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"8th Workshop on Three-Dimensional Modelling of Seismic Waves Generation, Propagation and their Inversion"

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Spectral-Element Method and Three-Dimensional Seismology

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The Spectral-Element Method (SEM) and three-dimensional seismology

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ICTP, Trieste, September 28, 2006

Note: the SPECFEM3D source code is freely available
for academic non-commercial research at
http://www.gps.caltech.edu/~jtromp/research/downloads/register.html

Part I
Theory and benchmarks

Brief history of numerical methods

Seismic wave equation : tremendous increase of computational power
$\Rightarrow$ development of numerical methods for accurate calculation of synthetic seismograms in complex 3D geological models has been a continuous effort in last 30 years.

Finite-difference methods : Yee 1966, Chorin 1968, Alterman and Karal 1968, Madariaga 1976, Virieux 1986, Moczo et al, Olsen et al..., difficult for boundary conditions, surface waves, topography, full Earth

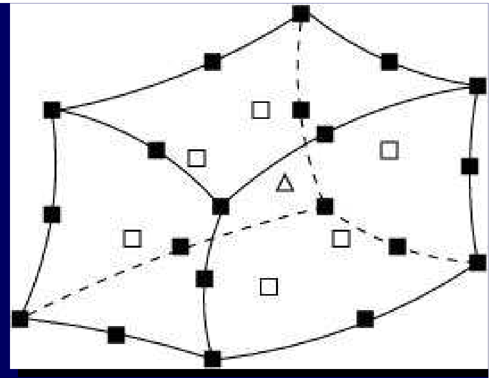
Boundary-element or boundary-integral methods (Kawase 1988, Sanchez-Sesma et al. 1991) : homogeneous layers, expensive in 3D

Spectral and pseudo-spectral methods (Carcione 1990) : smooth media, difficult for boundary conditions, difficult on parallel computers

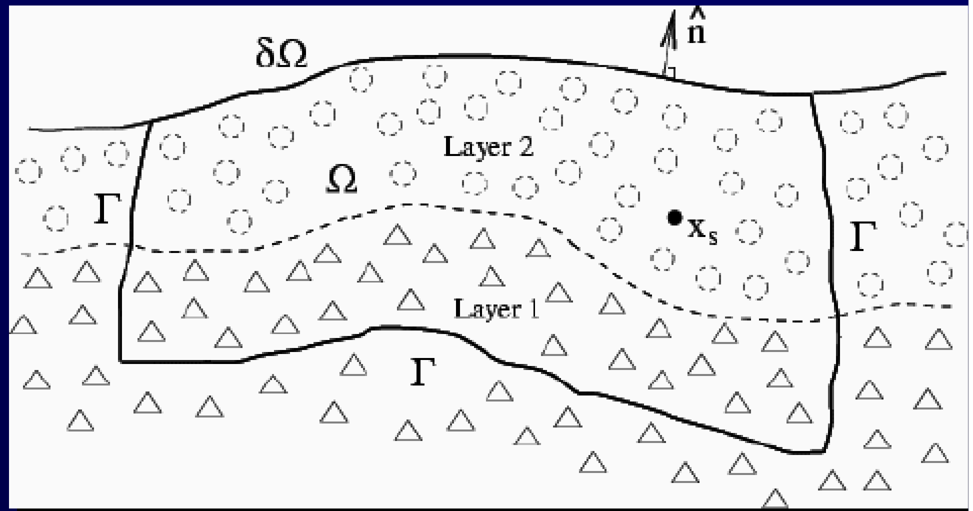
Classical finite-element methods (Lysmer and Drake 1972, Marfurt 1984, Bielak et al 1998) : linear systems, large amount of numerical dispersion

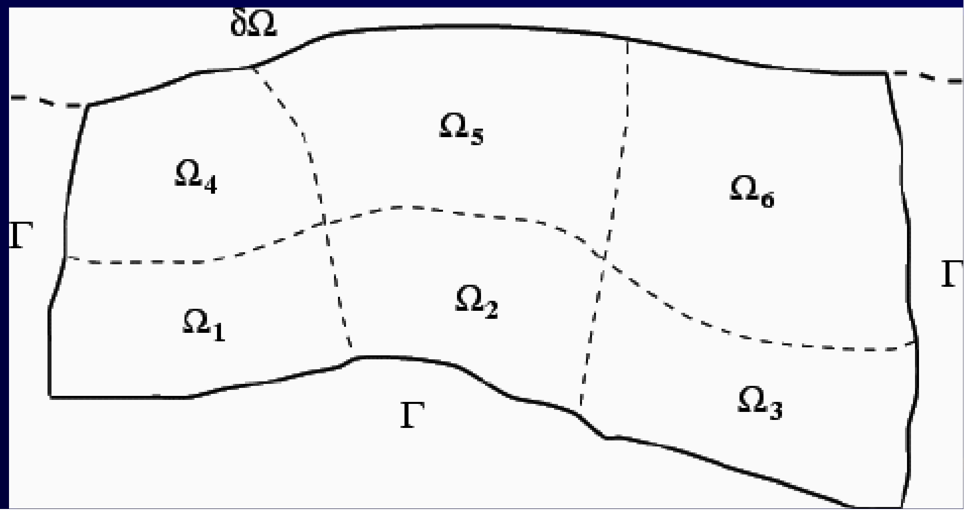
SEM technique for local or regional studies

Spectral-Element Method



- Developed in Computational Fluid Dynamics (Patera 1984)
- Accuracy of a pseudospectral method, flexibility of a finite-element method
- Extended by Komatitsch and Tromp, Chaljub et al., Capdeville et al.
- Large curved “spectral” finite-elements with high-degree polynomial interpolation
- Mesh honors the main discontinuities (velocity, density) and topography
- Very efficient on parallel computers, no linear system to invert (diagonal mass matrix)





Equations of Motion (solid)

Differential or *strong* form (e.g., finite differences):

$$\rho \partial_t^2 \mathbf{s} = \nabla \cdot \mathbf{T} + \mathbf{f}$$

We solve the integral or *weak* form:

$$\int \rho \mathbf{w} \cdot \partial_t^2 \mathbf{s} \, \mathrm{d}^3\mathbf{r} = -\int \nabla \mathbf{w} : \mathbf{T} \, \mathrm{d}^3\mathbf{r}$$

$$+ \mathbf{M} : \nabla \mathbf{w}(\mathbf{r}_s) S(t)$$

+ attenuation (memory variables) and ocean load

Equations of Motion (Fluid)

Differential or *strong* form:

$$\rho \partial_t \mathbf{v} = -\nabla p \qquad \partial_t p = -\kappa \nabla \cdot \mathbf{v}$$

We use a generalized velocity potential χ the integral or *weak* form is:

$$p = \partial_t \chi$$

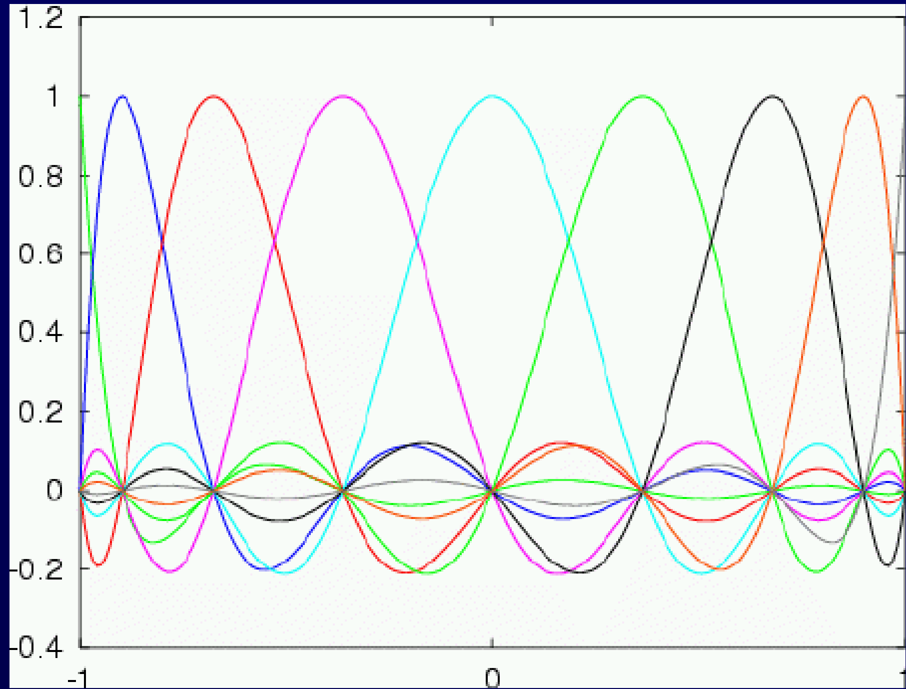
$$\int \kappa^{-1} w \partial_t^2 \chi \mathrm{d}^3\mathbf{r} = -\int \rho^{-1} \nabla w \cdot \nabla \chi \mathrm{d}^3\mathbf{r}$$

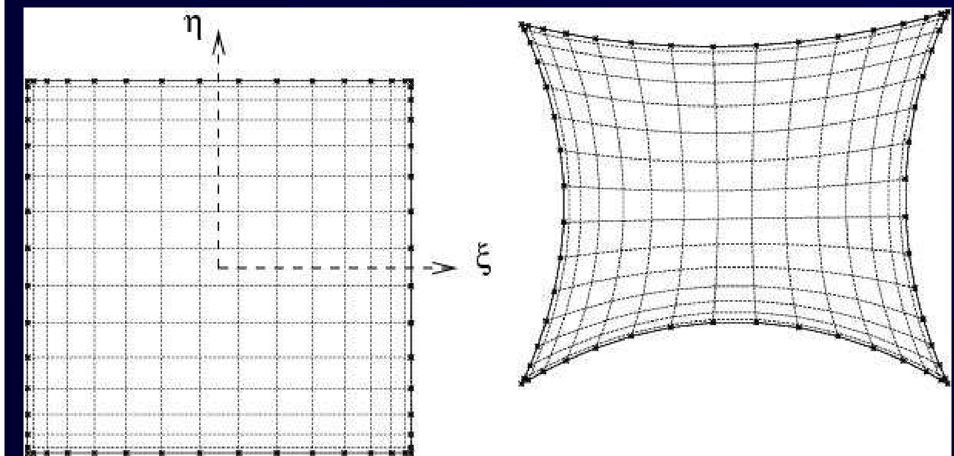
⇒ cheap (scalar potential)

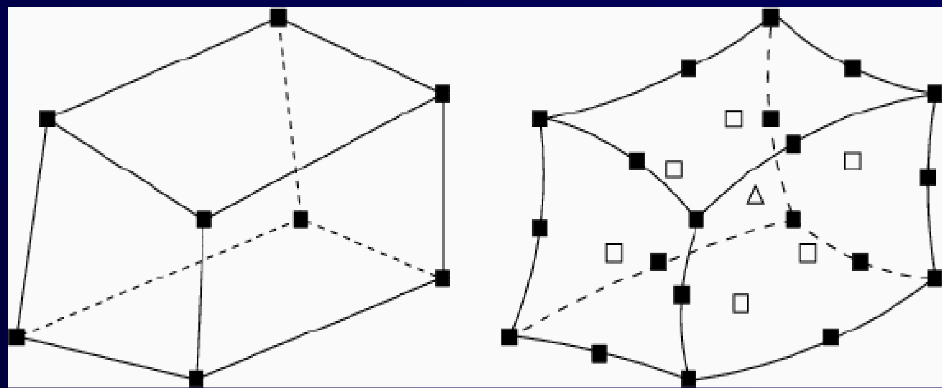
⇒ natural coupling with solid

Finite Elements

- High-degree pseudospectral finite elements
- N = 5 to 8 usually
- *Exactly* Diagonal mass matrix
- No linear system to invert

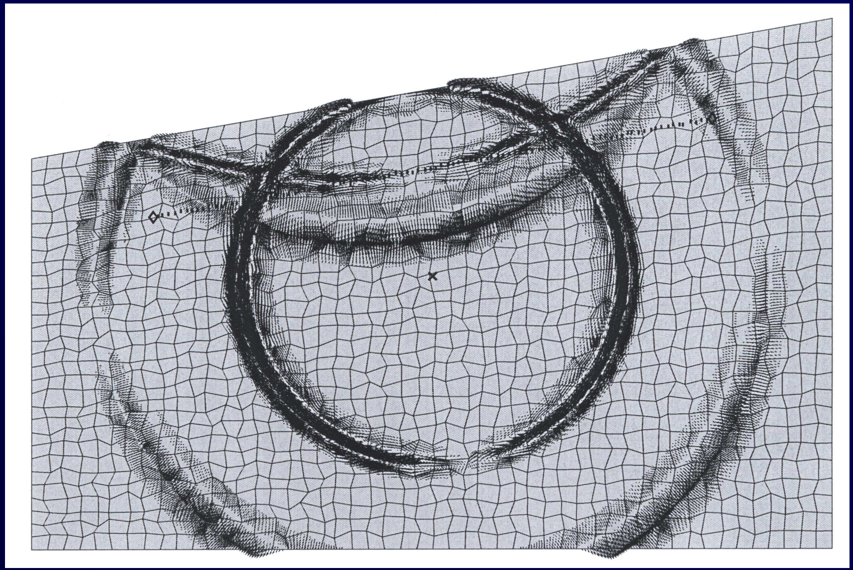


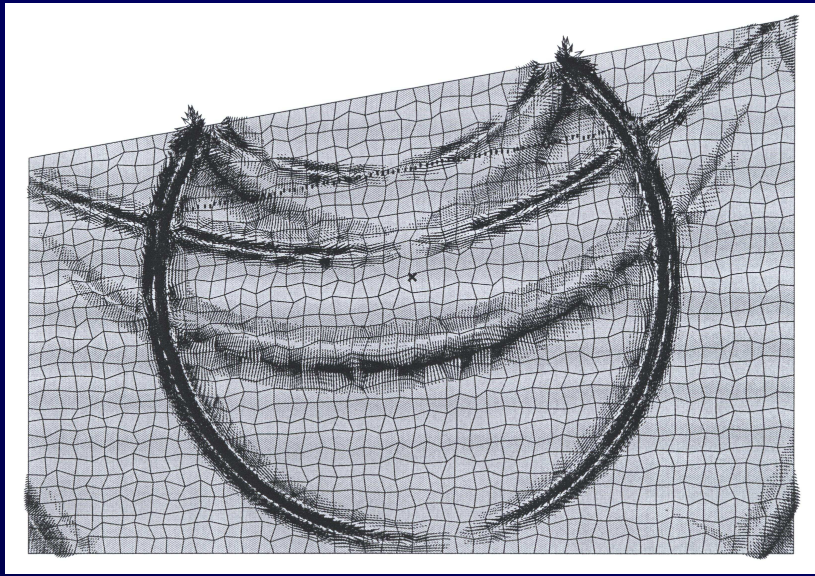


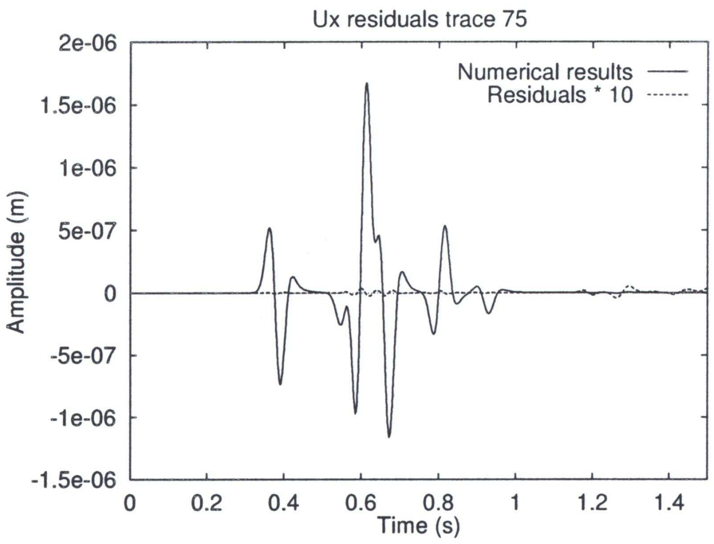


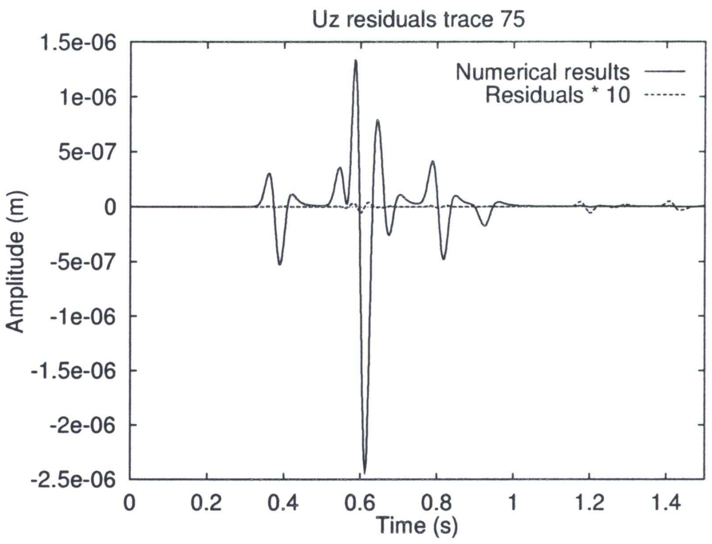
Benchmarks of the SEM at the regional scale

Distorted mesh for Lamb's problem



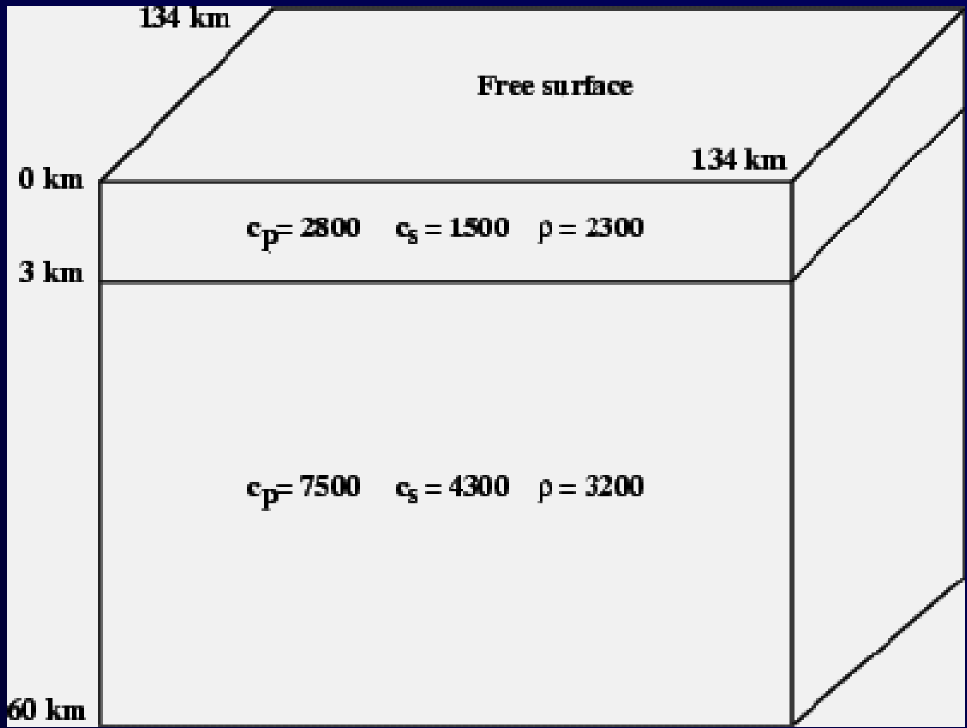






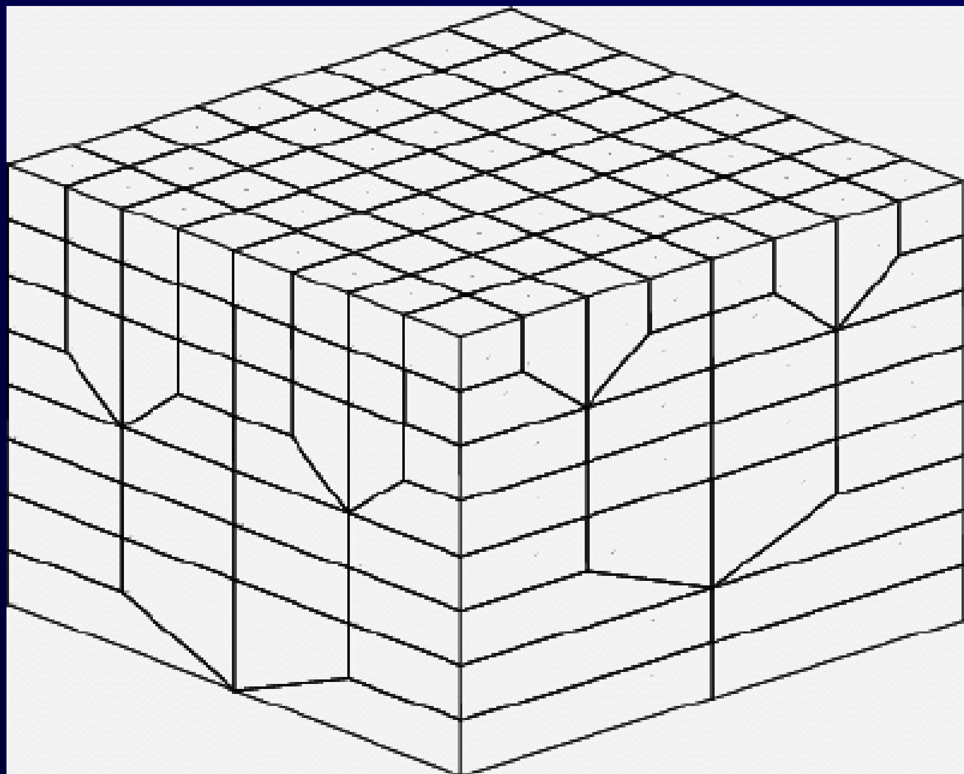
Validation on 3-D models

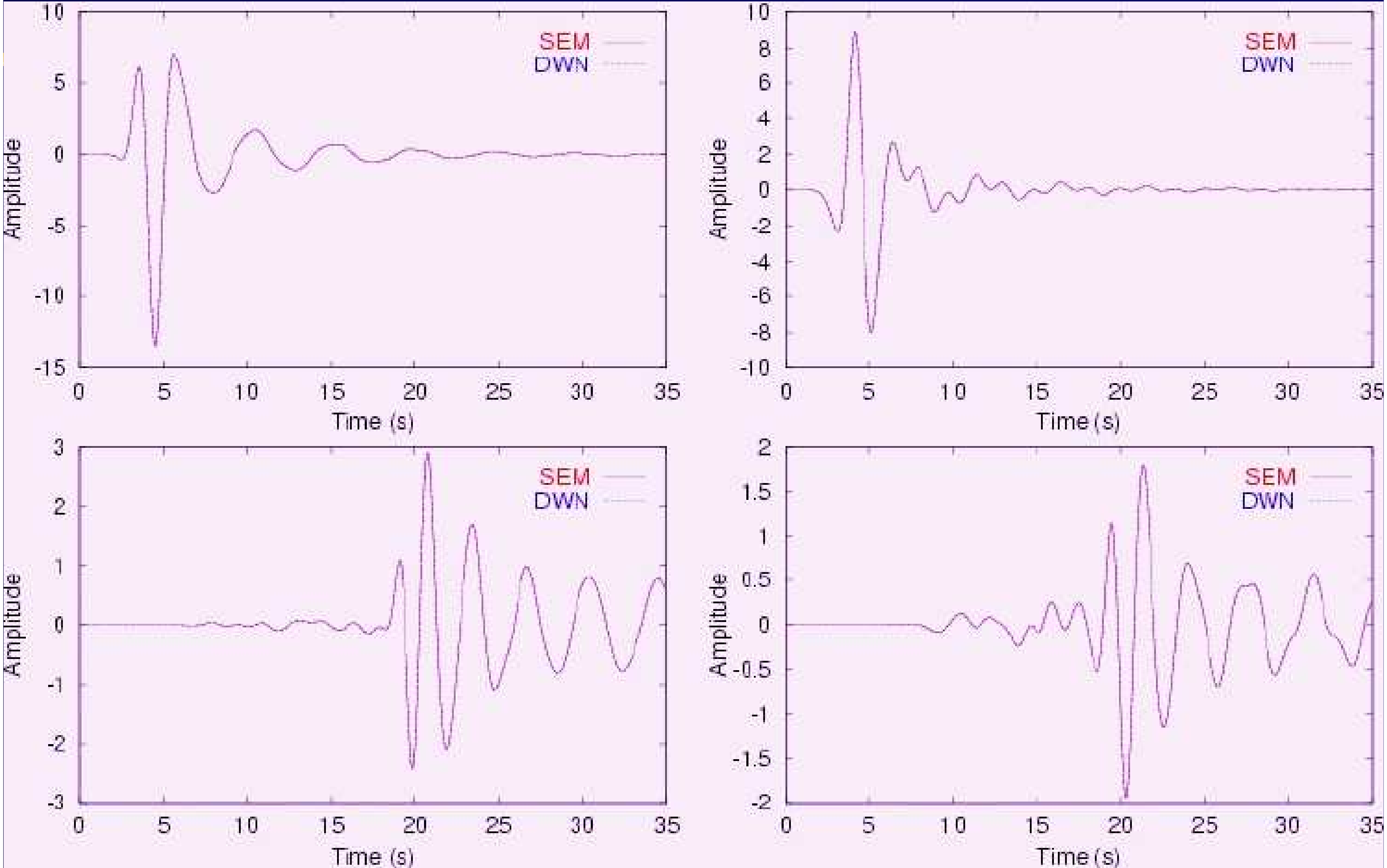
- Layer over a half-space: difficult to accurately model surface waves
- Very precise reference solution: DWNM



Mesh coarsening with depth

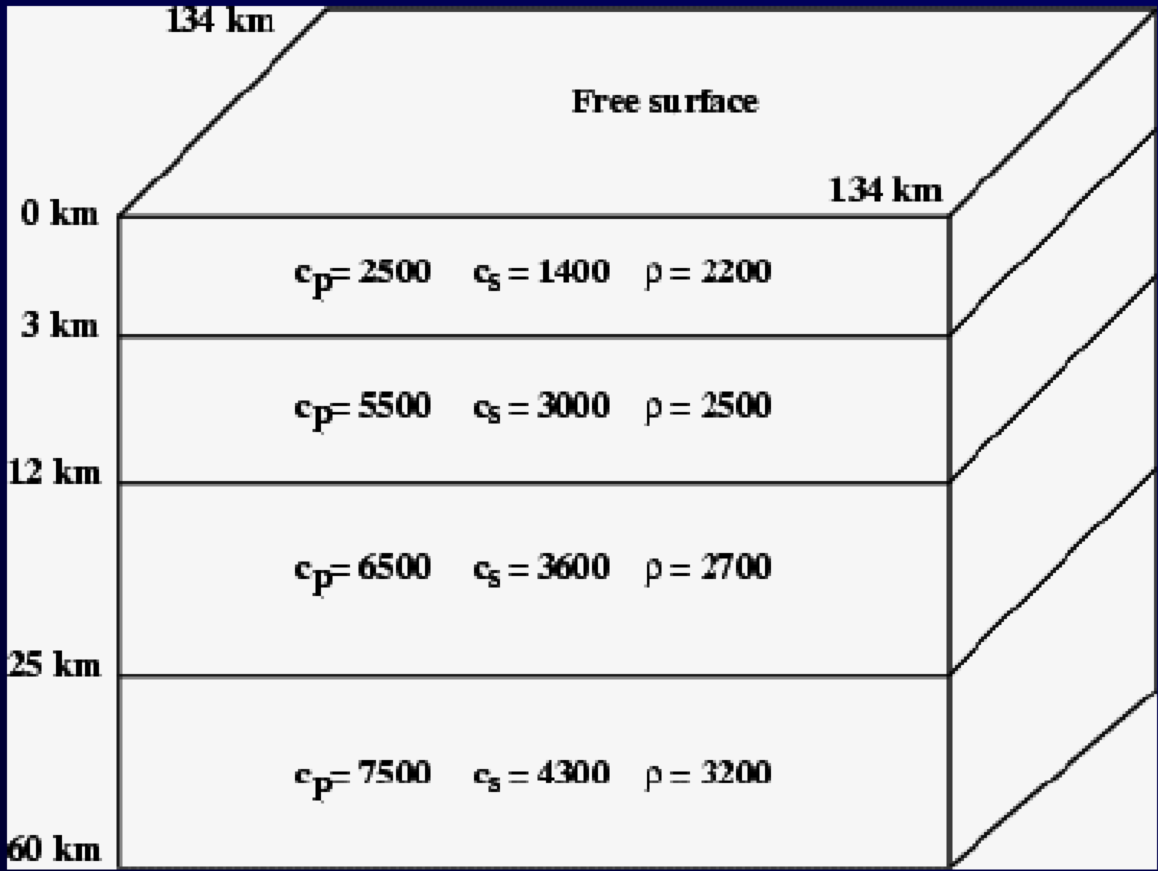
- Adapt the mesh to the velocity structure
- Save a lot of memory and CPU time

