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SMR.1759 - 11

Fourth Stig Lundqvist Conference on
Advancing Frontiers of Condensed Matter Physics

3 - 7 July 2006

Bose-Einstein Condensation with Entangled Order Parameter

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Ref: YS and Q. Niu, Phys. Rev. Lett. 96, 140401 (2006)

Fourth Stig Lundqvist Conference, ICTP, Trieste, 4 July 2006

Two-component BEC

A mixture of A-atoms and B-atoms:

$$\psi \approx \phi(\mathbf{r}_{a1}) \cdots \phi(\mathbf{r}_{aN_a}) \otimes \phi(\mathbf{r}_{b1}) \cdots \phi(\mathbf{r}_{bN_b})$$

$$|\psi\rangle = \frac{1}{\sqrt{N_a N_b}} (a^\dagger)^{N_a} (b^\dagger)^{N_b} |0\rangle = |\psi\rangle_a \otimes |\psi\rangle_b$$

A-atoms and B-atoms separately condense, with separate order parameters (classically coupled).

Mean field theory.

Similar is a mixture of one species of atoms with two spin states, the numbers of which are conserved respectively.

Spin-1 condensate

$$|\psi\rangle \sim [(a_0^\dagger)^2 - 2a_{-1}^\dagger a_1^\dagger]^{N/2}|0\rangle$$

- Similar is the spin-1/2 BEC (Kuklov-Svistunov)
- Non mean-field state.
- But the particles are all identical! Each particle can flip spins.
- Practically very difficult to realize, as the energy difference with the symmetry breaking mean-field state vanishes as $N \to \infty$

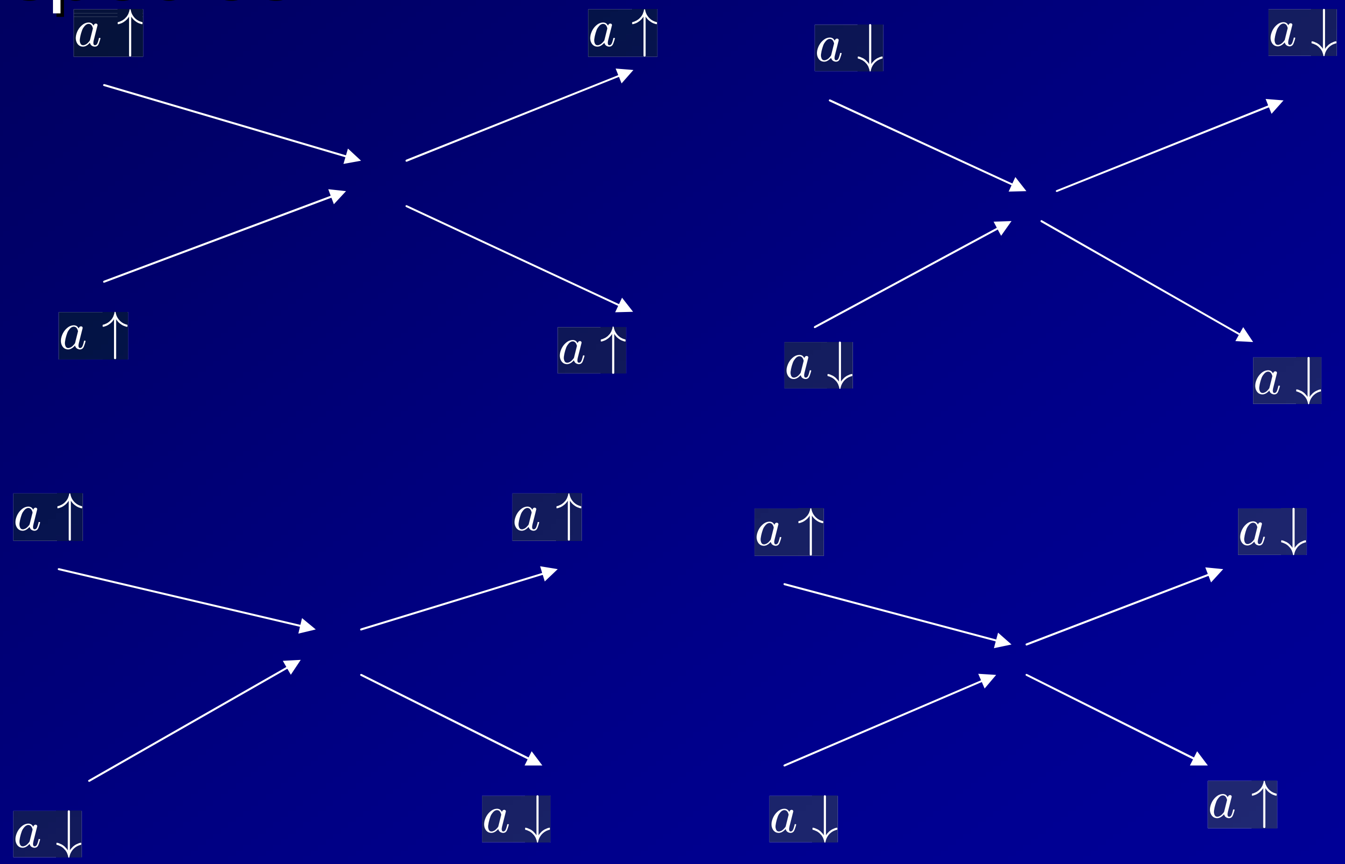
2 species $\times$ 2 spin states

- Each atom can flip the spin, but cannot transit between the atom species.
- $N_{i\uparrow}$ and $N_{i\downarrow}$ (i=a,b) are not conserved.

