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Uncertainty of shallow source inversion from surface waves

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I. Uncertainty of moment tensor determination.

We consider surface waves radiated by an instant point source in medium with weak lateral inhomogeneity. For the spectrum of displacement $\mathbf{u}(\mathbf{r},\omega)$ at a point $\mathbf{r}$ we have (see Woodhouse, 1974; Babich *et. al.*, 1976; Levshin *et. al.*, 1989; Bukchin, 1990):

$$\mathbf{u}(\mathbf{r},\omega) = \mathbf{q}(\omega)P(\mathbf{M},h,\omega,\varphi)\exp[-i\psi(\mathbf{r},\omega)]. \tag{1}$$

Here $\mathbf{q}(\omega)$ – a complex vector depending on the structure model, $\mathbf{M}$ - moment tensor, h - source depth, ω - angular frequency, φ - azimuth, $\psi(\mathbf{r},\omega)$ – propagation phase. Factor P determines the radiation pattern of the source. For Love wave

$$\begin{aligned}P(\mathbf{M},h,\omega,\varphi) &= \xi V^{(\tau)}(\omega,h)[0.5(M_{33}-M_{22})\sin 2\varphi + M_{23}\cos 2\varphi] \\ &+ i\frac{\partial V^{(\tau)}(\omega,h)}{\partial z}(M_{12}\sin\varphi - M_{13}\cos\varphi)\end{aligned} \tag{2}$$

For Rayleigh wave

$$\begin{aligned}P(\mathbf{M},h,\omega,\varphi) &= \frac{\partial V^{(z)}(\omega,h)}{\partial z}M_{11} - \xi V^{(r)}(\omega,h)(M_{22}\cos^2\varphi + M_{33}\sin^2\varphi + M_{23}\sin 2\varphi) \\ &+ i[\xi V^{(z)}(\omega,h) + \frac{\partial V^{(r)}(\omega,h)}{\partial z}](M_{12}\cos\varphi + M_{13}\sin\varphi).\end{aligned} \tag{3}$$

Here $V(z)$, $V(r)$ and $V(\tau)$ – vertical, radial and transversal components of correspondent eigenfunctions. Notations for moment tensor elements are as follows: 1 corresponds to the vertical axis, 2 is directed to the north, and 3 is directed to the east.

The terms $\frac{\partial V^{(\tau)}(\omega,h)}{\partial z}$ for Love wave and $[\xi V^{(z)}(\omega,h) + \frac{\partial V^{(r)}(\omega,h)}{\partial z}]$ for Rayleigh wave are proportional to the stresses on a horizontal plane. They are vanishing at the free surface. If the source depth h is much smaller than the wave length, then as one can see from formulae (1-3) the elements of moment tensor M_{12} and M_{13} do not affect on the radiation pattern of surface waves, and they can not be resolved from observed spectra. But in the cases of both Sumatra earthquakes these two moment tensor elements are dominant, and Harvard solution as well as few others was obtained by surface wave inversion.

Can these solutions be considered as reliable?

We will show that the answer is positive.

The fact is that the elements M_{12} and M_{13} do not affect on the radiation pattern of surface waves while their absolute values do not exceed absolute values of other elements significantly. For example, a source with other components equal to zero will radiate surface wave for any nonzero depth. For simplicity we consider a double couple with zero strike angle (nonzero strike angle cause rotation of the source radiation pattern around vertical axis by this angle). Then moment tensor elements can be expressed as follows:

$$M_{11} = M_0 \sin\lambda \sin 2\delta,\ M_{22} = 0,\ M_{33} = -M_0 \sin\lambda \sin 2\delta,\ M_{23} = M_0 \cos\lambda \sin\delta,$$
$$M_{12} = -M_0 \cos\lambda \cos\delta,\ M_{13} = M_0 \sin\lambda \cos 2\delta. \qquad (4)$$

Here M_0 is the seismic moment, λ is the rake angle and δ is the dip angle of double couple. As one can see from formula (4) the absolute values of elements M_{12} and M_{13} start to exceed absolute values of other elements when the dip angle δ becomes small enough, what means that one of nodal planes is close to the horizontal plane.

Let us consider two partial cases of double couple: pure thrust (or normal fault) and pure slip. We'll investigate the dependence of surface wave radiation on the value of dip angle δ.

