The Abdus Salam
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"8th Workshop on Three-Dimensional Modelling of
Seismic Waves Generation, Propagation and their Inversion"

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Surface Waves and Upper Mantle Anisotropy - II

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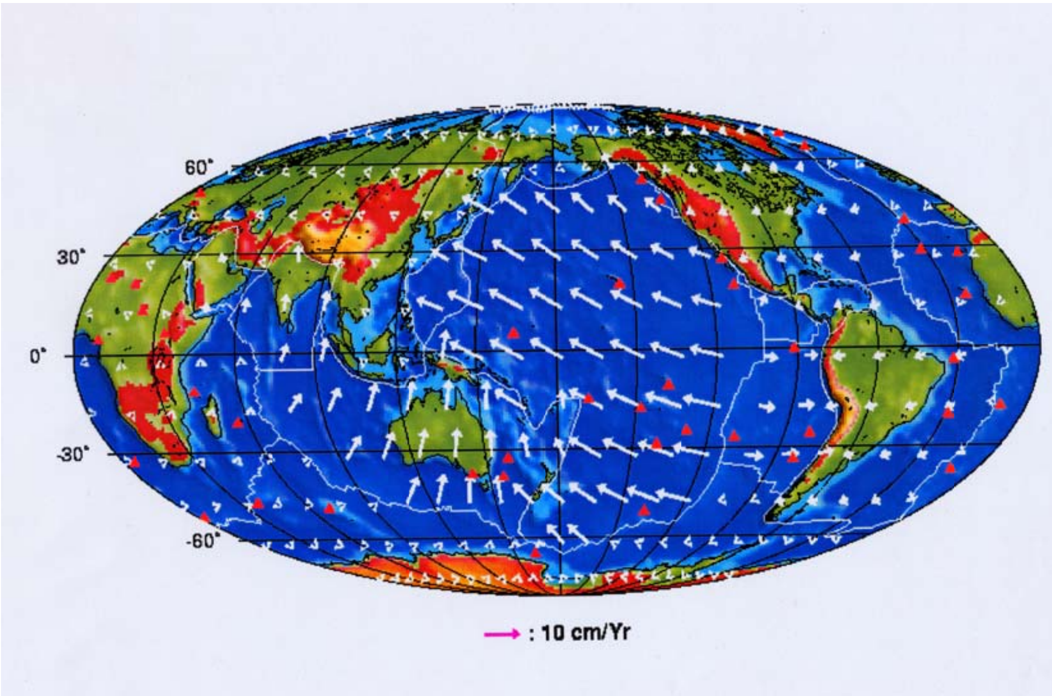
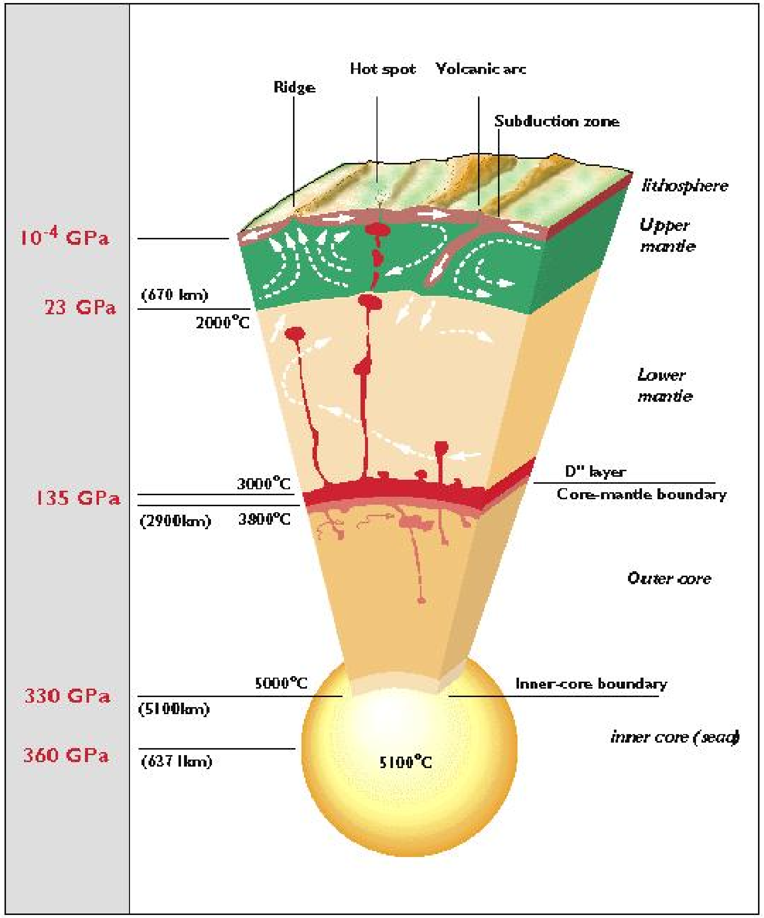
Structure of the Earth

Plate tectonics



Mantle Convection



Tomographic Technique

- Forward Problem: Theory **d=g(p)**

d data space, **p** parameter space

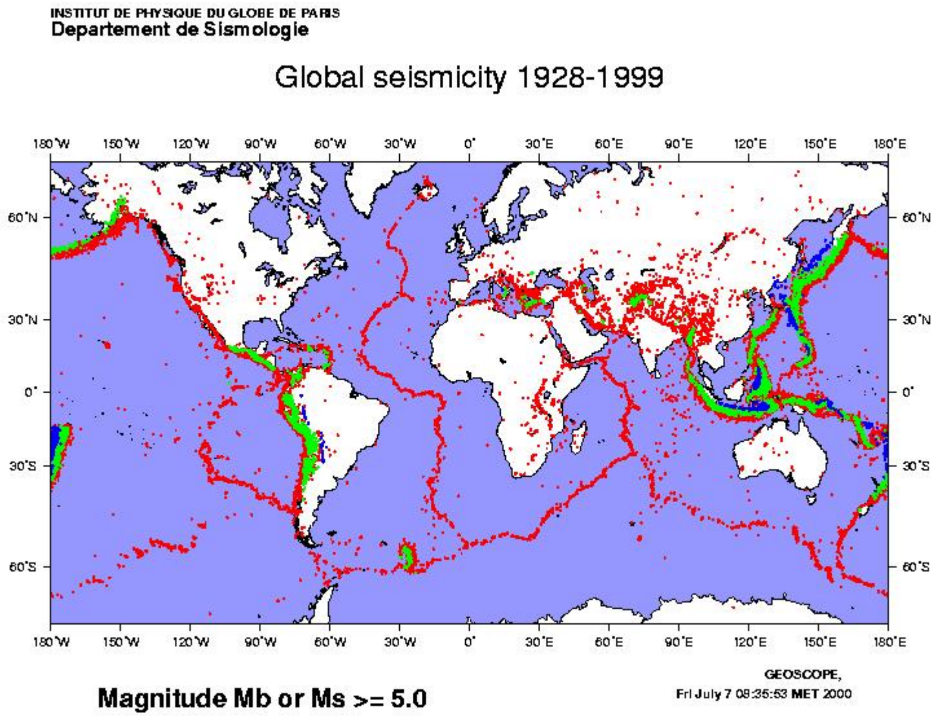
- Reference Earth model p_0:

$d_0 = g(p_0)$

- Kernels $\partial g/\partial p$
- Cd function (or matrix) of covariance of data

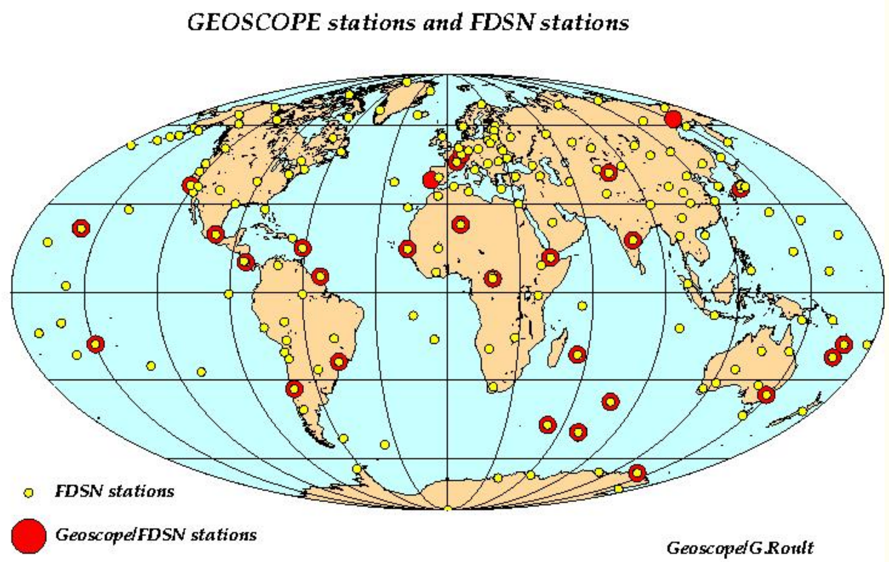
- Inverse Problem: $p-p_0 = \underline{g}^{-1}(d-d_0)$

- C_{p0} a priori Covariance function of parameters
- C_{pf} a posteriori Covariance function of parameters
- R Resolution



Receivers

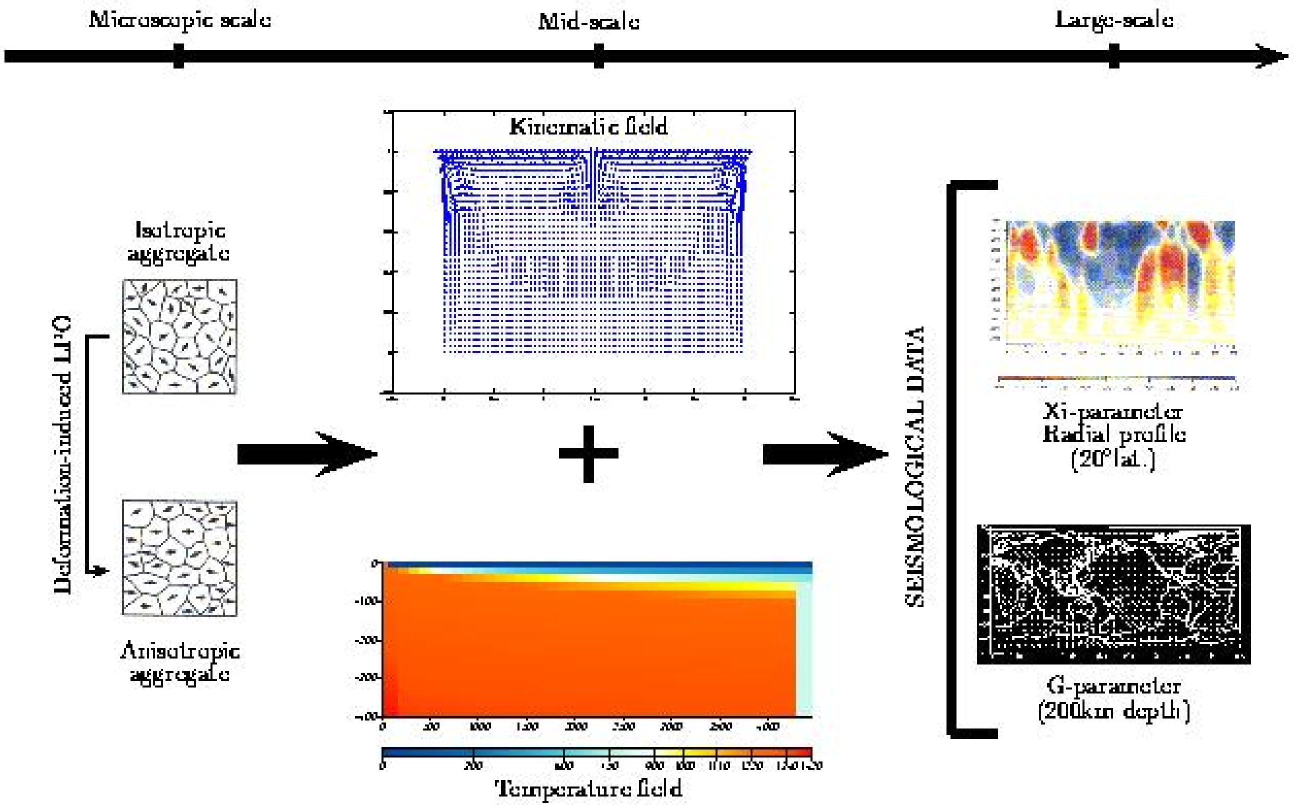
Seismic sources



Importance of seismic anisotropy

ANISOTROPY is the Rule not the Exception

Seismic Anisotropy is present at all scales

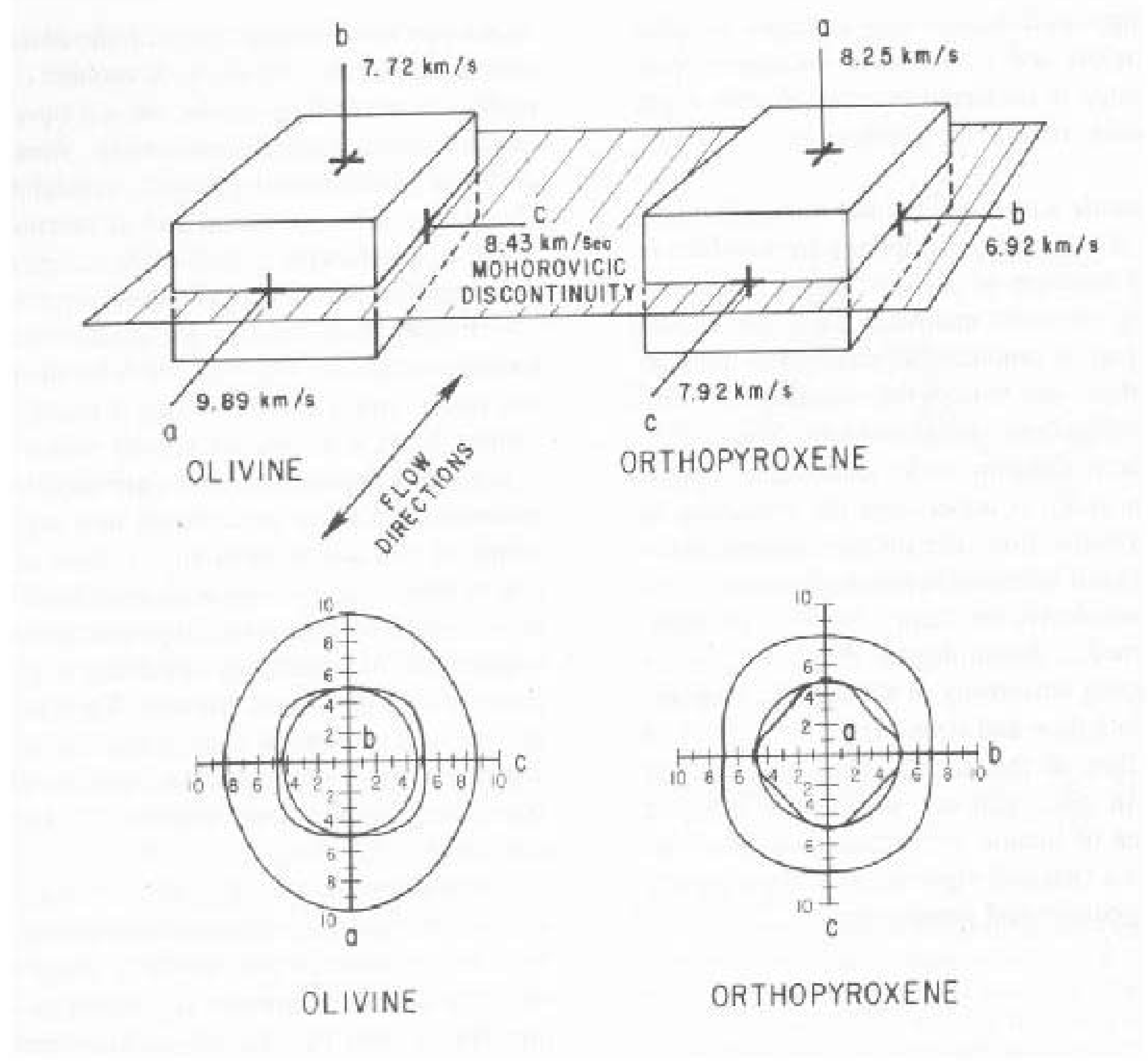


Mineralogical composition

Mapping Convection

(Montagner and Guillot, 2001)

