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3 - 7 July 2006

Mesoscopic anisotropic magnetoconductance fluctuations in ferromagnets

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Mesoscopic Anisotropic Magnetoconductance Fluctuations in Ferromagnets

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Fourth Stig Lundqvist Conference
ICTP, Trieste
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For details: S. Adam, M. Kindermann, S. Rahav and P.W. Brouwer, Phys. Rev. B **73** 212408 (2006)

Outline:

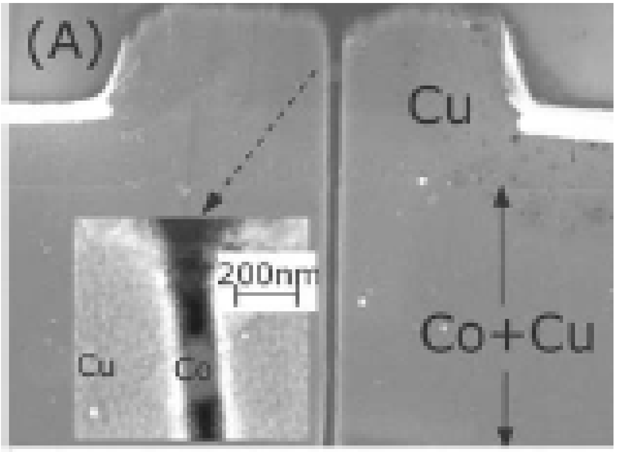
- **Motivation (recent experiments of phase coherent transport in ferromagnetic systems)**
- **Introduction to theory of disordered metals**
- **Analog of Universal Conductance Fluctuations in nanomagnets**

Motivation:

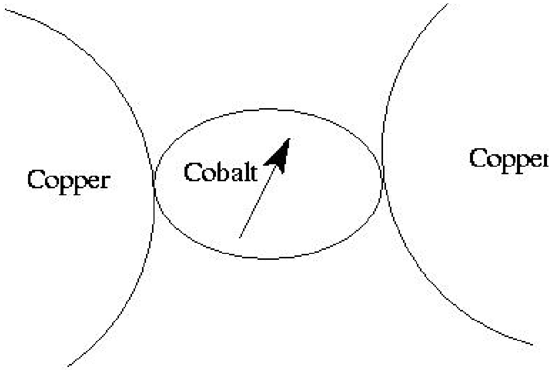
PRL 96, 146803 (2006) PHYSICAL REVIEW LETTERS week ending 14 APRIL 2006

Mesoscopic Resistance Fluctuations in Cobalt Nanoparticles

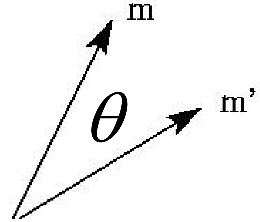
Y. G. Wei, X. Y. Liu, L. Y. Zhang, and D. Davidović
Georgia Institute of Technology, Atlanta, Georgia 30332, USA
(Received 15 September 2005; published 14 April 2006)



Picture taken from Davidovic group



Cu-Co interface are good contacts



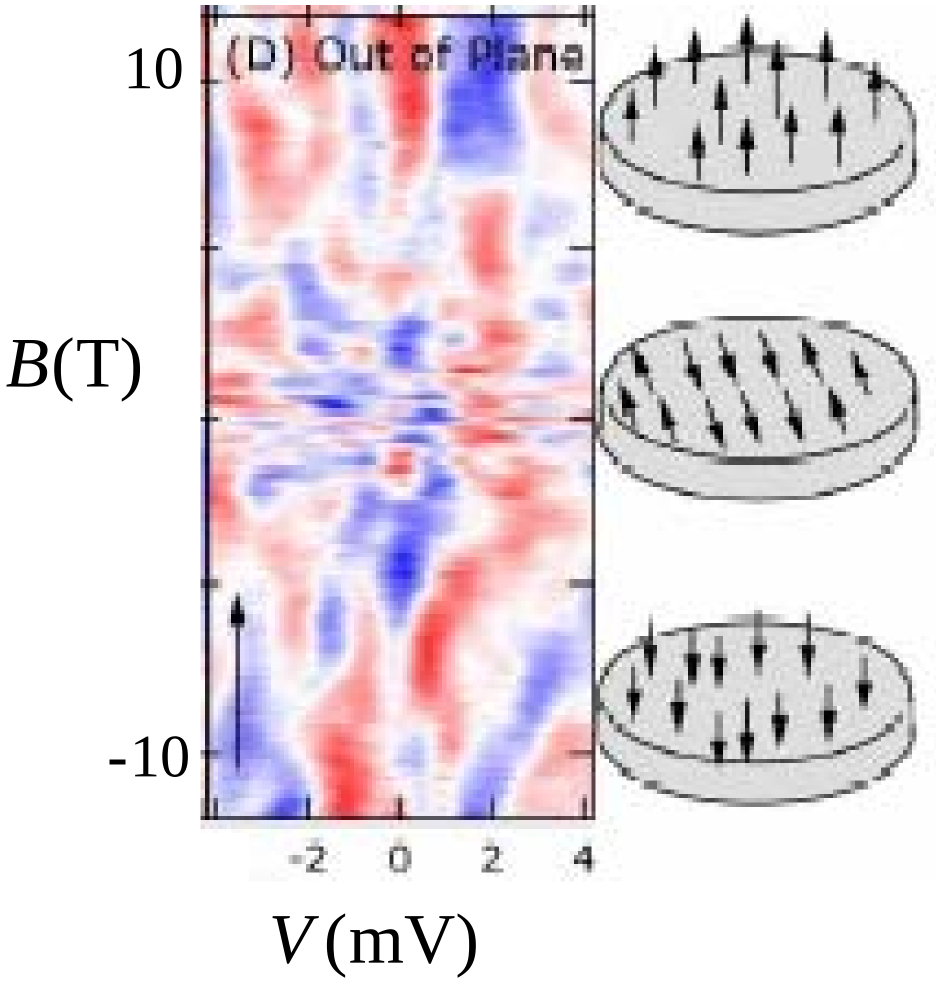
$$\ell \sim 5\ nm$$
$$L_\varphi \sim 30\ nm$$

$$\cos(\theta) = \vec{m} \cdot \vec{m}'$$

$$\ell << L_\phi, \text{ disordered regime}$$

$$C(\theta) = < G(\vec{m}) G(\vec{m}') > - < G(\vec{m}) >^2$$

Physical System we are Studying [2]



$$C(\theta) = < G(\vec{m})G(\vec{m}') > - < G(\vec{m}) >^2$$

$$\theta \in [0, \pi / 2]$$

- **Aharanov-Bohm contribution**
- **Spin-Orbit Effect**

Data/Pictures: Y. Wei, X. Liu, L. Zhang and D. Davidovic, PRL (2006)

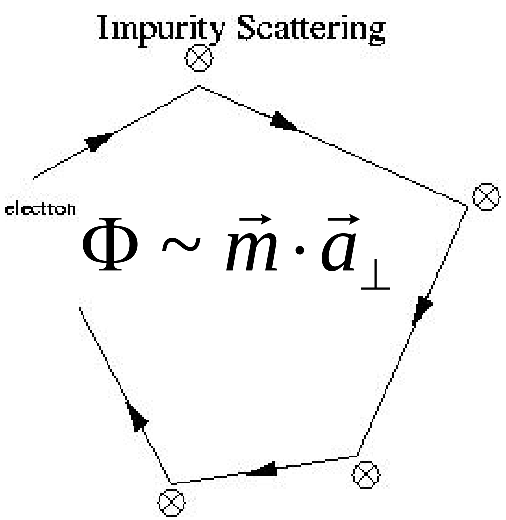
Physical System we are Studying [3]

$$C(\theta) = \langle G(\vec{m})G(\vec{m}') \rangle - \langle G(\vec{m}) \rangle^2$$

θ_c is correlation angle

Aharanov-Bohm contribution

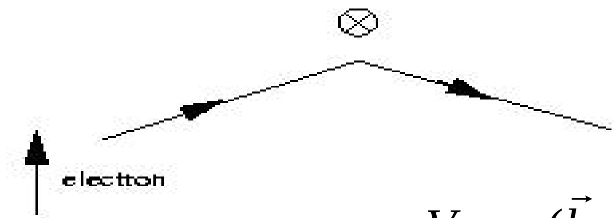
$$\theta_c \sim \pi/2$$



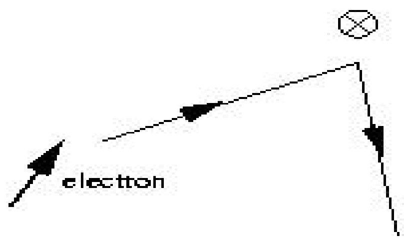
$$\Phi \sim \vec{m} \cdot \vec{a}_\perp$$

Spin-Orbit Effect

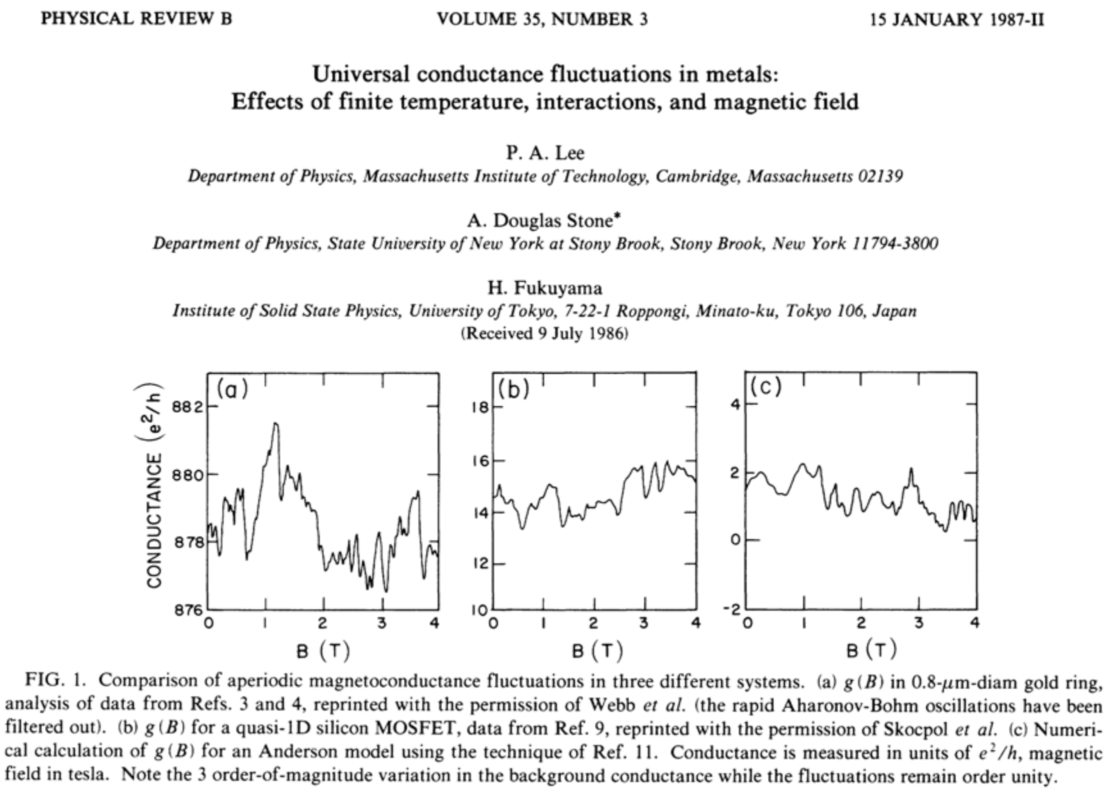
$$\theta_c \sim \pi/10$$



$$V_{SO} \sim (\vec{k} \times \vec{k}') \cdot \vec{S}$$
$$\sim (\vec{k} \times \vec{k}') \cdot \vec{m}$$



