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"8th Workshop on Three-Dimensional Modelling of
Seismic Waves Generation, Propagation and their Inversion"

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Introduction to the Earthquake Source Mechanics

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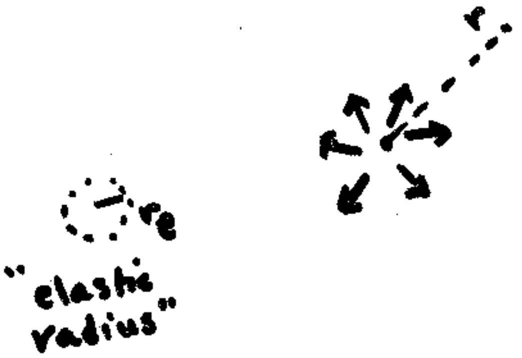
A simpe problem.



an Explosive point source
→ a pressure force $F(t)$
produces a spherical wave
of ampl. $u(r,t)$

Let $\phi(r,t)$ be the displ. potential
given by $u(r,t) = \dfrac{\partial \phi(r,t)}{\partial r}$.

$\Big\}$ 1-D problem

The explosive pt. source only generates P-waves
(in a homo. elastic medium)

ϕ satisfies the eqn.

$$\frac{\partial^2 \phi}{\partial r^2} + \frac{2}{r}\frac{\partial \phi}{\partial r} - \frac{1}{\alpha^2}\frac{\partial^2 \phi}{\partial t^2} = -4\pi F(t)\,\delta(r_e)$$

Gen. Soln. is the well-known D'Alembert's soln:

$$\phi(r,t) = -\frac{F(t - r/\alpha)}{r},$$

$\alpha \rightarrow$ P-wave speed

[Easy to test by substituting in orig. eqn.].

THUS, THE SOLUTION ϕ HAS THE SAME FUNCTIONAL FORM AS THE FORCE-TIME HISTORY.

∴ the (spherically symmetric) displ.

$$u(r,t) = \frac{\partial \phi(r,t)}{\partial r}$$

$$= \underbrace{\frac{F(t-r/\alpha)}{r^2}}_{\text{Near-field term}} + \underbrace{\frac{1}{r\alpha}\frac{\partial F(t-r/\alpha)}{\partial \tau}}_{\text{Far-field term}}$$

$\tau = (t - r/\alpha)$ = retarded time → causal (no motion till $t = r/\alpha$

① Decays rapidly w. r as $1/r^2$

② Form of displ. is SAME as original force $F(t)$

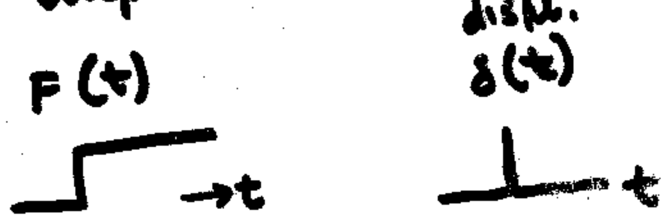
∴ $F(t)$ is a Heaviside func, then near-field displ. is also a step ie. permanent deformation occurs in the near-field.

① Decays w. distance as $1/r$ ie. more slowly than near-field term & ∴ dominates at larger distances.

②. V.V. IMP

Form of displ. is TIME-DERIVATIVE OF $F(t)$

∴ a step-func. $F(t)$ produces a delta func in displ ie. an impulse



i.e. the seismogram will see a transient displ (no perm. displ)

The body force equivalent to any source is a set of forces that produces the <u>same</u> motions in the body as the source:

For an explosive pt. source:

3 mutually $\perp^{r}$ dipoles

For an earthquake:

The "double couple".

<u>Note</u>: Elastic wave propagation does not modify the shape of a wave initiated at the source in a homogeneous, elastic medium, except through known transmission effects

What is another difference between "single" & "double" couple :

For single couple, the fault plane is obvious.

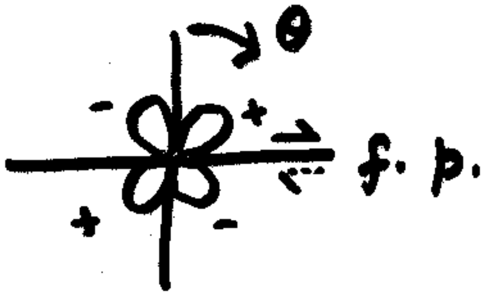
The double couple has a "fault" plane + an "auxiliary" plane – the 2 "nodal" planes + which is the fault plane has to be determined.

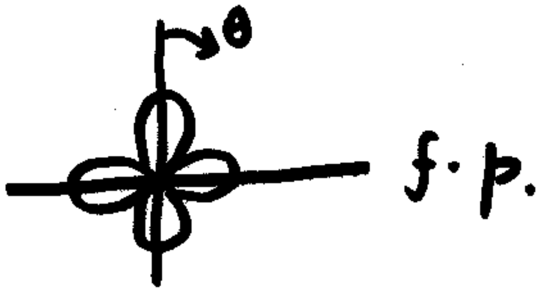
"Nodal" planes mean planes where the displacement is zero.

P

S

" radiation patterns"





