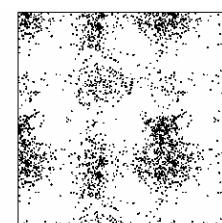
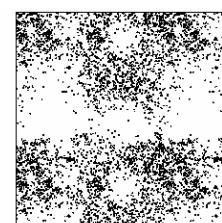
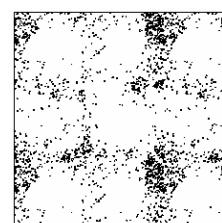
simulation box



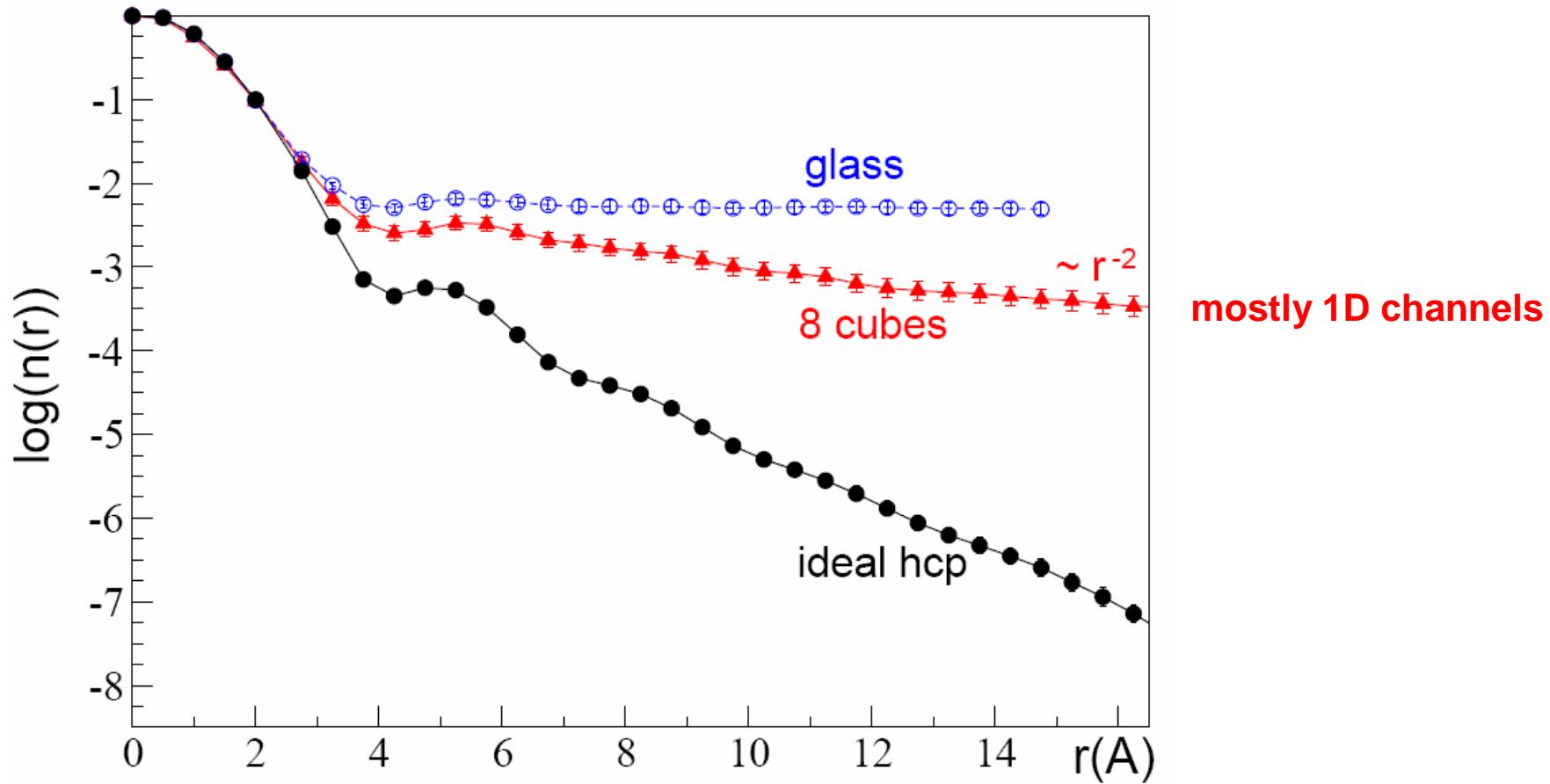
across x-axis



across y-axis



across z-axis



## **Summary:**

**Ideal *hcp* crystals are not superfluid.**

**Large gaps for vacancies and interstitials at all pressures.**

**Non-equilibrium vacancies is an unstable, phase separating system.**

**Some (not all!) grain boundaries and ridges in helium are superfluid.**

**Helium can form a meta-stable *superglass* phase (very stable even in contact with crystals!).**

**Open questions: can easily fill next five slides!**