



The Abdus Salam
International Centre for Theoretical Physics



SMR.1824 - 2

**13th International Workshop on
Computational Physics and Materials Science:
Total Energy and Force Methods**

11 - 13 January 2007

Pressure Induced Complexity in Light Alkalies

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These are preliminary lecture notes, intended only for distribution to participants

Pressure Induced Complexity in Light Alkalies



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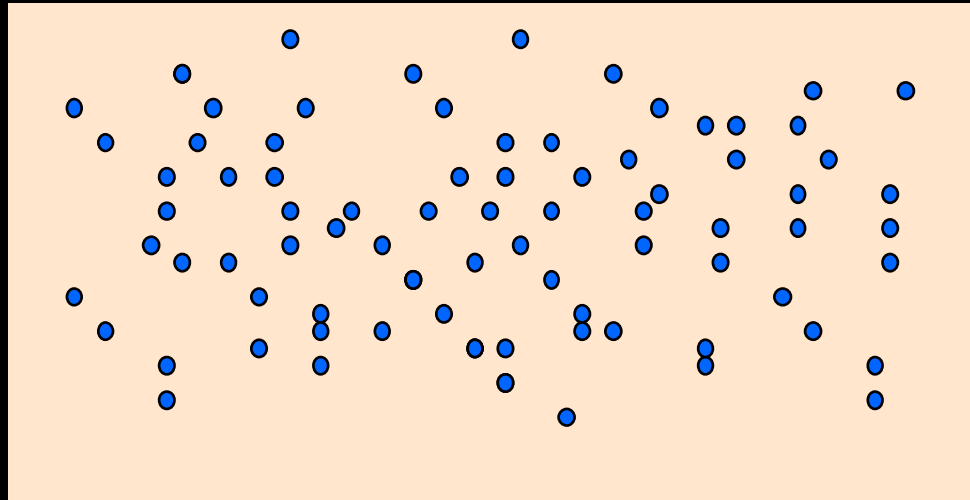
*13th International Workshop on
Computational Physics and Materials Science:
Total Energy and Force Methods*

Trieste, January 11

Outline

1. Introduction.
2. Fermi Surface Deformation in Lithium under Pressure:
 - bcc to fcc structural transition: Hume-Rothery mechanism.
 - fcc to *c/16* transition: Peierls-like mechanism.
3. Dynamic Screening and Optical Properties.
4. Anomalous Melting curve of Lithium?
5. Conclusions.

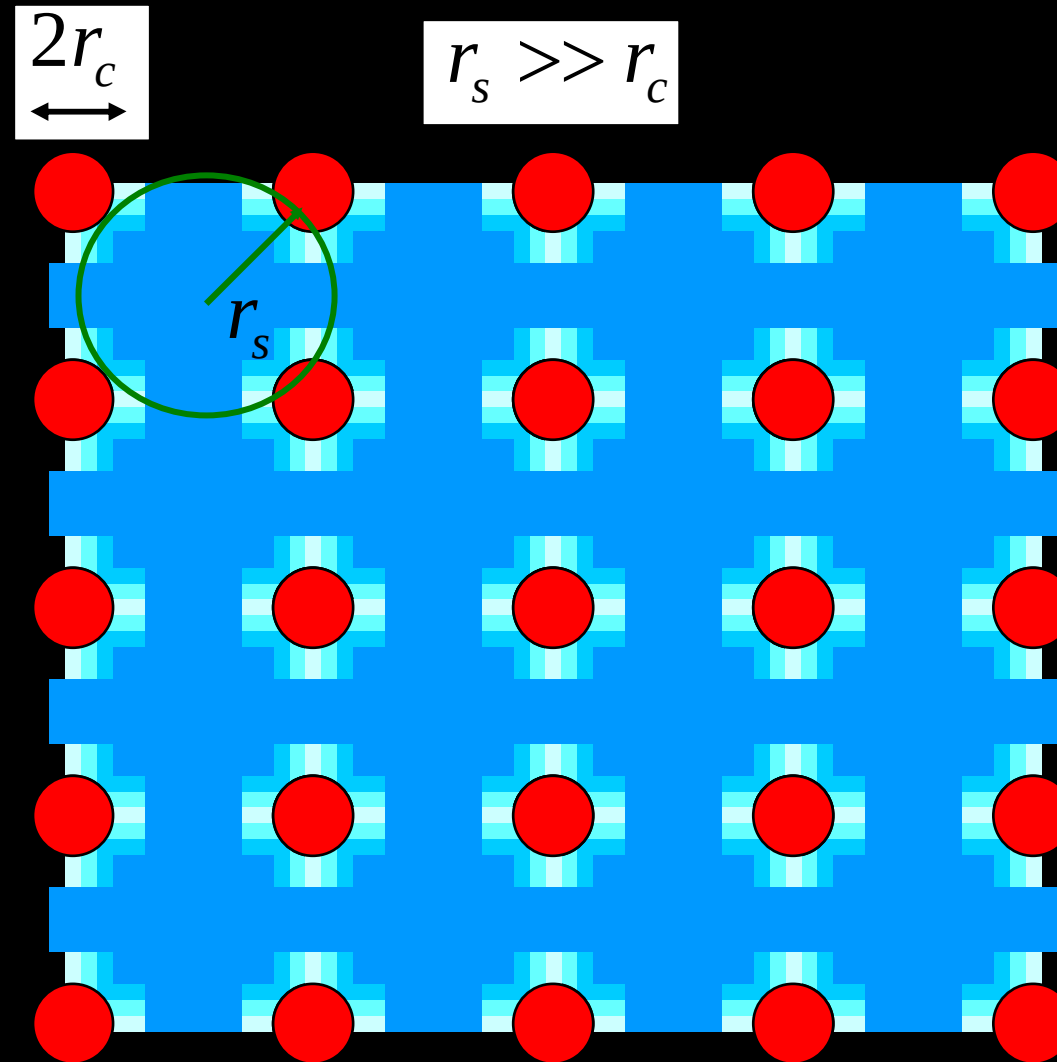
Elementary Energetics: The Electron Gas



$$\frac{E}{N} = \left\{ \frac{2.21}{r_s^2} - \frac{0.916}{r_s} + E_{\text{cor}} \right\} \text{ Ryd}$$

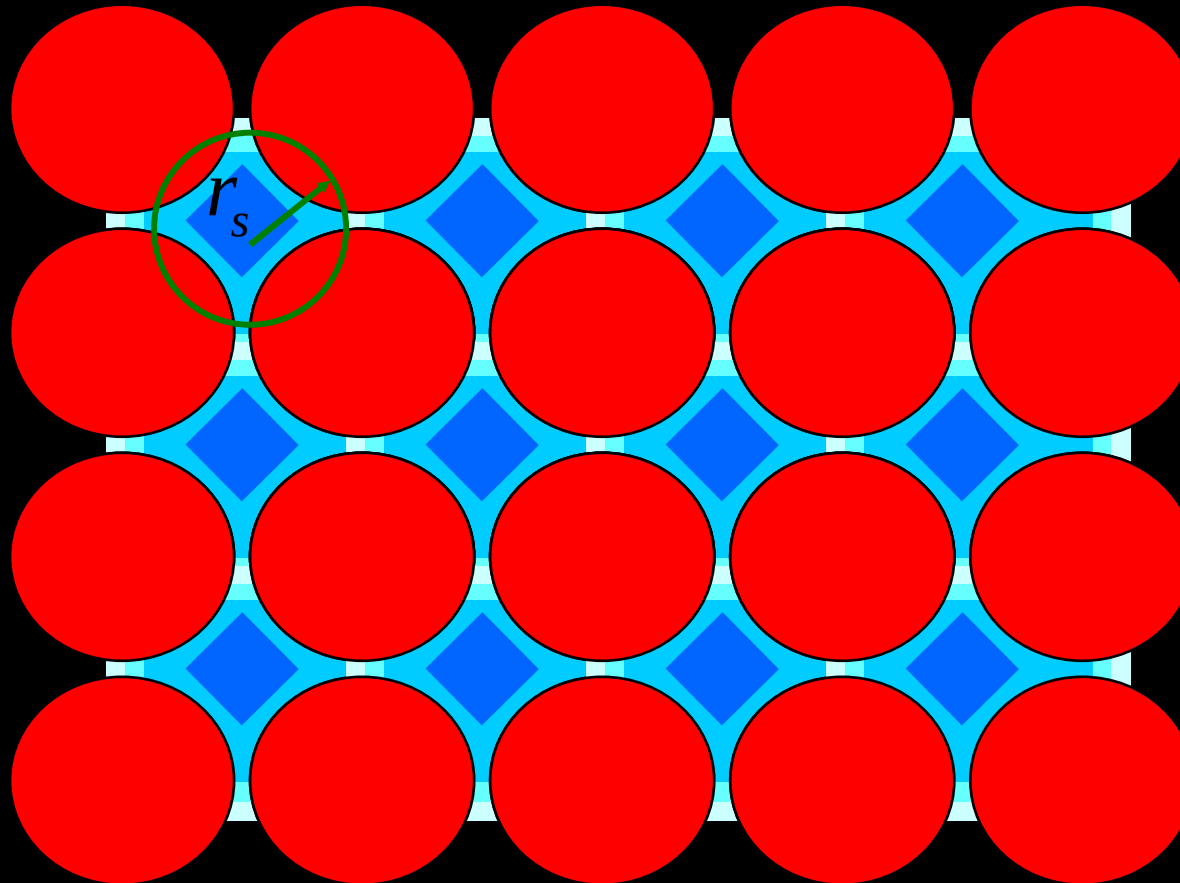
Weak dependence at high density

Corrections from Ion Cores: Interfering Length Scales



But suppose we force r_c to be similar or larger than r_s

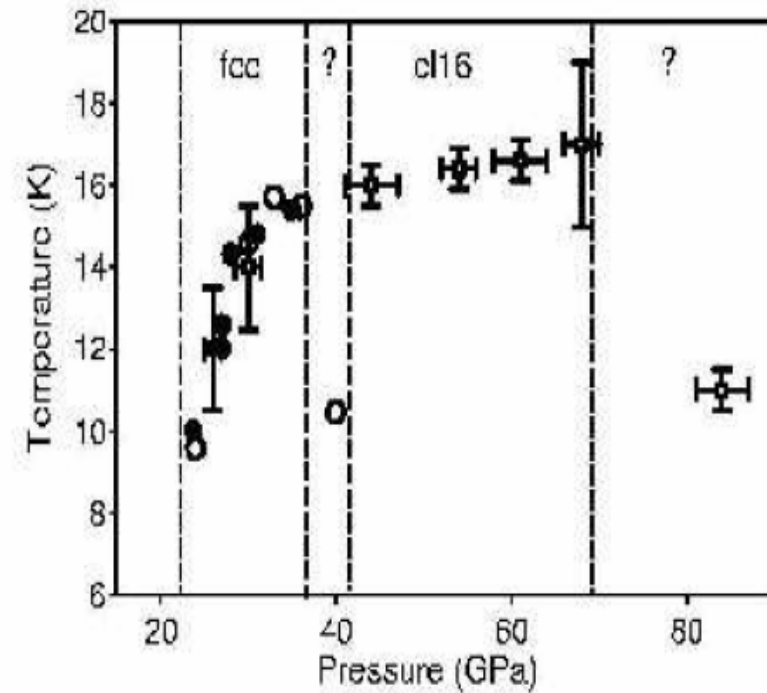
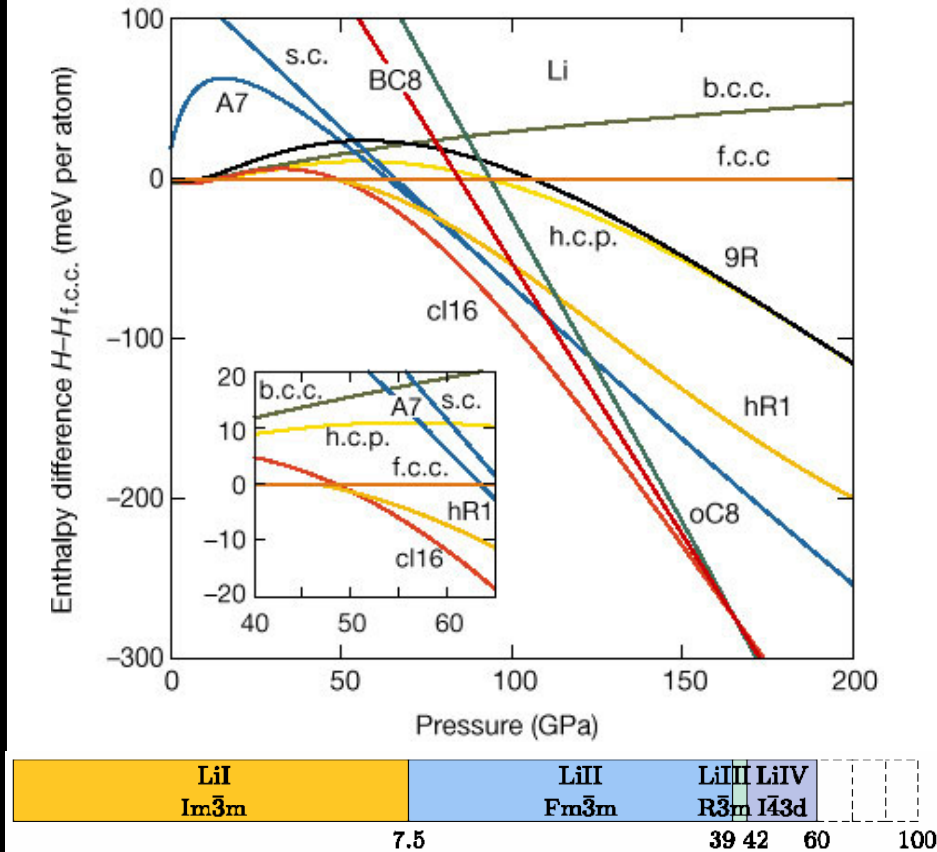
$$r_c \cong r_s$$



Electronic Localization at Interstitials!