Universal Conductance Fluctuations



PHYSICAL REVIEW B VOLUME 35, NUMBER 3 15 JANUARY 1987-II

Universal conductance fluctuations in metals:
Effects of finite temperature, interactions, and magnetic field

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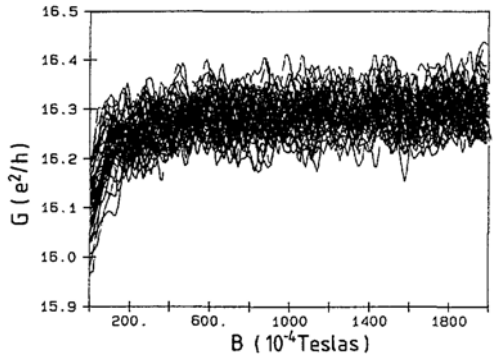
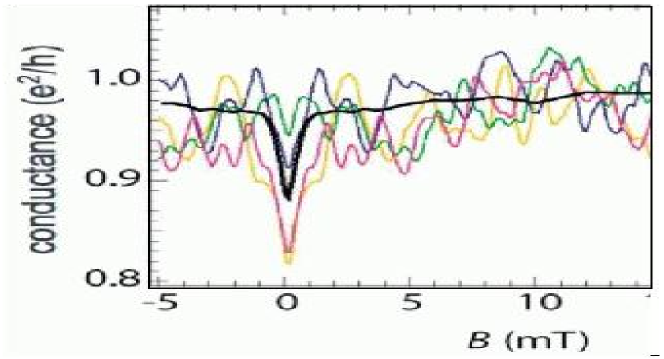
(Received 9 July 1986)

FIG. 1. Comparison of aperiodic magnetoconductance fluctuations in three different systems. (a) $g(B)$ in 0.8-μm-diam gold ring, analysis of data from Refs. 3 and 4, reprinted with the permission of Webb *et al.* (the rapid Aharonov-Bohm oscillations have been filtered out). (b) $g(B)$ for a quasi-1D silicon MOSFET, data from Ref. 9, reprinted with the permission of Skocpol *et al.* (c) Numerical calculation of $g(B)$ for an Anderson model using the technique of Ref. 11. Conductance is measured in units of e^2/h, magnetic field in tesla. Note the 3 order-of-magnitude variation in the background conductance while the fluctuations remain order unity.



[Mailly and Sanquer (1992)]

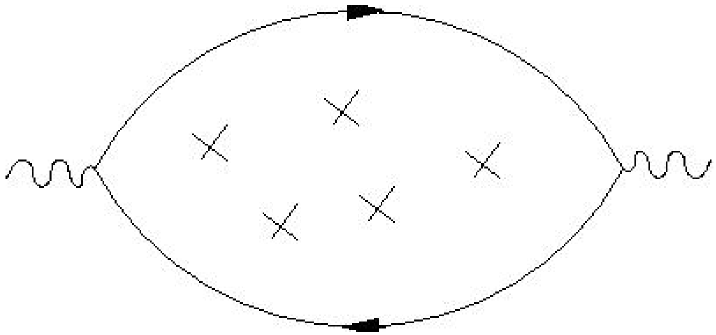


[C. Marcus]

Theory: Lee and Stone (1985), Altshuler (1985)

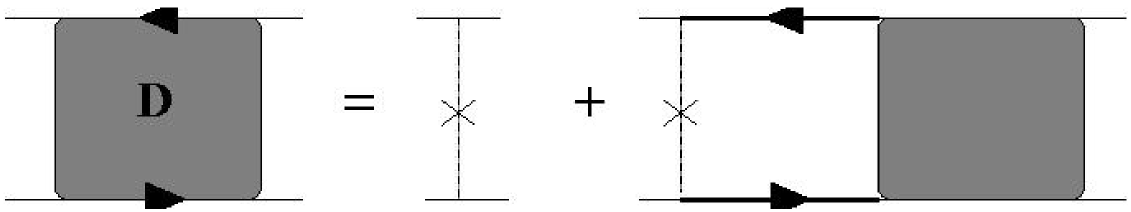
Review of Diagrammatic Perturbation Theory (Kubo Formula)

Conductance G ~



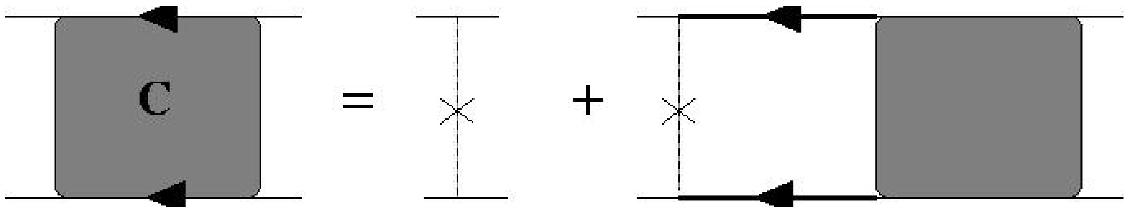
Diffuson

$$\frac{1}{\tau(Dq^2 - i\omega)}$$



Cooperon

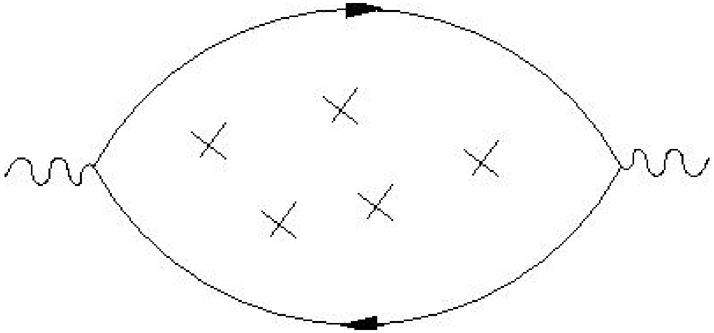
$$\frac{1}{\tau(D\tilde{q}^2 - i\omega)}$$

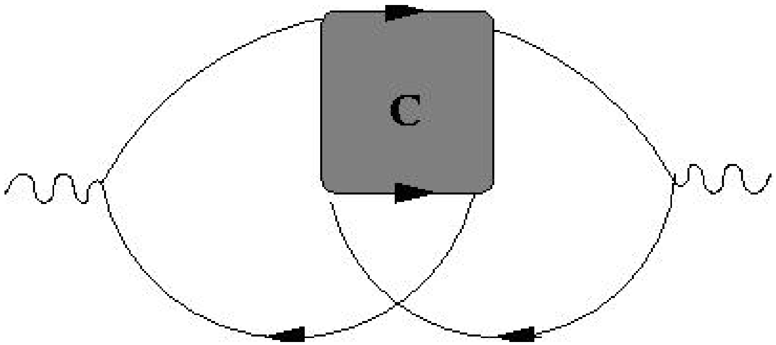


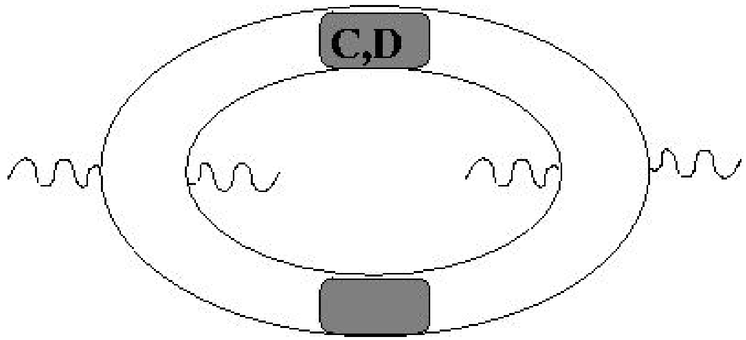
Cooperon defined similar to Diffuson upto normalization

Calculating Weak Localization and UCF

G







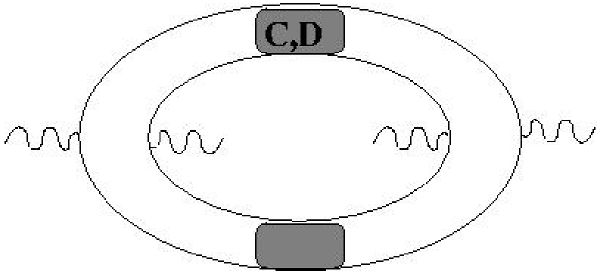
$< G >$

$< GG >_c$

Weak Localization

Universal Conductance Fluctuations

Calculation of UCF Diagrams



$$< GG > = \left(\frac{e^2}{h}\right)^2 \frac{3}{2L_z^2} Tr[JDJD + JCJC]$$

$$= \left(\frac{e^2}{h}\right)^2 \frac{3}{2} \sum_{C,D} \left(\frac{4}{\pi^4}\right) \sum_n \frac{1}{\lambda_n^2}$$

