The Abdus Salam
International Centre for Theoretical Physics

United Nations Educational, Scientific and Cultural Organization

International Atomic Energy Agency

H4.SMR/1775-15

"8th Workshop on Three-Dimensional Modelling of Seismic Waves Generation, Propagation and their Inversion"

25 September - 7 October 2006

Surface Waves and Upper Mantle Anisotropy

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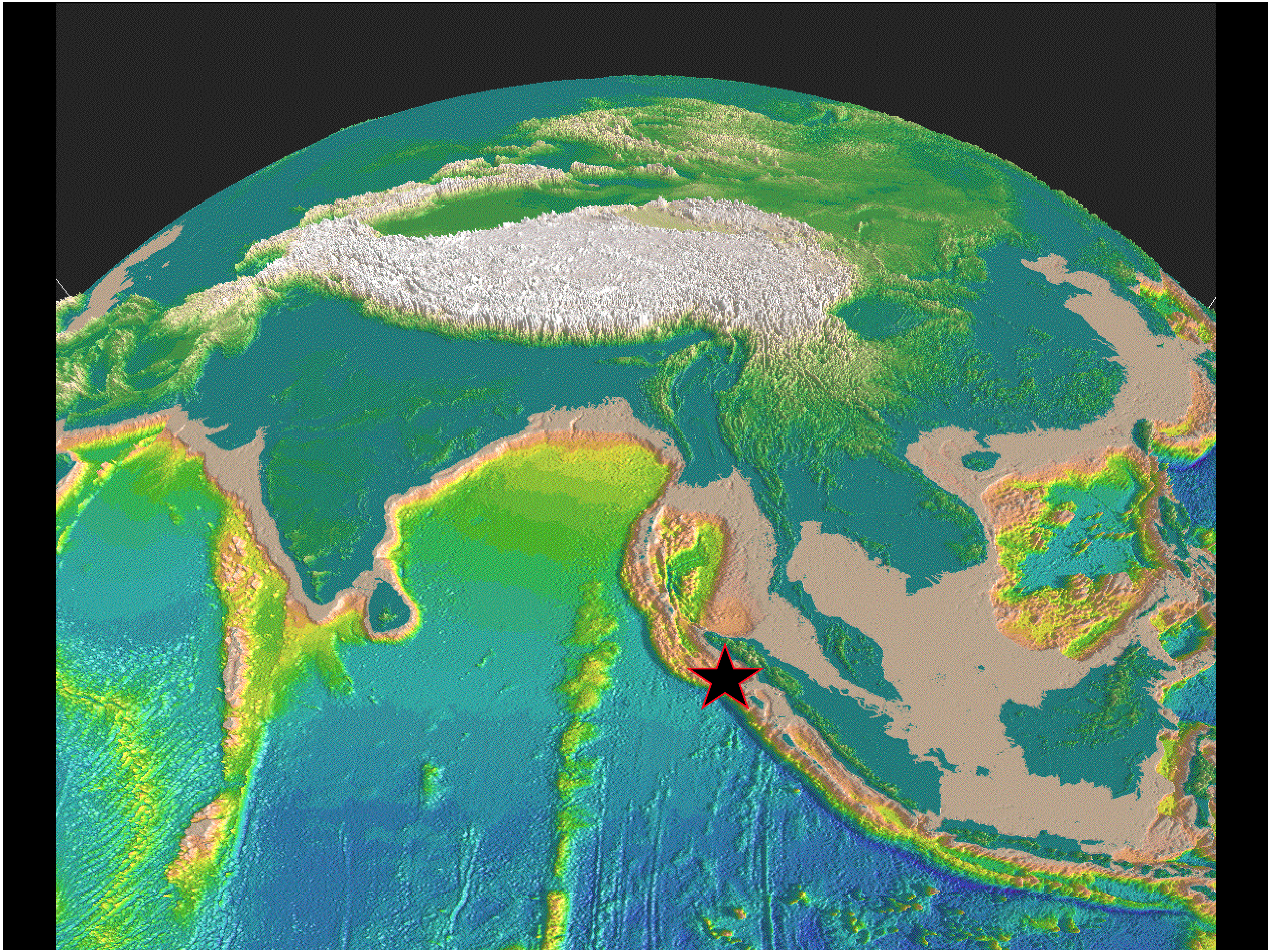
SURFACE WAVES and UPPER MANTLE ANISOTROPY

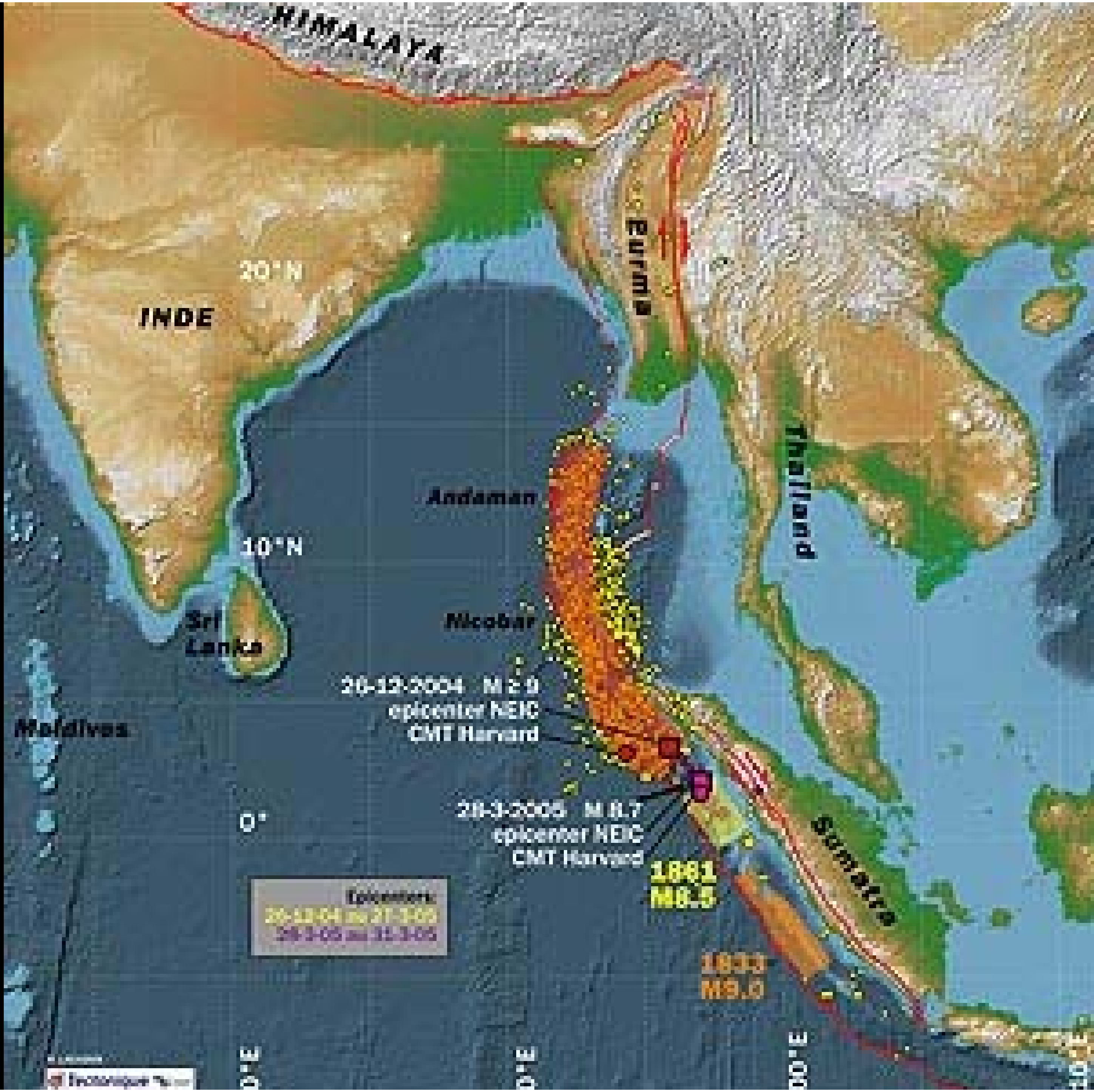
Jean-Paul Montagner

Dept. Sismologie, I.P.G., Paris; France

Overview
Large scale Seismology: an observational field

- Data (Seismic source) + Instrument (Seismometer) -> Observations (seismograms)
- Historical evolution: Ray theory, Normal mode theory, Numerical techniques (SEM, NM-SEM)
- Scientific Issues: Earthquakes (Sumatra-Andaman), Anisotropic structure of the Earth
- NM-SEM and time reversal
- Tomographic Technique
- Seismic Experiment: Plume detection

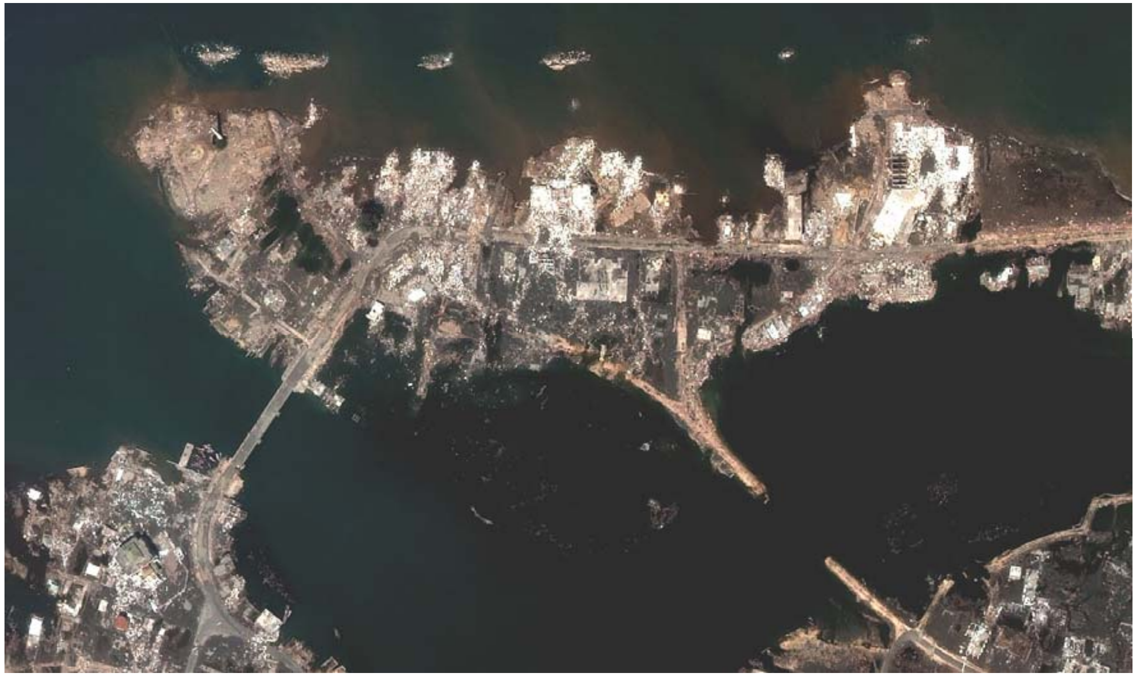



HIMALAYA
Burma
20°N
INDE
Thailand
Andaman
10°N
Nicobar
Sri Lanka
26-12-2004 M 9
epicenter NEIC
CMT Harvard
Maldives
28-3-2005 M 8.7
epicenter NEIC
CMT Harvard
0°
Sumatra
1861
M8.5
Epicenters:
1833
M9.0

Banda Aceh

before

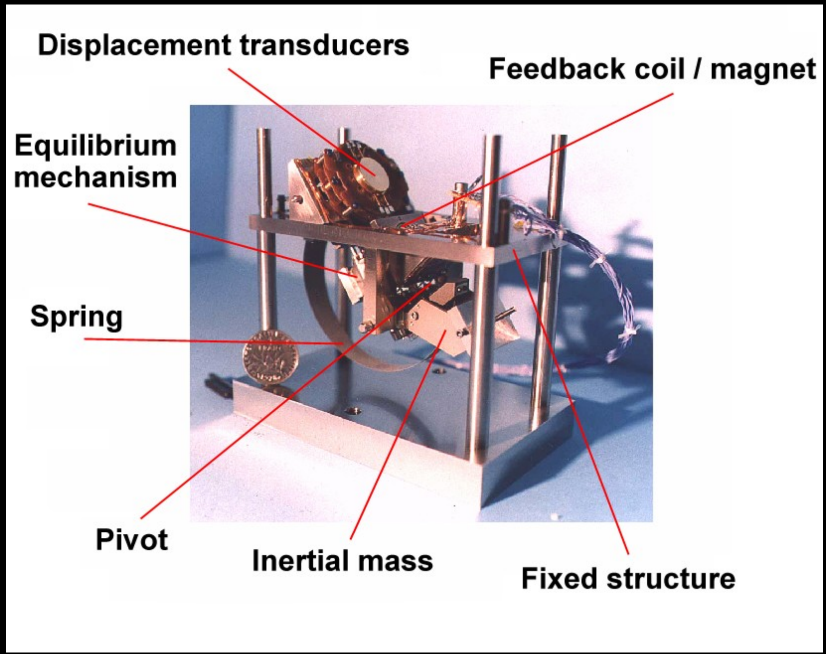
after



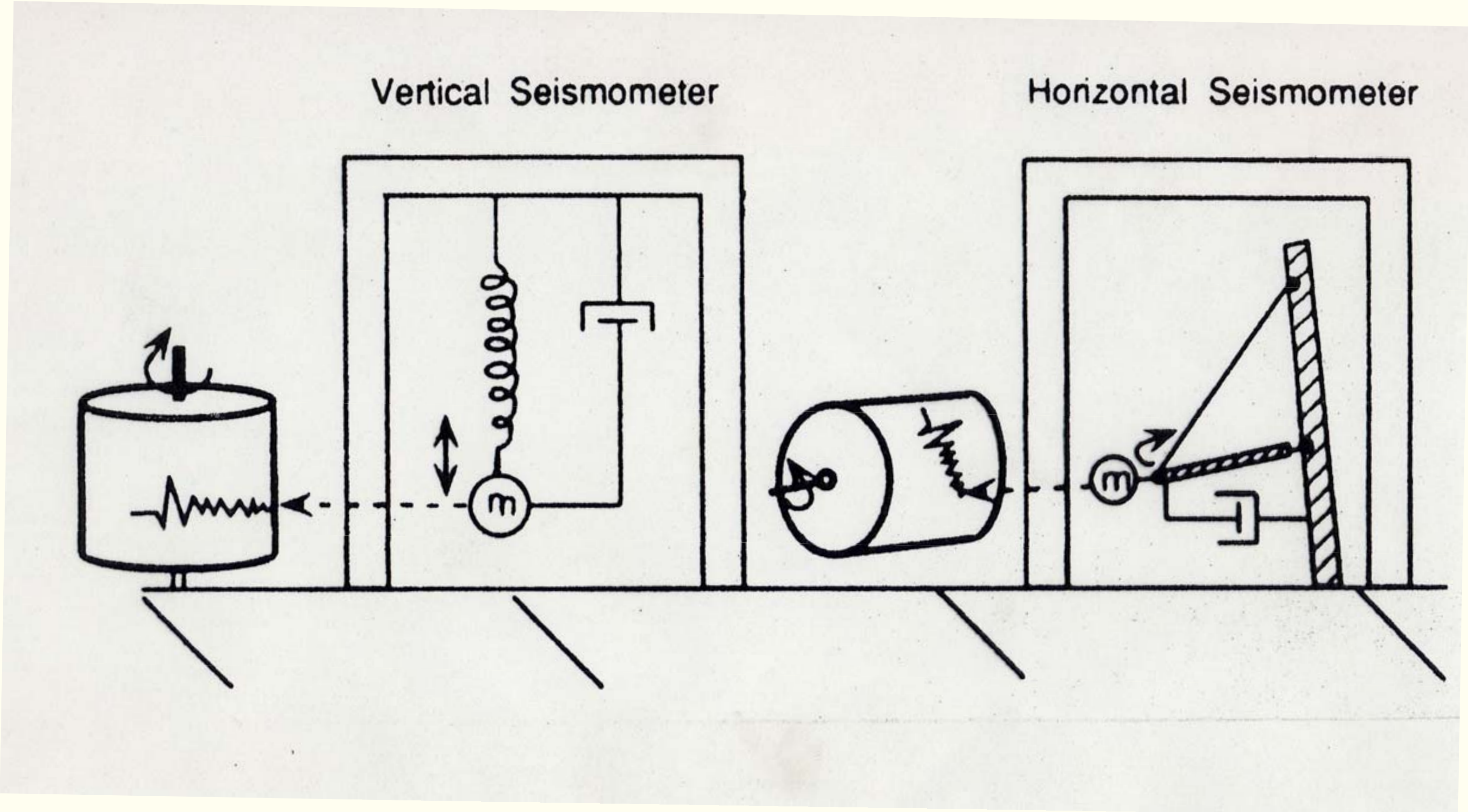
Seismic Instruments

- Seismoscope (China -100BC)

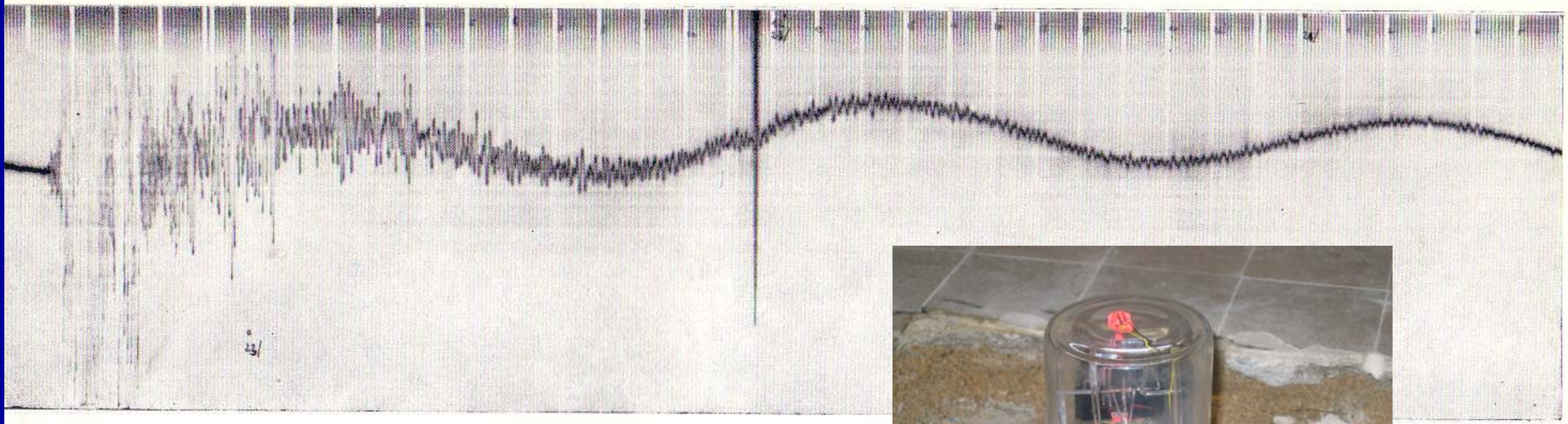
Broadband Seismometer (1mHz-20Hz) (Cacho, 1998)



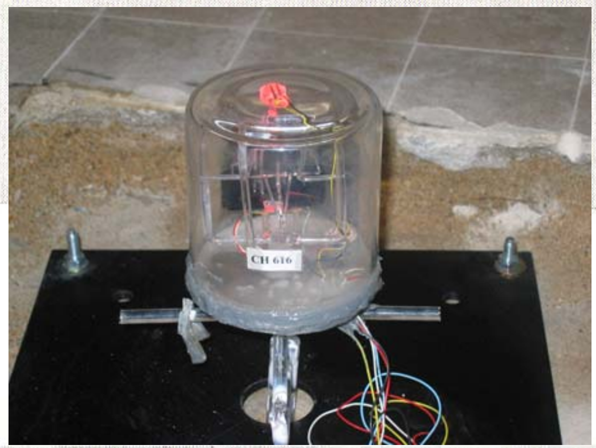
Principle of a Seismometer

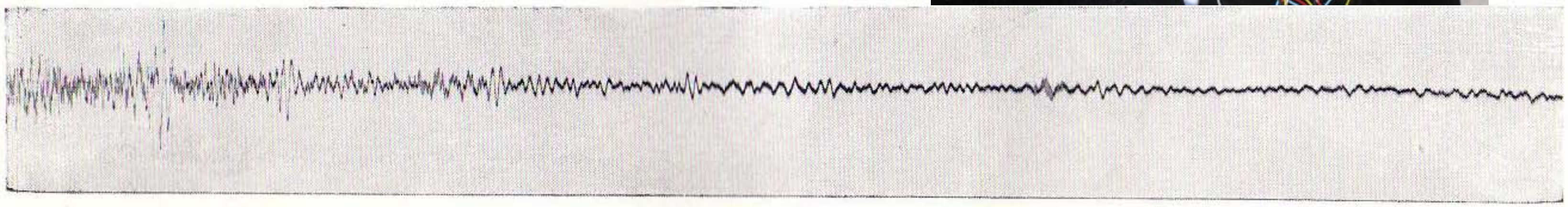


Chile Earthquake (22 may 1960) recorded at Paris (IPGP)



1a





1b

FIG. 1. — *a*) Enregistrement du pendule E, nº 1 (voir tableau I).
b) Enregistrement du pendule B, nº 4 (voir tableau I). (Un intervalle de 5 minutes est représenté sur cette figure par 1,45 mm.)

Chile earthquake (may 22 1960) recorded at Paris (IPGP)

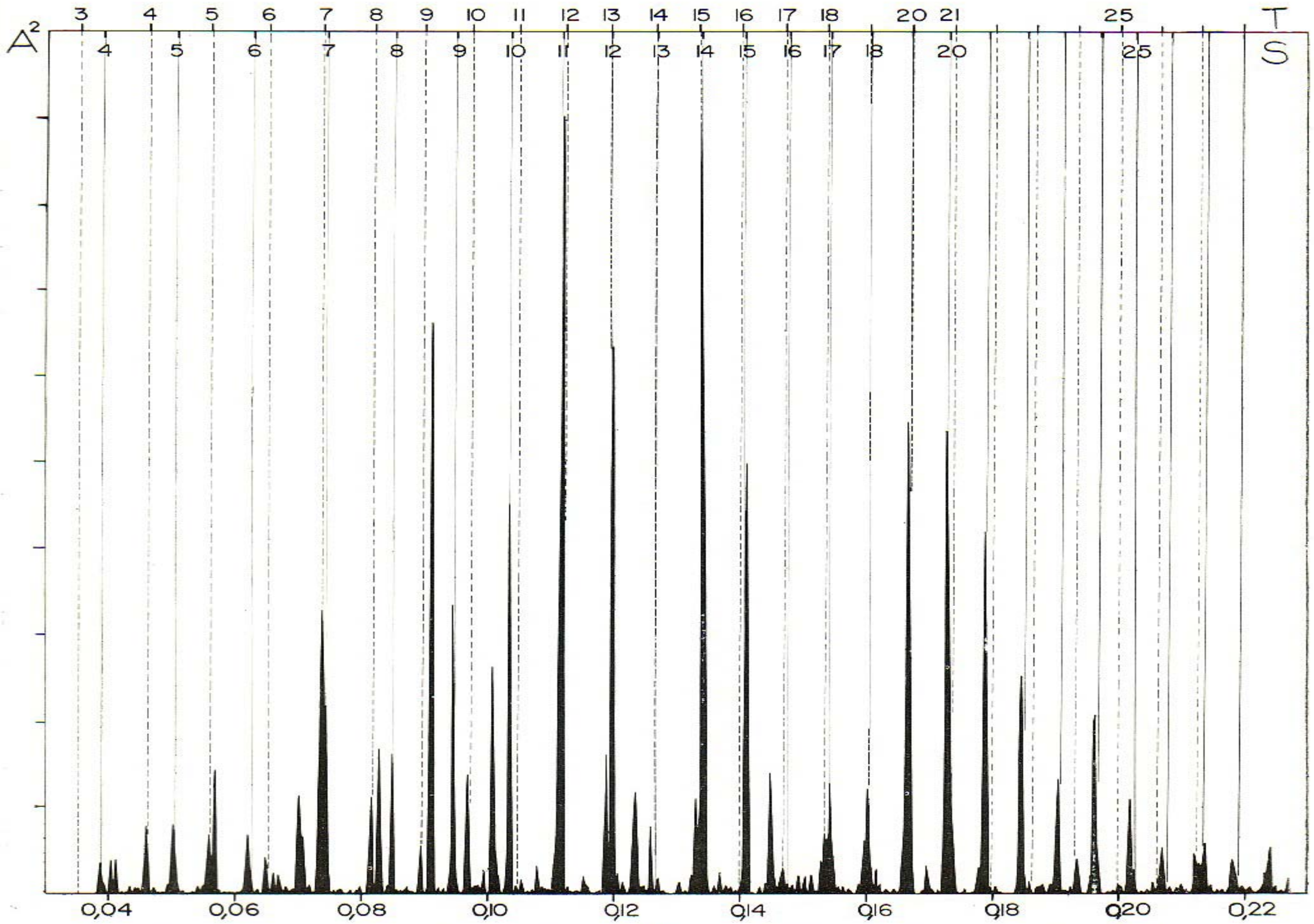
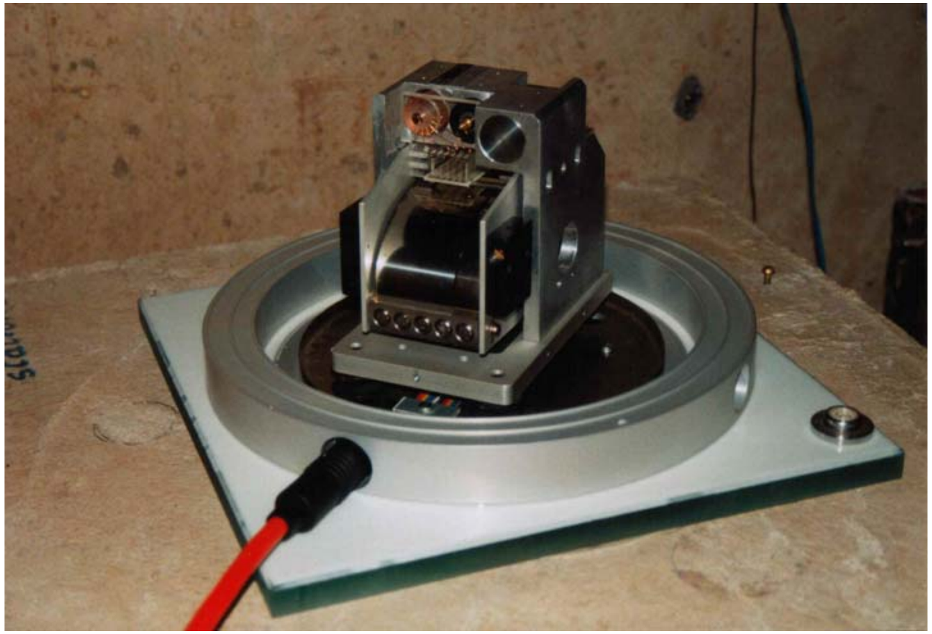


FIG. 3. — Spectre de l'enregistrement du pendule E (nº 1) — En haut : positions des pics théoriques pour les oscillations sphéroïdales S et les oscillations de torsion T du modèle de Gutenberg continental.