II. Pure thrust (or normal fault).

In the case of a pure thrust (or normal) fault the rake angle λ is equal to 90° (or -90°), $\cos\lambda = 0$, and $\sin\lambda = 1$ ($\sin\lambda = -1$). As can be seen from relations (4) there are only three nonzero elements of moment tensor in this case: $M_{11} = -M_{33} = \pm M_0 \sin 2\delta$, and $M_{13} = \pm M_0 \cos 2\delta$.

The equations (2) and (3) for such a source takes form

$$P(\mathbf{M}, h, \omega, \varphi) = -0.5\xi V^{(3)}(\omega, h) \sin 2\varphi\ M_0 \sin 2\delta$$
$$- i \frac{\partial V^{(3)}(\omega, h)}{\partial z} \cos\varphi\ M_0 \cos 2\delta \qquad (5)$$

and

$$P(\mathbf{M}, h, \omega, \varphi) = [\frac{\partial V^{(1)}(\omega, h)}{\partial z} + \xi V^{(2)}(\omega, h) \sin^2 \varphi]\ M_0 \sin 2\delta$$
$$+ i[\xi V^{(1)}(\omega, h) + \frac{\partial V^{(2)}(\omega, h)}{\partial z}] \sin\varphi\ M_0 \cos 2\delta \qquad (6)$$

for Love and Rayleigh waves correspondingly.

It follows from equations (5) and (6) that if the depth of such a source is so small that coefficients at the terms $\cos\varphi M_0 \cos 2\delta$ and $\sin\varphi M_0 \cos 2\delta$ are small, and at the same time the δ value is far from 0° or 90°, then the dependence of Love and Rayleigh wave spectra on δ is defined by the factor $\sin 2\delta$. It follows from this that surface wave radiation pattern (the dependence of radiated surface wave amplitudes and phases on the azimuth of radiation) is practically the same for any value of dip angle δ belonging to the interval [δ_0 , 90°- δ_0], where δ_0 is a small threshold value. The value δ_0 depends on the structure, on the period and on the source depth. The radiation pattern is similar to the case of

$\delta = 45°$ when $\cos 2\delta = 0$. All double couples with the same value of the product $M_0 \sin 2\delta$ radiate the same surface wave field, and dip angle cannot be uniquely determined from surface wave spectra.

In the case if dip angle δ belongs to one of two half-open intervals $[0, \delta_0)$ or $(90° - \delta_0, 90°]$ the surface wave radiation depends on the δ vale because the term proportional to $\cos 2\delta$ becomes significant as the value of $\sin 2\delta$ becomes small. Consequently the value of dip angle can be uniquely determined from observed surface wave spectra.

This behavior of spectrum is schematically illustrated by figure 1. The integral over azimuth of the modulus of difference between spectrum correspondent to the current value of δ and spectrum correspondent to $\delta = 45°$, is named 'integral change of spectrum'.

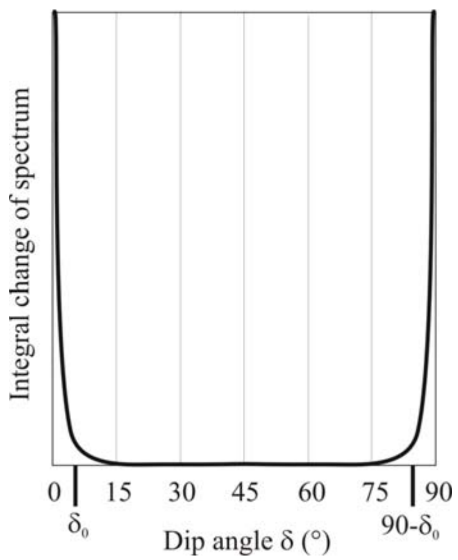

Fig. 1. Shallow thrust radiation pattern dependence on dip angle value.

An example of normalized diagrams for radiation of amplitude spectra of fundamental Love and Rayleigh modes for shallow trust for a set of values of dip angle δ are presented at the figure 2. These diagrams are calculated for period 200s for values of strike angle for Sumatra earthquake, 26.12.2004. To model the structure at the source neighborhood we used 3SMAC model for the crust and PREM model beneath.