$\propto \sin 2\theta$

$\propto \cos 2\theta$

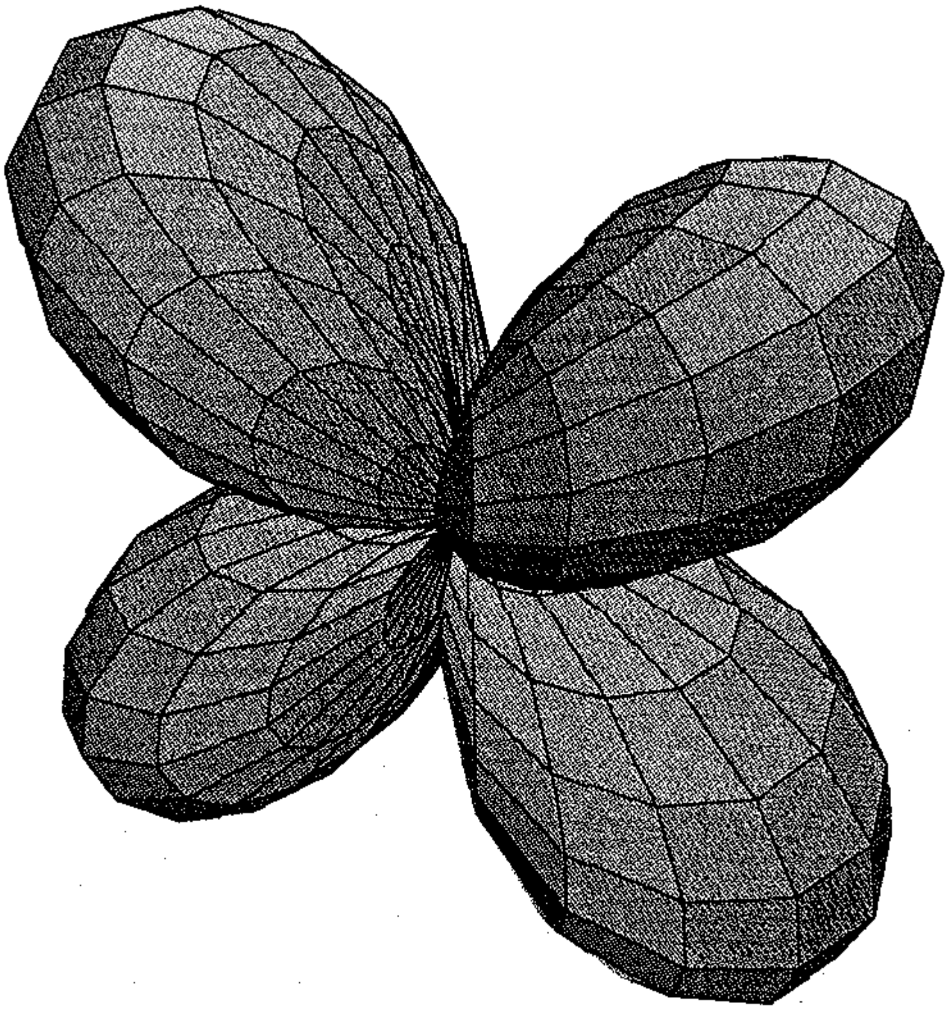
The radiation patterns are actually 3-dimensional + have both polar angles θ + ϕ in it [Eqn 8.64 of handout]

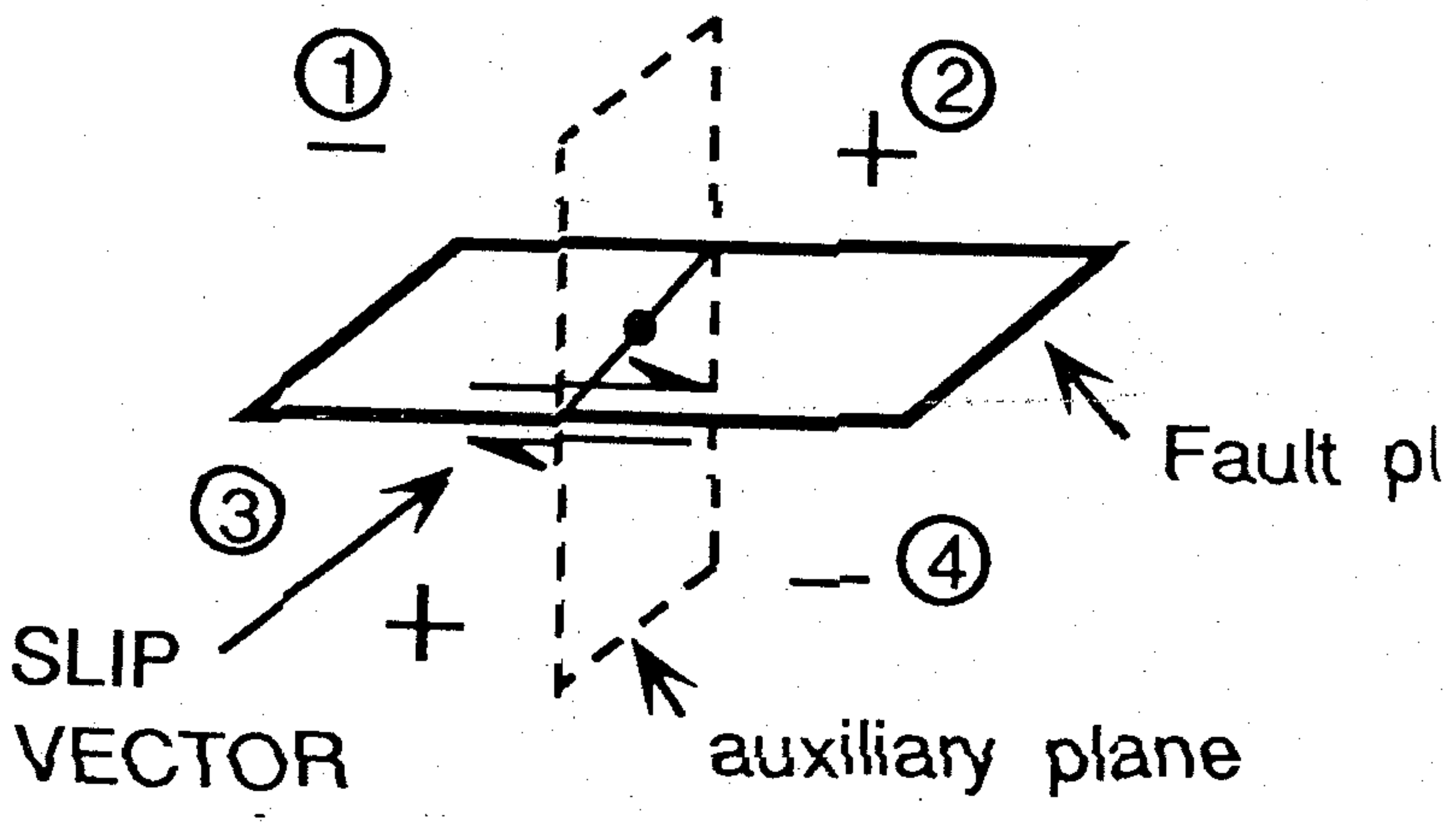
The single couple was considered seriously ∵ the P-wave radiation pattern was the same as the double couple.

But S-wave radiation pattern is DIFFERENT

+ finally the data was used to show that the eq. = "double couple" force system.