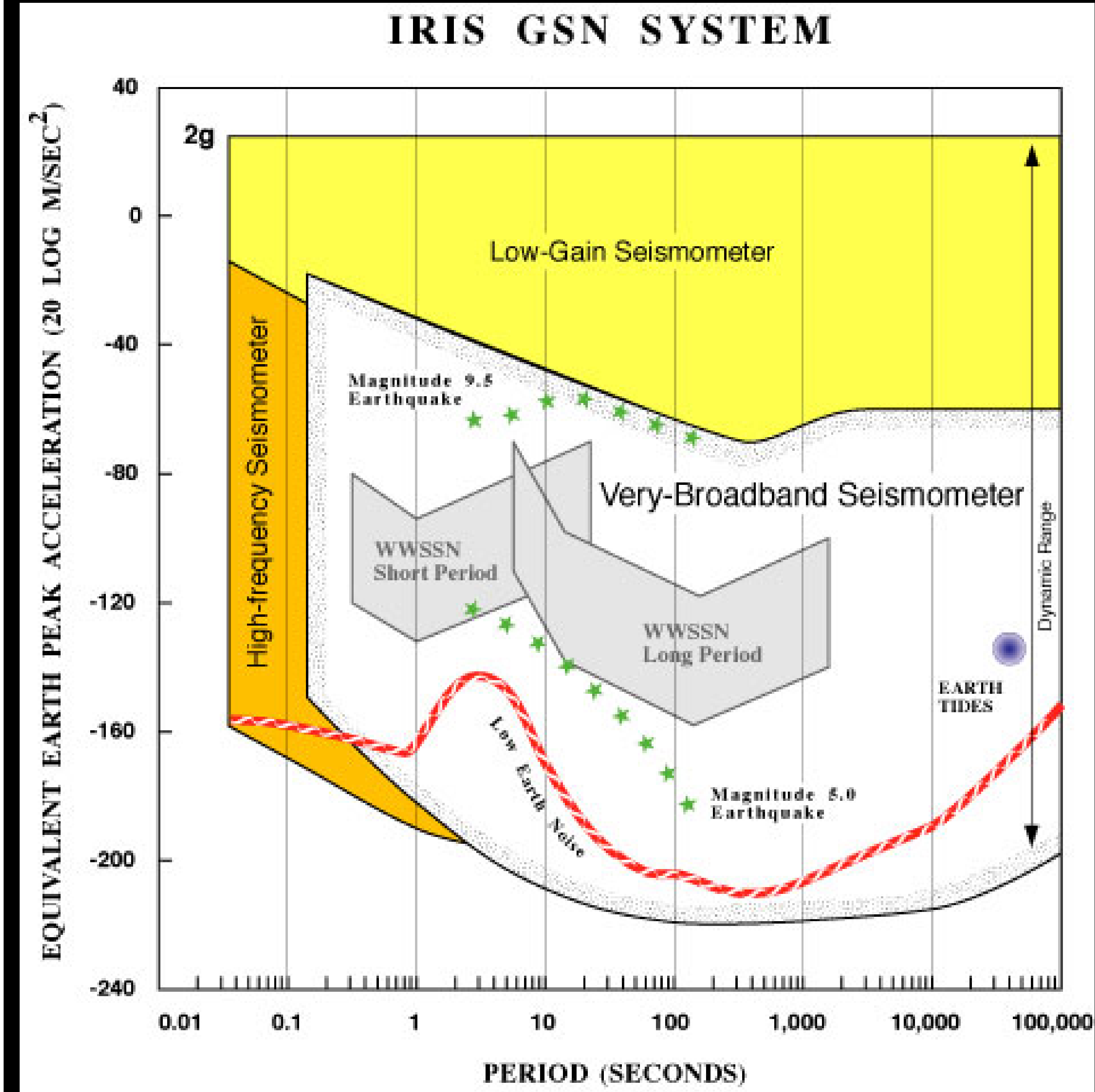
First observations of free oscillations of the Earth

Frequency Peaks not well understood

Theory was incomplete



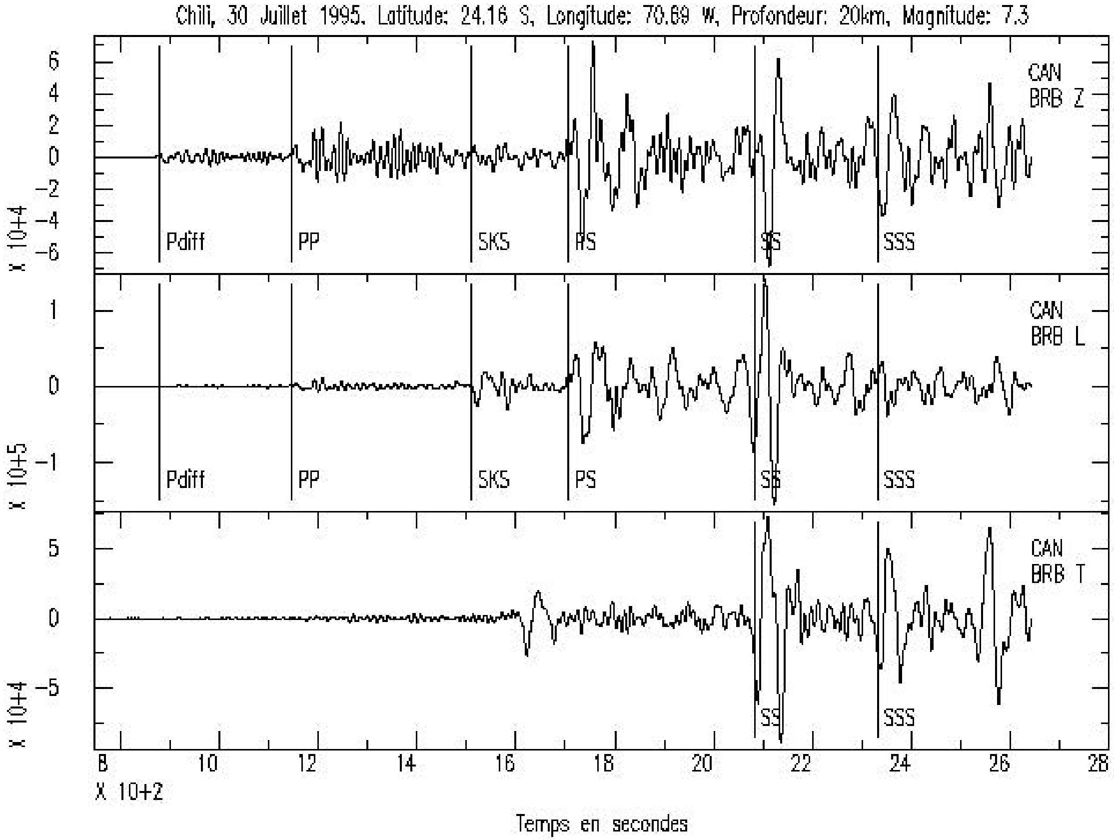
Broadband Seismometer
Streckeisen STS1: 0.05s< T< 5000s



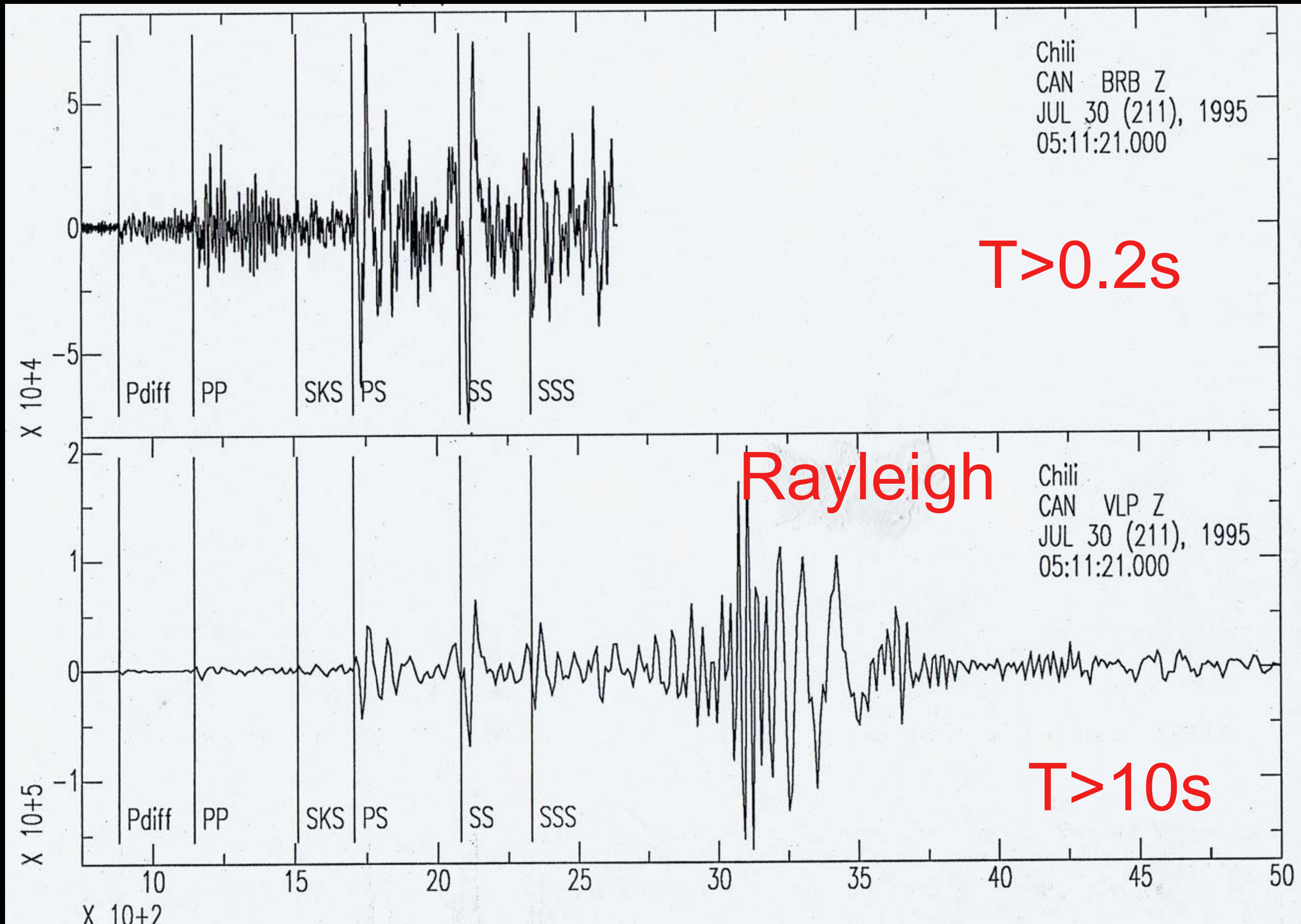
Butler et al., 2004

3 components
frequency range: 1mHz-20Hz
Period range: 0.05-1000s

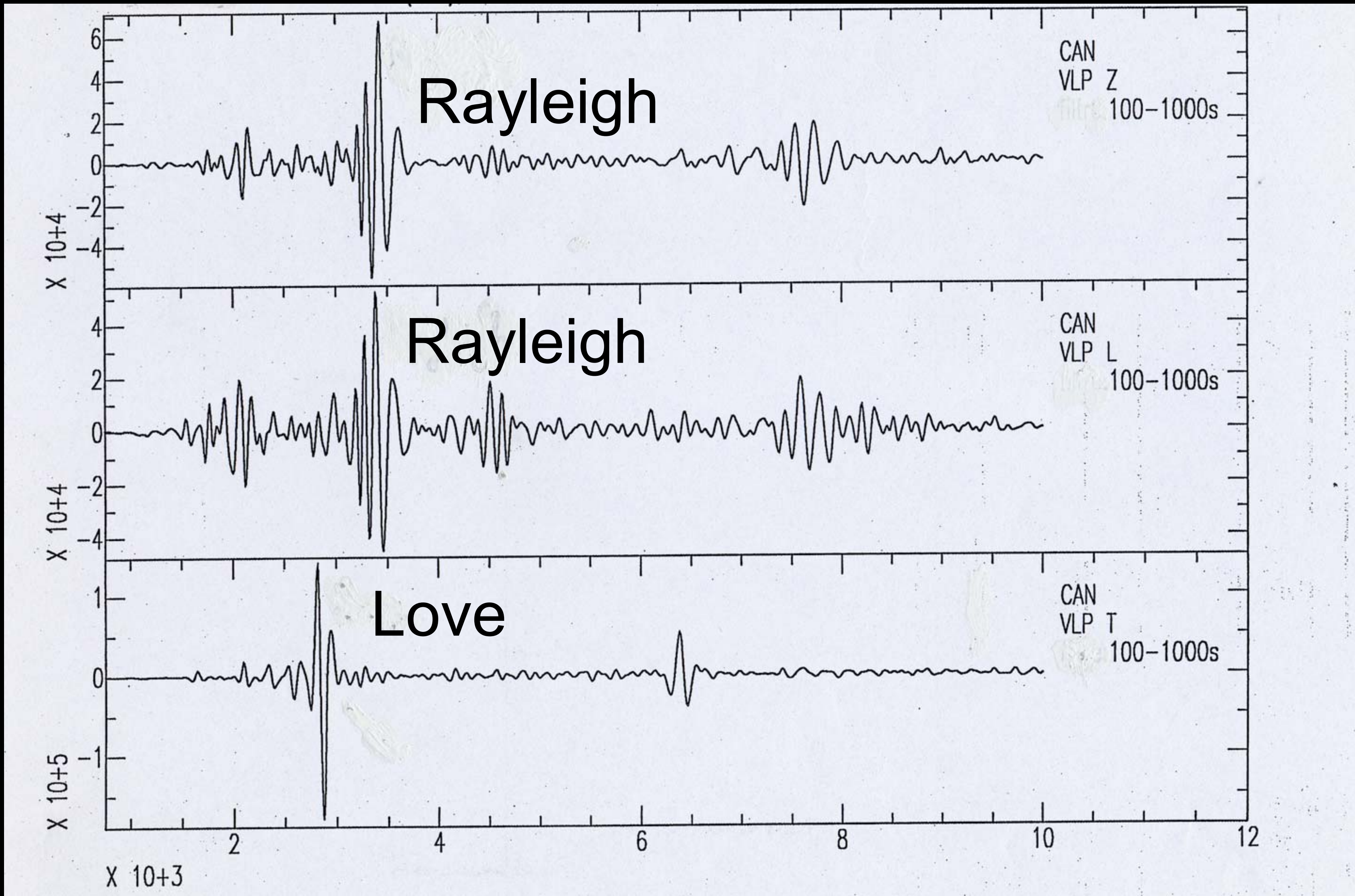
Chile July 30, 1995, Ms=8.3



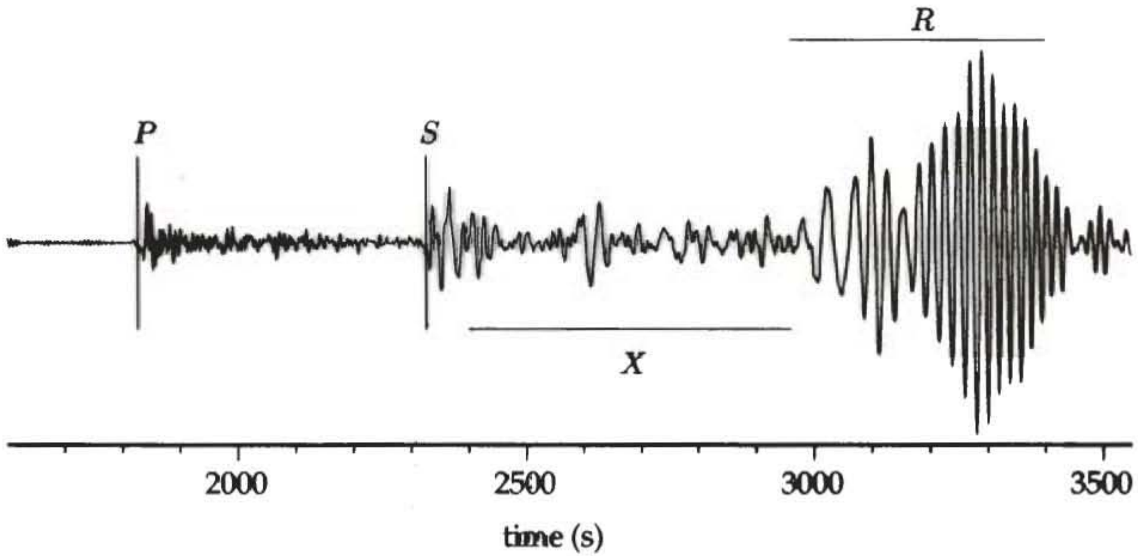
Chile earthquake magnitude= 7.3
Epicentral distance = 12,300km-depth 20km



Chile Earthquake Jul. 1995

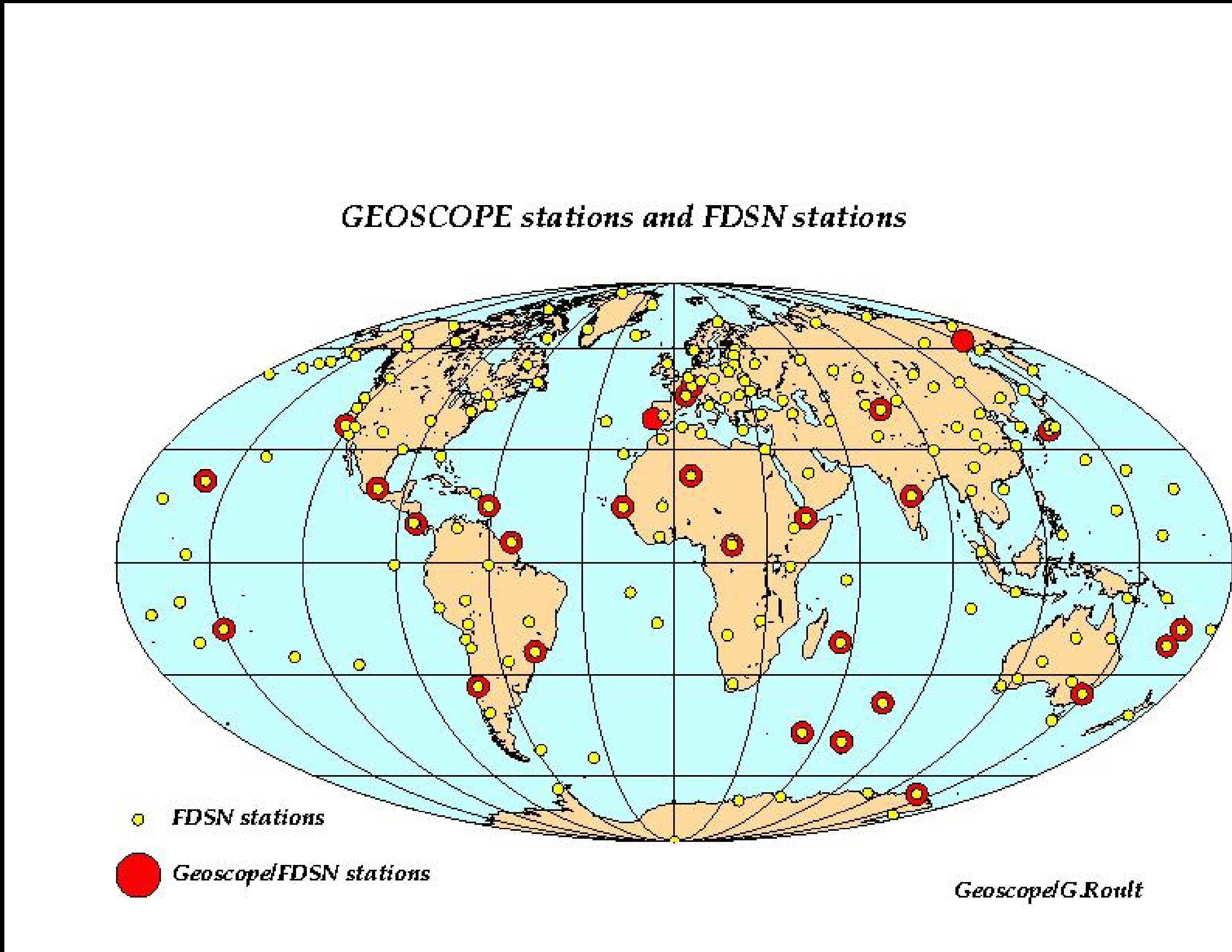


- ⇀ Dispersive waves,
- ⇀ Good global coverage,
- ⇀ Large scale heterogeneities (min. 600 km).



Vertical component of displacement field recorded at DRV station corresponding to the New-Guinea 05/16/1999 earthquake.

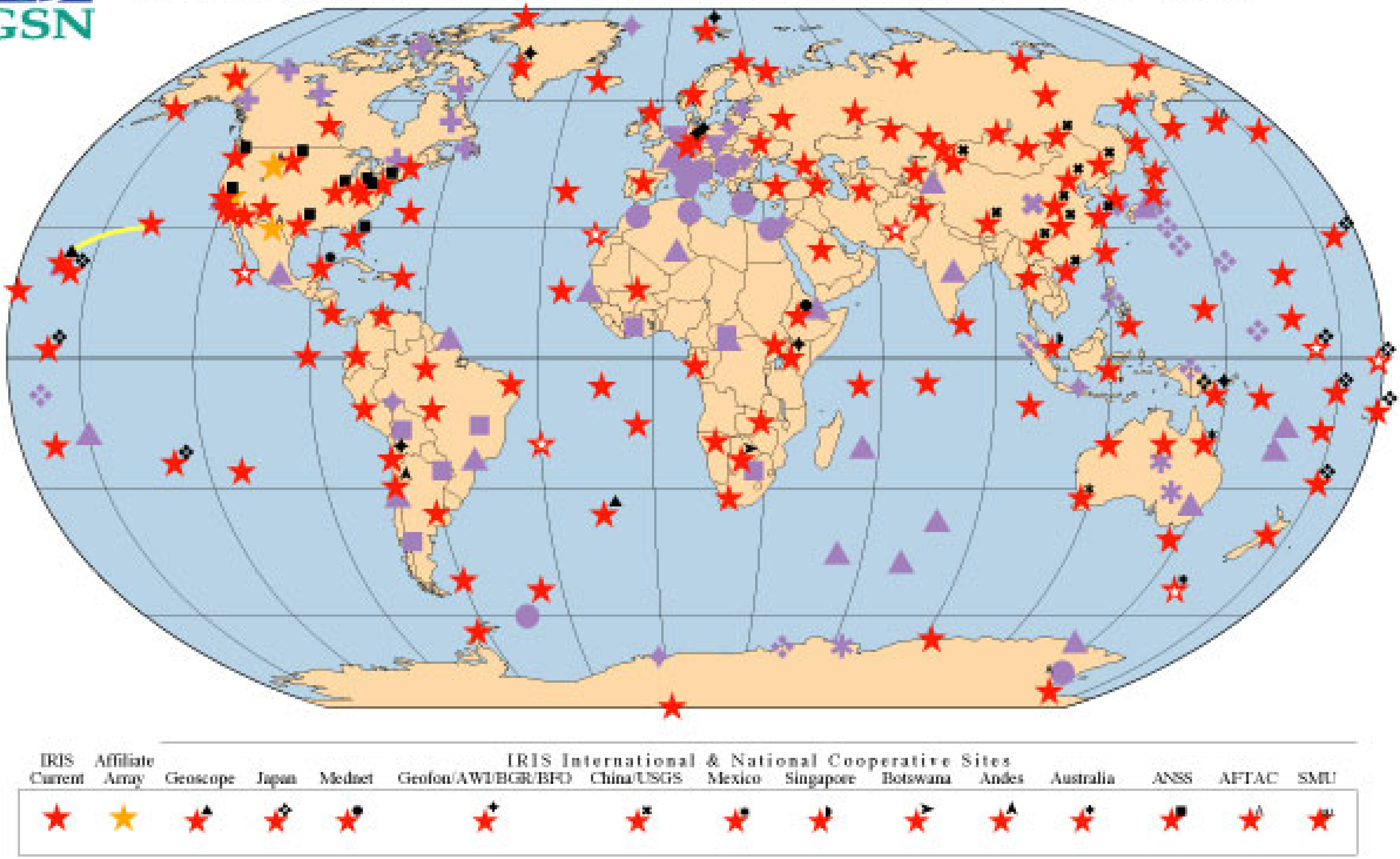
Global Networks



GSN

GLOBAL SEISMOGRAPHIC NETWORK

& FEDERATION OF DIGITAL BROADBAND SEISMIC NETWORKS



Ocean Bottom Observatories

=> International Ocean network (I.O.N.)

- 2/3 of the Earth are covered by water.
- seafloor seismometers enable:
  - To investigate oceanic regions with a better resolution
  - To fill gaps in the global coverage

NERO (joint French-Japanese Project)

I.O.N.

International Ocean Network

ION (International Ocean network) France, Italy, Japan, UK, U.S.