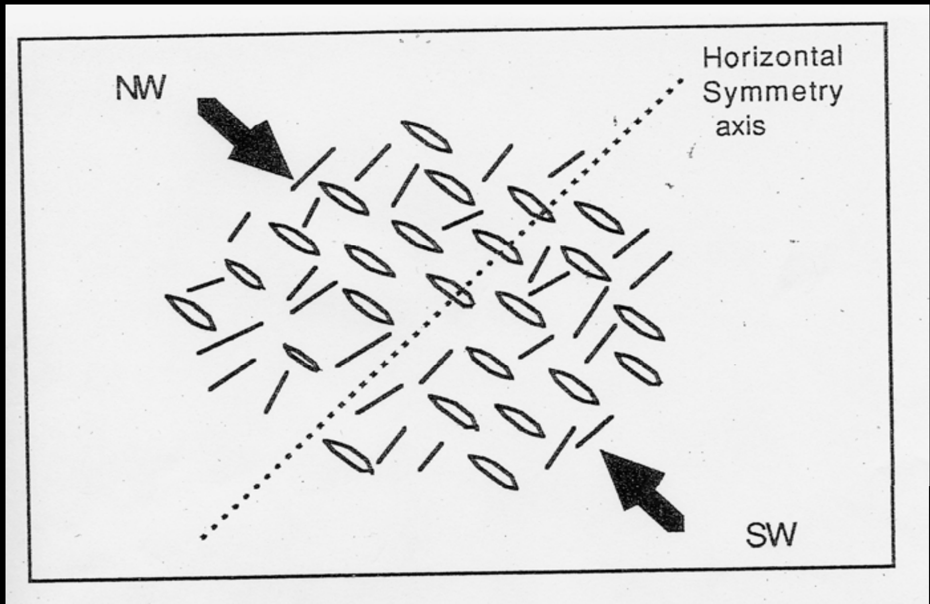
L.P.O. Lattice Preferred Orientation



From Christensen and Lundquist, 1982

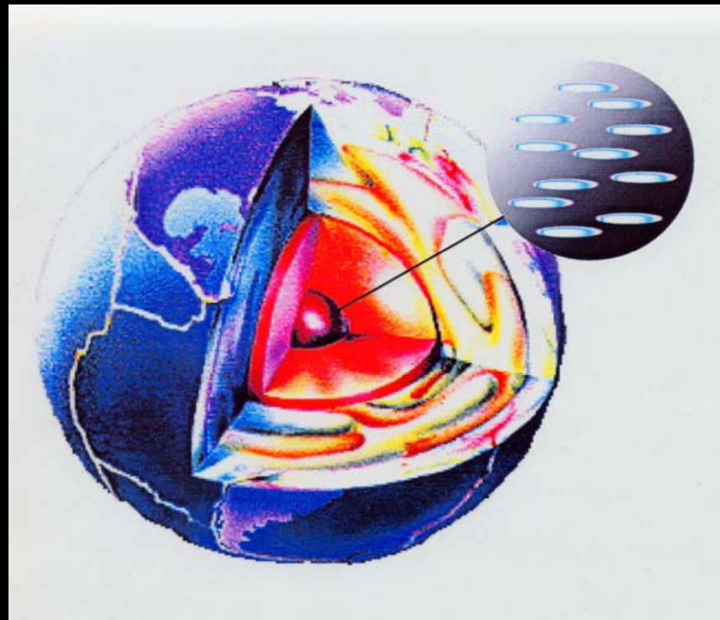
Cracks, fluid inclusions

Crust



(Babuska and Cara, 1991)

Inner core



(Singh et al., 2001)

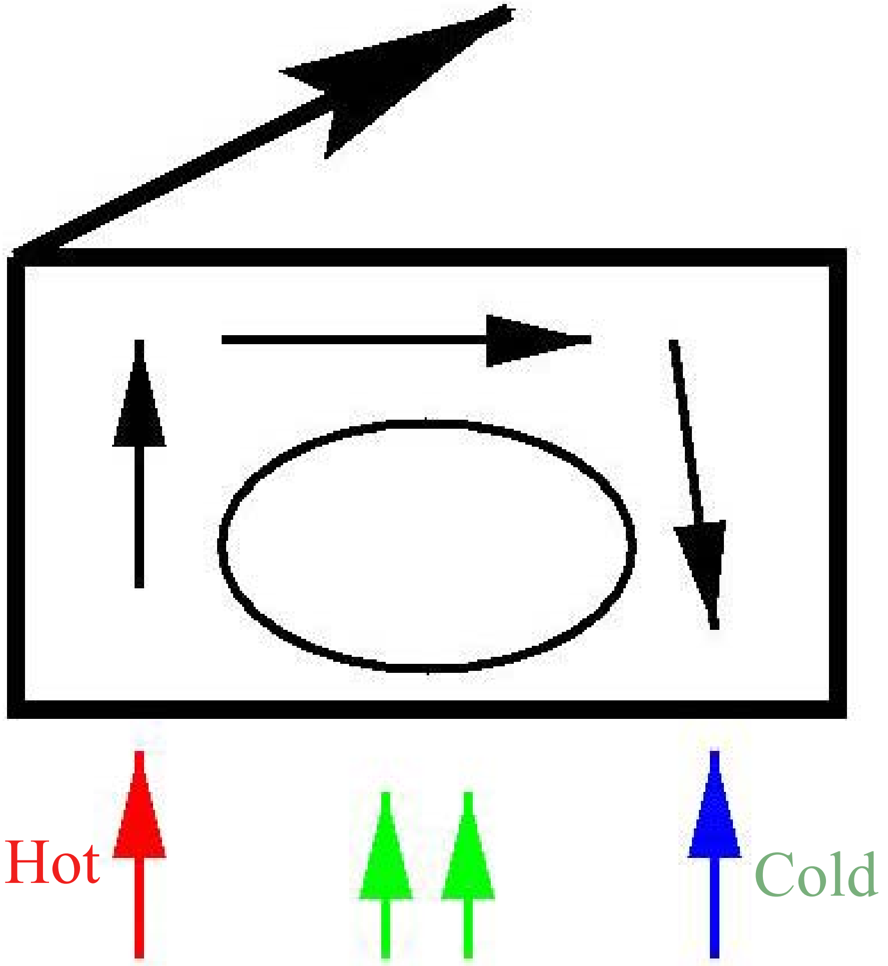
Importance of seismic anisotropy
ANISOTROPY is the Rule not the Exception

Anisotropy is present at all scales

-From microscopic scale up to macroscopic scale

-Efficient mechanisms of alignment (L.P.O.: lattice preferred orientation S.P.O.: shape preferred orientation; fine layering)

Montagner & Guillot, 2001



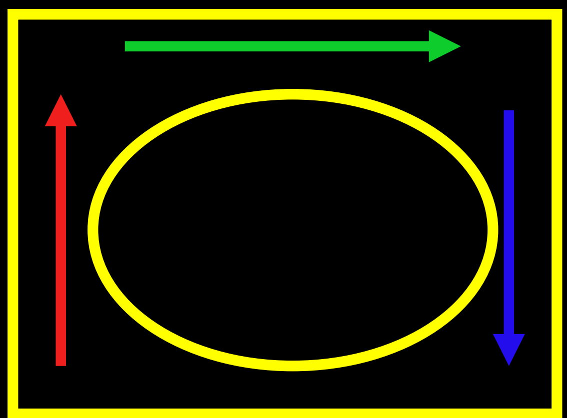
$\Delta\alpha$ Effect of Mineral Orientation

ΔT Effect of Temperature Heterogeneities

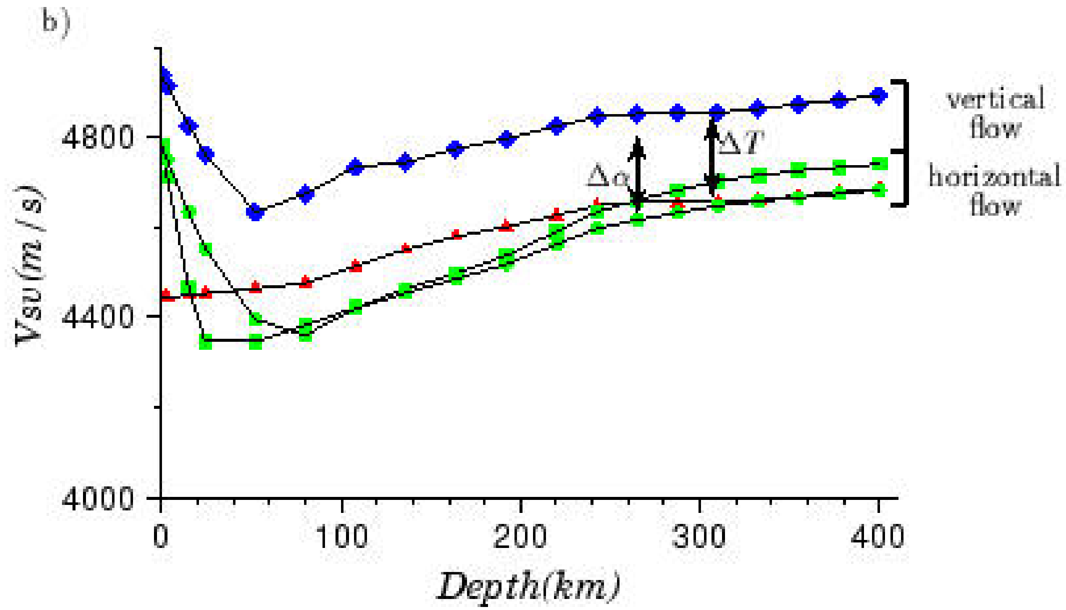
$\Delta\alpha$: Anisotropy Effect

ΔT: Temperature Effect

$$\Delta\alpha \approx \Delta T$$



Olivine (60%) +Opx (40%)



Montagner & Guillot, 2002

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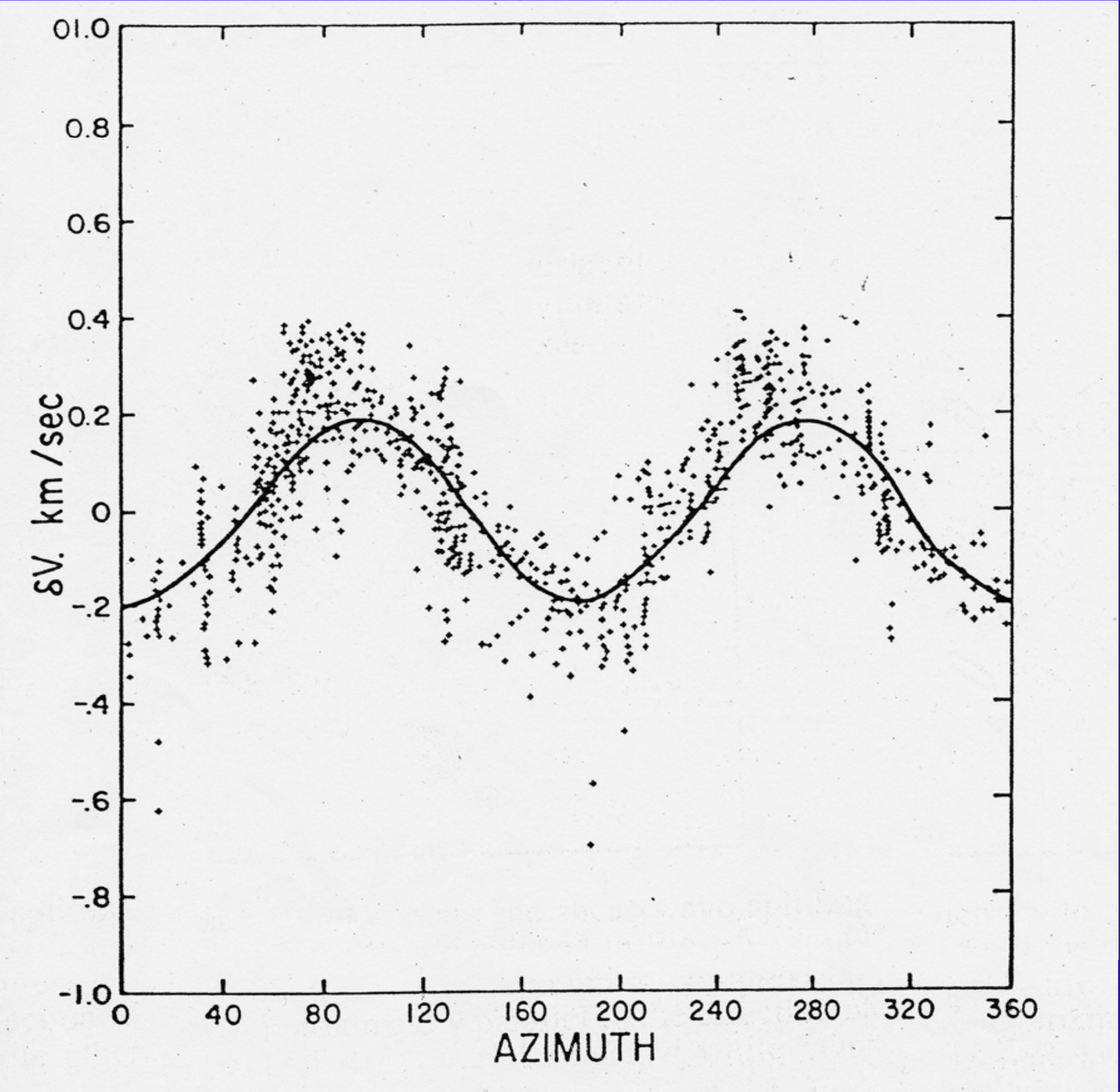
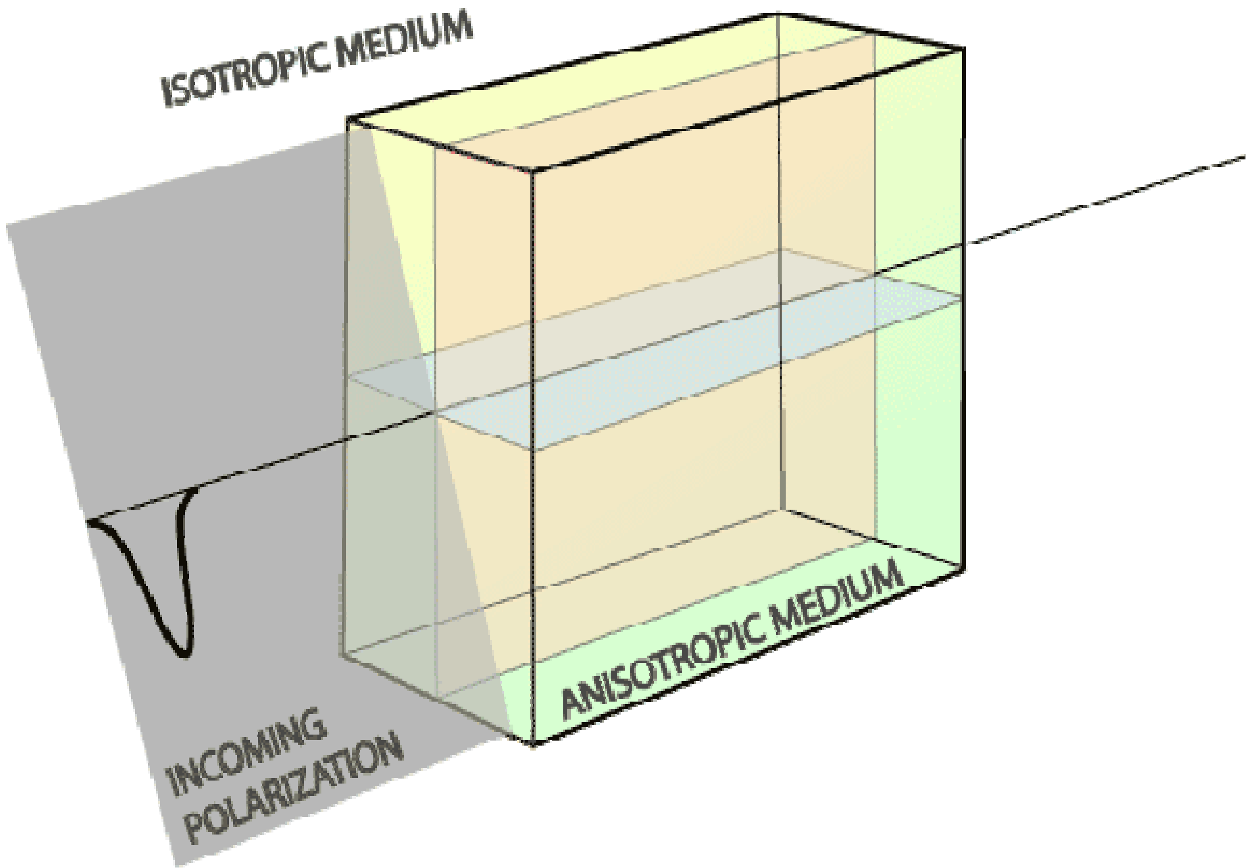
(L.P.O.: lattice preferred orientation
S.P.O.: shape preferred orientation; fine layering)