Only consider single particle orbital ground state; ignore depletion.

Scattering channels

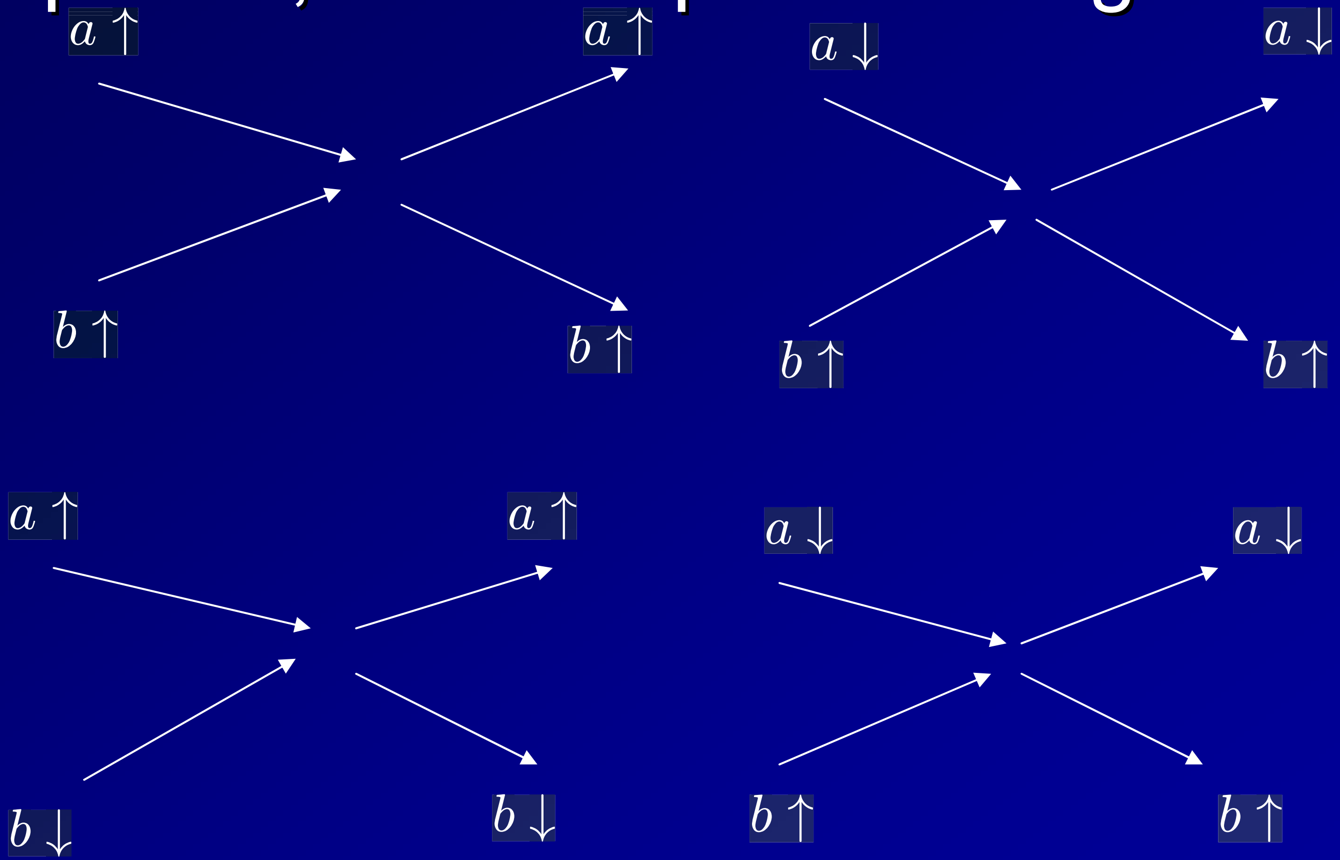
Intra-species:



Similar is the b-species.

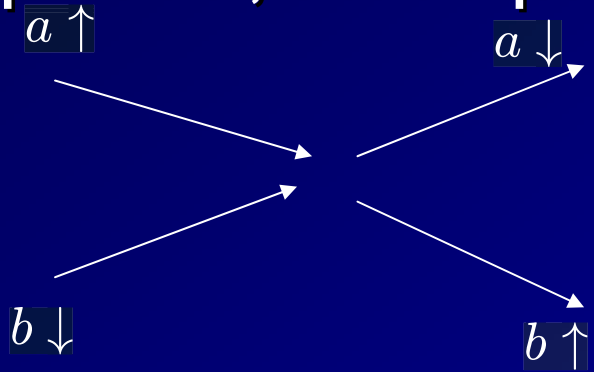
Scattering channels (continued)

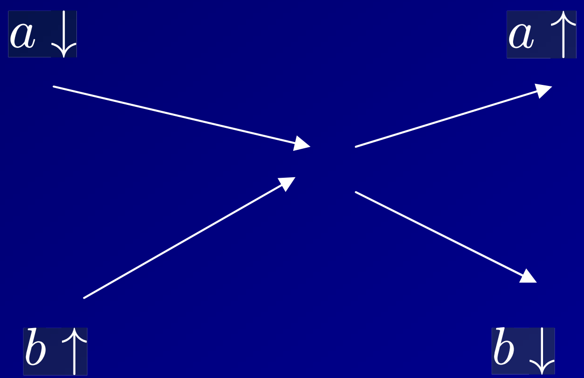
Inter-species, without spin-exchange



Scattering channels (continued)