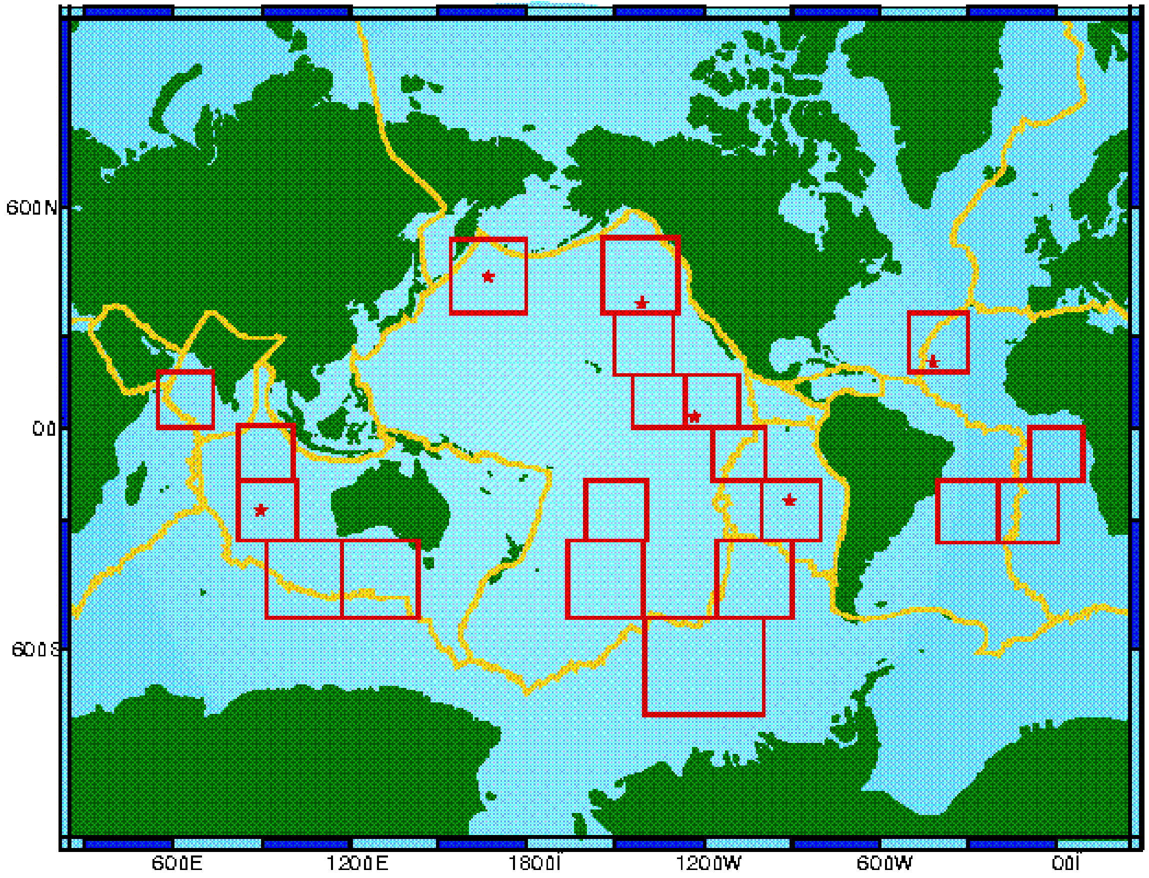
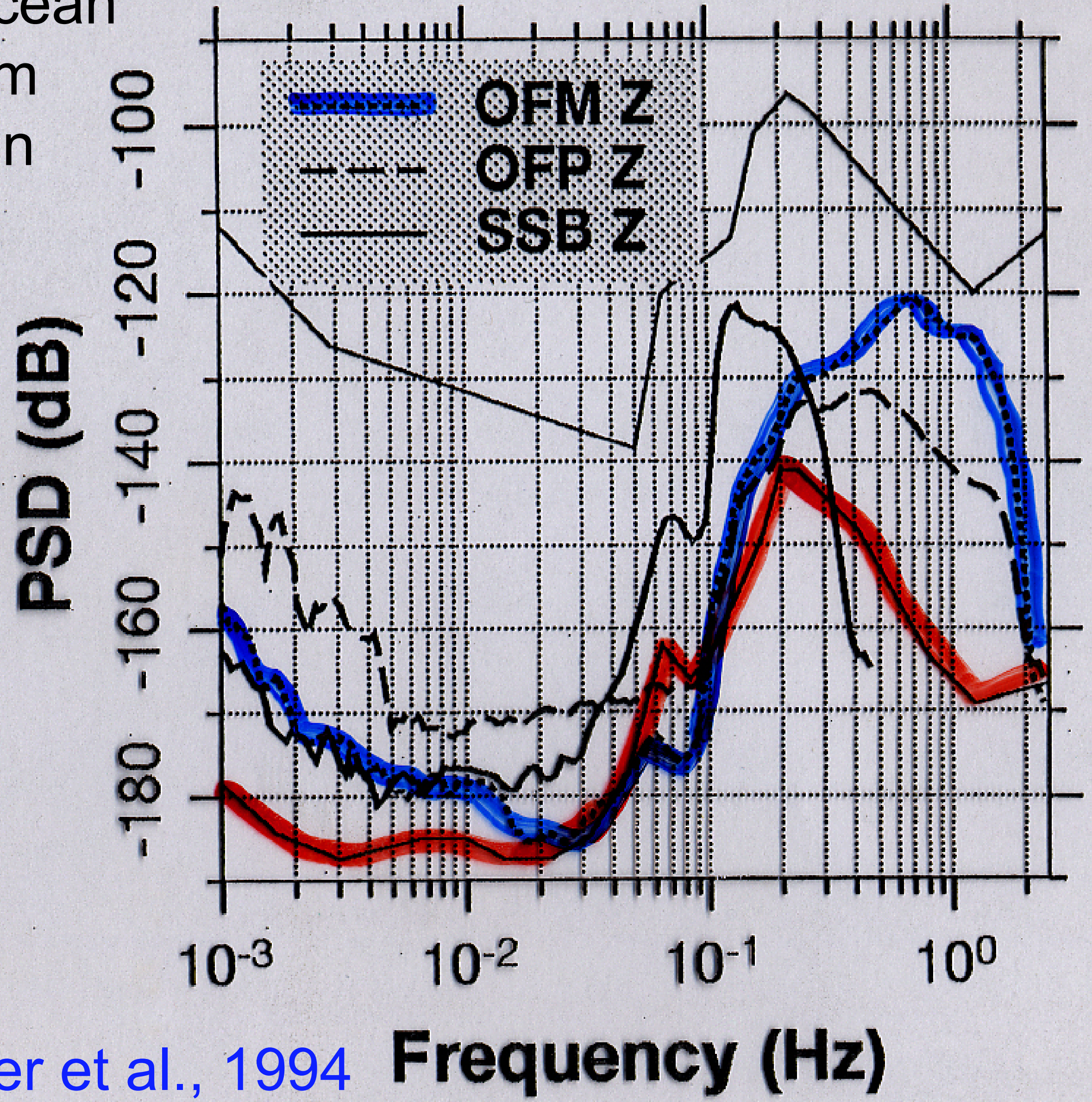


Figure 1: This map shows twenty regions which would require a seafloor seismic observatory in order to have 128 GSN stations evenly spaced around the globe (red boxes). The six starred boxes have been selected as preliminary test sites. The yellow lines mark plate boundaries.

OFM: Ocean bottom station



Montagner et al., 1994

IPGP
INSTITUT DE PHYSIQUE
DU GLOBE DE PARIS

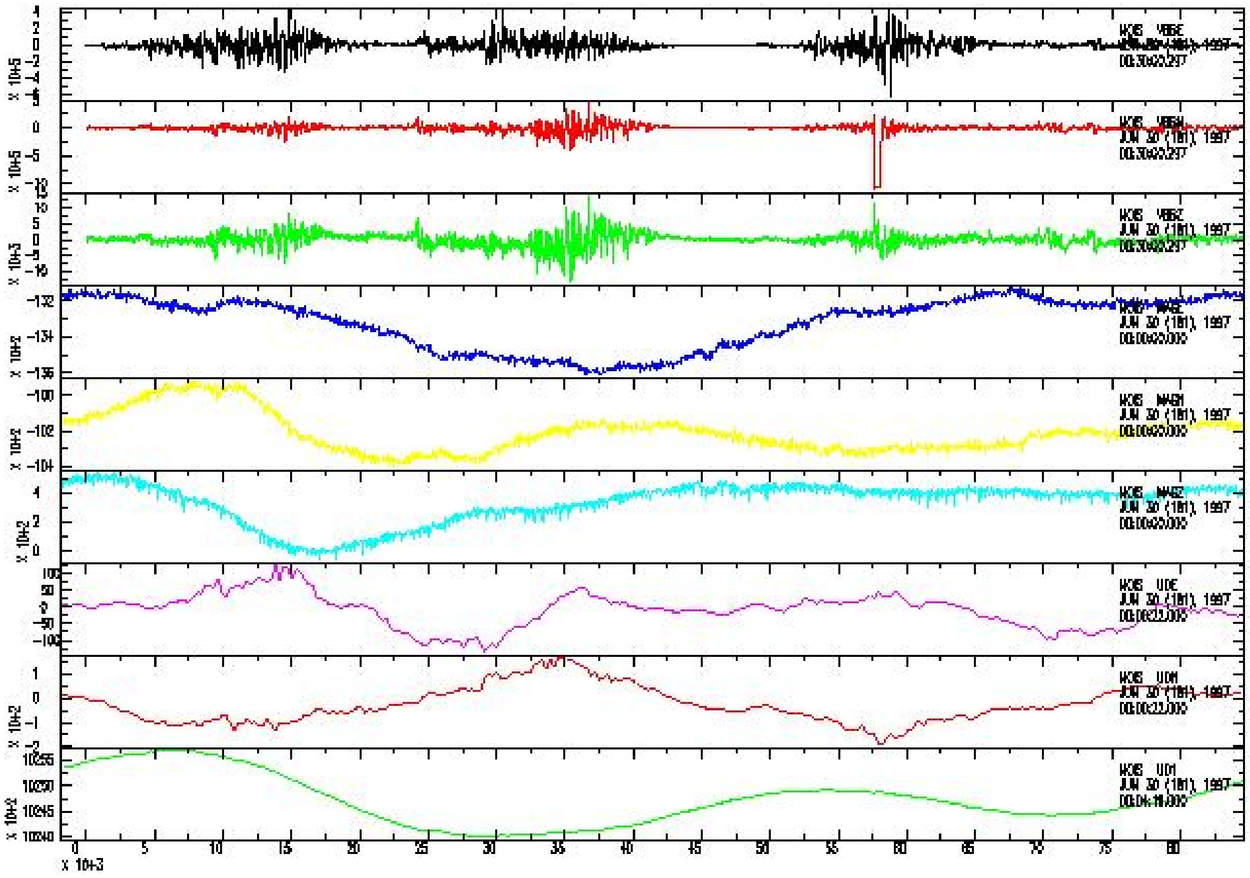
M.O.I.S.E (June-Sept. 1997)

(Monterey bay Ocean bottom International Experiment)

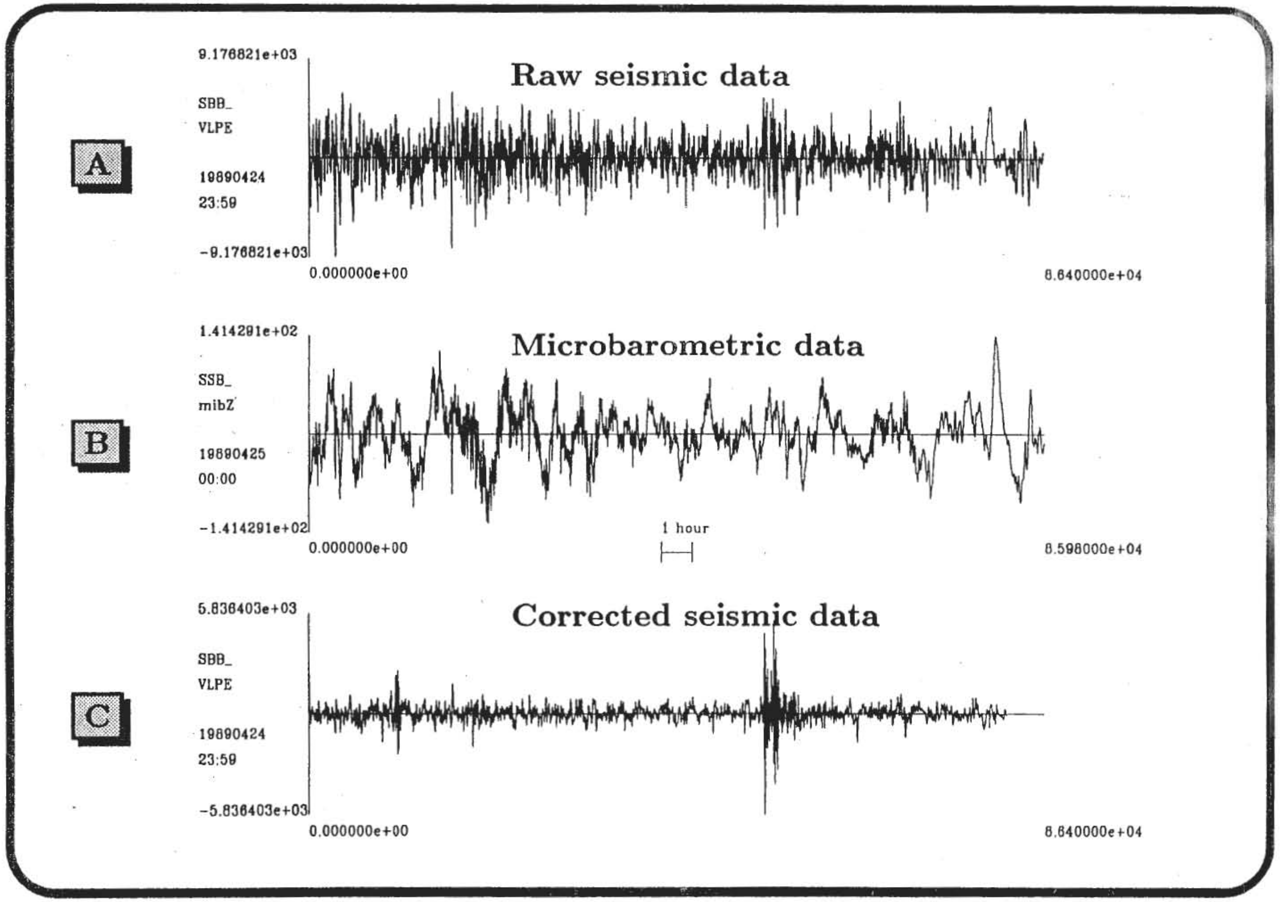
MBARI, UC Berkeley, IPG-Paris, UBO-Brest

IPGP
INSTITUT DE PHYSIQUE
DU GLOBE DE PARIS

Multiparameter signals



Deconvolution of the seismic signal from the pressure influence



Beauduin et al.,1996

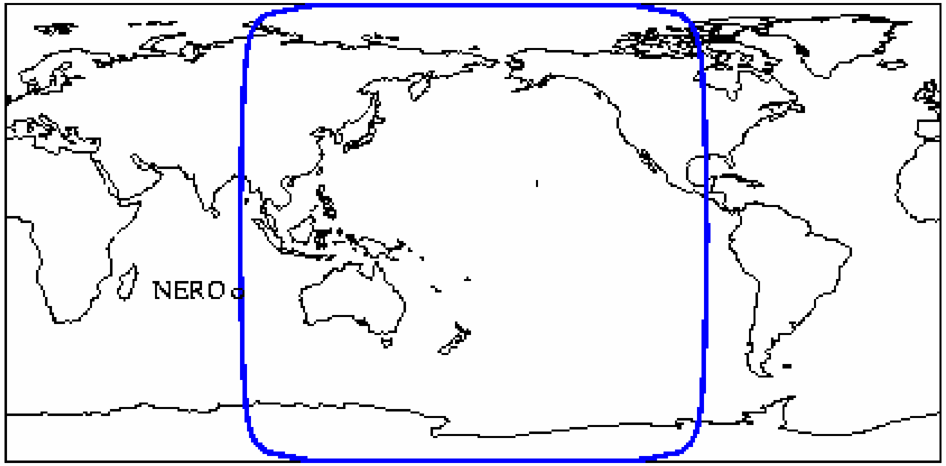
INSTITUT DE PHYSIQUE DU GLOBE DE PARIS

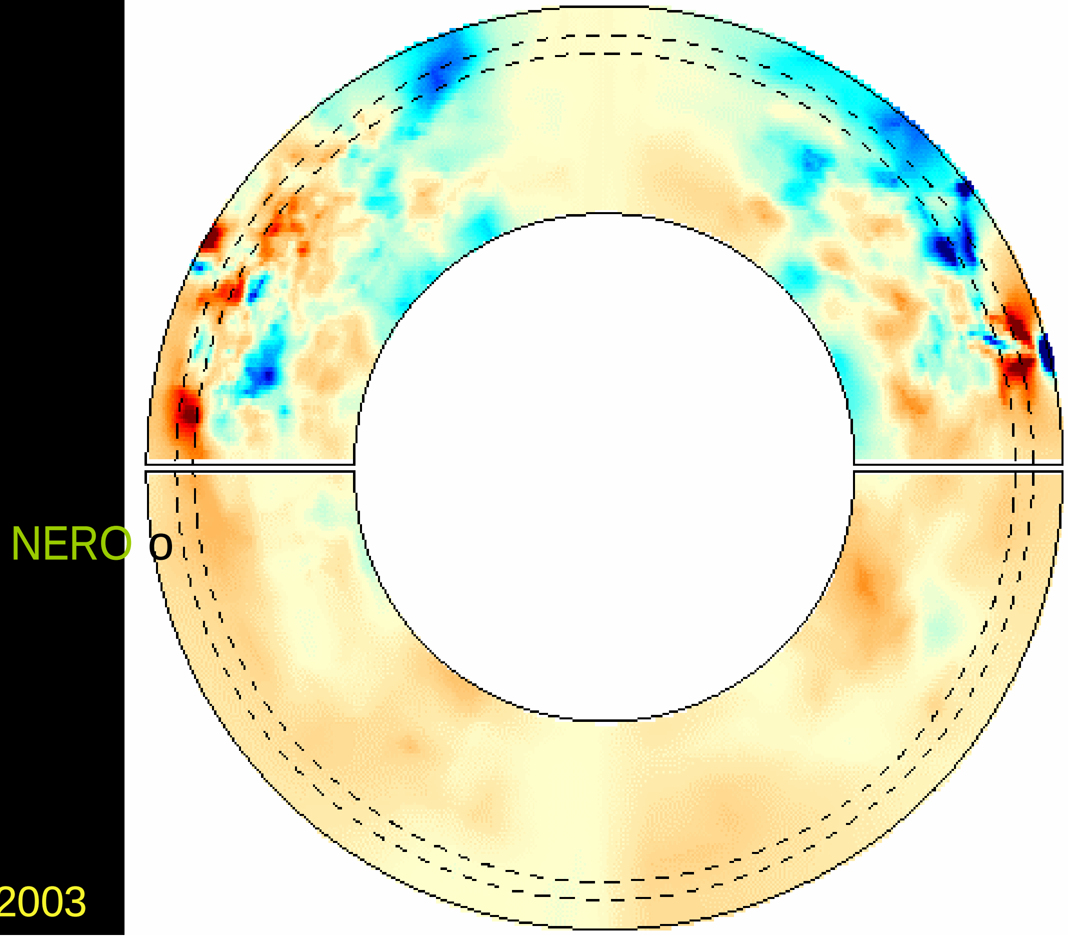
NERO: Scientific Interest Global scale

- To fill a gap in global station coverage

-To improve global tomographic model resolution

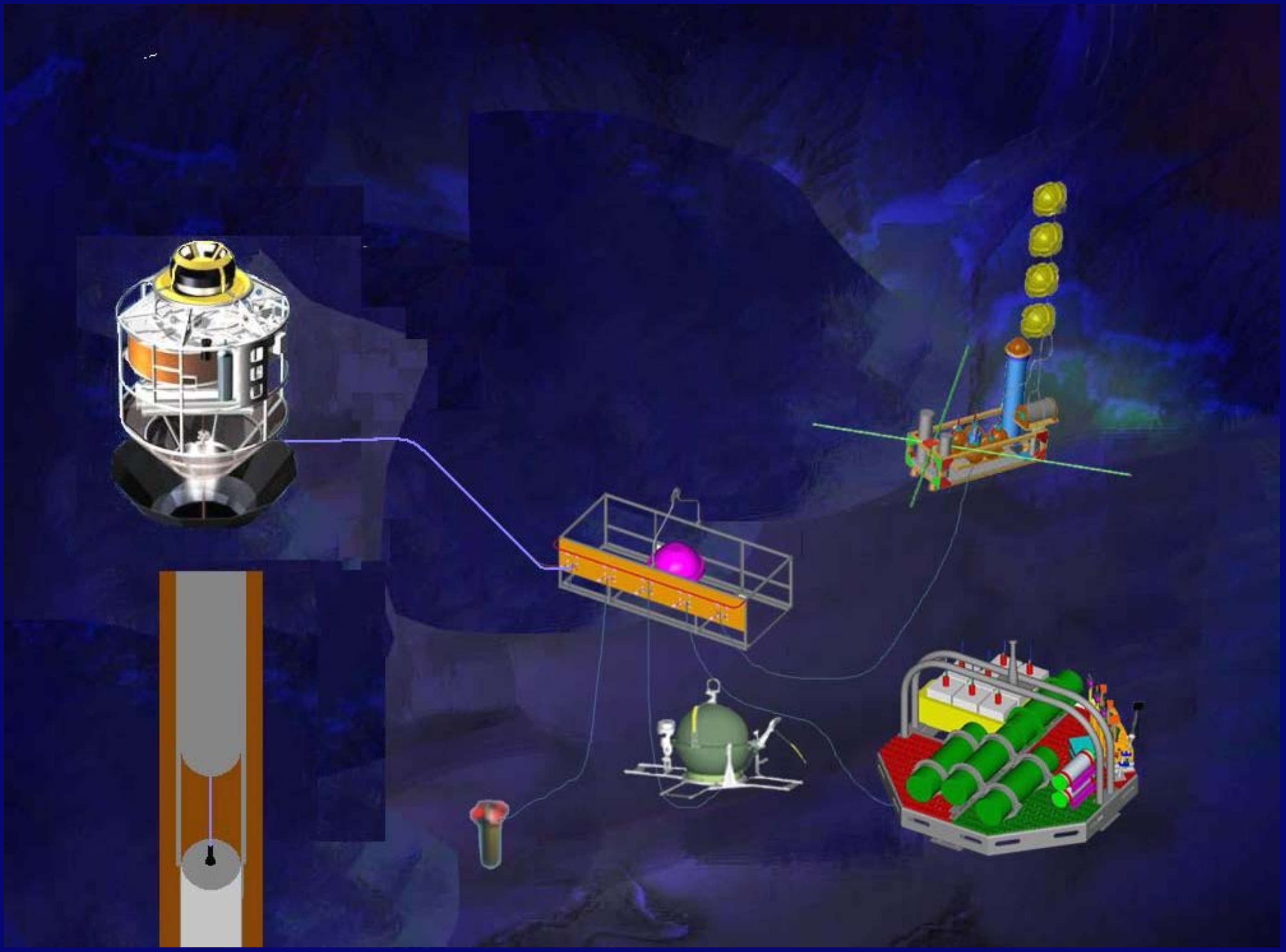
- To improve azimuthal distribution in determination of large earthquakes focal





Karason & van der Hilst, 2003

NERO observatory (in 2008)



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Hypothesis: Elastic Medium

$$\sigma_{ij} = C_{ijkl}\,\varepsilon_{kl}$$

Where ε_{kl} is the strain tensor, σ_{ij} the stress tensor

C_{ijkl} the elastic tensor: 81 elastic moduli

Symmetries of ε_{kl}, σ_{ij} and of the strain energy

W= 1/2 $\sigma_{ij}\,\varepsilon_{ij}$ => 21 independent elements

Isotropic case:

$C_{ijkl} = \lambda\,\delta_{ij}\,\delta_{kl} + \mu\,(\delta_{ik}\,\delta_{jl} + \delta_{il}\,\delta_{jk})$

λ , μ are Lamé parameters

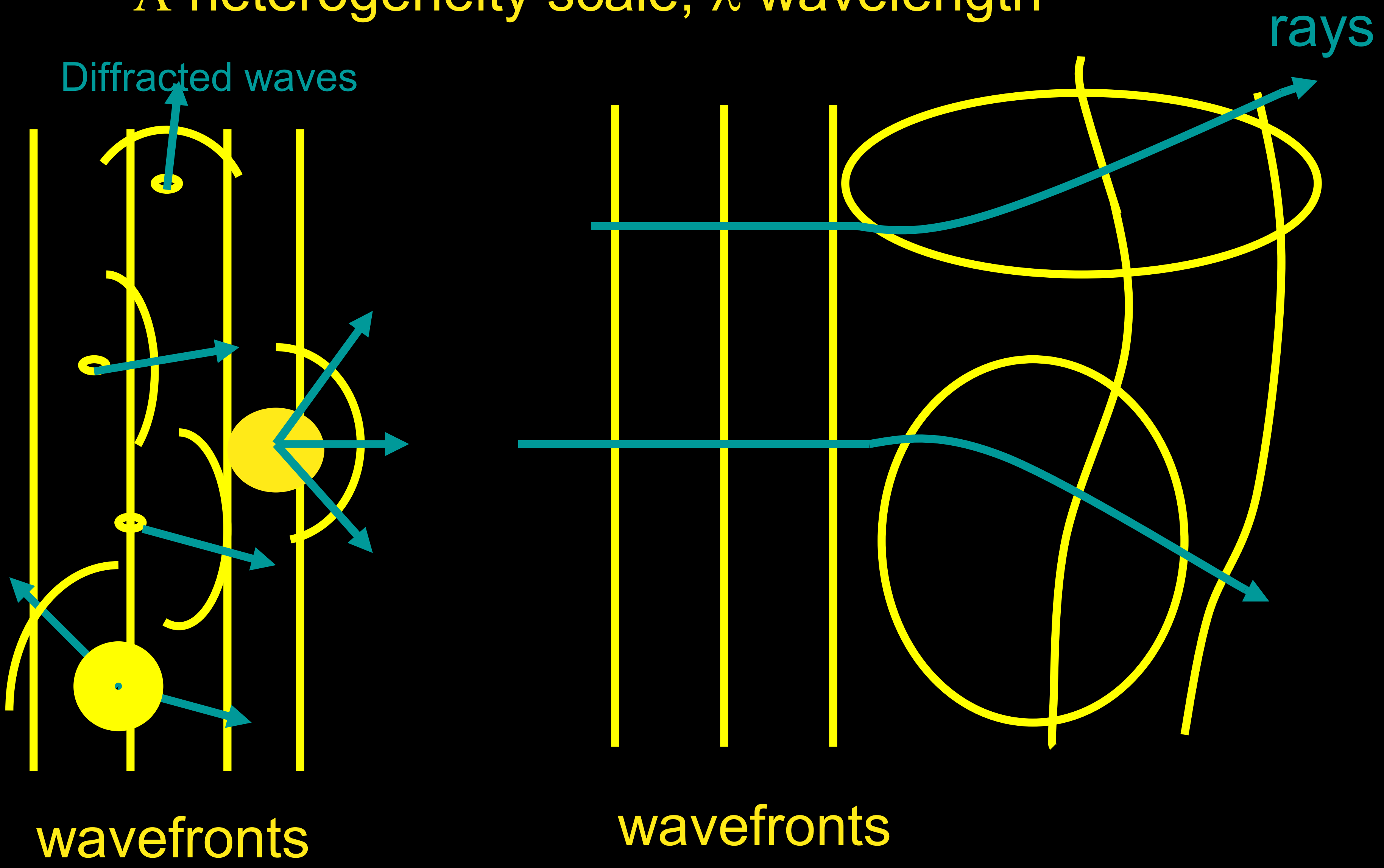
Elastodynamic equation

$$\partial_j(C_{ijkl}\,\partial_k u_l) - \rho\,\partial_{tt} u_i = 0$$

In the isotropic case, 2 solutions:
S-wave
P wave

In heterogeneous media, comparison between
Wavelength λ and scale of heterogeneity Λ