P Wave Radiation Pattern



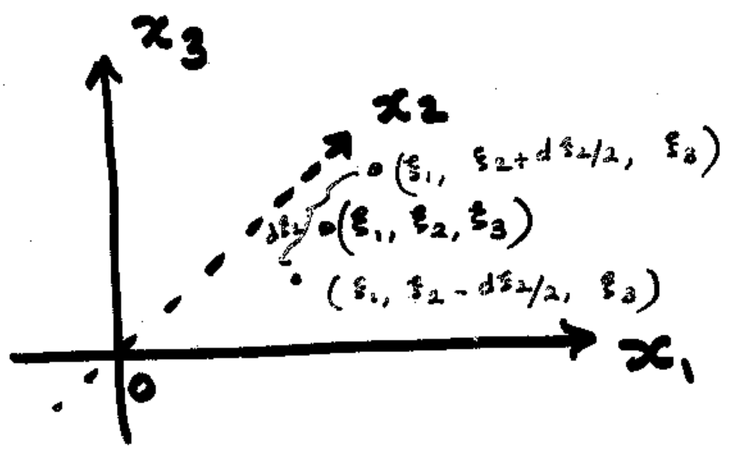


URE 8.6 Sense of initial *P*-wave moti
pect to the fault plane and auxiliary p

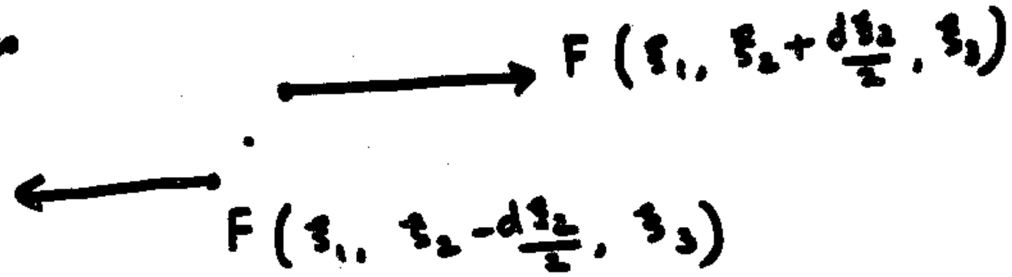
To construct the double couple soln:

Start w. single force solution:

Let it be $\bar{u}(\underbrace{\xi_1, \xi_2, \xi_3}_{\text{place where force is applied}}; \underbrace{x_1, x_2, x_3}_{\text{place where displ. is measured}})$



Equal + opp. forces



Then resulting displ is

$$\bar{u}\left(\xi_1, \xi_2 + \frac{d\xi_2}{2}, \xi_3\right) - \bar{u}\left(\xi_1, \xi_2 - \frac{d\xi_2}{2}, \xi_3\right)$$

Expanding by Taylor series;

$$\bar{u}(\xi_1, \cancel{\xi_2}, \xi_3) + \frac{d\xi_2}{2} \cdot \frac{\partial \bar{u}}{\partial \xi_2}(\xi_1, \xi_2, \xi_3) + \cancel{\cdots}$$

$$-(u \cdots \cancel{\ } \cdots \quad - \cdots\cdots \quad + \cancel{\cdots} \quad) = \frac{\partial \bar{u}}{\partial \xi_2} \cdot d\xi_2 + O(d\xi_2^3)$$

Similarly displ. 2nd couple in the $\perp^r$ dir. can be written down

$\therefore$ Adding: we can write down the total displ. due to a double couple.

In example above, time was not used but if we want to write the elastodynamic soln. (which is what we really want), then we can use the dynamic soln. for a single force + get the reqd. double couple soln. in the <u>same</u> <u>way</u>.

A STATIC RESULT:

The displacement field due to a shear dislocation can be given by the displ. field due to a distribution of equivalent double couples that are placed in the medium without any dislocation

→ the fault is finite now!

THE DYNAMICS: Double couple solns. can be obtd. using the dynamic single force soln. of Stokes? (eqn 8.45 + Yanovskaya's class).

Remember: If the force at the eq. source is a STEP. FUNC., then

displ. will be δ- funcs. + so on.

∴ Finally:

Seismogram (i.e. displ) in the far-field

→ derivative of the time history at source i.e. seismogram is sensitive to particle <u>velocities</u> at the source rather than particle displ. !!! (comp. w. explosive case done earlier).

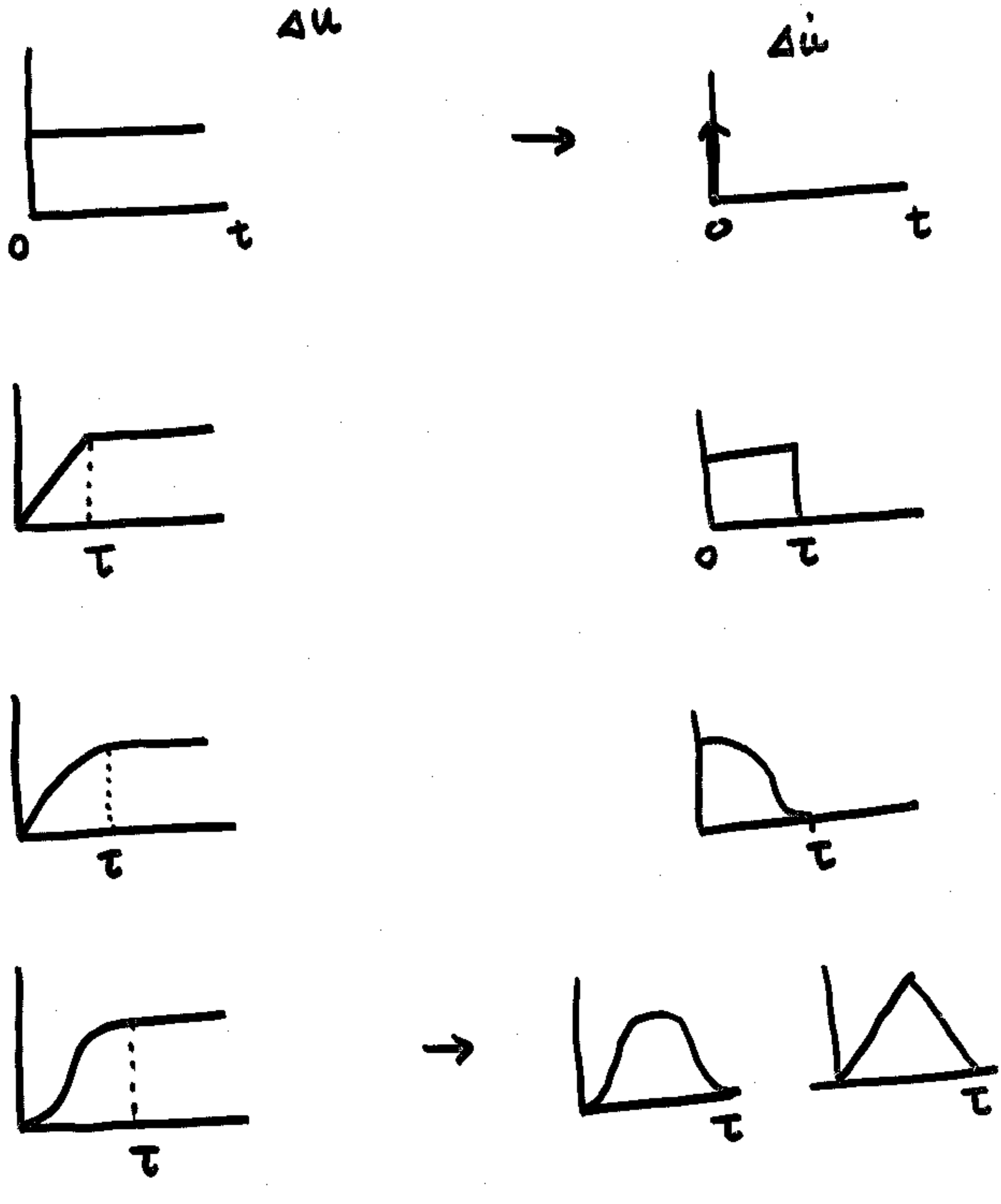
V.V. IMP result.