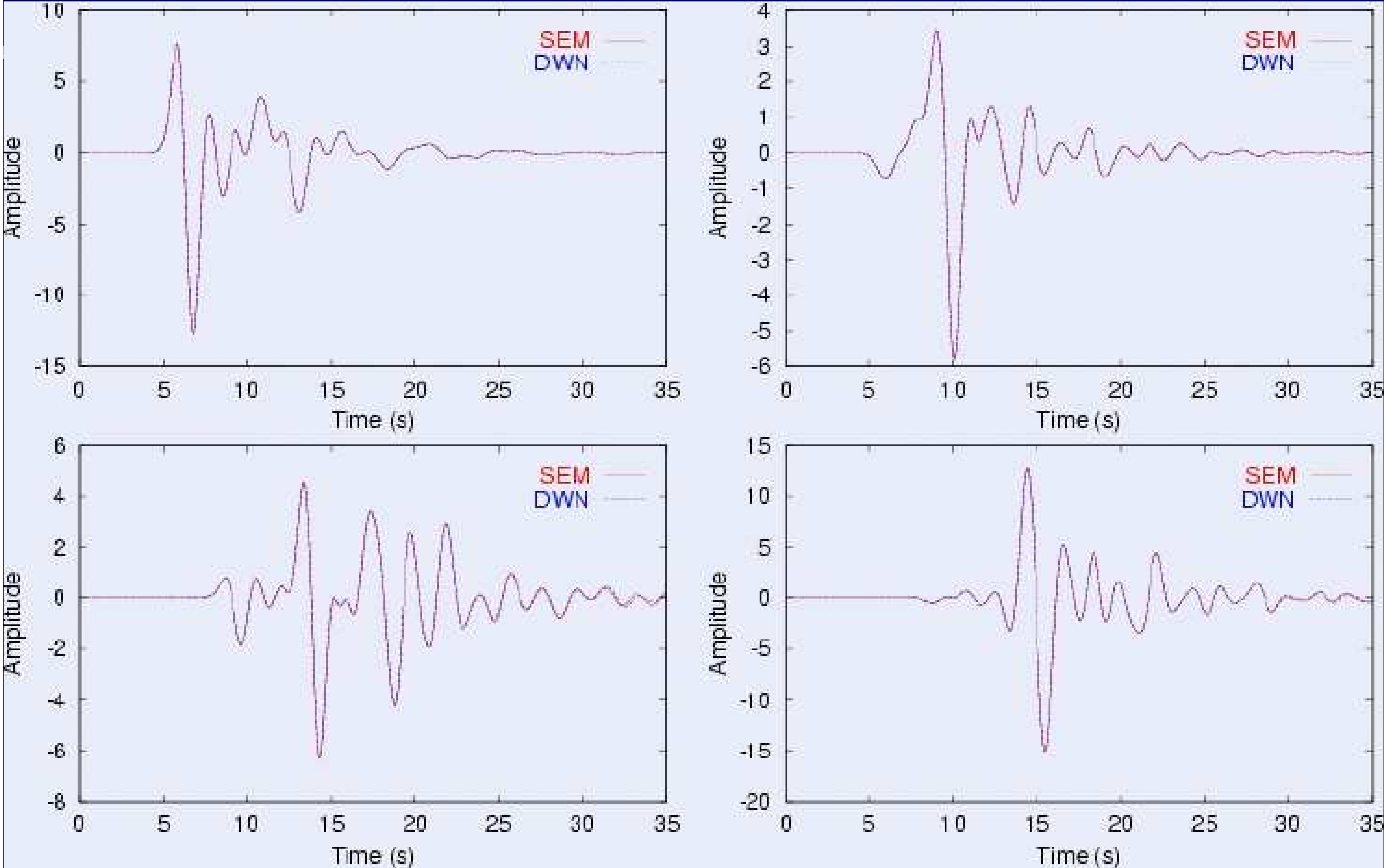



SEM
DWN
Amplitude
Time (s)

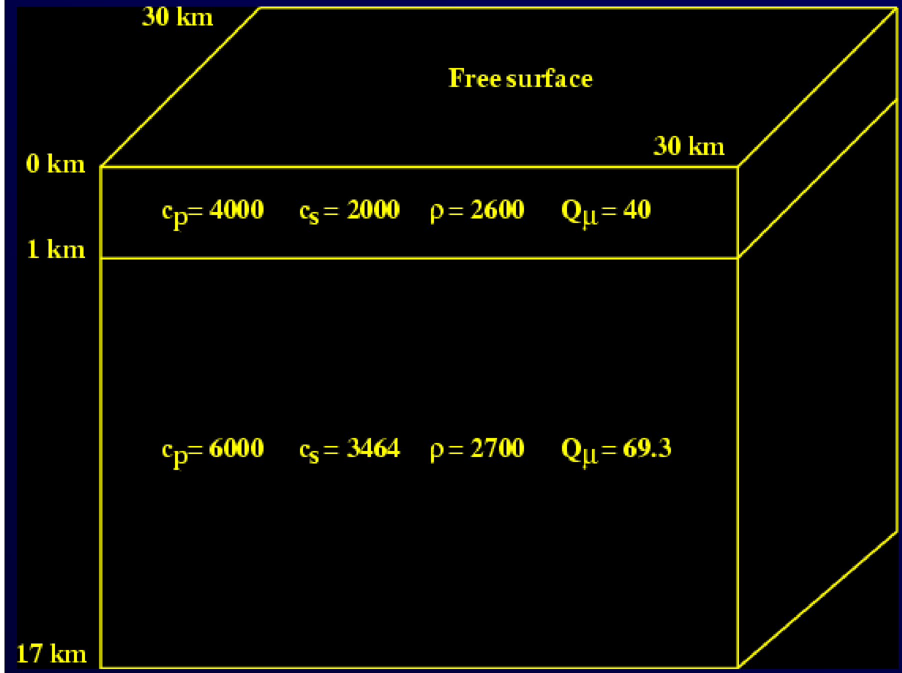
Layer-cake structure

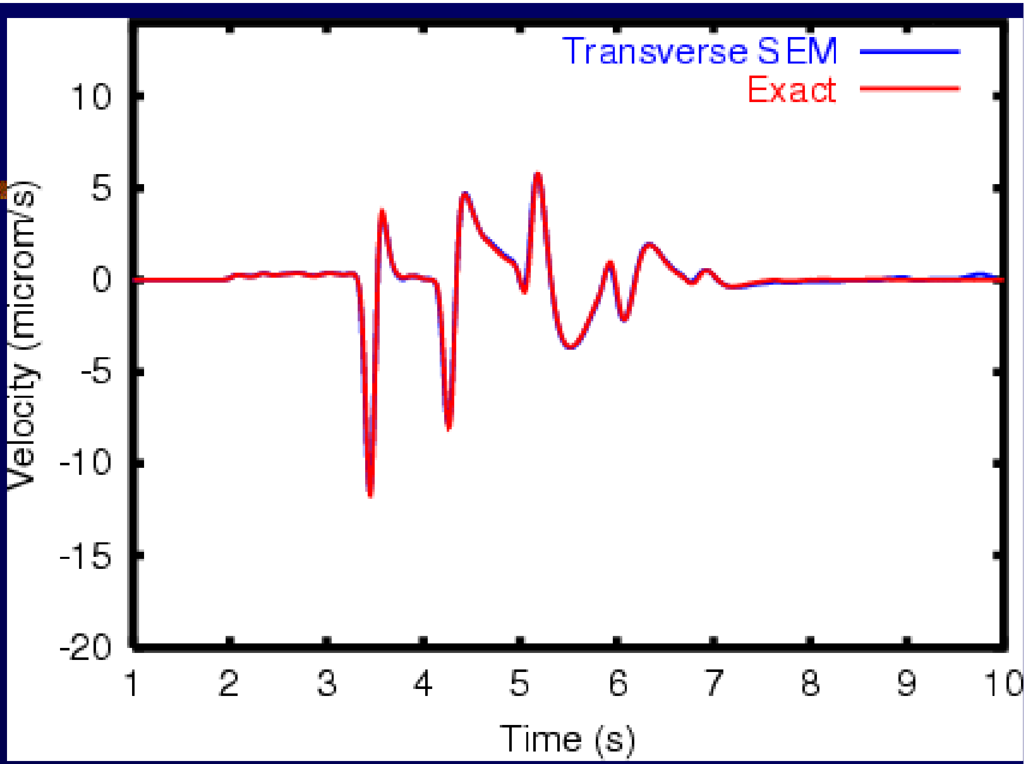
- Body waves and multiples
- Accurate absorbing conditions on edges

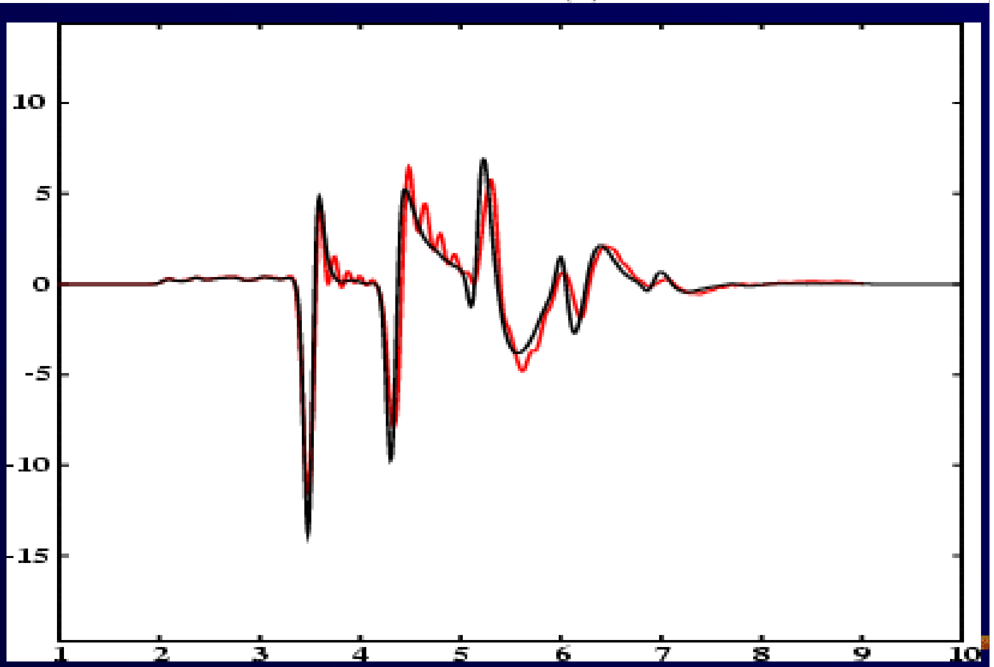



SEM
DWN
Amplitude
Time (s)

SCEC LOH.3



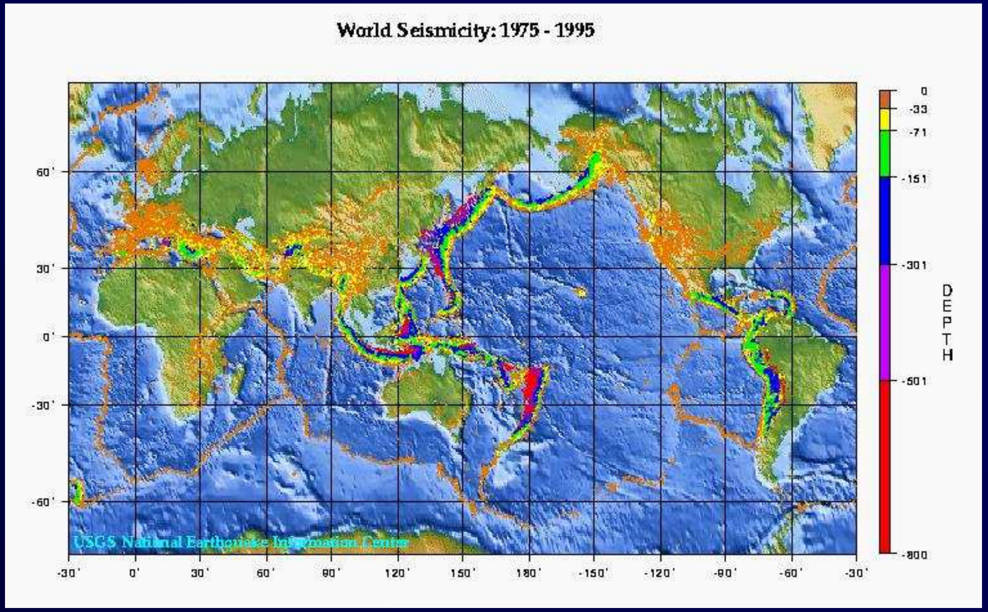


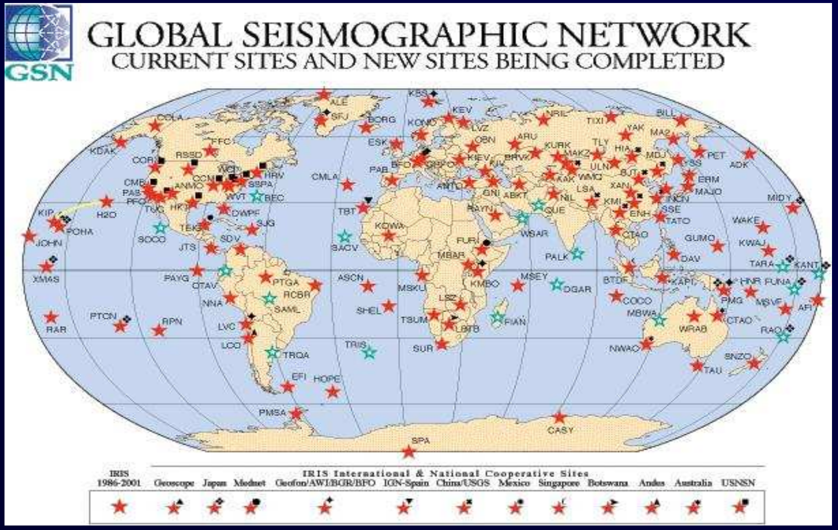


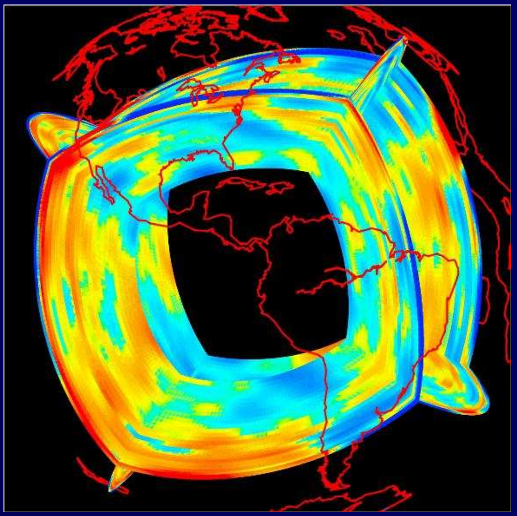
- Tester ondes de surface, et atténuation avec Q constant
- Comparer aux méthodes classiques de différences finies
- Solution de référence f-k de Apsel et Luco (1983)

SEM technique for the global Earth

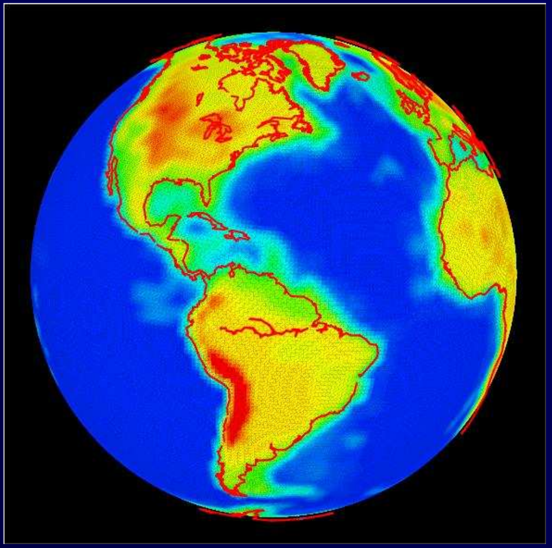
Global 3D Earth







Modèle de manteau S20RTS (Ritsema et al. 1999)



Crust 5.2 (Bassin et al. 2000)

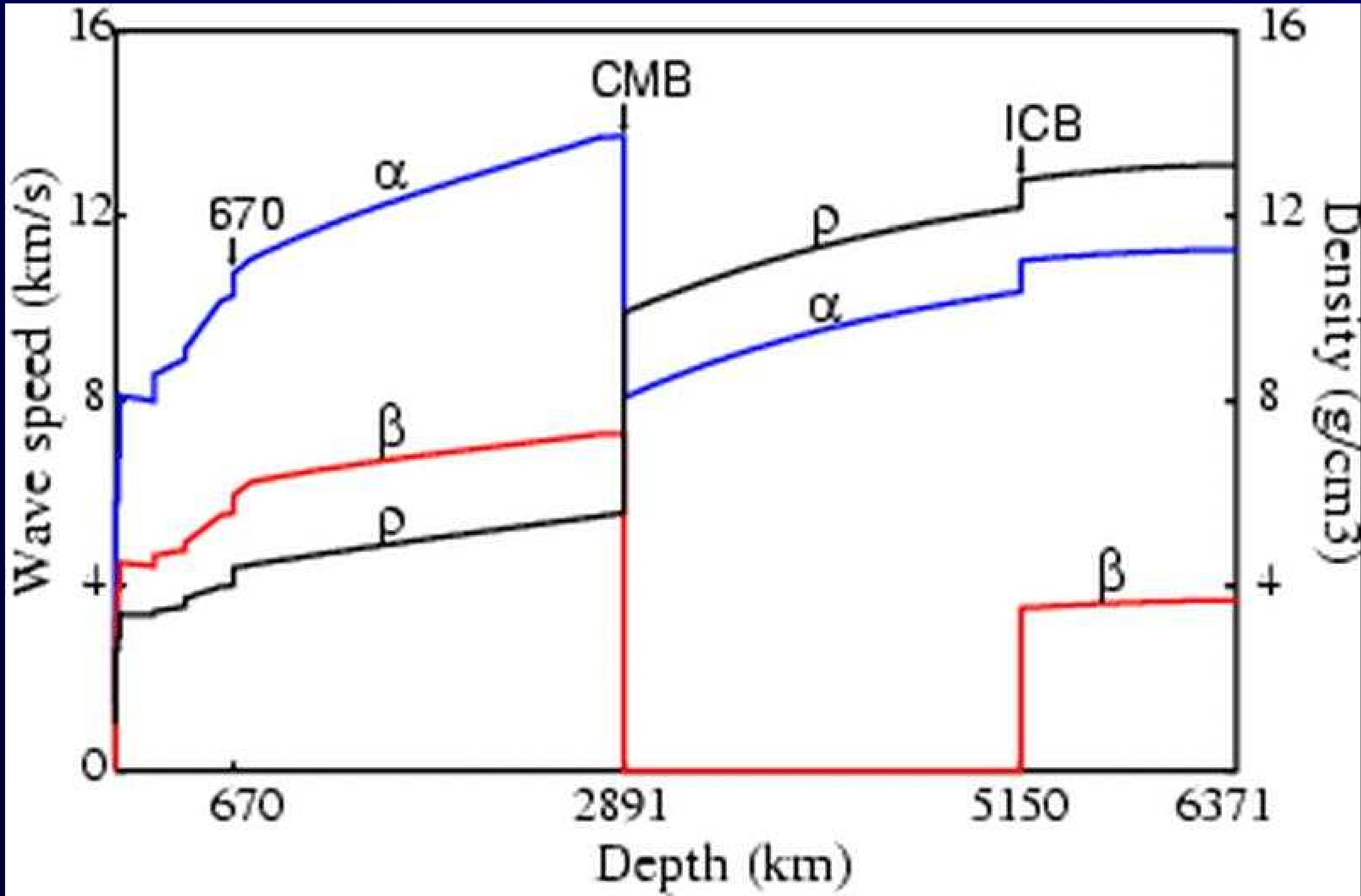
Introduction (Global Earth)

- Need accurate numerical modeling to study Earth structure (global scale)
- Very large models at high frequency (3D Earth)
- Complexity: classical methods (ray tracing, finite difference, pseudo-spectral) *do not work* for this problem (surface waves, anisotropy, fluid/solid interfaces, Earth’s crust etc.)

The Challenge of the Global Earth

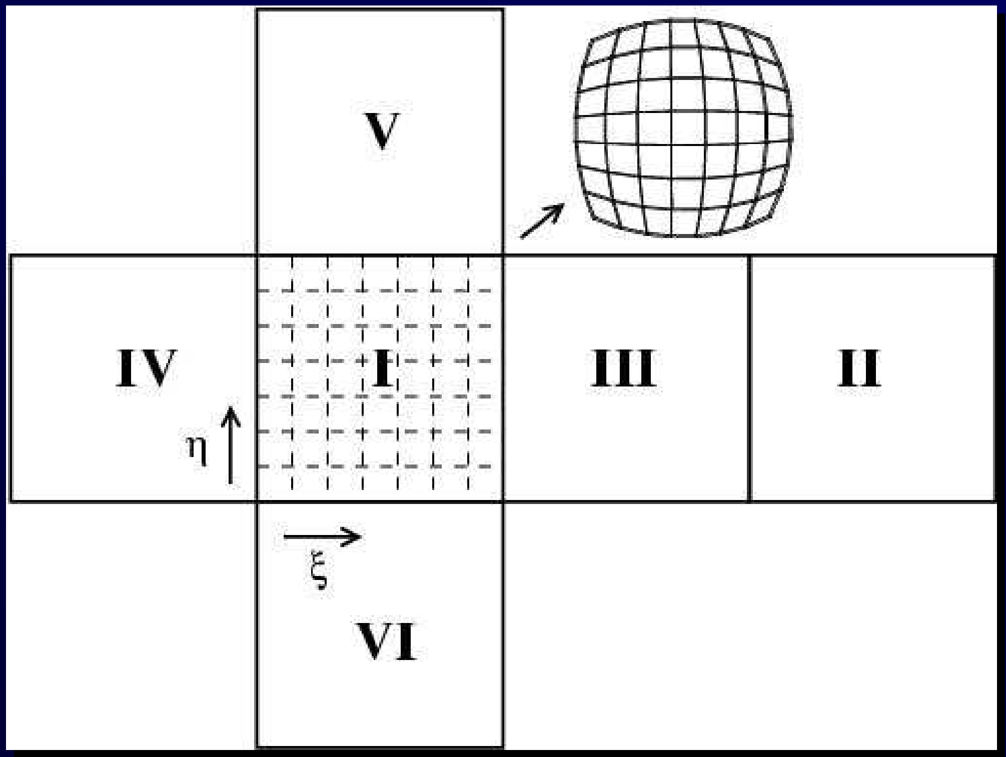
- A slow, thin, highly variable crust
- Sharp radial velocity and density discontinuities
- Fluid-solid boundaries (outer core of the Earth)
- Anisotropy
- Attenuation
- Ellipticity, topography and bathymetry
- Rotation
- Self-gravitation
- 3-D mantle and crust models (lateral variations)

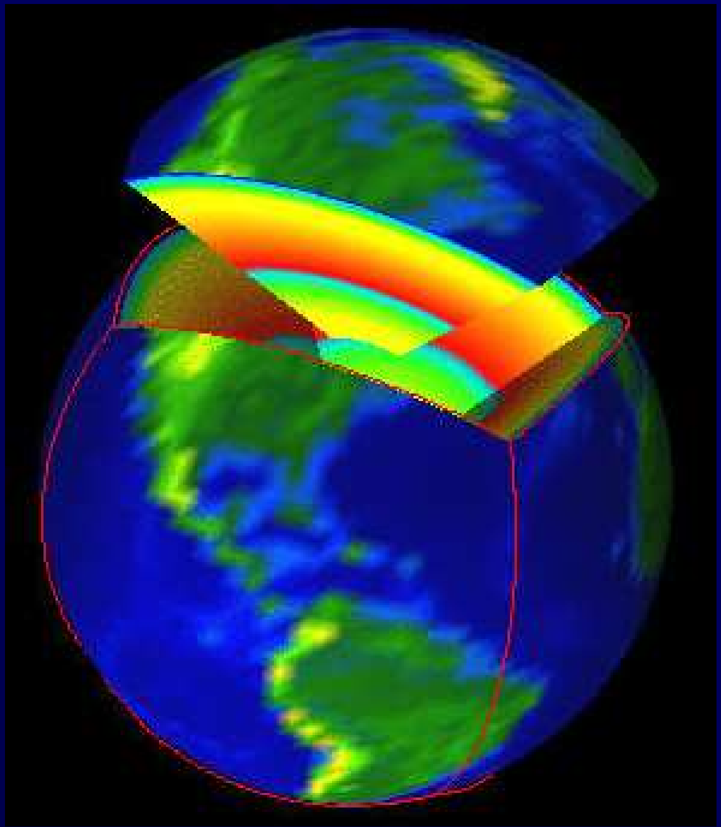
Sharp Contrasts in Earth Model



Thin crust, fluid outer core, high Poisson’s ratio in inner core

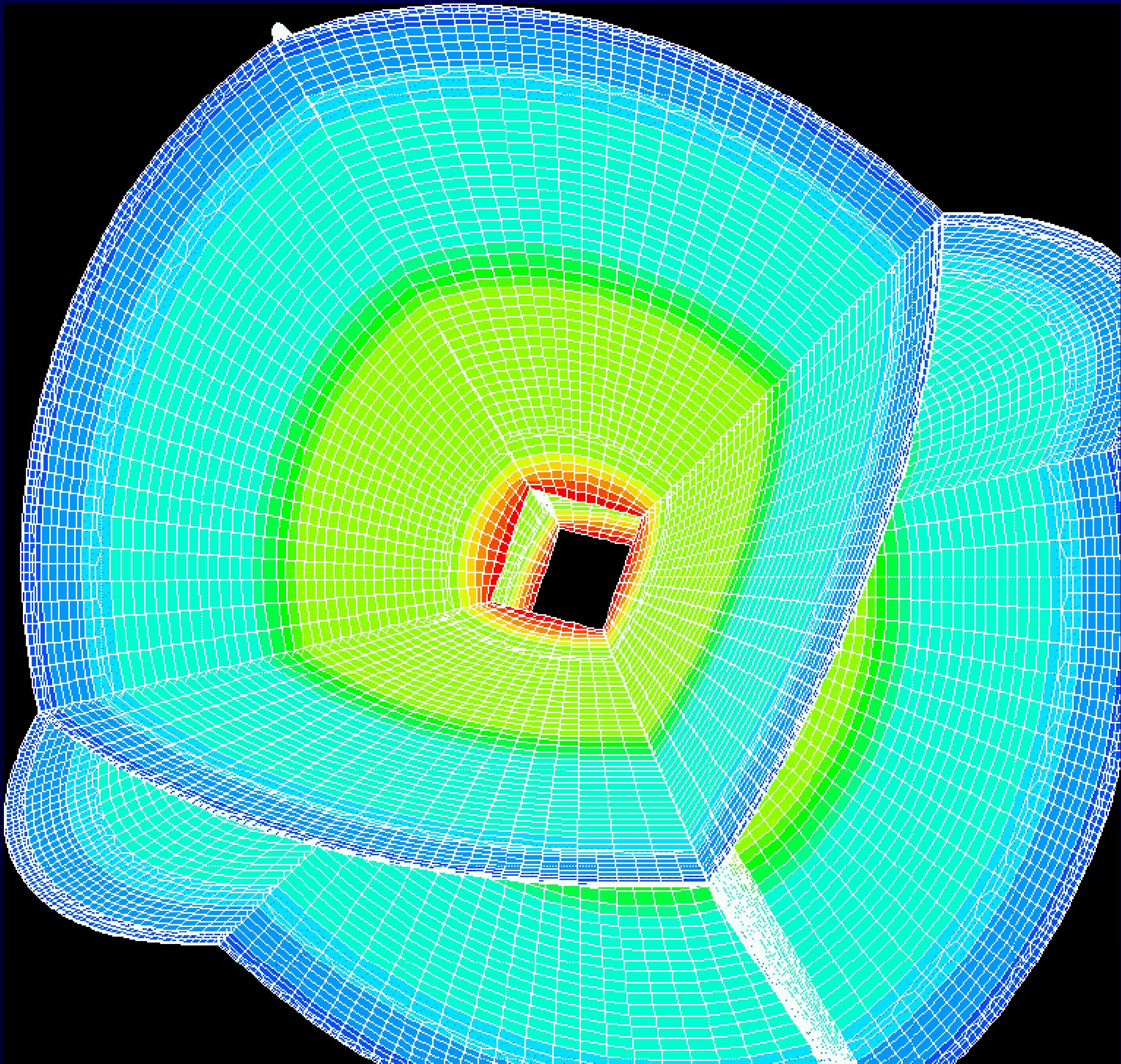
The Cubed Sphere

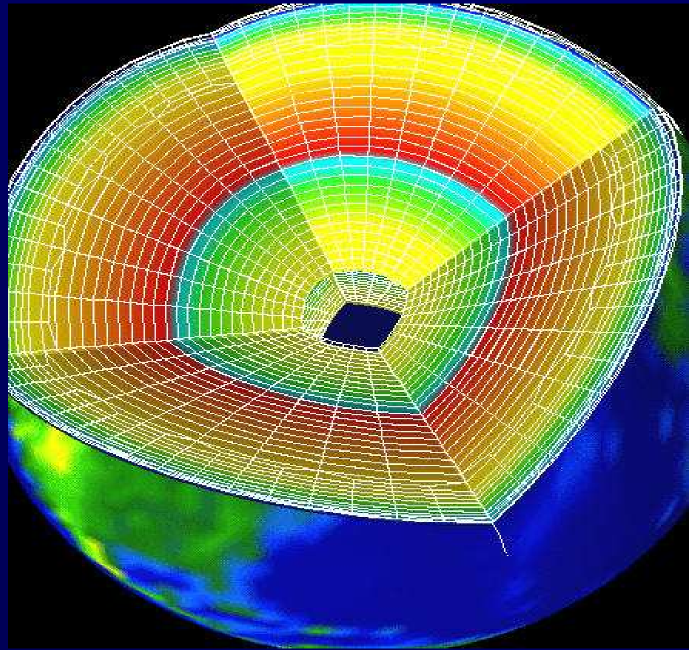


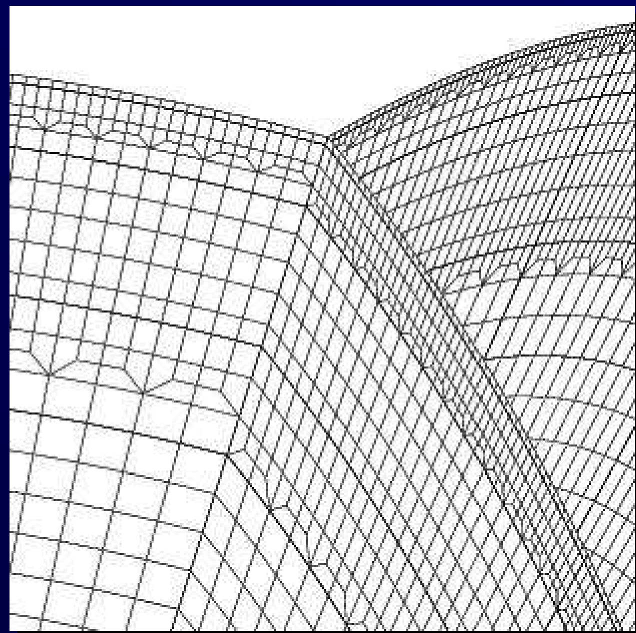


- “Gnomonic” mapping (Sadourny 1972)
- Ronchi et al. (1996), Chaljub (2000)
- Analytical mapping from six faces of cube to unit sphere