Λ heterogeneity scale, λ wavelength



$\lambda \sim$ [illegible] Λ or $\lambda \gg \Lambda$

$\lambda \ll \Lambda$

Duality wave - particle:

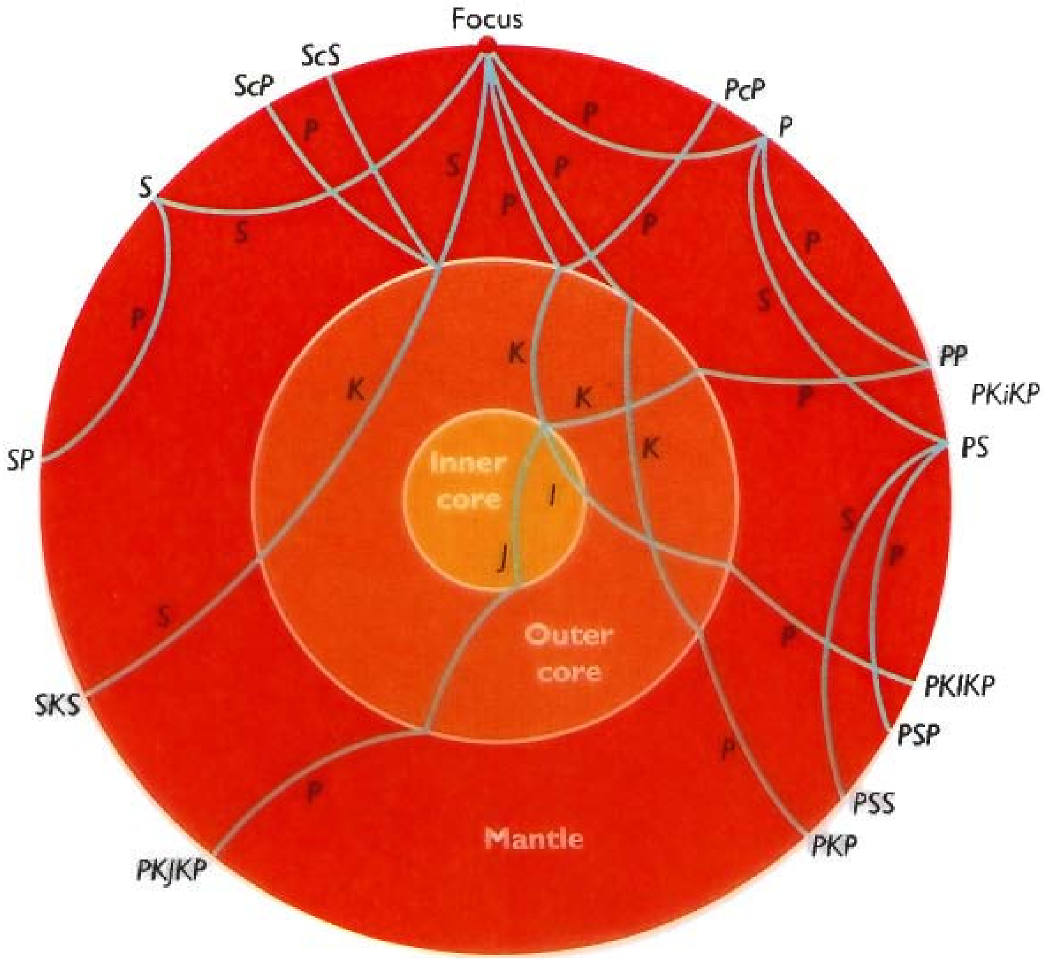
- λ seismic wavelength
- Λ scale heterogeneity

- Particle: Ray theory (XXth century)

$\lambda << \Lambda$

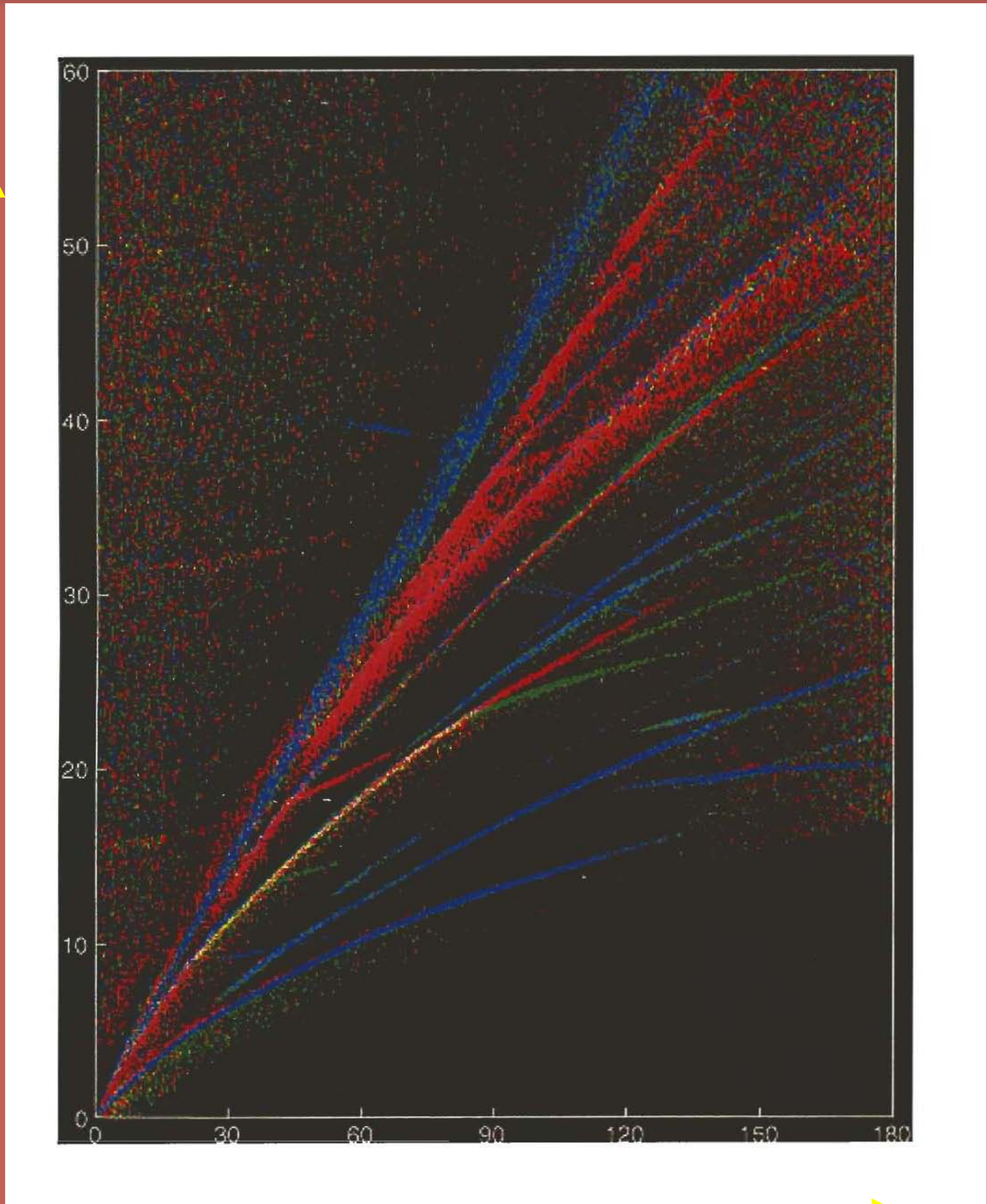
- Wave: Normal mode theory (>1970)

RAY PATHS INSIDE THE EARTH



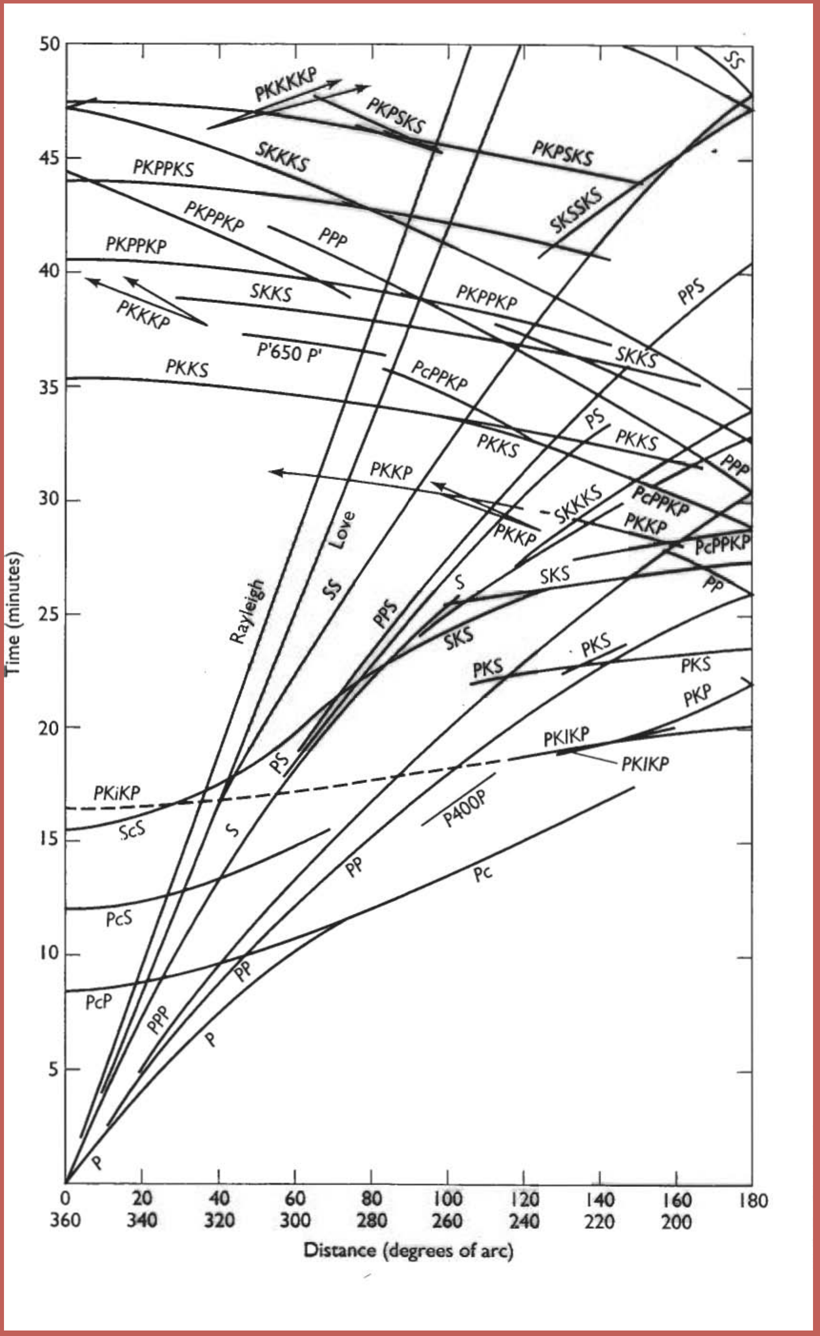
Bolt, 1993

Time



Epicentral distance

Shearer, 1997



Jeffreys, 1939

Duality wave - particle:

- λ seismic wavelength
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- Particle: Ray theory (XXth century)

$\lambda << \Lambda$

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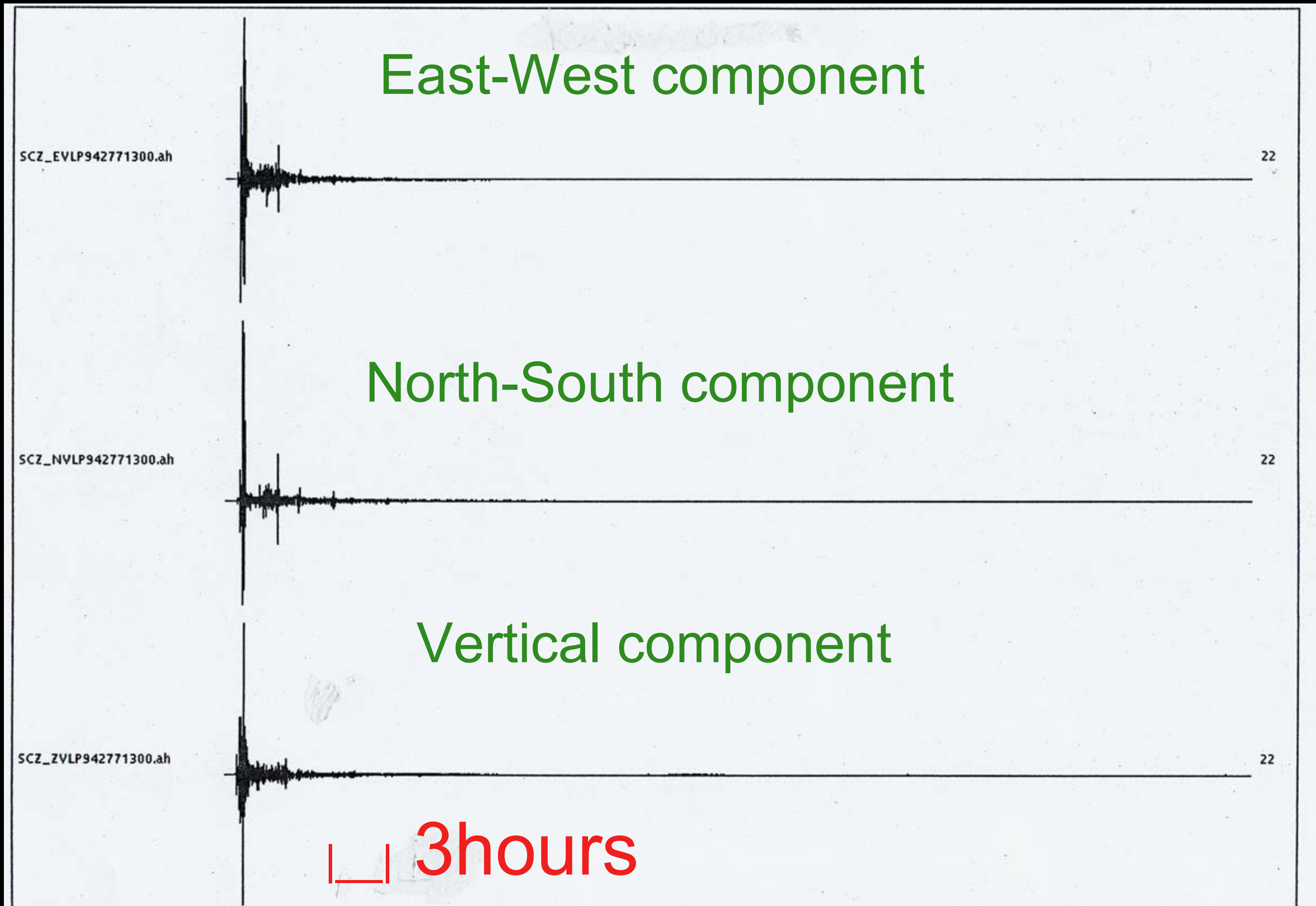
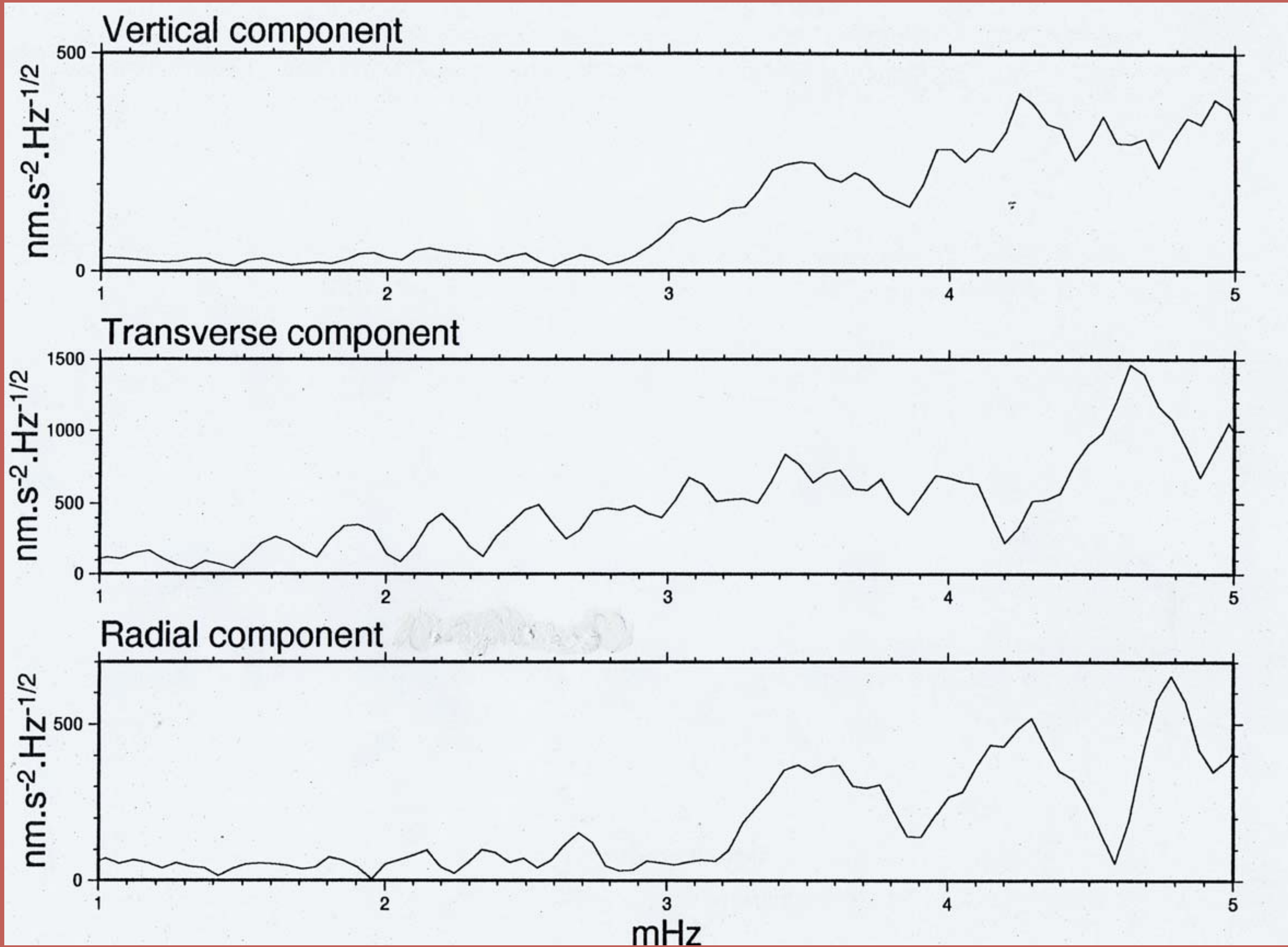
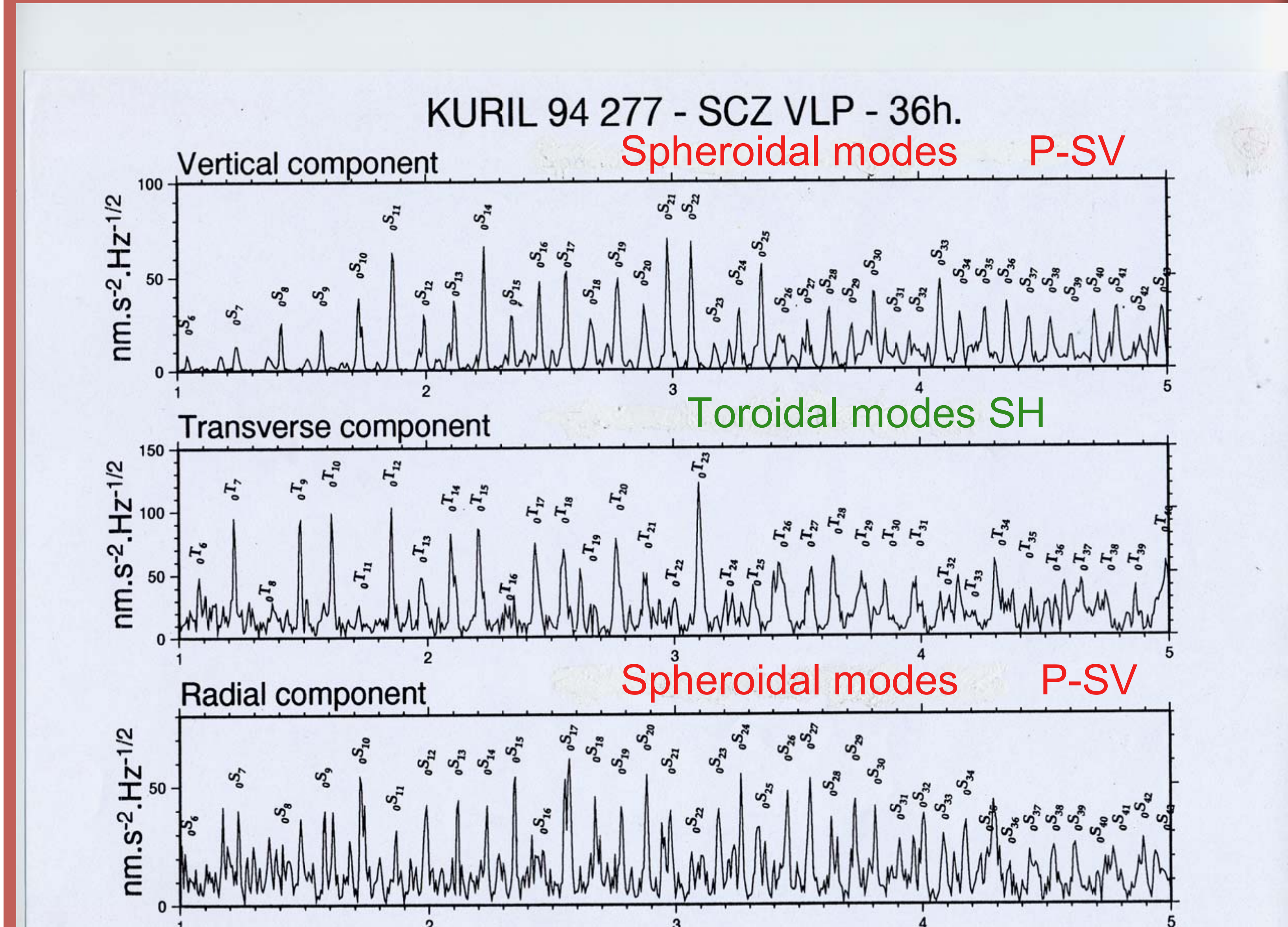
Kurils islands 1994-277 Ms=8.3



Kurils islands 1994-277 SCZ-VLP
Spectra 3 hours



Frequency (mHz)



Frequency (mHz)

Elasto-dynamic equation

$$\rho\partial_{tt}\mathbf{u}_{0i} = \partial_j\sigma_{\mathbf{ij}} + \rho\ \mathbf{g_i} + \mathbf{F_i}\ (\mathbf{+\ Fs_i\ +\ldots})$$

Which can be rewritten:

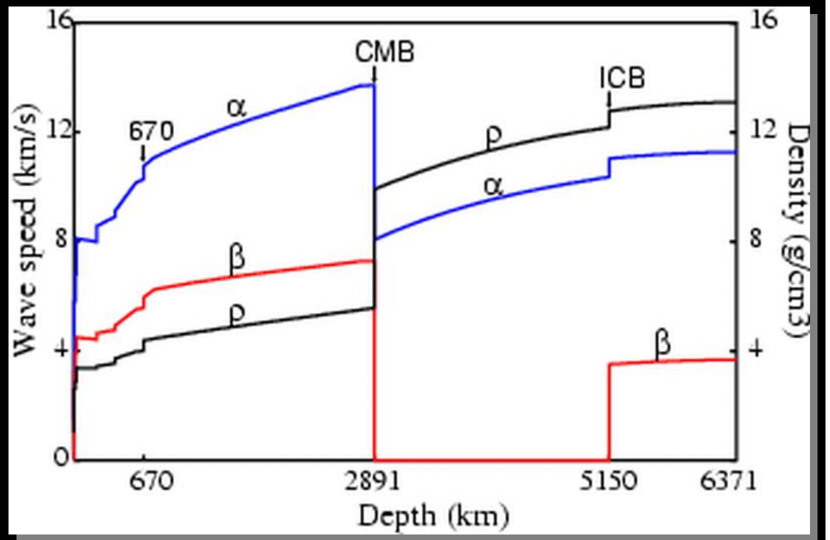
$$\rho\partial_{tt}\mathbf{u}_0 = \mathbf{H_0 u_0}\ \ (\mathbf{+\ Fs})$$

H_0 is an integro-differential operator

1D-Reference Earth Model:

$M_0(r)$, $\rho(r)$, $V_P(r)$, $V_s(r)$

(PREM, Dziewonski and Anderson, 1981
or IASP91, Kennett and Engdahl, 1991)



$\rho\partial_{tt}\mathbf{u}_0 = \mathbf{H_0 u_0}$ **(+ Fs)**

Eigenfrequencies: ${}_n\omega_l$

Eigenfunctions: ${}_nu_l^m$ (r,t)= |n,l,m>

3 quantum numbers (k={n,l,m}) => $\mathbf{u}_k$(r,t)

$\int \rho\, \mathbf{u}_k{}^{*} \,.\, \mathbf{u}_k\, d^3x = \delta_{ij}$

$\mathbf{H_0}\, \mathbf{u}_k = \rho\ {}_n\omega_l{}^2\, \mathbf{u}_k$

Displacement:

$\mathbf{u}(\mathbf{r},t) = \Sigma_{n,l,m}\ {}_n\mathbf{a}_l{}^m$ |n,l,m> $\exp(-i_n\omega_l t)$

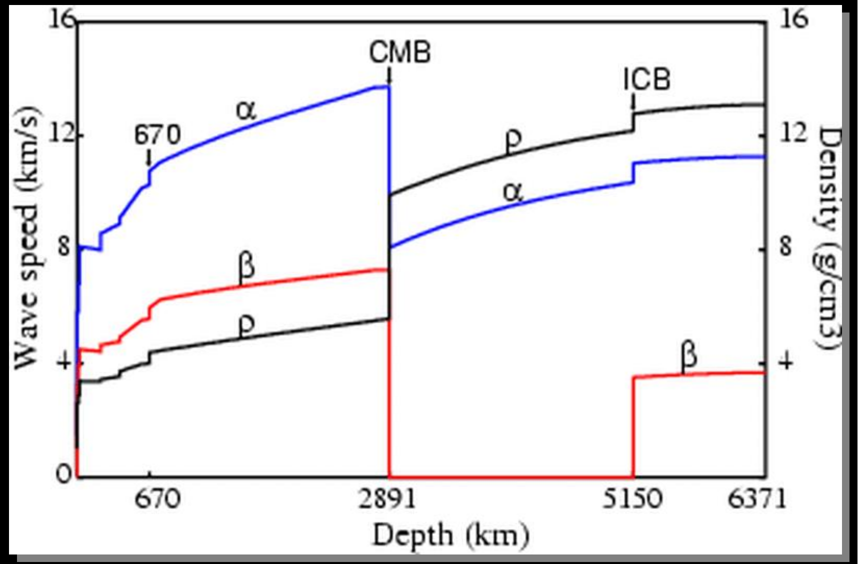
$\mathbf{u}_k(r,t)= \{U(r)\, \mathbf{e}_r + V(r)\mathbf{e}_\theta\, \partial_\theta + V(r)/\sin\theta\ \mathbf{e}_\phi\, \partial_\phi\}\, Y_l^m(\theta,\phi)$