$$M_0 = \mu \int_S u \cdot dS$$

<u>OR</u>

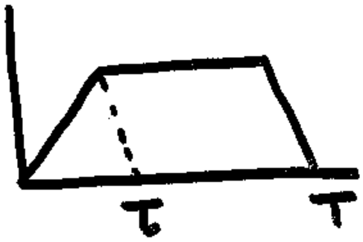
$$= \mu \cdot \int_t dt \int_{S(t)} \dot{u} \cdot dS$$

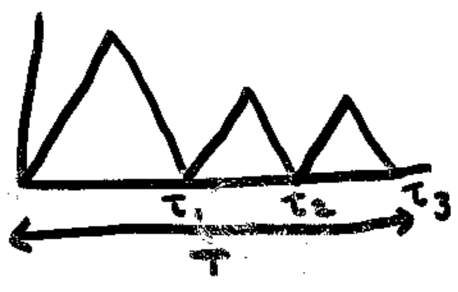
S.T. func → rel. to Mom. Rate func.



τ : Rise Time

T: Duration





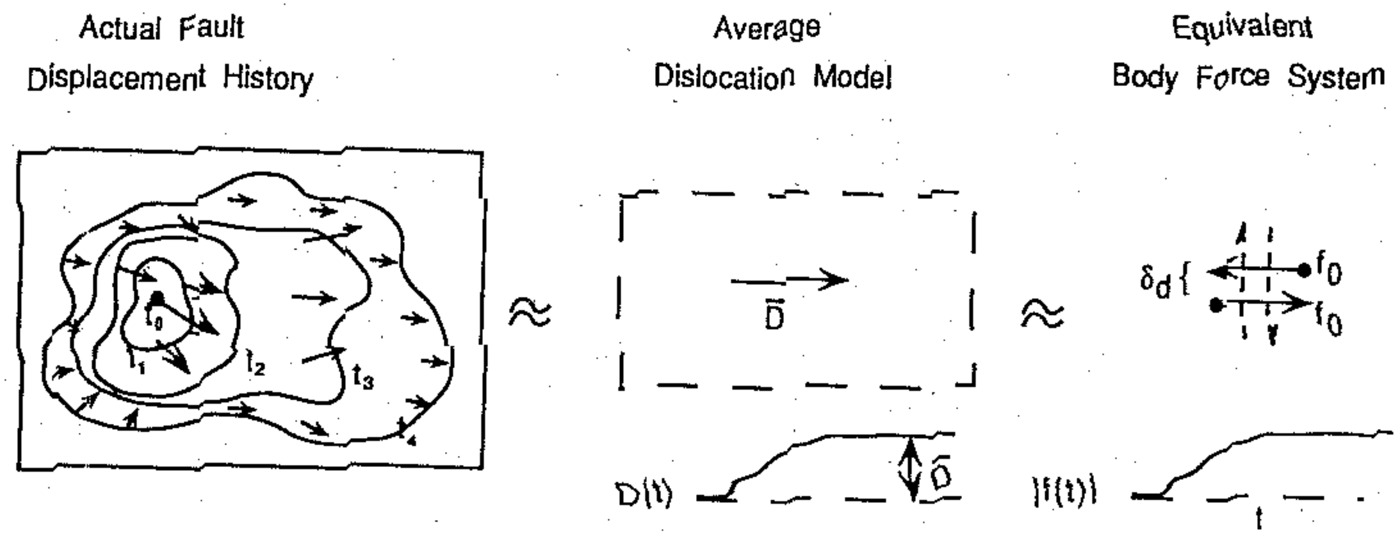


FIGURE 8.9 Concepts underlying equivalent body forces. Actual faulting involves complex cracking and frictional sliding over a surface in a short time that results in a space-time history of slipping motion. The finite spatial–temporal faulting process can be approximated by a dislocation model with dislocation time history $D(t)$. In turn, this dislocation model can be idealized by an equivalent force system that can be directly incorporated in the equations of motion.

Slip can (& does) vary over fault & $\dot{s} = 0$ at edges.