Complex Structures in Lithium under Pressure

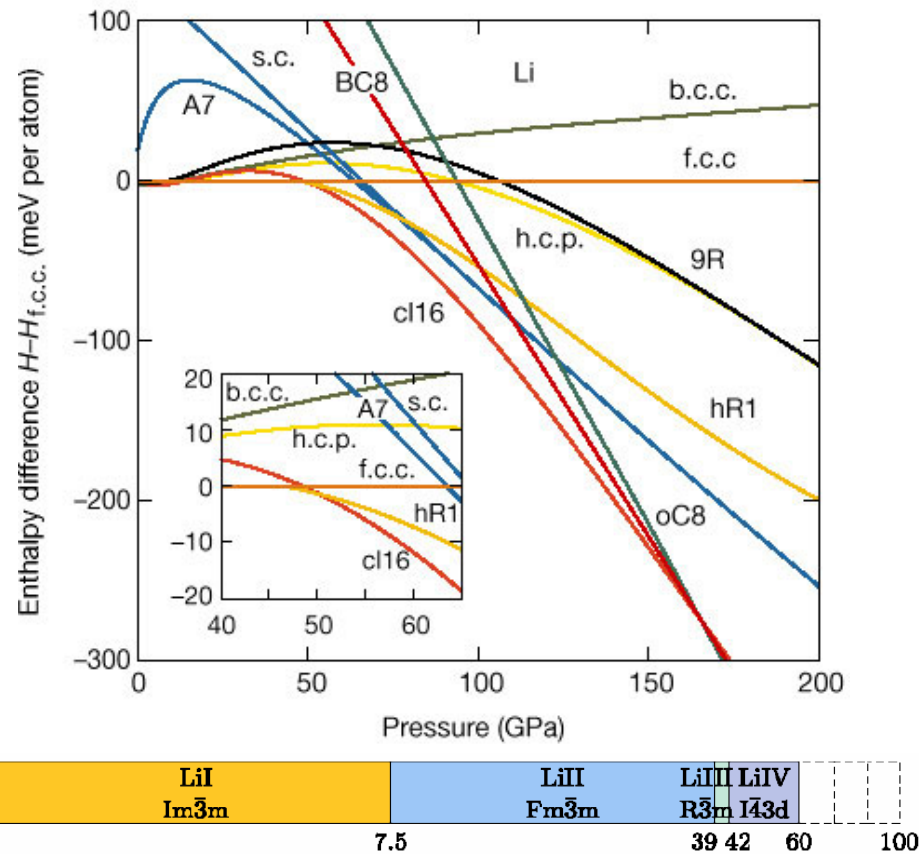
Lithium Superconducts at 17 K



J.B. Neaton and N.W. Ashcroft, *Nature*, **400**, 141 (1999);
 M. Hanfland, K. Syassen, *et al*, *Nature*, **408**, 174 (2000)

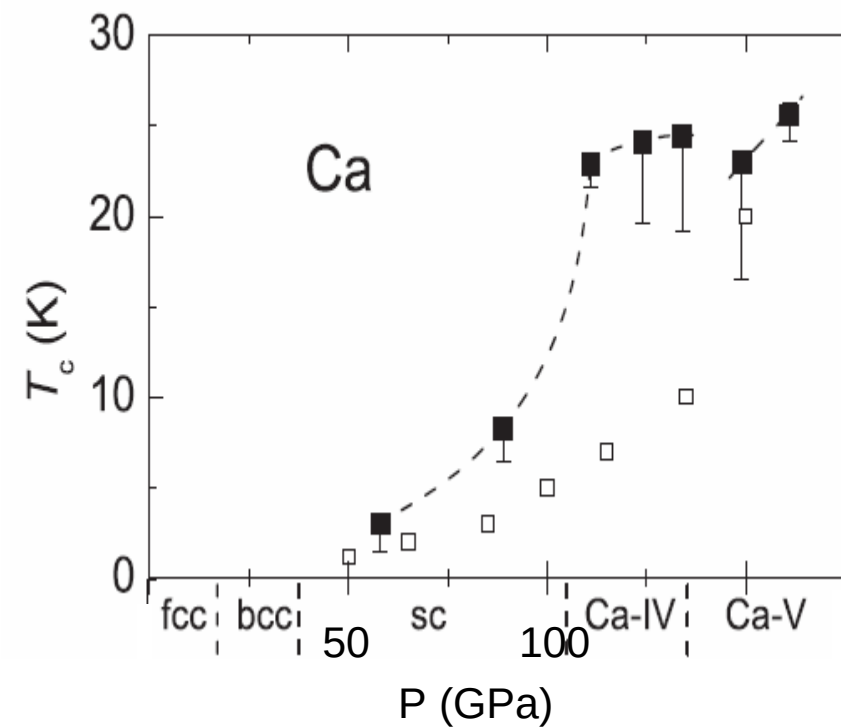
V.V. Struzhkin, M.L. Eremets, *et al*, *Science*, **298**, 1213 (2002);
 K. Shimizu, H. Ishikawa, *et al*, *Nature*, **419**, 597 (2002);
 S. Deemyad, *et al*, *Phys. Rev. Lett.*, **91**, 167001 (2003).

Complex Structures in Lithium under Pressure



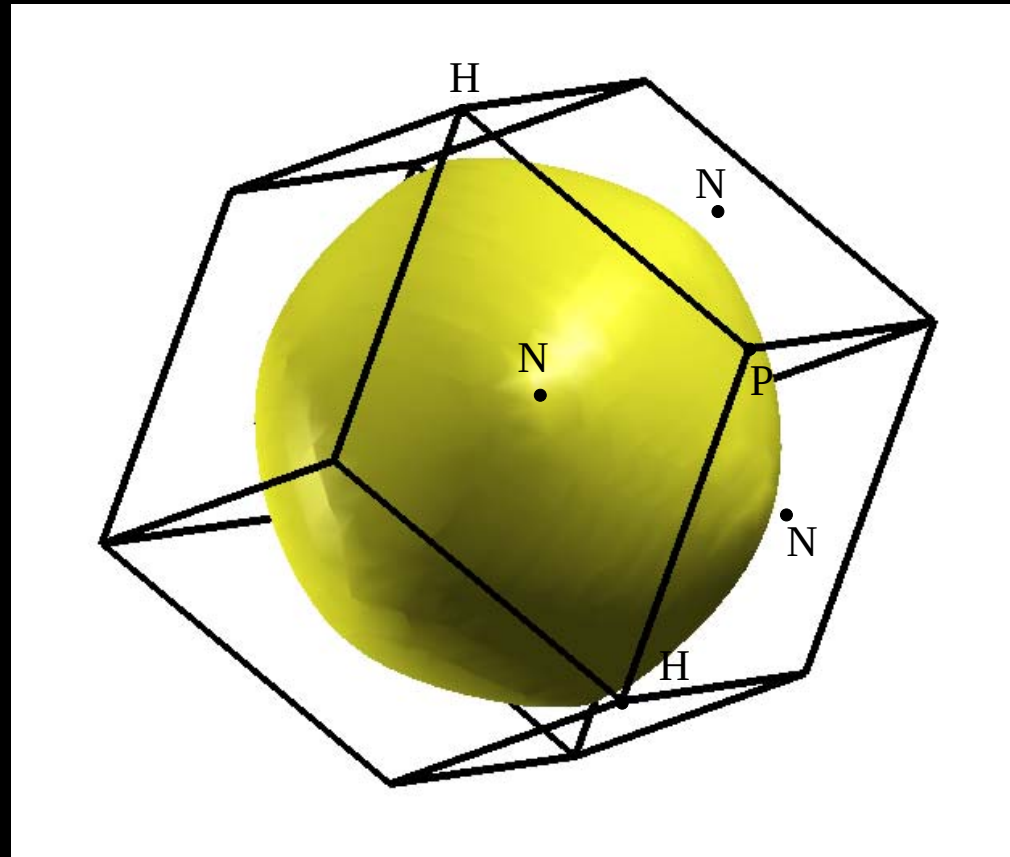
J.B. Neaton and N.W. Ashcroft, *Nature*, **400**, 141 (1999);
 M. Hanfland, K. Syassen, *et al*, *Nature*, **408**, 174 (2000)

Calcium Superconducts at 25 K



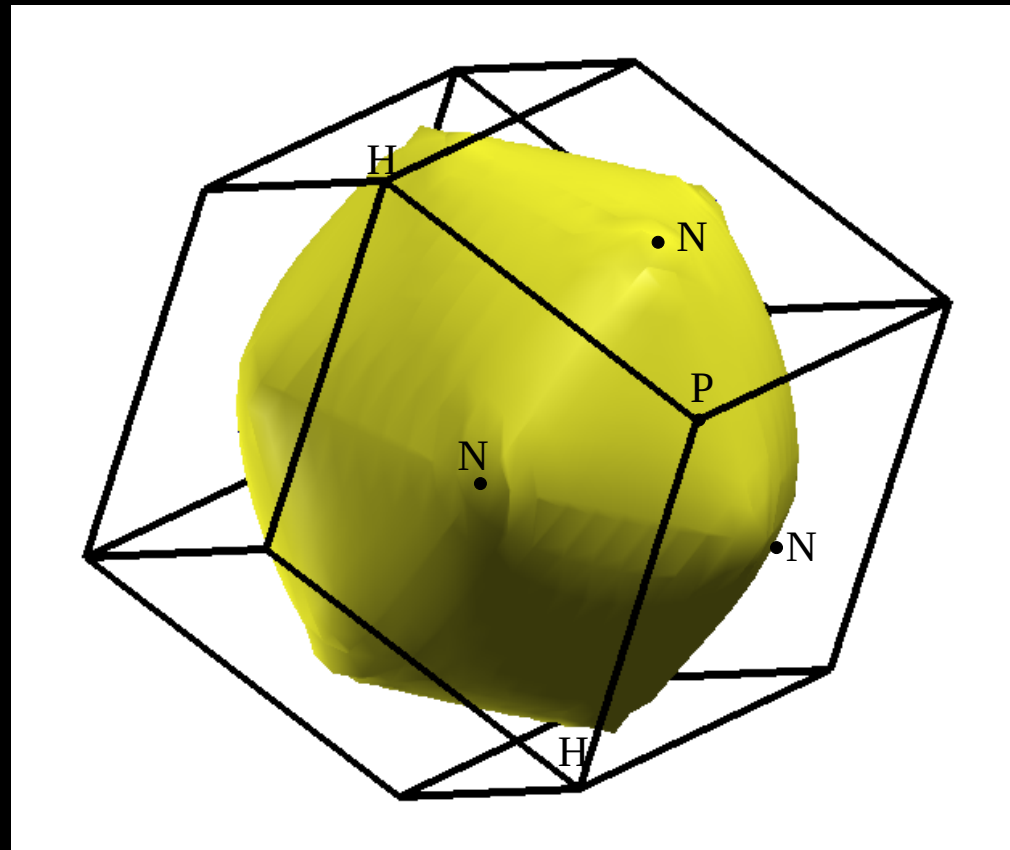
T. Yabuuchi, T. Matsuoka, Y. Nakamoto, and K. Shimizu,
J. Phys. Soc. Jpn. **75**, 083703 (2006).