As one can see from the figure 2, radiation patterns of Love waves are more sensitive to the dip angle value than those of Rayleigh waves, and the difference between Rayleigh diagrams for $\delta = 45°$ and $\delta = 0°$ is very small. Similar diagrams calculated for longer periods don't differ significantly from shown here. So, the values of dip angle less then 10° can be reasonably resolved from Love wave

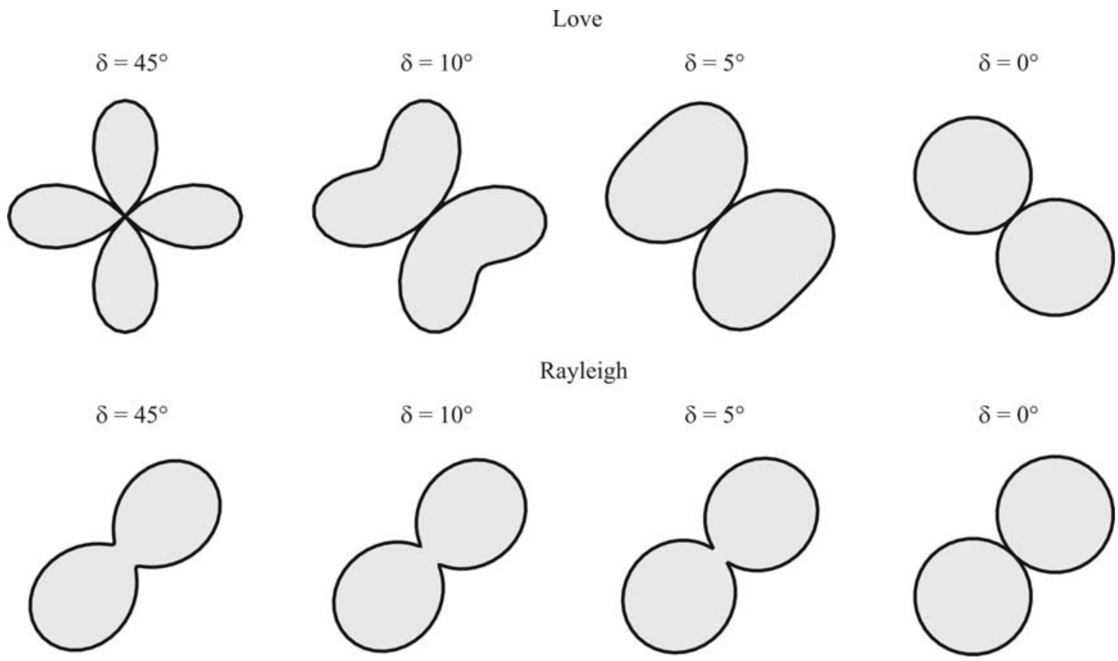

Fig. 2. Radiation of fundamental Love and Rayleigh mode amplitude spectra by shallow thrust double couple for period 200 s. Strike angle = 315°, rake angle = 90°, h = 30 km.

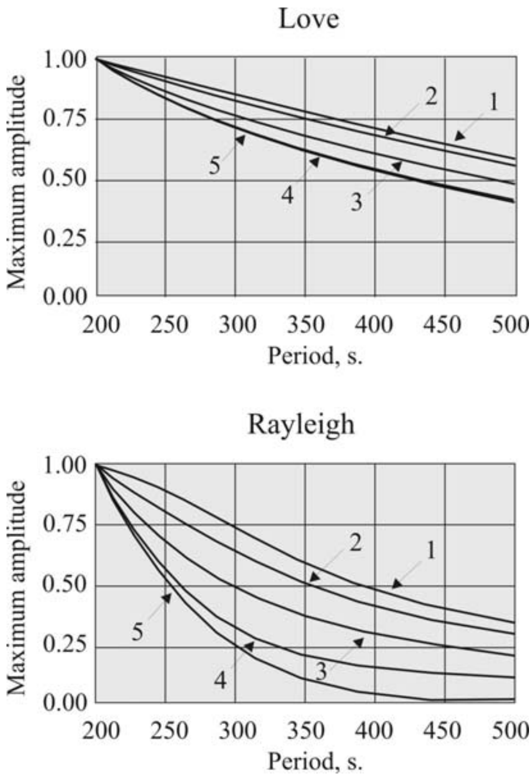

Fig. 3. Dependence on period of maximum value of radiated amplitude spectrum for fundamental Love and Rayleigh modes for shallow thrust double couple. Different curves correspond to different values of dip angle δ: 1 - δ= 45°, 2 - δ=10°, 3 - δ= 5°, 4 - δ= 2°, and 5 - δ= 0°. Slip angle of double couple is equal to 90°, h = 30 km. All curves are normalized to their maximum.