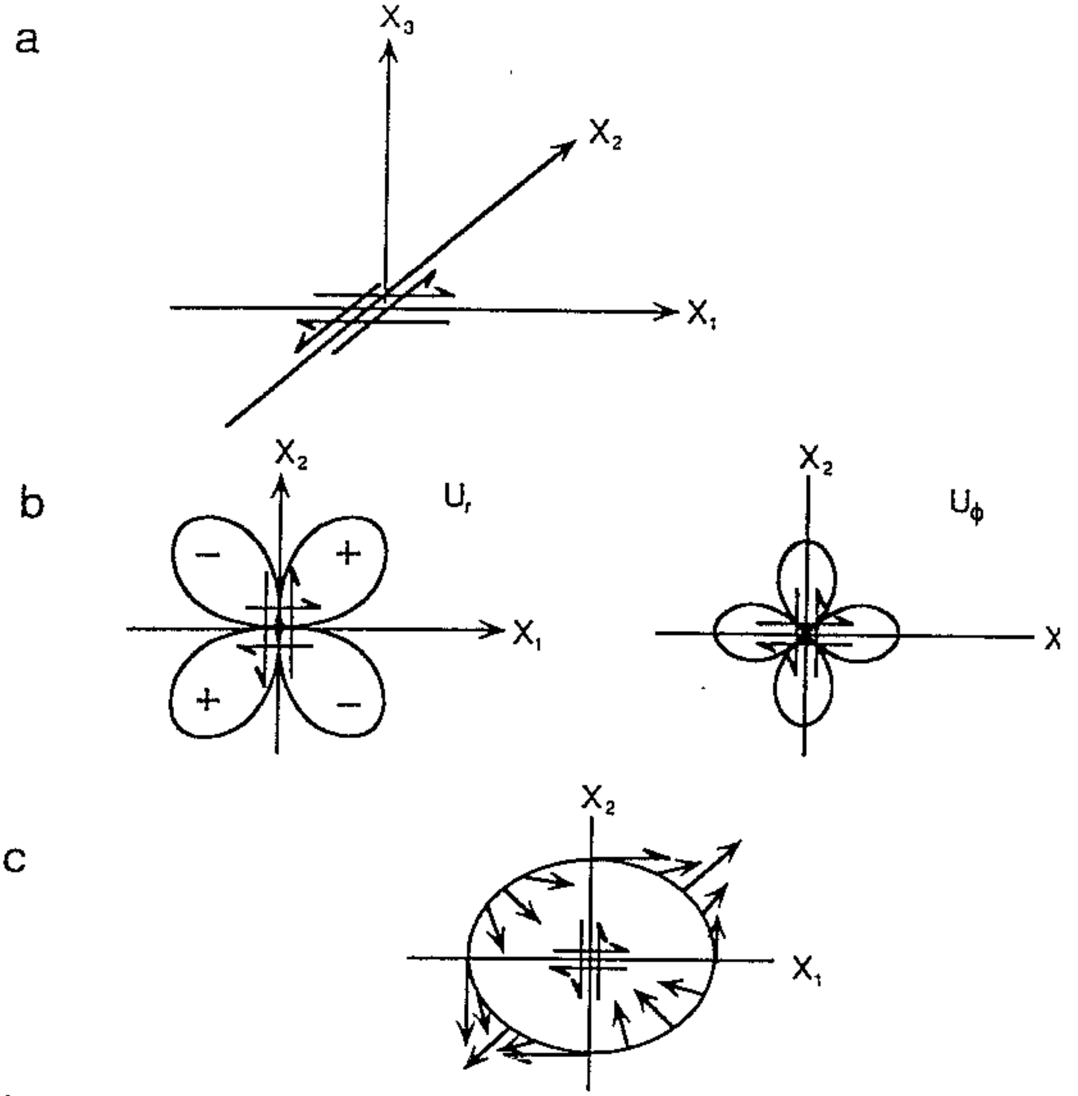


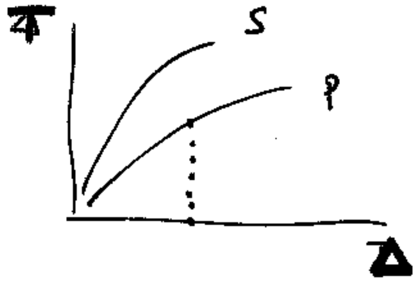
FIGURE 8.14 (a) A double couple in the x_1x_2 plane. (b) Azimuthal pattern of radial (u_r) and tangential (u_ϕ) displacements in the x_1x_2 plane. (c) The total displacement pattern in the x_1x_2 plane on a circle around the source, involving a combination of u_r and u_ϕ components.

~~1/r term: geom. spr. in homo. medium. For real geom. spreading:~~

$$E(\Delta) \propto \left| \frac{d^2T}{d\Delta^2} \right|$$

(Recall: $p = \frac{dT}{d\Delta}$).

Essential when computing theoretical seismograms (or practical ones!)



→ Eqn 8.66

→ GEOMETRIC SPREADING FACTOR

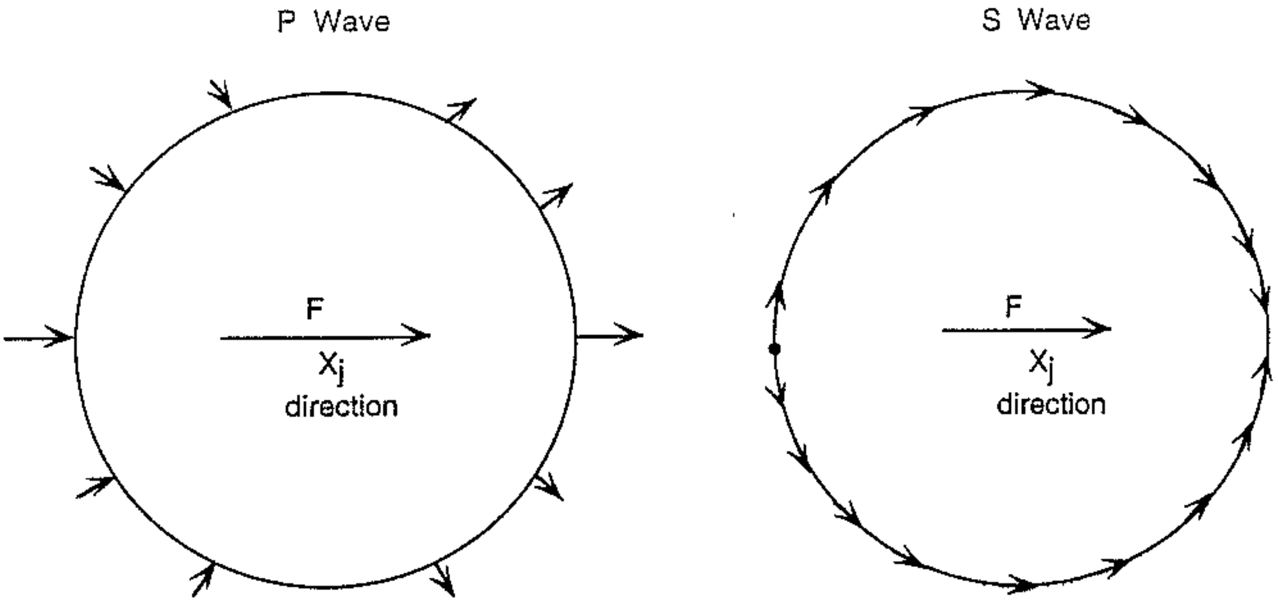


FIGURE 8.17 Sense of far-field displacements on P and S wavefronts produced by a single force in the x_j direction in an infinite, homogeneous, isotropic medium.

Fault orientation:

Strike-slip

Normal

Thrust

Can be found from moment tensor → Comp. functions

In stereographic projection!