Inter-species, with spin-exchange



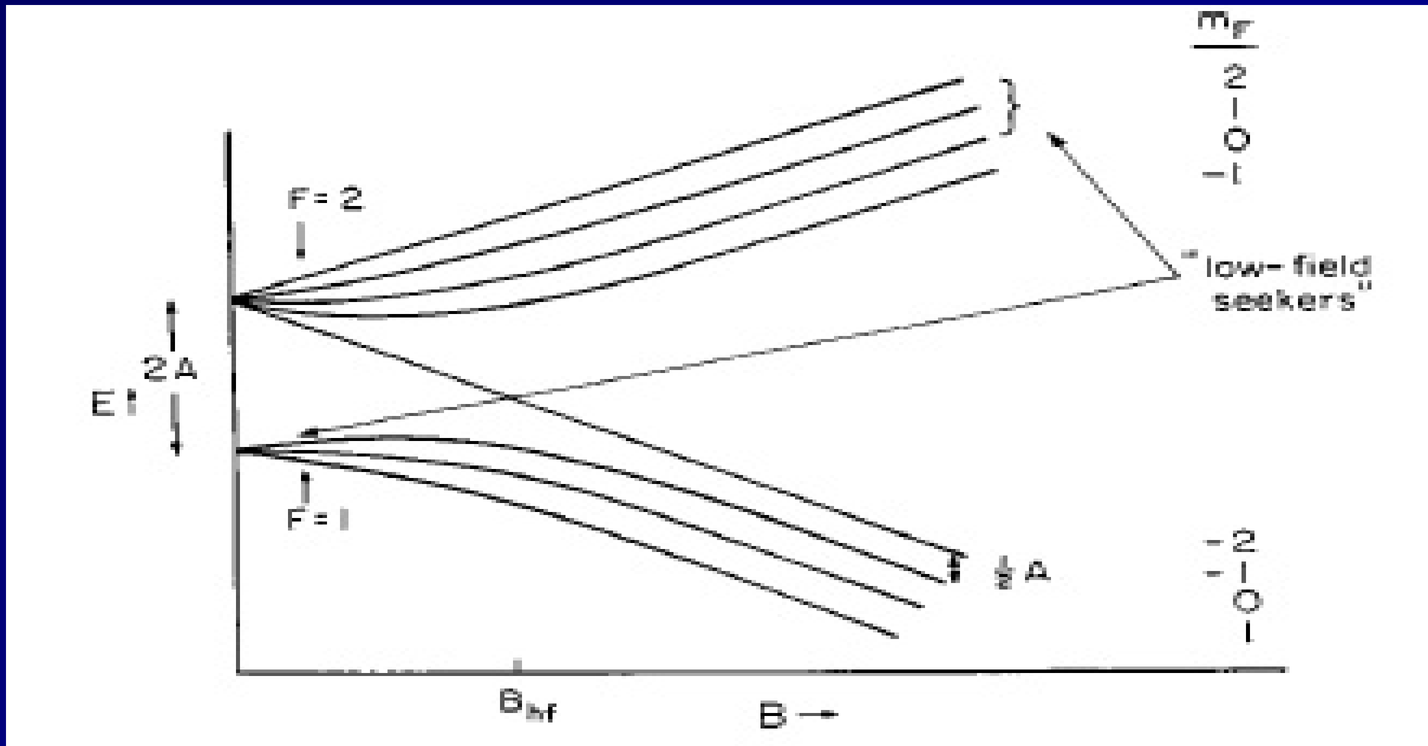


Requirements

- Energy conservation in each scattering
- Conservation of total z-component spin in each scattering

Experimental feasibility

- For given I and J, Hyperfine-Zeeman energy levels depend only on F, m_F, not on atom species.



(Copied from Leggett RMP)

Merits

- Interesting spinful BEC can thus also be realized in magnetic traps.
- Call for experiments on multichannel scattering between different species of atoms.
- But what is the goodness?

It realizes, in the ground state, entanglement between BECs.

Entanglement between BECs

- Like a pure two-particle entangled state, where each particle is not in any pure spin state, there is no simple BEC of either species; there is only a global simple BEC.
- BEC occurs in an entangled inter-species pair state.

[Y.S., Int. J. Mod. Phys. B 15, 3007 (2001)]

Hamiltonian

$$\mathcal{H} = \sum_{\sigma} f_{i\sigma} N_{i\sigma} + \frac{1}{2} \sum_{\sigma\sigma'} K^{(ii)}_{\sigma\sigma'} N_{i\sigma} N_{i\sigma'} + \sum_{\sigma\sigma'} K^{(ab)}_{\sigma\sigma'} N_{a\sigma} N_{b\sigma'} + \frac{K_e}{2} (a^\dagger_\uparrow a_\downarrow b^\dagger_\downarrow b_\uparrow + a^\dagger_\downarrow a_\uparrow b^\dagger_\uparrow b_\downarrow)$$

$$K^{(ij)}_{\sigma_1\sigma_2\sigma_3\sigma_4} \equiv (2\pi\hbar^2 \xi^{(ij)}_{\sigma_1\sigma_2\sigma_3\sigma_4} / \mu_{ij}) \int \phi^*_{i\sigma_1}(\mathbf{r}) \phi^*_{j\sigma_2}(\mathbf{r}) \phi_{j\sigma_3}(\mathbf{r}) \phi_{i\sigma_4}(\mathbf{r}) d^3 r$$