Anisotropy is observed on different kinds of seismic waves
-Body waves (Pn; S-wave splitting)
-Surface waves (Rayleigh-Love discrepancy, azimuthal anisotropy)

Pn- velocities

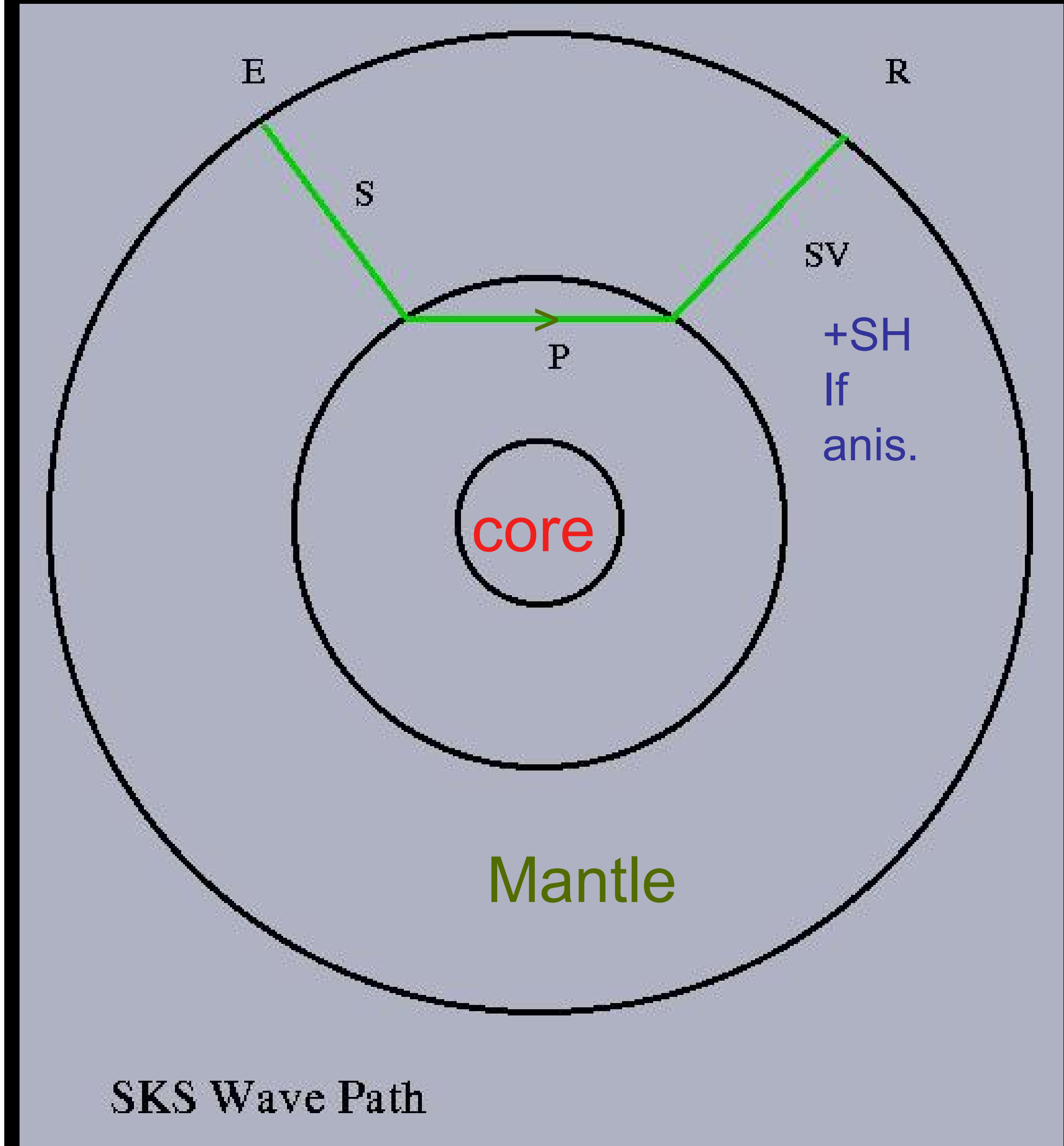


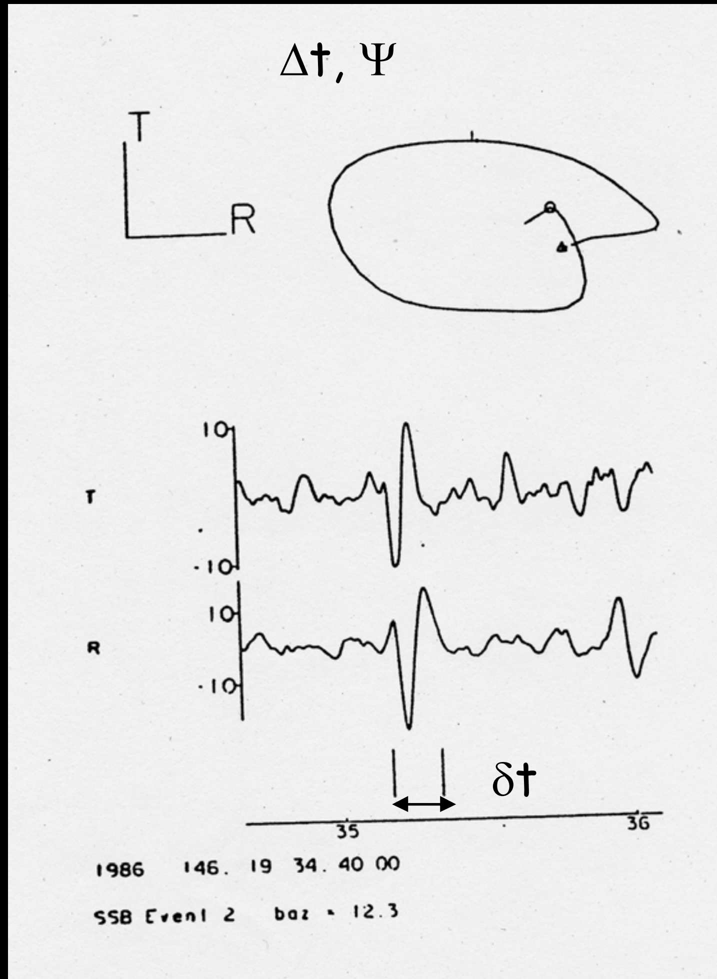
Shear Wave Splitting (Birefringence)



Animation courtesy of Ed Garnero

SKS- Splitting



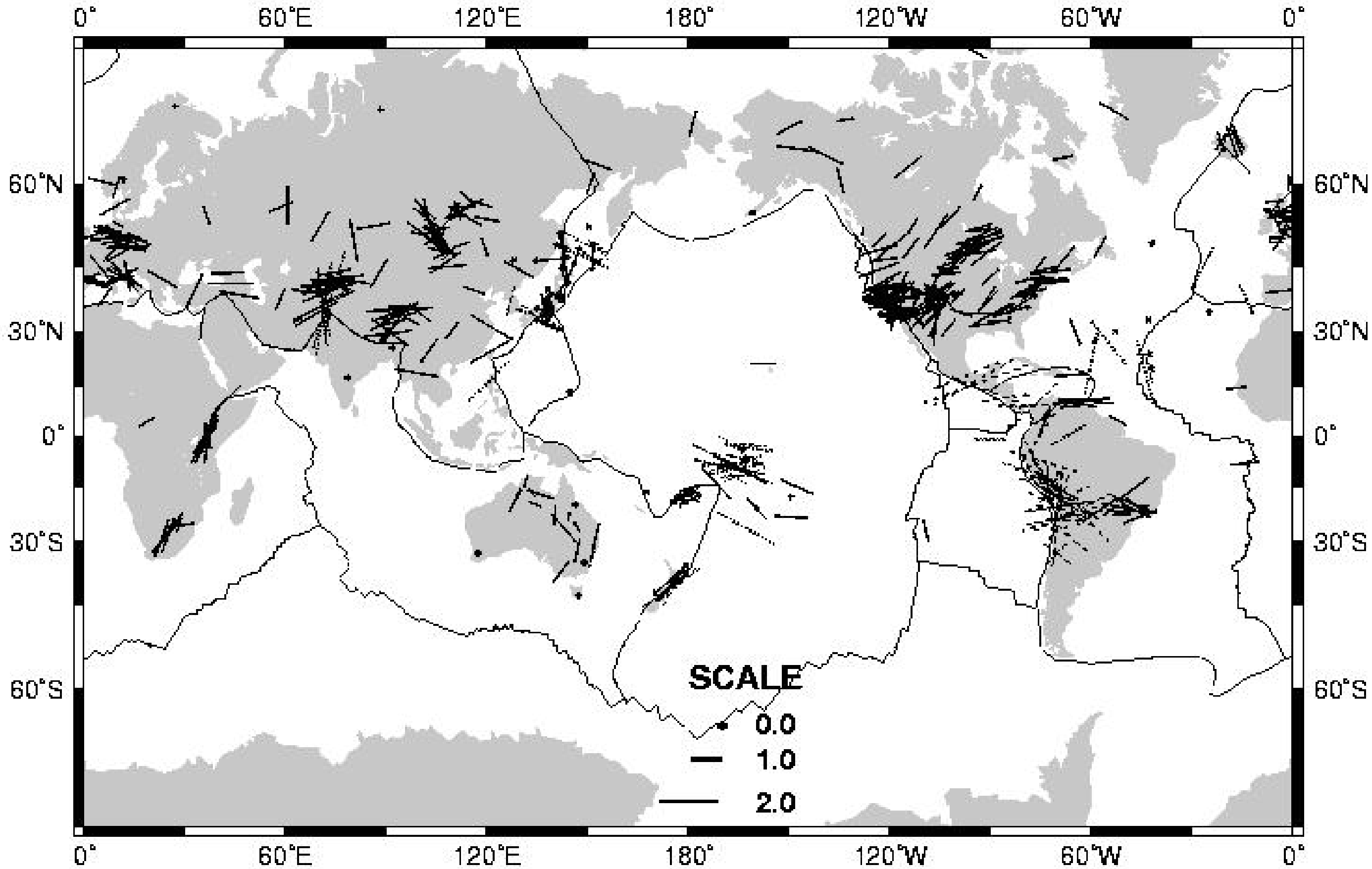


T

R

Vinnik et al., 1989

Compilation of S-wave splitting measurements



Savage, Rev. Geophys., 1999

SURFACE WAVES

Importance of seismic anisotropy
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- from microscopic scale to macroscopic scale
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(L.P.O.: lattice preferred orientation
S.P.O.: shape preferred orientation; fine layering)

Anisotropy is observed on different kinds of seismic waves
-Body waves (Pn; S-wave splitting)
-Surface waves (Rayleigh-Love discrepancy, azimuthal anisotropy)

ANISOTROPY REFLECTS AN INNER ORGANIZATION

ANISOTROPY IS NOT A SECOND ORDER EFFECT

Effect of anisotropy on surface waves

Effect on eigenfrequency for multiplet k={n,l,m}

$$\frac{\delta\omega_k}{\omega_k} = \frac{\int_\Omega \varepsilon_{ij}^* \, \delta C_{ijkl} \, \varepsilon_{kl} \, d\Omega}{\int_\Omega \rho_0^* \, u_r^* \, u_r \, d\Omega} = \left.\frac{\delta V}{V}\right|_k$$

ε strain tensor, u displacement, δC_{ijkl} elastic tensor perturbation

Phase velocity pertubation $V(T,\theta,\phi,\Psi)$ at point r (θ,ϕ) (Smith&Dahlen, 1973)

$$\delta V(T,\theta,\phi,\Psi)/V = \alpha_0(T,\theta,\phi) + \alpha_1(T,\theta,\phi)\cos 2\Psi + \alpha_2(T,\theta,\phi)\sin 2\Psi + \alpha_3(T,\theta,\phi)\cos 4\Psi + \alpha_4(T,\theta,\phi)\sin 4\Psi$$

Ψ Azimuth (angle between North and wave vector)

E

The first order perturbation in Love wave phase velocity $\delta C_L(k,\Psi)$ can be expressed as:

$$\delta C_L(k,\Psi) = \frac{1}{2C_{0L}(k)}[L_1(k) + L_2(k)cos2\Psi + L_3(k)sin2\Psi + L_4(k)cos4\Psi + L_5(k)sin4\Psi$$

where

$$L_0(k) = \int_0^\infty \rho W^2 dz$$

$$0\Psi \leftarrow L_1(k) = \frac{1}{L_0}\int_0^\infty (W^2 dN + \frac{W'^2}{k^2} dL)dz$$

$$2\Psi \leftarrow \begin{cases} L_2(k) = \frac{1}{L_0}\int_0^\infty -G_c(\frac{W'^2}{k^2})dz \\ L_3(k) = \frac{1}{L_0}\int_0^\infty -G_s(\frac{W'^2}{k^2})dz \end{cases}$$

$$4\Psi \leftarrow \begin{cases} L_4(k) = \frac{1}{L_0}\int_0^\infty -E_c.W^2 dz \\ L_5(k) = \frac{1}{L_0}\int_0^\infty -E_s.W^2 dz \end{cases}$$

The same procedure holds for Rayleigh waves, starting from the displacement given previously.