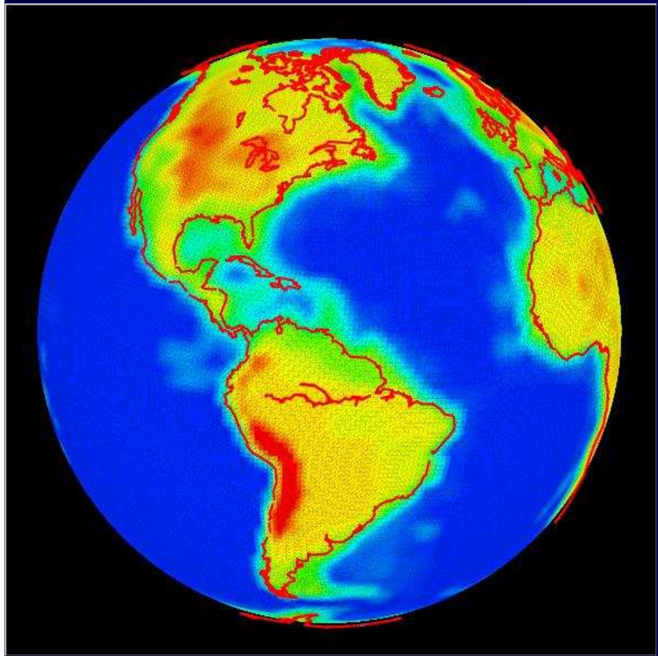
Final Mesh

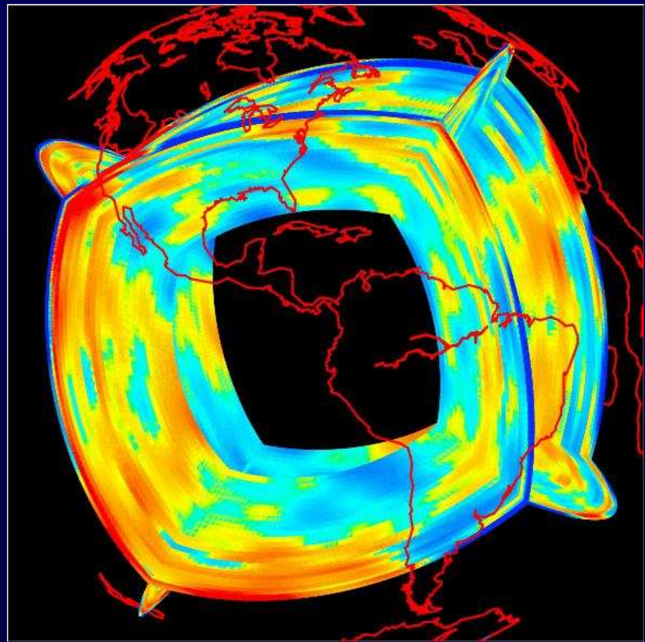


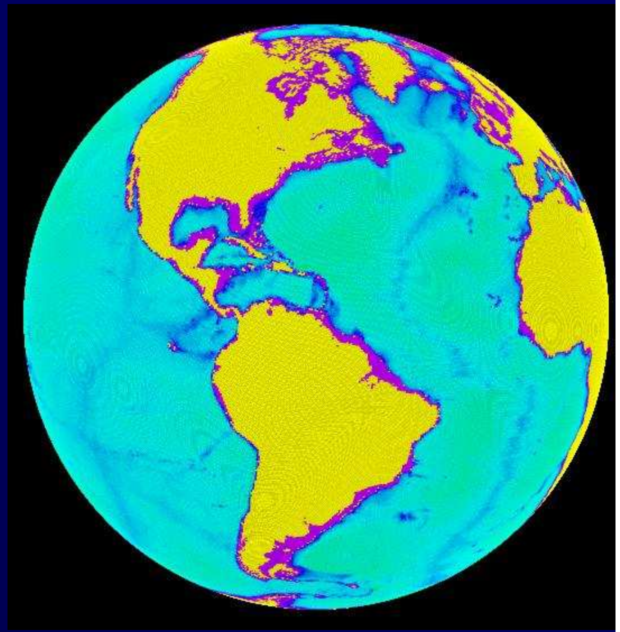




Global 3-D Earth





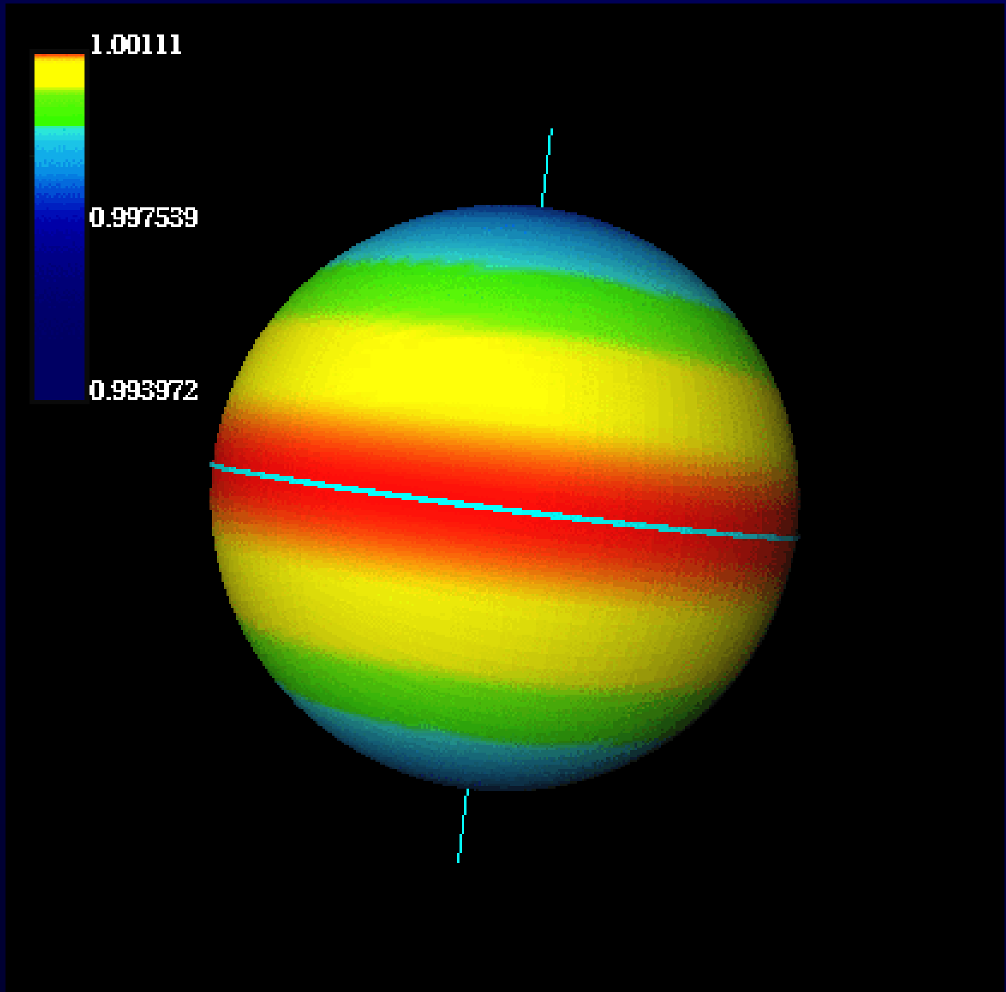


Crust 5.2 (Bassin et al. 2000)
Mantle model S20RTS (Ritsema et al. 1999)

Bathymetry and ocean load

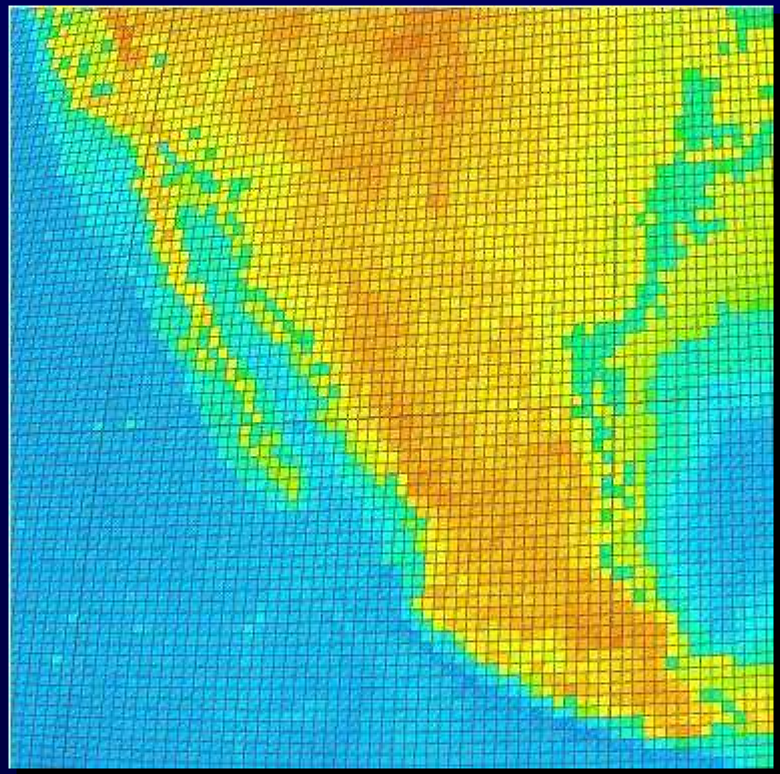
Small modification of the mesh, no problem

Global 3-D Earth



Ellipticity and topography

Small modification
of the mesh, no problem



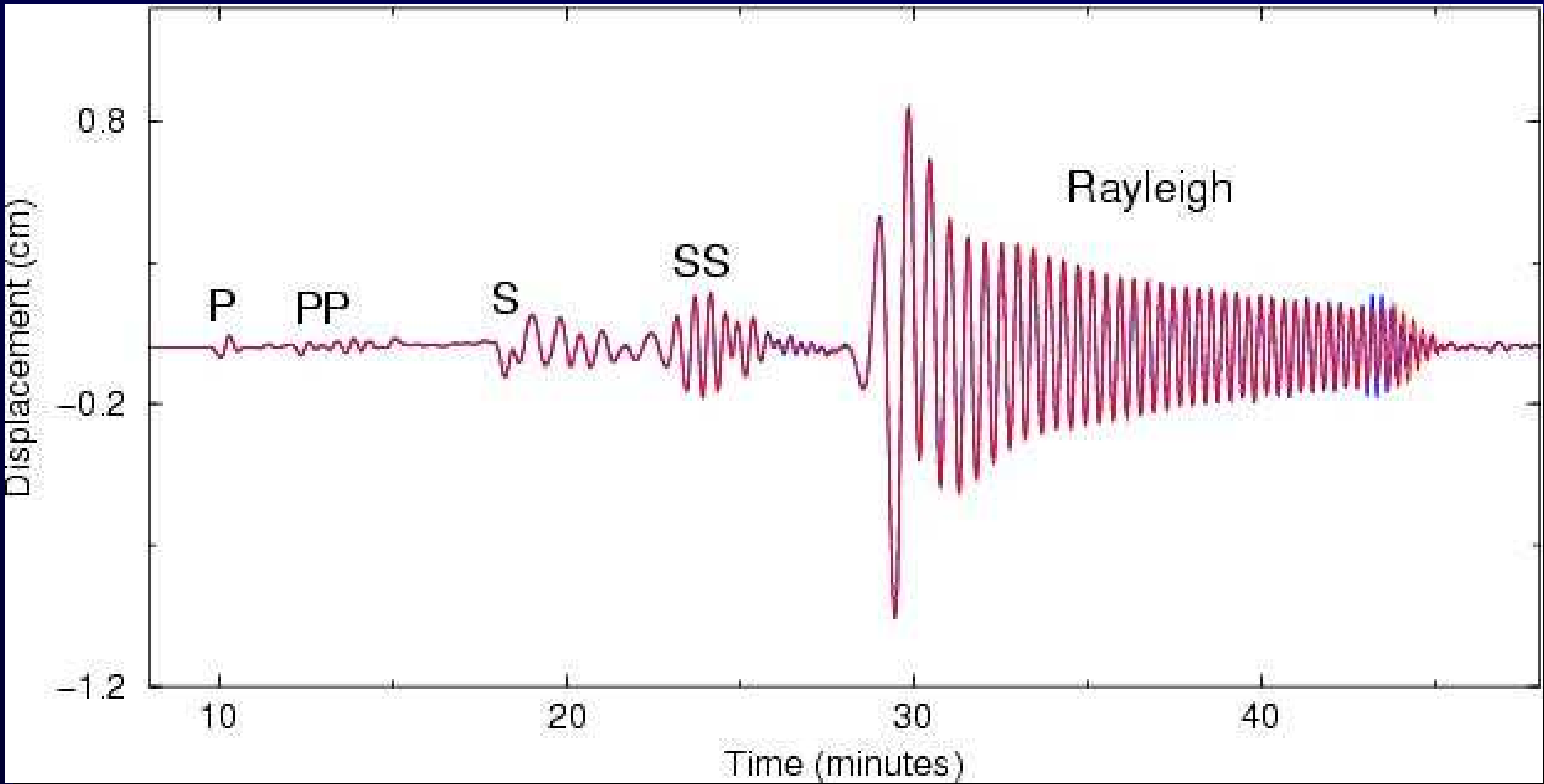
Other options for the global Earth

- Non-conforming meshes : Mortar method (Bernardi and Maday 1995, Chaljub 2000)

- Coupling with normal-modes : Capdeville, Vilotte and Montagner (2000)

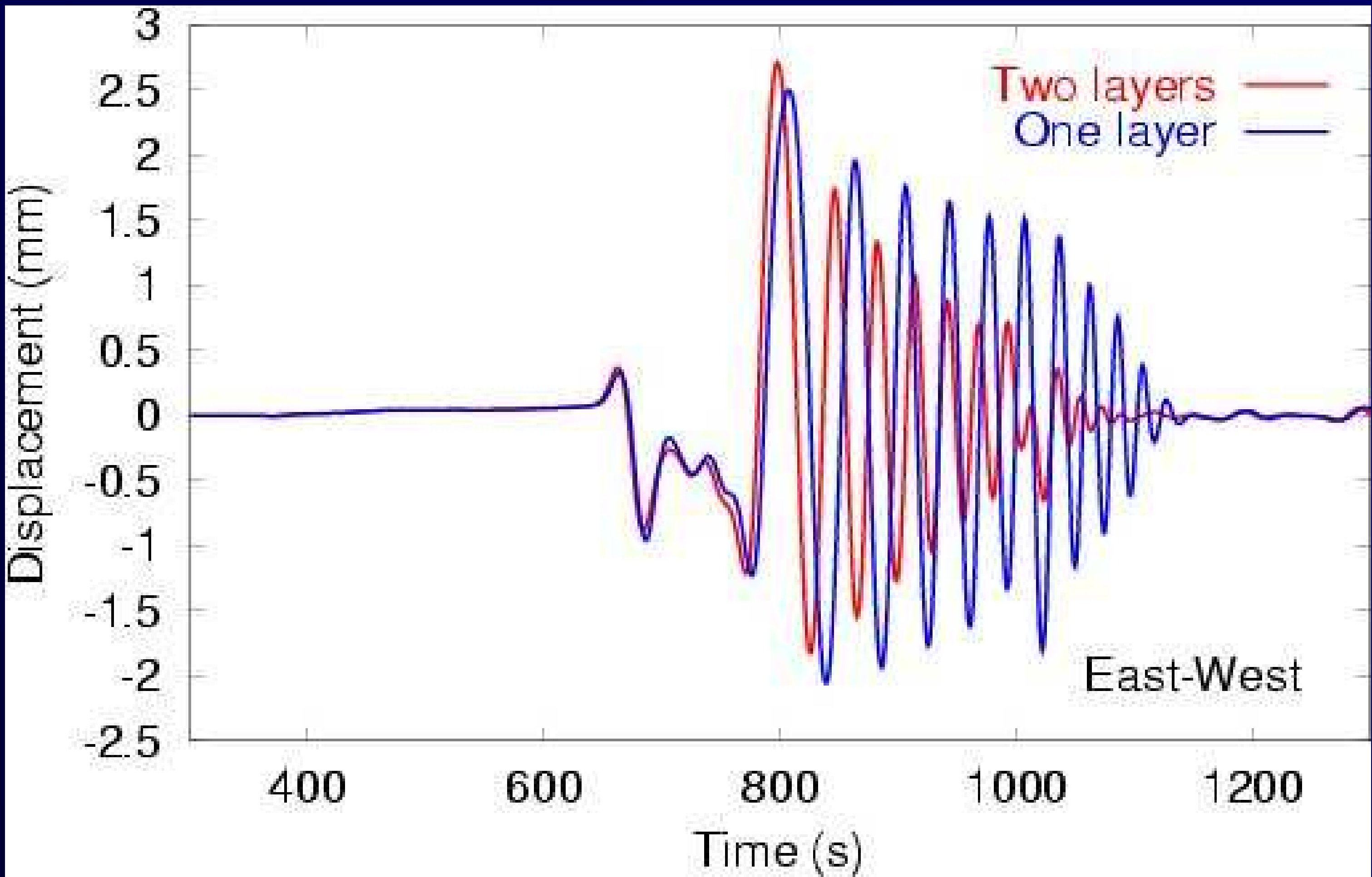
Benchmarks of the SEM at the global scale

Accurate surface waves



Excellent agreement with normal modes – Depth 15 km
Anisotropy included

Effect of the crust

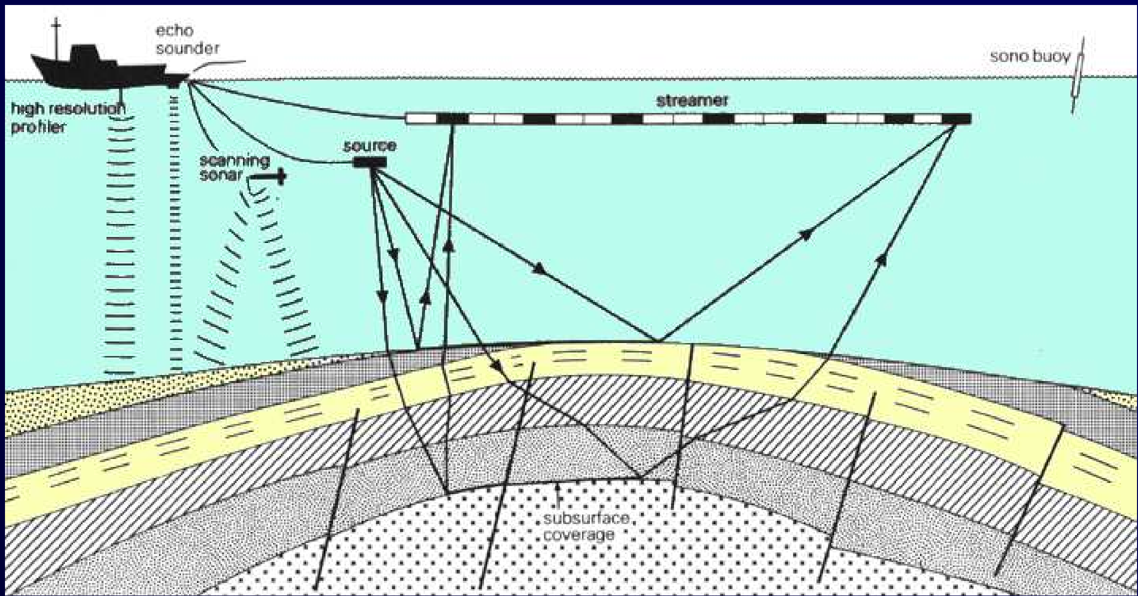


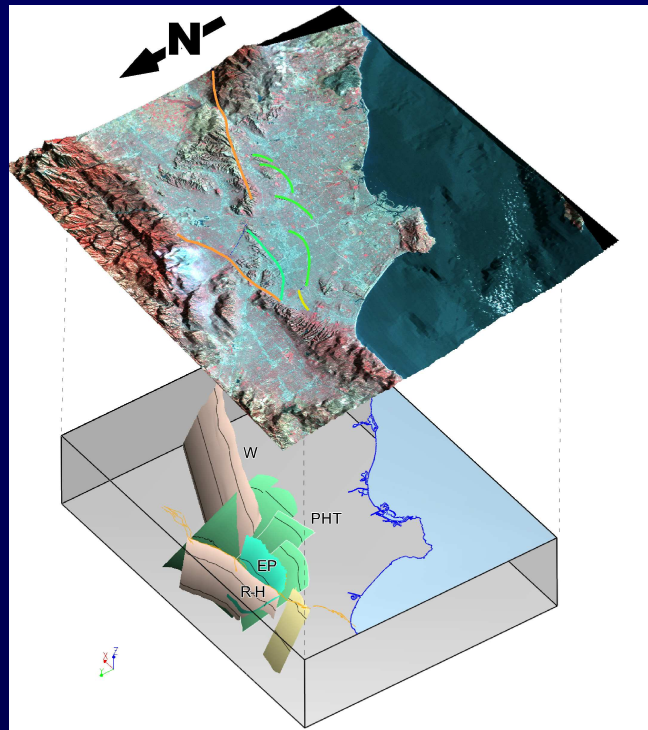
Large effect on surface waves – dispersion

Part II – More complex models or equations

Oil industry applications

Collaboration with the oil industry





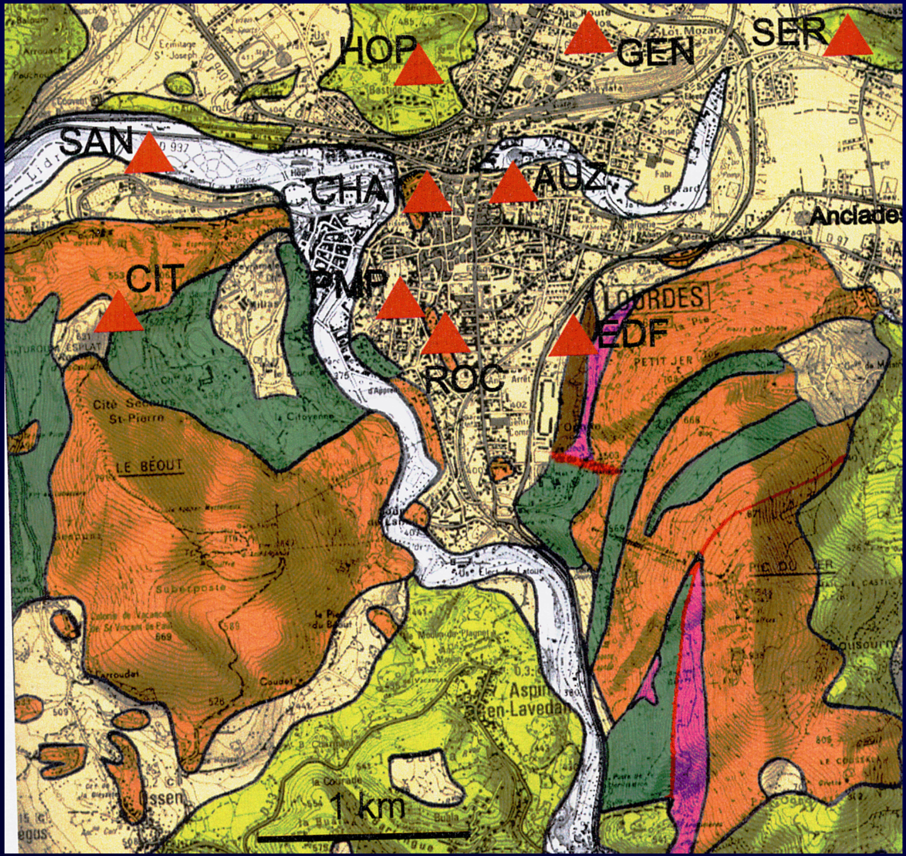
Dynamic geophysical technique of imaging subsurface geologic structures by generating sound waves at a source and recording the reflected components of this energy at receivers.

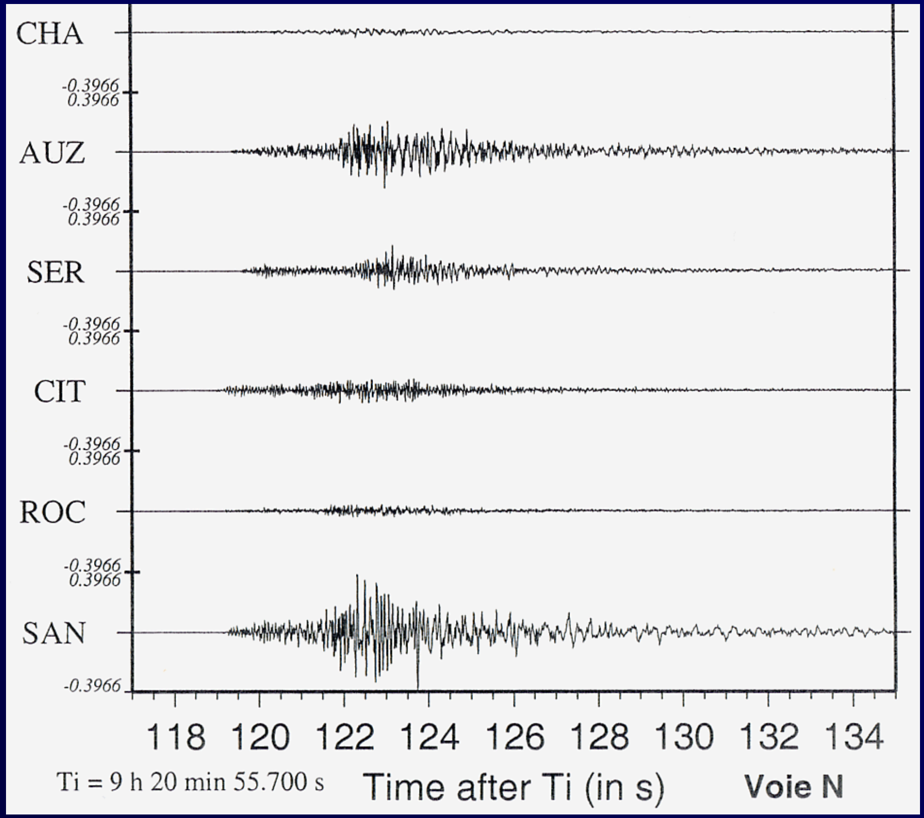
The Seismic Method is the *industry standard* for locating subsurface oil and gas accumulations.