Sum is over the Diffusion Equation Eigenvalues scaled by Thouless Energy

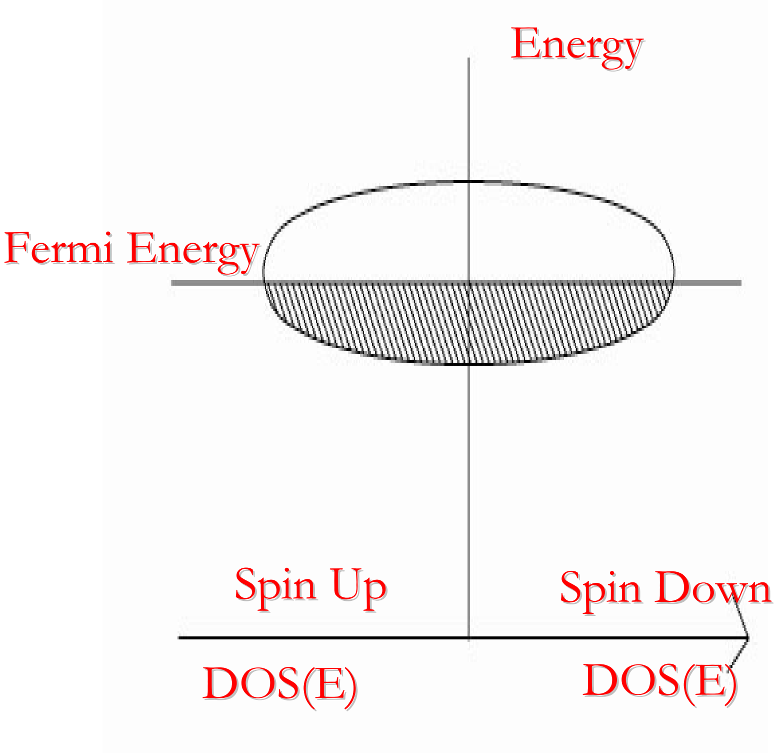
$$\lambda_n = n_z^2 + n_x^2 \left(\frac{L_z}{L_x}\right)^2 + n_y^2 \left(\frac{L_z}{L_y}\right)^2 - \frac{i\Delta E}{E_T}, \qquad \begin{array}{l} n_z = 1,2,\ldots \\ n_x, n_y = 0,1,2,\ldots \end{array} \qquad E_T = D\left(\frac{\pi}{L_z}\right)^2$$

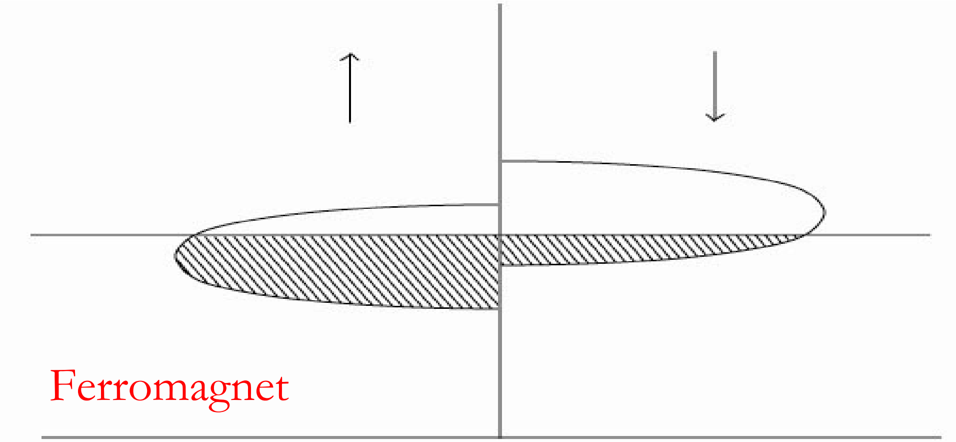
Quasi 1D can be done analytically,
and 3D can be done numerically: Var G = 0.272

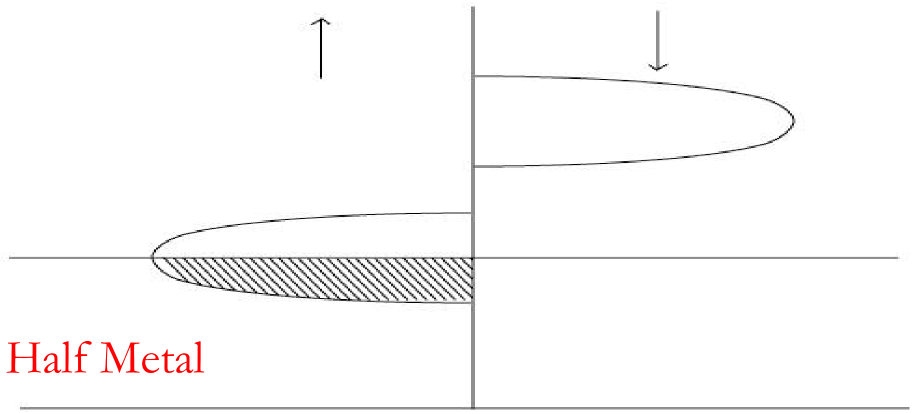
$$\sum_n \frac{1}{n^4} = \frac{\pi^4}{90} \qquad < GG > = \left(\frac{e^2}{h}\right)^2 \sum_{C,D} \frac{1}{15} \sim \frac{2}{15}$$

x 4 for spin

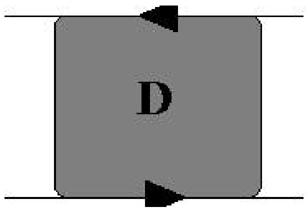
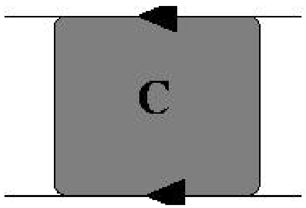
Effect of Spin-Orbit (Half-Metal example) [1]







Effect of Spin-Orbit
(Half-Metal example) [2]

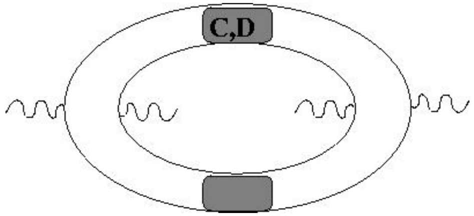
	Without S-O	With S-O
H	$V_{\vec{k}-\vec{k}'}$ $<V_q V_{q'}> = \frac{\delta(q-q')}{2\pi\upsilon\tau V}$	$V_{\vec{k}-\vec{k}'} - iV^{so}{}_{\vec{k}-\vec{k}'}\,\vec{m}\cdot(\vec{k}'\times\vec{k})/k_F^2$ $<V_q^{so} V_{q'}^{so}> = \frac{\delta(q-q')}{2\pi\upsilon\tau_{so} V}$
	$\frac{1}{\tau(Dq^2 - i\omega)}$	$\frac{1}{\tau\left(Dq^2 - i\omega + \frac{1-\vec{m}\cdot\vec{m}'}{\tau_{so}}\right)}$
	$\frac{1}{\tau(Dq^2 - i\omega)}$	$\frac{1}{\tau\left(Dq^2 - i\omega + \frac{1+\vec{m}\cdot\vec{m}'}{\tau_{so}}\right)}$

NOTE: For m=m', Spin-Orbit does not affect the Diffuson (classical motion) but large S-O kills the Copperon (interference)

Calculation of C(m,m') in Half-Metal

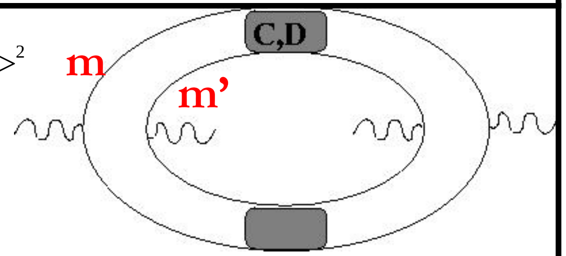
<table>
<tr><th>Without S-O</th><th>With S-O</th></tr>
<tr><td>

$< GG >$

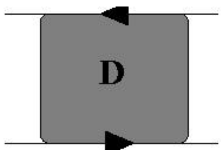


</td><td>

$C(\theta)=<G(\vec{m})G(\vec{m}')>-<G(\vec{m})>^2$

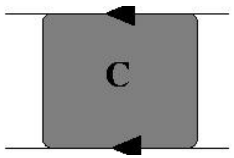


</td></tr>
<tr><td>

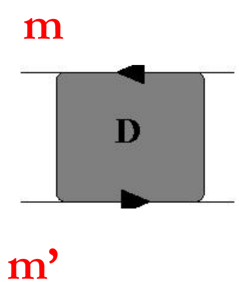


$= \dfrac{1}{\tau(Dq^2-i\omega)}$

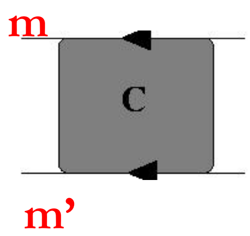
$=$



</td><td>



$(\theta)=$



$(\pi-\theta)$

$= \dfrac{1}{\tau\left(Dq^2-i\omega+\dfrac{1-\cos\theta}{\tau_{so}}\right)}$

</td></tr>
<tr><td>

$$\sum_n \frac{1}{n^4}=\frac{\pi^4}{90}$$

</td><td>

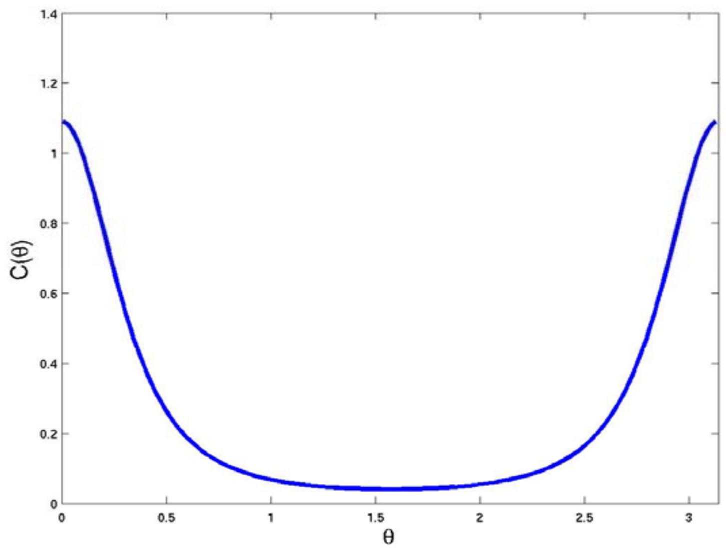
$$\sum_n \frac{1}{(n^2+A)^2}=\frac{\pi^4}{4x^4}[-2+x\coth(x)+x^2\sinh^{-2}(x)]$$