$$K^{(ii)}_{\sigma\sigma} \equiv K^{(ii)}_{\sigma\sigma\sigma\sigma}$$

$$K^{(ii)}_{\sigma\bar{\sigma}} \equiv 2K^{(ii)}_{\sigma\bar{\sigma}\bar{\sigma}\sigma} = 2K^{(ii)}_{\sigma\bar{\sigma}\sigma\bar{\sigma}}$$

$$K^{(ab)}_{\sigma\sigma'} \equiv K^{(ab)}_{\sigma\sigma'\sigma'\sigma}$$

$$K_e \equiv 2K^{(ab)}_{\uparrow\downarrow\uparrow\downarrow} = 2K^{(ab)}_{\downarrow\uparrow\downarrow\uparrow}$$

$$f_{i\sigma} \equiv \epsilon_{i\sigma} - K^{(ii)}_{\sigma\sigma}/2$$

$$\epsilon_{a\uparrow} - \epsilon_{a\downarrow} = \epsilon_{b\downarrow} - \epsilon_{b\uparrow}$$

Spin representation

$$\mathbf{S}_a = \sum_{\sigma,\sigma'} a^\dagger_\sigma \mathbf{s}_{\sigma\sigma'} a_{\sigma'}, \ \mathbf{S}_b = \sum_{\sigma,\sigma'} b^\dagger_\sigma \mathbf{s}_{\sigma\sigma'} b_{\sigma'}$$

- The Hamiltonian becomes that of two big spins $S_a = N_a/2$ and $S_b = N_b/2$

$$\frac{\mathcal{H}}{J_z} = \frac{K_e}{J_z}(S_{ax}S_{bx} + S_{ay}S_{by}) + S_{az}S_{bz} + B_a S_{az} + B_b S_{bz} + C_a S_{az}^2 + C_b S_{bz}^2 + \frac{E_0}{J_z}$$

- Coefficients are functions of K’s.

Conserved Quantities

- N_a and N_b , hence S_a and S_b

  $$N_i = N_{i\uparrow} + N_{i\downarrow}$$

- Total $S_z = (N_{a\uparrow} - N_{a\downarrow} + N_{b\uparrow} - N_{b\downarrow})/2$

Isotropic point

$$\mathcal{H} = J_z \mathbf{S}_a \cdot \mathbf{S}_b$$

Ground states:

$$|G_{S_z}\rangle = |S_a - S_b, S_z\rangle = A(a_\uparrow^\dagger)^{n_\uparrow}(a_\downarrow^\dagger)^{n_\downarrow}(a_\uparrow^\dagger b_\downarrow^\dagger - a_\downarrow^\dagger b_\uparrow^\dagger)^{N_b}|0\rangle$$

$$n_\uparrow = N_a/2 - N_b/2 + S_z,\ n_\downarrow = N_a/2 - N_b/2 - S_z$$

Degenerate but unique for a given S_z.

For $N_a = N_b = N$:

$$|G_0\rangle = (\sqrt{N+1}N!)^{-1}(a_\uparrow^\dagger b_\downarrow^\dagger - a_\downarrow^\dagger b_\uparrow^\dagger)^N|0\rangle$$

Concept of quantum entanglement

$$|\Psi\rangle \neq |\psi\rangle_A \otimes |\psi\rangle_B$$

- E.g. $\frac{1}{2}(|\uparrow\rangle|\downarrow\rangle - |\downarrow\rangle|\uparrow\rangle)$
- The most important concept distinguishing quantum mechanics from classical theory.
- Can be quantified as

$$S = \log \rho_A, \quad \rho_A = Tr_B|\Psi\rangle\langle\Psi|,$$

thanks to quantum information theory.

Using entanglement to characterize the non-mean field nature

$$|G_0\rangle = (\sqrt{N+1})^{-1}\sum_{m=0}^{N}(-1)^{N-m}|m\rangle_{a\uparrow}|N-m\rangle_{a\downarrow}|N-m\rangle_{b\uparrow}|m\rangle_{b\downarrow}$$

- Consider its occupation entanglement

  Method: YS, Phys.Rev.A 67, 024301 (03);
  J.Phys.A 37,6807 (04).

- The subsystems are the single particle basis states envolved.
- Entanglement entropy: von Neumann entropy of the reduced density matrix of a subsystem, which measures the entanglement with the rest of the system.

$N+1$

Using entanglement to characterize the non-mean field nature (continued)

$$|G_0\rangle = (\sqrt{N+1})^{-1} \sum_{m=0}^{N} (-1)^{N-m} |m\rangle_{a\uparrow} |N-m\rangle_{a\downarrow} |N-m\rangle_{b\uparrow} |m\rangle_{b\downarrow}$$

- For each single particle basis state, the occupation number is $N+1$-valued, so the base of the entanglement entropy is set to be $N+1$.
- $|G_0\rangle$ is an equal superposition of quart-orthogonal states, consequently the entanglement entropy for each single particle basis state is 1.