$$\delta C_R(k,\Psi) = \frac{1}{2C_{0R}(k)}[R_1(k) + R_2(k)cos2\Psi + R_3(k)sin2\Psi + R_4(k)cos4\Psi + R_5(k)sin4\Psi$$

where

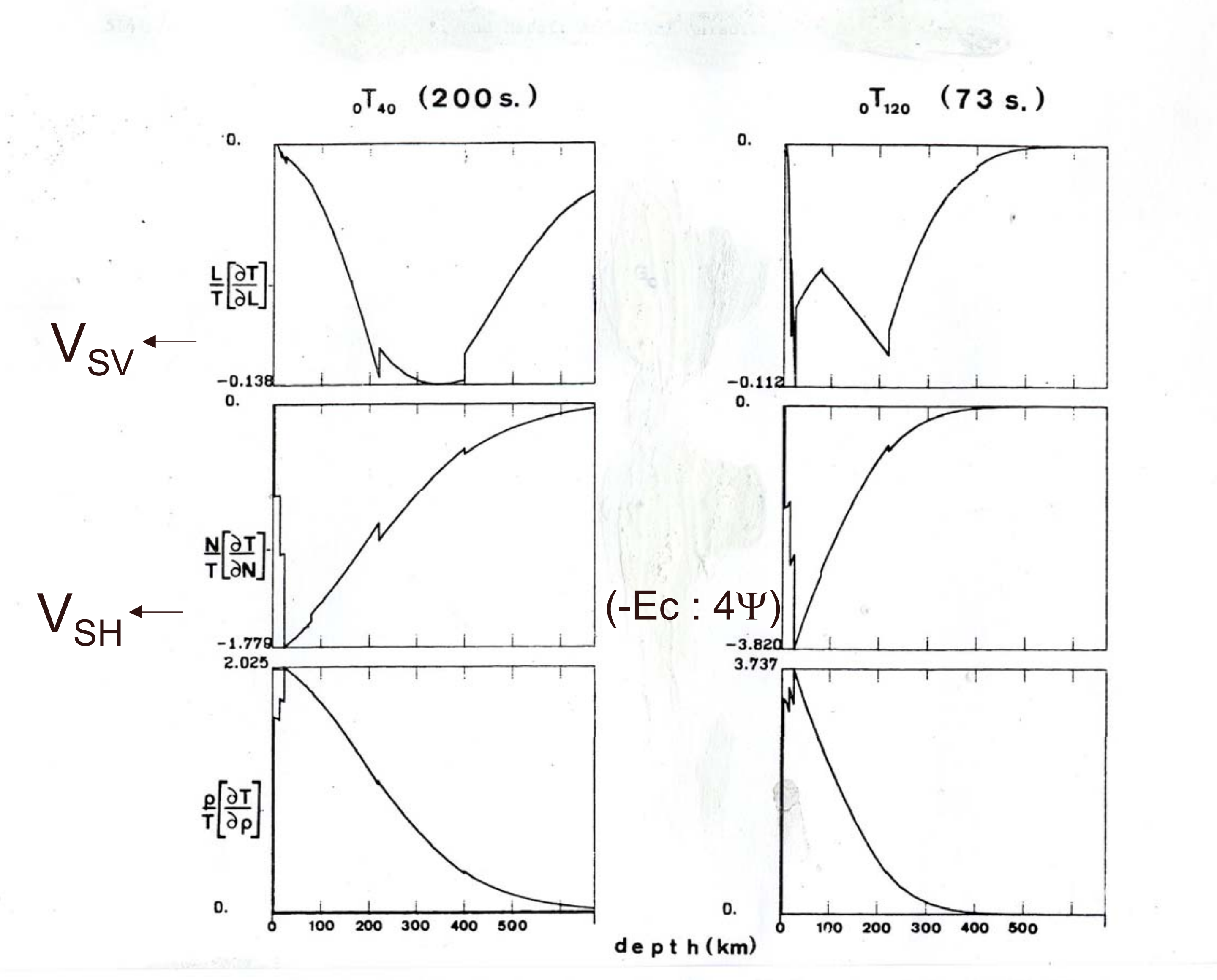
$$R_0(k) = \int_0^\infty \rho(U^2 + V^2)dz$$
$$R_1(k) = \frac{1}{R_0}\int_0^\infty [V^2 dA + \frac{U'^2}{k^2}.dC + \frac{2U'V}{k}.dF + (\frac{V'}{k} - U)^2 dL]dz$$
$$R_2(k) = \frac{1}{R_0}\int_0^\infty [V^2.B_c + \frac{2U'V}{k}.H_c + (\frac{V'}{k} - U)^2 G_c]dz$$
$$R_3(k) = \frac{1}{R_0}\int_0^\infty [V^2.B_s + \frac{2U'V}{k}.H_s + (\frac{V'}{k} - U)^2 G_s]dz$$
$$R_4(k) = \frac{1}{R_0}\int_0^\infty E_c.V^2 dz$$
$$R_5(k) = \frac{1}{R_0}\int_0^\infty E_s.V^2 dz$$

The 13 depth-functions $A, C, F, L, N, B_c, B_s, H_c, H_s, G_c, G_s, E_c, E_s$ are linear combinations of the elastic coefficients C_{ij} and are explicitly given as follows:

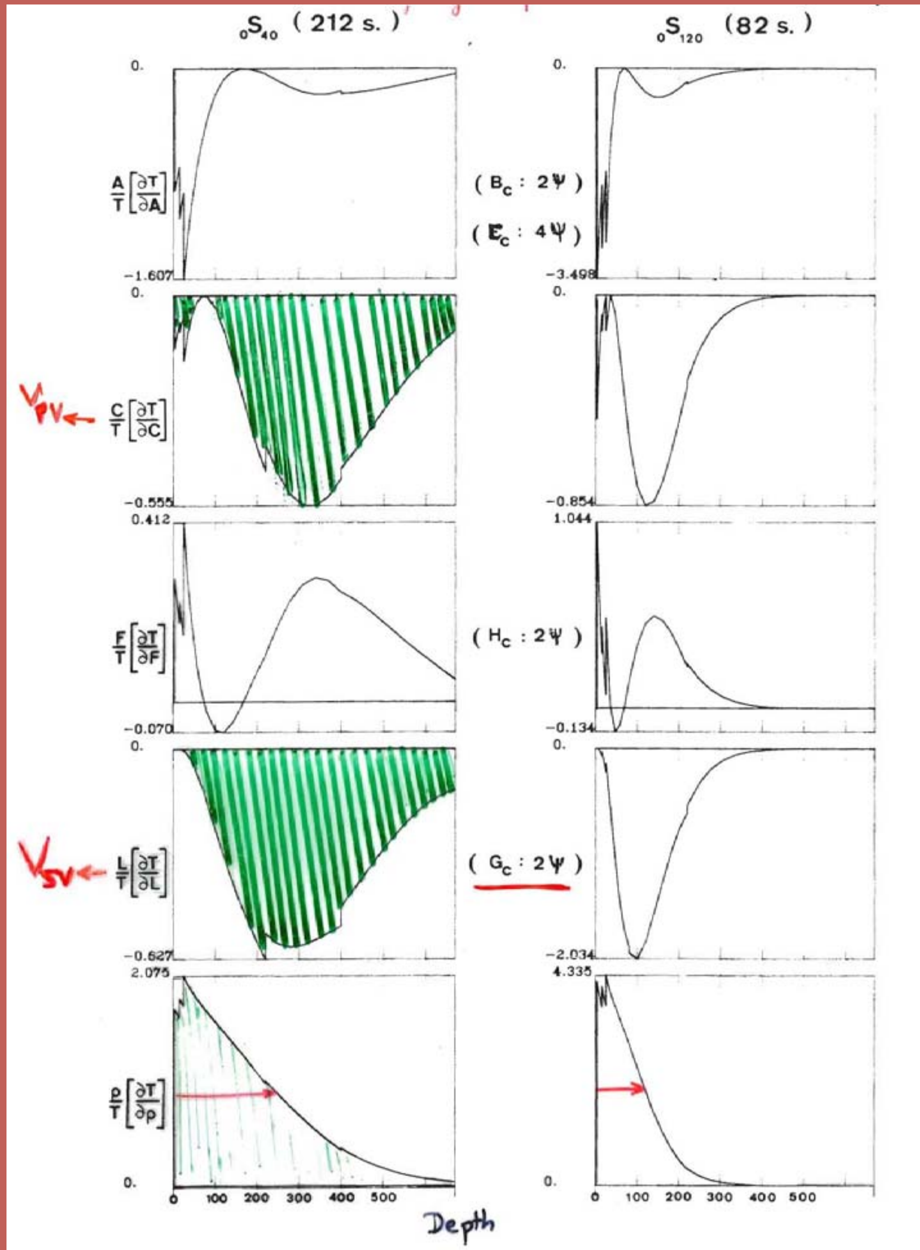
Table 1: Calculation of the various $c_{ij}\epsilon_i\epsilon_j$ for Love waves
$\alpha = cos\Psi$; $\beta = sin\Psi$

n	ij	$c_{ij}\epsilon_i\epsilon_j$
1	11	$c_{11}\alpha^2\beta^2.k^2W^2$
1	22	$c_{22}\alpha^2\beta^2.k^2W^2$
1	33	0
2	12	$-c_{12}\alpha^2\beta^2.k^2W^2$
2	13	0
2	23	0
2	24	
4	14	$c_{14}(-i\alpha^2\beta).\frac{kWW'}{2}$
4	15	$c_{15}(i\alpha^2\beta).\frac{kWW'}{2}$
4	16	$c_{16}(-\alpha\beta)(\alpha^2-\beta^2).\frac{k^2W^2}{2}$
4	24	$c_{24}(-i\alpha^2\beta).\frac{kWW'}{2}$
4	25	$c_{25}(-i\alpha\beta^2).\frac{kWW'}{2}$
4	26	$c_{26}(\alpha\beta)(\alpha^2-\beta^2).\frac{k^2W^2}{2}$
4	34	0
4	35	0
4	36	0
4	44	$c_{44}\alpha^2.\frac{W'^2}{4}$
8	45	$c_{45}(-\alpha\beta).\frac{W'^2}{4}$
8	46	$c_{46}(-i\alpha)(\alpha^2-\beta^2).\frac{kWW'}{2}$
4	55	$c_{55}\beta^2.\frac{W'^2}{4}$
8	56	$c_{56}(i\beta)(\alpha^2-\beta^2).\frac{kWW'}{2}$
4	66	$c_{66}(\alpha^2-\beta^2).\frac{k^2W^2}{4}$

Love wave partial derivatives



Rayleigh wave partial derivatives



13 parameters

0Ψ term	$A = \frac{3}{8}(C_{11} + C_{22}) + \frac{1}{4} C_{12} + \frac{1}{2} C_{66}$ $C = C_{33}$ $F = \frac{1}{2}(C_{13} + C_{23})$ $L = \frac{1}{2}(C_{44} + C_{55})$ $N = \frac{1}{8}(C_{11} + C_{22}) - \frac{1}{4} C_{12} + \frac{1}{2} C_{66}$	V_{PH} V_{PV} V_{SV} V_{SH}
	cos	sin
2Ψ term	$B_c = \frac{1}{2}(C_{11} - C_{22})$ $G_c = \frac{1}{2}(C_{55} - C_{44})$ $H_c = \frac{1}{2}(C_{13} - C_{23})$	$B_s = C_{16} + c_{26} \rightarrow B$ $G_s = C_{54} \rightarrow G$ $H_s = C_{36} \rightarrow H$
4Ψ term	$E_c = \frac{1}{8}(C_{11} + C_{22}) - \frac{1}{4} C_{12} - \frac{1}{2} C_{66}$	$E_s = \frac{1}{2}(C_{16} - C_{26}) \rightarrow E$

Transversely
Isotropic
Medium
With vertical
Symmetry axis

Isotropic medium: 2 parameters
VTI: 5 parameters (A,C,F,L,N)
+ 8 (from surface waves)

Montagner and Nataf, 1986

13 parameters

- Best Resolved parameters for Surface Waves

$\mathbf{L} = \rho V_{SV}^2$ Isotropic part of V_{SV}.

$\xi = \frac{N}{L} = \frac{V_{SH}^2}{V_{SV}^2}$ Radial Anisotropy.

G, Ψ_G Azimuthal Anisotropy of V_{SV}, also related to SKS splitting (when horizontal symmetry axis).

+ a priori information (from mineralogy, …)

- Body Waves (*Crampin*, 1984)

$$\rho V_{qSV}^2 = L + G_c cos 2\Psi + G_s sin 2\Psi$$

$$\rho V_{qSH}^2 = N - E_c cos 4\Psi - E_s sin 4\Psi$$

Geodynamic Interpretation

S- Velocity

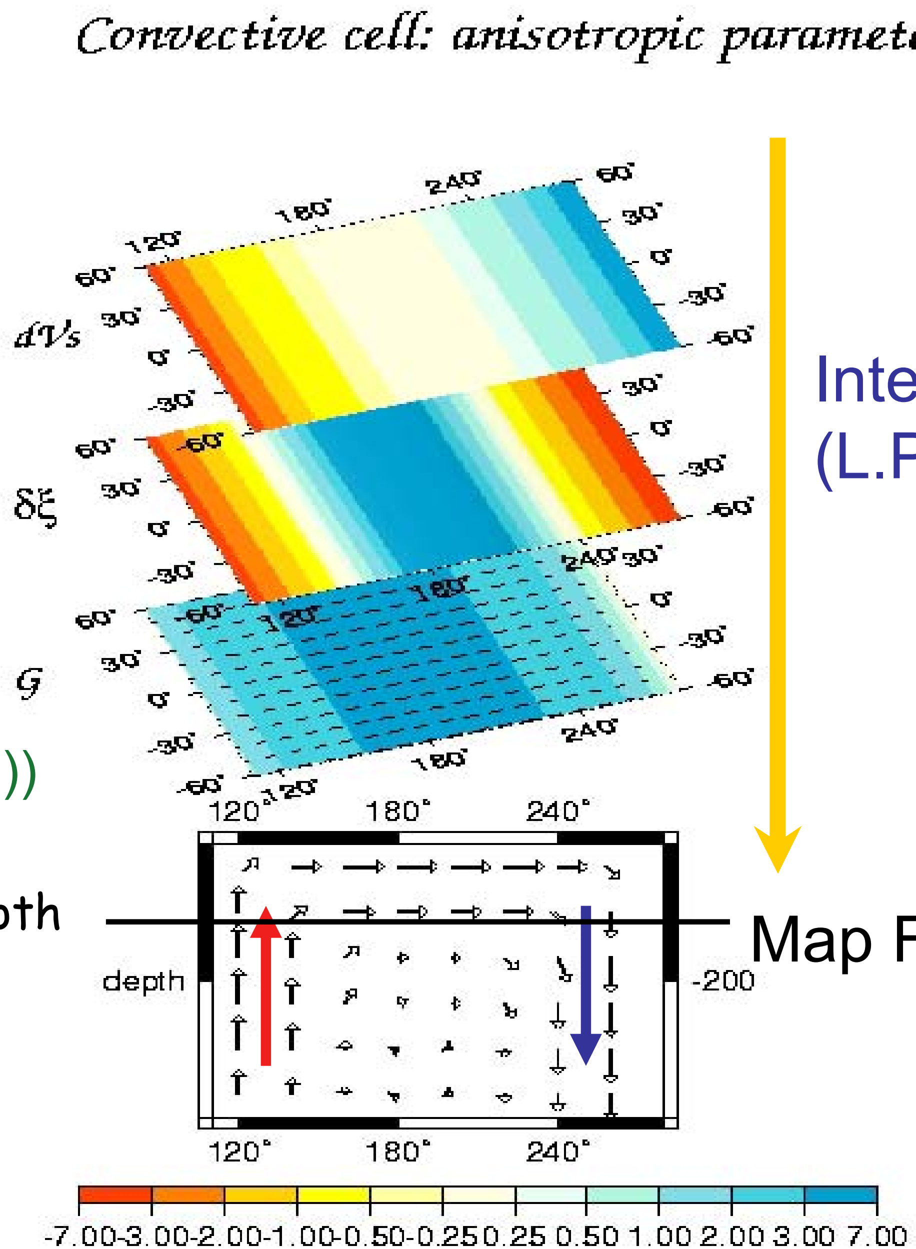
Radial Anisotropy

$\xi = (V_{SH}^2 - V_{SV}^2)/V_{SV}^2$

Azimuthal Anisotropy

$V_{SV} \approx V_{SV0} + {}^{1}/_{2} G\cos(2(\Psi - \Psi_G))$

At a given depth

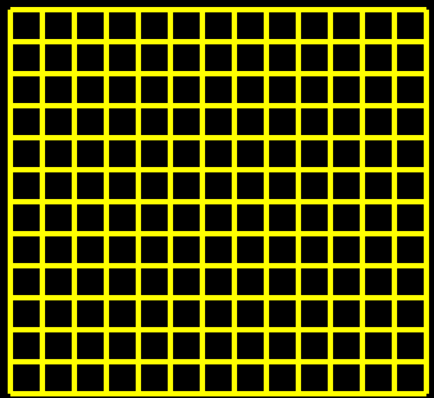


Interpretation (L.P.O.)