$A=\dfrac{1\mp\cos(\theta)}{E_T\tau_{so}}$ $\quad$ $x=\pi\sqrt{\dfrac{1\mp\cos(\theta)}{E_T\tau_{so}}}$

</td></tr>
</table>

Results for Half-Metal

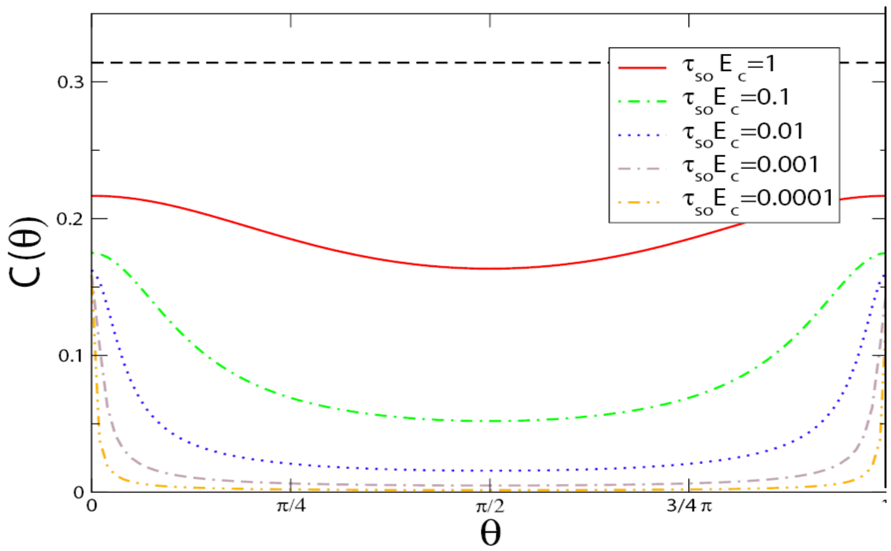
D=1, Analytic Result



$$C(\theta)=\frac{3}{2}\left(\frac{e^2}{h}\right)^2\left[F\left(\pi\sqrt{\frac{1-\cos\theta}{E_T\tau_{s0}}}\right)+F\left(\pi\sqrt{\frac{1+\cos\theta}{E_T\tau_{s0}}}\right)\right]$$

$$F(x)=\frac{-2+x\coth(x)+x^2\sinh^{-2}(x)}{x^4}$$

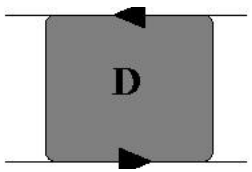
D=3, Done Numerically



$\theta \leftrightarrow \pi-\theta$, changes definition of $\uparrow,\downarrow$

We can estimate correlation angle for parameters and find about five UCF oscillations for 90 degree change

Full Ferromagnet

	Half Metal	Ferromagnet
$H_{\alpha\beta}$	$V_{\vec{k}-\vec{k}'} - iV^{so}_{\vec{k}-\vec{k}'}\ \vec{m}\cdot(\vec{k}'\times\vec{k})/k_F^2$ $< V_q^{so} V_{q'}^{so} > = \dfrac{\delta(q-q')}{2\pi\upsilon\tau_{so} V}$	$V_{\vec{k}-\vec{k}'} + E_z\sigma^z_{\alpha\beta}$ $-iV^{so}_{\vec{k}-\vec{k}'}\ (\vec{m}\sigma^z + e_1\sigma^x + e_2\sigma^y)_{\alpha\beta}\cdot(\vec{k}'\times\vec{k})/k_F^2$
 	$\dfrac{1}{\tau\left(Dq^2 - i\omega + \dfrac{1-\vec{m}\cdot\vec{m}'}{\tau_{so}}\right)}$	$\sum_{\gamma=\uparrow,\downarrow} K_{\alpha\gamma} D(\omega,\mathrm{q},\theta)_{\gamma\beta} = \delta_{\alpha\beta}\dfrac{1}{2\pi\nu_\alpha\tau_\alpha}.$ $\hat{K}_{\alpha\alpha} = \tau_\alpha\left[D_\alpha q^2 + i\omega + \dfrac{2}{\tau_{\alpha\perp}} + \dfrac{1-\cos\theta}{\tau_{\alpha\parallel}}\right];$ $K_{\uparrow\downarrow}K_{\downarrow\uparrow} = \dfrac{\tau_\uparrow\tau_\downarrow}{\tau_{\uparrow\perp}\tau_{\downarrow\perp}}(1+\cos\theta)^2.$
λ_n	$n^2 + A(\theta)$ are Eigenvalue s of Diffusion Equation	$\sum_\pm n^2 + a_\pm(\theta)$ are Eigenvalues of 2×2 Diffusion Equation
	$A = \dfrac{1-\cos\theta}{E_T\tau_{so}}$	$a_\pm(\theta) = \dfrac{1}{\tau_{\uparrow\perp}E_\uparrow} + \dfrac{1}{\tau_{\downarrow\perp}E_\downarrow} + \dfrac{\tau_{\uparrow\parallel}E_\uparrow + \tau_{\downarrow\parallel}E_\downarrow}{2\tau_{\uparrow\parallel}\tau_{\downarrow\parallel}E_\uparrow E_\downarrow}(1-\cos\theta))$ $\pm\sqrt{\dfrac{(1+\cos\theta)^2}{\tau_{\uparrow\perp}\tau_{\downarrow\perp}E_\uparrow E_\downarrow} + \left[\dfrac{1}{\tau_{\uparrow\perp}E_\uparrow} - \dfrac{1}{\tau_{\downarrow\perp}E_\downarrow} - \dfrac{\tau_{\uparrow\parallel}E_\uparrow - \tau_{\downarrow\parallel}E_\downarrow}{2\tau_{\uparrow\parallel}\tau_{\downarrow\parallel}E_\uparrow E_\downarrow}(1-\cos\theta)\right]^2}$

Results for C(m,m') in Ferromagnet

$$C(\theta)=\frac{3}{2}\left(\frac{e^2}{h}\right)^2\left[F\left(a_\pm(\theta)\right)+F\left(a_\pm(\pi-\theta)\right)\right] \qquad F(x)=\frac{-2+x\coth(x)+x^2\sinh^{-2}(x)}{x^4}$$

$$\begin{aligned} a_\pm(\theta) &= \frac{1}{\tau_{\uparrow\perp}E_\uparrow}+\frac{1}{\tau_{\downarrow\perp}E_\downarrow}+\frac{\tau_{\uparrow\|}E_\uparrow+\tau_{\downarrow\|}E_\downarrow}{2\tau_{\uparrow\|}\tau_{\downarrow\|}E_\uparrow E_\downarrow}(1-\cos\theta)) \\ &\pm\sqrt{\frac{(1+\cos\theta)^2}{\tau_{\uparrow\perp}\tau_{\downarrow\perp}E_\uparrow E_\downarrow}+\left[\frac{1}{\tau_{\uparrow\perp}E_\uparrow}-\frac{1}{\tau_{\downarrow\perp}E_\downarrow}-\frac{\tau_{\uparrow\|}E_\uparrow-\tau_{\downarrow\|}E_\downarrow}{2\tau_{\uparrow\|}\tau_{\downarrow\|}E_\uparrow E_\downarrow}(1-\cos\theta)\right]^2} \end{aligned}$$

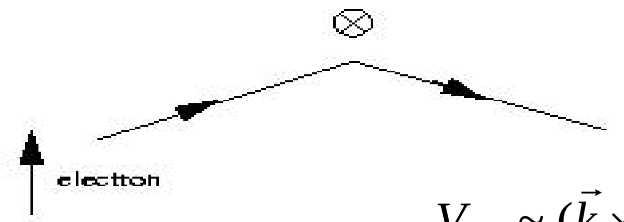
Limiting Cases for m = m'

m=m'	SO	C	D	spin	Total
Normal Metal	-	1/15	1/15	4	8/15
Half Metal	No	1/15	1/15	1	2/15
Half Metal	Strong	0	1/15	1	1/15
Ferromagnet	Weak	1/15	1/15	2	4/15
Ferromagnet	Strong	0	1/15	1	1/15

Conclusions:

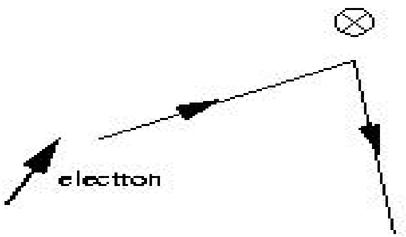
Showed how spin-orbit scattering causes Mesoscopic Anisotropic Magnetoconductance Fluctuations in half-metals (This is the analog of UCF for ferromagnets)