Fermi Surface of *bcc* Lithium at $P=0$ GPa

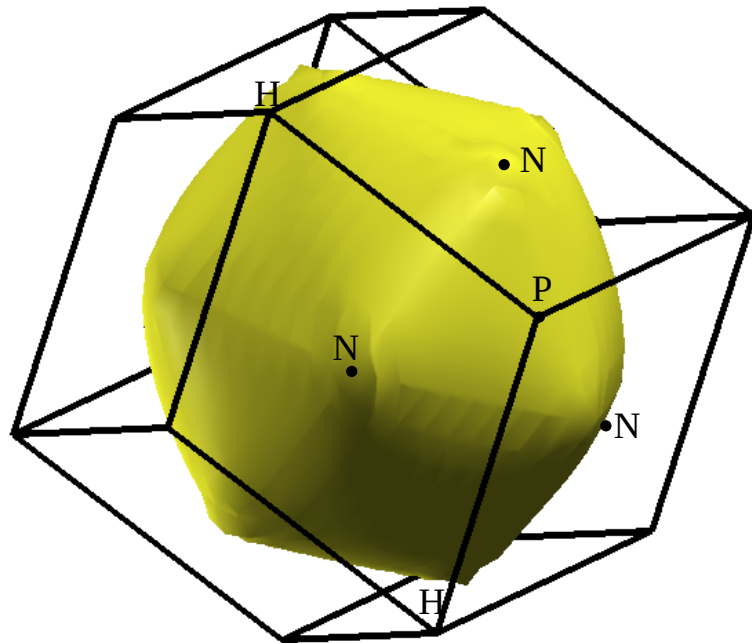


Fermi Surface of *bcc* Lithium at $P=8$ GPa

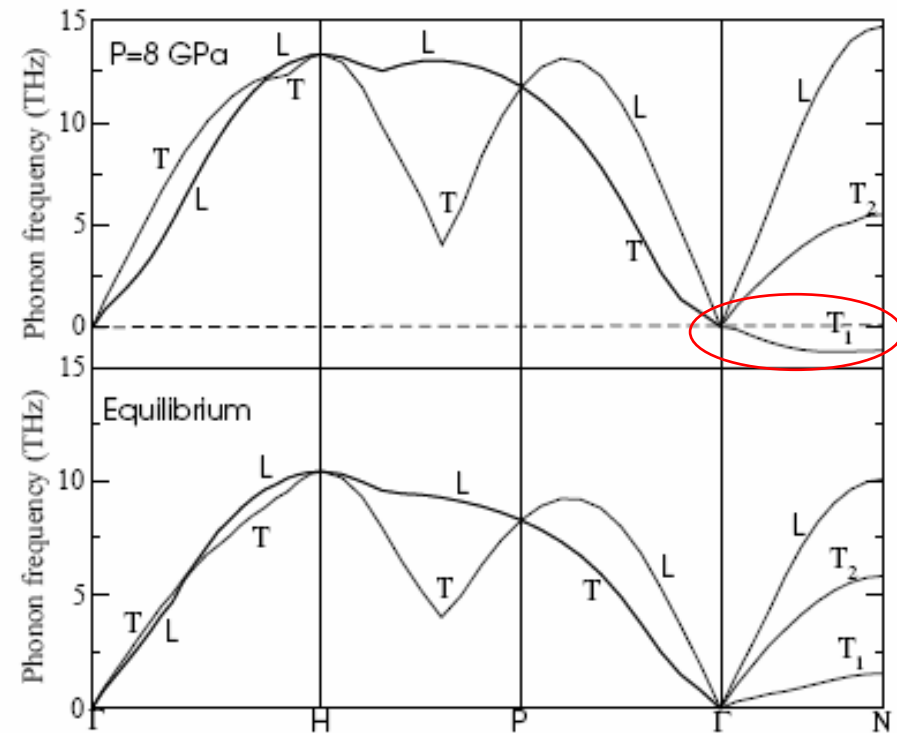
A. Rodriguez-Prieto, A. Bergara, V.M. Silkin, and P.M. Echenique,
Phys. Rev. B **74**, 172104 (2006)



Fermi Surface of *bcc* Lithium at $P=8$ GPa



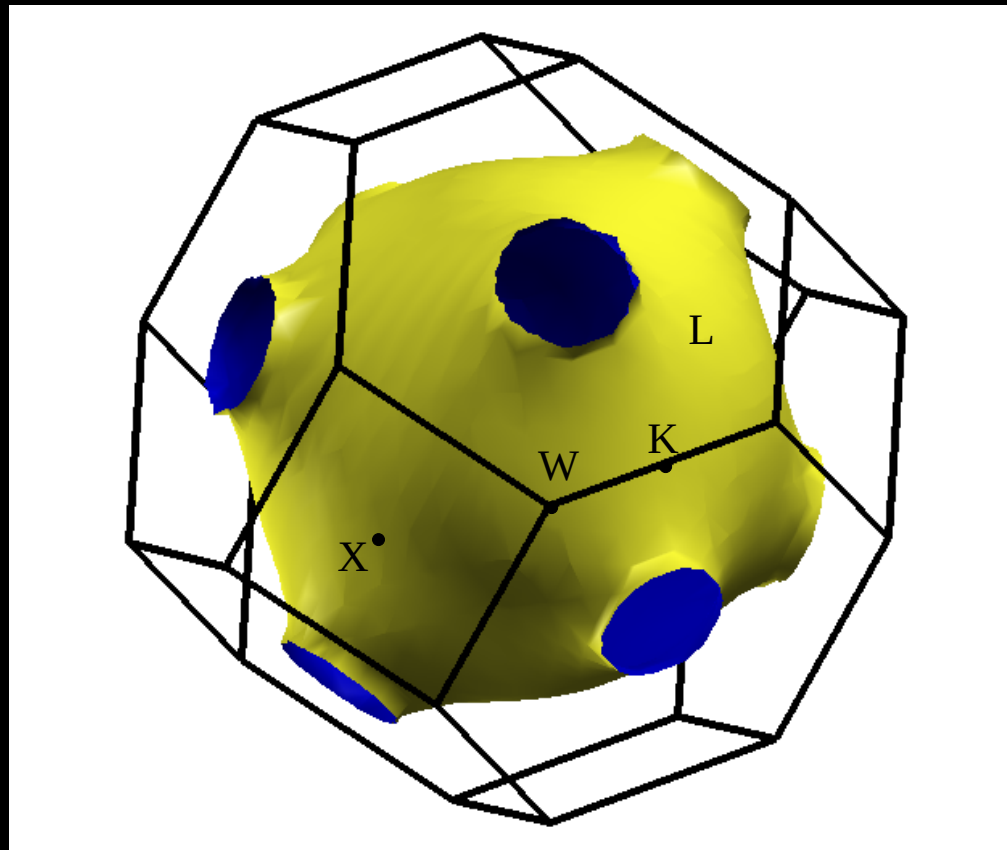
Phonon Frequencies



At 5 GPa the Fermi surface starts touching the BZ at the N point, corresponding to the experimental *bcc* to *fcc* transition via a Hume-Rothery mechanism.

Fermi Surface of *fcc* Lithium at $P=30$ GPa

A. Rodriguez-Prieto, A. Bergara, V.M. Silkin, and P.M.Echenique,
Phys. Rev. B **74**, 172104 (2006)

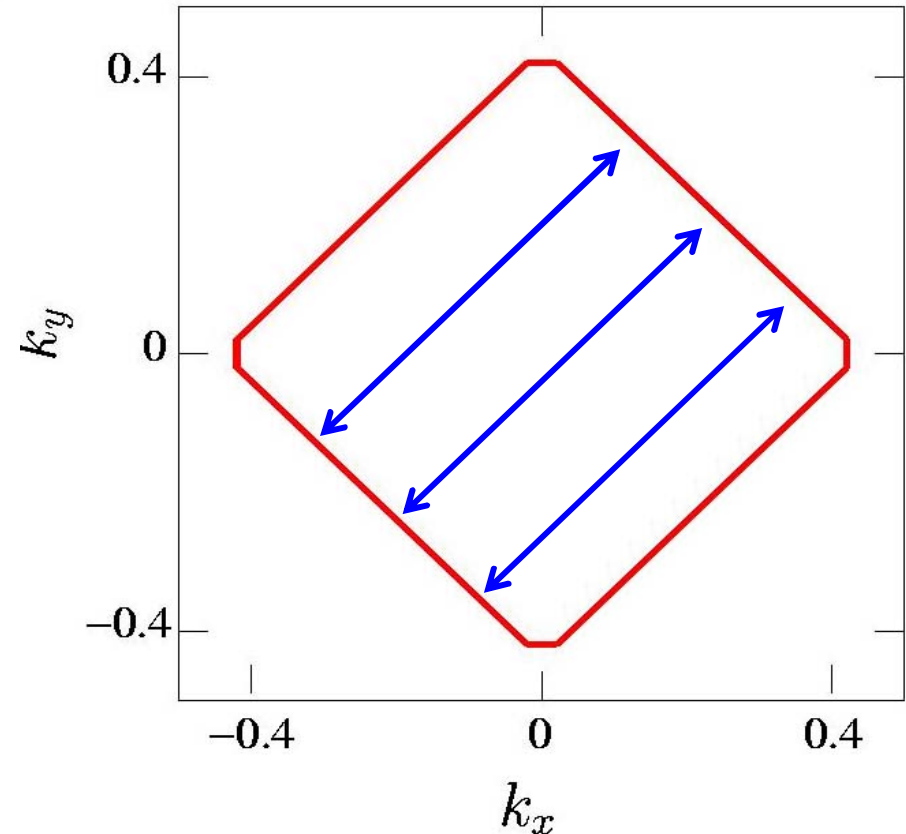
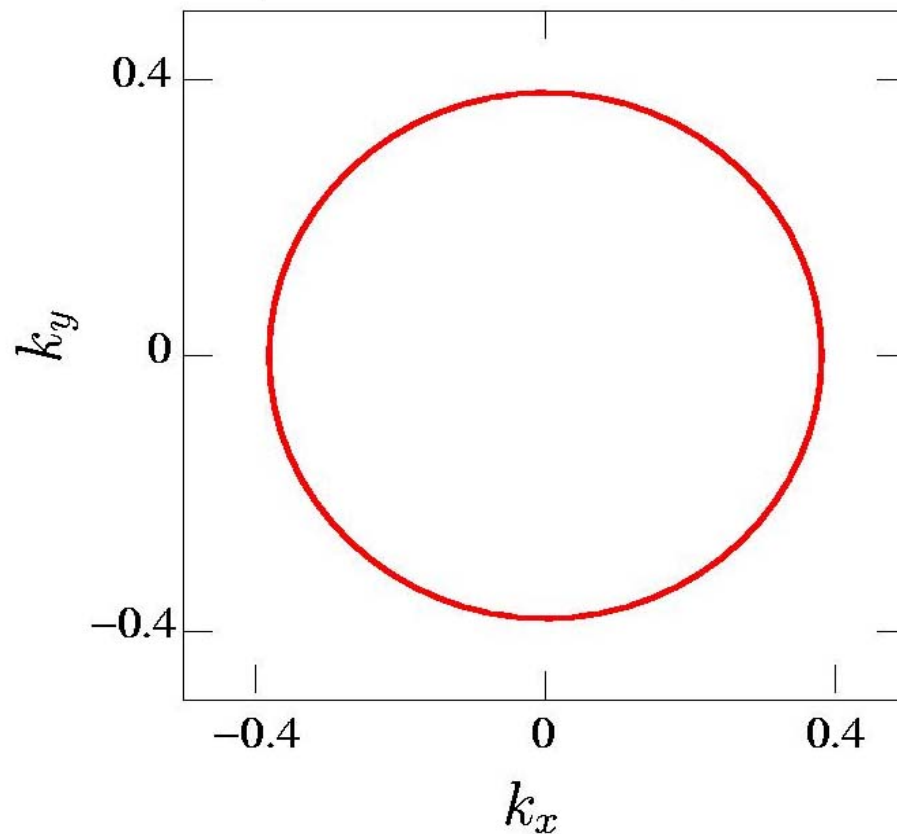


Lithium Monolayer

A. Rodriguez-Prieto and A. Bergara, Phys. Rev. B **72**, 125406 (2005)

Fermi *line* (Equilibrium)

Fermi *line* (sq, $r_s=2.1$ a.u.)



From Itinerant to **Hubbard** Description for Electrons !!

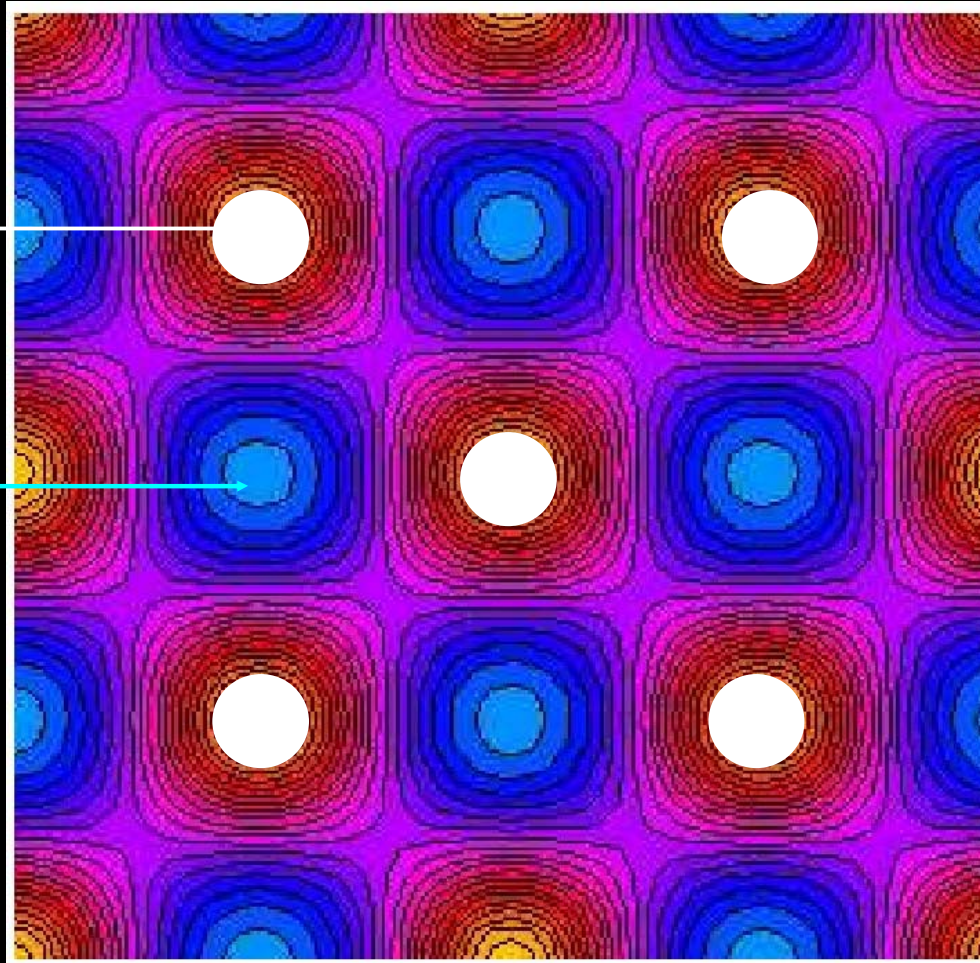
Lithium Monolayer

A. Rodriguez-Prieto and A. Bergara, Phys. Rev. B **72**, 125406 (2005)

Electronic Charge Density (ρ , $r_s=2.1$ a.u.)