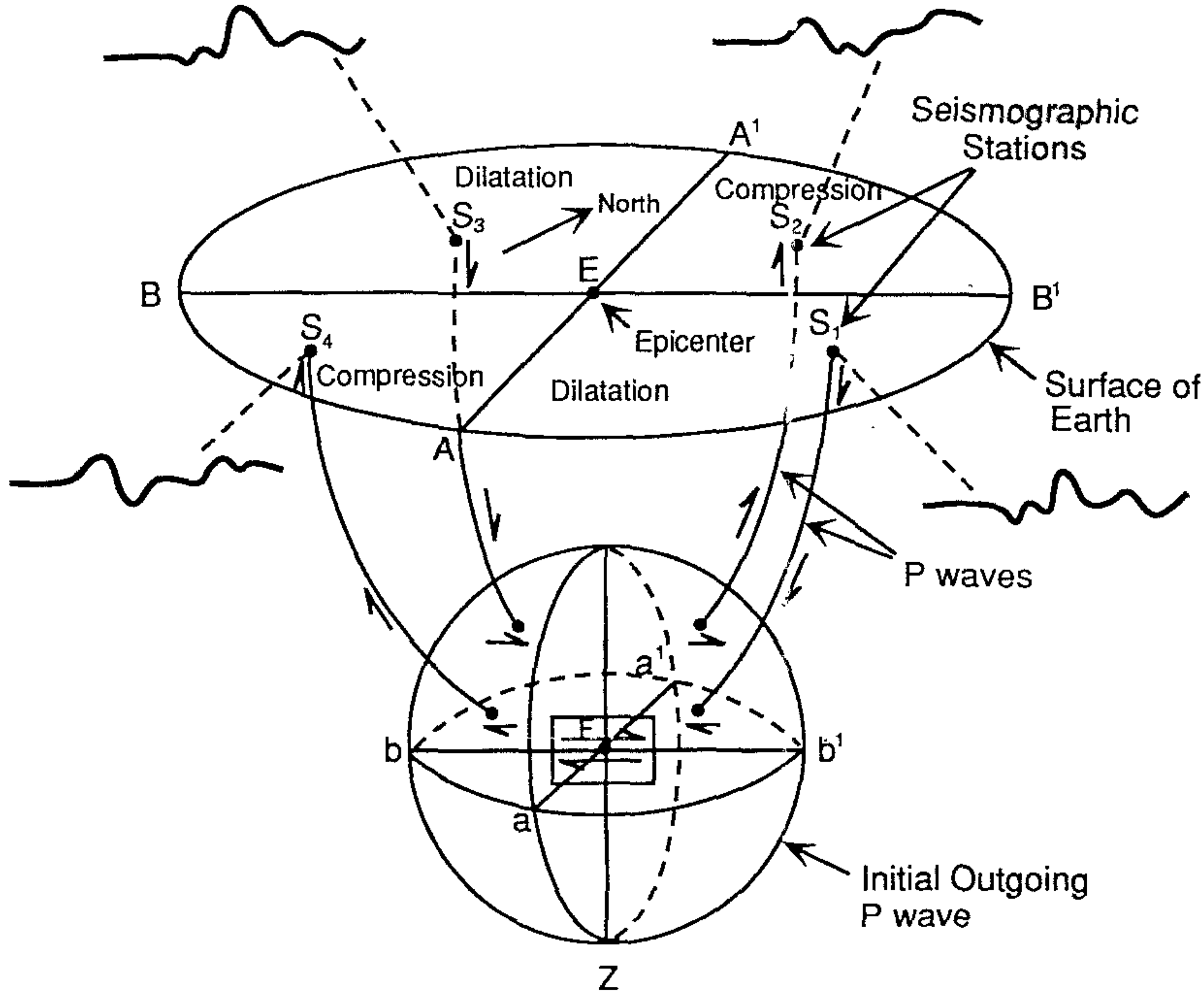
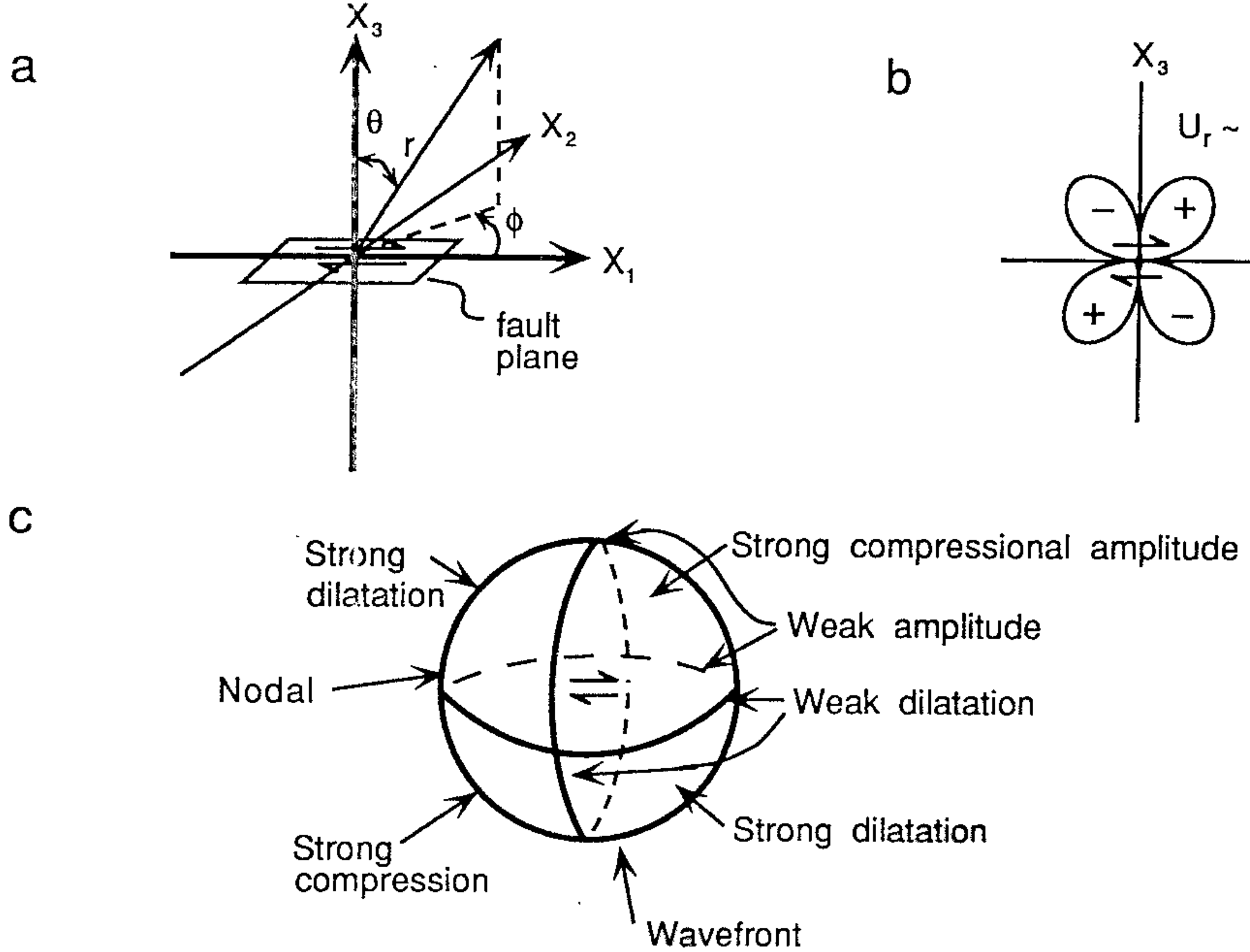