This effect can be probed experimentally

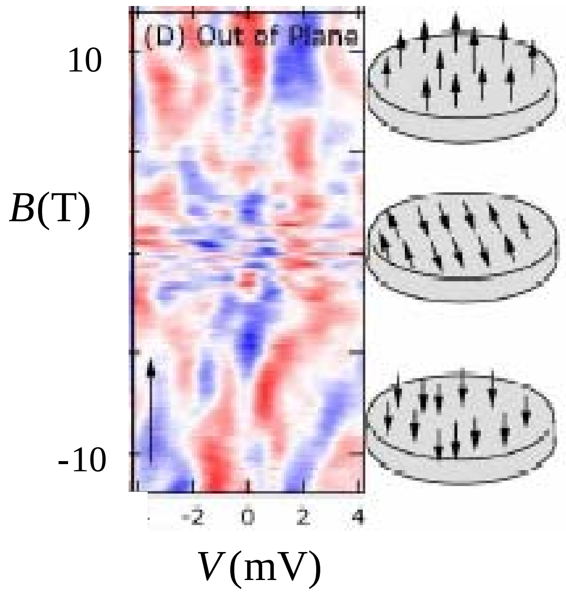


$$V_{SO} \sim (\vec{k} \times \vec{k}') \cdot \vec{S}$$

$$\sim (\vec{k} \times \vec{k}') \cdot \vec{m}$$



$$5\,\theta_c \sim \pi/2$$



Backup Slides

Magnetic Properties of Nanoscale Conductors

Shaffique Adam

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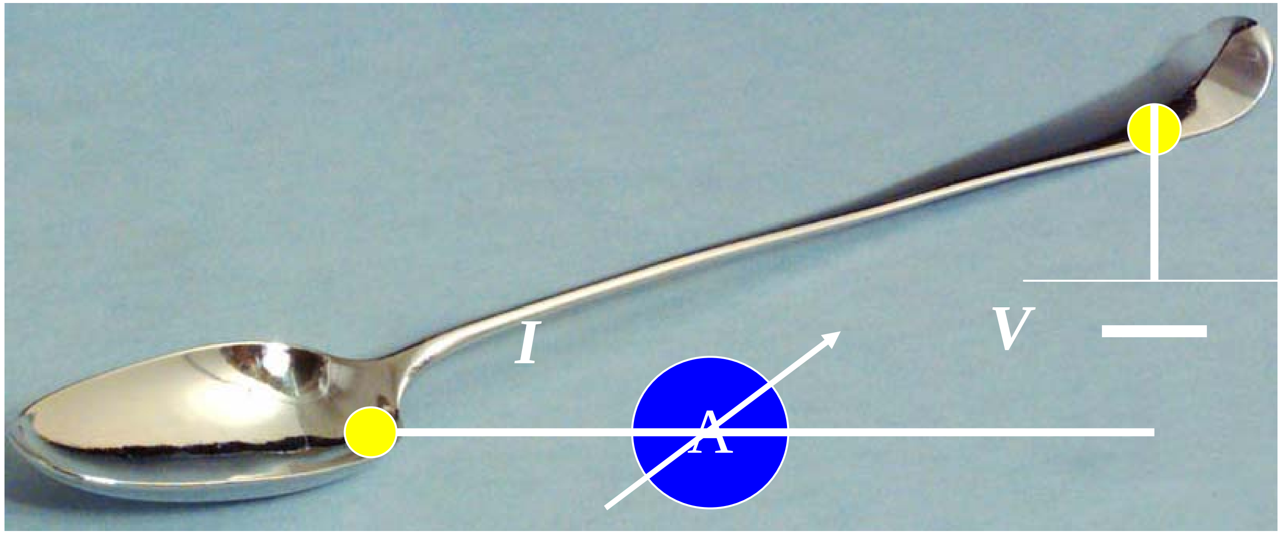
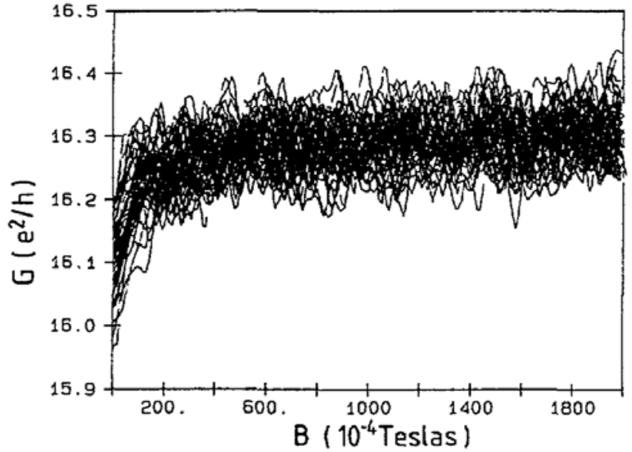
Introduction to Phase Coherent Transport



Smaller and colder!

Image Courtesy (L. Glazman)

Conductance: $G = 1/R$



[Mailly and Sanquer, 1992]

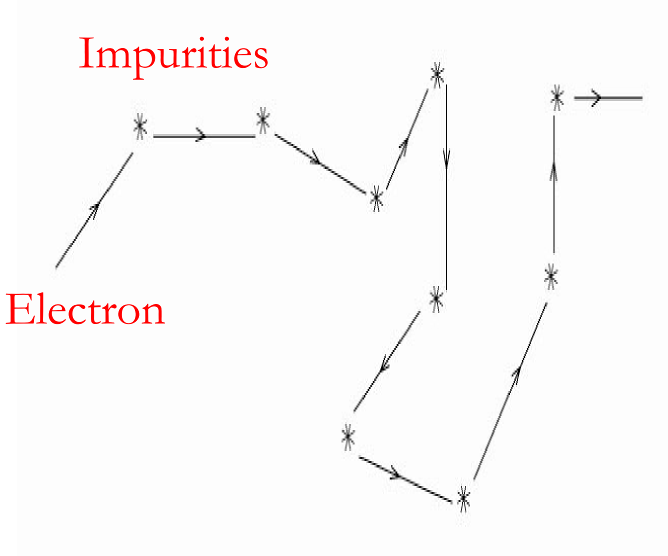
Sample dependent fluctuations are reproducible (not noise)

Ensemble Averages

Need a theory for the mean <G> and fluctuations <GG>

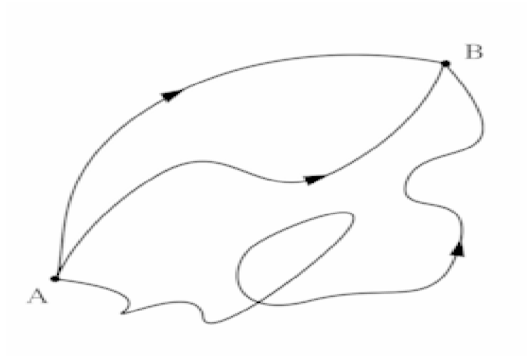
Introduction to Phase Coherent Transport [2]

Electron diffusing in a dirty metal



$\mu = path$

$\phi_\mu \sim path, flux$



$$A_\mu = C_\mu e^{i\phi_\mu}$$

$$P_{A\to B} = \left|\sum_\mu A_\mu\right|^2 = \sum_{\mu,\nu} A_\mu A_\nu^*$$

$$P_{A\to B} = \sum_\mu |A_\mu|^2 + \sum_{\mu\neq\nu} A_\mu A_\nu^*$$

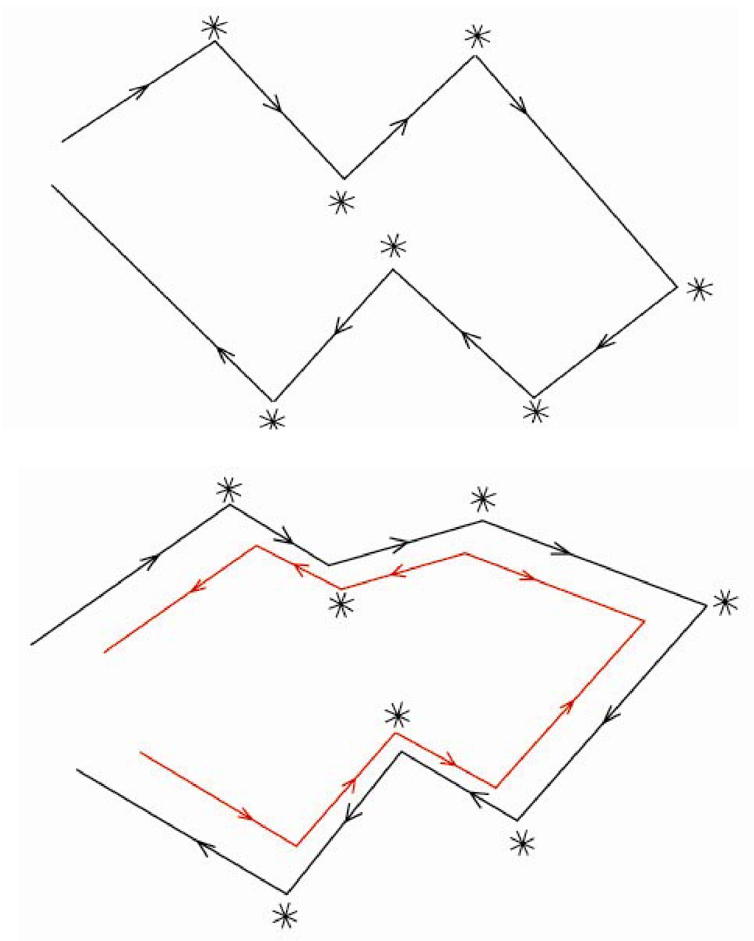
Classical Contribution — Quantum Interference

Diffuson — Cooperon

Backup Slide

Introduction to Phase Coherent Transport [3]

Weak Localization in Pictures



For no magnetic field, the phase depends only on the path.

Every possible path has a twin that is exactly the same, but which goes around in the opposite direction.

Because these paths have the same flux and picks up the same phase, they can interfere constructively.

Therefore the probability to return to the starting point in enhanced (also called enhanced back scattering).

In fact the quantum probability to return is exactly twice the classical probability

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Introduction to Phase Coherent Transport [4]

Weak Localization in Equations

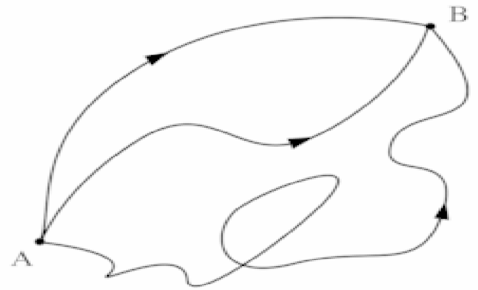
$$A_\mu = C_\mu e^{i\phi_\mu}$$

$$P_{A\to B} = |\sum_\mu A_\mu|^2 = \sum_{\mu,\nu} A_\mu A_\nu^*$$

$$P_{A\to B} = \sum_\mu |A_\mu|^2 + \sum_{\mu\neq\nu} A_\mu A_\nu^*$$

Classical Contribution — Diffuson

Quantum Interference — Cooperon



$\mu = path$

$\bar{\mu} = reversepath$

$\phi_\mu \sim path, flux$

$$A_\mu = C_\mu e^{i\phi_\mu}$$

$$P_{A\to B} = |\sum_\mu A_\mu|^2 = \sum_{\mu,\nu} A_\mu A_\nu^*$$

$$P_{A\to B} = \sum_\mu |A_\mu|^2 + \sum_\mu A_\mu A_{\bar{\mu}}^* + \sum_{\mu\neq\bar{\mu},\nu} A_\mu A_\nu^*$$

$$P_{A\to B} = \sum_\mu |A_\mu|^2 + \sum_\mu |A_\mu|^2 + \sum_{\mu\neq\bar{\mu},\nu} A_\mu A_\nu^*$$

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$$C(\theta) = < G(\vec{m})G(\vec{m}') > - < G(\vec{m}) >^2$$