$+ \{W(r)\, \mathbf{e}_\theta\, \partial_\phi - W(r)\, \mathbf{e}_\phi\, \partial_\theta\}\, Y_l^m(\theta,\phi)$

1D-Reference Earth Model:

$M_0(r)$, $\rho(r)$, $V_P(r)$, $V_s(r)$

(PREM, Dziewonski and Anderson, 1981)



$\rho\partial_{tt}u_0 + \mathbf{H_0 u_0 = 0}$

Eigenfrequencies: ${}_n\omega_l$

Eigenfunctions: ${}_n u_l^m$ (r,t)= |n,l,m>

2 kinds of mode: Toroïdal ${}_nT_l$, Spheroïdal ${}_nS_l$

Degeneracy of eigenfrequencies ${}_n\omega_l$: 2 l +1

Spherical eigenfrequencies

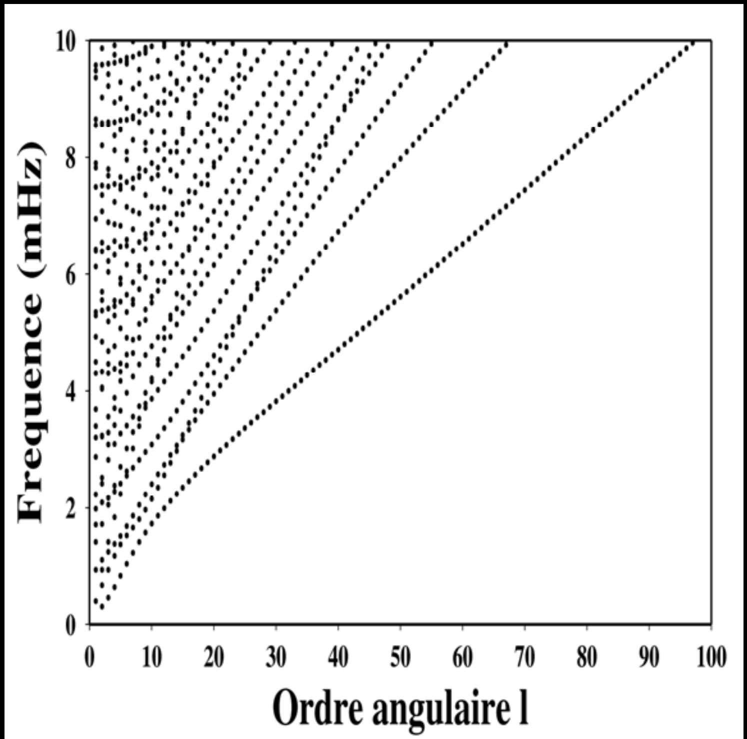
Spheroïdal Modes $_nS_l$

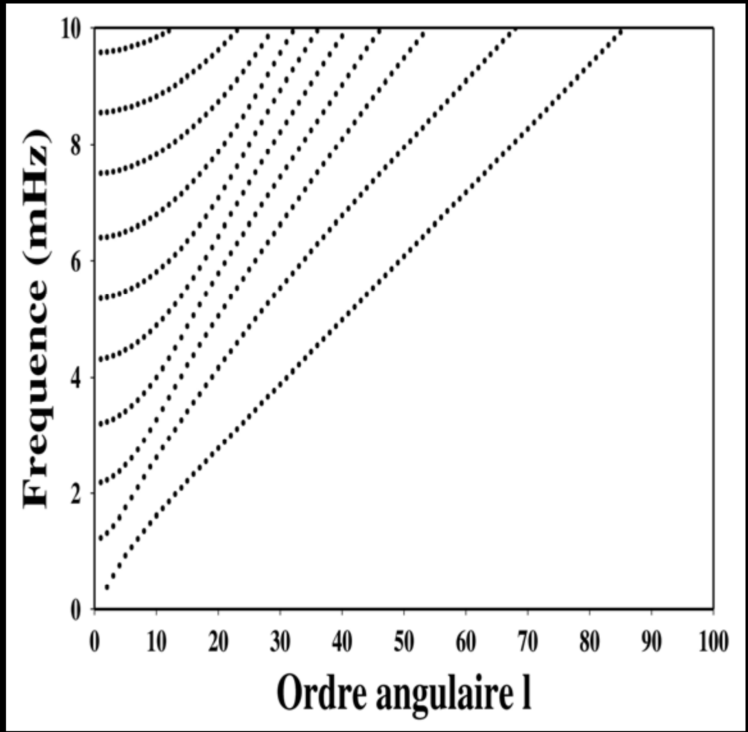
(P-SV / Rayleigh)

Toroïdal modes $_nT_l$

(SH / Love)

Dispersion Branches





multiplet : (n,l) = 2l+1 singlets

singlet : (n,l,m)

n : radial order

l : angular order

m : azimuthal order

Spheroidal Modes

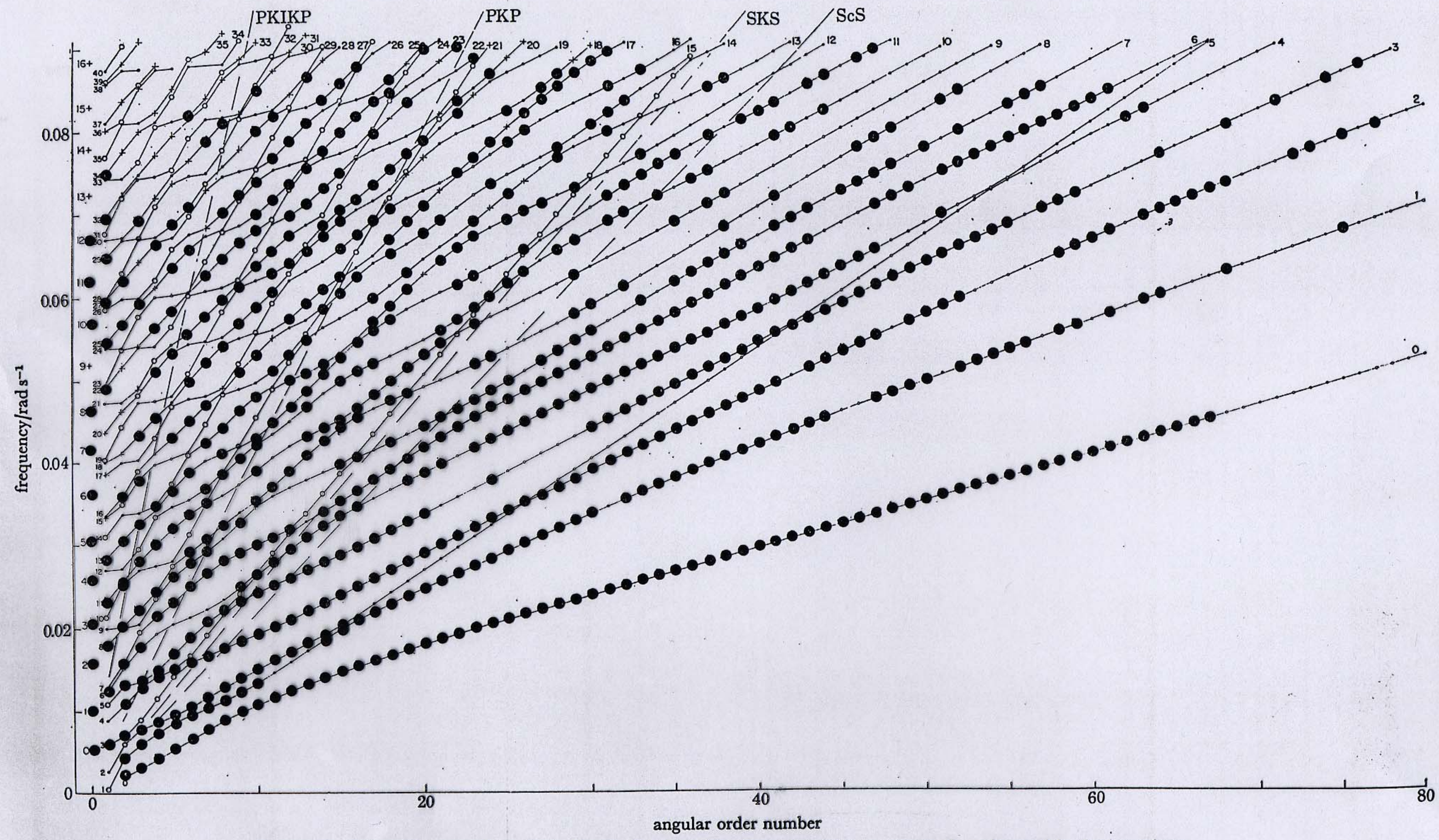
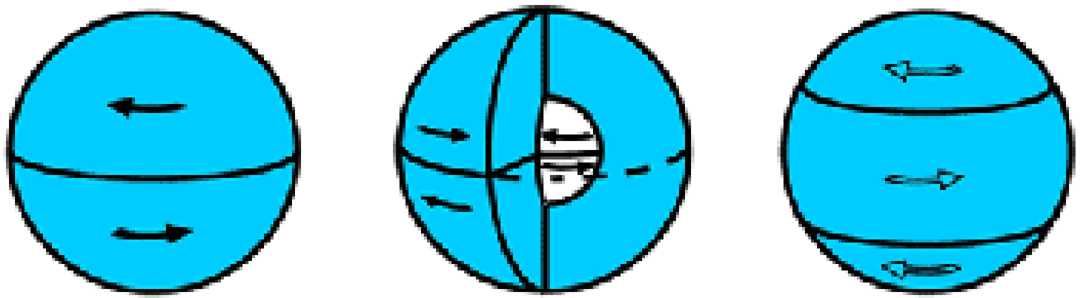
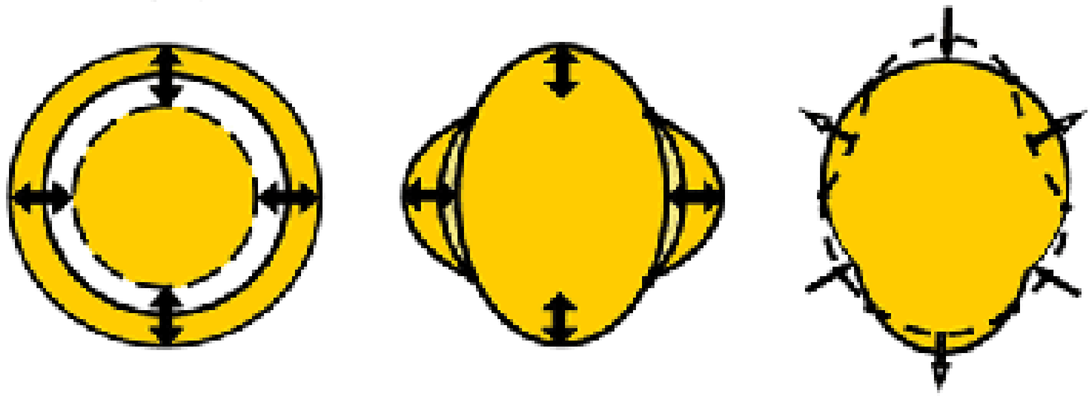


Figure 17. Spheroidal normal modes in the (ω, l) plane. The large dots indicate observed modes used in the inversions. For further details we refer the reader to §3 of Alaska II. •, $CE < 0.5$; +, $CE \geqslant 0.5$; ∘ core modes.

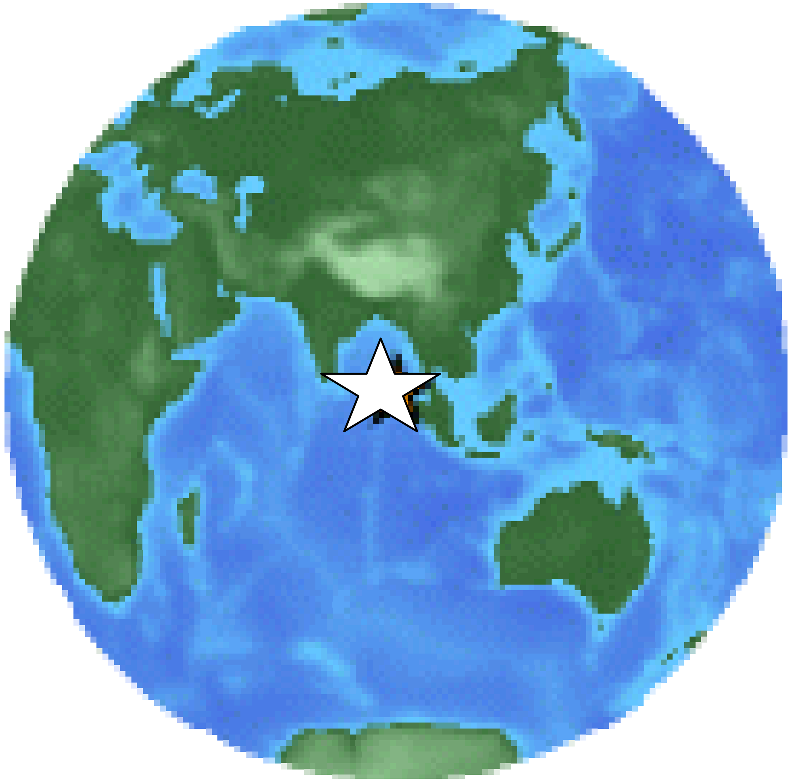


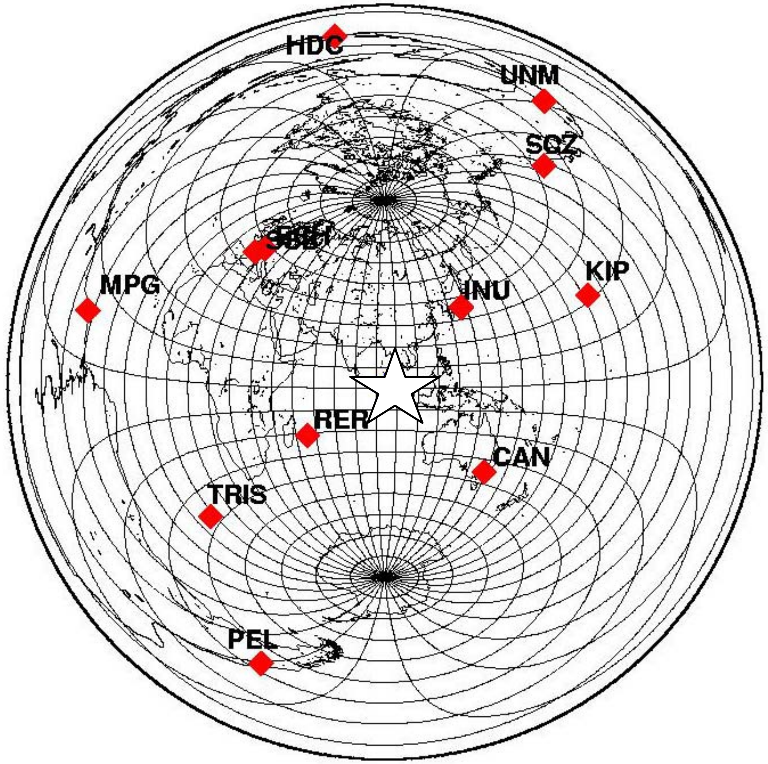
Toroidal modes ${}_0T_2$ (44.2 min), ${}_1T_2$ (12.6 min) and ${}_0T_3$ (28.4 min)



Spheroidal modes ${}_0S_0$ (20.5 min), ${}_0S_2$ (53.9 min) and ${}_0S_3$ ((35.6 min)

GEOSCOPE and Source Investigations



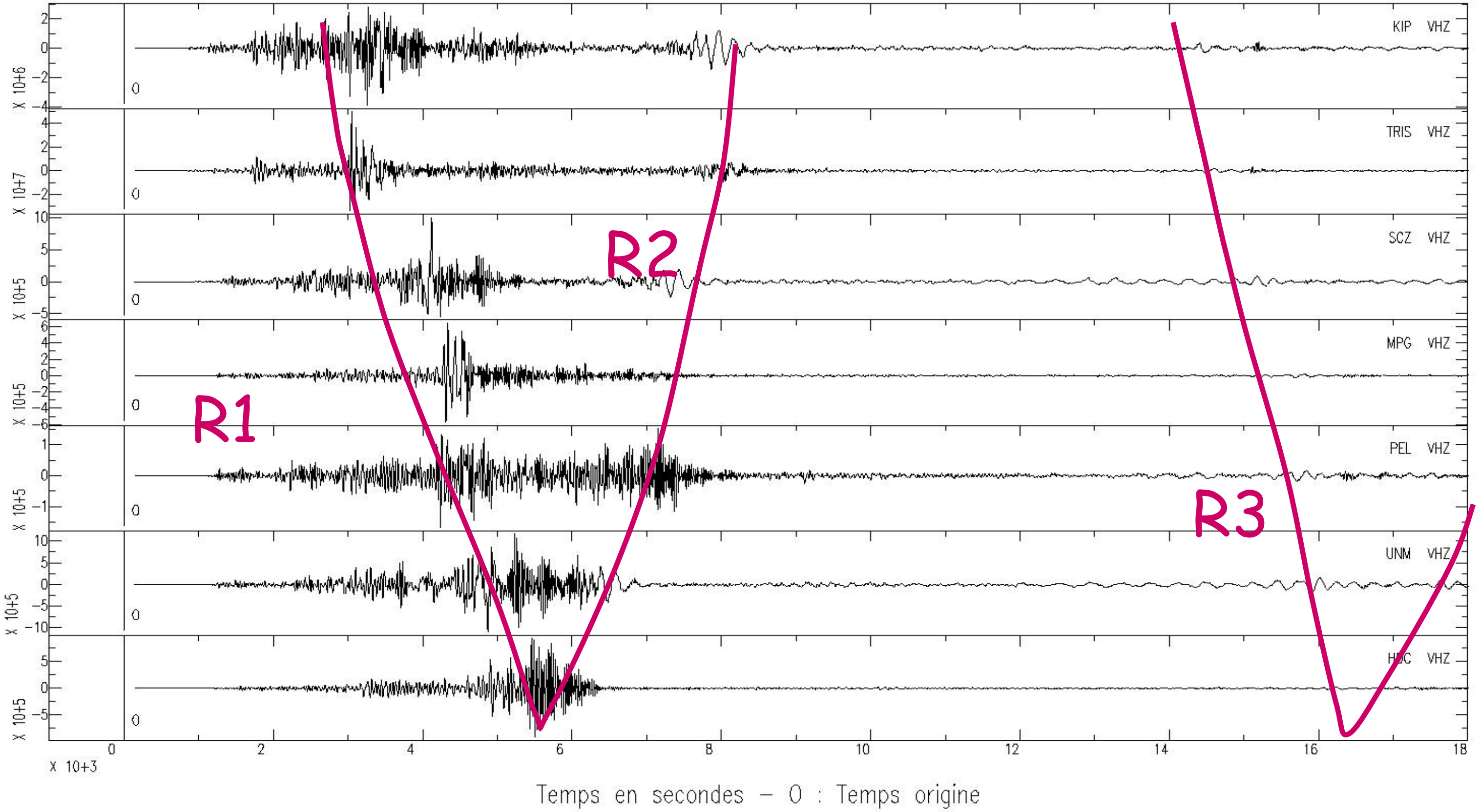


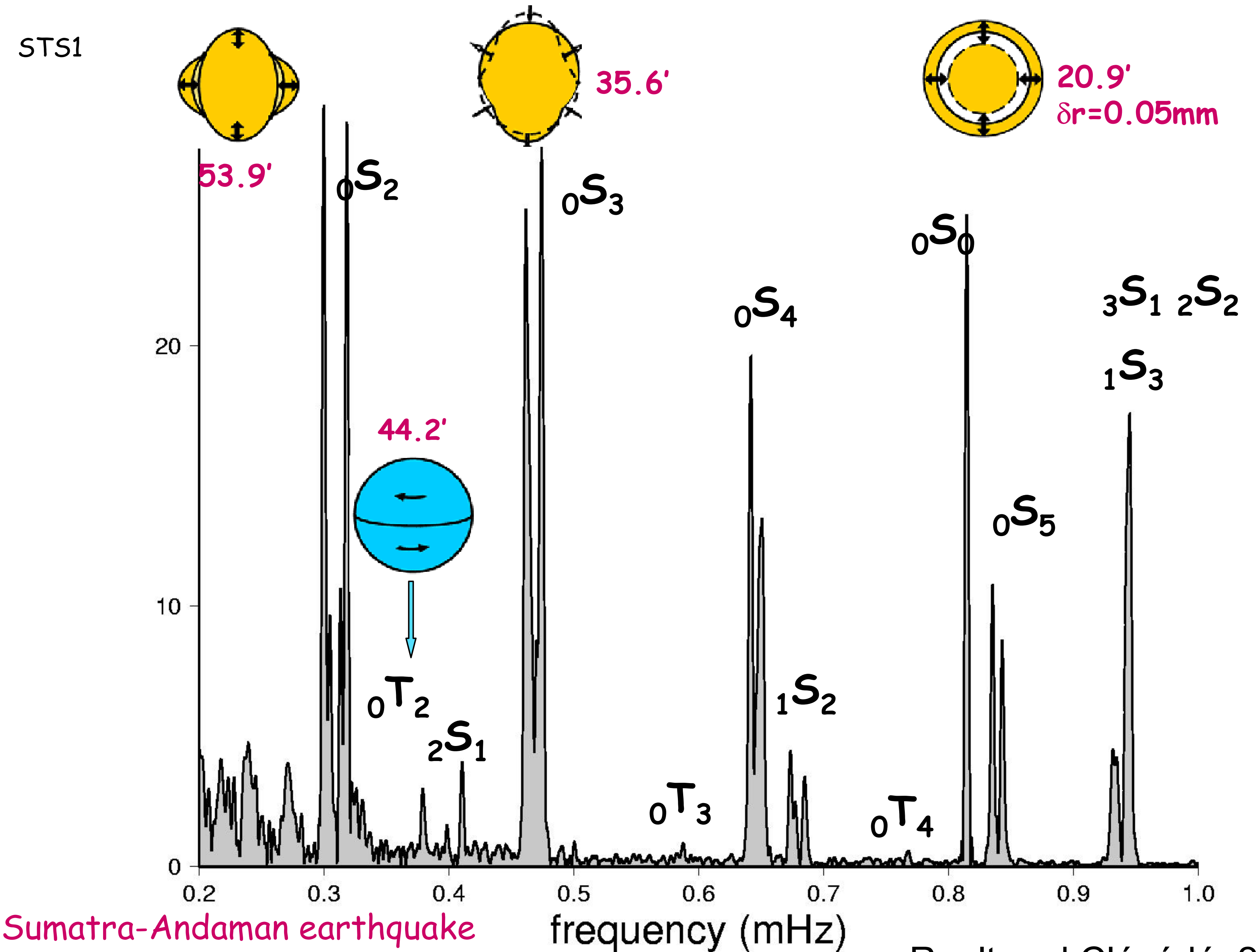
Study of Sumatra earthquake (26 december 2004)

(Roult and Clévédé, 2005 ; Park et al., Science, 2005)

Sumatra Earthquake

26 december 2004, Mw=9.3, Ho=00h58'50"

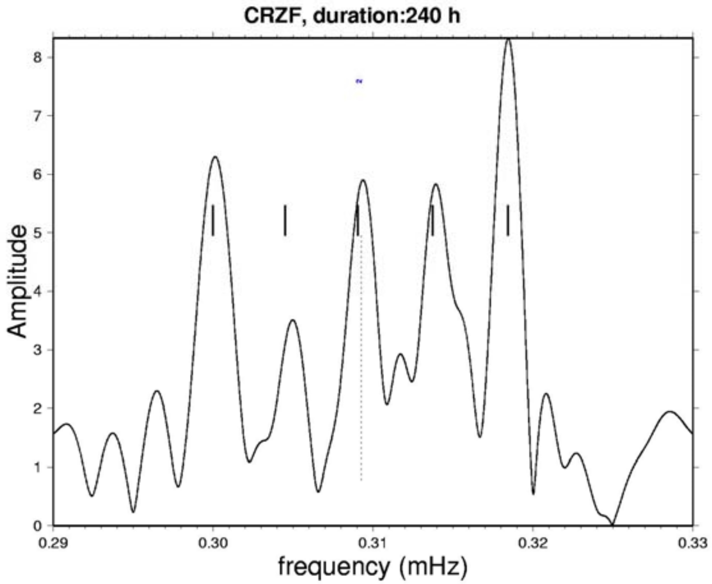


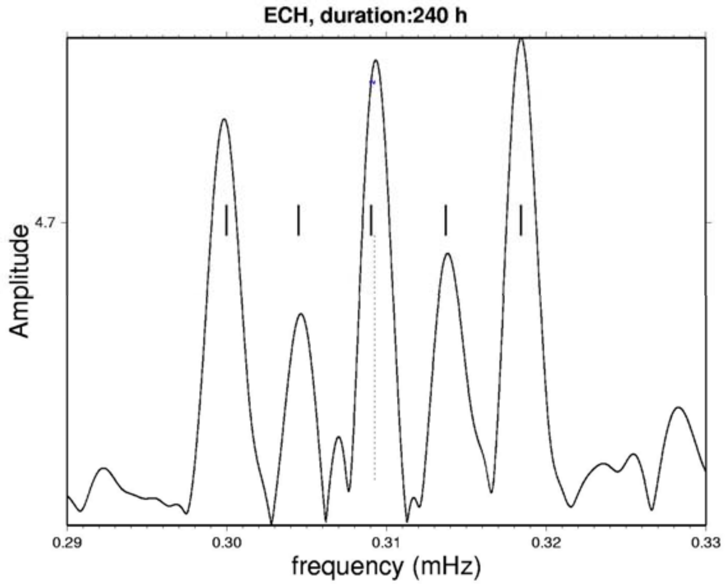


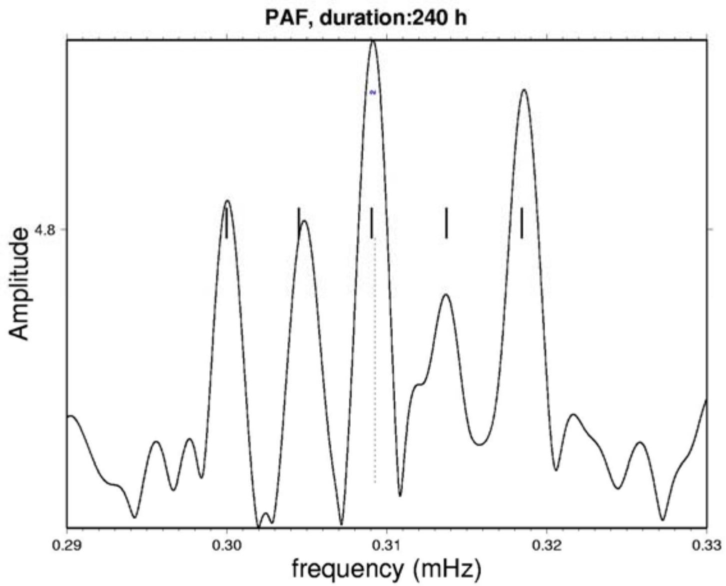
Sumatra-Andaman earthquake
26 December 2004