Entanglement between the two species

- The basis of A species is chosen to be $(a\uparrow, a\downarrow)$
- The occupation [always (m,N-m)] is still N+1-valued.
- Consequently the entanglement between the two species is 1.

$$|G_0\rangle = (\sqrt{N+1})^{-1} \sum_{m=0}^{N} (-1)^{N-m} |m\rangle_{a\uparrow} |N-m\rangle_{a\downarrow} |N-m\rangle_{b\uparrow} |m\rangle_{b\downarrow}$$

Entanglement as a kind of pairing

- Note $(a_\uparrow^\dagger b_\downarrow^\dagger - a_\downarrow^\dagger b_\uparrow^\dagger)^{N_b} = [\sqrt{2}\int d^3r_a d^3r_b \psi_a^\dagger(\mathbf{r}_a)\psi_b^\dagger(\mathbf{r}_b)\phi(\mathbf{r}_a,\mathbf{r}_b)]^{N_b}$

$\psi_a(\mathbf{r}) = \sum_\sigma a_\sigma \phi_{a\sigma}(\mathbf{r}_a)|\sigma\rangle_a,$ $\psi_b(\mathbf{r}) = \sum_\sigma b_\sigma \phi_{b\sigma}(\mathbf{r}_b)|\sigma\rangle_b$

$$\phi(\mathbf{r}_a,\mathbf{r}_b) \equiv \frac{1}{\sqrt{2}}[\phi_{a\uparrow}(\mathbf{r}_a)|\uparrow\rangle_a\phi_{b\downarrow}(\mathbf{r}_b)|\downarrow\rangle_b - \phi_{a\downarrow}(\mathbf{r}_a)|\downarrow\rangle_a\phi_{b\uparrow}(\mathbf{r}_b)|\uparrow\rangle_b]$$

- $|G_{S_z}\rangle$ is thus a condensation of interspecies pairs in the same two-particle entangled state $\phi(\mathbf{r}_a,\mathbf{r}_b)$
- $\phi(\mathbf{r}_a,\mathbf{r}_b)$ is the entangled order parameter.

Entangled pairing lowers the energy

A simple example:

$$h(\mathbf{r}_a) + h(\mathbf{r}_b) + U_1(\mathbf{r}_a - \mathbf{r}_b) + U_2(\mathbf{r}_a - \mathbf{r}_b)(|\uparrow\downarrow\rangle\langle\uparrow\downarrow| + |\downarrow\uparrow\rangle\langle\downarrow\uparrow|)$$

$$U_2 > 0$$

$$\phi_a(\mathbf{r}_a)\phi_b(\mathbf{r}_b)(|\uparrow\rangle|\downarrow\rangle - |\downarrow\rangle|\uparrow\rangle)/\sqrt{2}$$

has lower energy than

$$\phi_a(\mathbf{r}_a)\phi_b(\mathbf{r}_b)|\sigma\rangle|\sigma'\rangle$$

Detection of the entanglement (1)

- (Of course) fluctuations of $N_{i\sigma}$

$$\sqrt{\langle N_{a\sigma}^2 \rangle - \langle N_{a\sigma} \rangle^2} / \langle N_{a\sigma} \rangle \approx 1/\sqrt{3}$$

- Can be obtained from density fluctuation, which is self-averaging, and can be studied in a single image $\rho_{i\sigma}(\mathbf{r}_i) = N_{i\sigma} |\phi_{i\sigma}(\mathbf{r}_i)|^2$

$$\sqrt{\langle \rho_{i\sigma}(\mathbf{r}_i)^2 \rangle - \langle \rho_{i\sigma}(\mathbf{r}_i) \rangle^2} / \langle \rho_{i\sigma}(\mathbf{r}_i) \rangle = \sqrt{\langle N_{i\sigma}^2 \rangle - \langle N_{i\sigma} \rangle^2} / \langle N_{i\sigma} \rangle$$

- Free expansion of the condensate does not affect entanglement

Detection of the entanglement (2)

- Nonvanishing of the connected correlations $C_{\sigma,\sigma'} \equiv \langle N_{a\sigma} N_{b\sigma'} \rangle - \langle N_{a\sigma} \rangle \langle N_{b\sigma'} \rangle$

$$C_{\sigma,\sigma} = -N(N+2)/12,\ C_{\sigma,\bar{\sigma}} = N(N+2)/12$$

$$g(\mathbf{r}_a, \sigma; \mathbf{r}_b, \sigma') \equiv \langle \rho_{a\sigma}(\mathbf{r}_a) \rho_{b\sigma'}(\mathbf{r}_b) \rangle - \langle \rho_{a\sigma}(\mathbf{r}_a) \rangle \langle \rho_{b\sigma'}(\mathbf{r}_b) \rangle$$

$$g(\mathbf{r}_a, \sigma; \mathbf{r}_b, \sigma') / \langle \rho_{a\sigma}(\mathbf{r}_a) \rangle \langle \rho_{b\sigma'}(\mathbf{r}_b) \rangle = C_{\sigma,\sigma'} / \langle N_{a\sigma} \rangle \langle N_{b\sigma'} \rangle$$

Detection of entanglement (3)

- Measuring spin of an A-atom,

$$P_\sigma = \langle a_\sigma^\dagger a_\sigma \rangle / \sum_{\sigma'} \langle a_{\sigma'}^\dagger a_{\sigma'} \rangle$$

- Joint measurement of the spins of an A-atom and a B-atom which leave the trap

$$P_{\sigma,\sigma'} = \langle b_{\sigma'}^\dagger a_\sigma^\dagger a_\sigma b_{\sigma'} \rangle / \sum_{\sigma_a,\sigma_b} \langle b_{\sigma_b}^\dagger a_{\sigma_a}^\dagger a_{\sigma_a} b_{\sigma_b} \rangle.$$

Detection of entanglement (3) (continued)

- Mean-field (non-entangled) state:

$$(\sqrt{N_1!N_2!N_3!N_4!})^{-1} {a^\dagger_{\hat{\mathbf{n}}}}^{N_1} {a^\dagger_{-\hat{\mathbf{n}}}}^{N_2} {b^\dagger_{\hat{\mathbf{m}}}}^{N_3} {b^\dagger_{-\hat{\mathbf{m}}}}^{N_4} |0\rangle,$$

$$P_{\sigma_a,\sigma_b} = P_{\sigma_a} P_{\sigma_b}$$

Detection of entanglement (3) (continued)