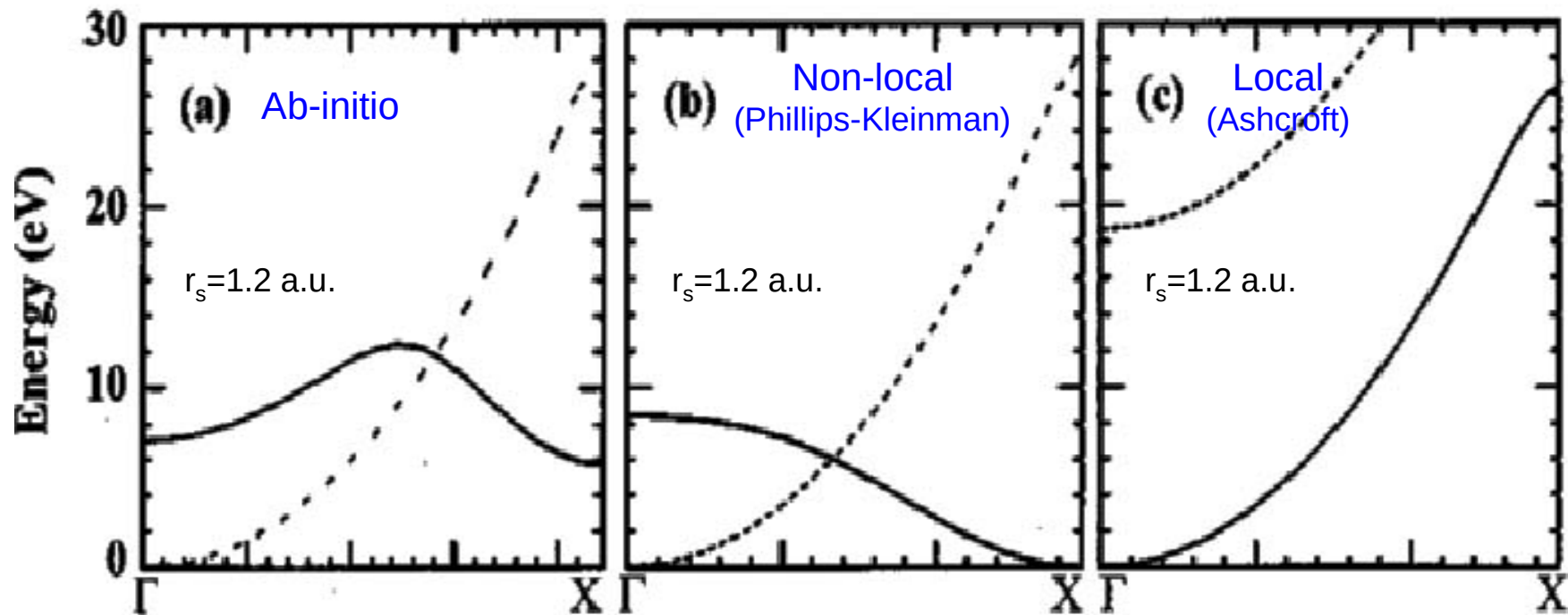
Li ions

Electronic
localization



Increasing Non-locality with Pressure

A. Bergara, J.B. Neaton, and N.W. Ashcroft, PRB **62**, 8495 (2000)



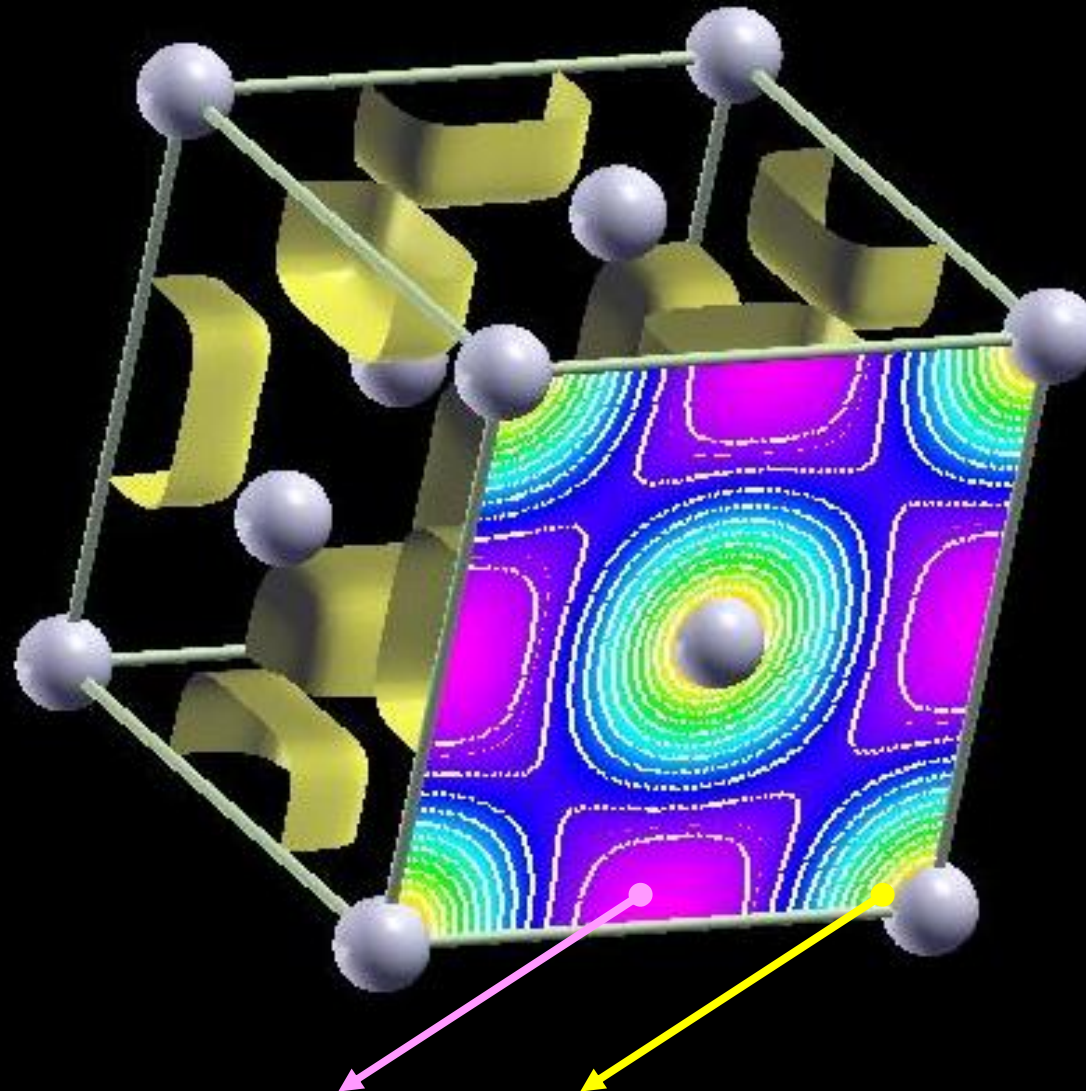
(b) Phillips-Kleinman Non-local Pseudopotential:

$$V = V_{\text{loc}}(r) + \sum_{c,\mathbf{R}} (\varepsilon - E_c) |\psi_{c,\mathbf{R}}\rangle \langle \psi_{c,\mathbf{R}}|$$

(c) Ashcroft Empty-core Local Pseudopotential:

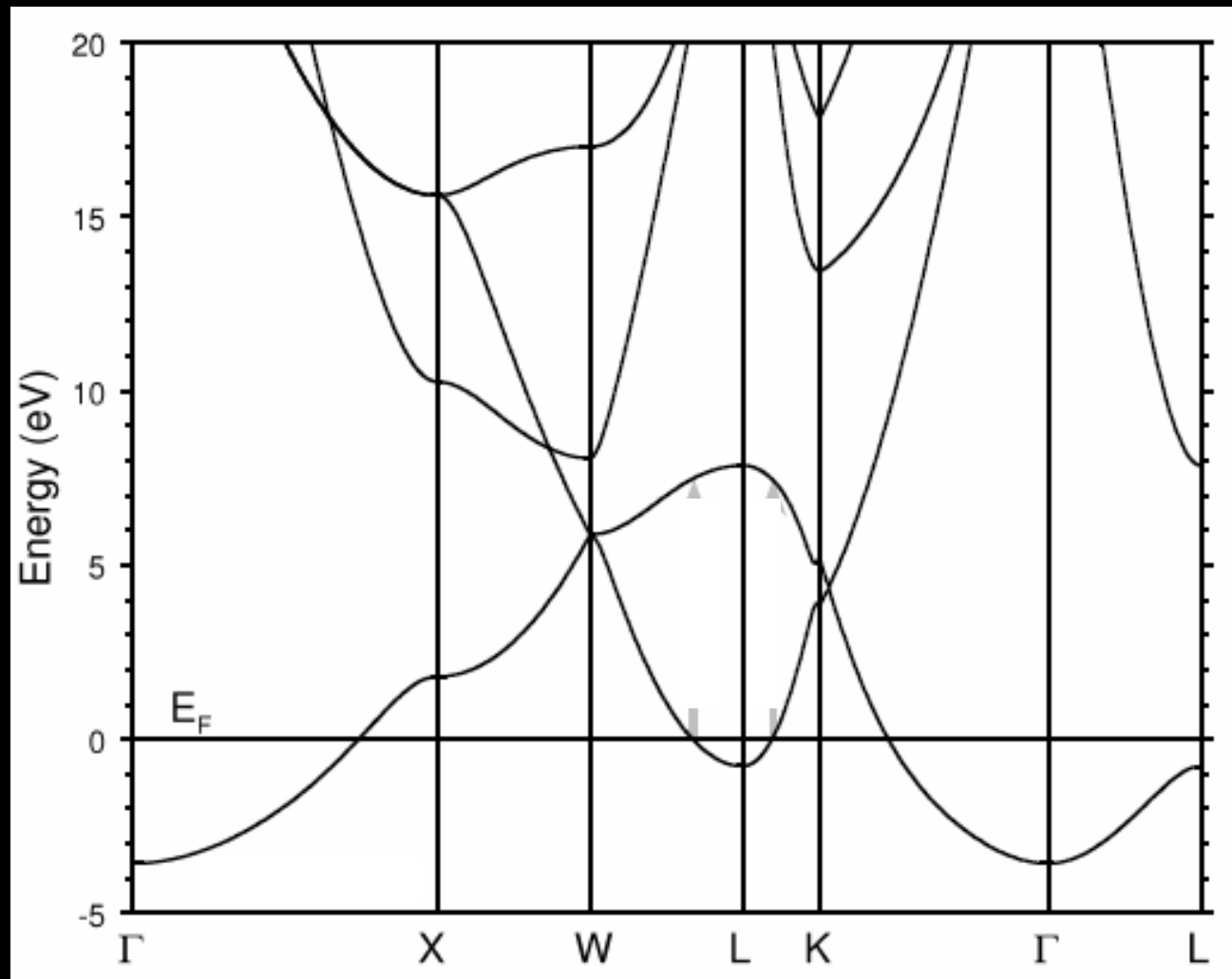
$$V(\mathbf{G}) = \frac{-4\pi Z}{\mathbf{G}^2 \Omega} \cos \mathbf{G} \mathbf{R}_c$$

Charge Density of *fcc* Lithium at $P=30$ GPa



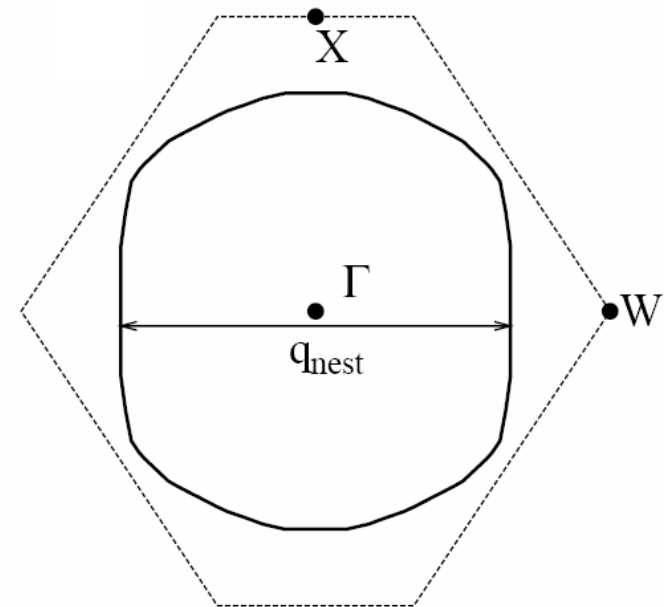
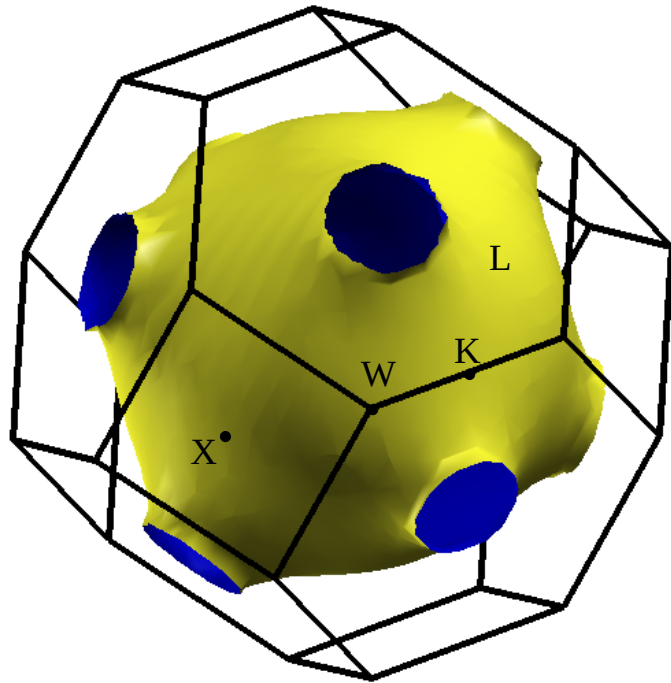
Maximum and minimum electronic charge