amplitude spectra measured for different azimuths, and cannot be resolved from Rayleigh wave amplitude spectra.

But the resolution of dip angle depends not only on the sensitivity of normalized diagrams to its change. It is important if the diagram scale (maximum amplitude) dependence on period is sensitive to the dip angle value. We calculated such functions for different values of dip angle for periods from 200 to 500 seconds for the same values of slip angle and source depth as for diagrams discussed above. The results are given in figure 3. As it is clear from figure 3, the dependence of amplitude functions on δ is much stronger for Rayleigh waves than for Love waves. Taking into account the both considered factors we conclude that small values of dip angle can be resolved from observed Love wave amplitude spectra as well as from Rayleigh wave amplitude spectra. The estimate for threshold value δ_0 is about 10°. Use of observations of both types will improve the dip angle resolution.

III. Pure strike-slip.

Let us consider in similar way a strike-slip double couple. In this case the rake angle λ is equal to 0° for leftlateral slip and to 180° for rightlateral slip. Correspondingly $\sin\lambda = 0$, and $\cos\lambda = 1$ ($\cos\lambda = -1$). In this case as it follows from relations (4) there are only two nonzero elements of moment tensor in this case: $M_{23} = \pm M_0 \sin\delta$ and $M_{12} = \mp M_0 \cos\delta$. We'll consider leftlateral slip. The equations (2) and (3) for such a source takes form

$$P(\mathbf{M}, h, \omega, \varphi) = \xi V^{(3)}(\omega, h) \cos 2\varphi \;\, M_0 \sin\delta - i \frac{\partial V^{(3)}(\omega, h)}{\partial z} \sin\varphi \;\, M_0 \cos\delta \tag{7}$$

and

$$P(\mathbf{M}, h, \omega, \varphi) = -\xi V^{(2)}(\omega, h) \sin 2\varphi \;\, M_0 \sin\delta - i[\xi V^{(1)}(\omega, h) + \frac{\partial V^{(2)}(\omega, h)}{\partial z}] \cos\varphi \;\, M_0 \cos\delta \tag{8}$$

for Love and Rayleigh waves correspondingly.

It follows from equations (7) and (8) that if the depth of such a source is so small that coefficients at the terms $\sin\varphi\, M_0\cos\delta$ and $\cos\varphi\, M_0\cos\delta$ are small, and at the same time the δ value is far from 0°, then the dependence of Love and Rayleigh wave spectra on δ is defined by the factor $\sin\delta$. It follows from this that surface wave radiation pattern is practically the same for any value of dip angle δ belonging to the interval [δ_0 , 90°], where δ_0 is a small threshold value. The value δ_0 depends on the structure, on the period and on the source depth. The radiation pattern is similar to the case of δ = 90° (pure strike-slip on a vertical fault) when $\cos\delta = 0$. All double couples with the same value of the product $M_0\sin\delta$

radiate the same surface wave field, and as in the case of pure thrust the dip angle cannot be uniquely determined from surface wave spectra.

In the case if dip angle δ belongs to the half-open interval $[0,\delta_0)$ the surface wave radiation depends on the δ vale because the term proportional to $\cos\delta$ becomes significant as the value of $\sin\delta$ becomes small. Consequently the value of dip angle can be uniquely determined from observed surface wave spectra. This behavior of spectrum is schematically illustrated by figure 4.