- Non-mean-field (entangled) BEC:

$$P_{\sigma_a,\sigma_b} \neq P_{\sigma_a} P_{\sigma_b}$$

- E.g., for $|G_0\rangle$, $P_{\sigma_a} = P_{\sigma_b} = 1/2$,

$$P_{\uparrow\downarrow} = P_{\downarrow\uparrow} = (2N+1)/6N,$$

$$P_{\uparrow\uparrow} = P_{\downarrow\downarrow} = (N-1)/6N$$

Feedback effect on single-particle orbits

$$\{-\frac{\hbar^2}{2m_a}\nabla^2 + U_{a\sigma}(\mathbf{r}) + [N(N-1)/3]g^{(aa)}_{\sigma\sigma}|\phi_{a\sigma}(\mathbf{r})|^2$$
$$+ [N(N-1)/6]g^{(aa)}_{\uparrow\downarrow}|\phi_{a\bar{\sigma}}(\mathbf{r})|^2 + [N(N-1)/6]g^{(ab)}_{\sigma\sigma}|\phi_{b\sigma}(\mathbf{r})|^2$$
$$+ [N(2N+1)/6]g^{(ab)}_{\sigma\bar{\sigma}}|\phi_{b\bar{\sigma}}(\mathbf{r})|^2\}\phi_{a\sigma}(\mathbf{r})$$
$$- [N(N+2)/12]g_e\phi^*_{b\bar{\sigma}}(\mathbf{r})\phi_{b\sigma}(\mathbf{r})\phi_{a\bar{\sigma}}(\mathbf{r})$$
$$= \mu_{a\sigma}\phi_{a\sigma}(\mathbf{r})$$

- Interference term (proportional to g_e) emerges.

How the entanglement survives the coupling anisotropy and the nonvanishing of

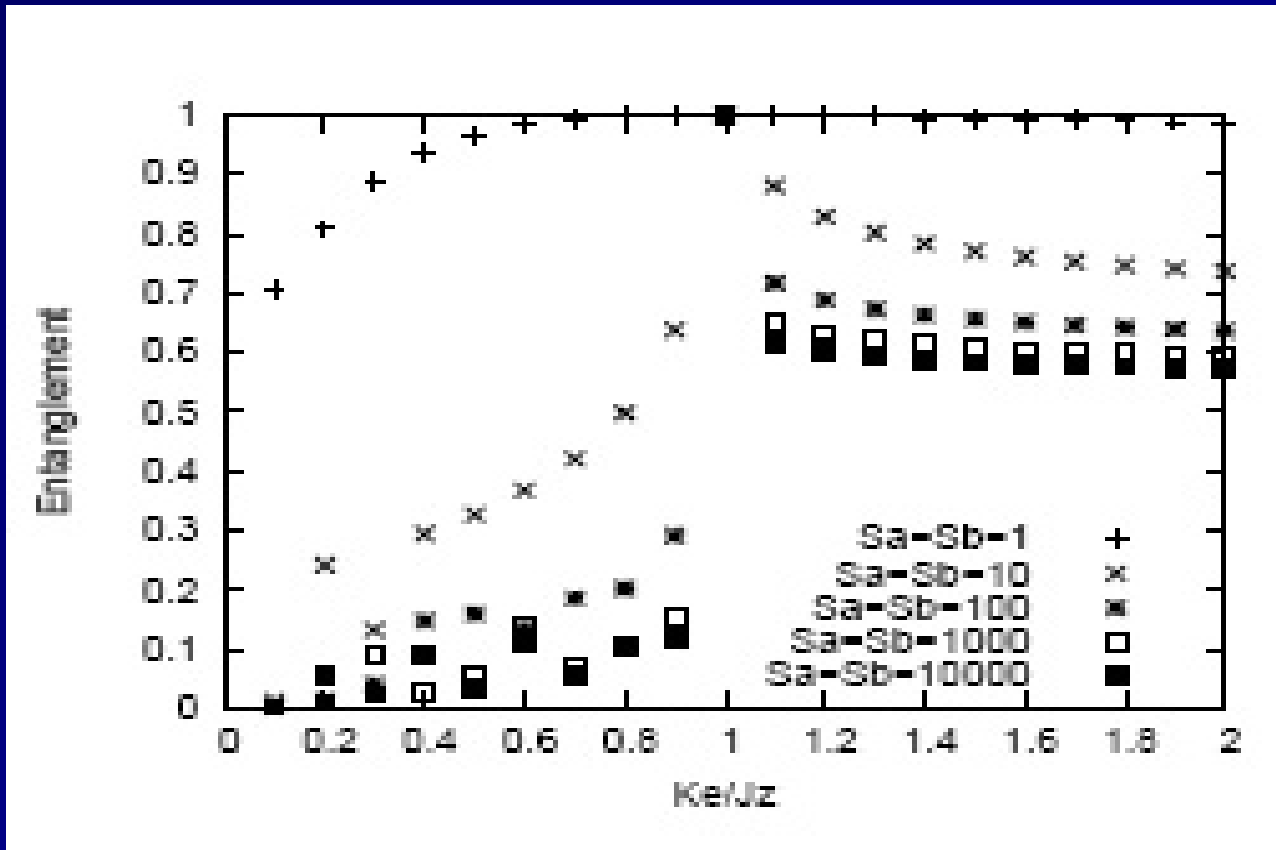
$$B_a,\ B_b,\ C_a\ ,\ C_b$$

$$\frac{\mathcal{H}}{J_z} = \frac{K_e}{J_z}(S_{ax}S_{bx} + S_{ay}S_{by}) + S_{az}S_{bz} + B_aS_{az} + B_bS_{bz} + C_aS_{az}^2 + C_bS_{bz}^2 + \frac{E_0}{J_z}$$