θ_c is correlation angle

Aharanov-Bohm contribution

Spin-Orbit Effect

$$\theta_c \sim \frac{\Phi_0}{\Phi} \sim L^{-2}$$

$$\theta_c \sim \sqrt{\tau_{so} E_T} \sim \sqrt{\frac{\tau_{so}}{\tau}}\frac{\ell}{L} \sim L^{-1}$$

$$\theta_c \sim \pi/2$$

$$\theta_c \sim \pi/10$$

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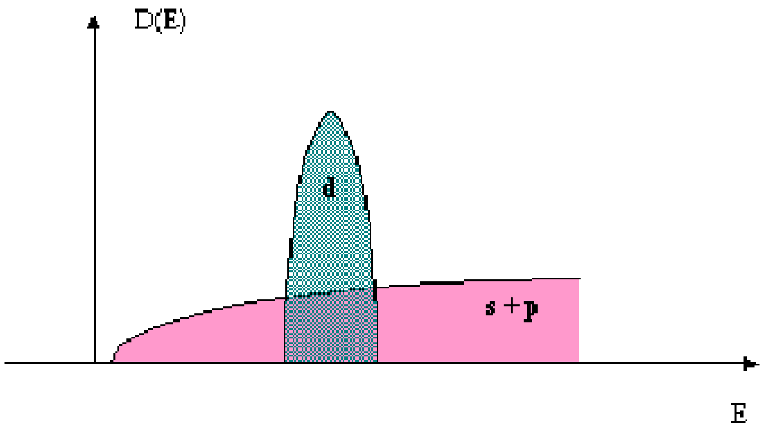
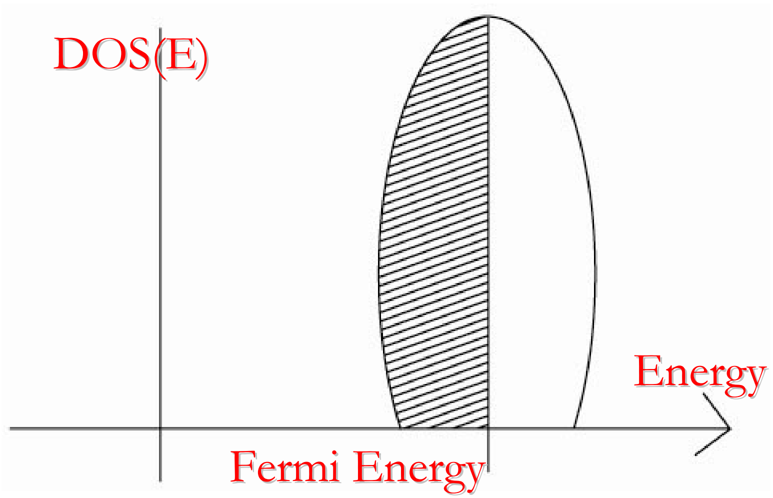
Density of States quantifies how closely packed are energy levels.

DOS(E) dE = Number of allowed energy levels per volume in energy window E to E +dE

DOS can be calculated theoretically or determined by tunneling experiments





Fermi Energy is energy of adding one more electron to the system (Large energy because electrons are Fermions, two of which can not be in the same quantum state).

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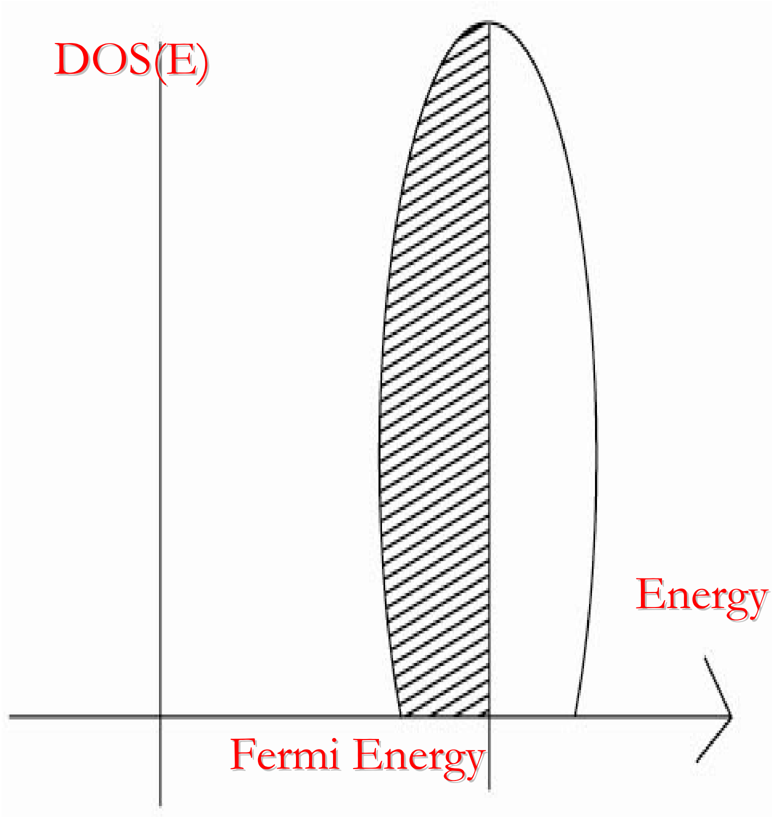

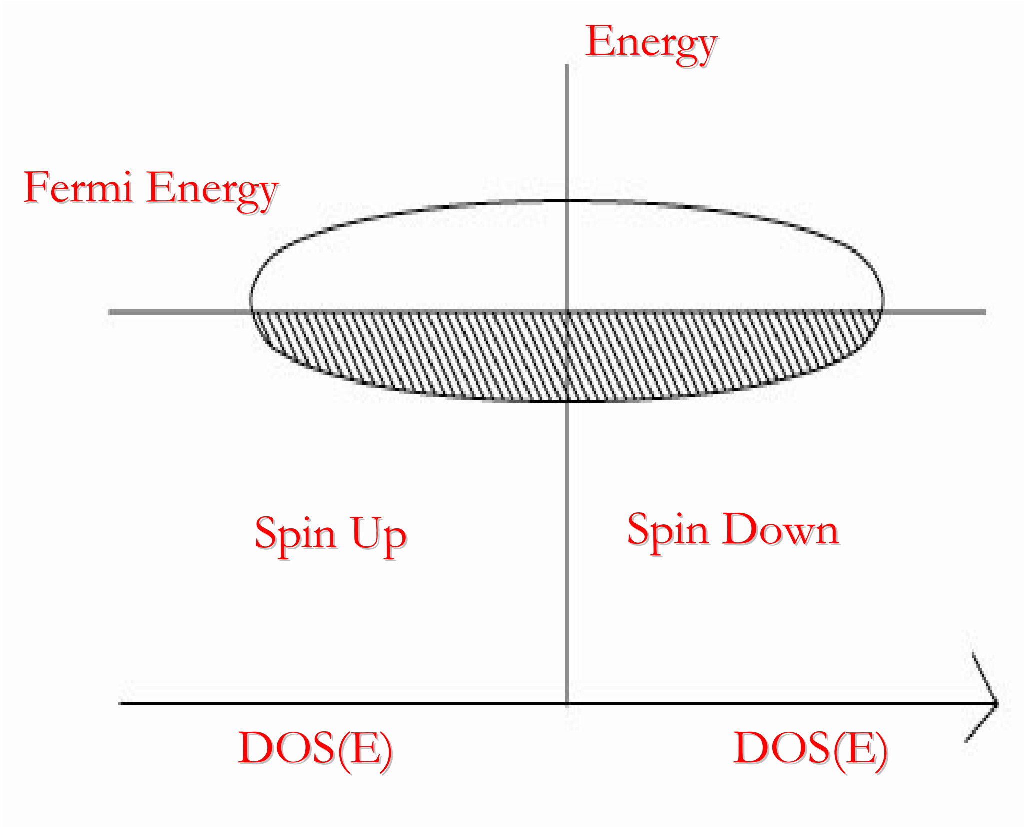

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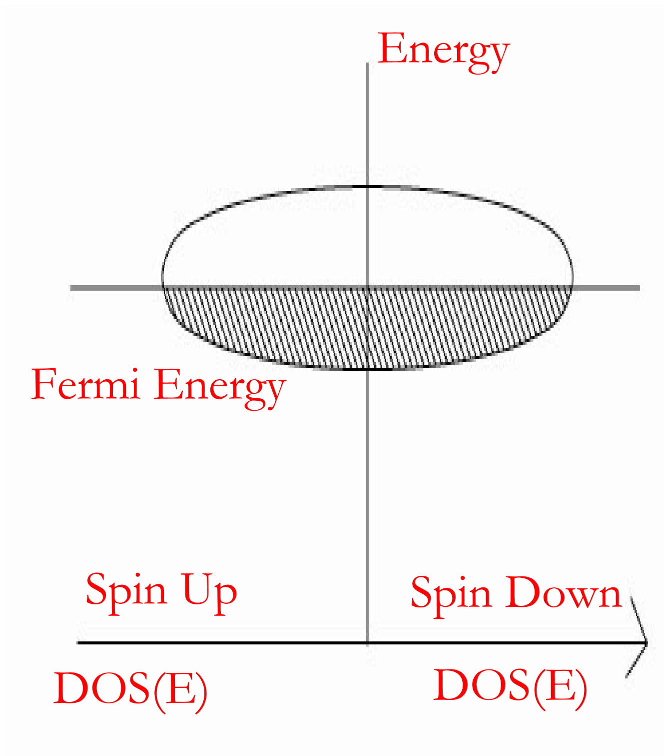
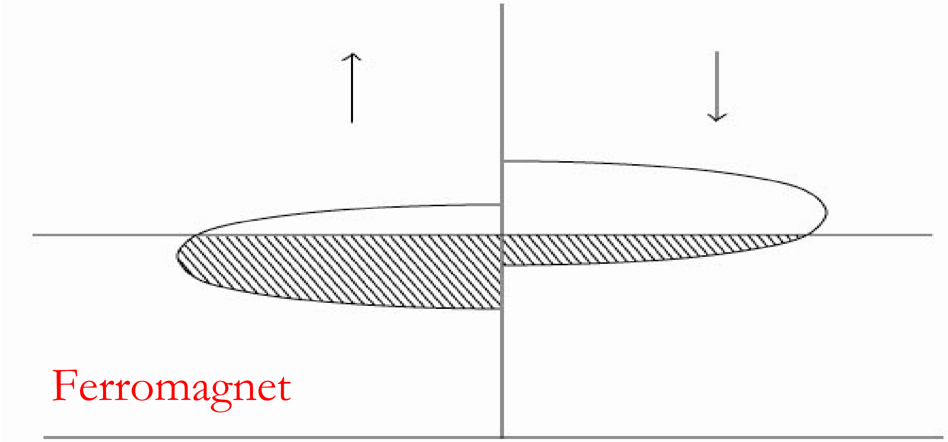
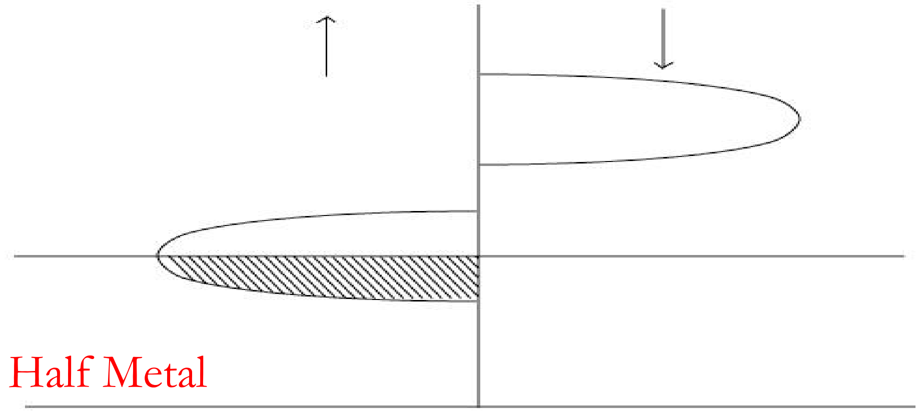
- Magnetic Field shifts the spin up and spin down bands