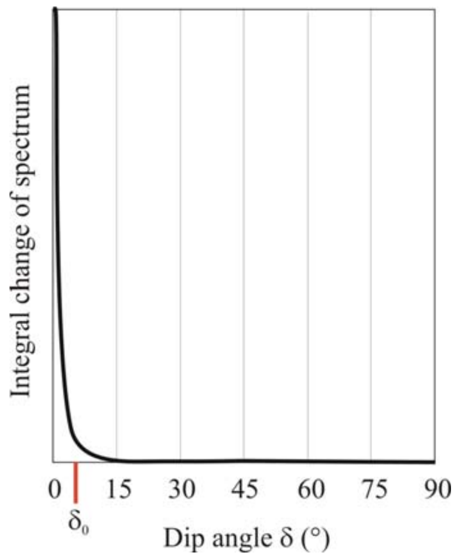

Fig. 4. Shallow strike-slip radiation pattern dependence on dip angle value.

An example of normalized diagrams for radiation of amplitude spectra of fundamental Love and Rayleigh modes for shallow strike-slip for period 200s are given for a set of values of dip angle δ at the figure 5. The strike angle, source depth values and structure were used the same as for considered thrust source. As one can see from figure 5, radiation patterns of both wave types in this case are changing significantly while dip angle takes values from 0° to 90°. Love waves are more sensitive to dip angle values in the band from 0° to 5°, but Rayleigh waves resolve better the values of dip angle from 5° to 15°-20°.

The dependence on period of maximum value of radiated surface wave amplitude spectra is presented for different values of dip angle varying from 0° to 90° at figure 6. As can be seen from figures 5 and 6 the threshold value δ_0 is about 15°-20°.

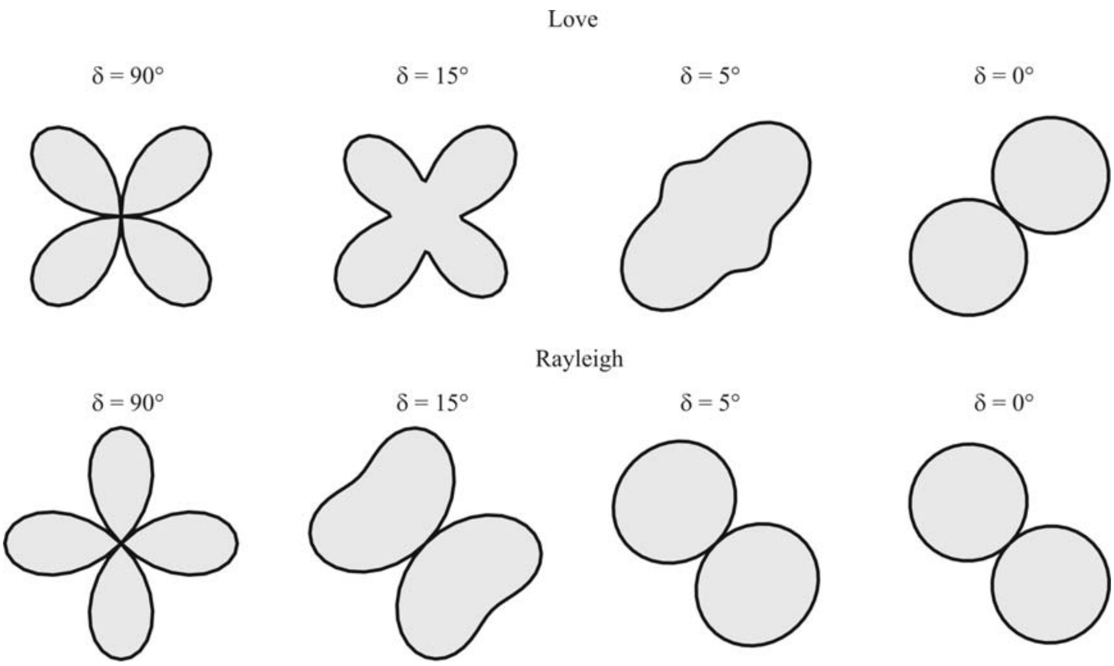

Fig. 5. Radiation of fundamental Love and Rayleigh mode amplitude spectra by shallow strike-slip double couple for period 200 s. Strike angle = 315°, slip angle = 0°, h = 30 km.