Persistence of entanglement in a wide parameter regime (1)

- Coupling anisotropy K_e/J_z , $B_a = B_b = C_a = C_b = 0$

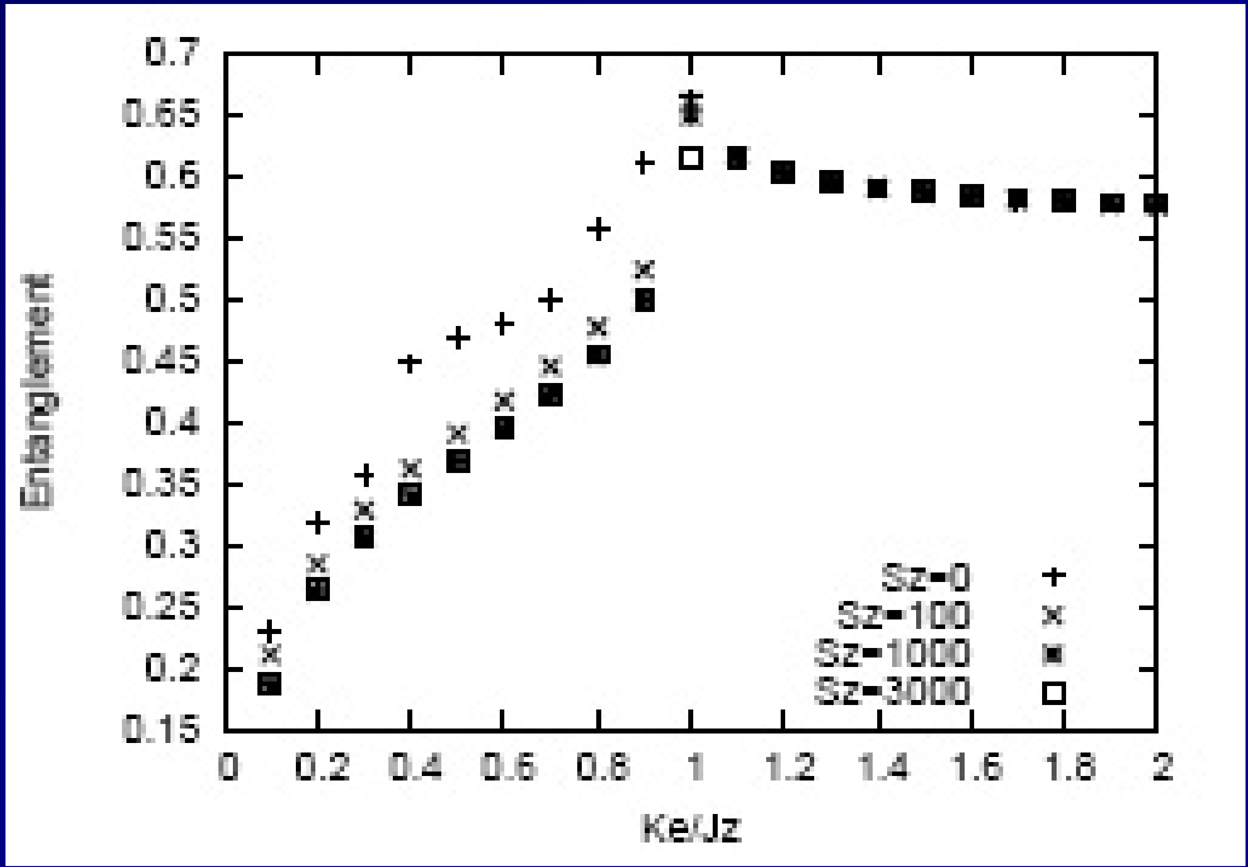
  $S_z = 0$



Persistence of entanglement in a wide parameter regime (2)