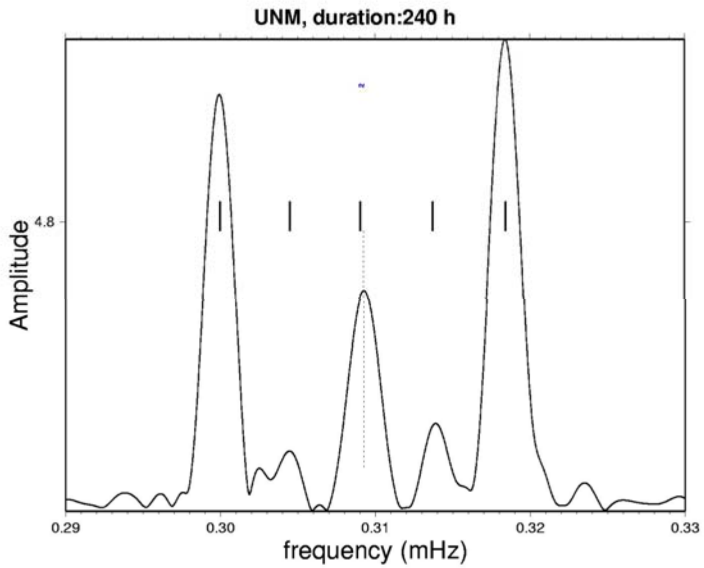
Roult and Clévédé, 2005

mode $_0S_2$ => splitting 5 singlets

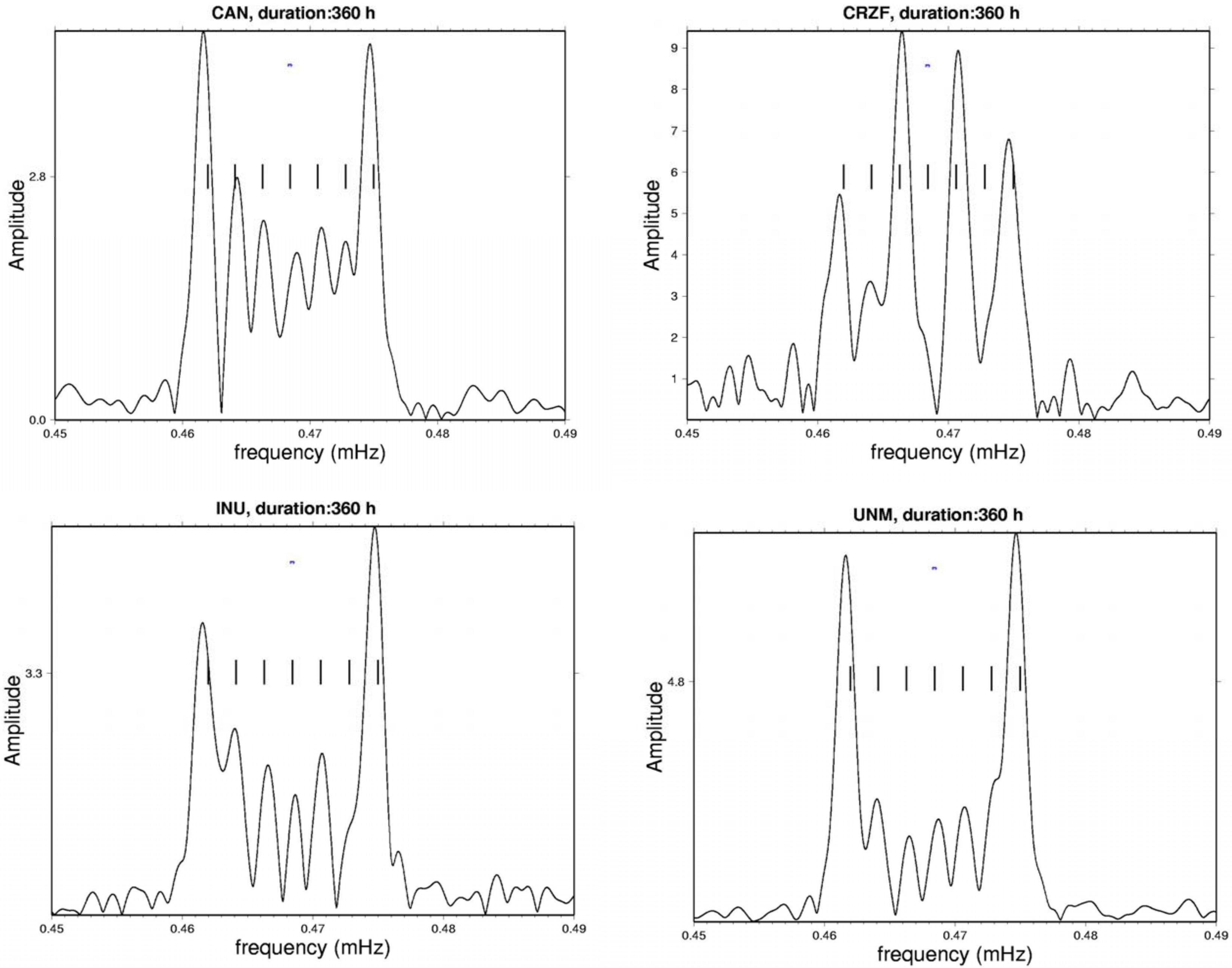








mode $_0S_3$

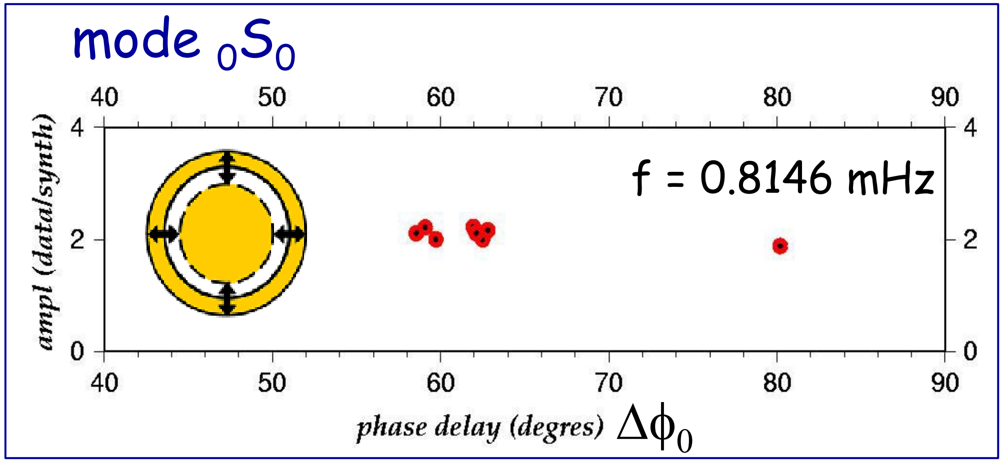


CAN, duration:360 h
CRZF, duration:360 h
INU, duration:360 h
UNM, duration:360 h
Amplitude
frequency (mHz)
2.8
0.0
3.3
4.8
1
2
3
4
5
6
7
8
9
0.45
0.46
0.47
0.48
0.49

Estimation of the Sumatra earthquake size

(Park, ….Roult & Clévédé)

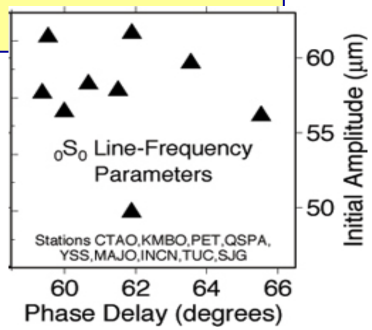
$$\Delta\phi_0 = \omega * (t_0^{lf} - t_0^{hf})$$



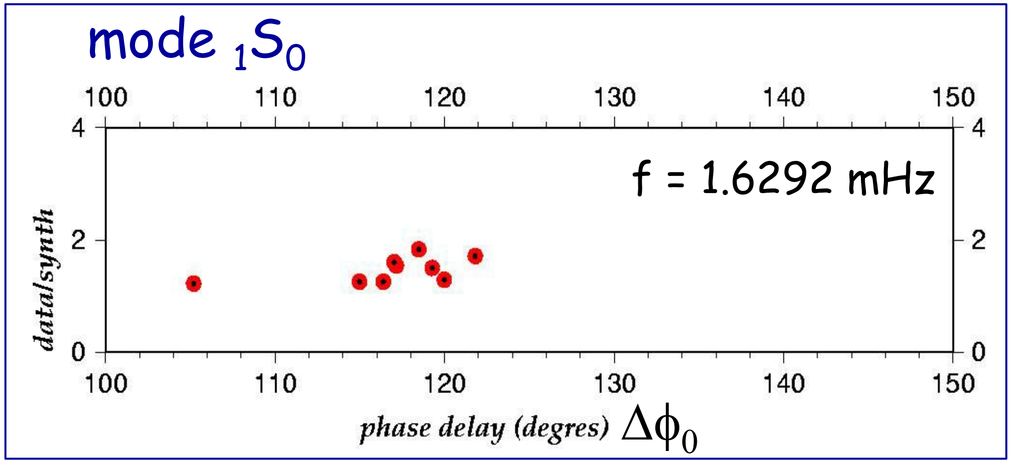
$\Delta\phi_0$ = 60° pour $_0S_0$,

$\Delta\phi_0$ ~119° pour $_1S_0$

Δt ~ 200s, Δ ~ 1200km



J.Park

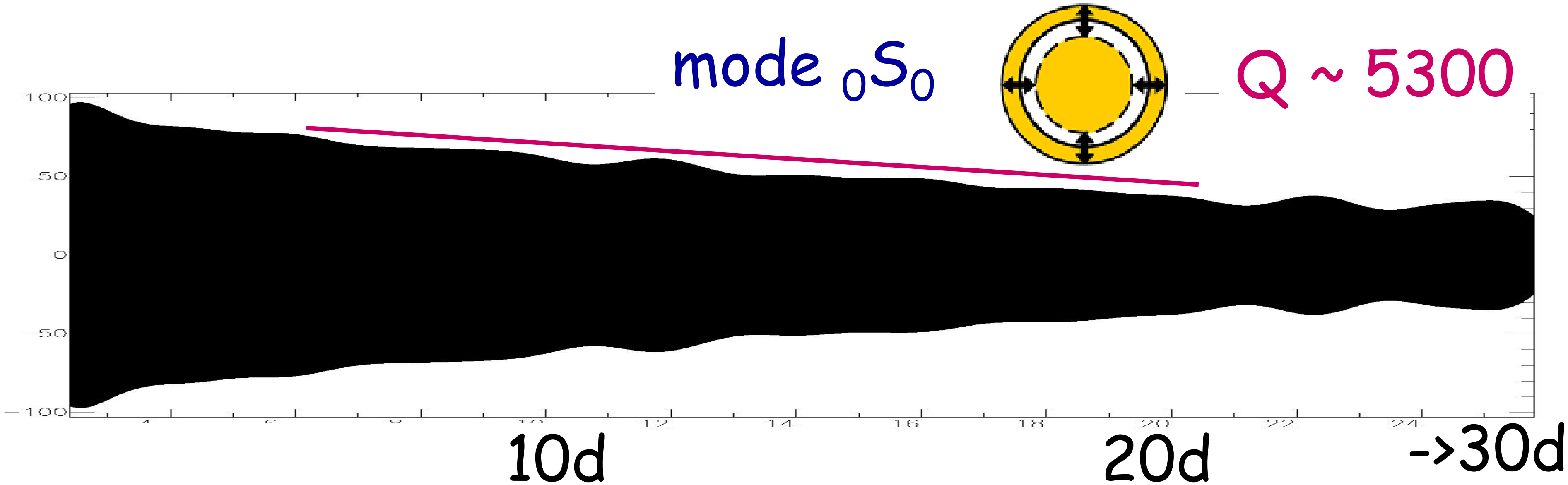


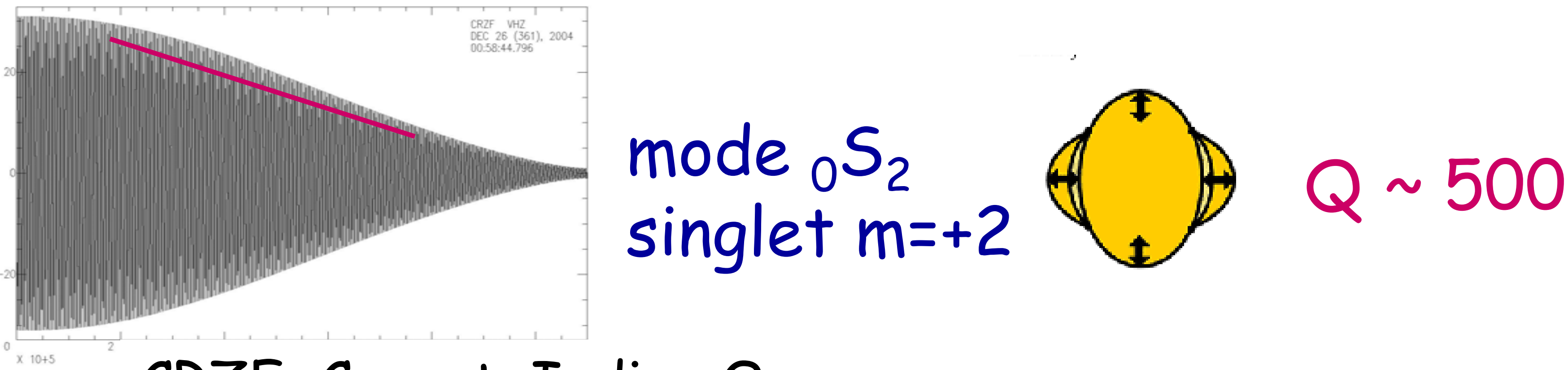
rapport data/synth

~2.01 pour $_0S_0$

~1.60 pour $_1S_0$

Attenuation of some modes





CRZF, Crozet, Indian Ocean

Seismic Source

$$\rho\partial_{tt}\mathbf{u} + \mathbf{H_0 u} = \mathbf{F_s}$$

Displacement in point **r** at time t due to a force system $\mathbf{F_s}$ at point source $\mathbf{r_s}$

eigenfrequencies: ${}_n\omega_l$

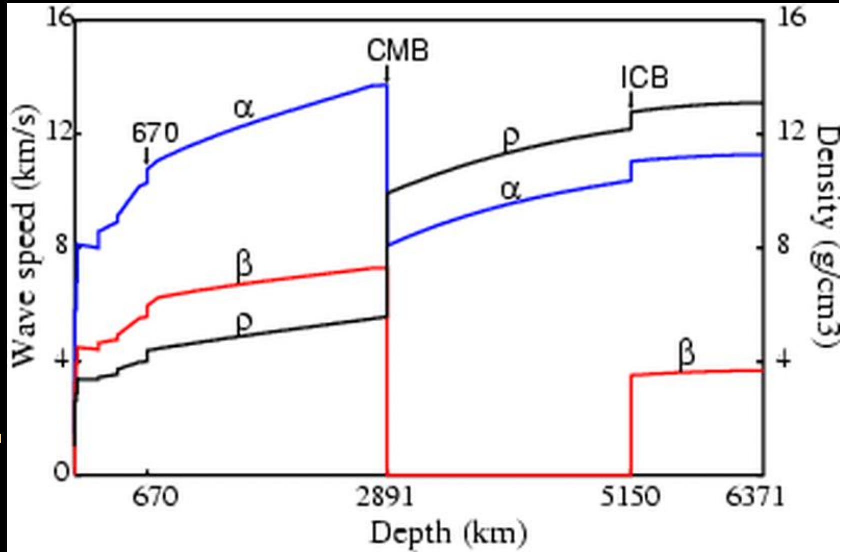
eigenfunctions: ${}_n u_l^m$ (r,t)= |n,l,m>

$$\mathbf{u}(\mathbf{r},t) = \Sigma_{n,l,m}\ {}_n\mathbf{a}_l^m\ |n,l,m> \exp(-i\,{}_n\omega_l t)$$

Eigenfunction basis is a complete basis => any wave can be modelled by normal mode summation including surface waves and body waves.

1D- Reference Earth Model

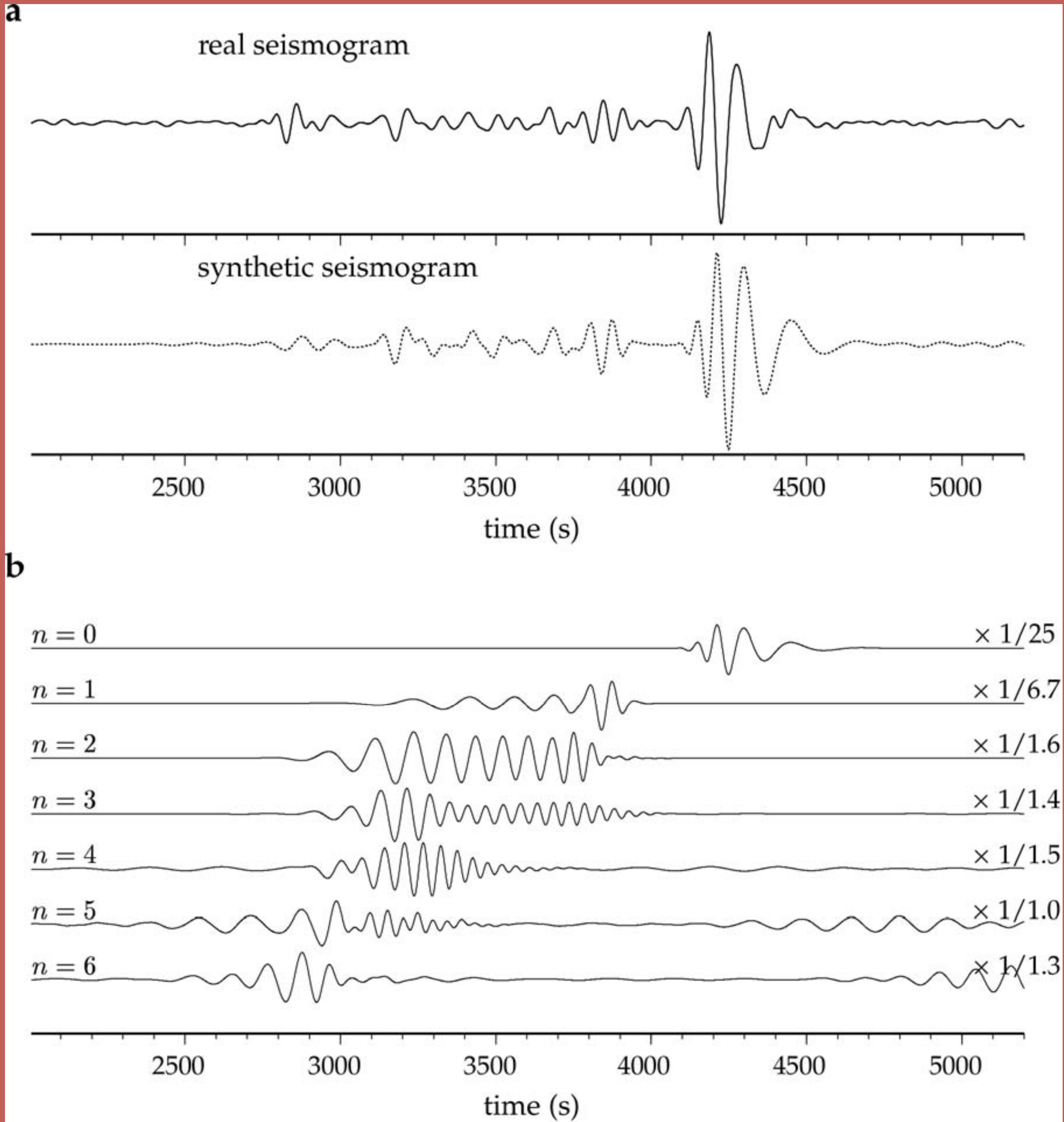
- Synthetic Seismograms by normal mode summation $\mathbf{u_k}$(k={n,l,m}).



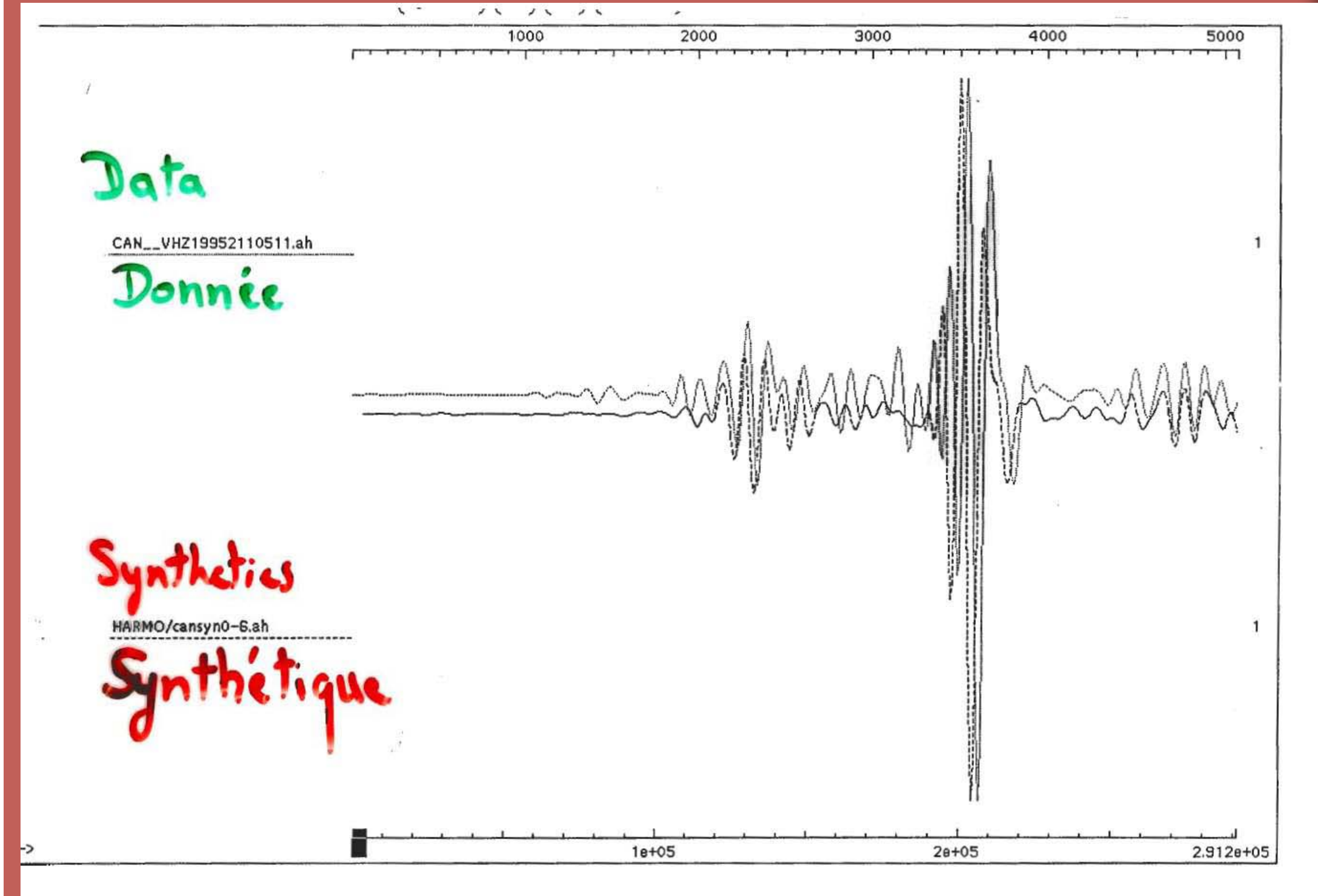
$$\mathbf{u}(\mathbf{r},t) = \Sigma_k\, u_k(\,r\,) \cos \omega_k t\, / \omega_k^2 \exp(-\omega_k t / 2Q_k)\, (\mathbf{u_k}.\mathbf{F})_s$$