Site effect applications

Échelle locale (effets de site)

- Variations locales très significatives, non reproduites par un calcul 1D (Dubos et Souriau)



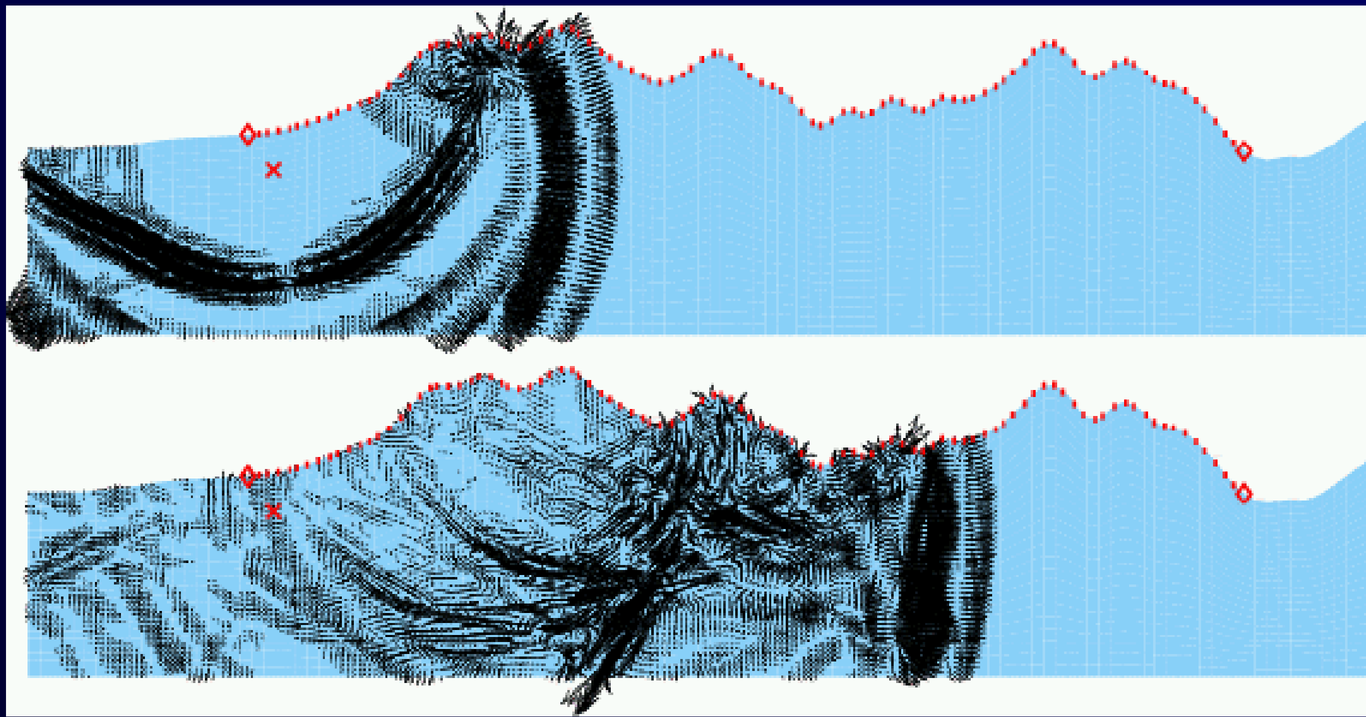


Valorisation du réseau accélérométrique permanent (RAP)

Topography

Topography

- Use flexibility of mesh generation
- Accurate free-surface condition

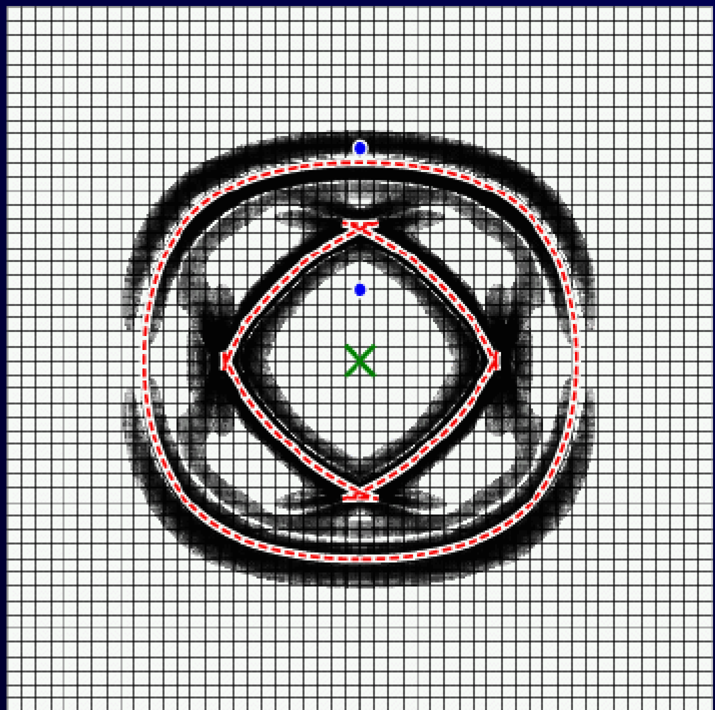


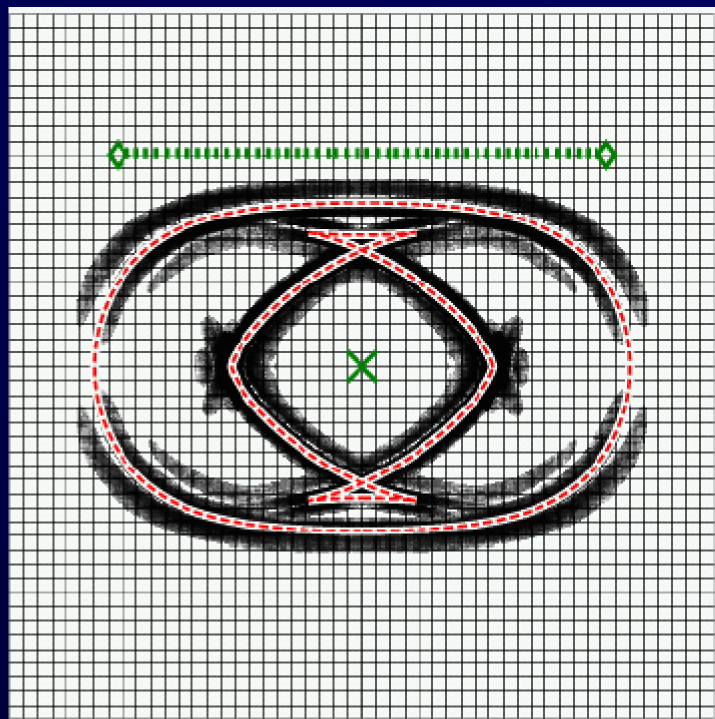
Anisotropy

Anisotropy

- Easy to implement up to 21 coefficients
- No interpolation necessary
- Tilted axes can be modeled
- Attenuation can also be included

Cobalt

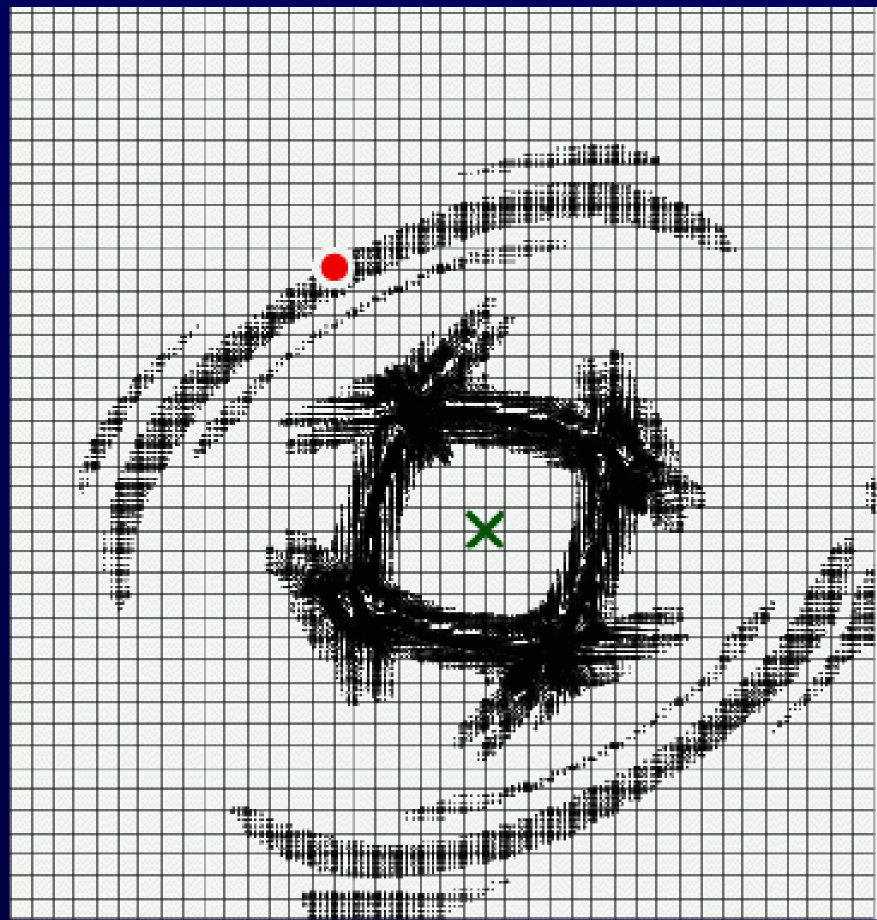




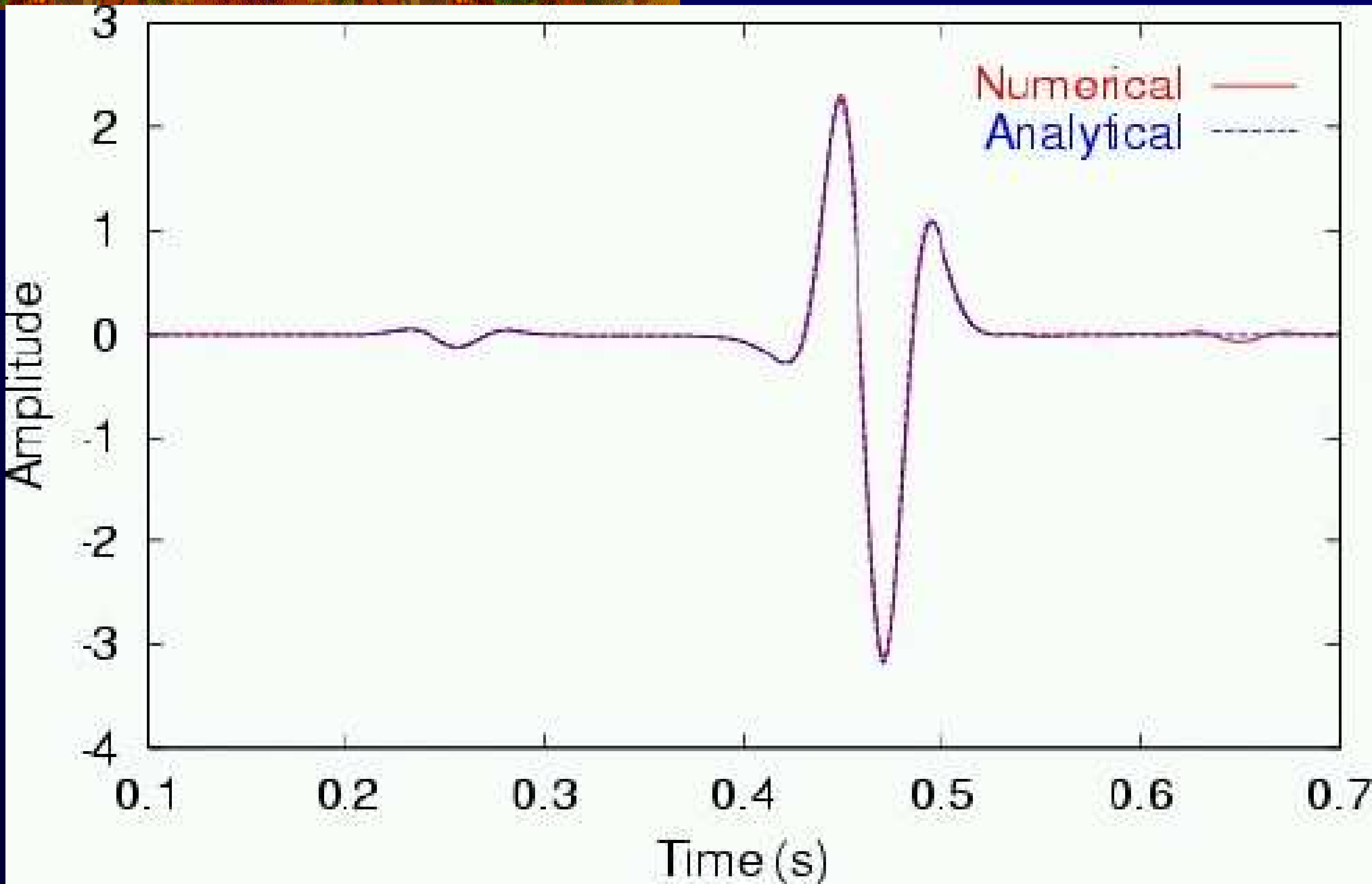
Zinc

Anisotropy – Tilted 3D case

- Transversely isotropic with rotated axis
- Most of the 21 coefficients $\neq 0$
- Carcione (1988): analytical solution



Tilted 3D case: analytical solution



- Excellent fit for both *qP* and *qS*
- Small phase reflected off model edge

Attenuation

Attenuation

- Constitutive relationship:

$$\mathbf{T}(t) = \int_{-\infty}^{t} \partial_t \mathbf{c}(t - t') : \nabla \mathbf{s}(t')\, \mathrm{d}t'$$

Difficult in time domain methods because of convolution

- Use L standard linear solids to make an absorption-band model:

$$\mu(t) = \mu_R \left[1 - \sum_{\ell=1}^{L} \left(1 - \tau_\ell^{\varepsilon} / \tau_\ell^{\sigma}\right) e^{-t/\tau_\ell^{\sigma}} \right] H(t)$$

Attenuation

- Constitutive relationship becomes:

$$\mathbf{T} = \mathbf{c}_U : \nabla \mathbf{s} - \sum_{\ell=1}^{L} \mathbf{R}_\ell$$

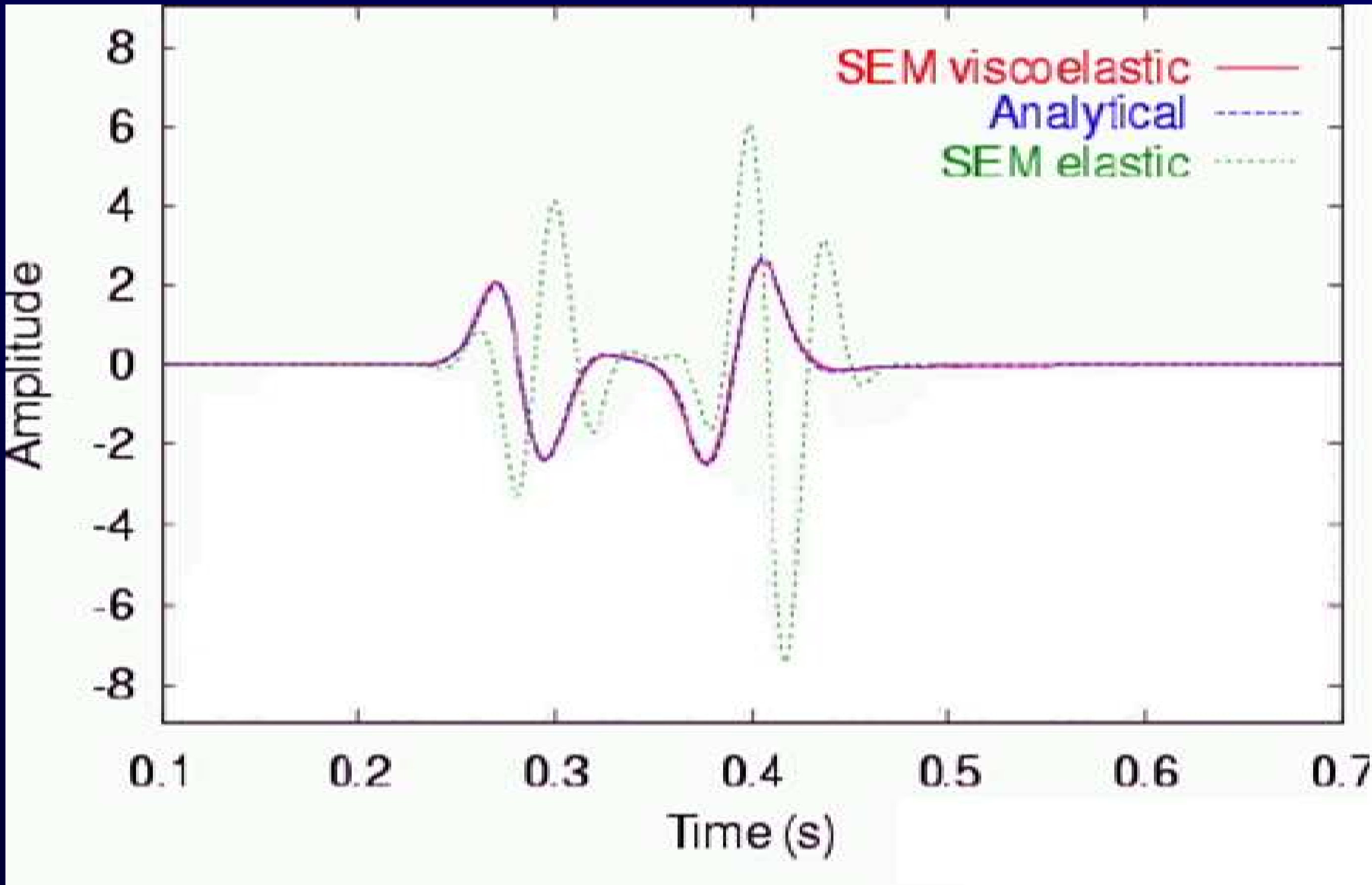
- Memory variable equation:

$$\partial_t \mathbf{R}_\ell = -\left(\mathbf{R}_\ell - \delta\mu_\ell \mathbf{D}\right) / \tau_\ell^\sigma$$

where $\mathbf{D}$ is the strain deviator:

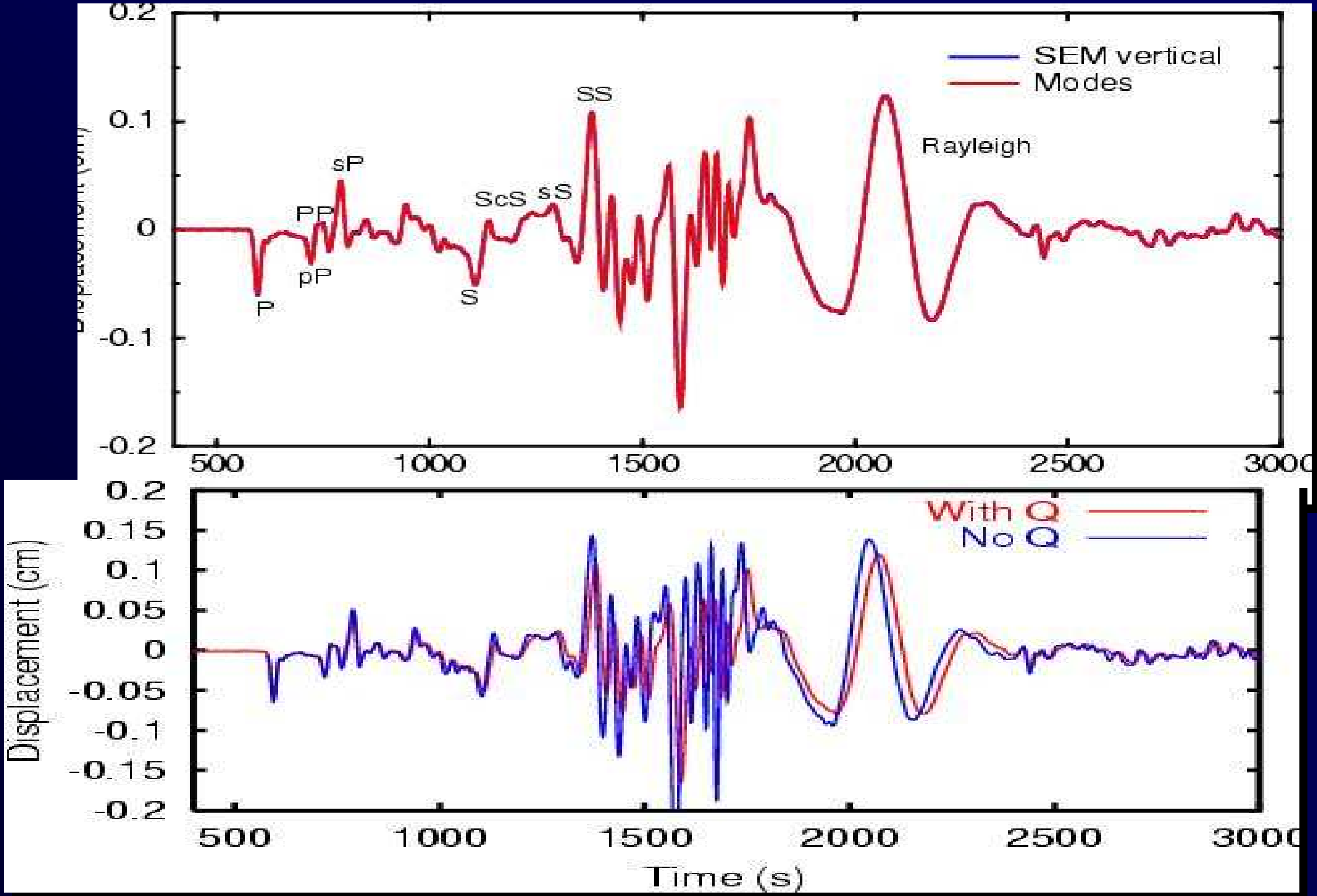
$$\mathbf{D} = \frac{1}{2}\left[\nabla \mathbf{s} + (\nabla \mathbf{s})^T\right] - \frac{1}{3}(\nabla \cdot \mathbf{s})\mathbf{I}$$

Attenuation



- Problématique en temps – Variables à mémoire
- Difficulté: facteur de qualité *Q* constant
- Implémenter avec le minimum de mémoire possible

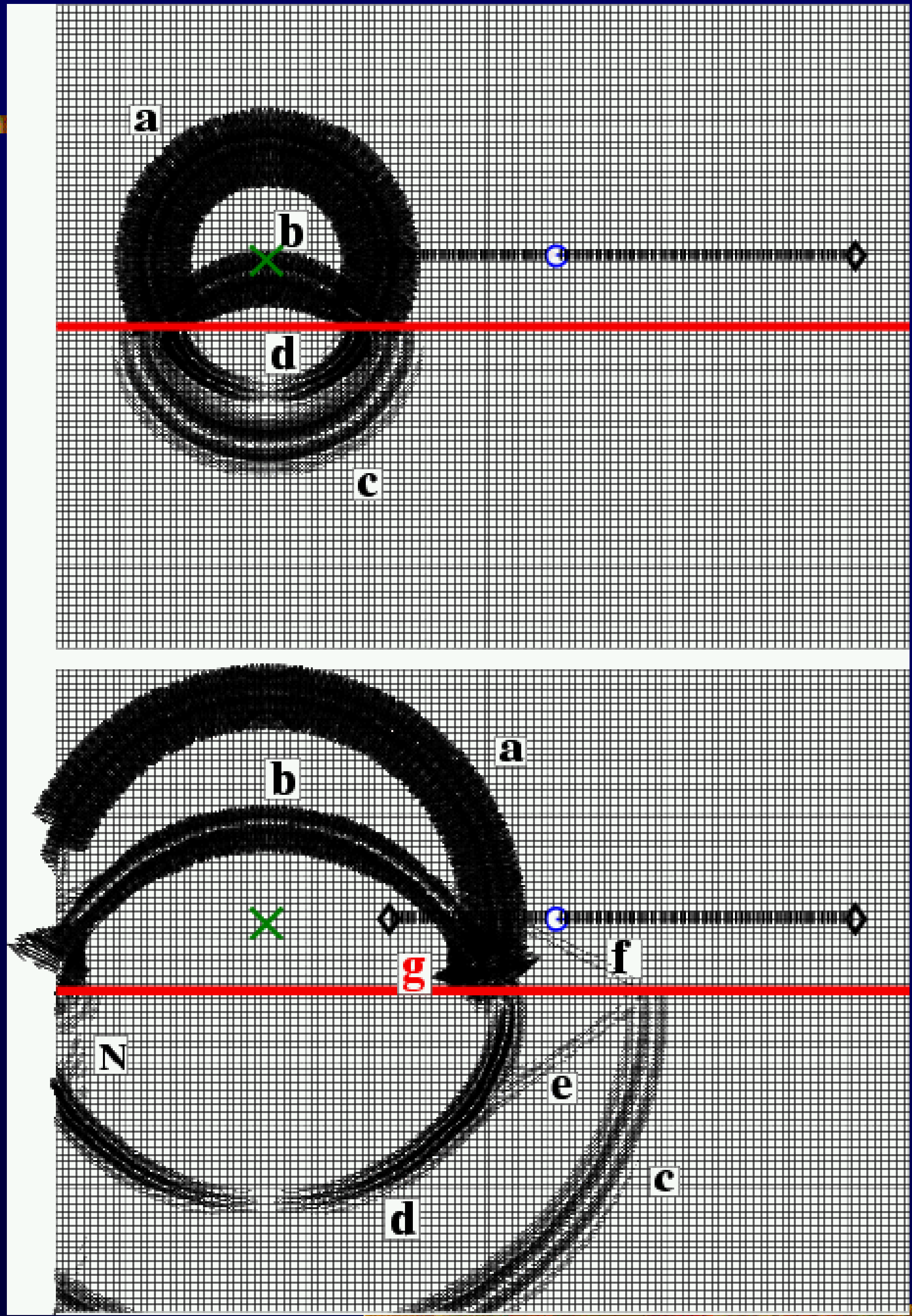
Effect of Attenuation



Fluid-solid coupling

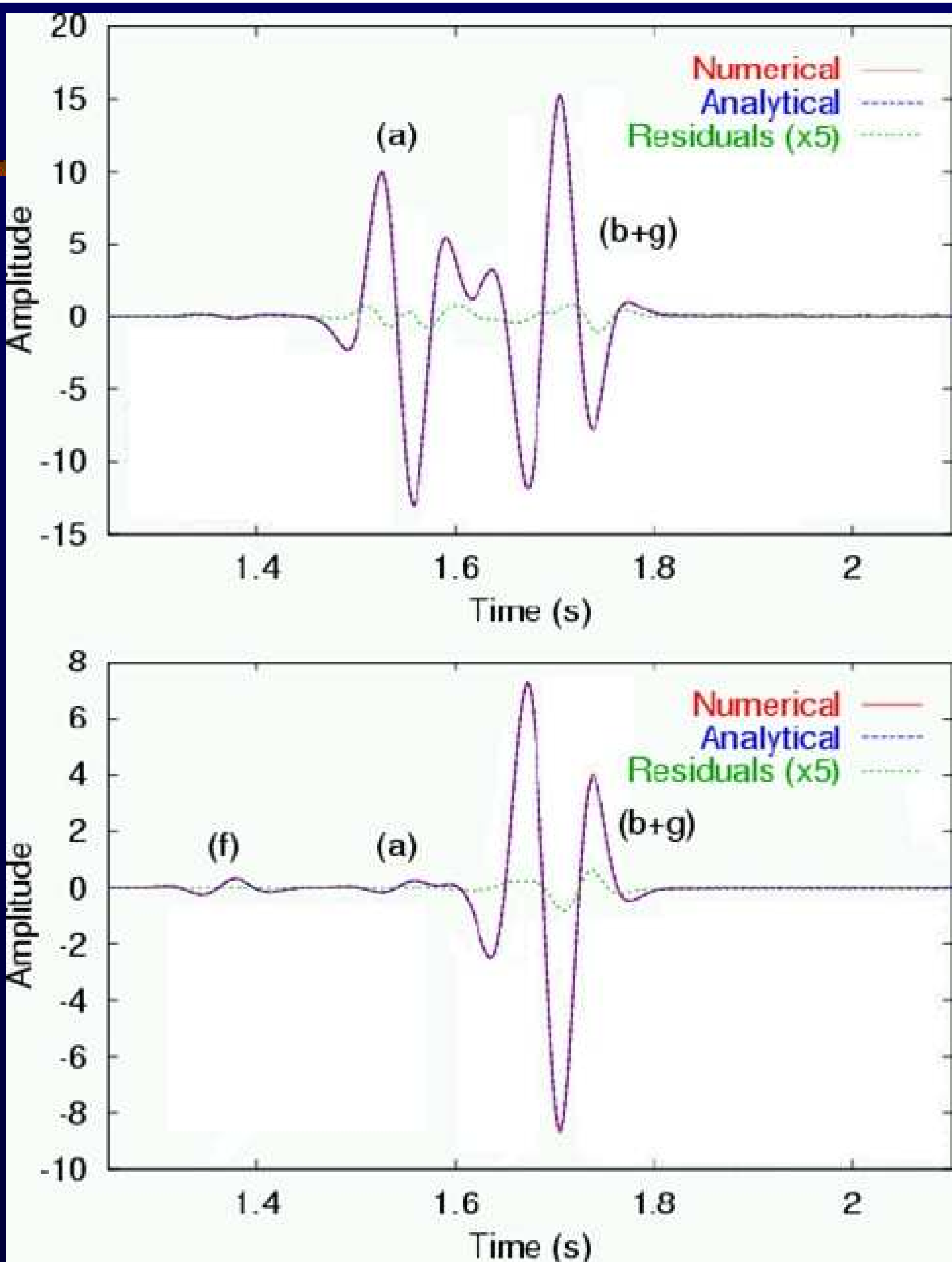
Fluid/solid boundaries

- Difficult with classical finite elements
- We use a velocity potential in the fluid
- Keep diagonal mass matrix