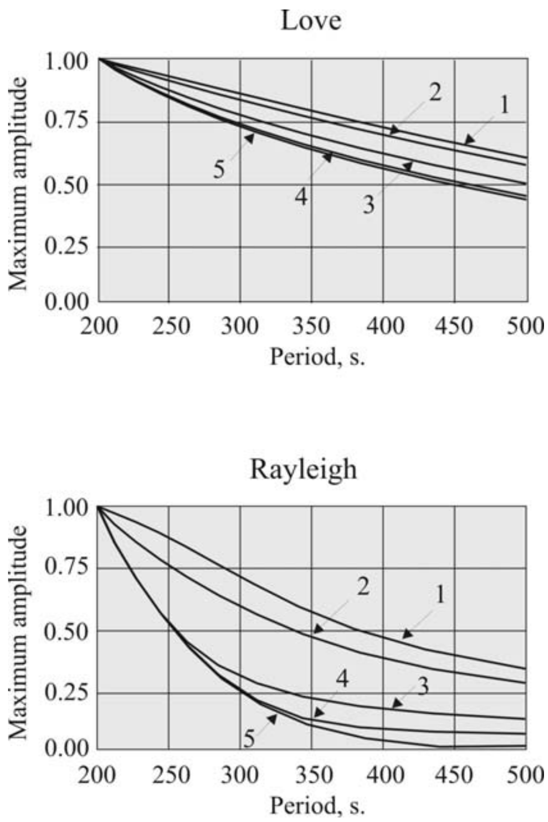

Fig.6. Dependence on period of maximum value of radiated amplitude spectrum for fundamental Love and Rayleigh modes for shallow strike-slip double couple. Different curves correspond to different values of dip angle δ: 1 - δ= 90°, 2 - δ=15°, 3 - δ= 5°, 4 - δ= 2°, and 5 - δ= 0°. Slip angle of double couple is equal to 0°, h = 30 km. All curves are normalized to their maximum.

IV. General case of double couple.

Let us consider now a general case of shallow double couple with rake angle λ and dip angle δ. It can be presented as a sum of a thrust (or normal) fault and a strike-slip with weights $\sin\lambda$ and $\cos\lambda$ correspondingly and with the same dip angle δ. It follows from relations (2), (3) and (4) that the factor P in this case can be presented in form

$$\begin{aligned}
&P(\mathbf{M},h,\omega,\varphi)= \\
&-\xi V^{(3)}(\omega,h)M_0[0.5\sin 2\varphi\ \sin 2\delta\sin\lambda-\cos 2\varphi\ \sin\delta\cos\lambda] \\
&-i\frac{\partial V^{(3)}(\omega,h)}{\partial z}M_0(\cos\varphi\ \cos 2\delta\sin\lambda+\sin\varphi\ \cos\delta\cos\lambda)
\end{aligned} \tag{9}$$

and

$$\begin{aligned}
&P(\mathbf{M},h,\omega,\varphi)=[\frac{\partial V^{(1)}(\omega,h)}{\partial z}+\xi V^{(2)}(\omega,h)\sin^2\varphi]M_0\sin 2\delta\sin\lambda \\
&-\xi V^{(2)}(\omega,h)M_0\sin 2\varphi\ \sin\delta\cos\lambda \\
&+i[\xi V^{(1)}(\omega,h)+\frac{\partial V^{(2)}(\omega,h)}{\partial z}]M_0(\sin\varphi\ \cos 2\delta\sin\lambda-\cos\varphi\ \cos\delta\cos\lambda).
\end{aligned} \tag{10}$$

for Love and Rayleigh waves correspondingly.

Let δ_0 be the larger of two threshold values estimated for thrust and for strike-slip components for given structure, source depth and period, and let δ belongs to the segment [δ_0, 90°-δ_0]. Then the terms in the imaginary part of P in equations (9) and (10) correspondent to the thrust component as well as to the strike-slip component are much less then correspondent terms in the real part, and the equations take form

$$P(\mathbf{M},h,\omega,\varphi)=-\xi V^{(3)}(\omega,h)M_0[0.5\sin 2\varphi\ \sin 2\delta\sin\lambda-\cos 2\varphi\ \sin\delta\cos\lambda] \tag{11}$$

and

$$\begin{aligned}
&P(\mathbf{M},h,\omega,\varphi)=[\frac{\partial V^{(1)}(\omega,h)}{\partial z}+\xi V^{(2)}(\omega,h)\sin^2\varphi]M_0\sin 2\delta\sin\lambda \\
&-\xi V^{(2)}(\omega,h)M_0\sin 2\varphi\ \sin\delta\cos\lambda.
\end{aligned} \tag{12}$$