Map Flow

2 D tomography: N cells

Isotropic Inversion:



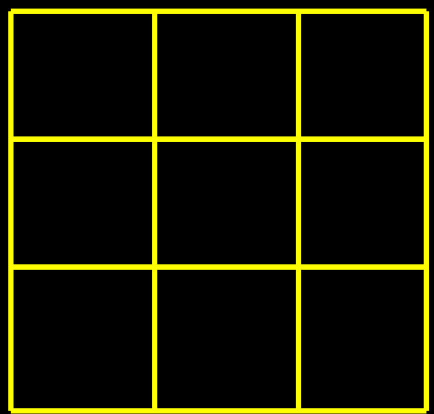
N independent parameters

0-Ψ term

VR1

Variance reduction

Anisotropic inversion



3N’ = N

0+2 Ψ term

VR2

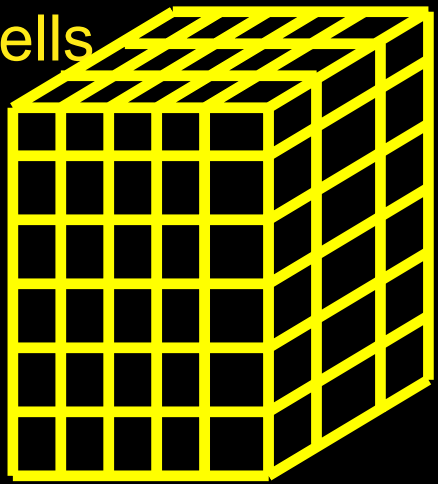
VR2>VR1 => the anisotropic model can be simpler than the isotropic model

Parameter Space

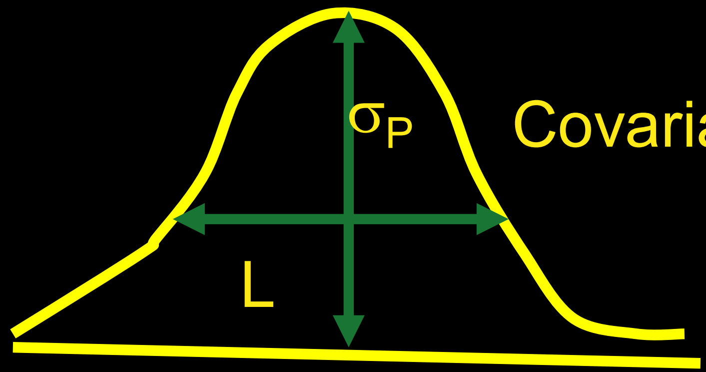
Physical parameters: ρ + 13 physical parameters

Geographical parameterization: $\mathbf{p}(r,\theta,\phi)$

Cells



Continuous parameterization



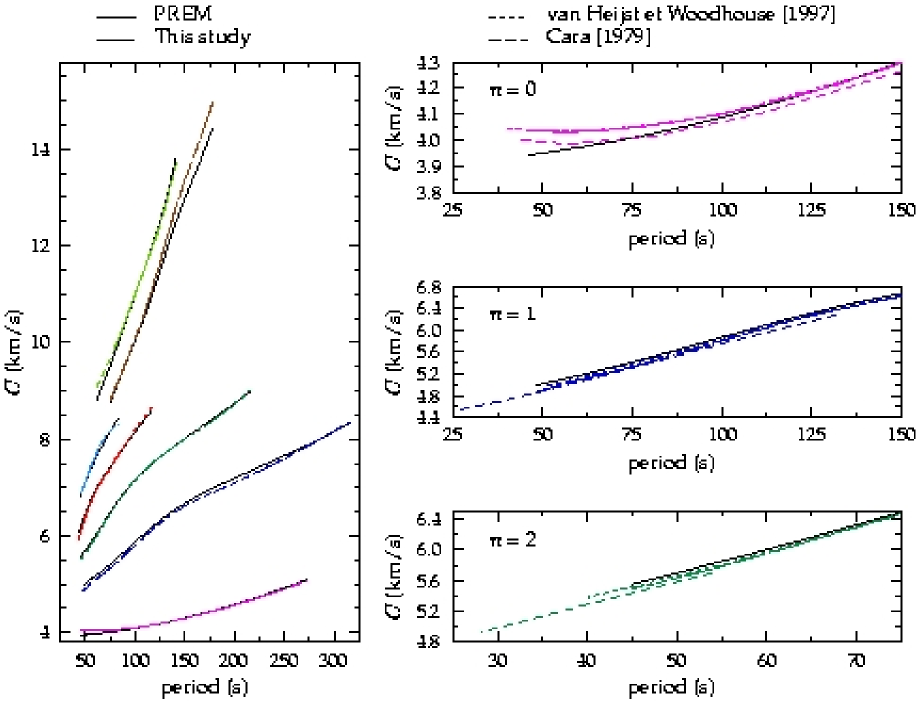
Covariance fonction

- Spherical harmonic expansion

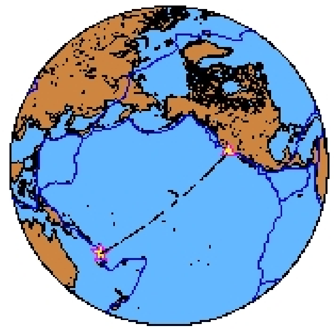
- Lateral resolution:

Hor 1000km, Rad 50km => 500*60*14 ≈ 420,000 parameters

Calculation of dispersion curves: Fundamental modes and higher modes

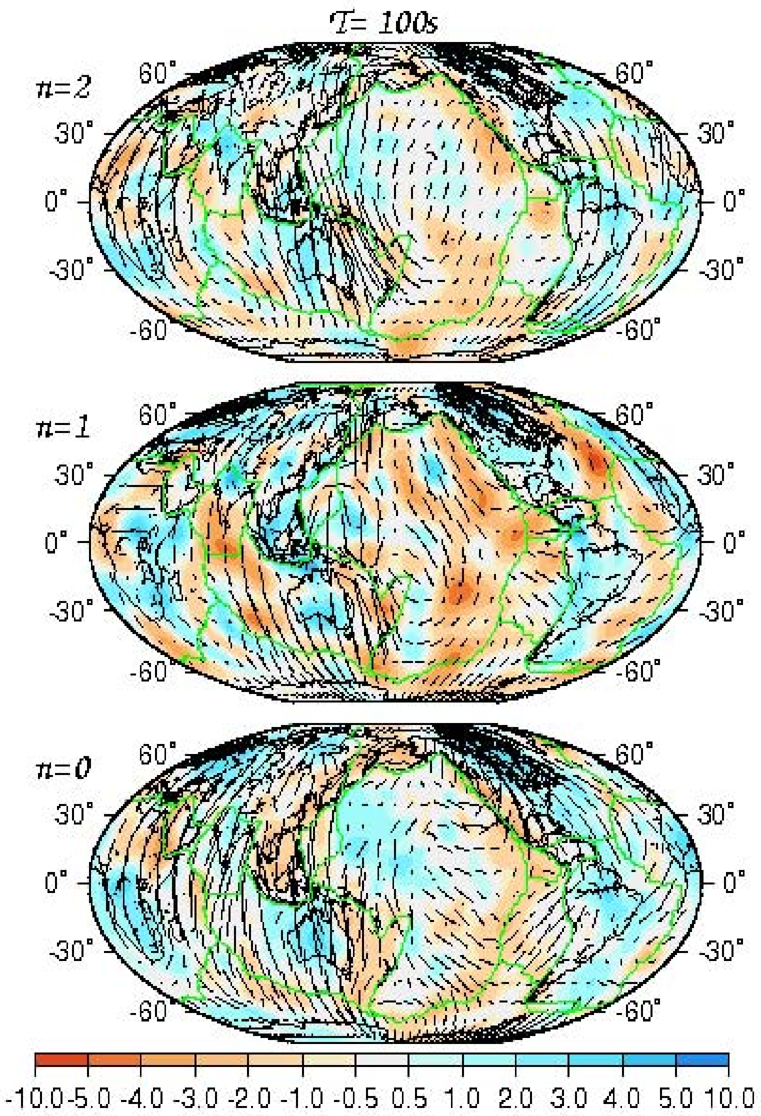


Comparison with previous results along the Vanuatu-California path.



Beucler et al., 2002

Phase velocity maps
At 100s



2nd overtone

1st overtone

Fundamental mode

Beucler and Montagner, 2006

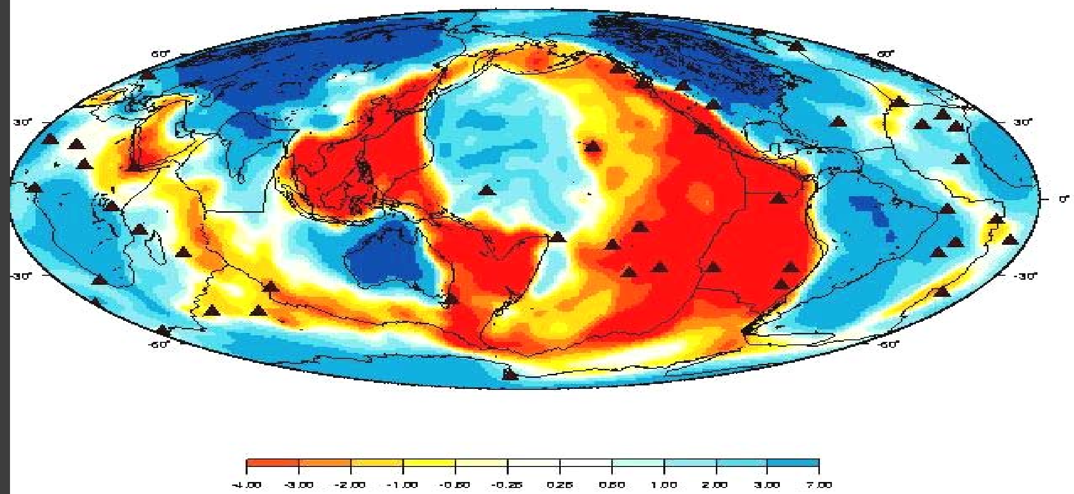
Global Tomography

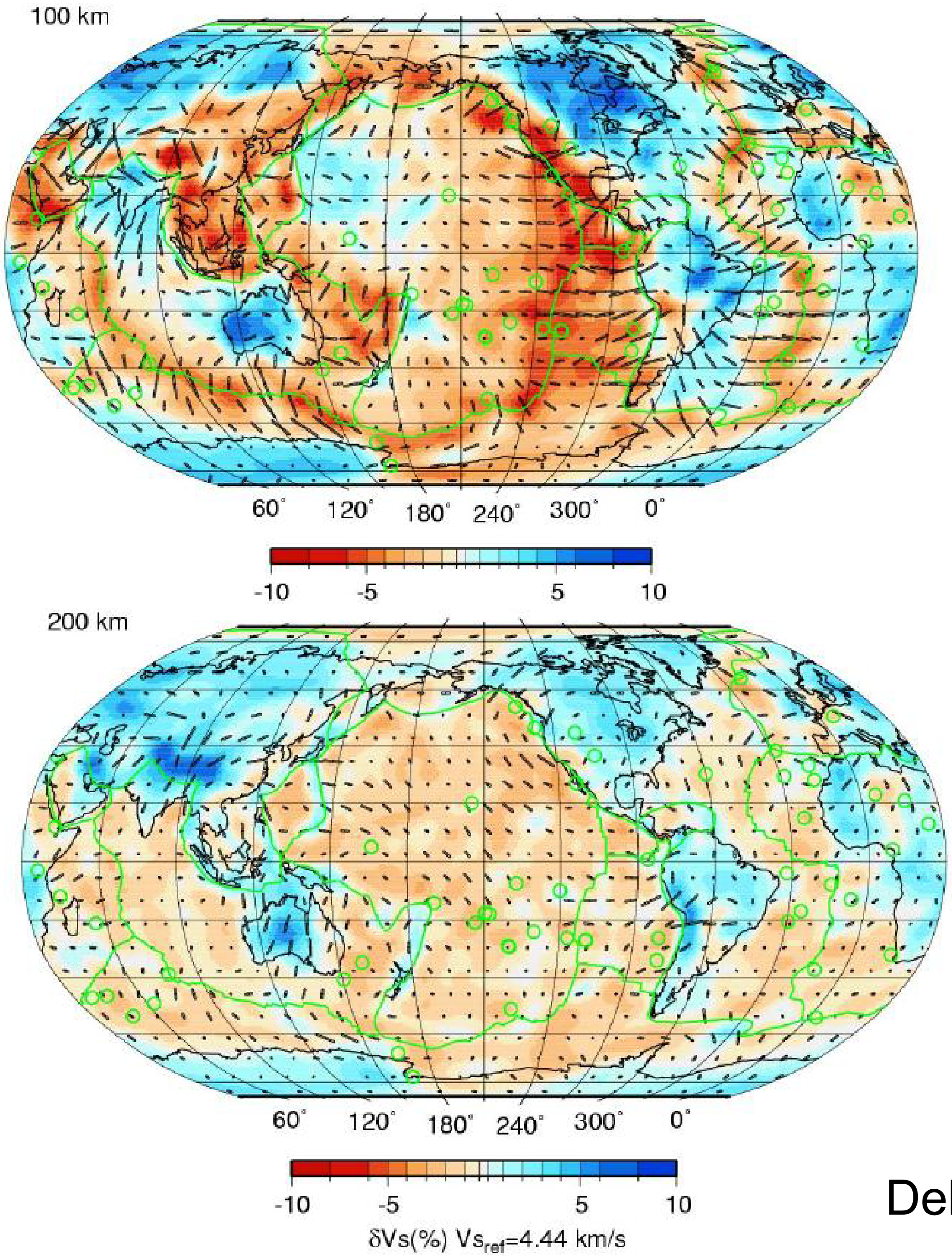
Scale $\Lambda \approx$ 2000km (degree 20)

Seismic wavelength $\lambda \leq$ 500km

⇨ Ray theory applies

Shear wave velocities - depth = 100km





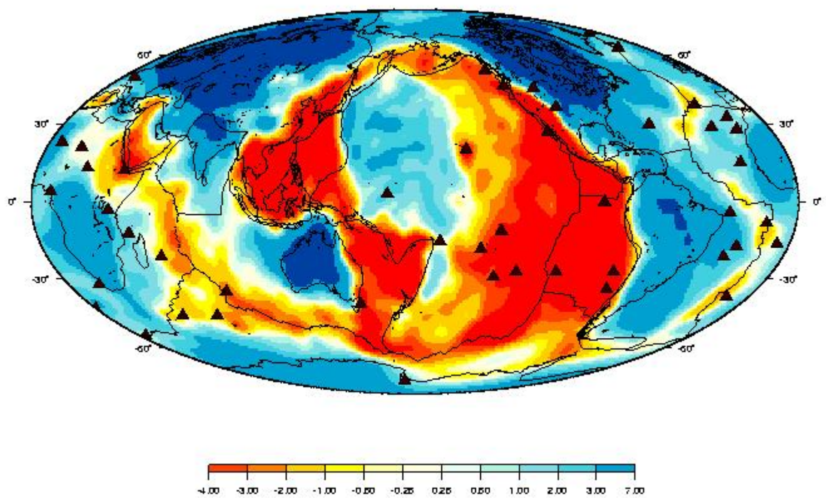
Debayle et al., 2005

CAN WE NEGLECT SMALL-SCALE HETEROGENEITIES?