Analytical solution

- Very good fit
- Validation of the method
- Refracted phases accurately modeled

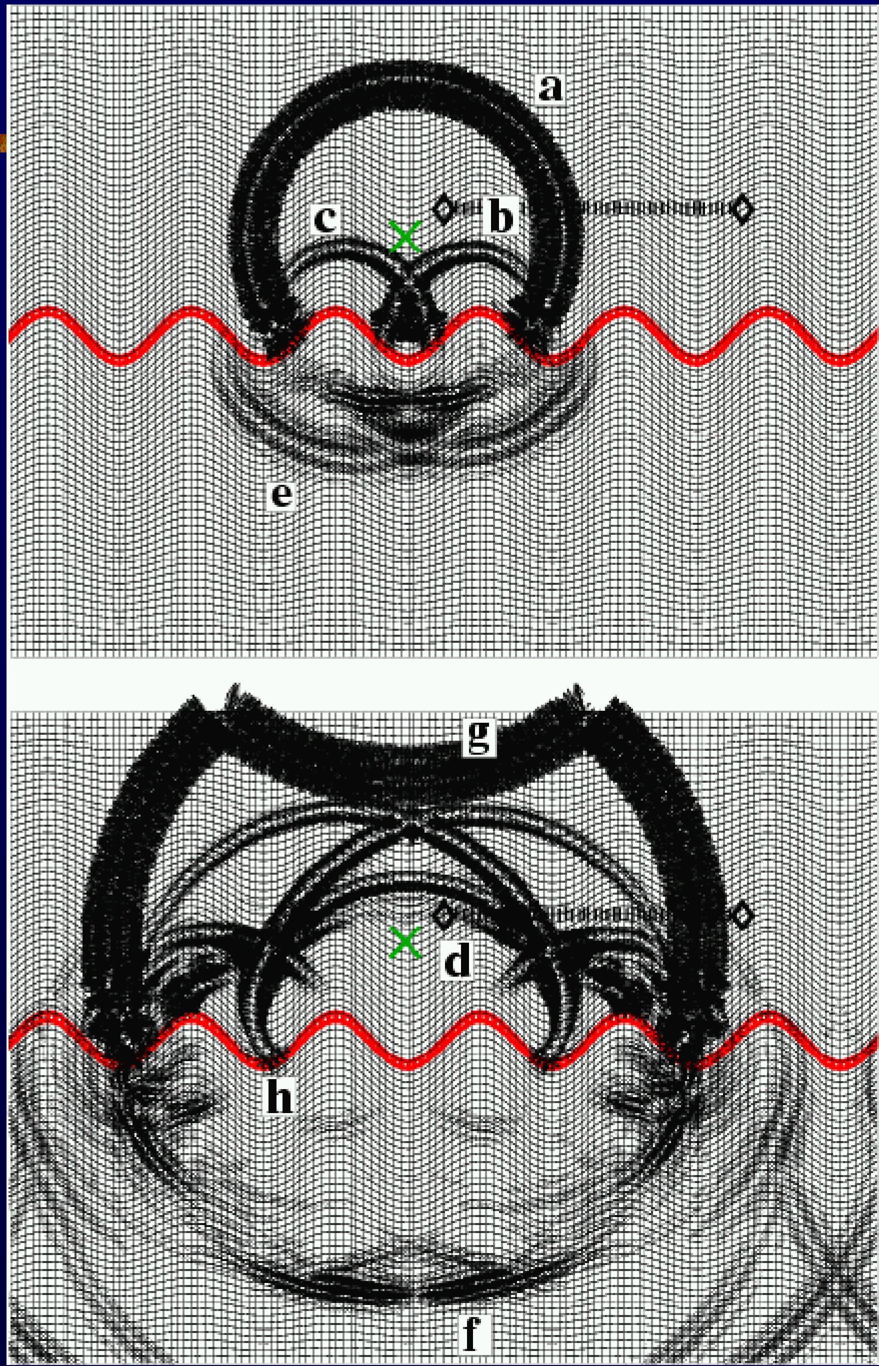


Bathymetry

- Use flexibility of mesh generation process

- Triplications

- Stoneley



Oceans (effect of the ocean load)

Effect of the Oceans

Modified weak form with ocean floor integral:

$$\int \rho \mathbf{w} \cdot \partial_t^2 \mathbf{s} \, \mathrm{d}^3\mathbf{r} = -\int \nabla \mathbf{w} : \mathbf{T} \, \mathrm{d}^3\mathbf{r}$$

$$+ \mathbf{M} : \nabla \mathbf{w}(\mathbf{r}_s) S(t) - \int_{\mathrm{F-S}} p \, \mathbf{w} \cdot \hat{\mathbf{n}} \, \mathrm{d}^2\mathbf{r}$$

Ocean load:

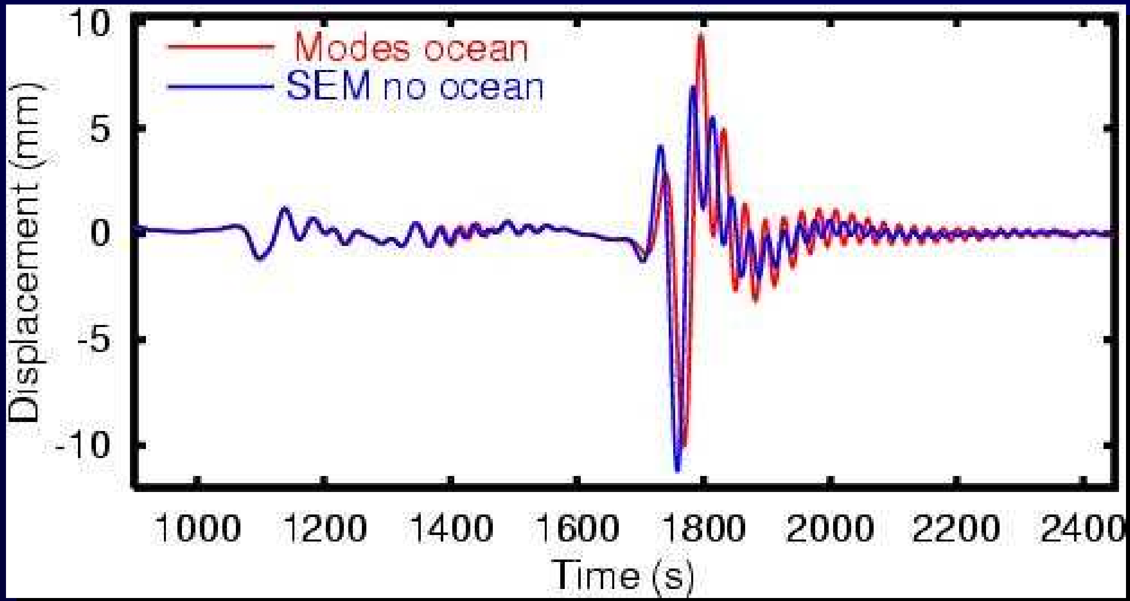
$$p = \rho_w h \hat{\mathbf{n}} \cdot \partial_t^2 \mathbf{s}$$

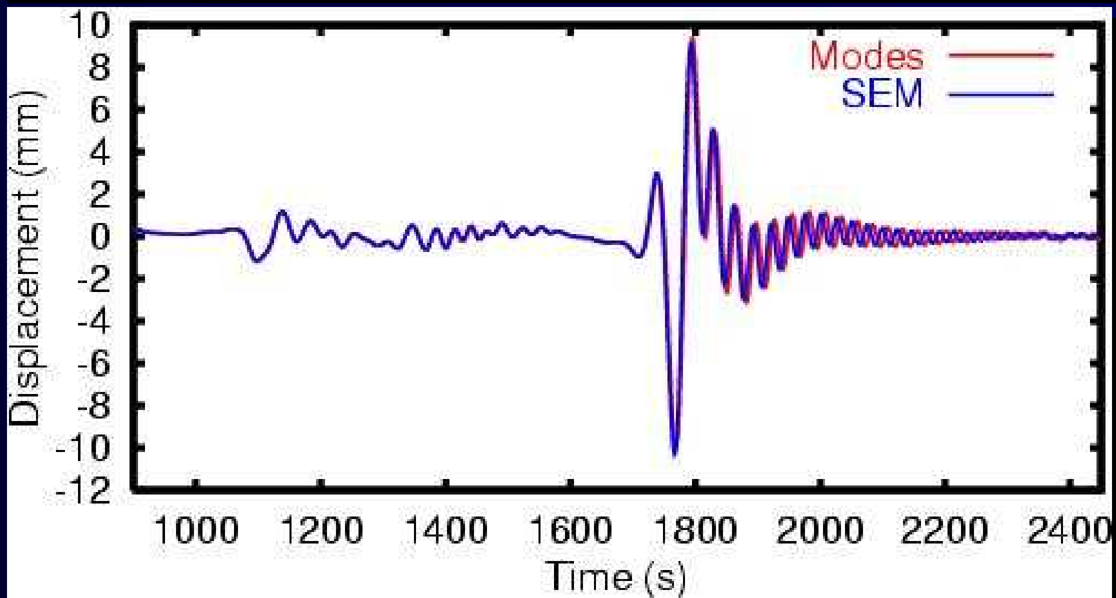
weight of column of water, zero thickness

Good approximation if wavelength >> thickness of oceans

(good at 20 s, not good at 1 s)

Effect of the Oceans





Depth 18 km

SEM without oceans

⇒ Effect of oceans on surface waves is significant for shallow events

SEM with ocean load

Gravity / rotation

Self-Gravitation and Rotation

- Strong form:

$$\rho\left(\partial_t^2\mathbf{s}+2\mathbf{\Omega}\times\partial_t\mathbf{s}\right)=\nabla\cdot\mathbf{T}+\nabla\left(\rho\mathbf{s}\cdot\mathbf{g}\right)$$

$$-\rho\nabla\phi-\nabla\cdot\left(\rho\mathbf{s}\right)\mathbf{g}+\mathbf{f}$$

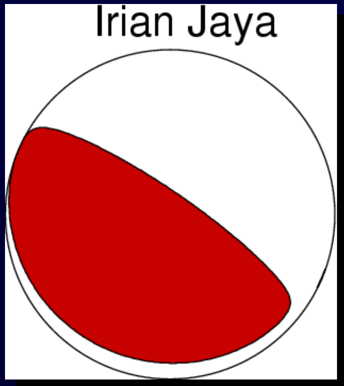
- Neglect mass redistribution - Cowling approximation (Valette 1986, Dalhen and Tromp 1998, Chaljub 2000):

$$\rho\left(\partial_t^2\mathbf{s}+2\mathbf{\Omega}\times\partial_t\mathbf{s}\right)=\nabla\cdot\mathbf{T}+\nabla\left(\rho\mathbf{s}\cdot\mathbf{g}\right)$$

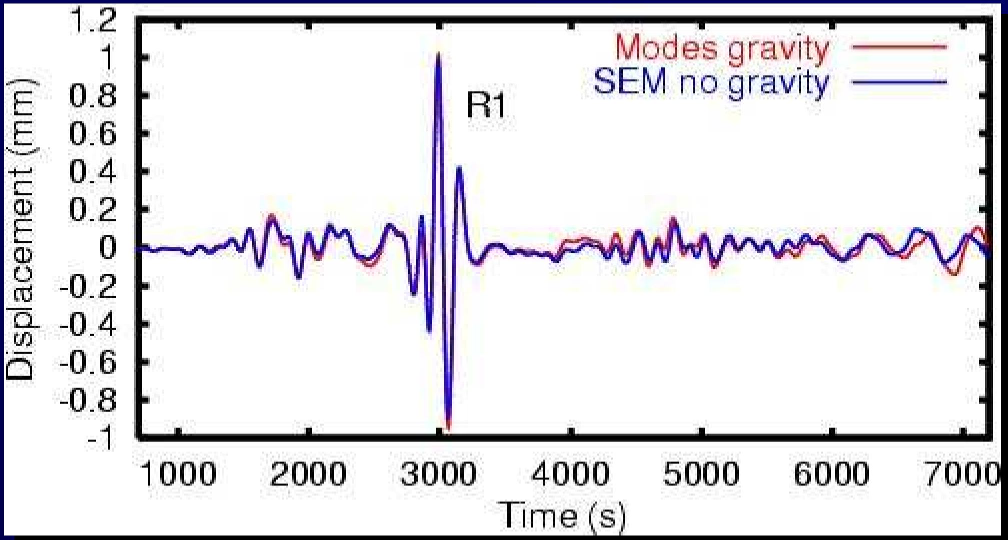
$$-\nabla\cdot\left(\rho\mathbf{s}\right)\mathbf{g}+\mathbf{f}$$

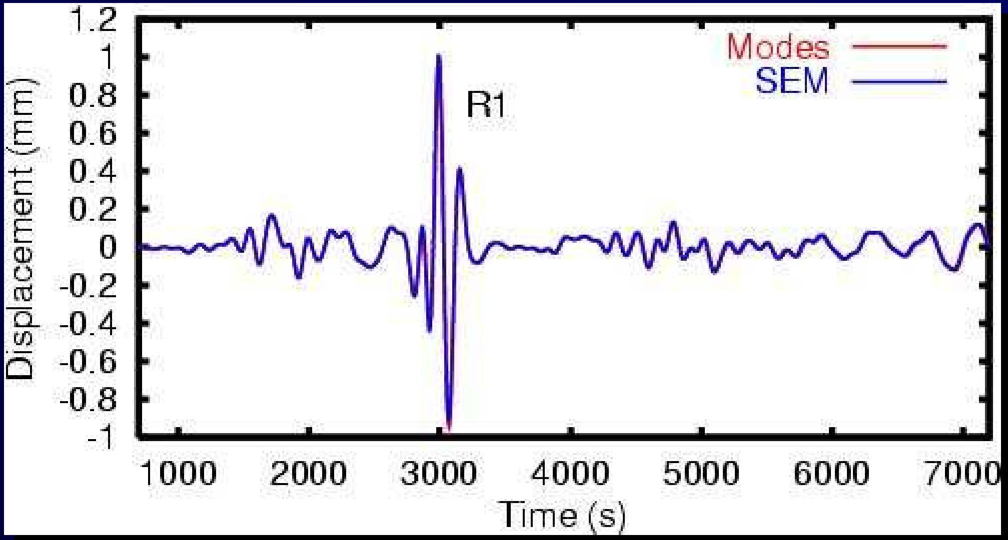
Effect of Self-Gravitation

- Main effect is long period oscillation
- Very well reproduced by the spectral-element method



Irian Jaya - depth 15 km

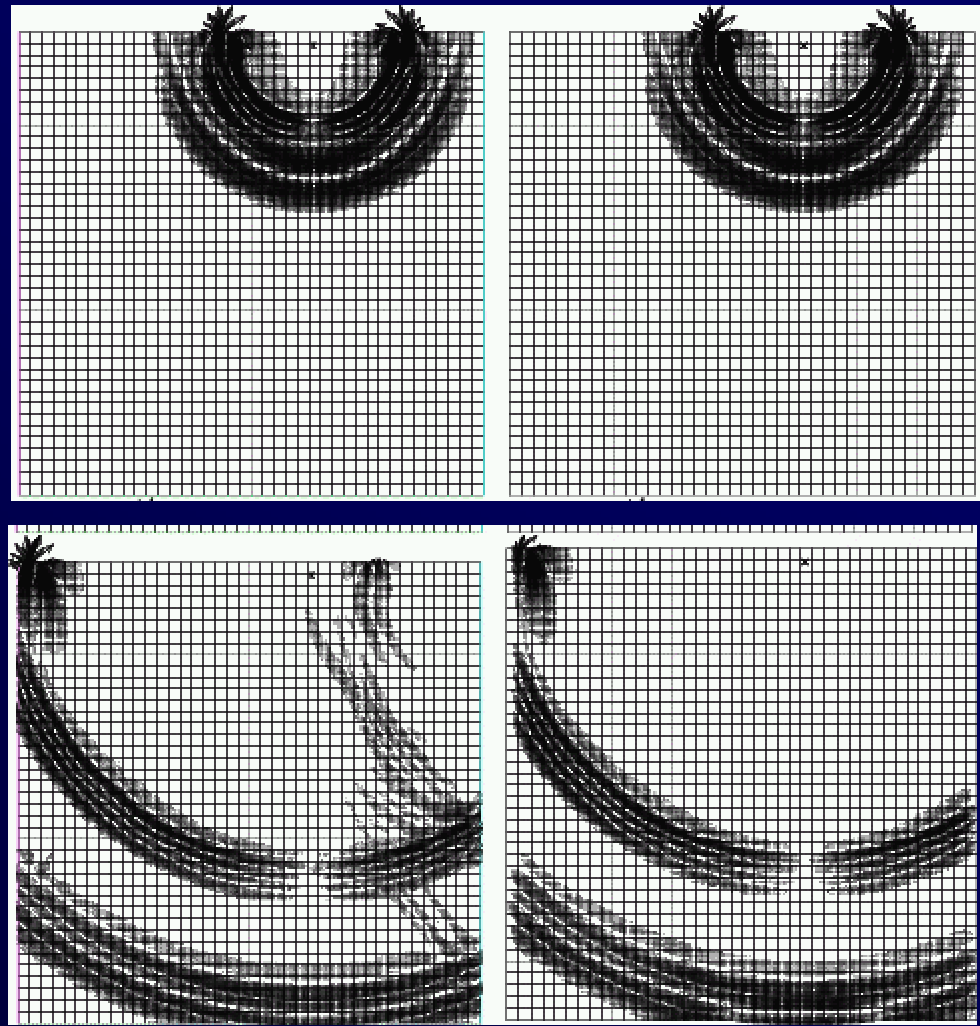




PML absorbing conditions

Absorbing conditions

- Used to be a big problem
- Bérenger 1994
- INRIA (Collino, Cohen)
- Extended to second-order systems by Komatitsch and Tromp (2003)

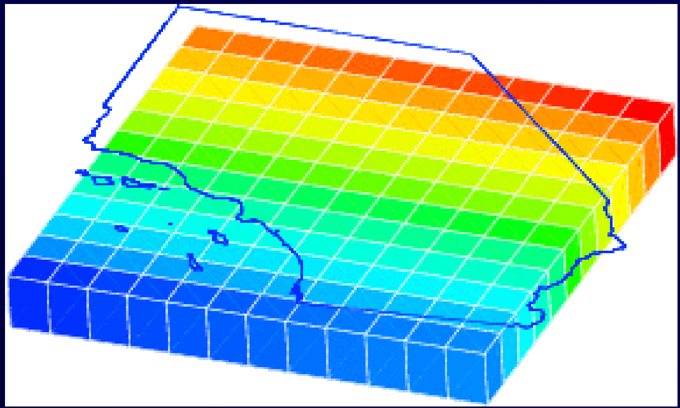


PML (Perfectly Matched Layer) $\Rightarrow$ Hélène Barucq

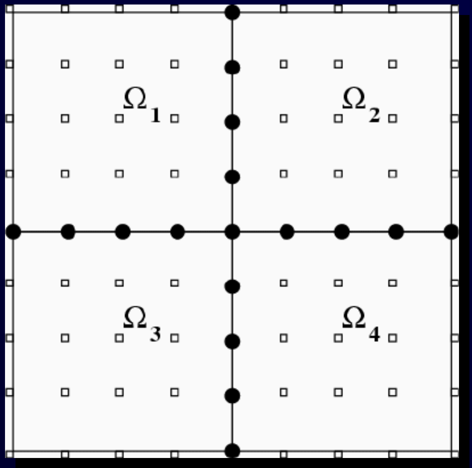
Parallel implementation

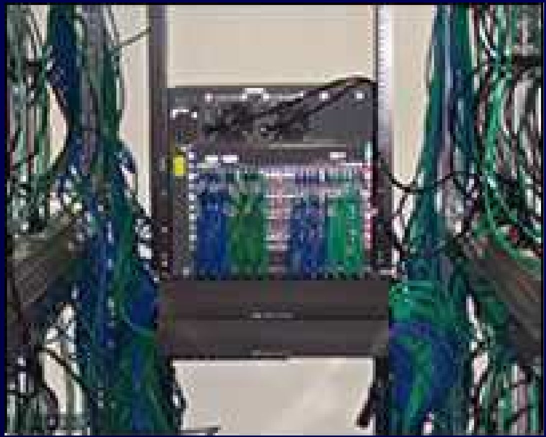
Construction du cluster

320 processeurs - Linux

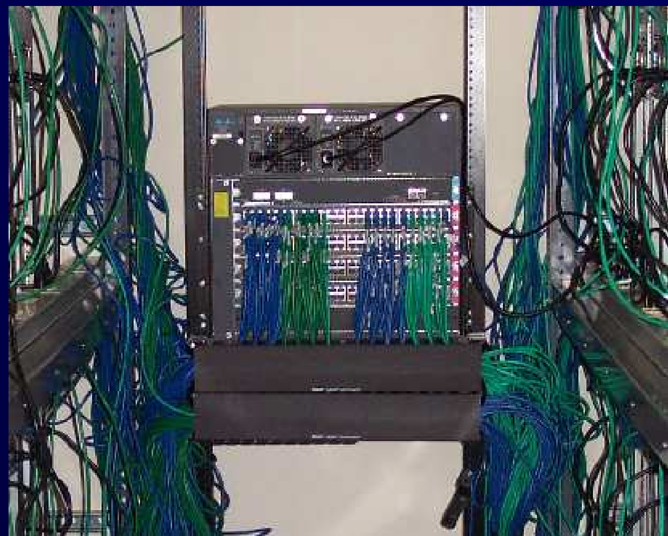


160 Gb de mémoire





Construction du cluster de Caltech



320 processeurs, 160 Gb de mémoire

Été 2000, maintenant « obsolète », renouvellement en cours

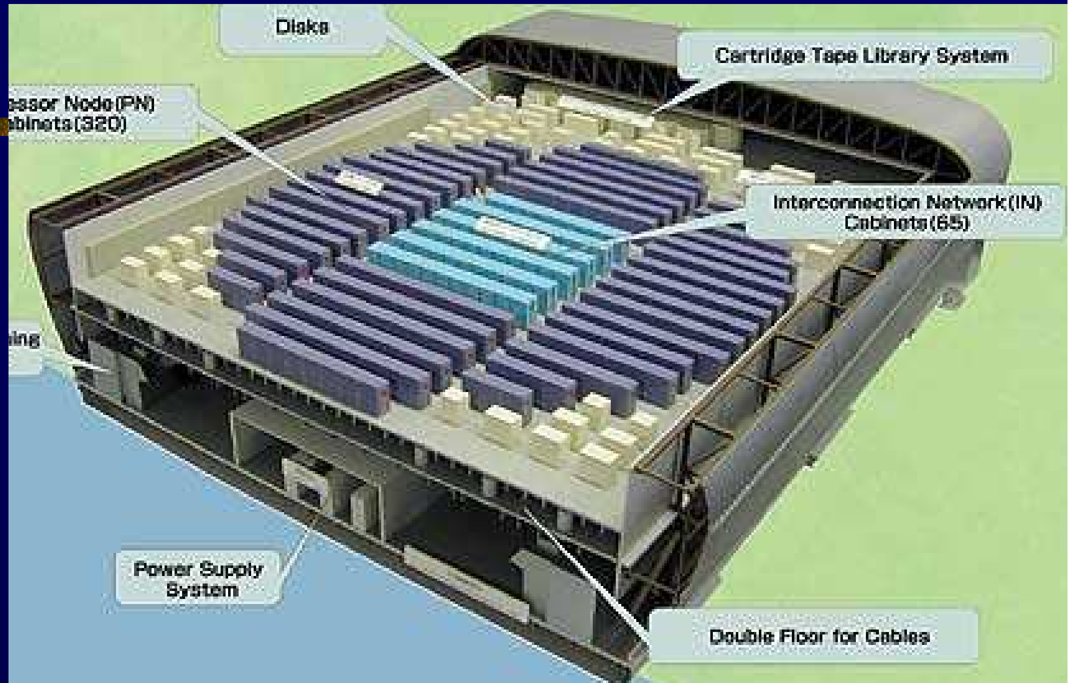
Earth Simulator - Japan

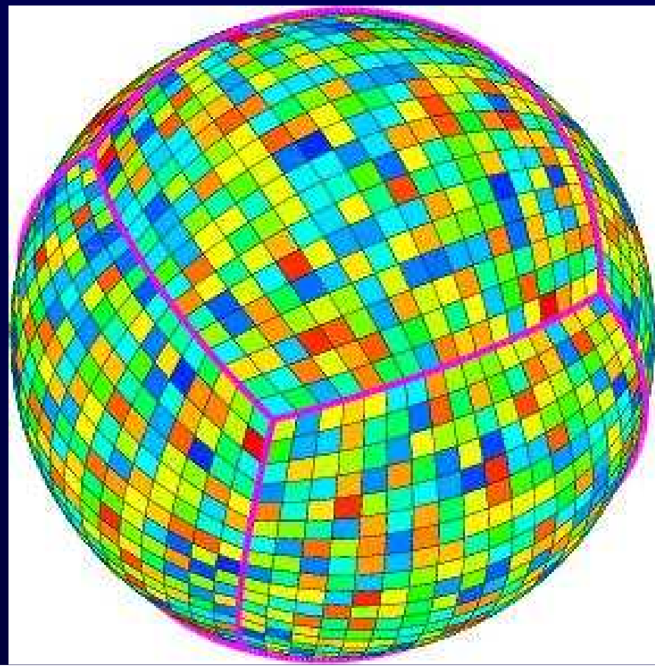
Earth Simulator Center
Japan Marine Science and Technology Center
Yokohama Institute For Earth Science

Pictures and data taken from
http://www.es.jamstec.go.jp/

640 processor nodes, each consisting of eight vector processors are connected as a high speed interconnection network.
The Earth Simulator is in 2004 the fastest supercomputer in the world

EarthSimulator





Performance on the Earth Simulator

- 5120 processors (640 blocks of 8), 10 terabytes of memory (10000 PCs of 1 Gb), NEC EV-6 vector processors (« vector pipe » of size 256)

- Three optimization levels: parallelism between blocks, inside each block, and vectorization