- Coupling anisotropy K_e/J_z $B_a = B_b = C_a = C_b = 0$

$$S_a = 12000,\ S_b = 10000$$

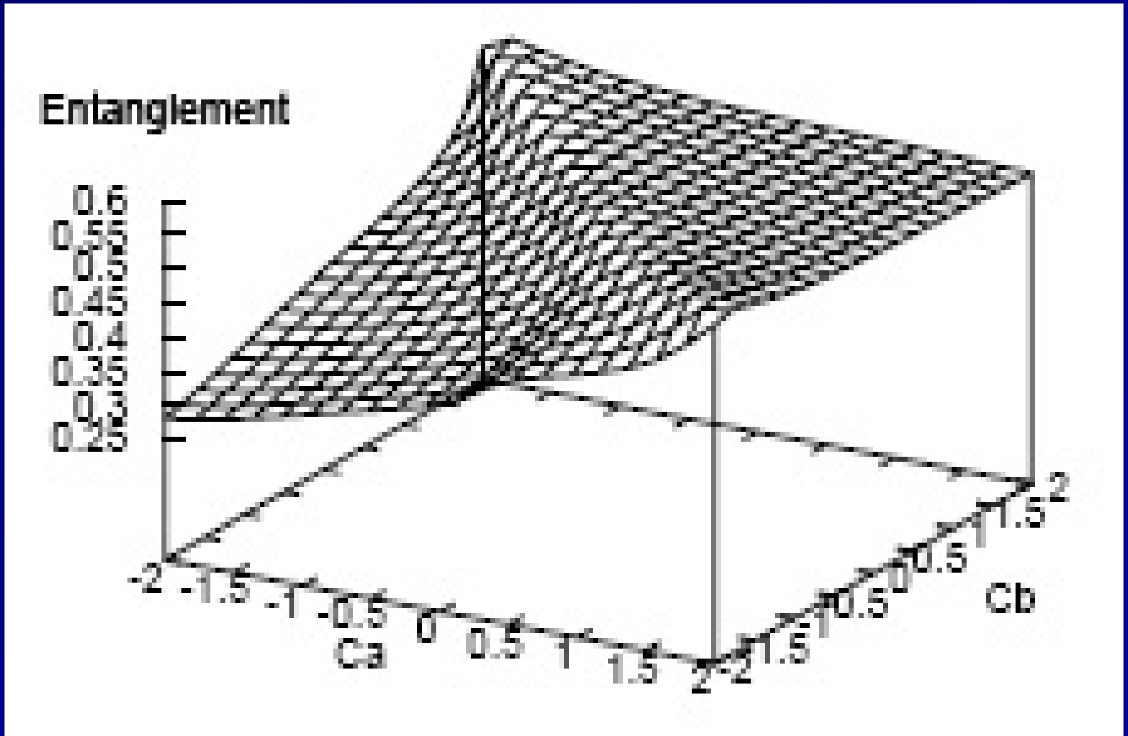


Persistence of entanglement in a wide parameter regime (3)

- C_a and C_b nonzero, $B_a = B_b = 0$, under typical values

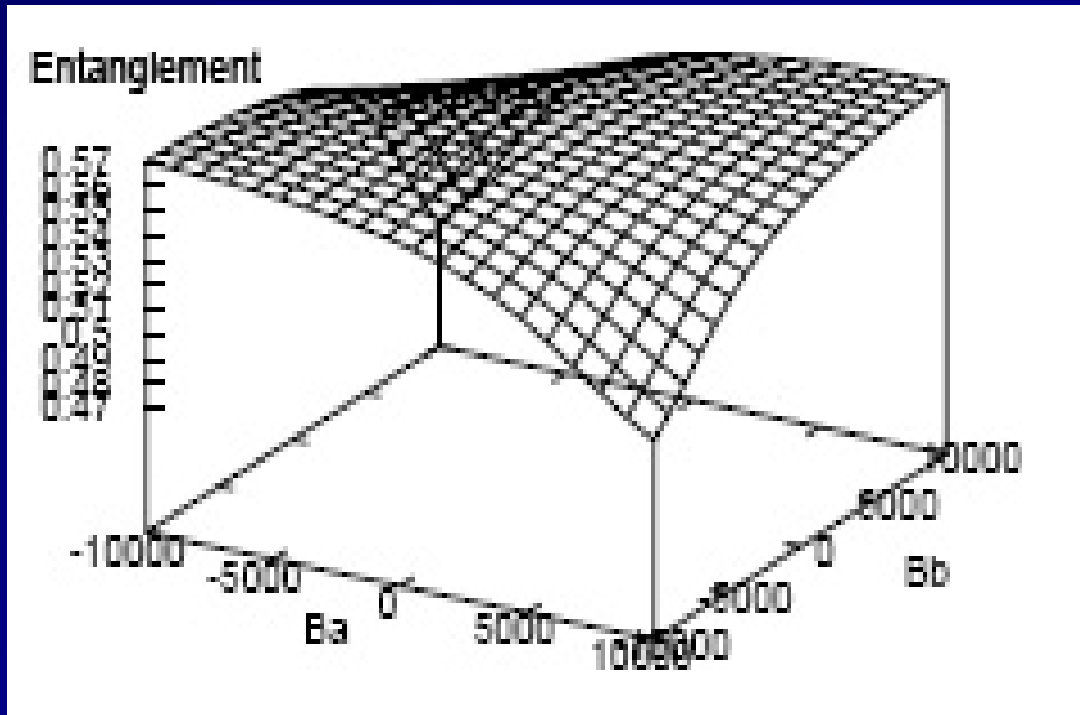
  $S_a = 12000,\ S_b = 10000,\ S_z = 1000,\ K_e/J_z = 1.2$

  J_z, C_aJ_z and C_bJ_z are of the same order of magnitude



Persistence of entanglement in a wide parameter regime (4)

Typically choose $C_a = 0.2$ and $C_b = 0.4$



Therefore, in a wide parameter regime, the ground state is of non-mean-field.

Energy difference with the mean-field state

- It is (of course) vanishing at the isotrpic limit.
- But it is estimated that

  when $|B_a - B_b|$ is of the order of N, or $C_a + C_b$ is of the order of -1, the energy difference with the lowest mean-field (symmetry breaking) state is of the order of N/V
- Finite in thermodynamic limit!
- In this regime, as far as K_e is larger or not much smaller than J_z, the entanglement is significantly nonvanishing.

Summary

- We proposed a non-mean-field ground state of BEC, occuring in an interspecies two-particle entangled stated.
- Hence the order parameter is entangled.
- Interspecies entanglement persists in a wide parameter regime.
- In part of the regime, the energy difference with the mean-field state is nonvanishing.
- Call for study of interspecies multichannel scattering.

Thank you for your attention!