Band structure (fcc, P=30 GPa)

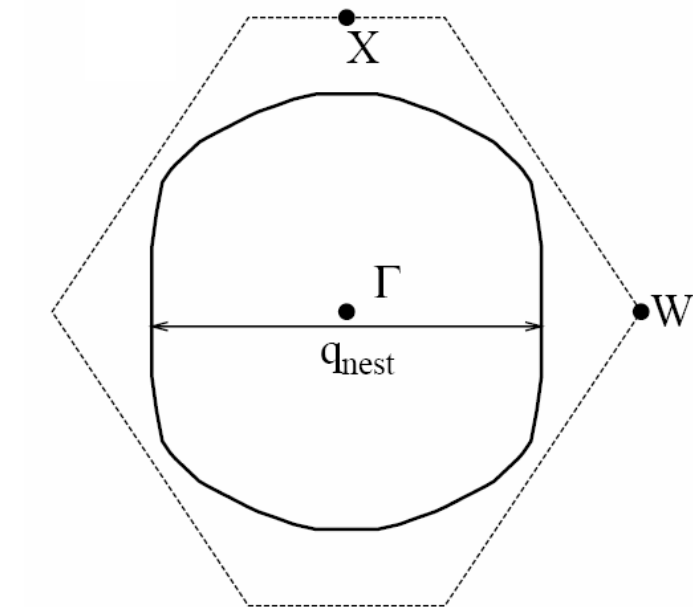


Fermi Surface of *fcc* Lithium at 30 GPa

Cross section along ΓXW

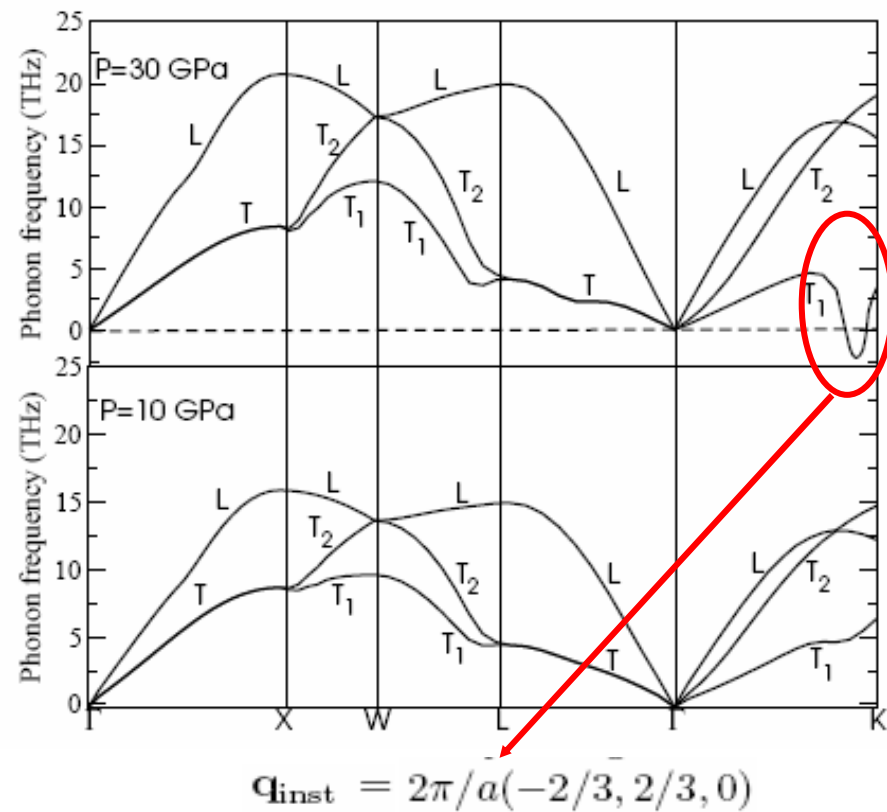


Fermi Surface section along ΓXW at 30 GPa



$$\mathbf{q}_{\text{nest}} = 2\pi/a(4/3, 2/3, 0) (\approx 2k_F)$$

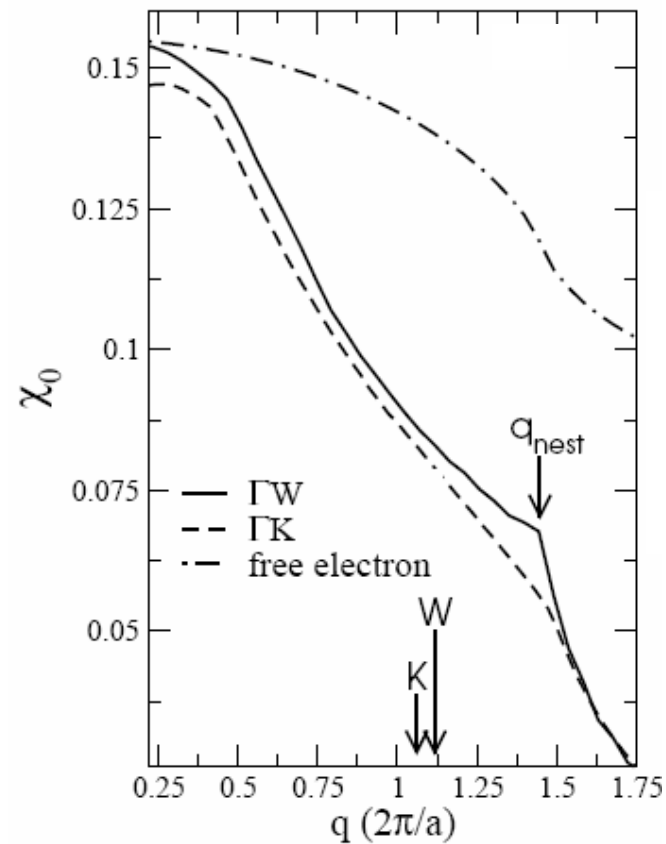
Phonon Frequencies



$$\mathbf{q}_{\text{inst}} = \mathbf{q}_{\text{nest}} + \mathbf{G} \text{ with } \mathbf{G} = 2\pi/a(-2, 0, 0)$$

Ab initio Electronic Susceptibility

A. Rodriguez-Prieto, A. Bergara, V.M. Silkin, and P.M.Echenique,
Phys. Rev. B **74**, 172104 (2006)



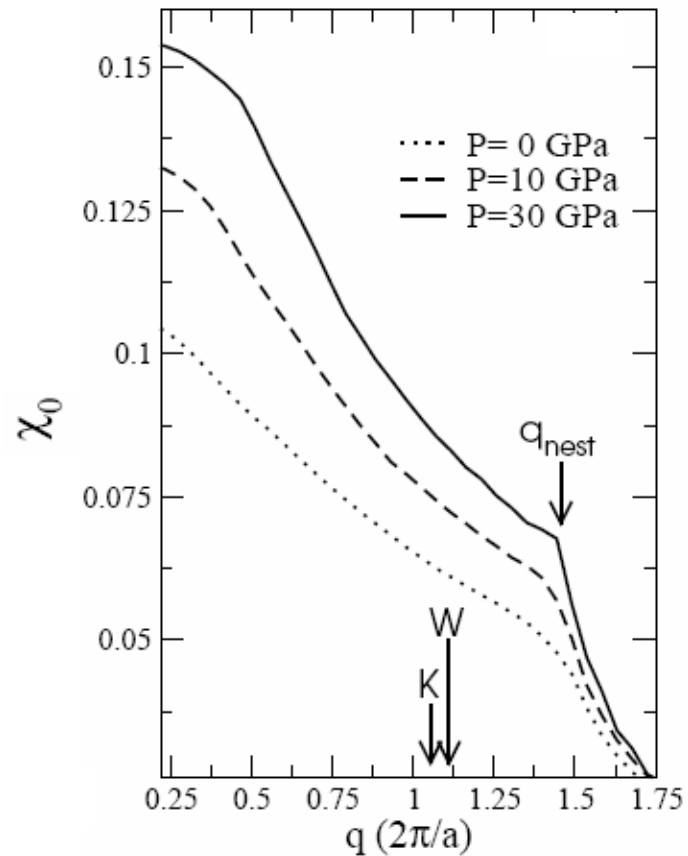
$$\chi_{\mathbf{G},\mathbf{G}'}^0(\mathbf{q},\omega) = \frac{1}{\Omega} \sum_{\mathbf{k}} \sum_{n,n'}^{BZ} \frac{f_{n\mathbf{k}} - f_{n'\mathbf{k}+\mathbf{q}}}{\varepsilon_{n\mathbf{k}} - \varepsilon_{n'\mathbf{k}+\mathbf{q}} + (\omega + i\eta)}$$

$$\times \langle \psi_{n\mathbf{k}} | e^{-i(\mathbf{q}+\mathbf{G})\cdot\mathbf{r}} | \psi_{n'\mathbf{k}+\mathbf{q}} \rangle$$

$$\times \langle \psi_{n'\mathbf{k}+\mathbf{q}} | e^{i(\mathbf{q}+\mathbf{G}')\cdot\mathbf{r}} | \psi_{n\mathbf{k}} \rangle$$

Ab initio Electronic Susceptibility

A. Rodriguez-Prieto, A. Bergara, V.M. Silkin, and P.M.Echenique,
Phys. Rev. B **74**, 172104 (2006)

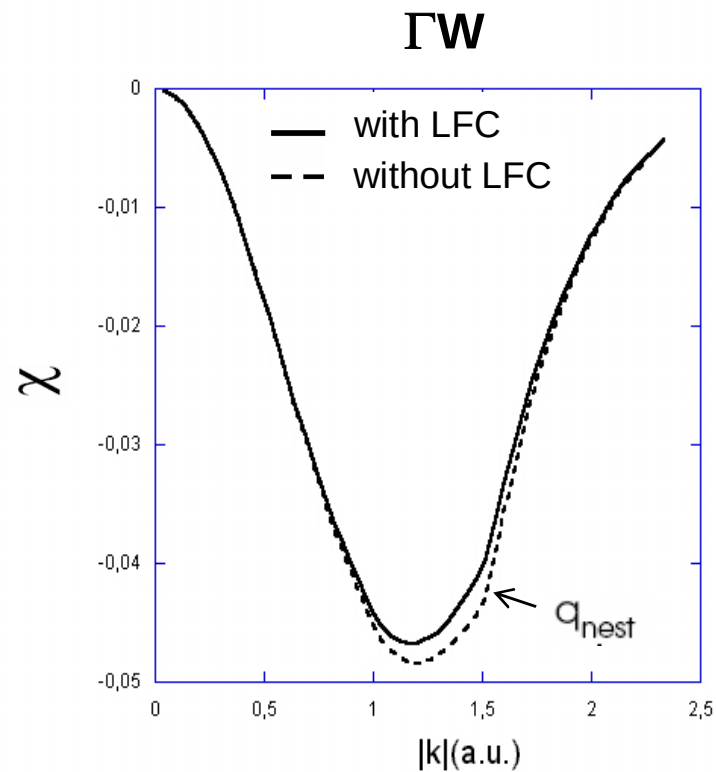


$$\chi_{\mathbf{G},\mathbf{G}'}^0(\mathbf{q},\omega) = \frac{1}{\Omega} \sum_{\mathbf{k}} \sum_{n,n'}^{BZ} \frac{f_{n\mathbf{k}} - f_{n'\mathbf{k}+\mathbf{q}}}{\varepsilon_{n\mathbf{k}} - \varepsilon_{n'\mathbf{k}+\mathbf{q}} + (\omega + i\eta)}$$

$$\times \langle \psi_{n\mathbf{k}} | e^{-i(\mathbf{q}+\mathbf{G})\cdot\mathbf{r}} | \psi_{n'\mathbf{k}+\mathbf{q}} \rangle$$

$$\times \langle \psi_{n'\mathbf{k}+\mathbf{q}} | e^{i(\mathbf{q}+\mathbf{G}')\cdot\mathbf{r}} | \psi_{n\mathbf{k}} \rangle$$