Source Term $(\mathbf{u_k}\,.\mathbf{F})_s = (\mathbf{M}:\varepsilon)_S$

M Seismic moment tensor, ε deformation tensor



Beucler et al., 2003


1000
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3000
4000
5000
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Synthetics
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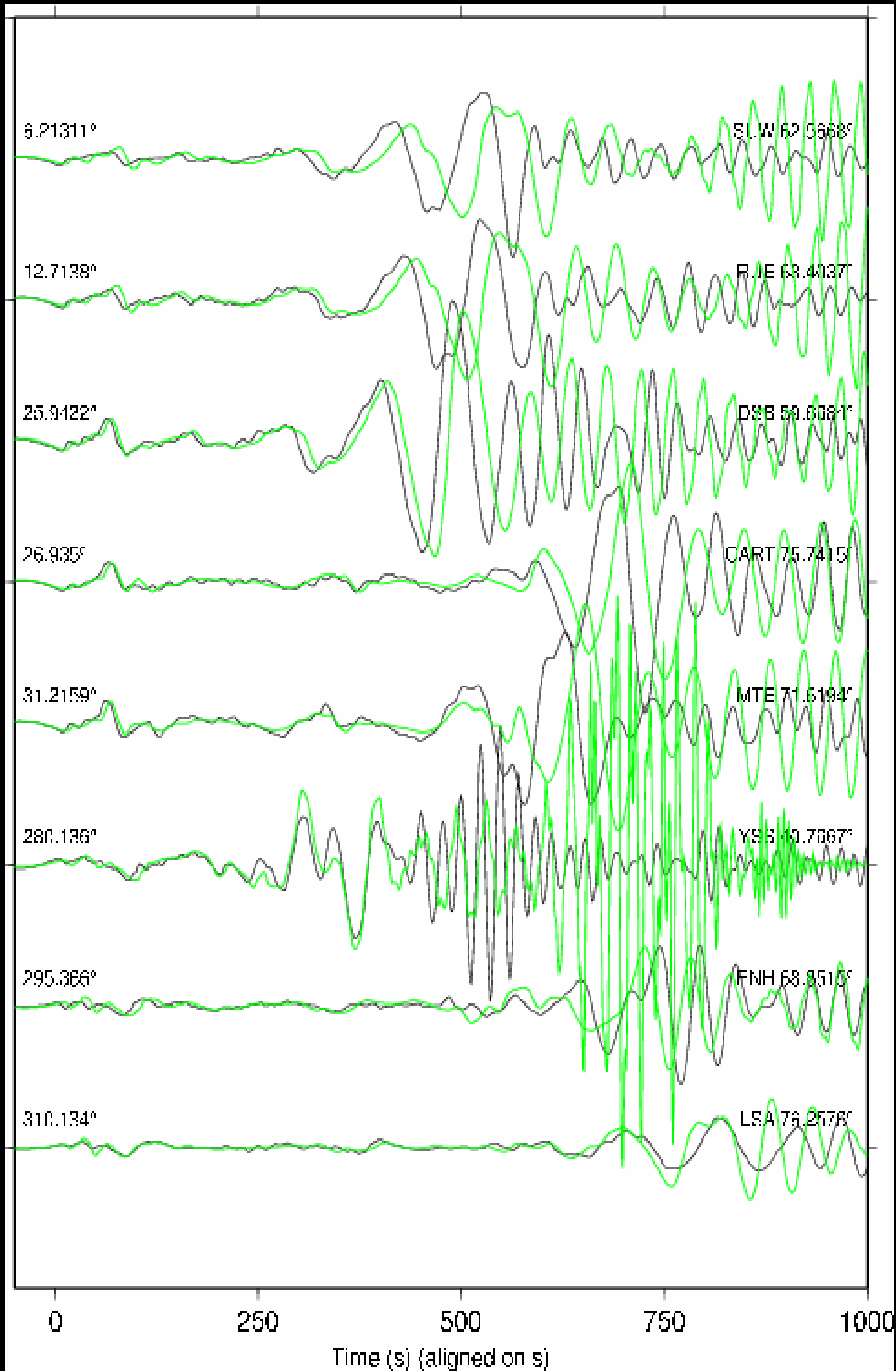
Synthétique
1
1e+05
2e+05
2.912e+05



Synthetic seismograms
By normal mode
summation

Denali-Alaska
earthquake (Nov. 2002)

Komatitsch and Tromp, 2003

Duality wave - particle:

λ seismic wavelength

Λ scale heterogeneity

Particle: **Ray** theory $\lambda \ll \Lambda$

Wave: Normal mode theory (NM) +
Perturbation theories (small amplitude of 3D-heterogeneities)

Numerical modelling of wave equation

Strong or weak forms: $\lambda \approx \Lambda$

-Spectral Element Method (SEM)

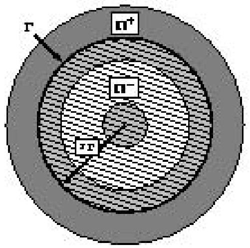
-Coupled SEM-NM method

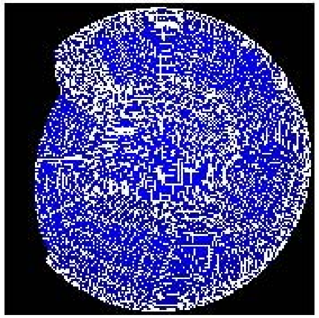
Spectral Element Method: D. Komatitsch (1999)

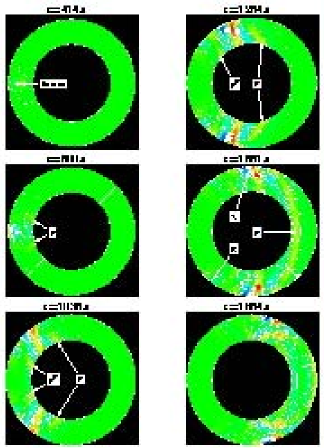
Coupled method of Spectral Elements and Modal Solution

Principle:

- Ω^+: Spectral Element area: 3D model
- Ω^-: Modal Solution area: 1D model







Capdeville et al., 2002

Overview
Large scale Seismology: an observational field

- Data (Seismic source) + Instrument (Seismometer)

-> Observations (seismograms)

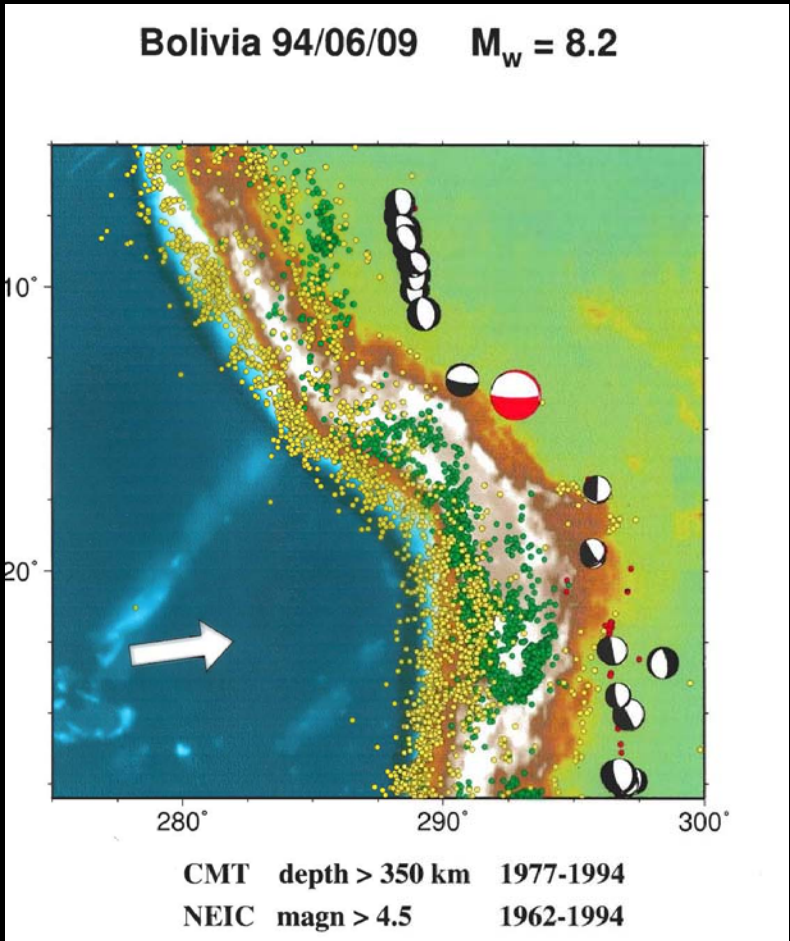
- Historical evolution: Ray theory, Normal mode theory, Numerical techniques (SEM, NM-SEM)
- **Scientific Issues: Earthquakes (Sumatra-Andaman), Anisotropic structure of the Earth**
- NM-SEM and time reversal
- Tomographic Technique
- Seismic Experiment: Plume detection

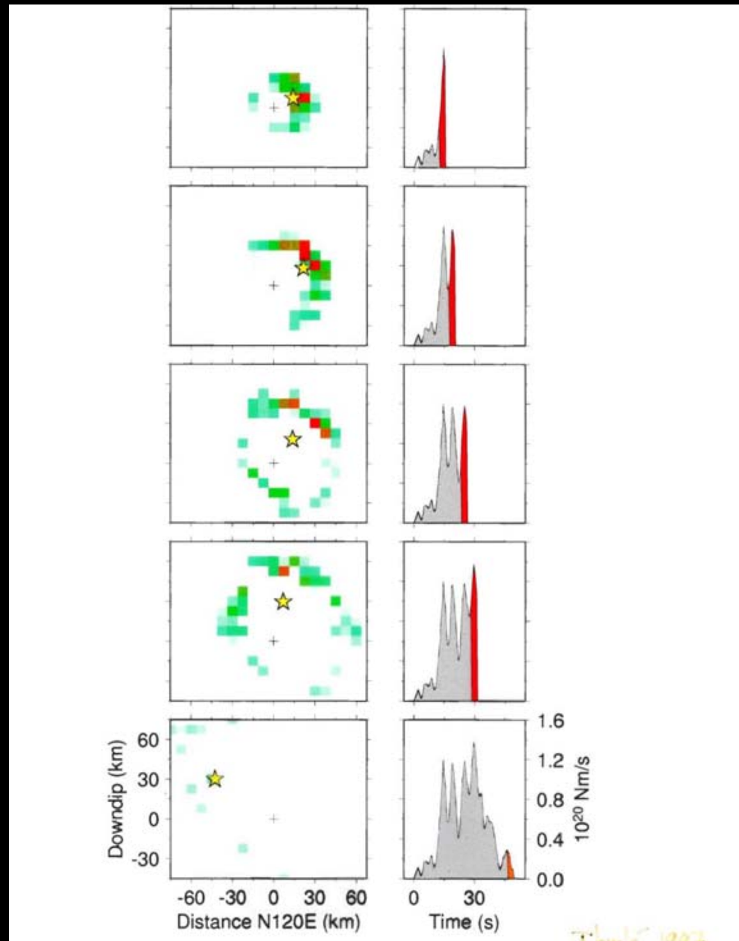
Seismic Source Studies

$$\mathbf{u}(\mathbf{r},t) = \Sigma_k\, u_k\,(\,r\,) \cos \omega_k t\,/\omega_k^2 \exp(-\omega_k t/2Q)\,(\mathbf{u_k.F})_S$$

Source Term $(\mathbf{u_k\,.F})_S = (\mathbf{M}{:}\varepsilon)_S$

M Seismic moment tensor, ε deformation tensor





Overview
Large scale Seismology:

- Data (Seismic source) + Instrument (Seismometer) -> Observations (seismograms)
- Historical evolution: Ray theory, Normal mode theory, Numerical techniques (SEM, NM-SEM)
- Scientific Issues: earthquakes (Sumatra-Andaman earthquake)
- **NM-SEM and time reversal**
- Anisotropic structure of the Earth
- Seismic Experiment: Plume detection

Time reversal

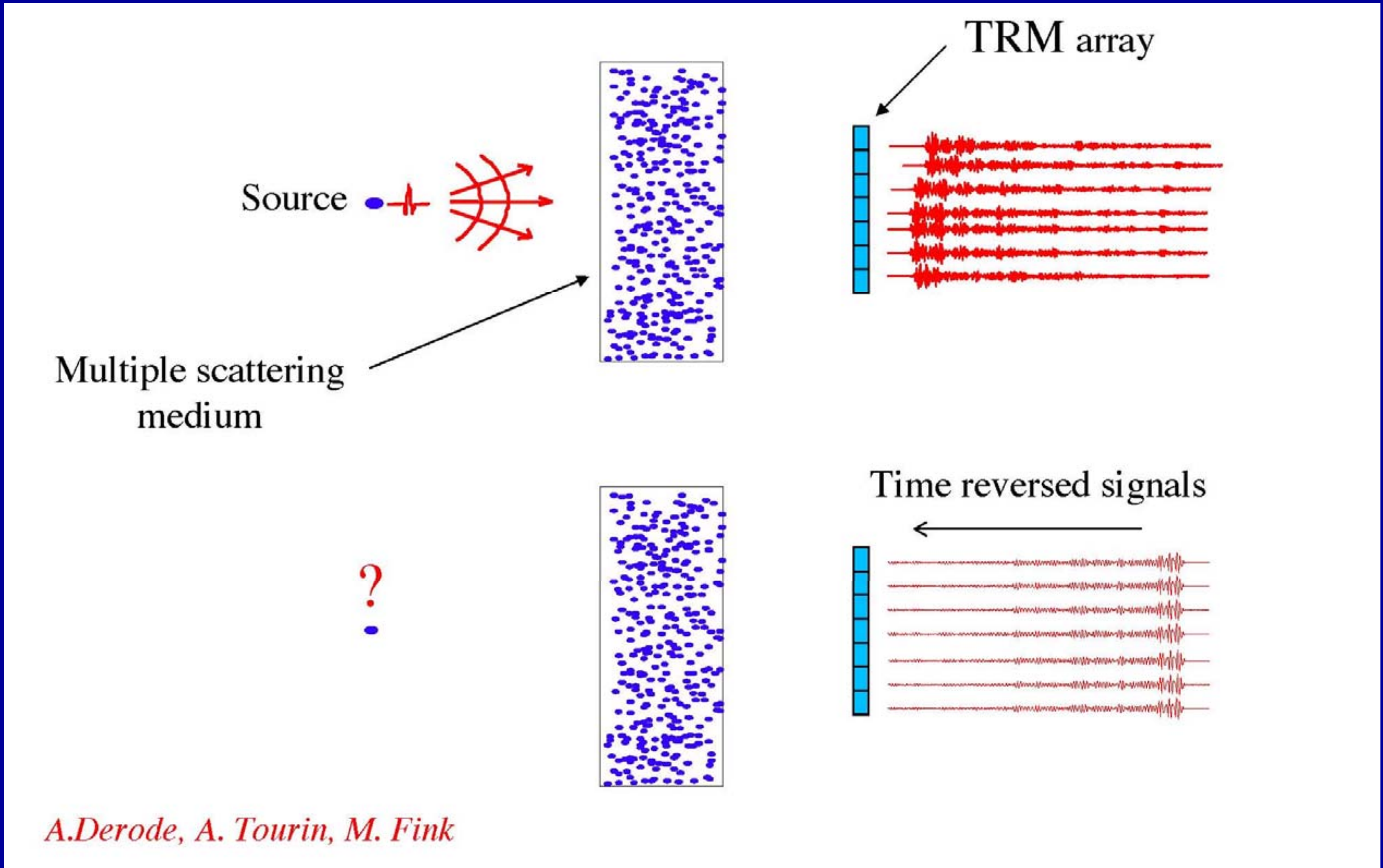
1. Seismic displacement field **u**(r,t) can be calculated everywhere by the SEM-NM method

$$\partial^2 u/\partial t^2 = H.u$$

2. In the absence of attenuation, rotation, time invariance and spatial reciprocity

if u(t) is a solution, u(-t) is also a solution.

If we send waves with reversed time:
How do they focus?



Refocusing at the source location by sending back signal (- t) through the SAME medium from a small number of emitters

Seismic Source Imaging by time reversal

Method Principle:

- Acoustic Source -> receivers
- Existence of transducers at the same time recorders and emitters sending back signal in the same medium

How to apply this concept to seismic waves within the Earth?

1C (scalar) ->3C (elastic case)?

Limited number of receivers?

Realistic Propagating Medium? 1D-3D Earth

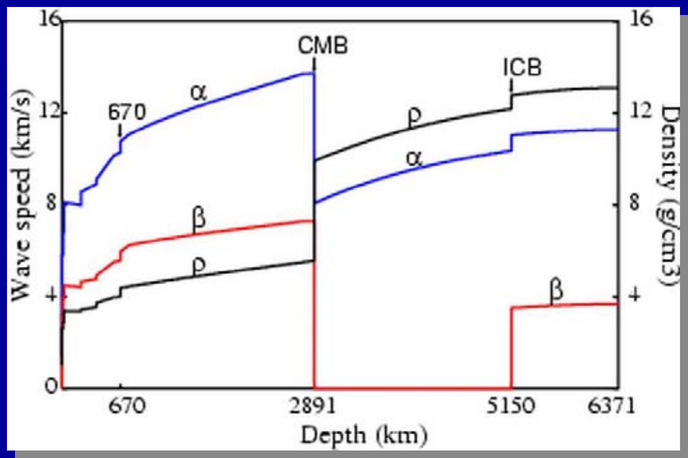
Time reversal

- Seismic displacement field **u**(r,t) calculated everywhere by the SEM-NM method

- It is possible to backpropagate **u(-t) or γ(-t)**

- Tests in 1D-model

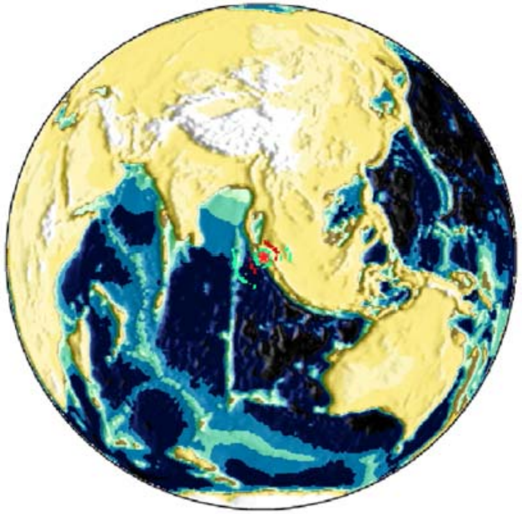
1D PREM



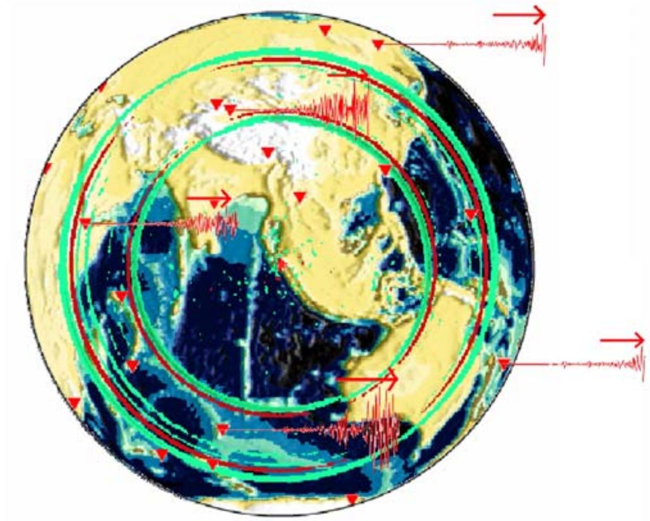
Very long periods
T> 200s

Larmat et al., 2005

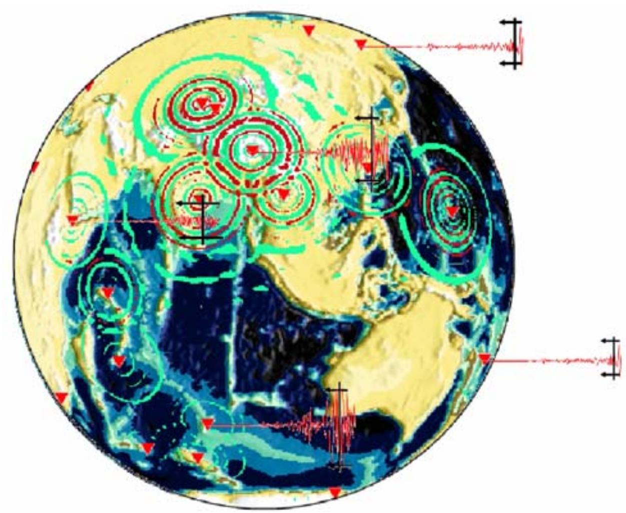
Event rupture



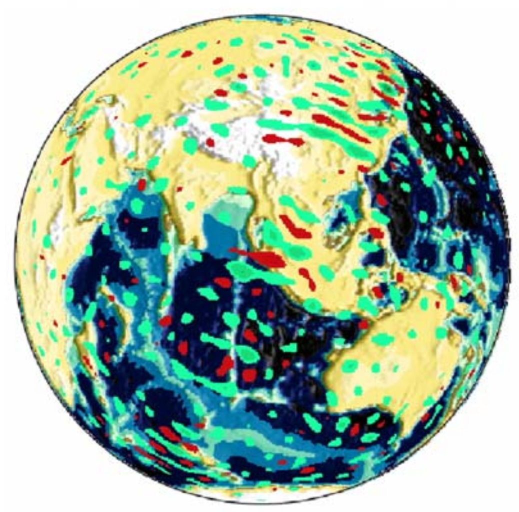
Seismogram recording



Time reversal experiment



Focusing



Synthetic test: Point source

Sumatra Earthquake 26/12/04 -NM-SEM

Sumatra

Synthetic Test

Normal modes:

Point Source

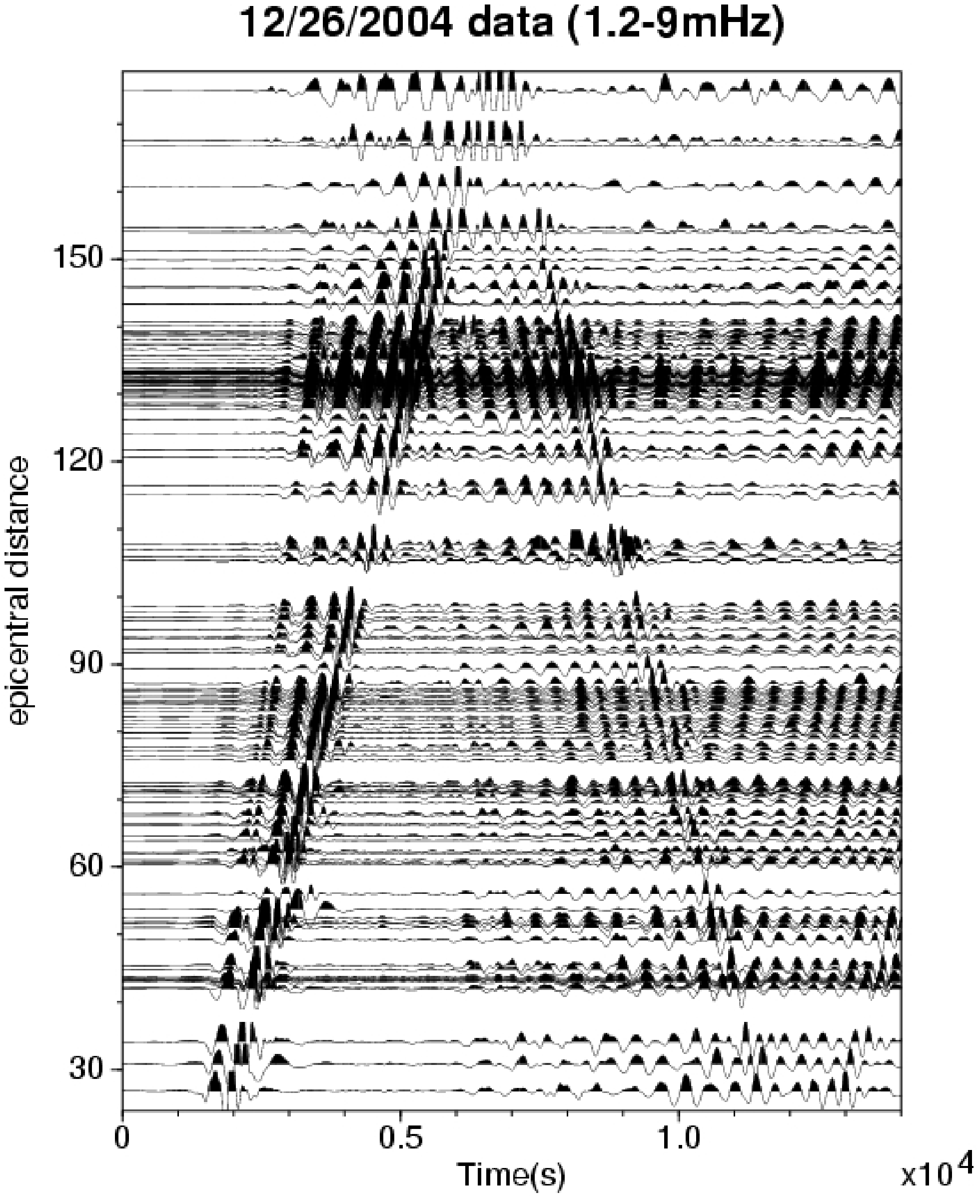
Y. Capdeville

Sumatra

Synthetic test

Normal modes:

Extended
source



The 121 real records we work with in this experiment (#11).