- MPI + vectorization (manual inlining of the code), no OpenMP

- Performance : vectorization 99.3 %, but short vectors

- 5 billion grid points (14.6 billion degrees of freedom) on 1944 processors (38 % of the machine)

- Performance 5 teraflops, memory 2.5 terabytes

- One person, 1 operation/sec: 160000 years, 6 billion people: 14 minutes

- SPECFEM3D won the Gordon Bell award at the SuperComputing’2003 conference

Part III
Some real 3D cases studied

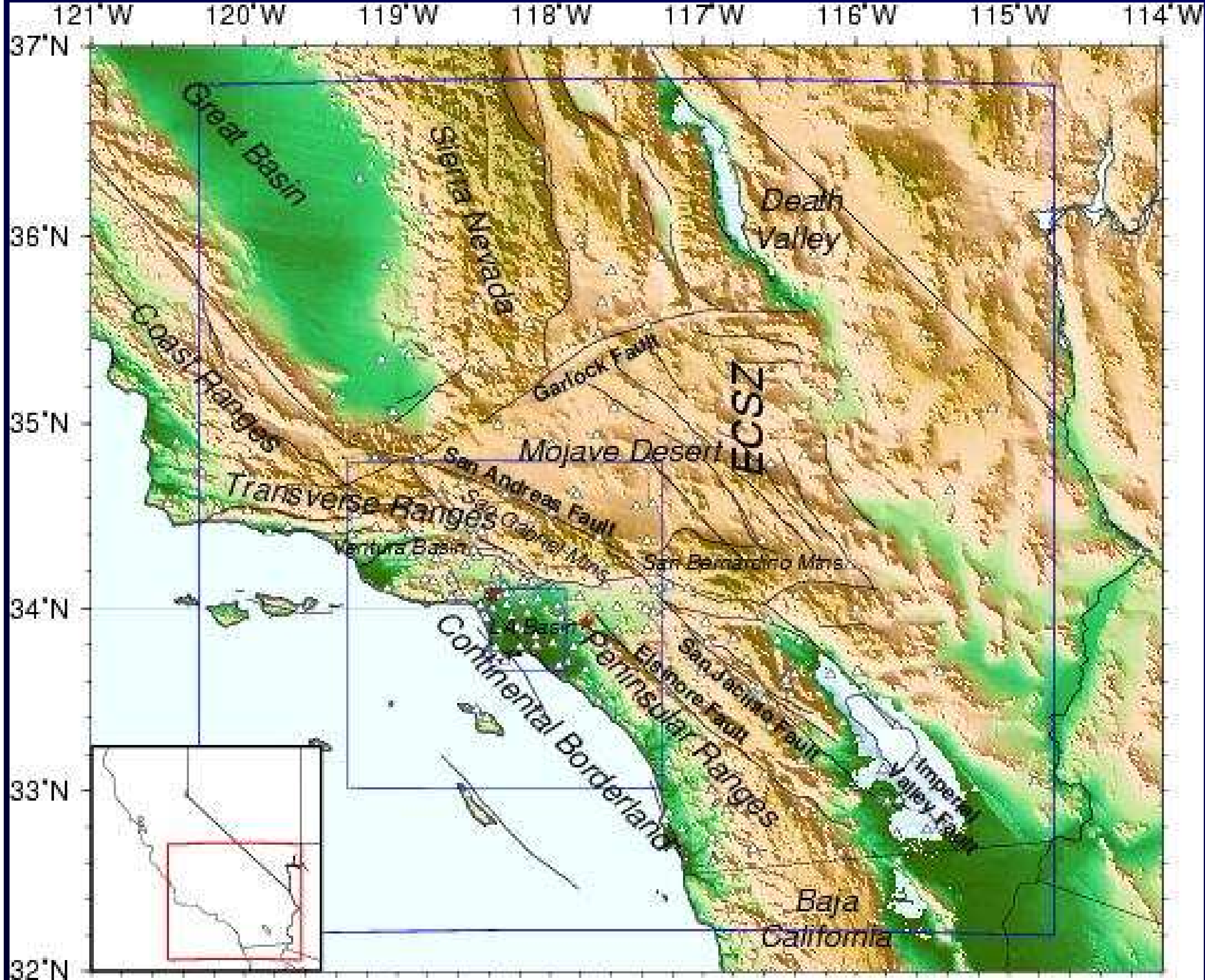
Los Angeles basin

The Basin Challenge

- Slow, laterally variable sedimentary layers
- Sharp transitions between sediments and basement, with complex shape (Magistrale et al. 1996, 2000, SCEC)
- Significant topography/bathymetry
- Shape of Moho (Zhu and Kanamori, 2000)
- Attenuation (very poorly constrained)
- Complex source models for large events (Wald et al.)
- Effect of oceans for Channel island stations (small)

Classically computed based on finite-difference (Olsen et al. 1996, Graves et al 1996, Peyrat et al 2001) or finite-element techniques (Bao et al., Bielak 1998, Moczo). Not all of above effects included.

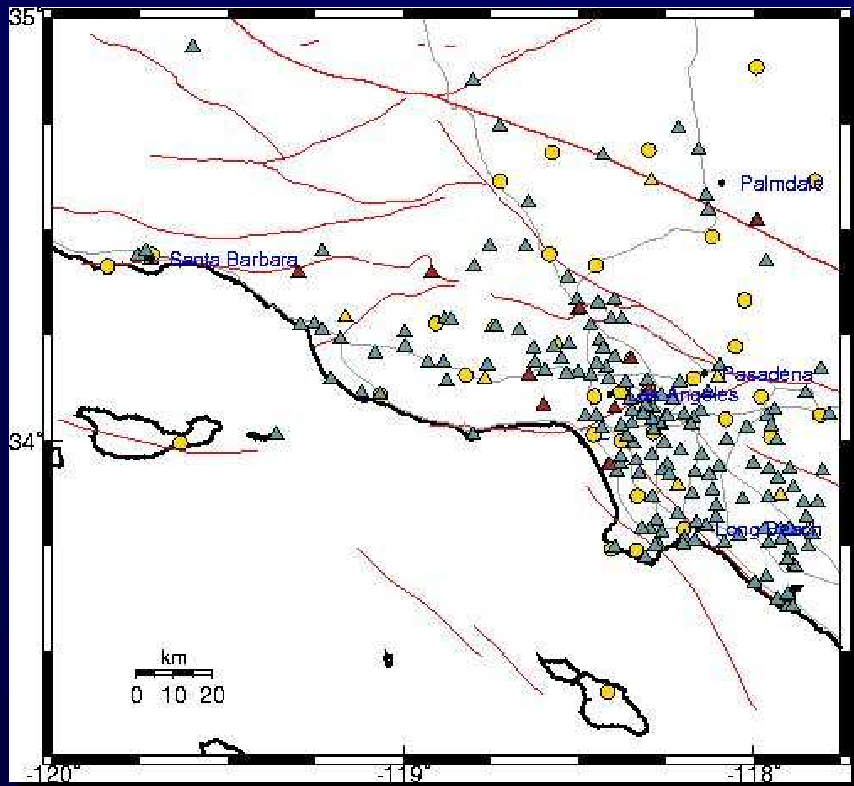
The Los Angeles region



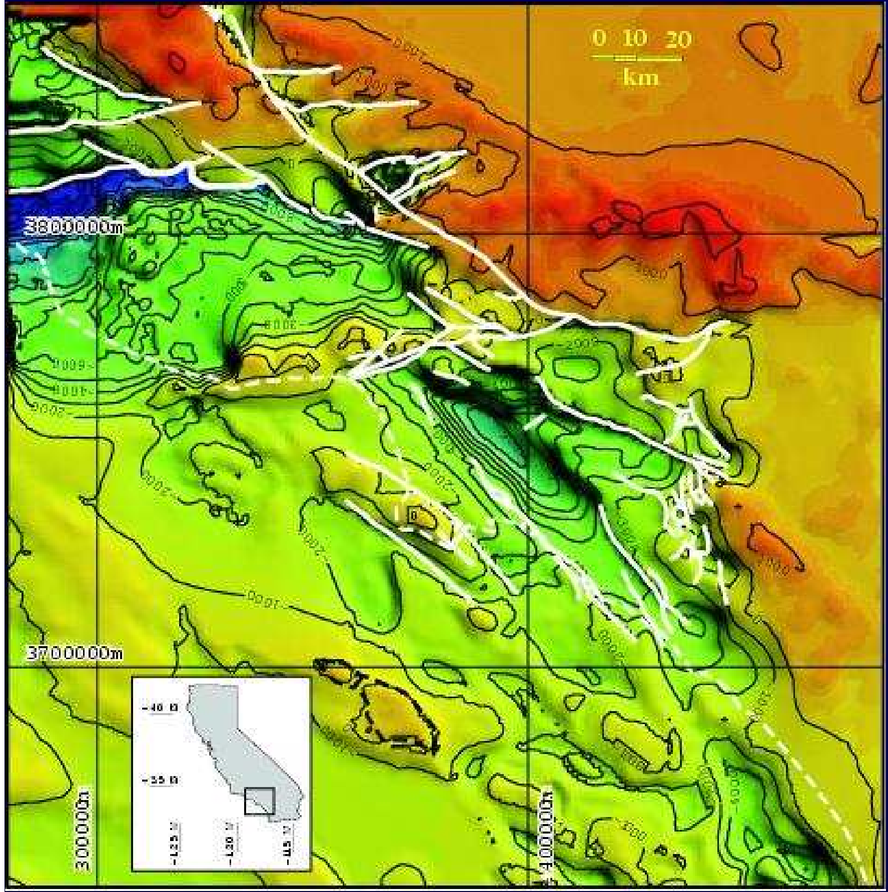
- Large region
- L.A. basin, San Fernando valley, Ventura basin
- Mountains, bathymetry
- ECSZ
- Blue rectangles
- Large number of stations (TriNet)

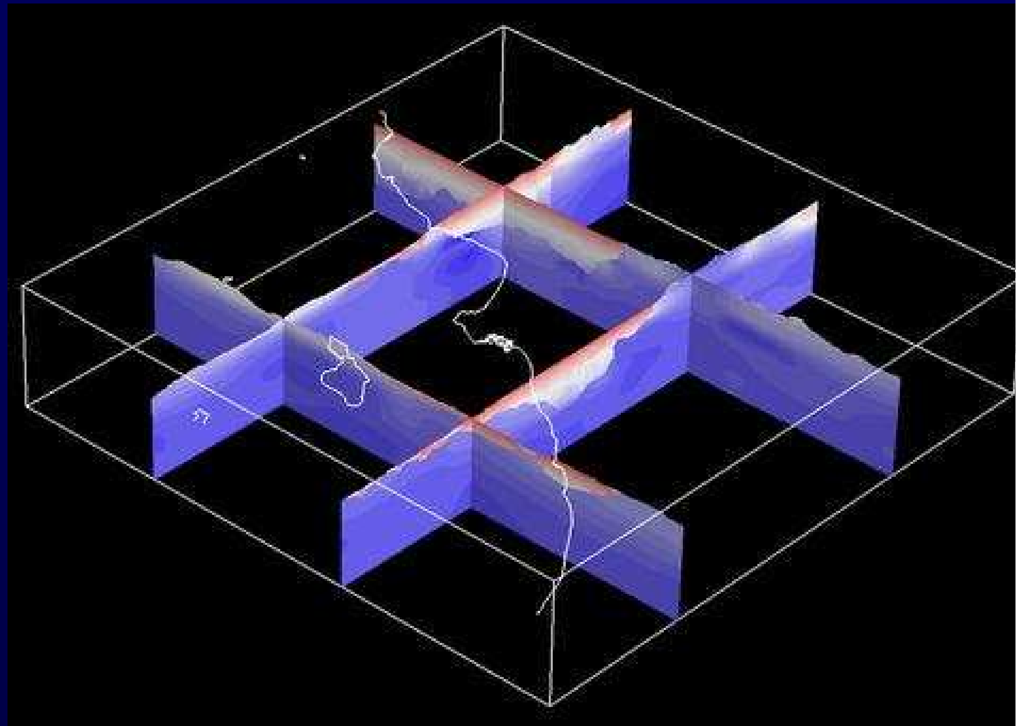
Introduction (Basins)

- Need accurate numerical methods to model seismic hazard – very densely populated areas
- Large and complex 3D models (e.g., L.A., Tokyo, Mexico)
- Wealth of high-quality data (TriNet)



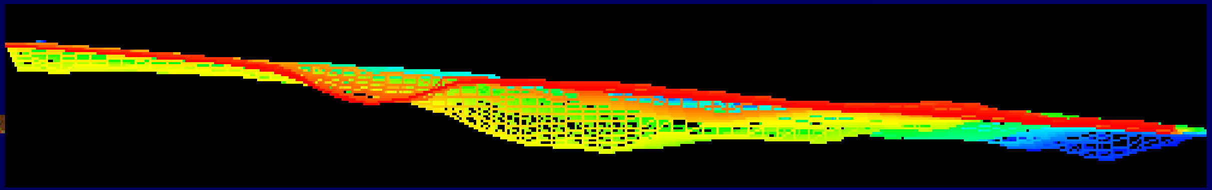
Harvard LA basin model

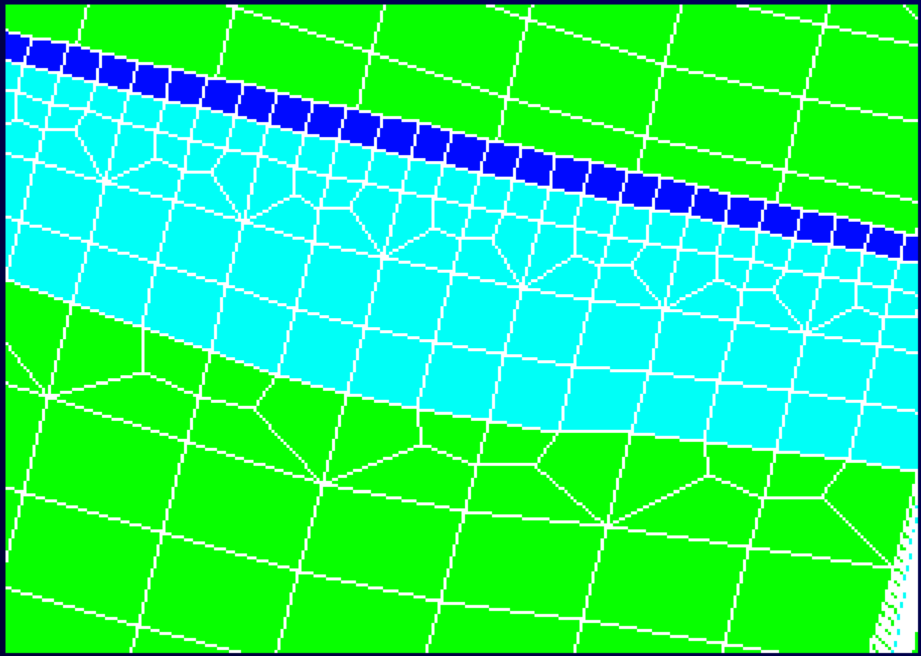


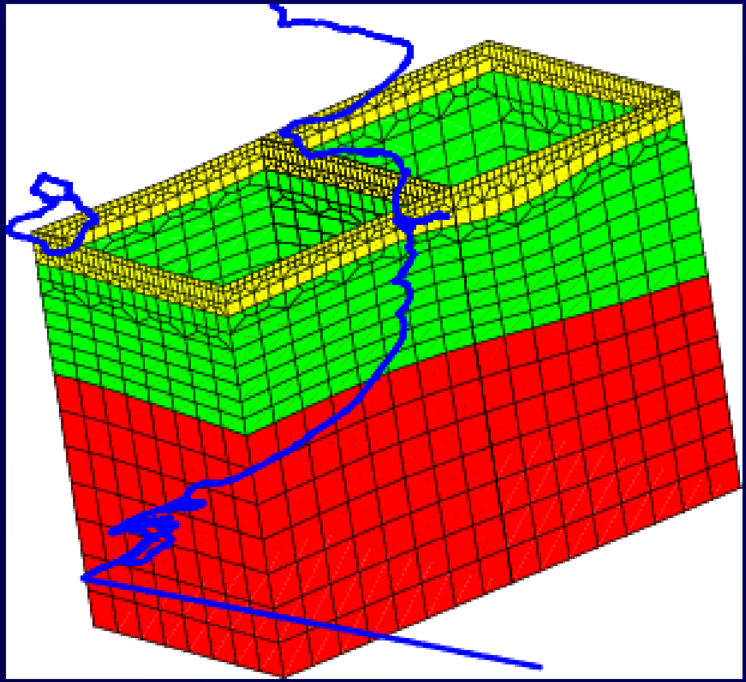


- 20,000 km of petroleum industry profiles
- 300+ well logs *(Süss and Shaw, JGR, 2003)*
- 85,000 direct velocity measurements

Final Mesh



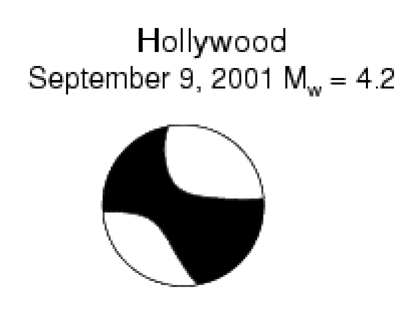




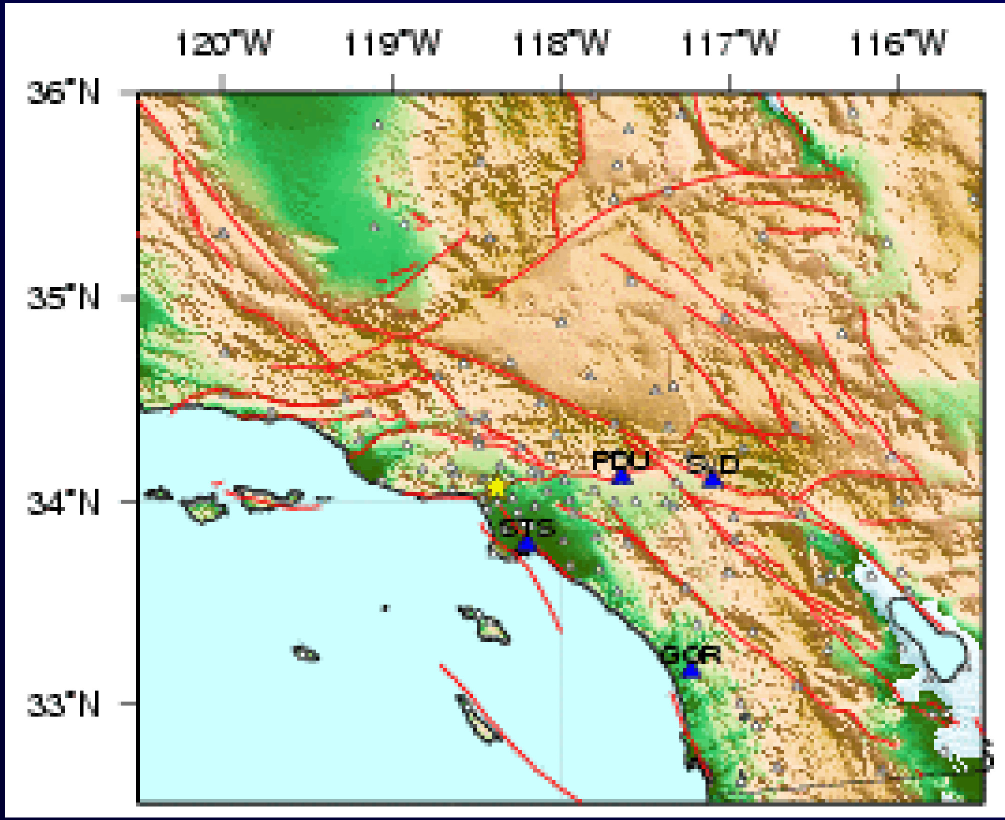
Difficulties:

- Adapt the mesh to topography, bathymetry, bottom part of basement, and 3D shape of Moho
- Implement coarsening with depth to save CPU and memory

Hollywood Earthquake

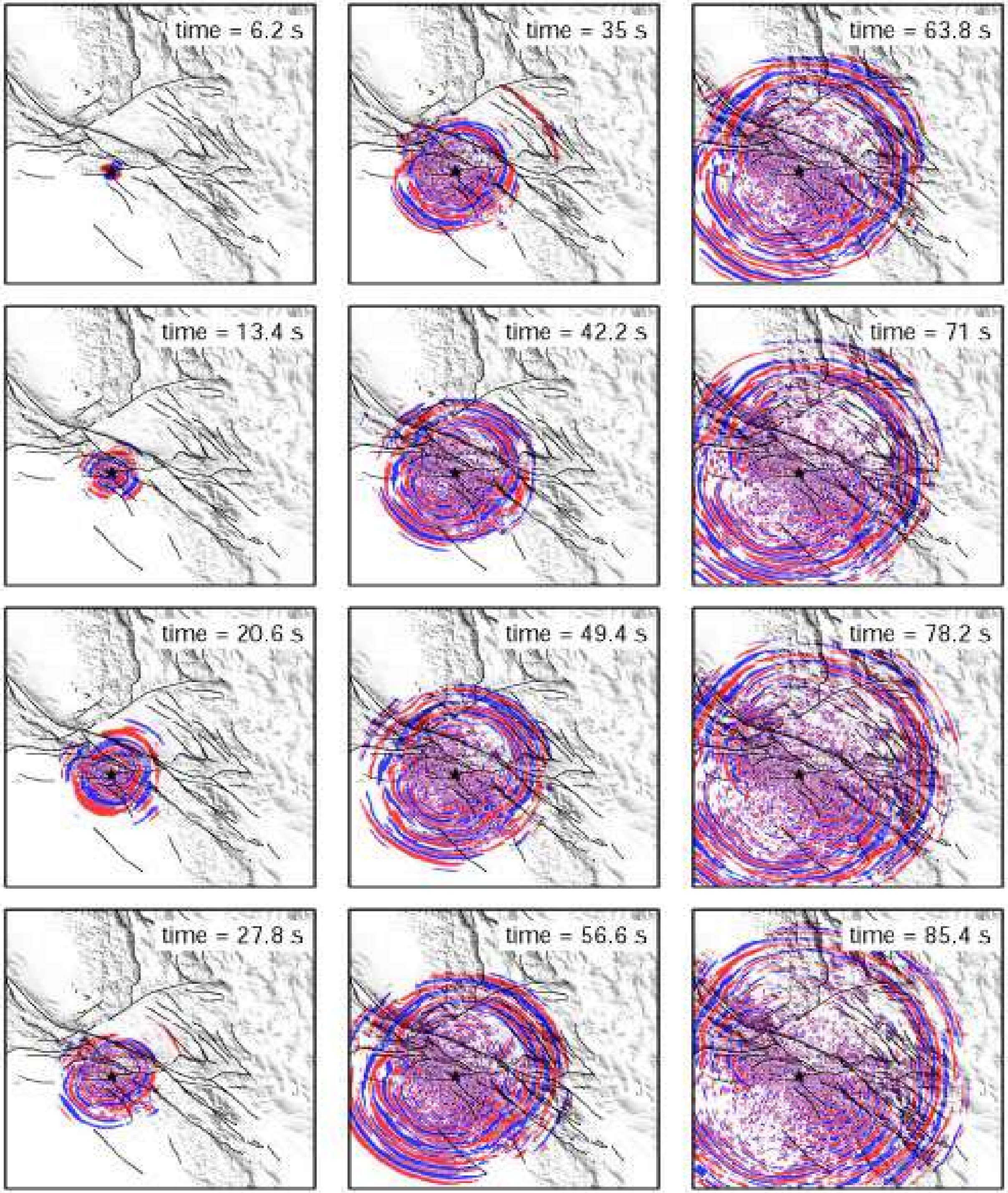


Small M 4.2 earthquake on Sept 9, 2001

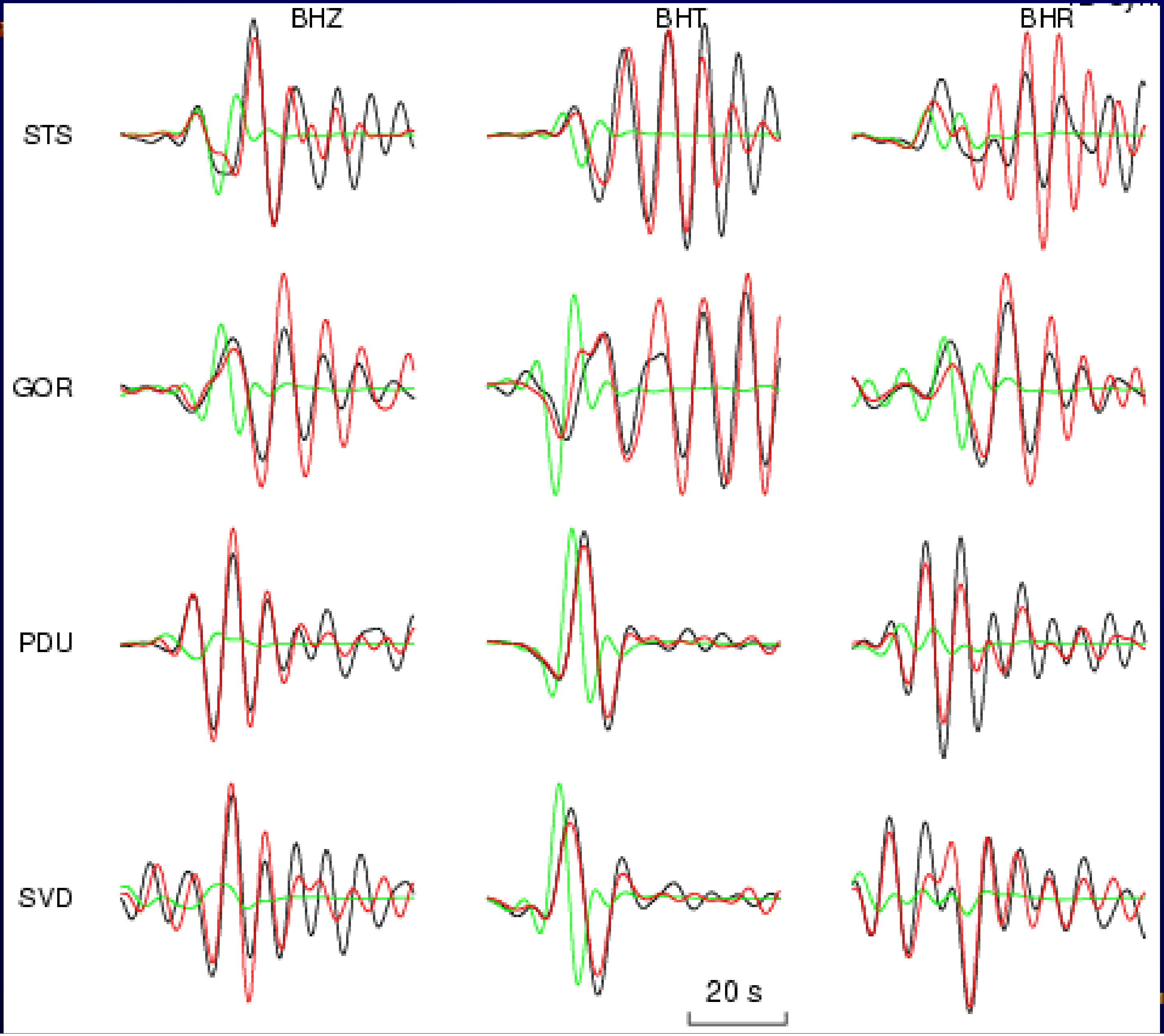


Amplification in basin

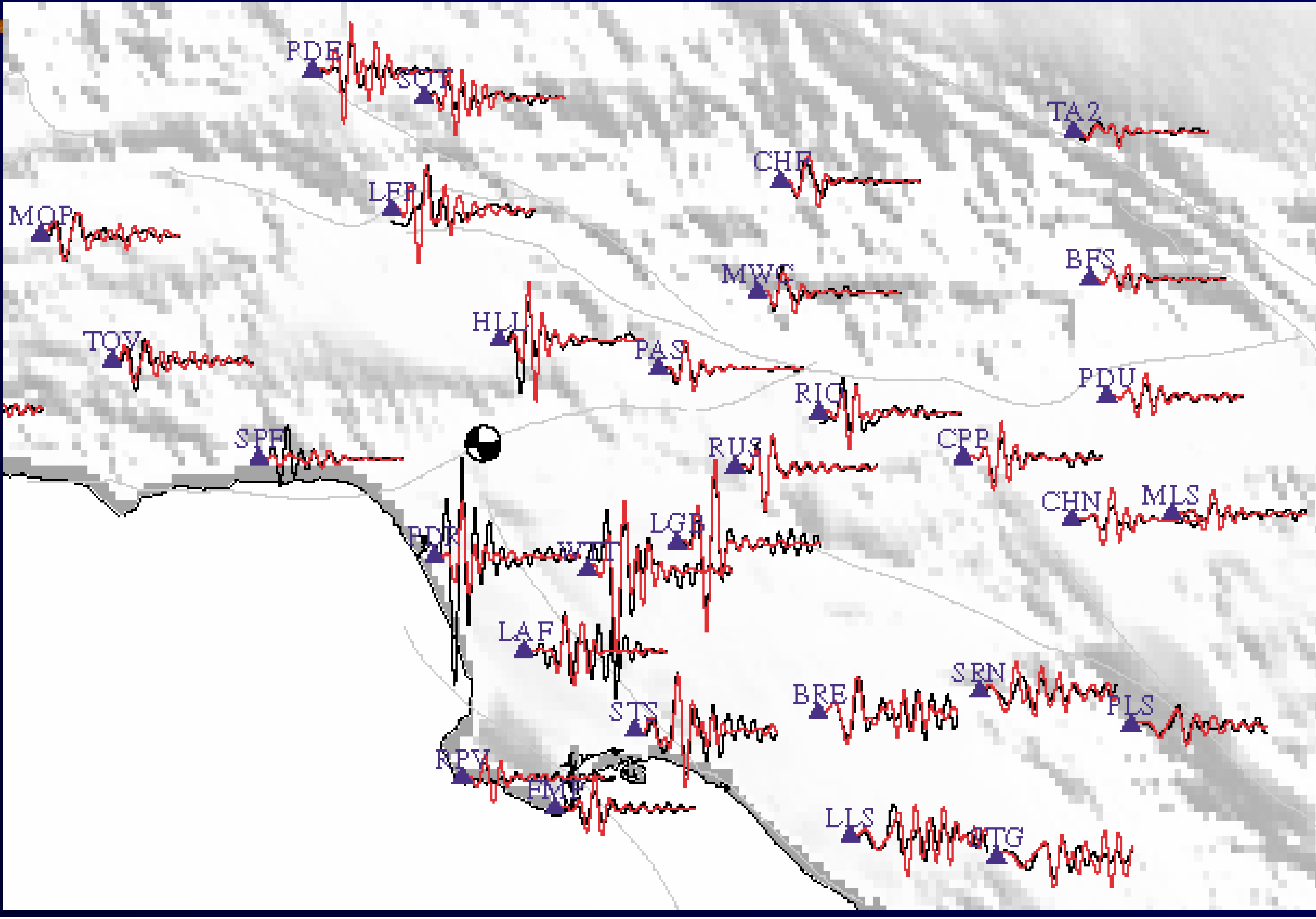
Snapshots


time = 6.2 s
time = 35 s
time = 63.8 s
time = 13.4 s
time = 42.2 s
time = 71 s
time = 20.6 s
time = 49.4 s
time = 78.2 s
time = 27.8 s
time = 56.6 s
time = 85.4 s