Ab initio Electronic Susceptibility



$$\chi = \chi^0 + \chi^0 (V + K^{xc}) \chi;$$

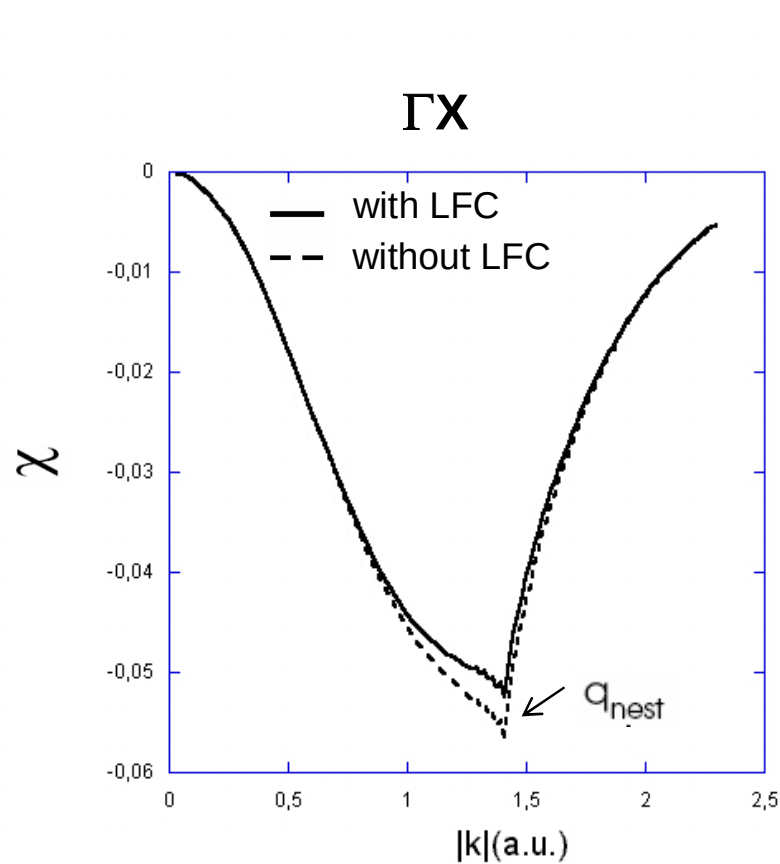
0 (RPA) ↑

$$\chi_{\mathbf{G}, \mathbf{G}'}^0(\mathbf{q}, \omega) = \frac{1}{\Omega} \sum_{\mathbf{k}} \sum_{n, n'}^{BZ} \frac{f_{n\mathbf{k}} - f_{n'\mathbf{k}+\mathbf{q}}}{\varepsilon_{n\mathbf{k}} - \varepsilon_{n'\mathbf{k}+\mathbf{q}} + (\omega + i\eta)}$$

$$\times \langle \psi_{n\mathbf{k}} | e^{-i(\mathbf{q}+\mathbf{G}) \cdot \mathbf{r}} | \psi_{n'\mathbf{k}+\mathbf{q}} \rangle$$

$$\times \langle \psi_{n'\mathbf{k}+\mathbf{q}} | e^{i(\mathbf{q}+\mathbf{G}') \cdot \mathbf{r}} | \psi_{n\mathbf{k}} \rangle$$

Ab initio Electronic Susceptibility



$$\chi = \chi^0 + \chi^0 (V + K^{xc}) \chi,$$

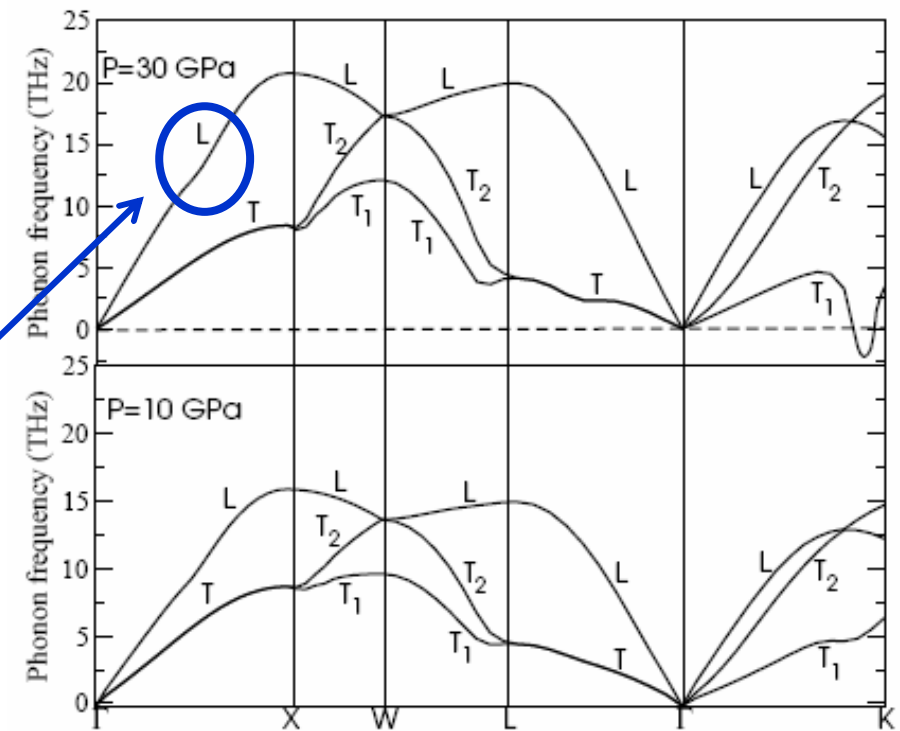
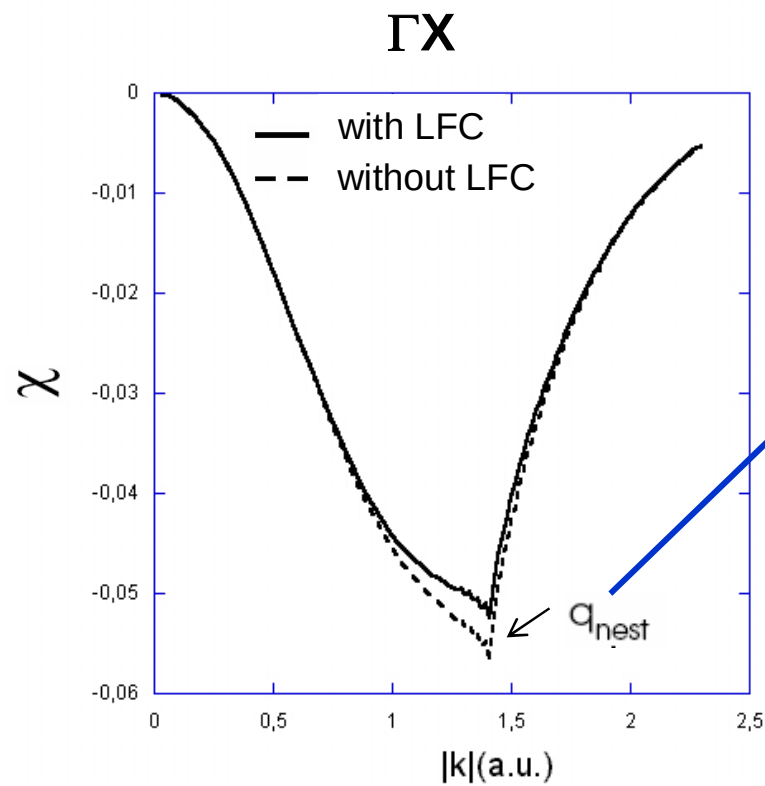
0 (RPA) ↑

$$\chi_{\mathbf{G},\mathbf{G}'}^0(\mathbf{q},\omega) = \frac{1}{\Omega} \sum_{\mathbf{k}} \sum_{n,n'} \frac{f_{n\mathbf{k}} - f_{n'\mathbf{k}+\mathbf{q}}}{\varepsilon_{n\mathbf{k}} - \varepsilon_{n'\mathbf{k}+\mathbf{q}} + (\omega + i\eta)}$$

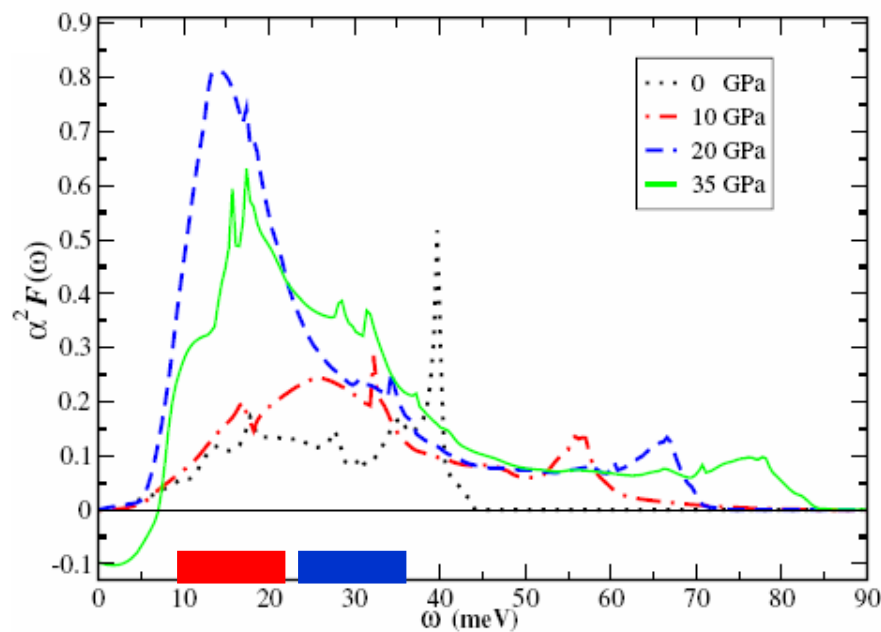
$$\times \langle \psi_{n\mathbf{k}} | e^{-i(\mathbf{q}+\mathbf{G})\cdot\mathbf{r}} | \psi_{n'\mathbf{k}+\mathbf{q}} \rangle$$

$$\times \langle \psi_{n'\mathbf{k}+\mathbf{q}} | e^{i(\mathbf{q}+\mathbf{G}')\cdot\mathbf{r}} | \psi_{n\mathbf{k}} \rangle$$

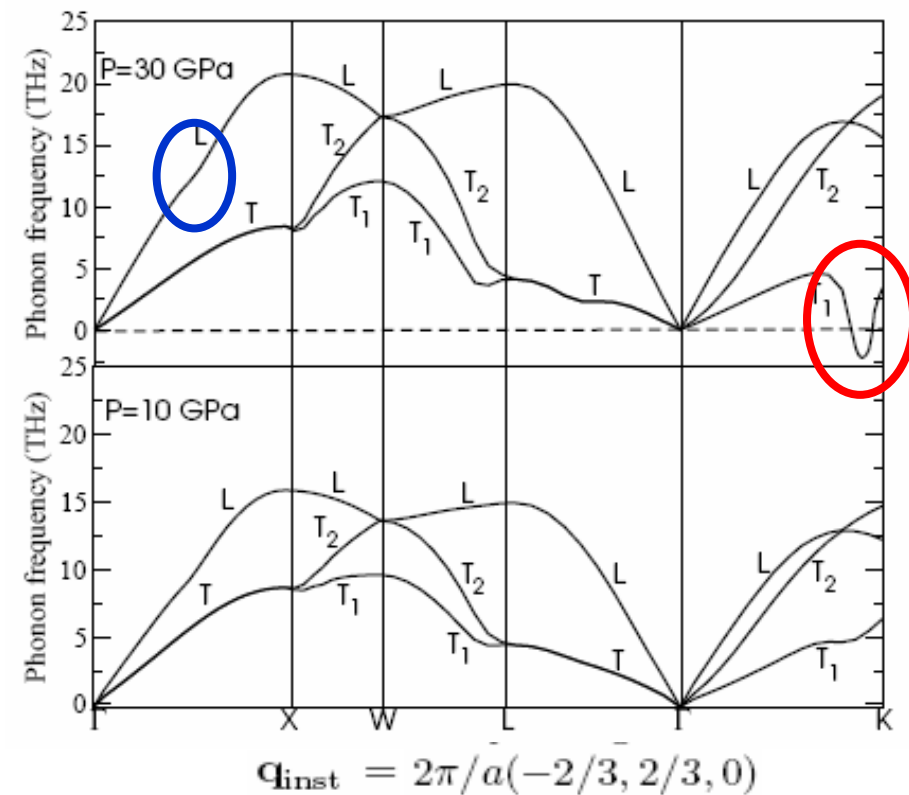
Ab initio Electronic Susceptibility



Electron-phonon Coupling for high T_c Superconductivity

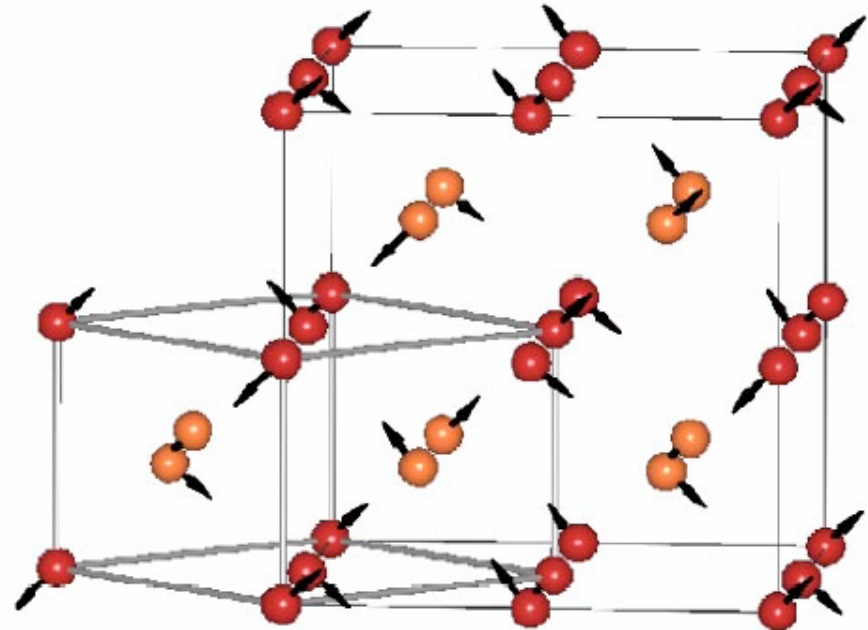
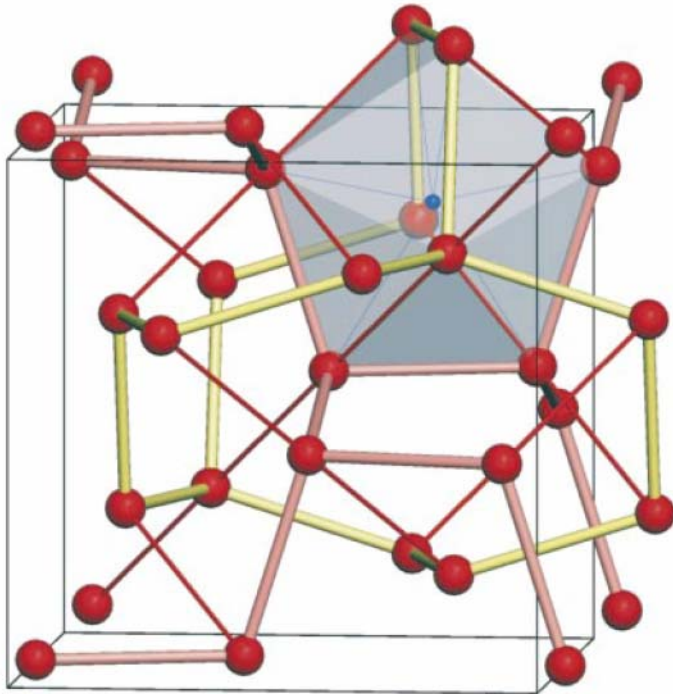