When λ values are close to 0° or 180° the value of $\sin\lambda$ is close to 0 and the surface wave radiation pattern is practically the same as in the case of pure strike-slip. When λ values are close to 90° or -90° the value of $\cos\lambda$ is close to 0 and the surface wave radiation pattern is practically the same as in the case of pure thrust or normal fault. In this both cases it was shown that surface wave amplitude and phase spectra doesn't depend on the value of dip angle belonging to the segment [δ_0, 90°-δ_0].

Let us consider a source with the value of slip angle λ significantly different from the values 0°,180° and ±90°. Let us rewrite the equations (11) and (12) in the form

$$P(\mathbf{M}, h, \omega, \varphi) = -\xi V^{(3)}(\omega, h) M_0 \sin\delta \cos\lambda \ [0.5 \sin 2\varphi \ \ \cos\delta \,\mathrm{tg}\,\lambda - \cos 2\varphi] \tag{13}$$

and

$$P(\mathbf{M}, h, \omega, \varphi) = M_0 \sin\delta \cos\lambda \ \{[\frac{\partial V^{(1)}(\omega, h)}{\partial z} + \xi V^{(2)}(\omega, h) \sin^2 \varphi] \cos\delta \,\mathrm{tg}\,\lambda - \xi V^{(2)}(\omega, h) \sin 2\varphi\} \tag{14}$$

for Love and Rayleigh waves correspondingly.

Functions P defined by equation (13) or by equation (14) for different values λ и δ satisfying the identity

$$\tan\lambda \cos\delta \equiv const \tag{15}$$

differ from each other by factor $\sin\delta\cos\lambda$ only.

Correspondingly all double couples with values of λ и δ satisfying the identity (15) and with seismic moment satisfying the identity

$$M_0 \sin\delta \cos\lambda \equiv const, \tag{16}$$

have the same surface wave radiation pattern.

As a result the focal mechanism for such a source cannot be uniquely determined from surface wave spectra.

Let in contrary the dip angle δ of one of nodal planes is so small that it belong to the half-open interval $[0,\delta_0)$, where δ_0 is the smaller of two threshold values estimated for thrust and for strike-slip components for given structure, source depth and period band. Then the values of $\sin\delta$ and $\sin 2\delta$ are so small that the imaginary part of function P in formulae (9) and (10) is comparable with its real part. In this case the radiated surface wave spectra essentially depend on the values of dip and slip angles, and the focal mechanism of such a source can be uniquely determined from observed surface wave spectra.

Conclusions

We have shown that focal mechanism and seismic moment of seismic source can be uniquely determined from records of surface waves, which wavelengths are much larger then the value of source depth, on condition that dip angle of one of two source nodal planes is small enough. The threshold value of this angle depends on the structure, on the period and on the source depth.

References

Babich, V.M., Chikachev, B.A., and Yanovskaya, T.B., 1976. Surface Waves in a Vertically Inhomogeneous Elastic Halfspace with Weak Horizontal Inhomogeneity, *Izv. Akad. Nauk SSSR, Fizika Zemli* 4, 24-31.

Bukchin, B. G., 1990. Determination of Source Parameters from Surface Waves Recordings Allowing for Uncertainties in the Properties of the Medium, *Izv. Akad. Nauk SSSR, Fizika Zemli* 25, 723-728.

Levshin, A.L., Yanovskaya, T.B., Lander, A.V., Bukchin, B.G., Barmin, M.P., Ratnikova, L.I., Its, E., N., 1989. Seismic Surface Waves in Laterally Inhomogeneous Earth, ed. V.I. Keilis-Borok, Kluwer. Publ., Dordrecht, 293 pp.

Ricard Y., H-C. Nataf, Montagner J-P. The 3SMAC model. Confrontation with seismic data. // *J. Geophys. Res*.1996. V.101. P. 8457-8472.

Woodhouse J.H. Surface waves in the laterally varying structure. // *Geophys. J. R. astr. Soc.* 1974. V. 90. №12. P. 713-728.