Spin DOS





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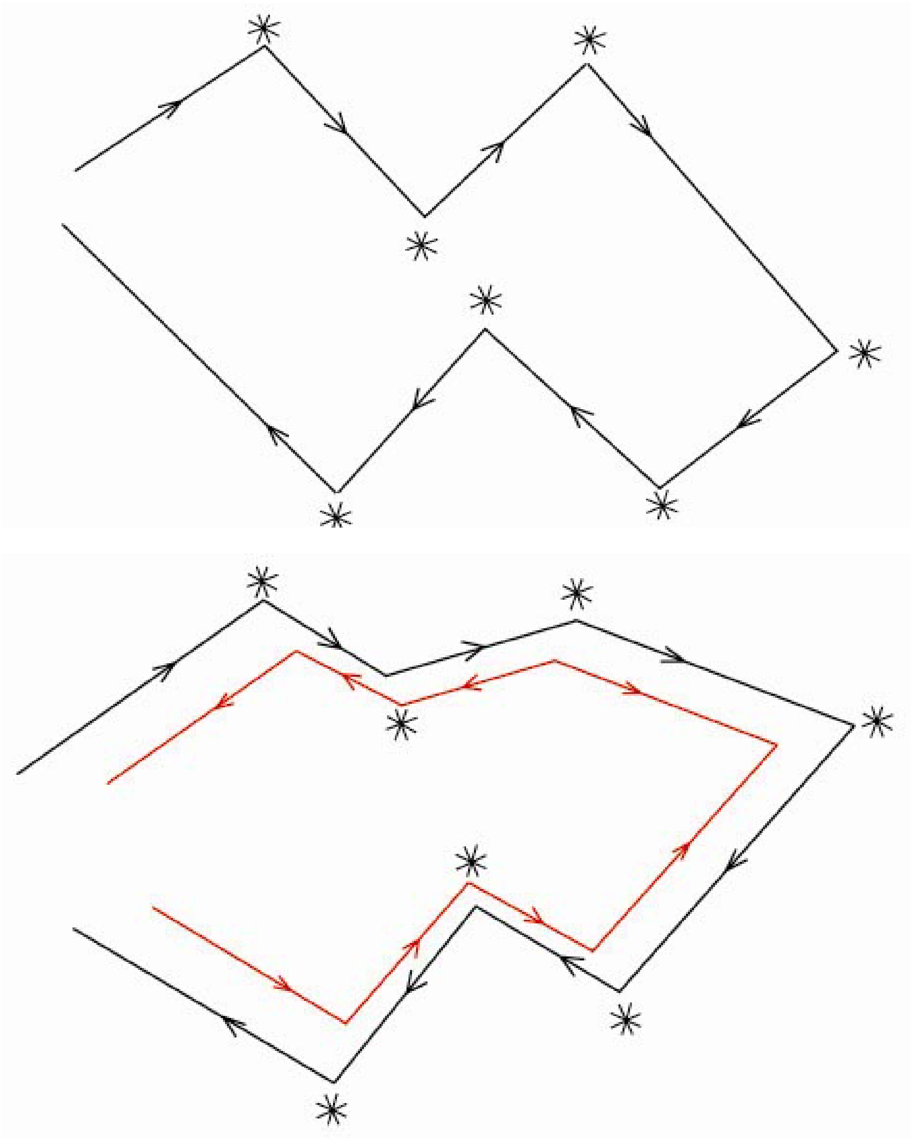
Weak localization (pictures)



For no magnetic field, the phase depends only on the path.

Every possible path has a twin that is exactly the same, but which goes around in the opposite direction.

Because these paths have the same flux and picks up the same phase, they can interfere constructively.

Therefore the probability to return to the starting point in enhanced (also called enhanced back scattering).

In fact the quantum probability to return is exactly twice the classical probability

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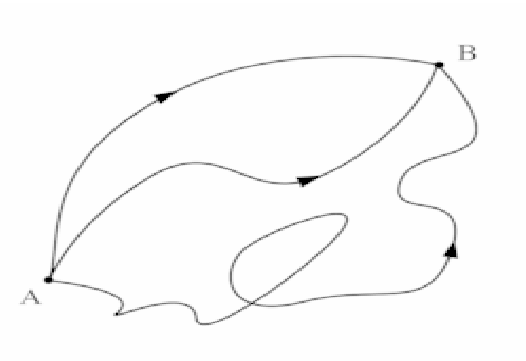
Weak localization (equations)



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Classical Contribution — Quantum Interference

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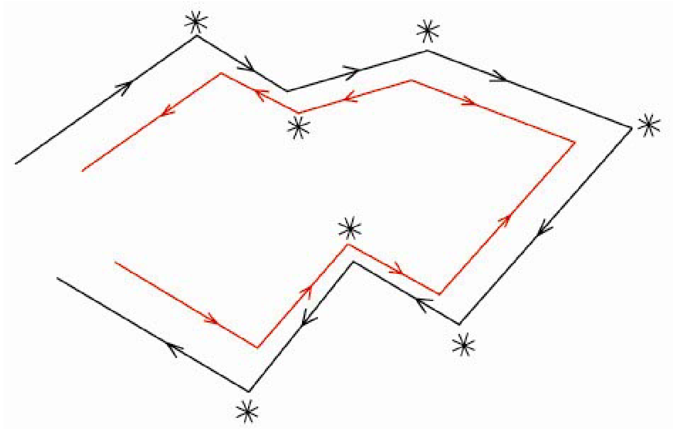
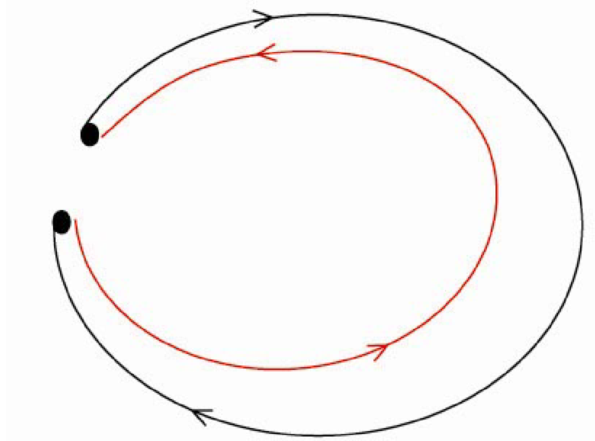
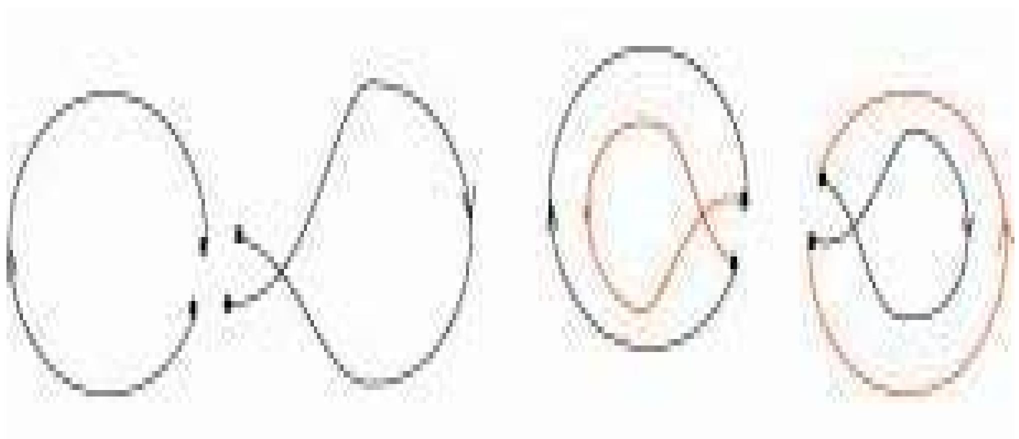
Weak Localization and UCF in Pictures

<G>





<G G>



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$$G = \lim_{\omega \to 0} \frac{e^2}{\omega} \int dt \exp^{i\omega t} \theta(t) \langle [\hat{I}(t), \hat{I}(0)] \rangle, \quad (1.15)$$

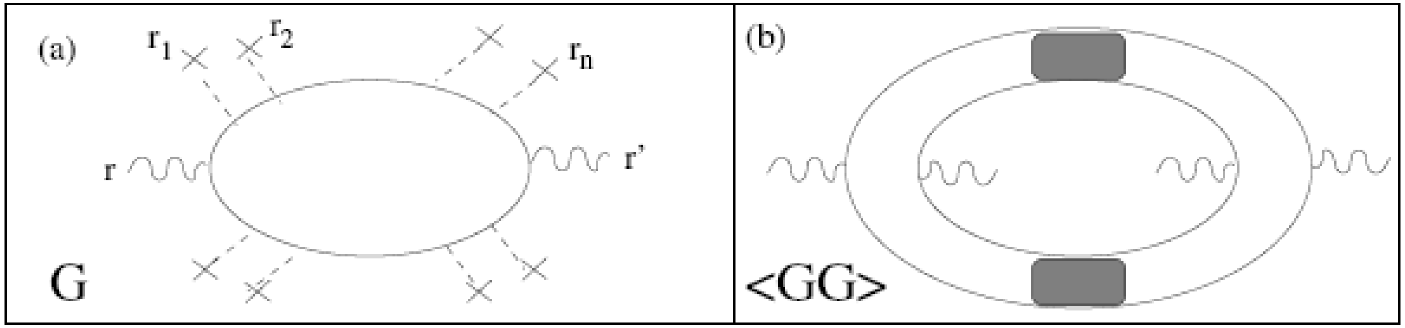

Figure 1.6: (a) Diagram for the conductance G before impurity averaging. Electrons propagate from $\mathbf{r}$ to $\mathbf{r}'$ while being scattered by impurities located at $\mathbf{r}_1, \mathbf{r}_2, \cdots \mathbf{r}_n$ which are represented by the dashed lines and crosses. (b) Diagram for the impurity averaged variance of conductance $\langle GG \rangle$, where the shaded area represents impurity averages involving both classical Diffusion modes and the Cooperon quantum corrections.