Phonon Frequencies



J.S. Tse, *et al* J. Phys.: Condens. Matter **17**, S911 (2005);
D. Kasinathan, *et al*, Phys. Rev. Lett. **96**, 047004 (2006);
G. Profeta *et al*, Phys. Rev. Lett. **96**, 047007 (2006).

c/16 Structure

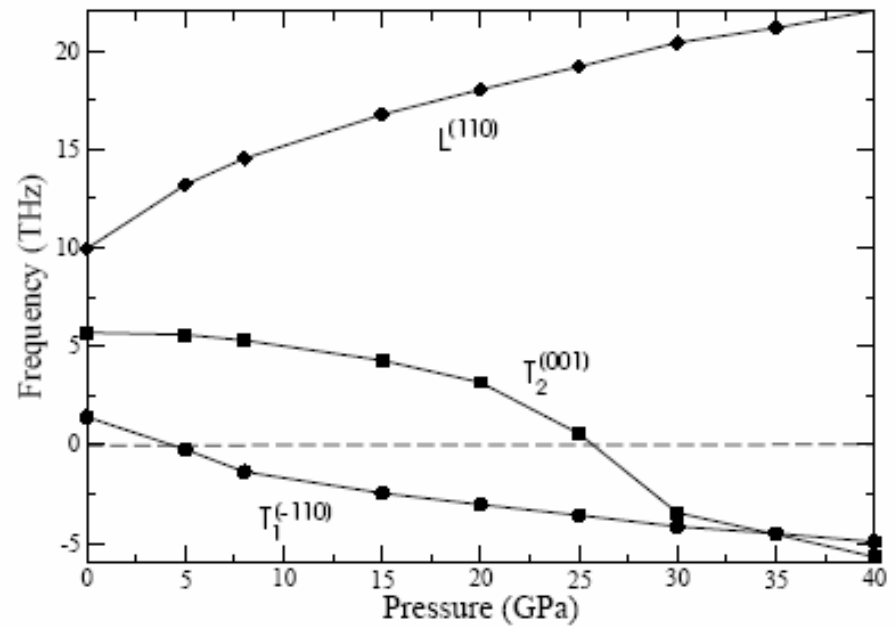
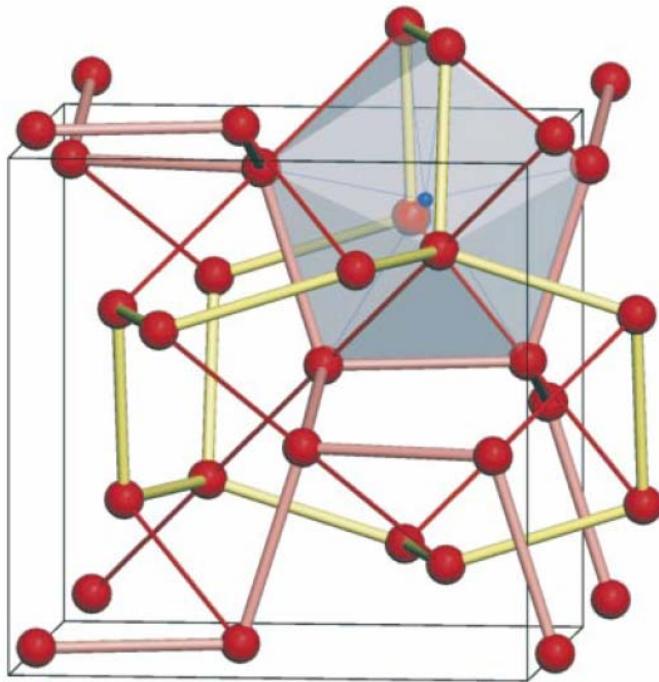


M. Hanfland, K. Syassen, N. E. Christensen, and D. L. Novikov, *Nature* **408**, 174 (2000)

H. Katzke and P. Toledano, *Phys. Rev. B* **71**, 184101 (2005)

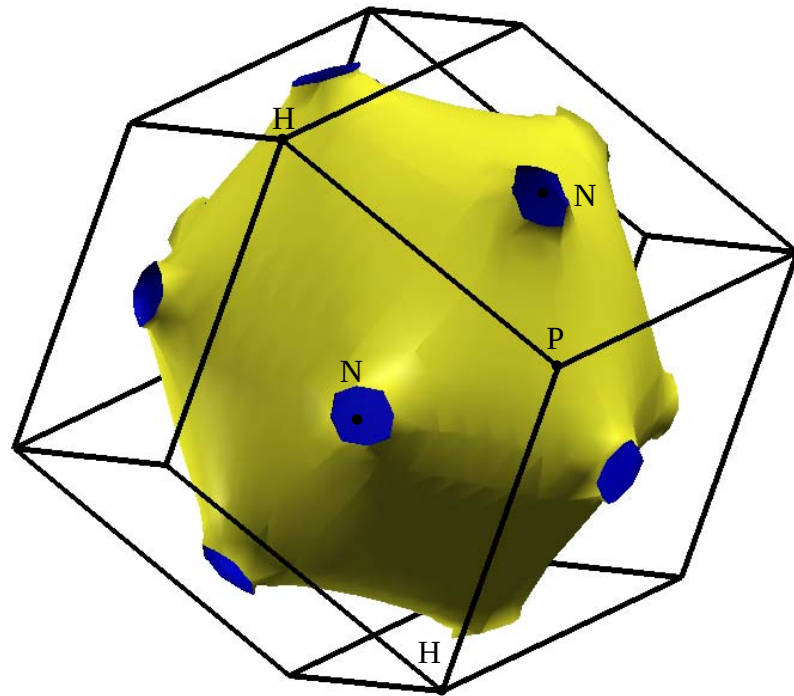
c/16 Structure

Phonon Frequencies of *bcc* Lithium at *N*

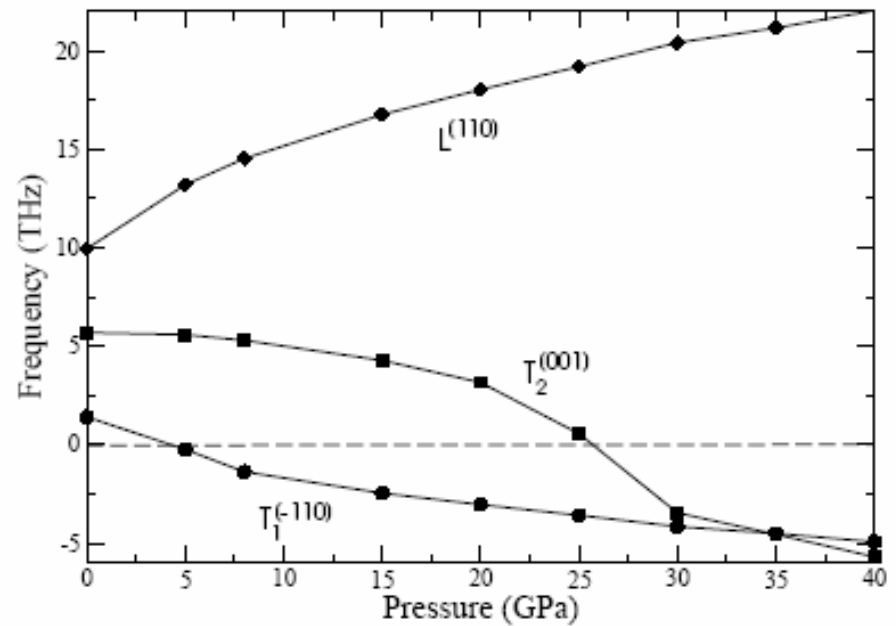


A. Rodriguez-Prieto, A. Bergara, and V.M. Silkin,
to be published in J. Phys. Soc. Jpn. (2007)

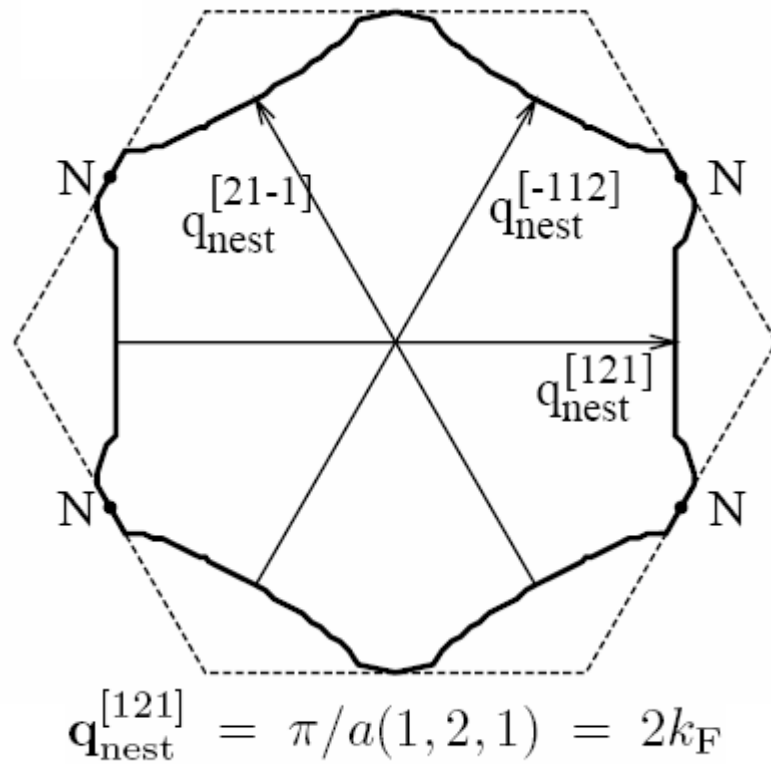
Fermi Surface of *bcc* Lithium at 40 GPa



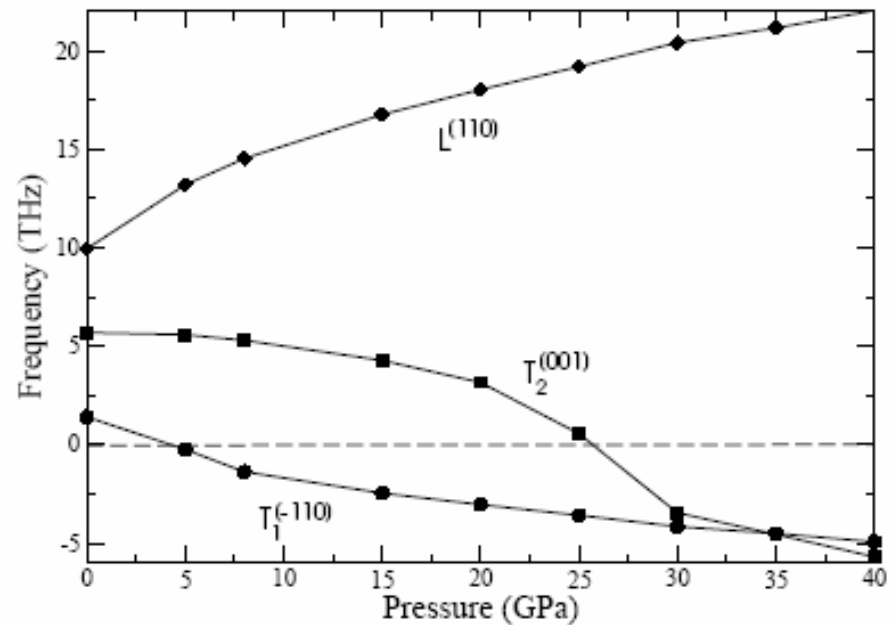
Phonon Frequencies of *bcc* Lithium at *N*