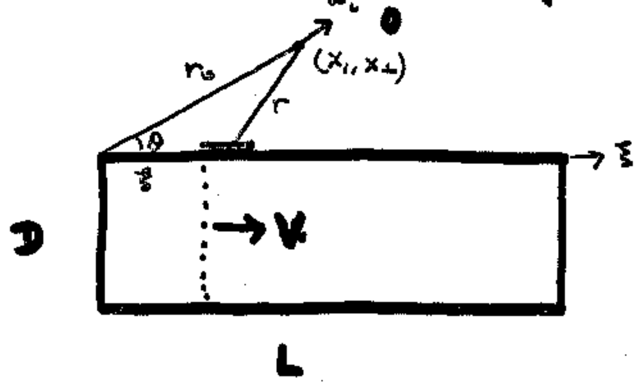
FIGURE 8.8 First motions of *P* waves at seismometers located in various directions about an earthquake allow determination of the fault orientation. *P* waves, for which upward


a
X_3
θ
r
X_2
φ
X_1
fault plane
b
X_3
$U_r \sim \sin 2\theta$
X_1
c
Strong dilatation
Nodal
Strong compression
Strong compressional amplitude
Weak amplitude
Weak dilatation
Strong dilatation
Wavefront

Using the previous results, we can write down the seismogram due to a finite source by super-imposing solns. for pt. sources.

$$u_i^P(x,t) \propto \int_\Sigma \dot{u}\left(\xi, t - r/\alpha\right) dS$$

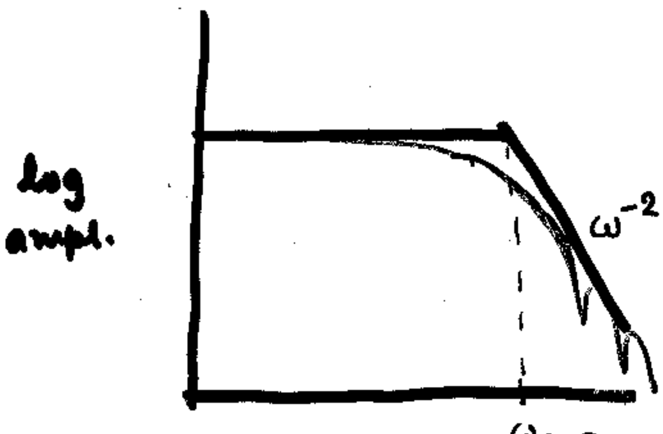
$$= D \int_0^L \dot{u}\left(t - r/\alpha - \frac{\xi}{v}\right) d\xi$$



v = rupture vel.
u = displ. on fault

(Haskell model type)

Take its F.T. to get the shape of the spectrum of the seismogram.



$$u_i^P(x,\omega) \propto \frac{\sin X}{X} e^{i\left(\frac{\omega r}{\alpha} + X - \pi/2\right)}$$

where $X = \frac{\omega L}{2\alpha}\left(\cos\theta - \frac{\alpha}{v}\right)$

ω_c → corner freq. rel. to the fault length L : is in fact used to find L for simple cases. Actually it is rel. to fault duration (even in the most complex cases).

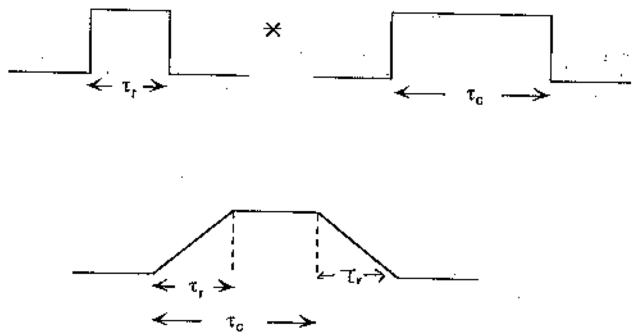


FIGURE 9.6 The convolution of two boxcars, one of length τ_r and the other of length τ_c ($\tau_c > \tau_r$). The result is a trapezoid with a rise time of τ_r, a top of length $\tau_c - \tau_r$, and a fall of width τ_r.

For this simple line source, or *Haskell ult model* (Haskell, 1964), the far-field - and *S*-wave displacements should be apezoidally shaped. Now consider the ea under a far-field *P* wave:

$$\int_{-\infty}^{\infty} u_r(r,t)\,dt$$

$$= \int_{-\infty}^{\infty} \frac{R^P \mu}{4\pi\rho\alpha^3} v_r \frac{w}{r} \dot{D}(t) * B(t;\tau_c)\,dt \qquad (9.16)$$

, rearranging terms,

$$\frac{4\pi r\rho\alpha^3}{R^P} \int_{-\infty}^{\infty} u_r(r,t)\,dt$$

$$= \int_{-\infty}^{\infty} \dot{D}(t)\mu w v_r B(t;\tau_c)\,dt. \qquad (9.17)$$

e right-hand side of (9.17) is the area of (t) (D) multiplied by the area of $\mu w v_r \cdot$ $t;\tau_c)$ $(\mu w L)$. Thus, the right-hand side equal to the seismic moment, $M_0 =$ DA. The left-hand side is the area under e displacement pulse corrected for reading, the radiation pattern, and the urce material constants. This equality provides a procedure for determining the seismic moment from far-field displacements. Figure 9.7 shows the *SH* displacement waveform from an earthquake near Parkfield, California. Note that its shape is roughly trapezoidal.