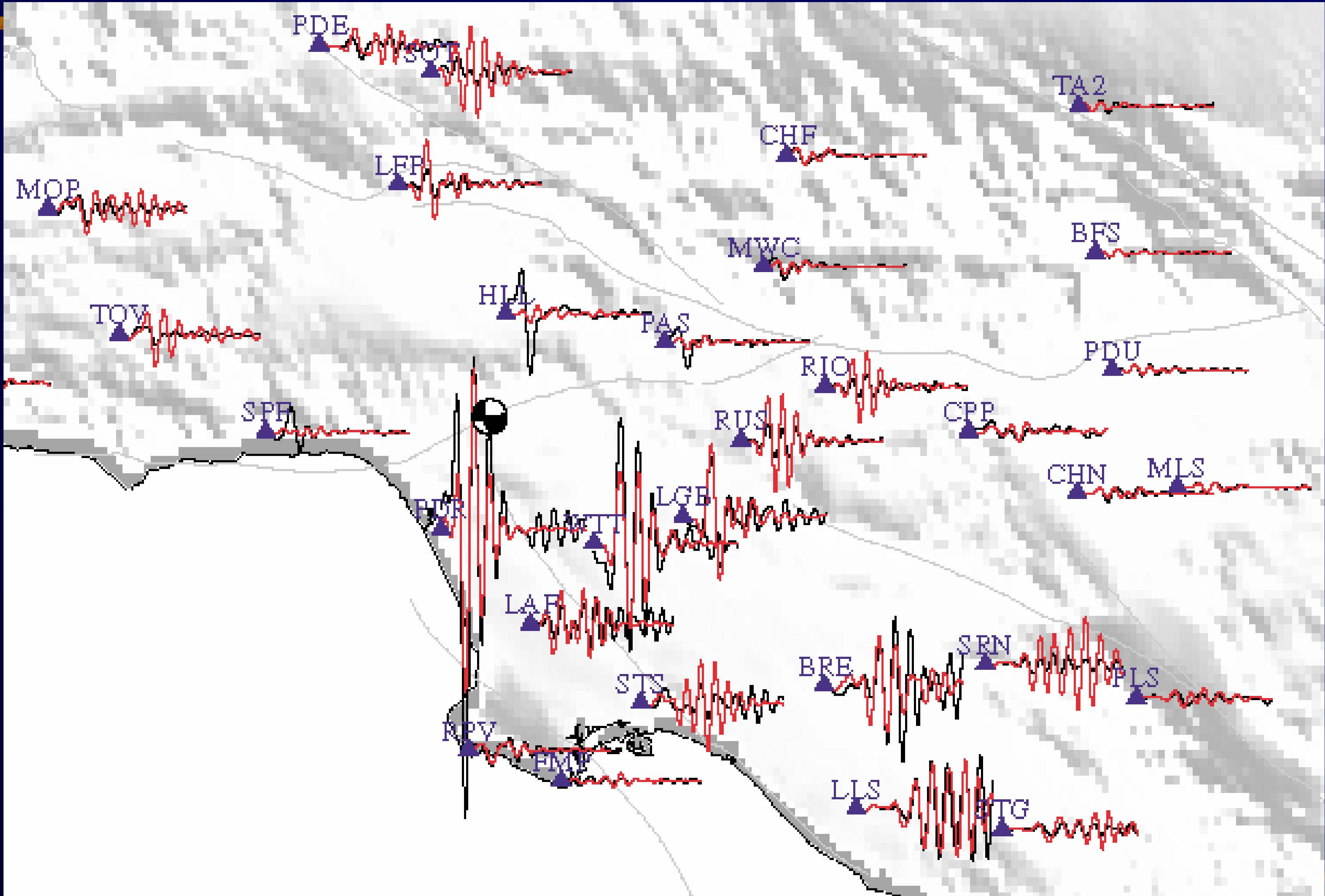
Data vs. 3D and 1D at 6 sec.



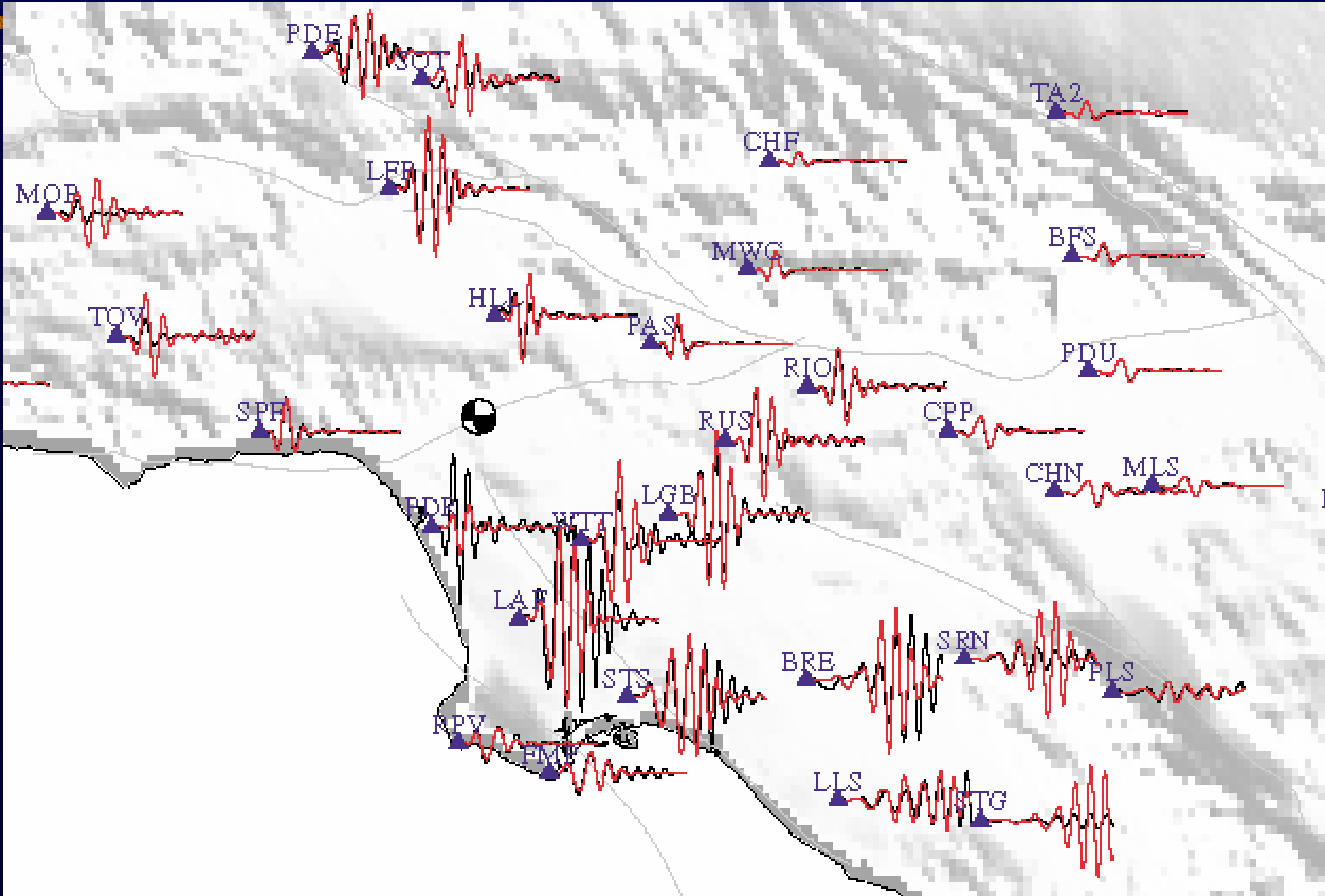
Hollywood vertical



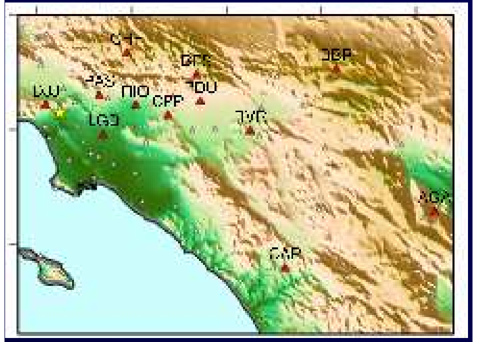
Hollywood radial

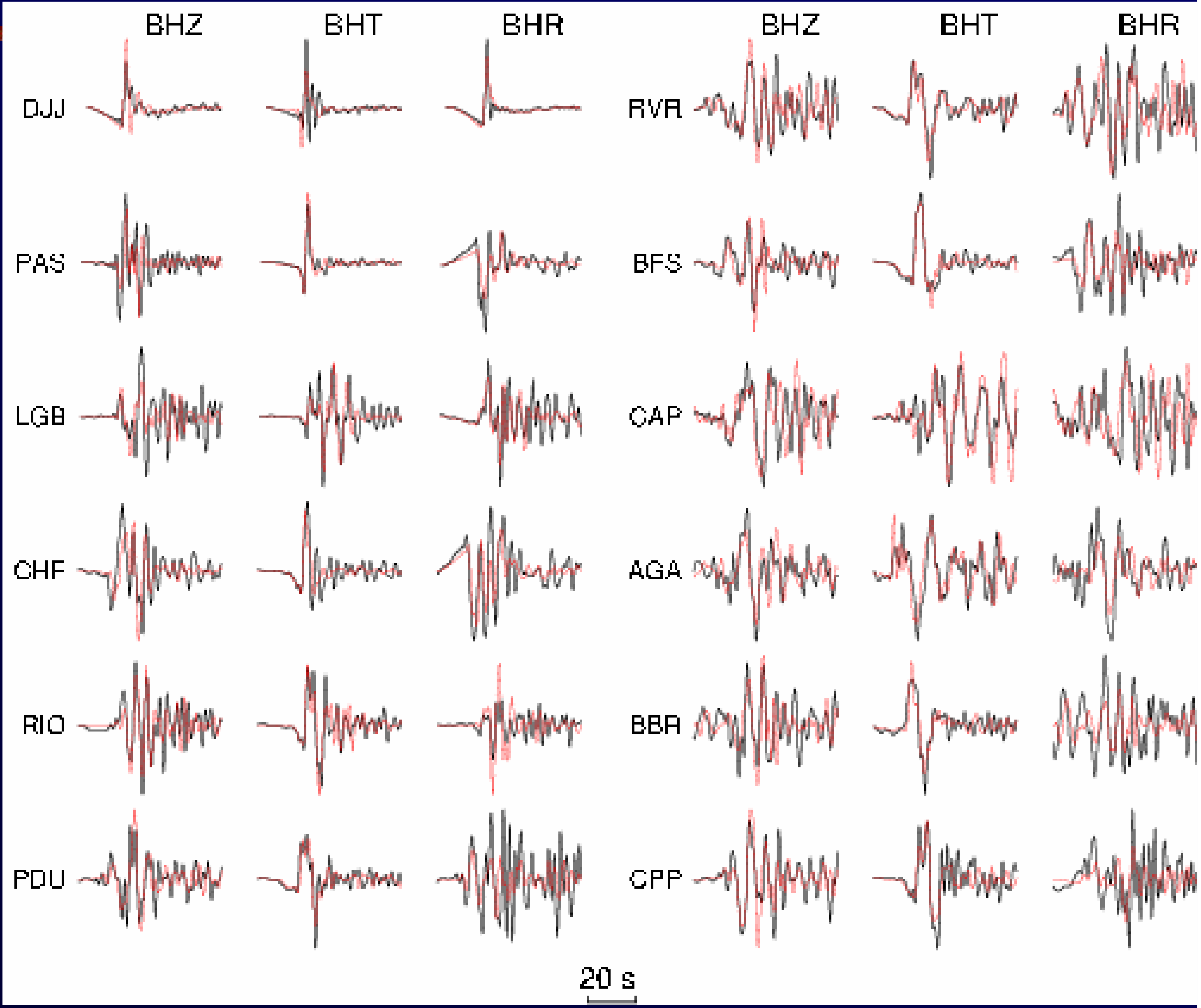


Hollywood transverse

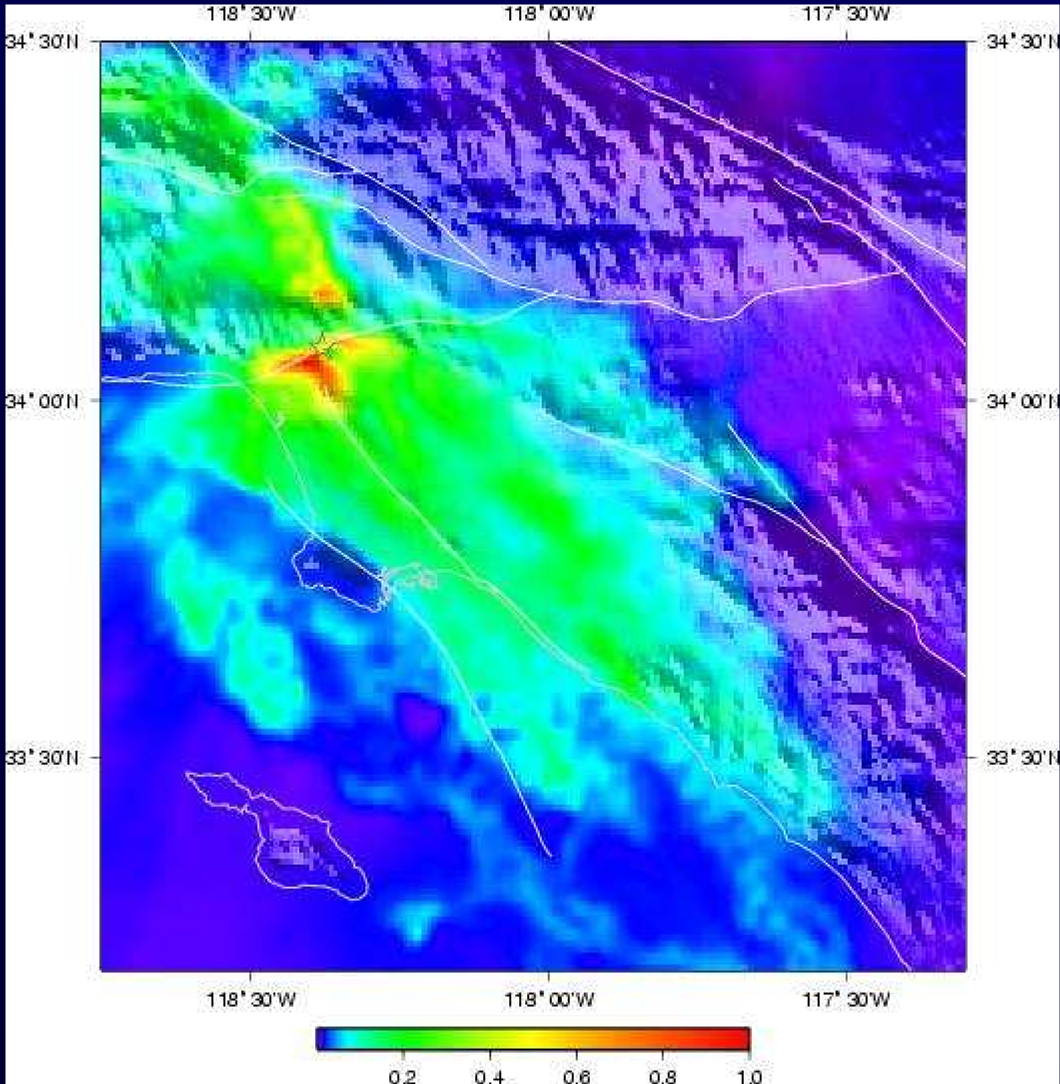


Hollywood 2 s at 12 stations



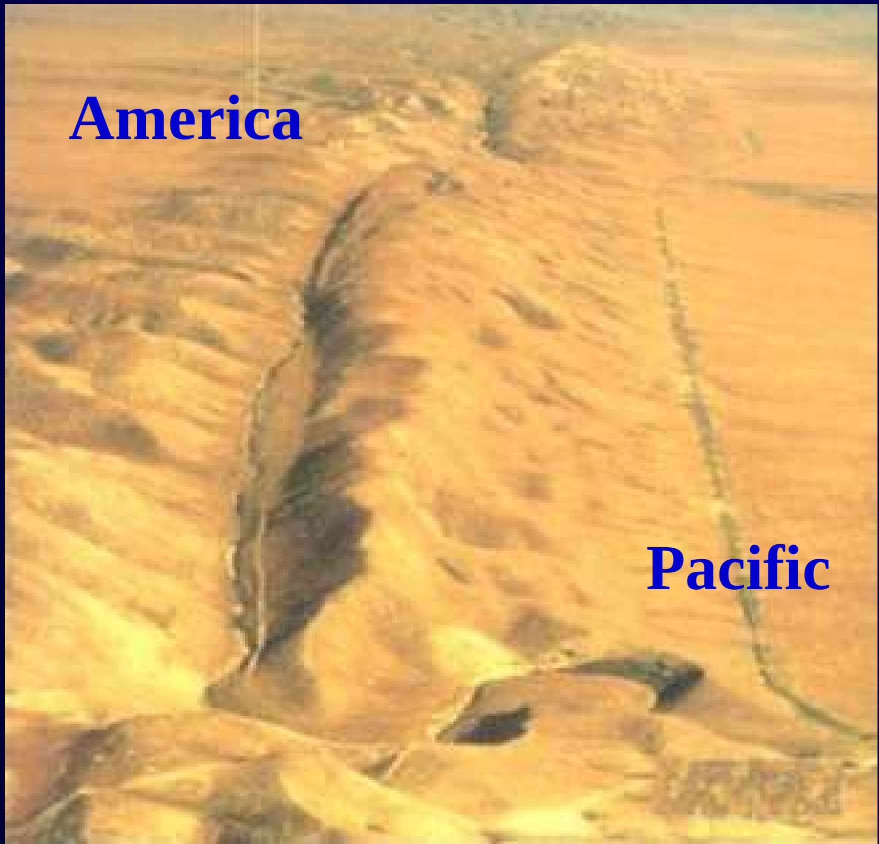


Peak ground acceleration



- Maximum of norm of acceleration
- Consistent with shape of basin
- Transfer from L.A. basin to San Fernando
- Almost no shaking in Palos Verdes
- Nothing in mountains

San Andreas – January 9, 1857

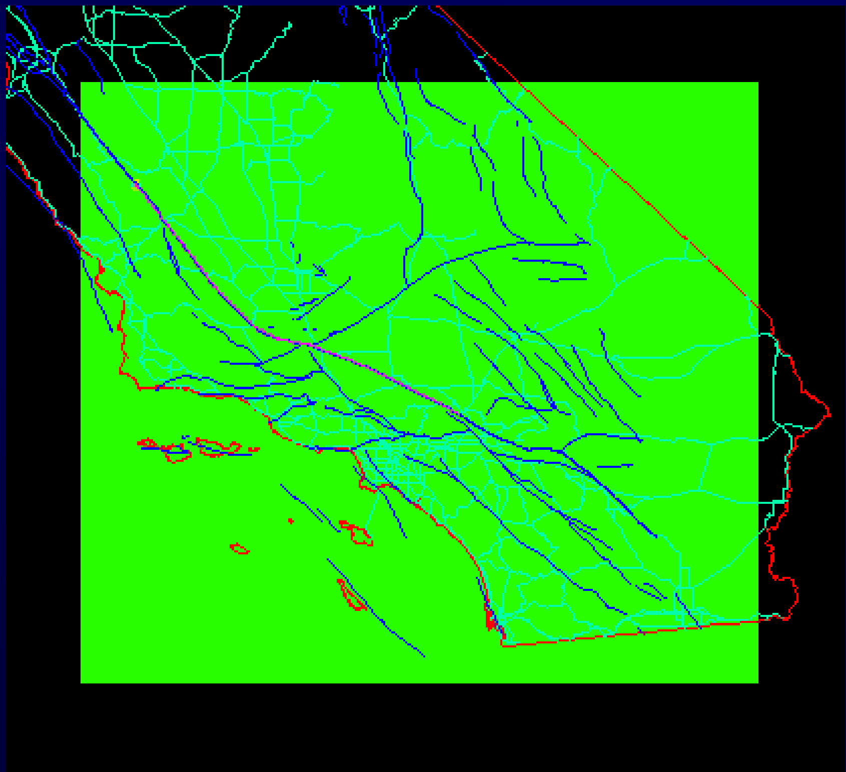


Carrizo Plain, USA, horizontal scale ≅ 200 m

Vertical scale approximately 1 km

Carrizo Plain, San Andreas Fault, California, USA

Earthquakes at the regional scale



Scale approximately 500 km

3D spectral-element method (SEM)

Conclusions (Basins)

- We have demonstrated the flexibility and accuracy of the spectral-element method for seismic wave propagation in 3D basins models
- Relatively easy to implement on parallel computers, and very efficient – e.g., PC Beowulf cluster
- Three components down to 2 seconds, good fit
- Can handle complex 3D models, attenuation, topography, 3D shape of Moho, oceans
- We are now limited by knowledge of model, not by the method

  $\Rightarrow$ Will give us the ability to test and improve models

  $\Rightarrow$ Will improve our ability to assess seismic hazard

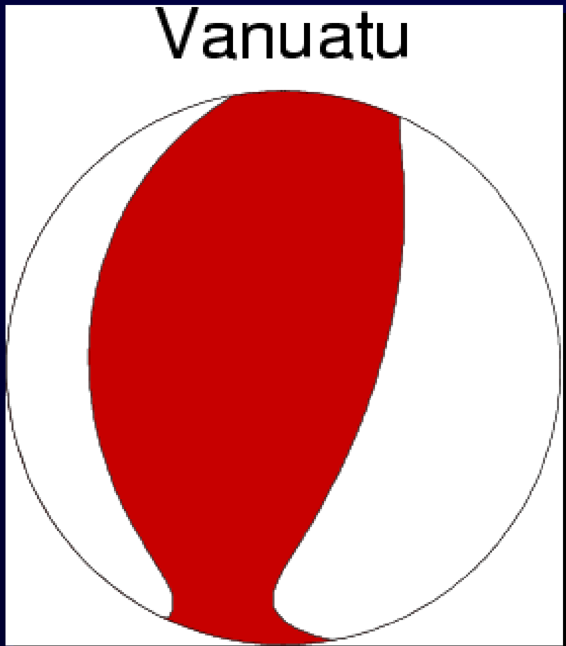
Global Earth: large earthquake in Vanuatu

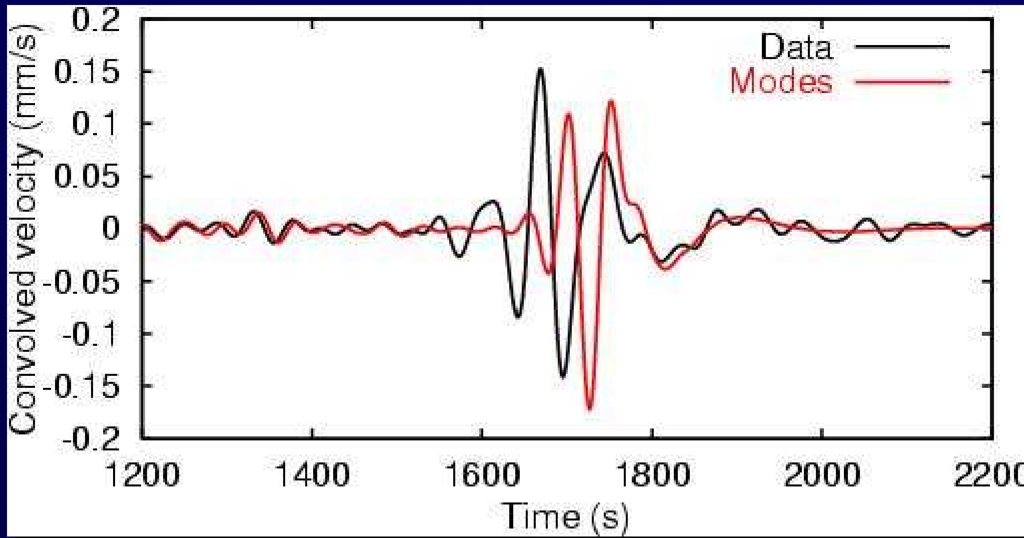
Vanuatu Earthquake in Japan

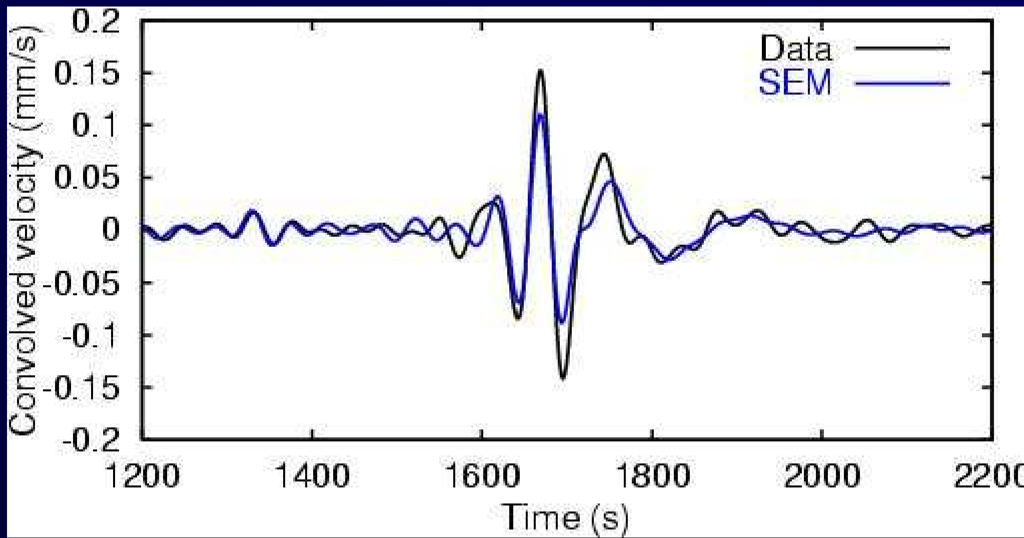
- Vertical component (Rayleigh wave)