Scale Λ >1000km

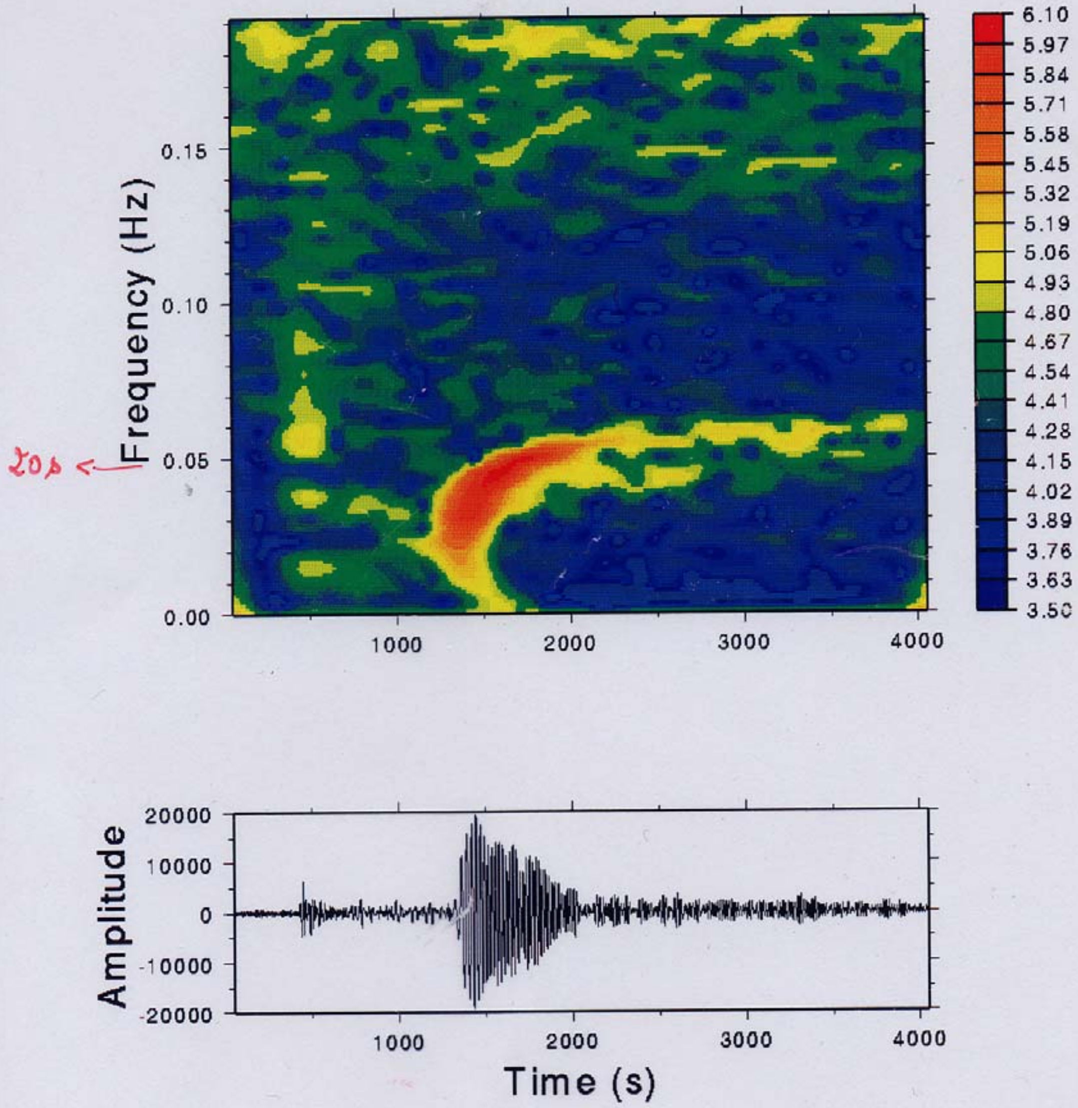
Seismic wavelength 20km ≤ λ ≤ 500km

S-wave velocity -
depth = 100km



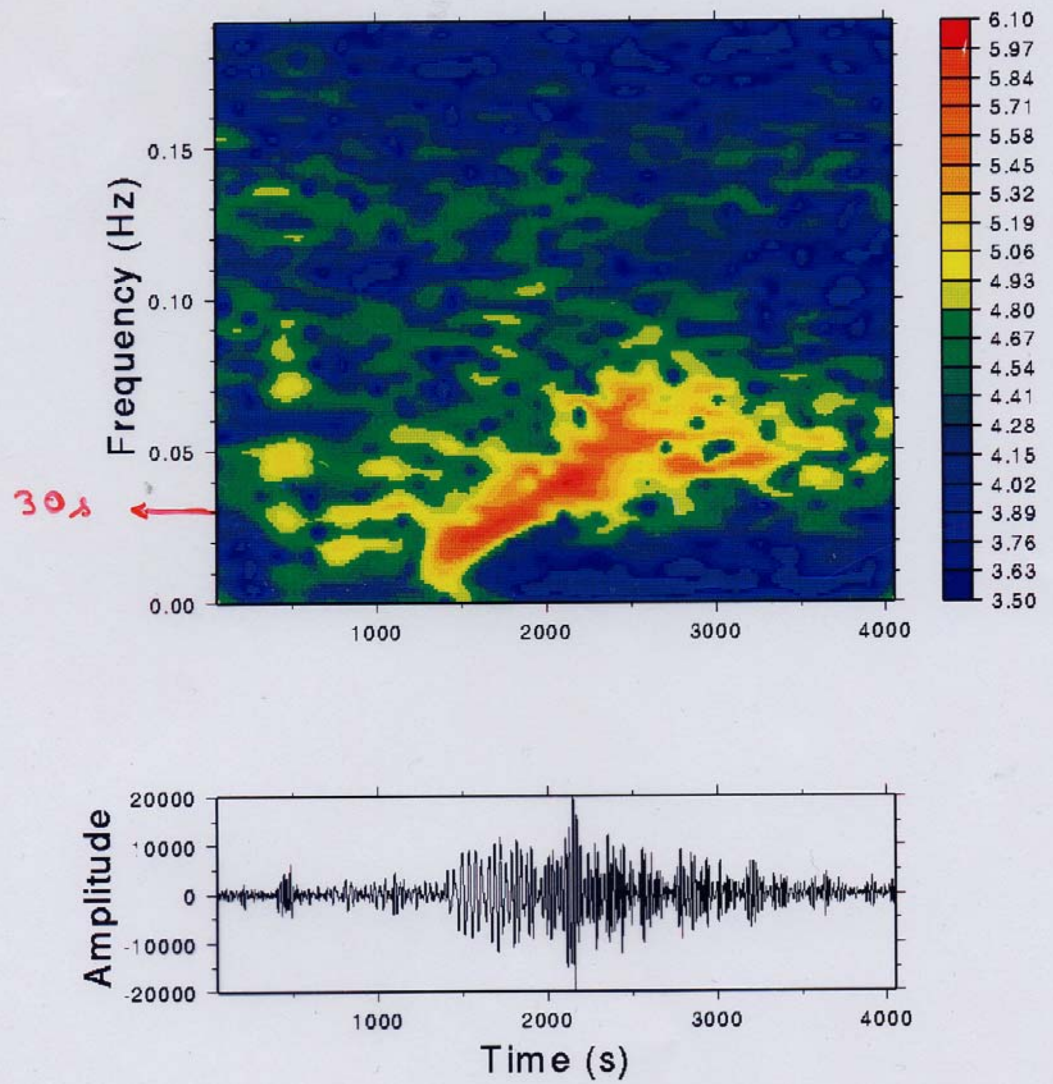
Spectrogram at station PPT (Z component, distance=82 degrees)

Aleutians 1992/03/02 depth=39km ms=6.8



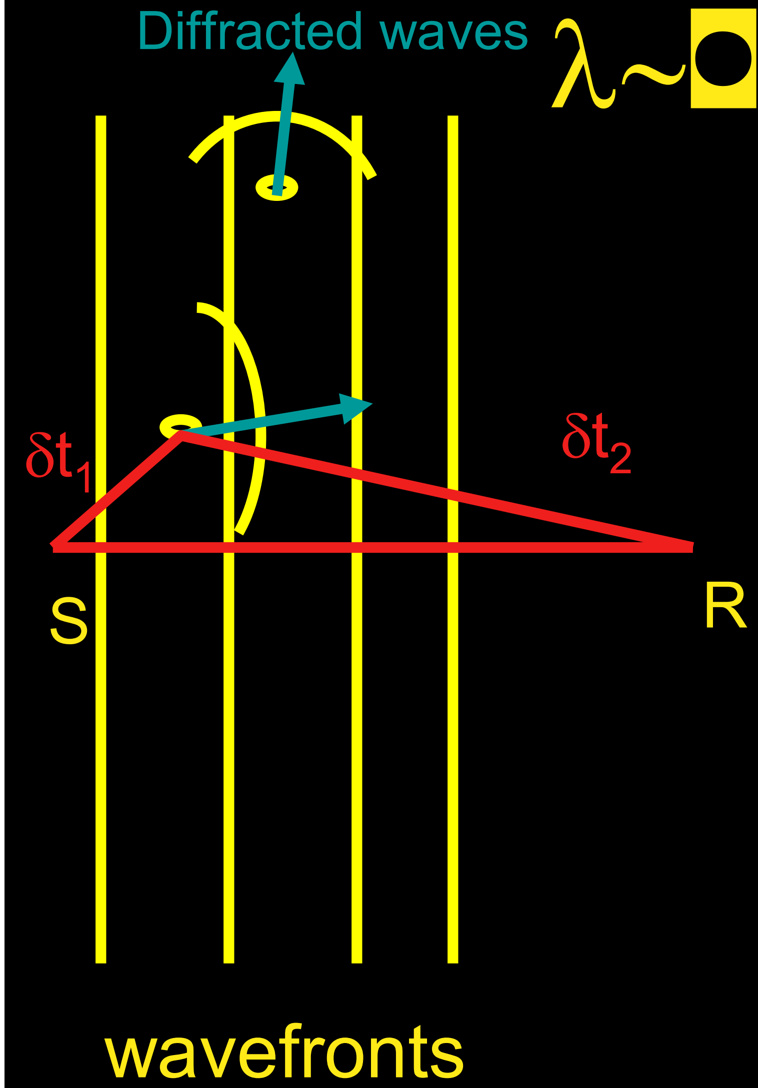
Spectrogram at station SSB (Z component, distance=80 degrees)

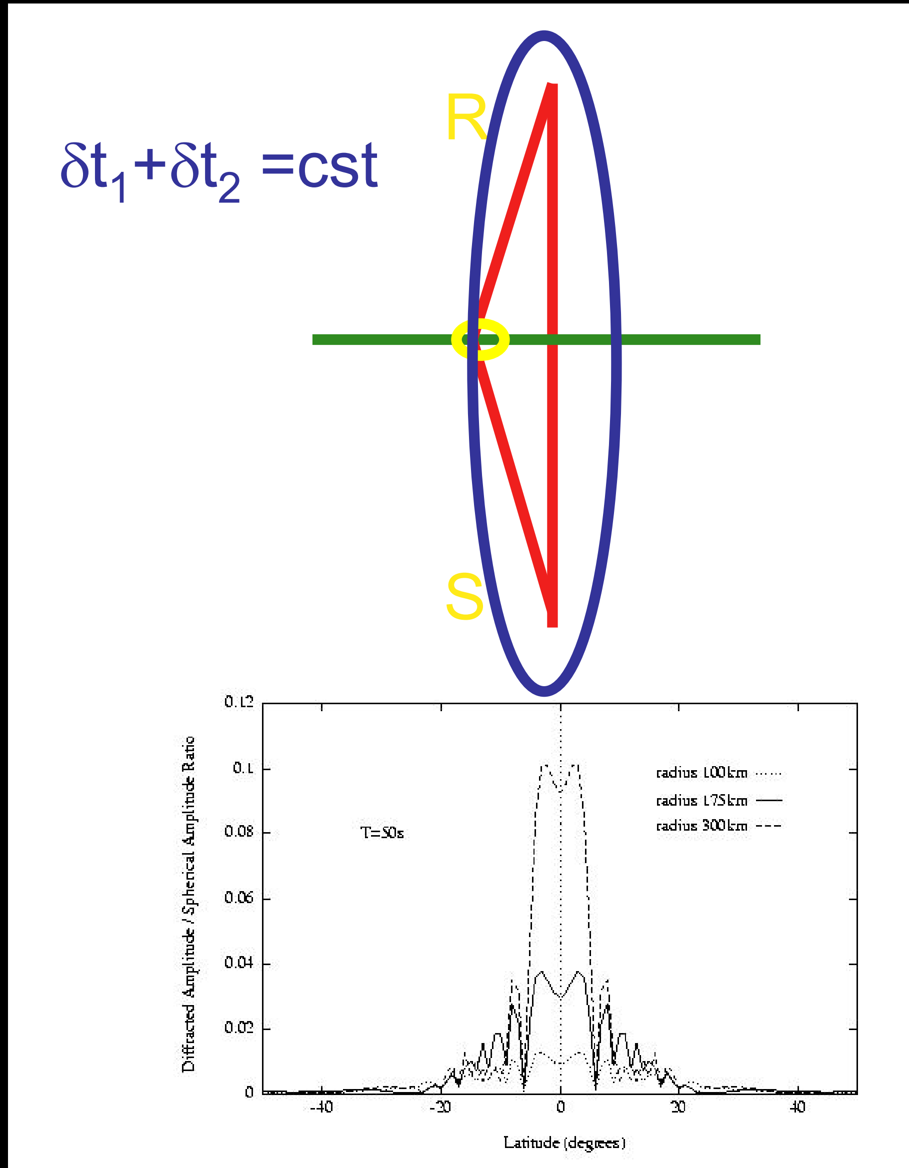
Aleutians 1992/03/02 depth=39km ms=6.8



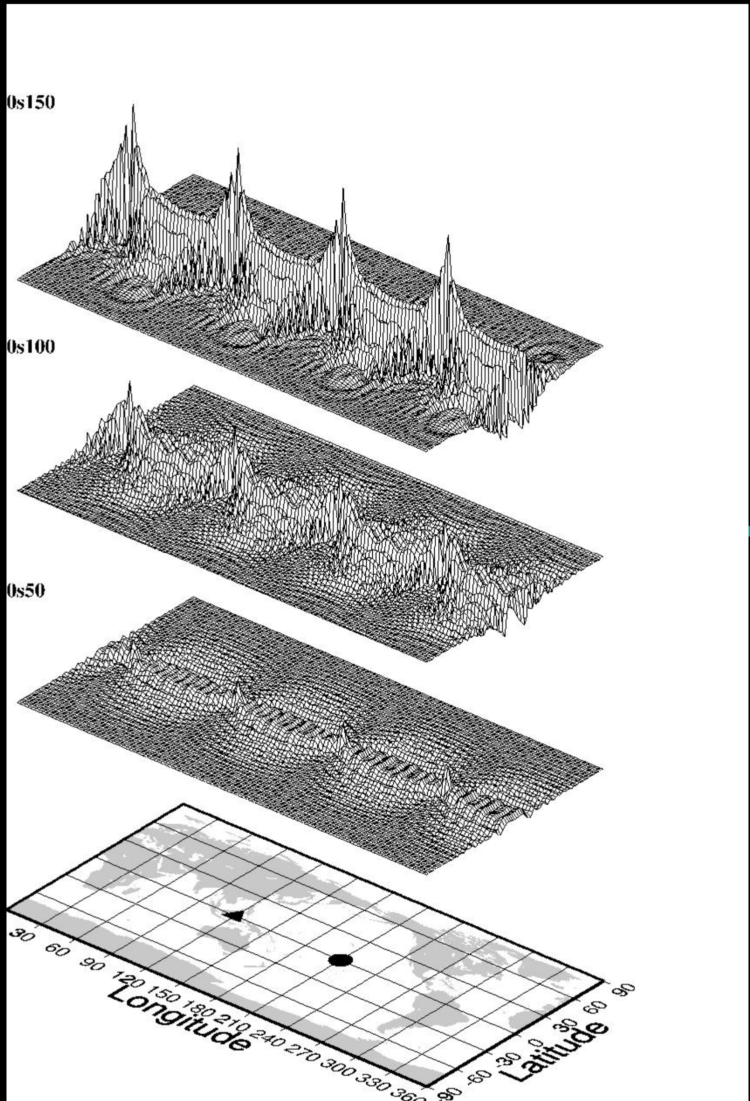
Λ heterogeneity scale, λ wavelength

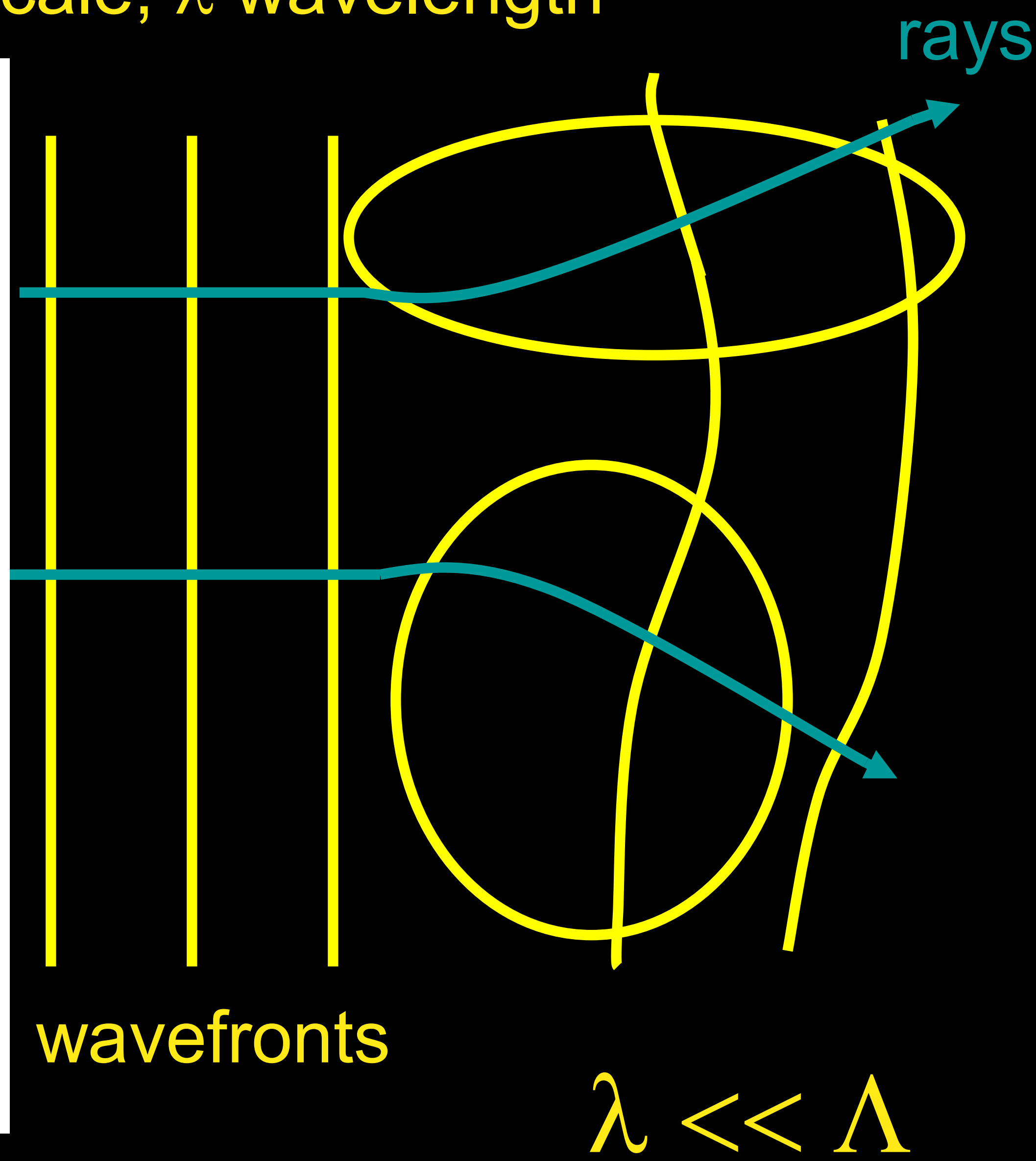
$\lambda \sim$ [illegible] Λ or $\lambda >> \Lambda$