Sumatra

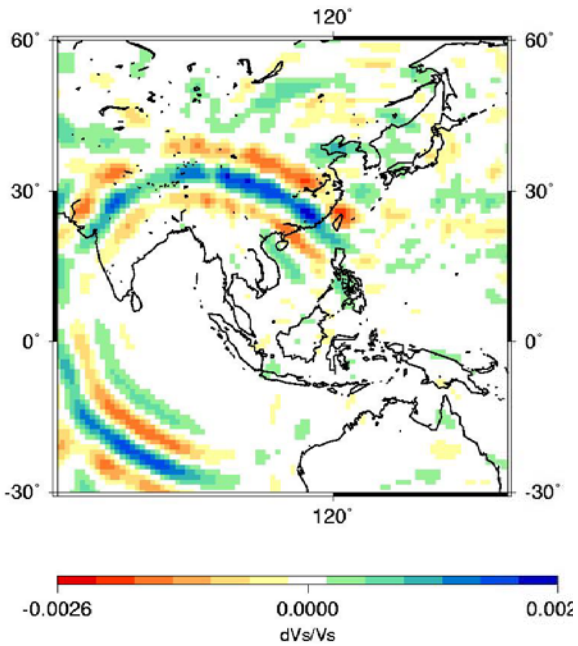
Normal mode
Time reversal

Real Data

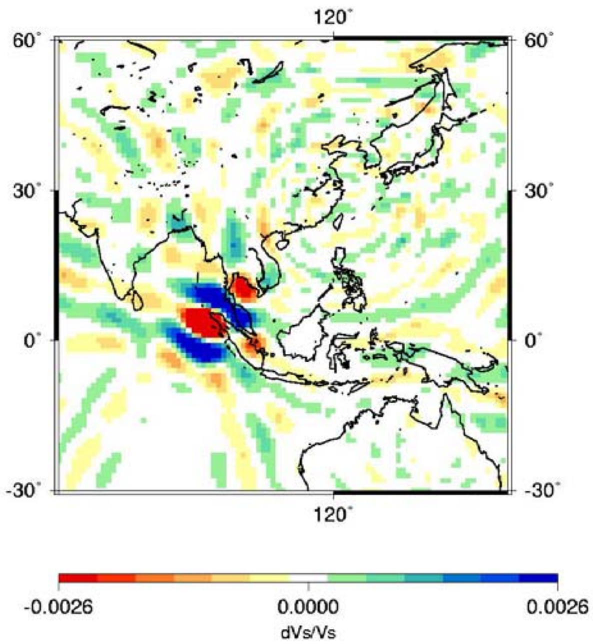
First conclusions

-Time reversal focus at the right time
($t_0 \approx -7000$s)
- and at the right place

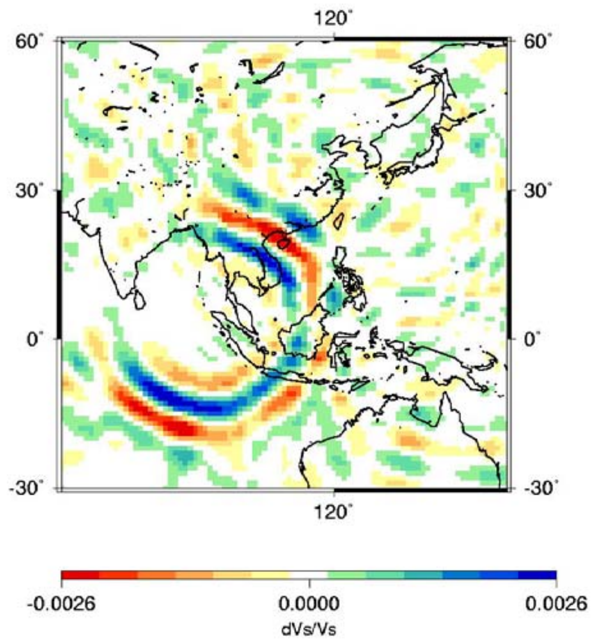
t= -6000s



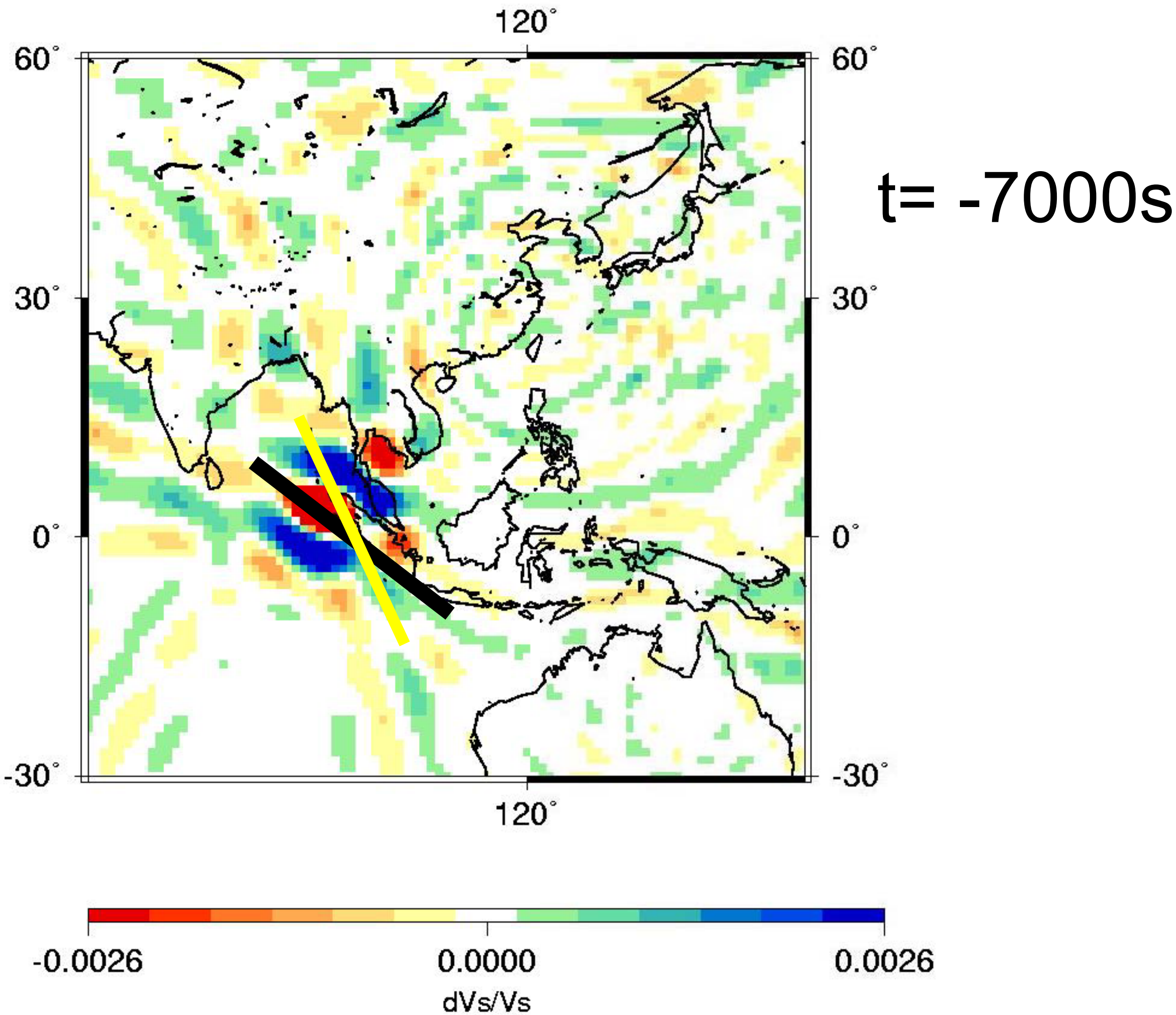
t= -7000s



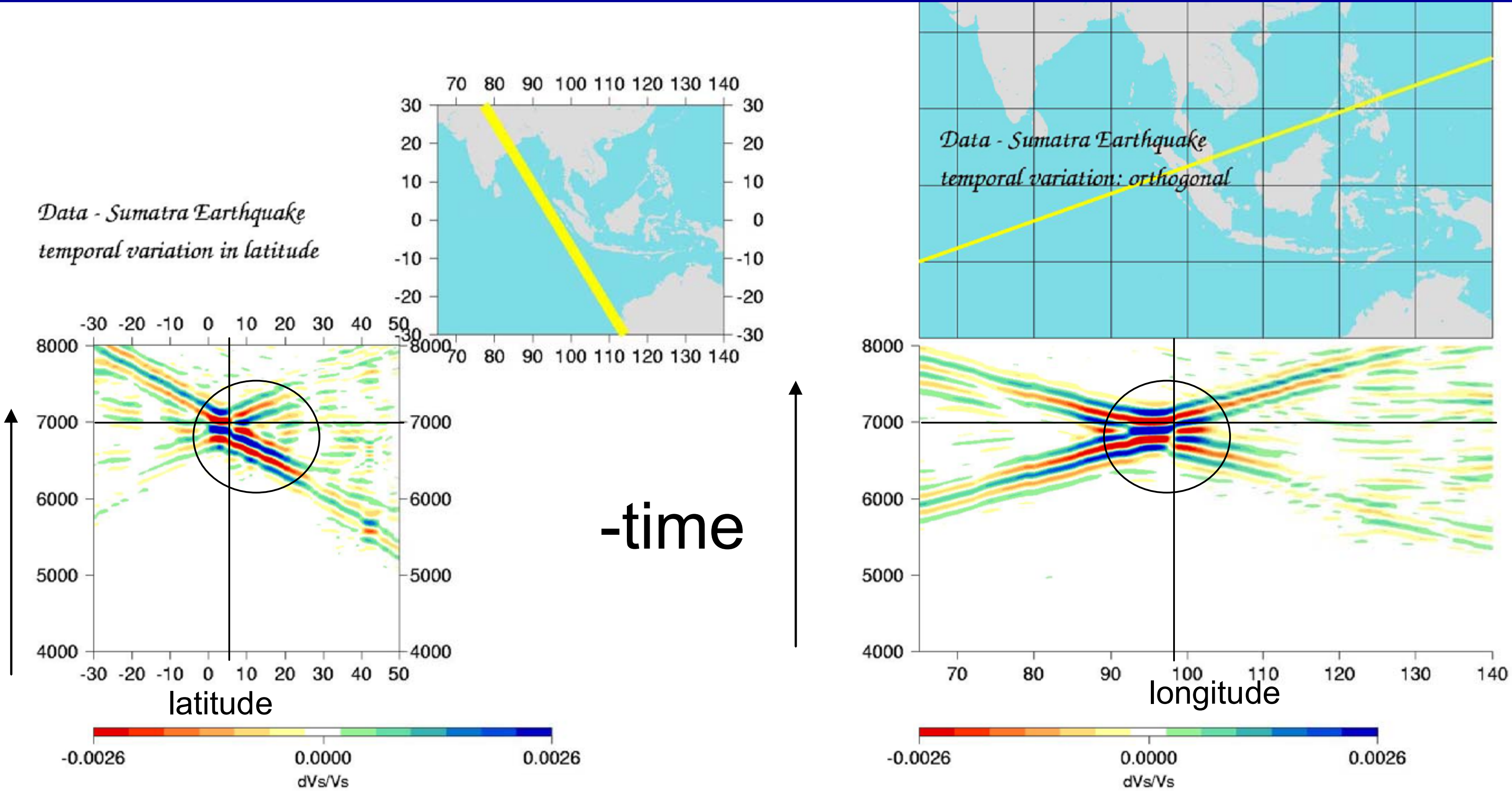
t= -7500s



Time reversal focus at the right time
($t_0 \approx$ -7000s)
And at the right place

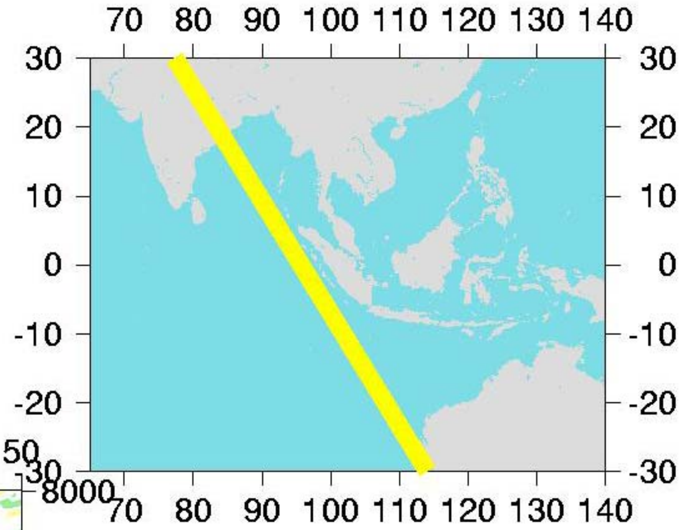


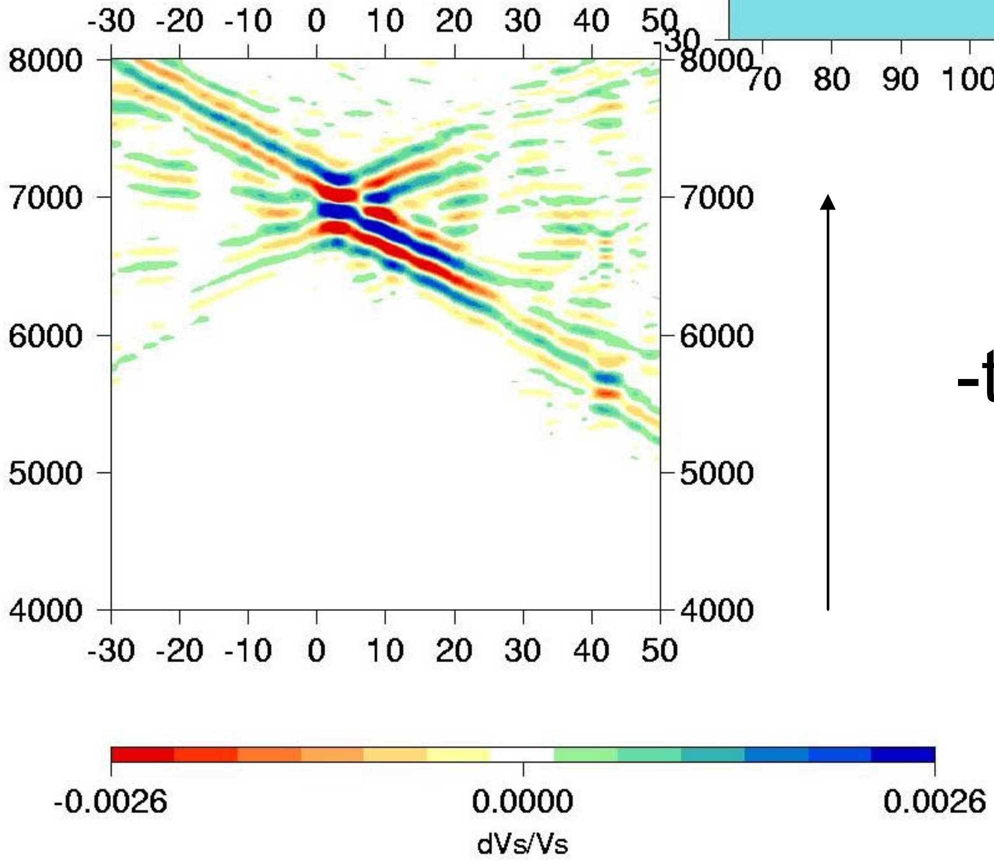
Can we get information about the history Of the seismic rupture?



Data - Sumatra Earthquake

temporal variation in latitude





-time

Source Rupture Imaging

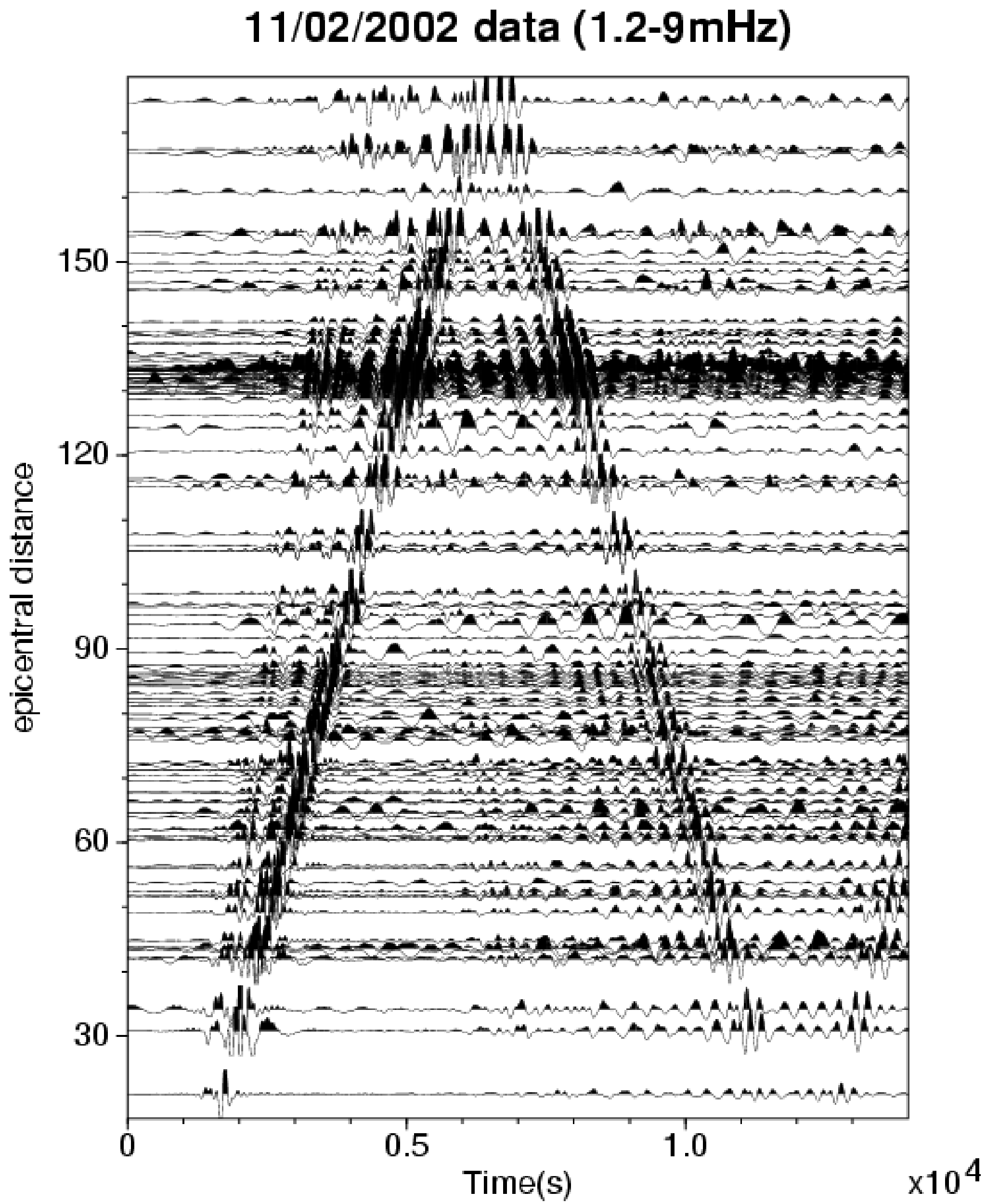
$\mathbf{u}(\mathbf{r},t) = \Sigma_k\, u_k\,(\,r\,) \cos \omega_k t\, / \omega_k^2 \exp(-\omega_k t/2Q_k)\, (\mathbf{u_k.F})_\mathbf{S}$

$\mathbf{u}(\mathbf{r}, \omega) = G\,(\mathbf{r},\mathbf{r_S}, \omega)\, S(\mathbf{r_S}, \omega)$

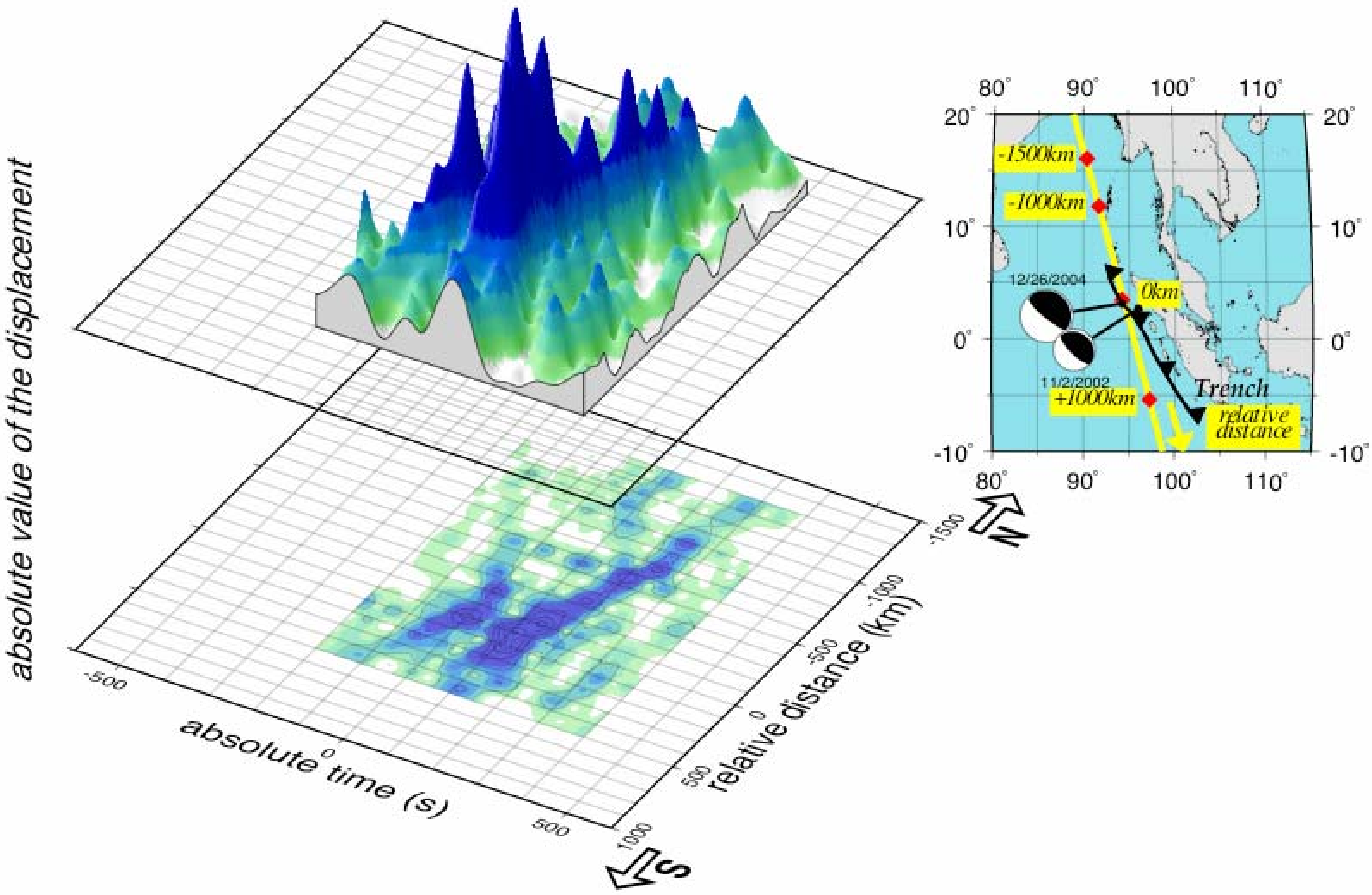
$G\,(\mathbf{r},\mathbf{r_S}, \omega)$ Fonction de Green

$S(\mathbf{r_S}, \omega)$ Fonction source

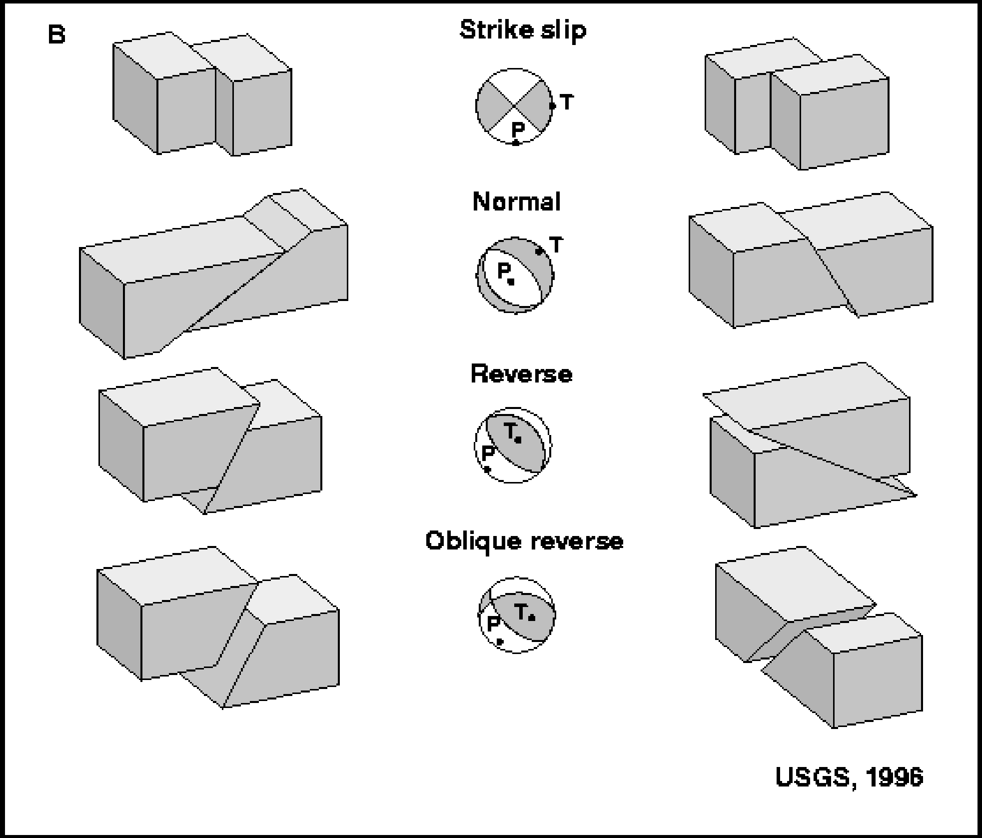
=> Reference source: delta function?

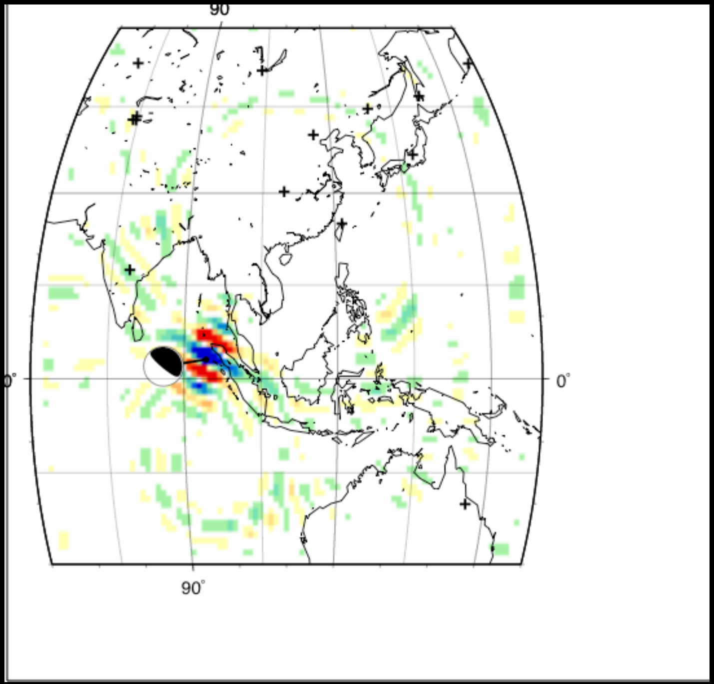


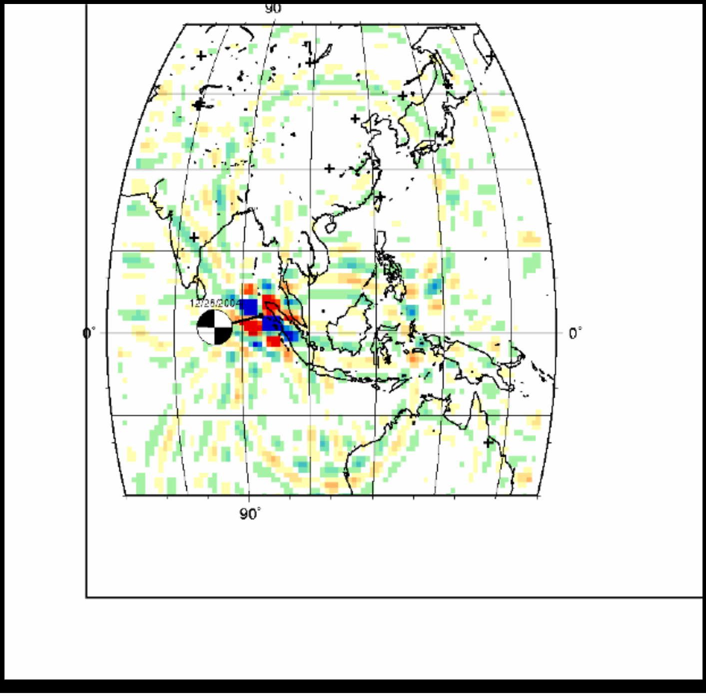
The 121 real records we work with in this experiment (#12).


absolute value of the displacement
absolute time (s)
-500
0
500
relative distance (km)
-1500
-1000
-500
0
500
1000
N
S
80°
90°
100°
110°
20°
10°
0°
-10°
-1500km
-1000km
0km
+1000km
12/26/2004
11/2/2002
Trench
relative distance

Different types of faults







TIME REVERSAL

- Application to real seismograms with broadband FDSN stations
- Spatio-temporal Imaging of seismic source
- Detection of unknown seismic sources (“quiet” , slow, glacial earthquakes, Seismic “Hum” of the Earth)
- Applications to seismic Tomography- Detection of mantle plumes…

Ocean Bottom Observatories