  Mostly oceanic path

  Depth 15 km





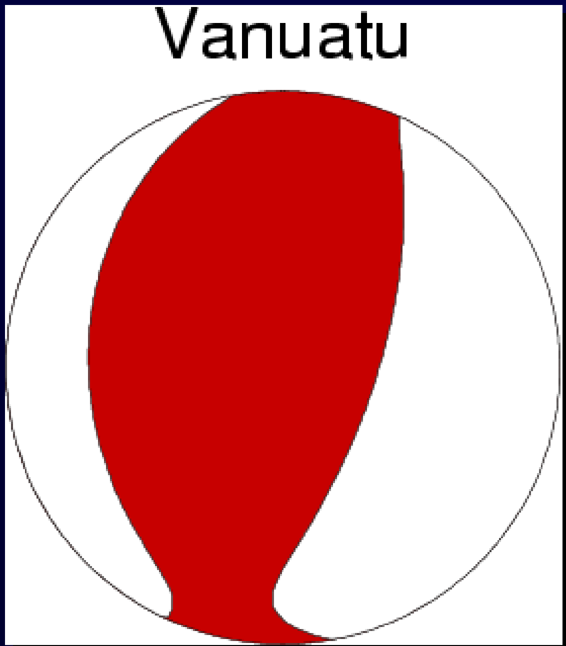


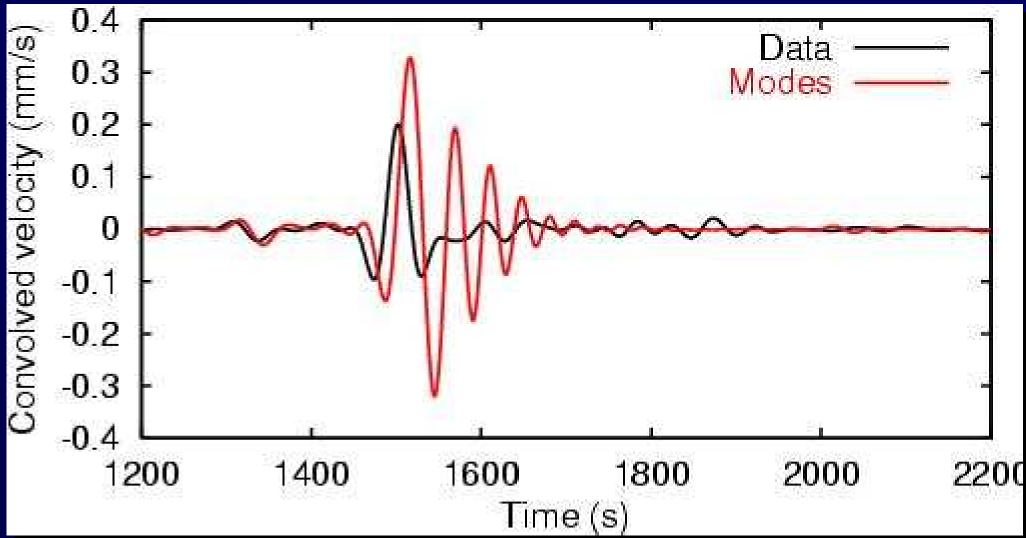
Vanuatu Earthquake in Japan

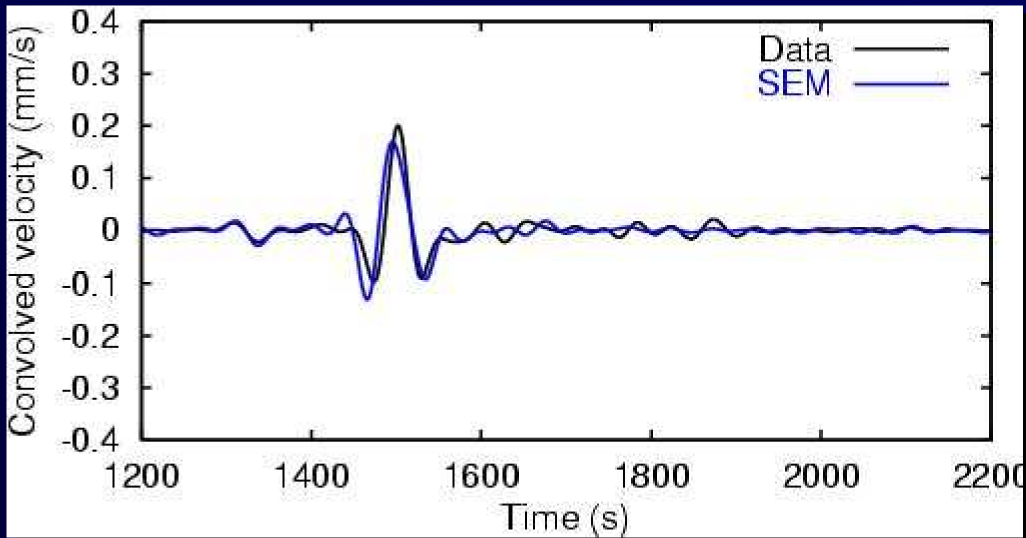
- Transverse component (Love wave)

  Mostly oceanic path

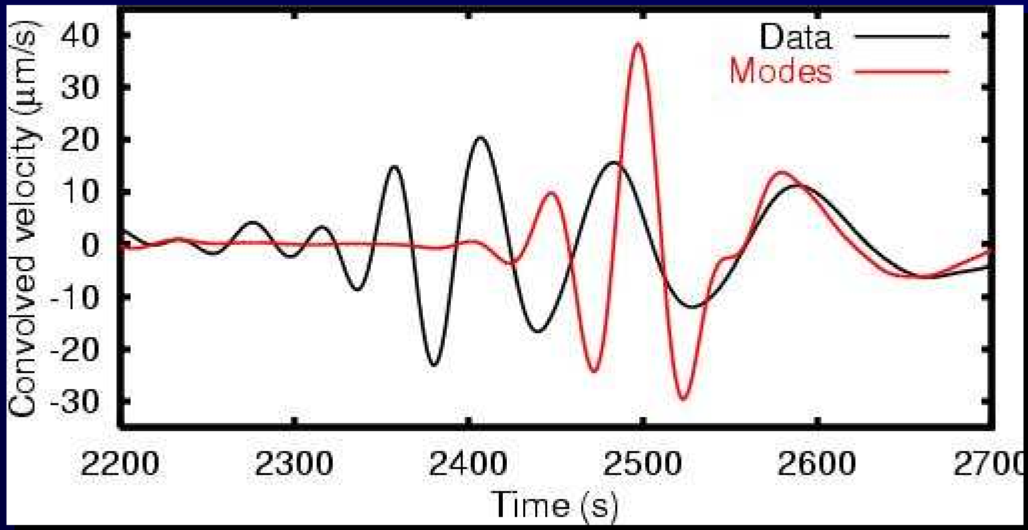
  Depth 15 km

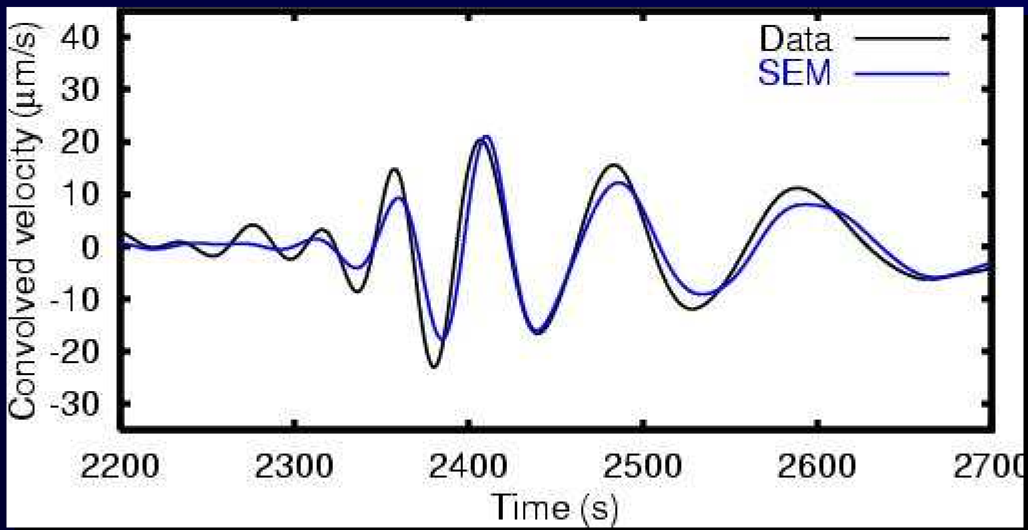






Vanuatu Earthquake in Pasadena





Mostly oceanic path

Delay is 85 s
for Rayleigh wave

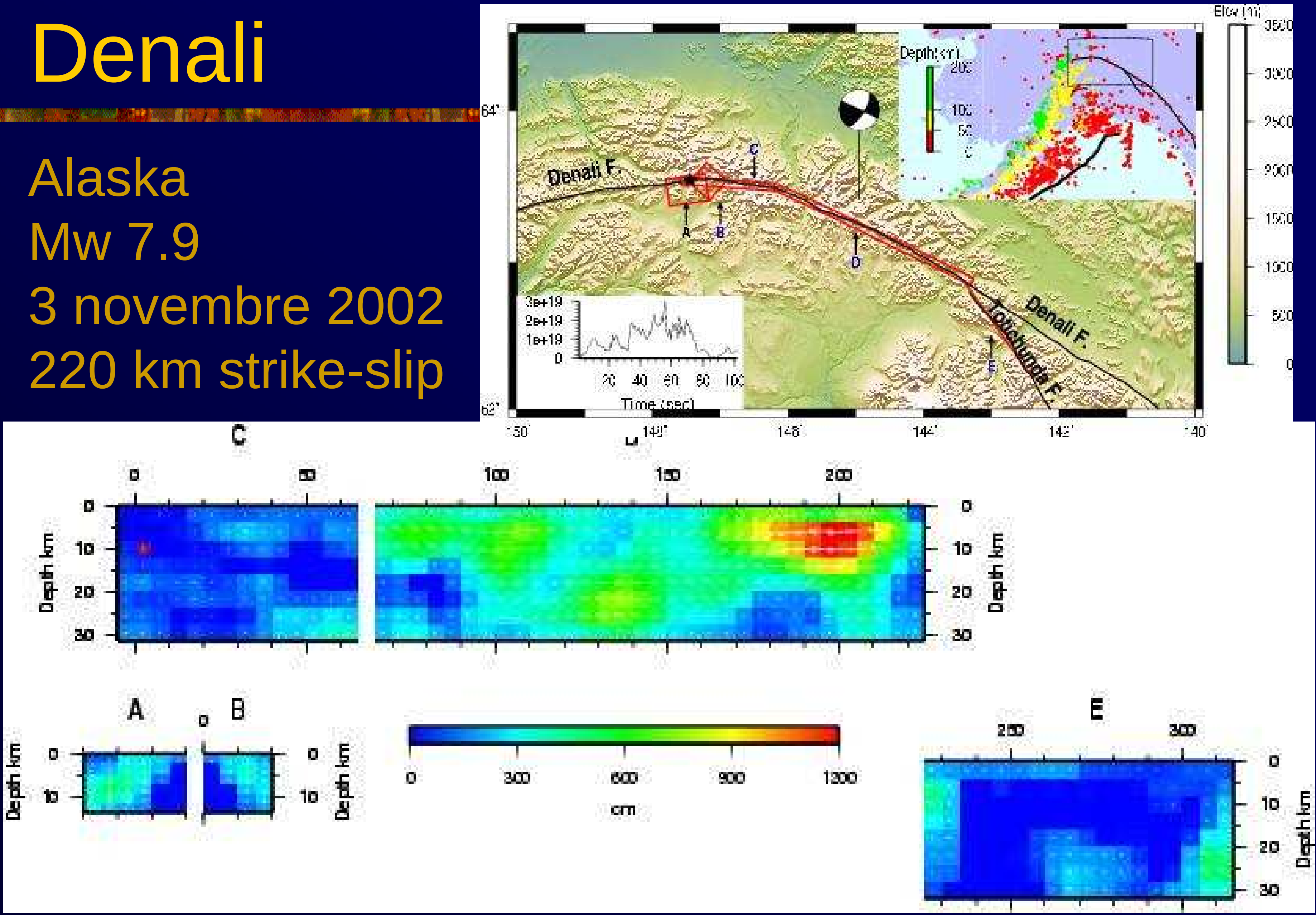
Conclusions (Global Earth)

- Large machines like the EarthSimulator allow us to compute global 3D models with full complexity down to a few seconds
- Earth models are not accurate enough
- Worse for surface waves, crustal model not well known
- Will ultimately need to perform tomographic inversion based upon fully 3D synthetics
- Relatively easy to implement on parallel computers, and very efficient – e.g., PC Beowulf cluster

Denali (Alaska) earthquake

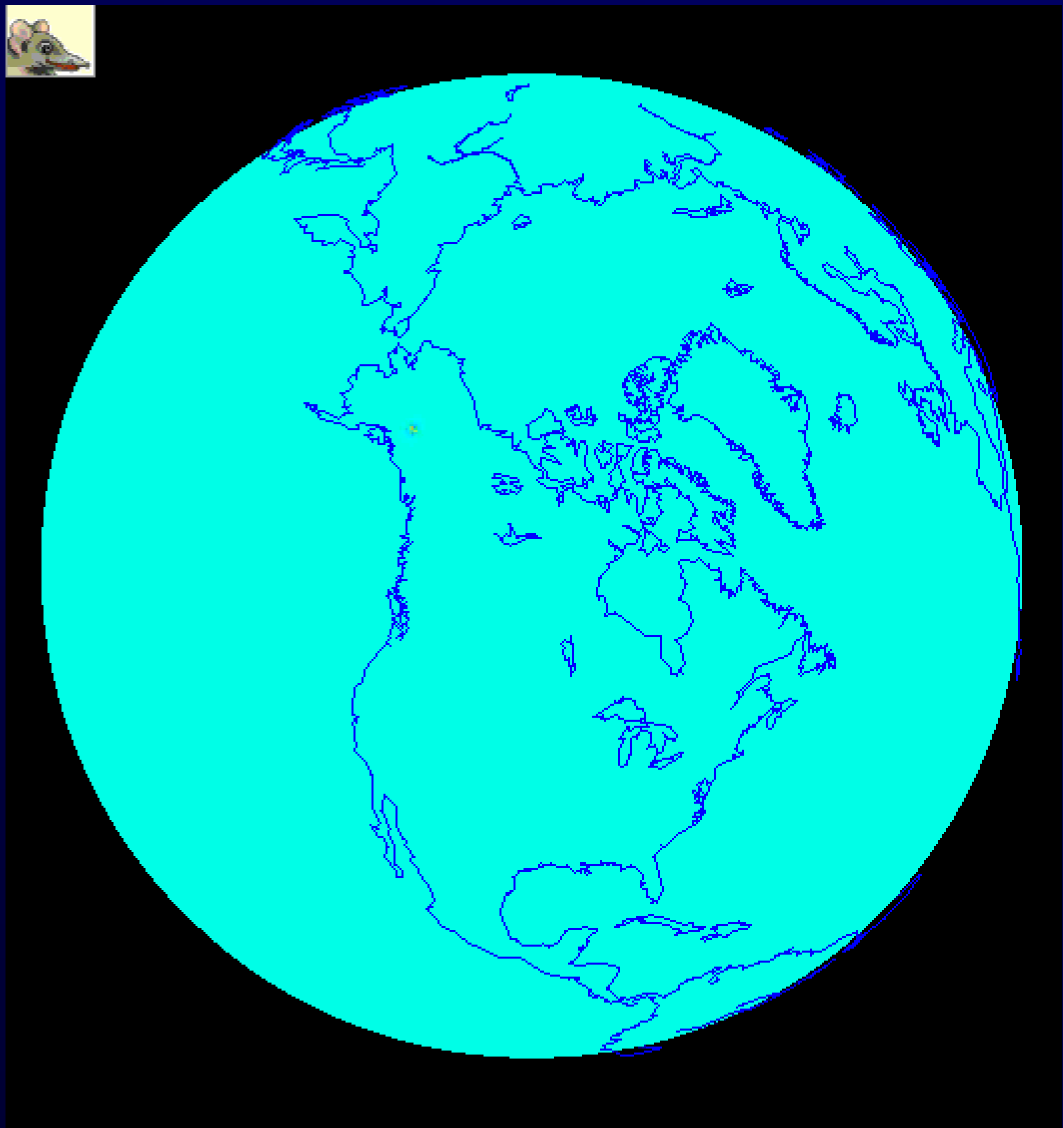
Denali

Alaska
Mw 7.9
3 novembre 2002
220 km strike-slip

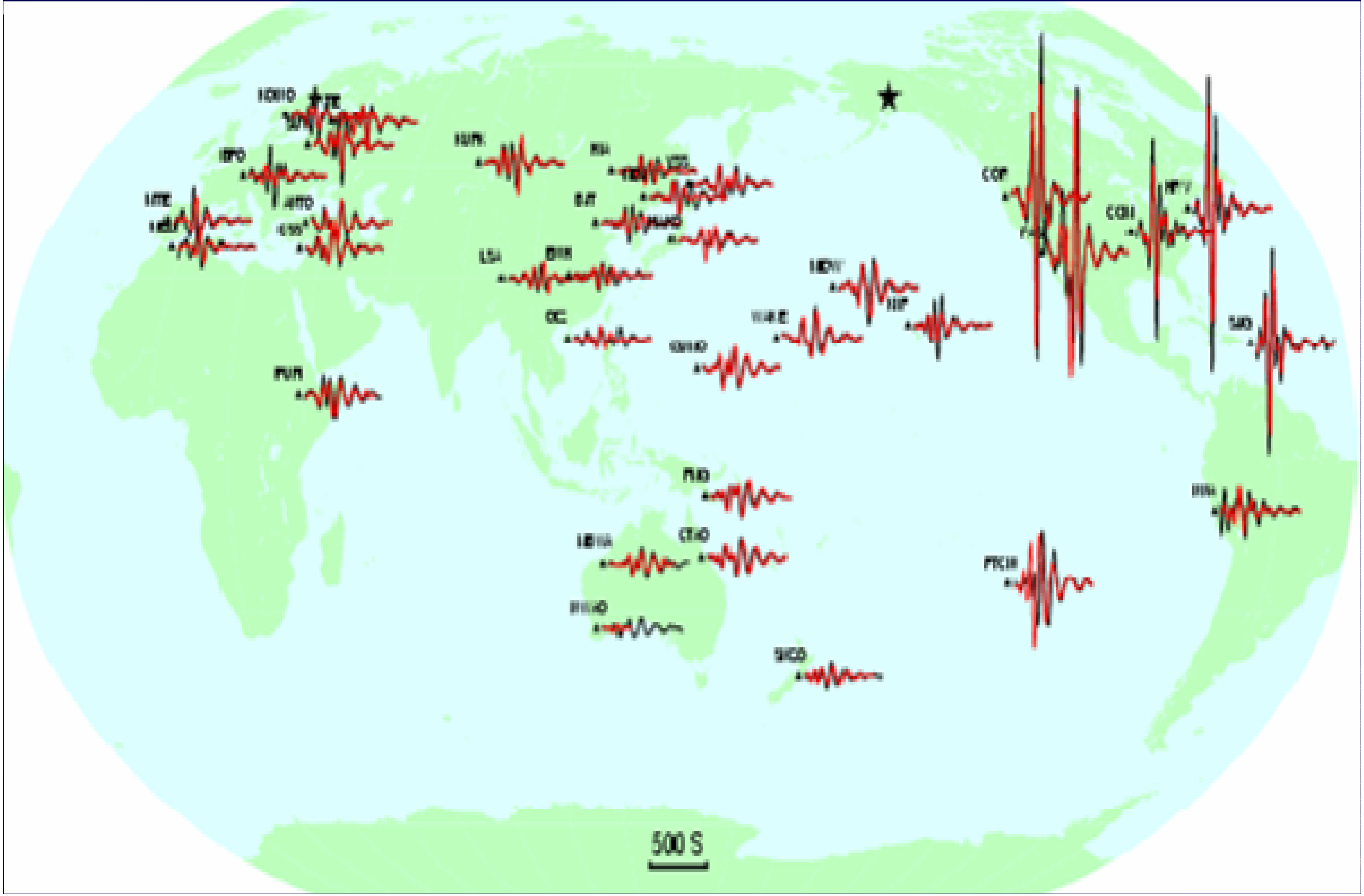


Inversion source (ondes de volume télésismiques + déplacements en surface) par Ji Chen et al.

Denali, Alaska – Rayleigh wave

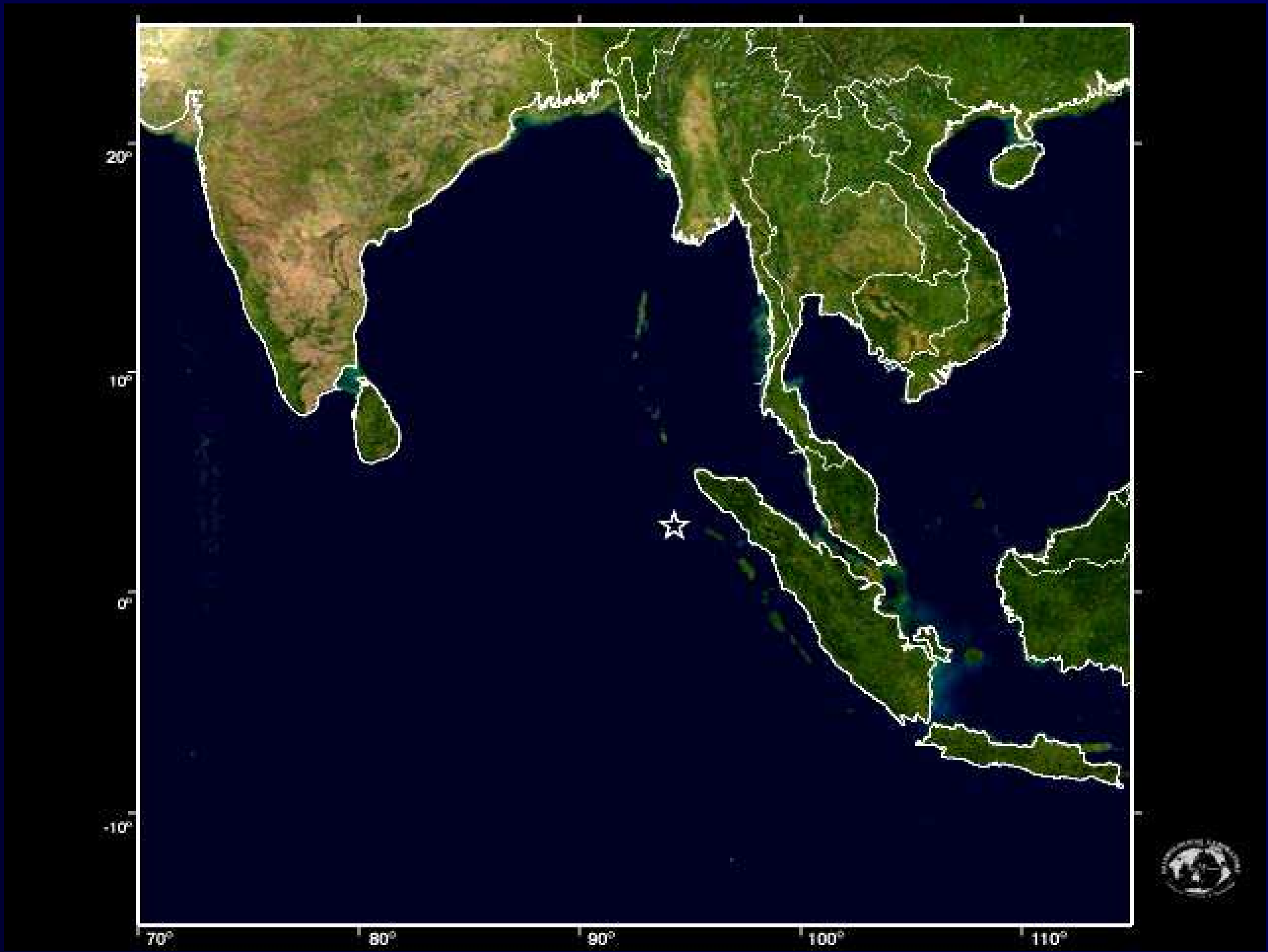


Rayleigh wave



Sumatra earthquake (but no tsunami)

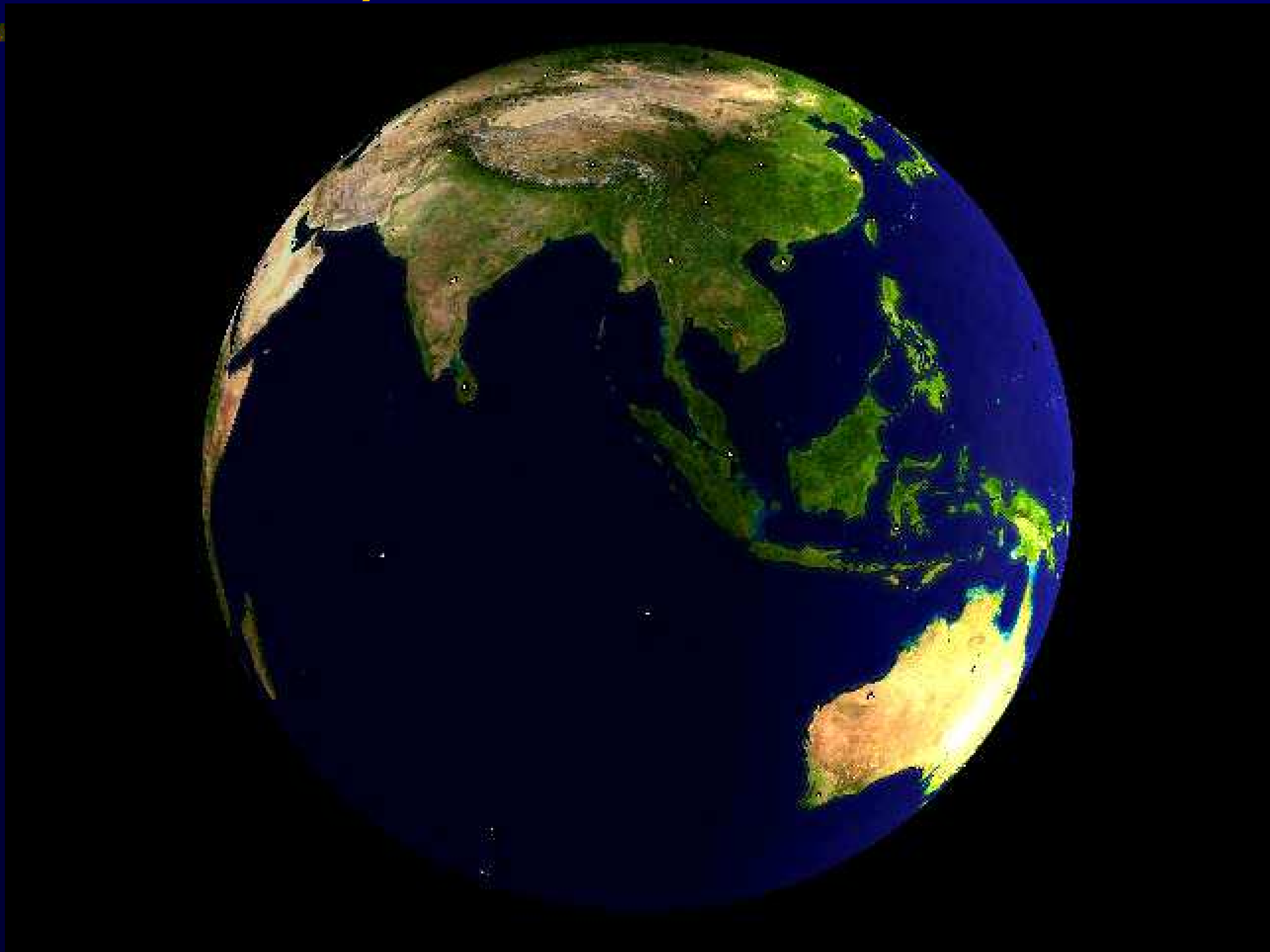
Dec 26, 2004 Sumatra event



vertical component of velocity at periods of
10 s and longer on a regional scale

From Tromp et al., 2005

Dec 26, 2004 Sumatra event



From Tromp et al., 2005

the two dispersive Rayleigh waves R1 and R2 at the scale of the globe (vertical component of velocity, periods of 18 s and longer), including the caustics and refocusing at the antipode and pode (3 hours worth of data)