Λ heterogeneity scale, λ wavelength





Typical scales Λ≈2,000km, λ≈ 500km

Universiteit Utrecht

SEISMOLOGICAL LABORATORY California Institute of Technology

Finite frequency sensitivity of surface waves to anisotropy using adjoint methods

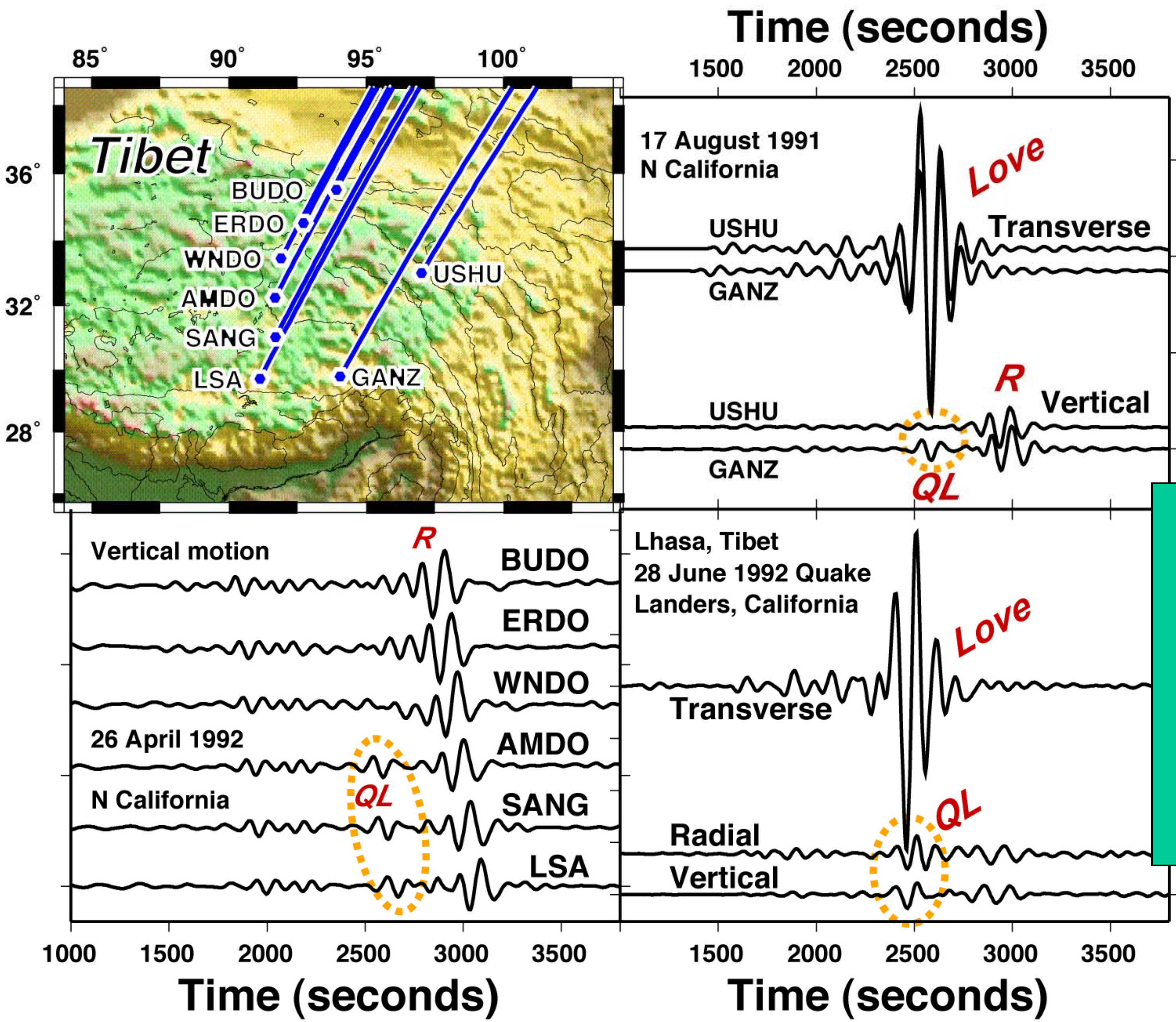
Anne Sieminski[1], Qinya Liu[2], Jeannot Trampert[1] and Jeroen Tromp[2]

(1) Utrecht University

(2) Caltech Seismological Lab

Effects on amplitude?

- Surface wave tomography of mantle anisotropy is based on ray theory
- Finite frequency tomography has been tested to improve the imaging of the isotropic mantle
- How do finite frequency surface waves "sense" anisotropy ?
- Computational tool : Adjoint Spectral Element Method (ASEM) (Tromp et al., 2005)

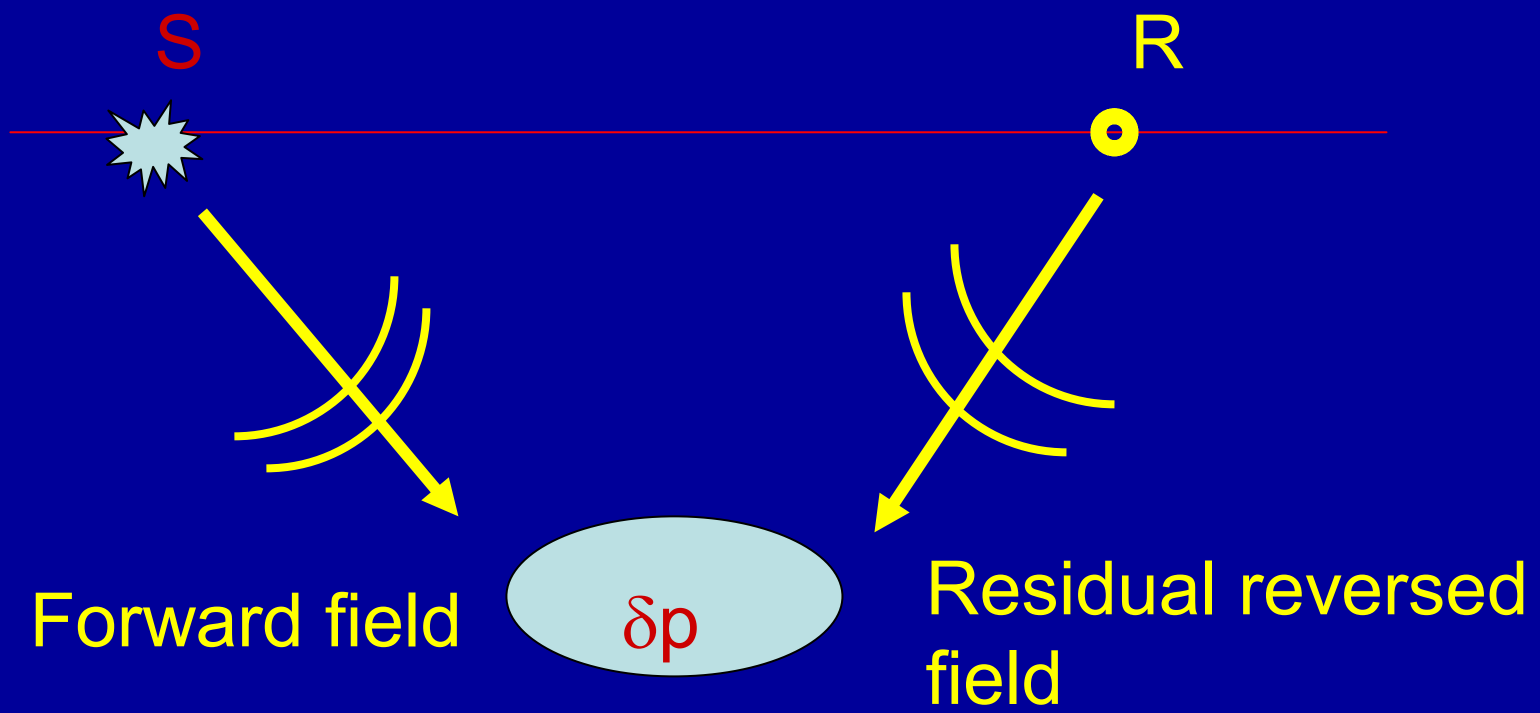


An Example of Using Quasi-Love waves to infer anisotropic gradients (Park, 2006)

Time Reversal- Adjoint Method

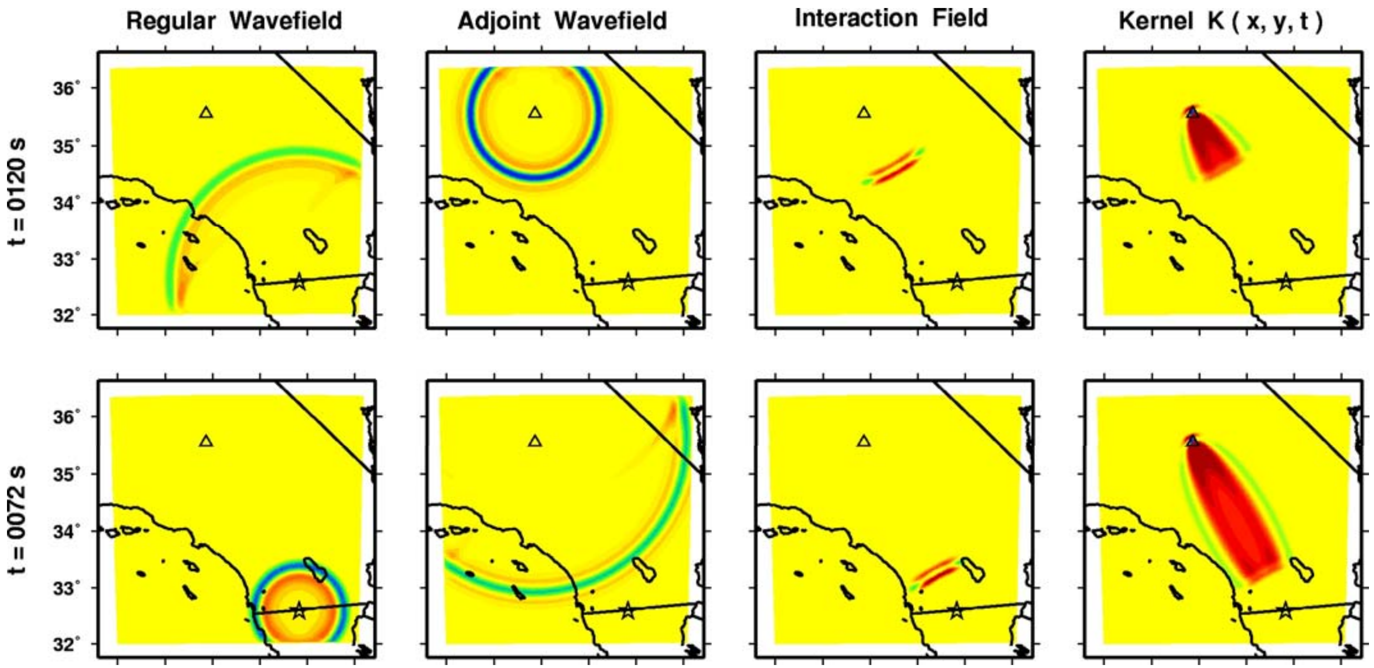
(Tarantola, 1984, Tromp et al., 2005):

Adjoint => Fréchet Derivatives



Sensitivity computation (1)

- Sensitivity kernel : $\delta T = \iiint K_{\delta m}^{\delta T}(x)\delta m(x)dx^3$
- Construction :

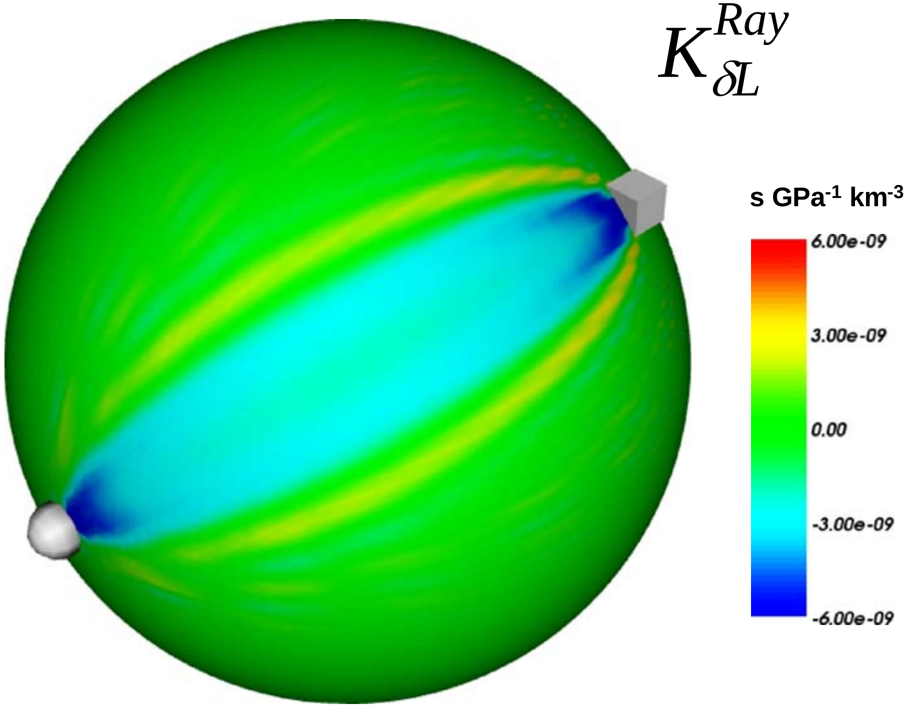


Tape et al. (2006)

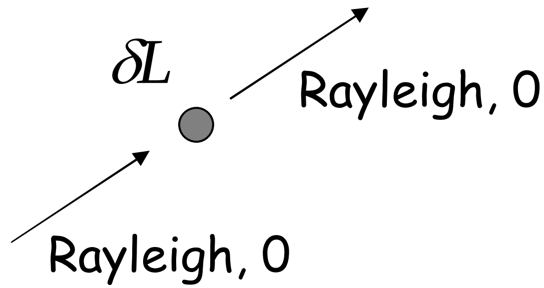
Sensitivity kernels (2)