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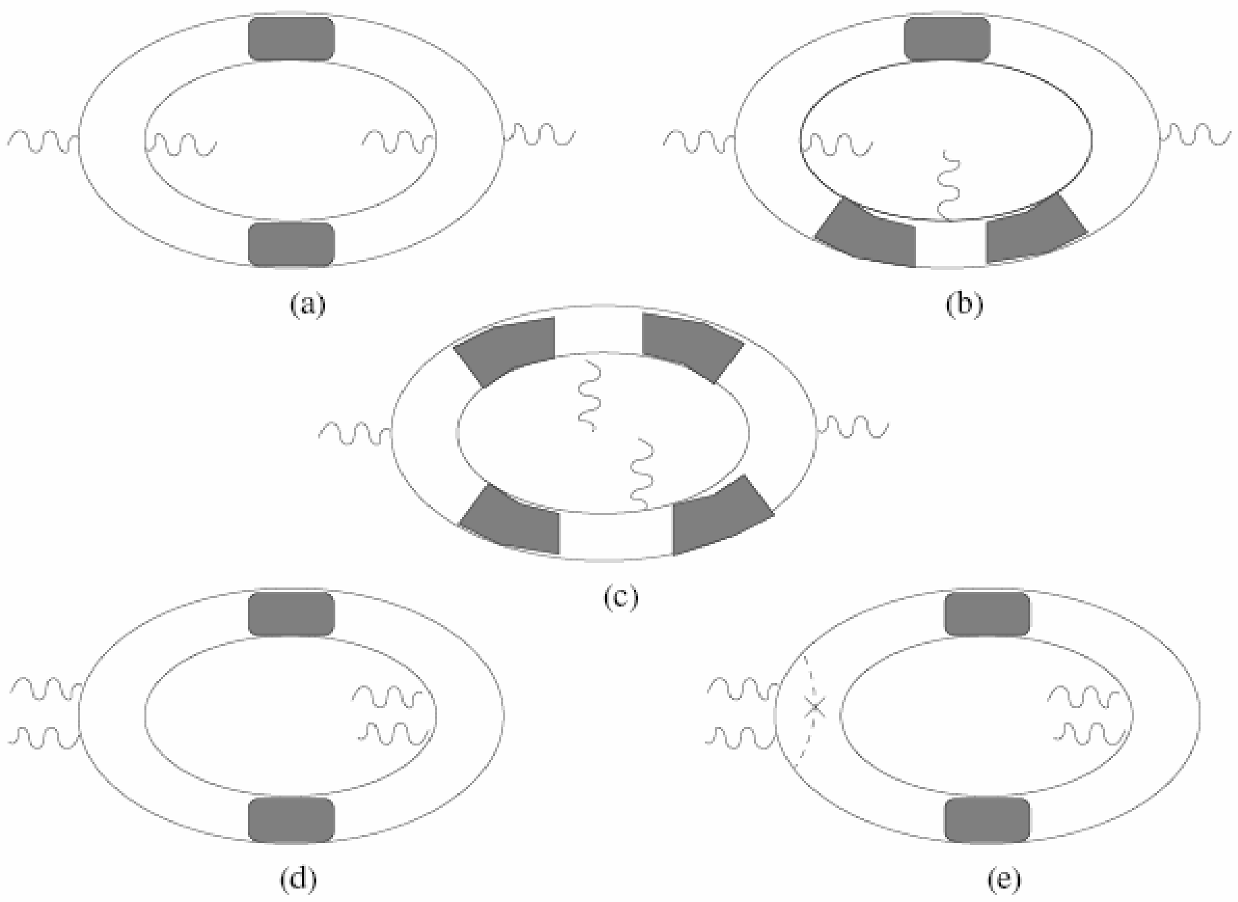

Figure B.1: Diagrams contributing to conductance correlations. For each of the diagrams, the shaded area represents Diffuson or Cooperon Ladders. The dashed line in diagram (e) represents an additional single impurity scattering.

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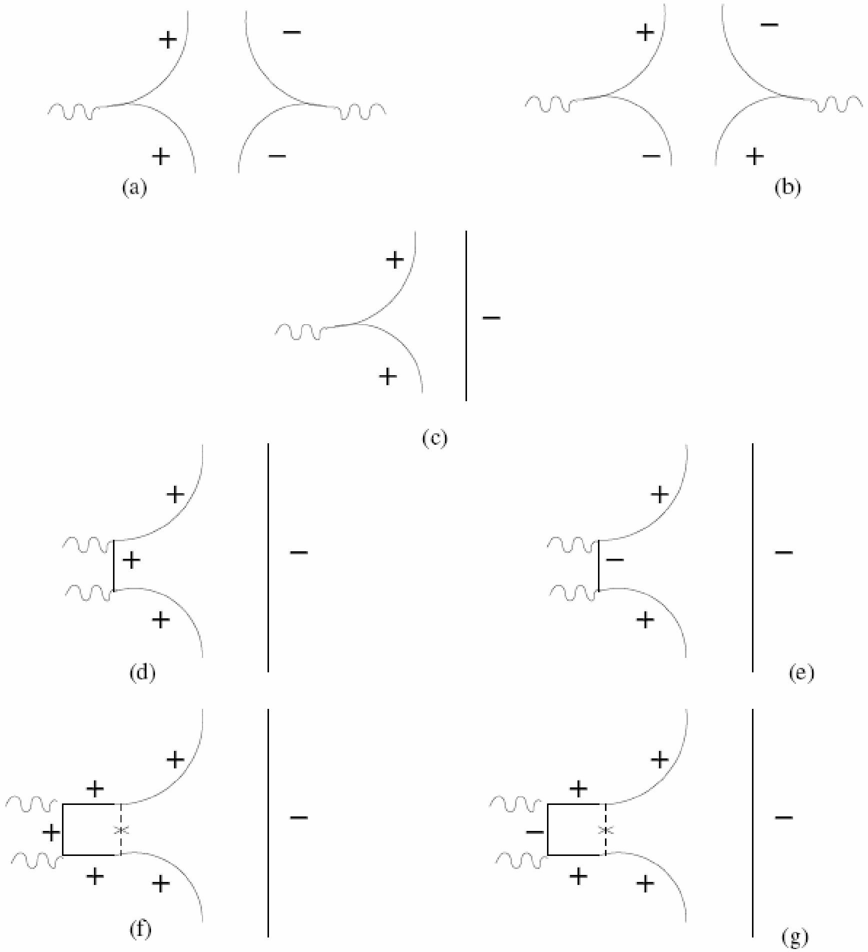

Figure B.2: Different types of current vertices found in the conductance correlation diagrams shown in Fig. B.1. Diagrams (a), (c), (d), (f) do not change their analyticity at the vertex, while (b) and (e) do.

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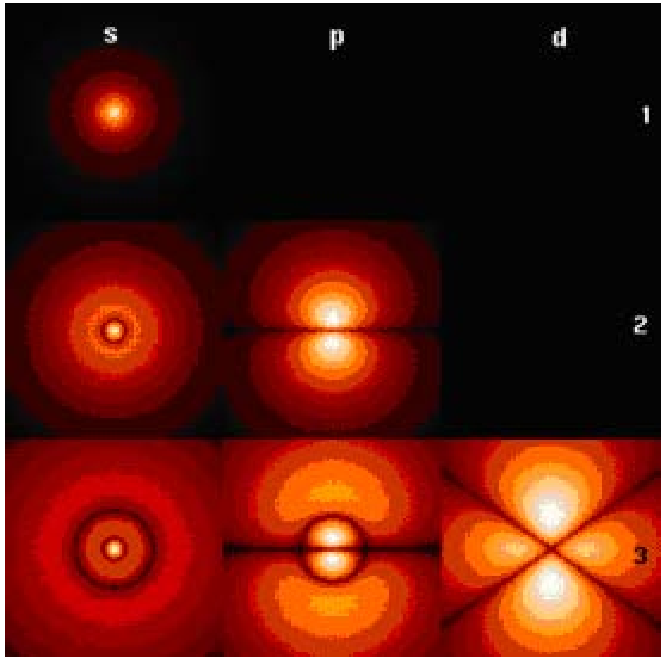
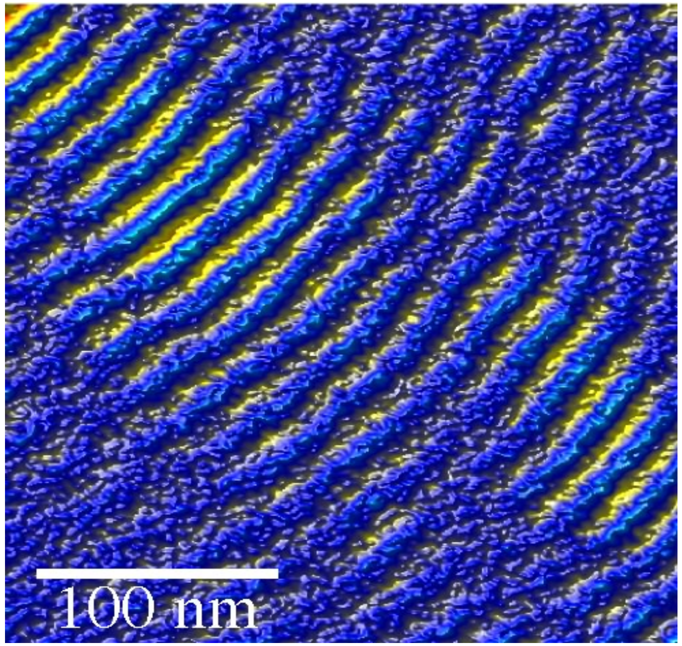
Introduction to Quantum Mechanics

Energy is Quantized
Wave Nature of Electrons (Schrödinger Equation)



Wavefunctions of electrons in the Hydrogen Atom (Wikipedia)



Scanning Probe Microscope Image of Electron Gas (Courtesy A. Bleszynski)