9.1.1 Directivity

In the simple Haskell source model, the boxcar associated with the propagation of the rupture had a length τ_c for a station at an azimuth perpendicular to the strike of the ribbon fault. Obviously, τ_c depends on the dimensions of the fault and on v_r, but it also depends on the orientation of the observer relative to the fault. In general, the rupture velocity is less than the *S*-wave velocity of the faulted material; the body waves generated from a breaking segment of the fault will arrive at a station before the body waves arrive from a segment that ruptures later. On the other hand, when the path to the station is not perpendicular to the fault, the body waves generated from different segments of the fault will have different travel path lengths to the recording station and thus unequal travel times. Figure 9.8 shows a fault of length

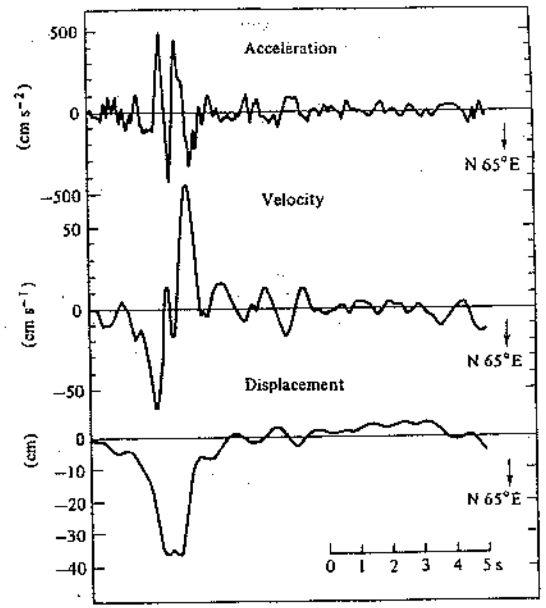


FIGURE 9.7 A recording of the ground motion near the epicenter of an earthquake a Parkfield, California. The station is located on a node for *P* waves and a maximum for *SH*. Th displacement pulse is the *SH* wave. Note the trapezoidal shape. [From Aki, *J. Geophys. Res* 73, 5359–5375, 1968; © copyright by the American Geophysical Union.]

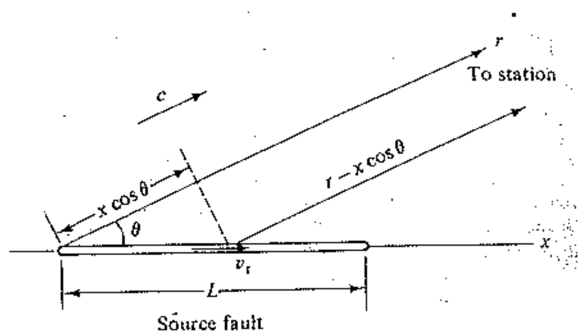


FIGURE 9.8 Geometry of a rupturing fault and the path to a remote recording statio (From Kasahara, 1981.)

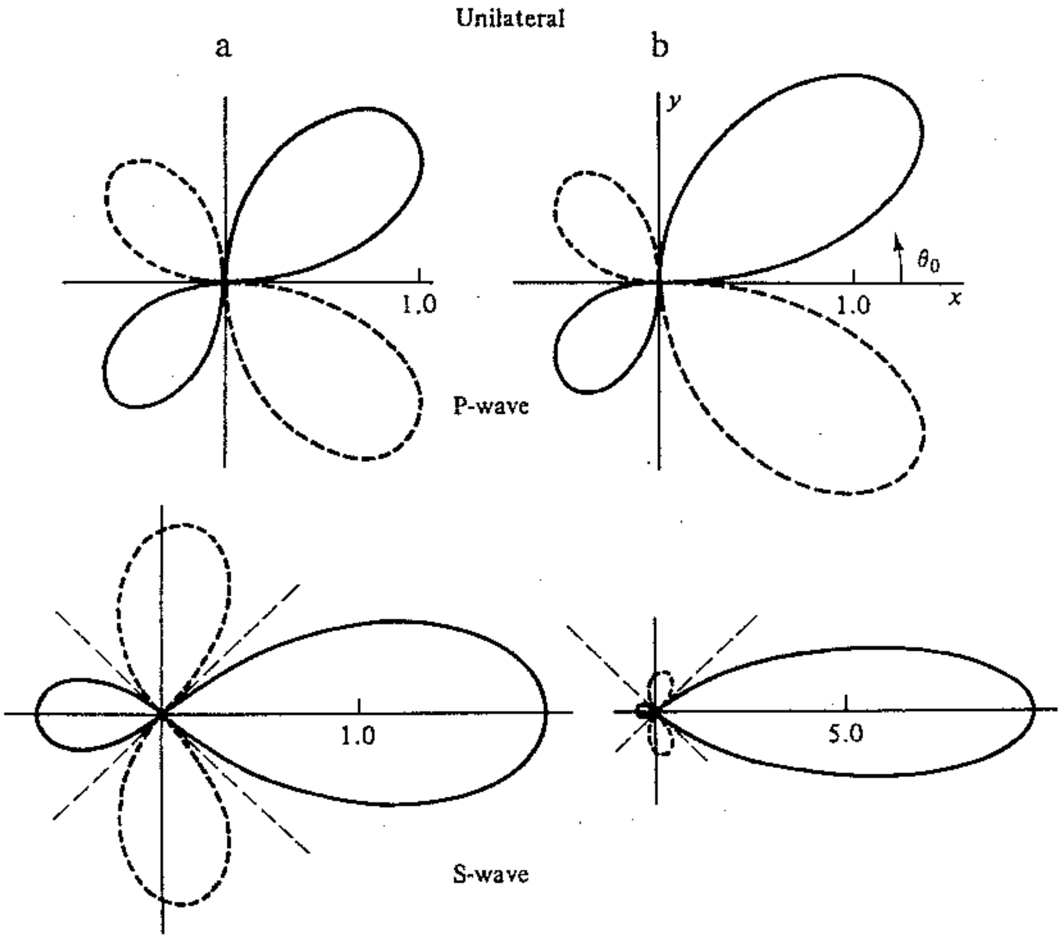


FIGURE 9.10 The variability of P- and SH-wave amplitude for a propagating fault (from left to right). For the column on the left $v_r/v_s = 0.5$, while for the column on the right $v_r/v_s = 0.9$. Note that the effects are amplified as rupture velocity approaches the propagation velocity. (From Kasahara, 1981.)