- Off the ray path :

$$K_{\delta L}^{Ray}$$



Interpretation with
Born scattering theory

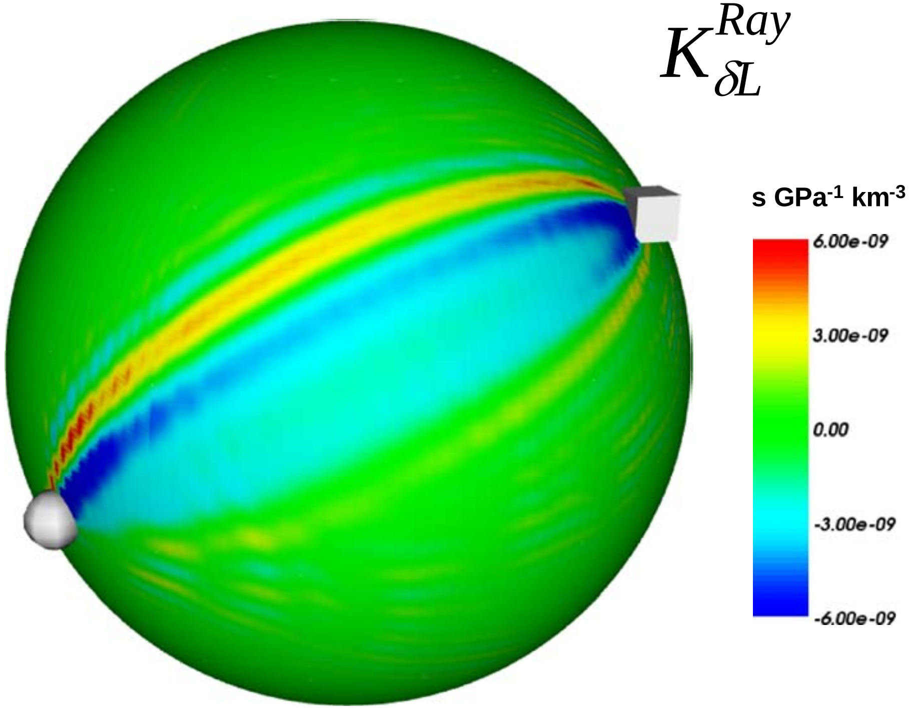


at 150 km depth (mode 0 ; 100s < T < 180s ; Δ = 120°)

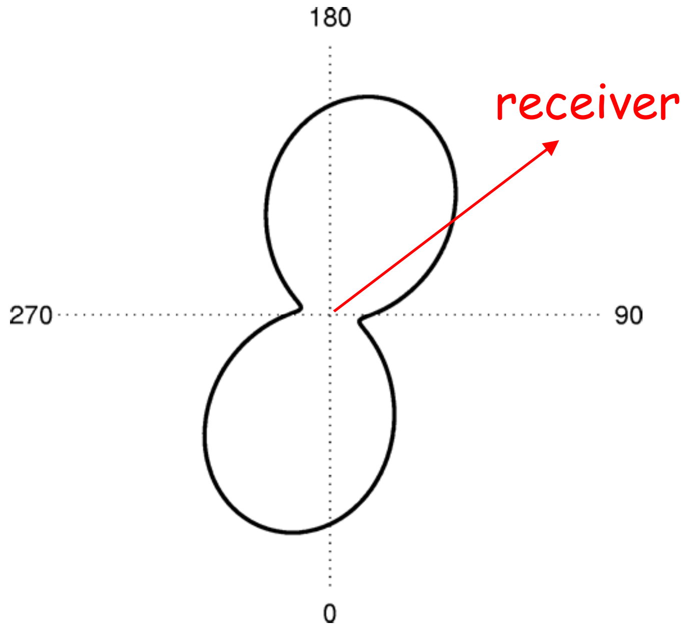
Sensitivity kernels (3)

- Off the ray path :

$K_{\delta L}^{Ray}$



Source radiation

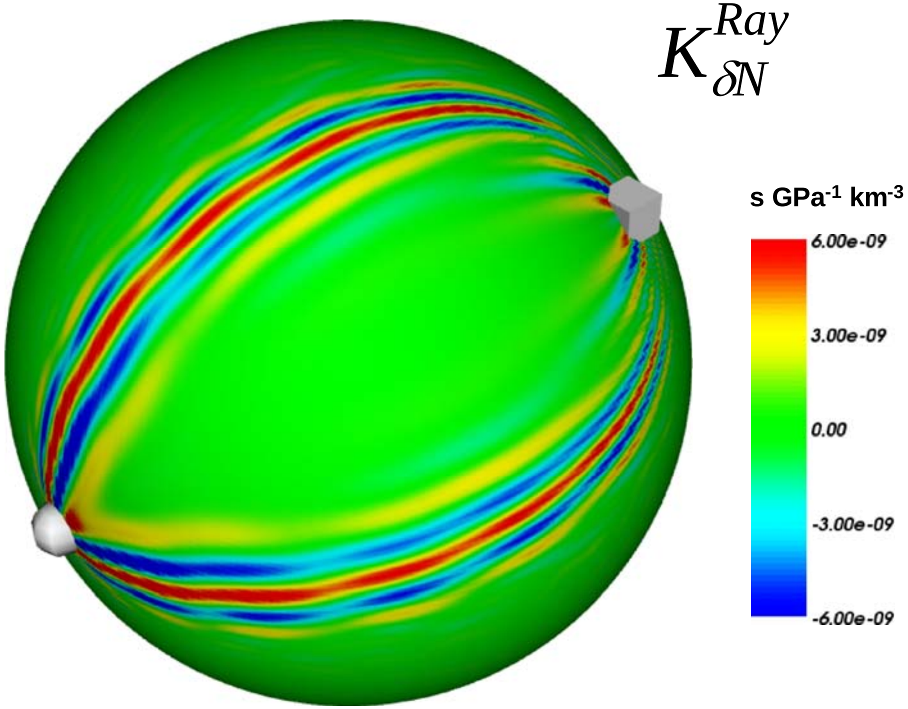


at 150 km depth (mode 0 ; 100s < T < 180s ; Δ = 120°)

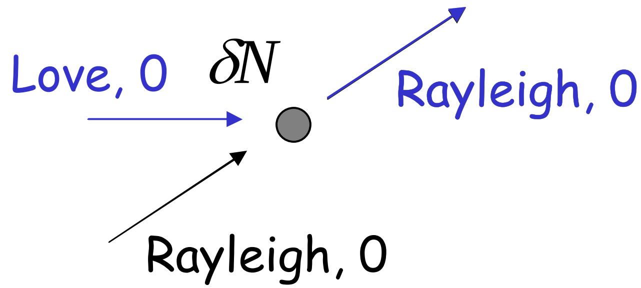
Sensitivity kernels (4)

- Love-Rayleigh coupling effect :

$K_{\delta N}^{Ray}$



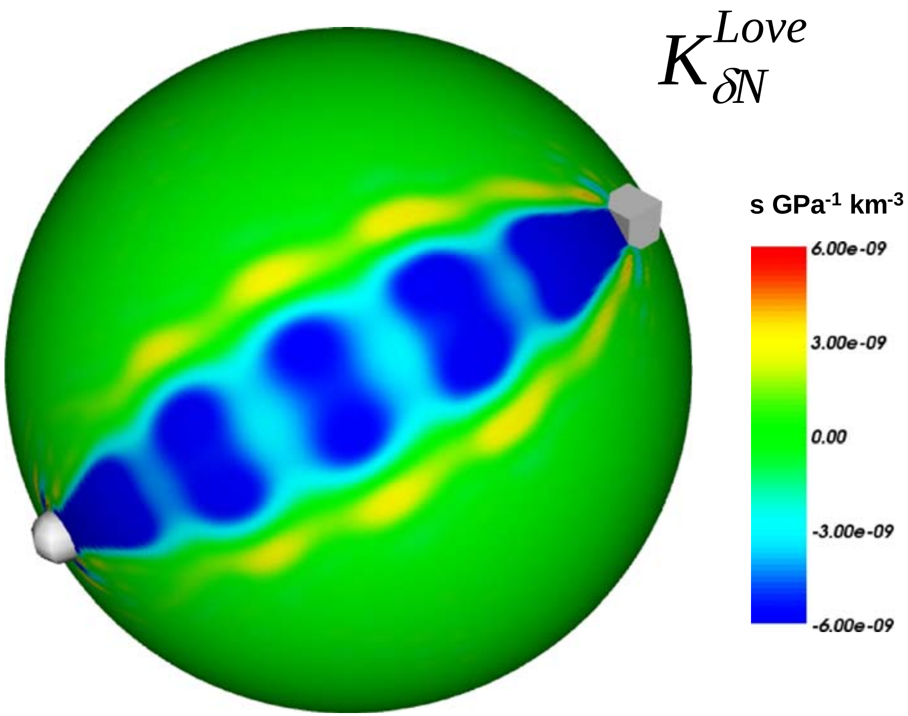
Interpretation with Born scattering theory



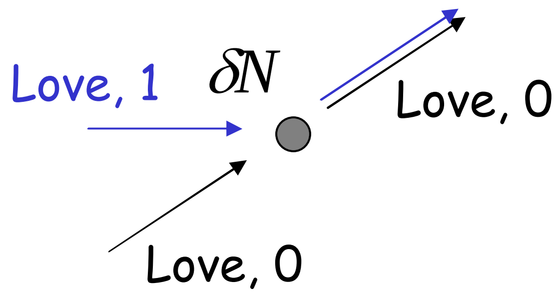
at the surface (mode 0 ; 100s < T < 180s ; Δ = 120°)

Sensitivity kernels (5)

- Cross-branch coupling effect :



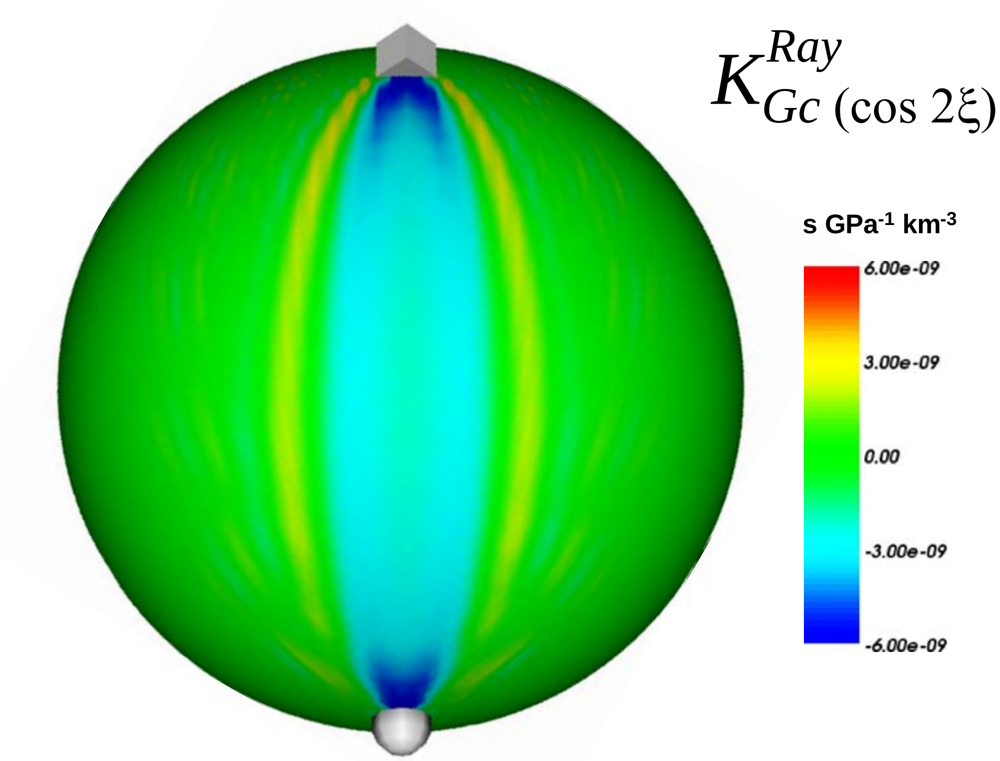
Interpretation with
Born scattering theory



at the surface (mode 0 ; 100s < T < 180s ; Δ = 120°)

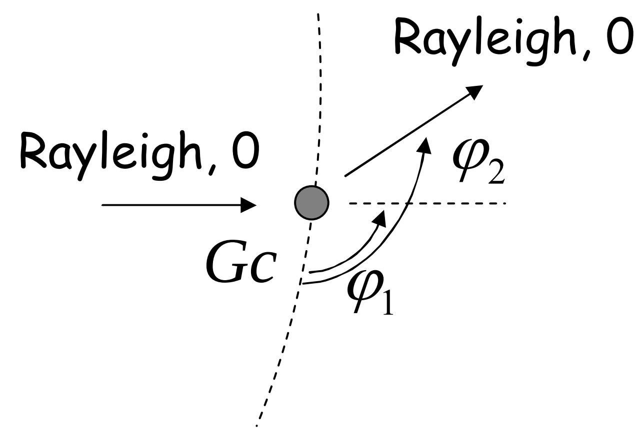
Sensitivity kernels (6)

- Path-dependency (azimuthal anisotropy) :



In Born scattering theory

$$K_{Gc}^{Ray} \sim \cos(\varphi_1 + \varphi_2)$$

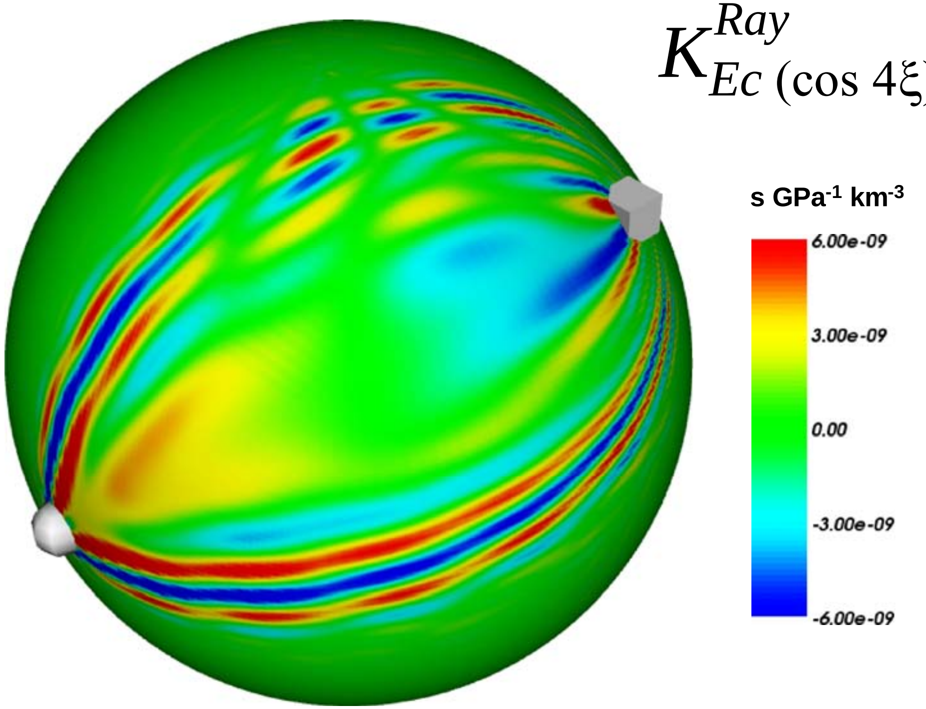


at the surface (mode 0 ; 100s < T < 180s ; Δ = 120°)

Sensitivity kernels (7)

- Love-Rayleigh coupling and path-dependency :

$$K_{Ec}^{Ray}\,(\cos 4\xi)$$



at the surface (mode 0 ; 100s < T < 180s ; Δ = 120°)

Conclusion

- Surface wave amplitude sensitivity to anisotropy is controlled by :

    - significant mode coupling effects

    - strong path-dependency (azimuthal anisotropy)

    - source radiation

$\Rightarrow$ Most of the sensitivity kernels do not show a simple elliptical pattern

$\Rightarrow$ Consequences for tomography ?