Fermi Surface Cross Section of *bcc* Lithium at 40 GPa

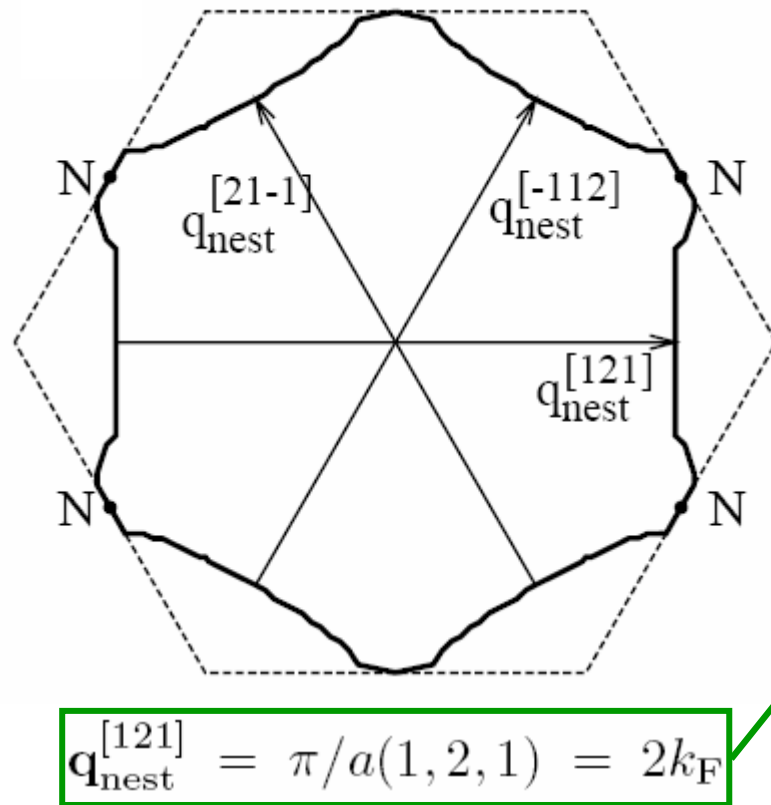


Phonon Frequencies of *bcc* Lithium at *N*

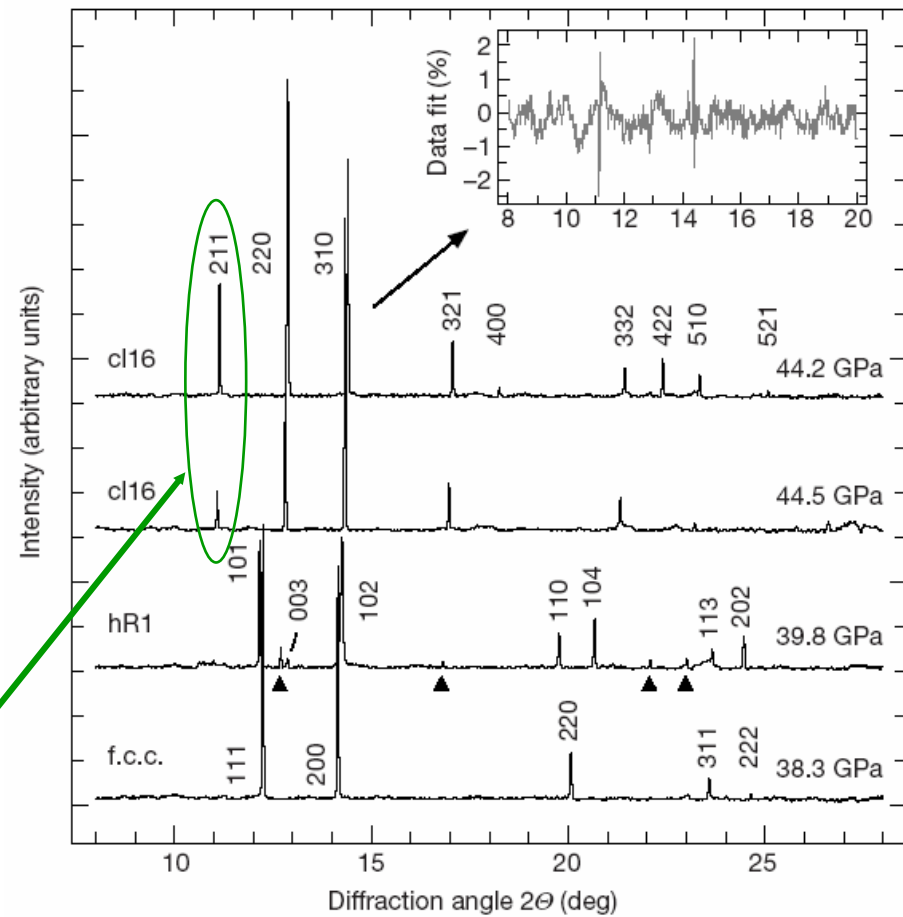


$$\mathbf{q}_{\text{nest}}^{[121]} + \mathbf{G} = \frac{\pi}{a}(1, 0, 1) = \Gamma N \quad \text{with} \quad \mathbf{G} = \frac{\pi}{a}(0, -2, 0)$$

Fermi Surface Cross Section of bcc Lithium at 40 GPa



X-ray diffraction diagram of Lithium



Dynamic Response Function of Lithium under Pressure

$$\epsilon^{-1} = 1 + V\chi$$

$$\chi = \chi^0 + \chi^0(V + K^{xc})\chi,$$

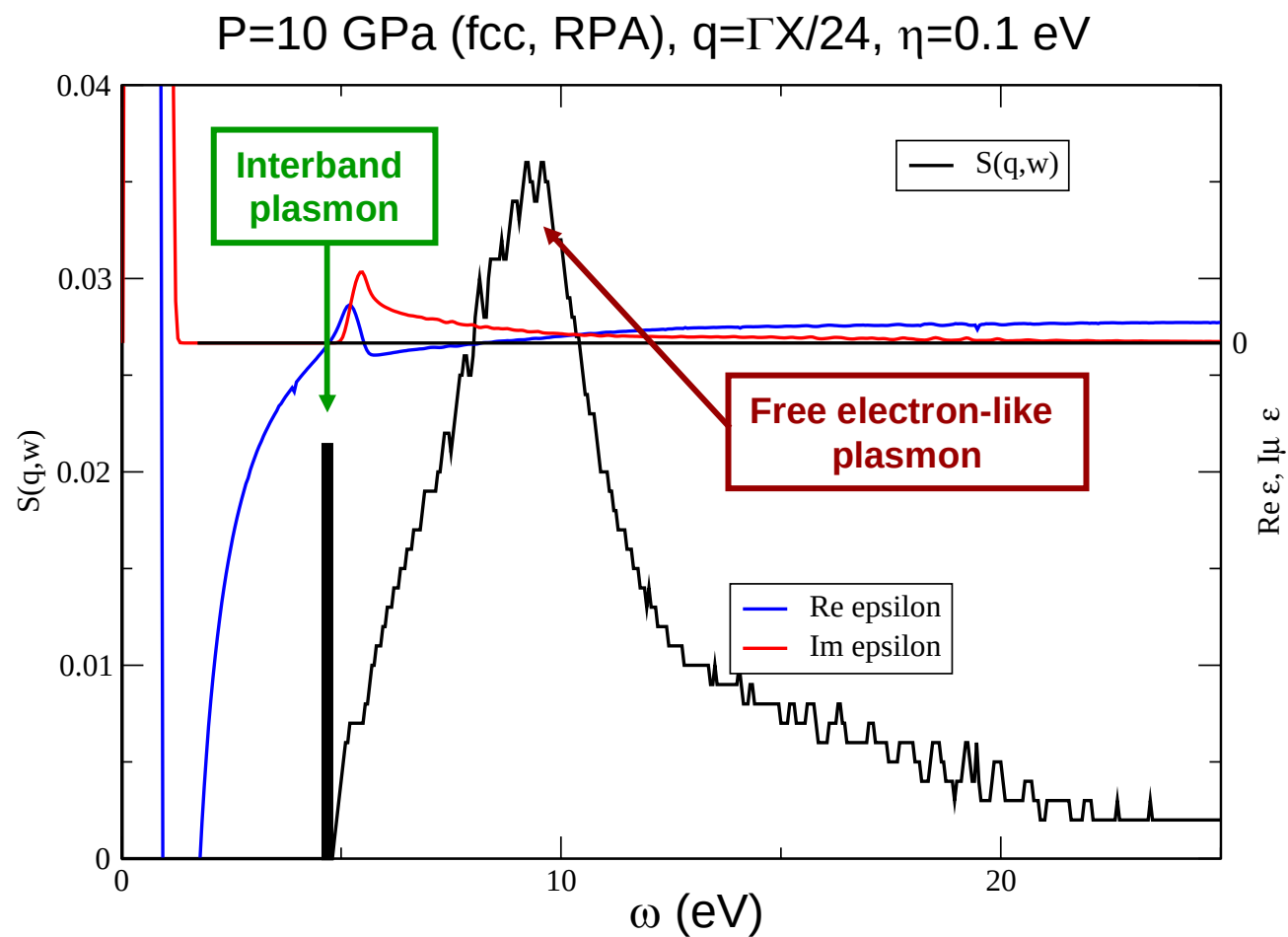
$\downarrow = 0 \text{ (RPA)}$

$$\chi_{\mathbf{G},\mathbf{G}'}^0(\mathbf{q},\omega) = \frac{1}{\Omega} \sum_{\mathbf{k}} \sum_{n,n'} \frac{f_{n\mathbf{k}} - f_{n'\mathbf{k}+\mathbf{q}}}{\epsilon_{n\mathbf{k}} - \epsilon_{n'\mathbf{k}+\mathbf{q}} + (\omega + i\eta)}$$

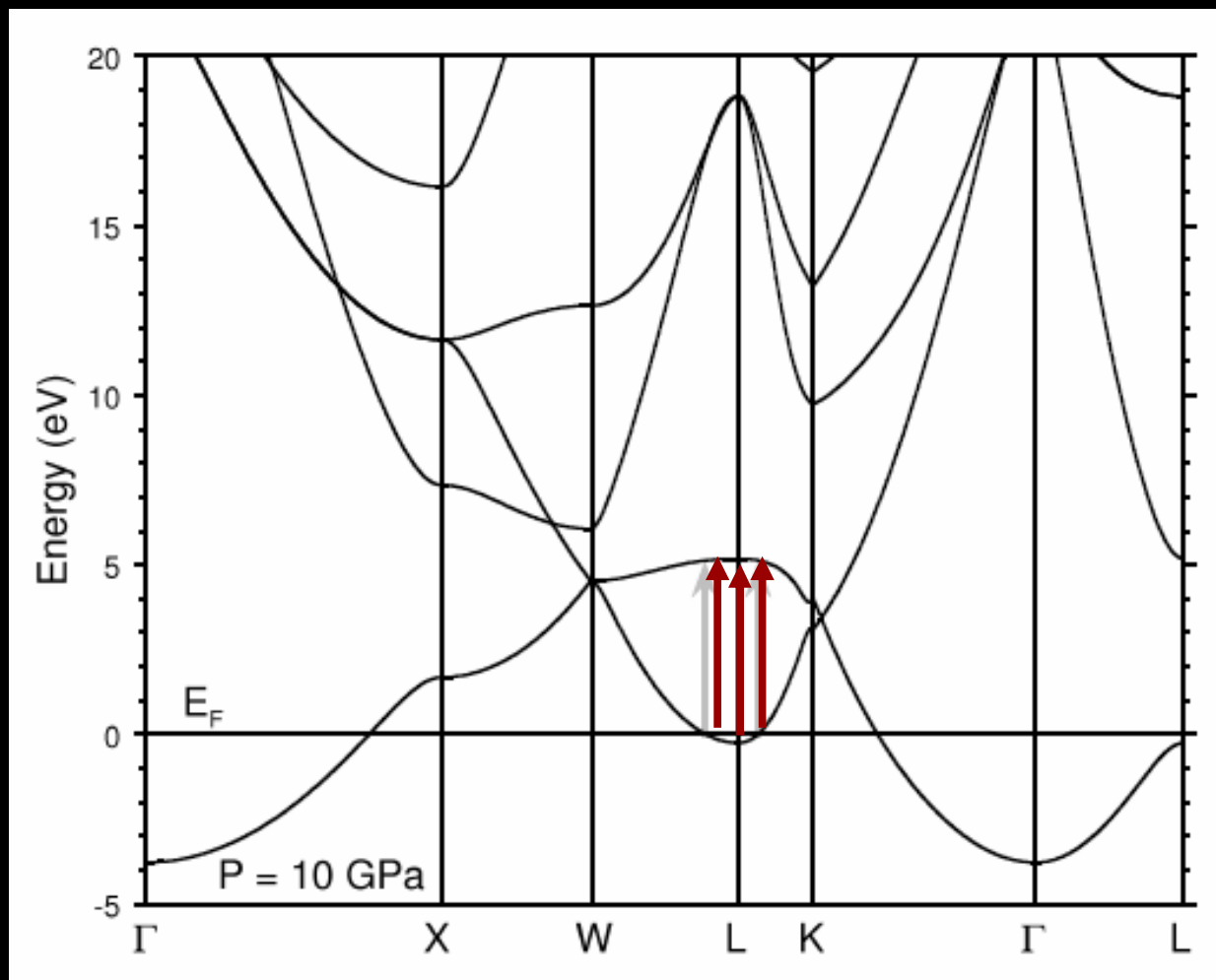
$$\times \langle \psi_{n\mathbf{k}} | e^{-i(\mathbf{q}+\mathbf{G})\cdot\mathbf{r}} | \psi_{n'\mathbf{k}+\mathbf{q}} \rangle$$

$$\times \langle \psi_{n'\mathbf{k}+\mathbf{q}} | e^{i(\mathbf{q}+\mathbf{G}')\cdot\mathbf{r}} | \psi_{n\mathbf{k}} \rangle$$

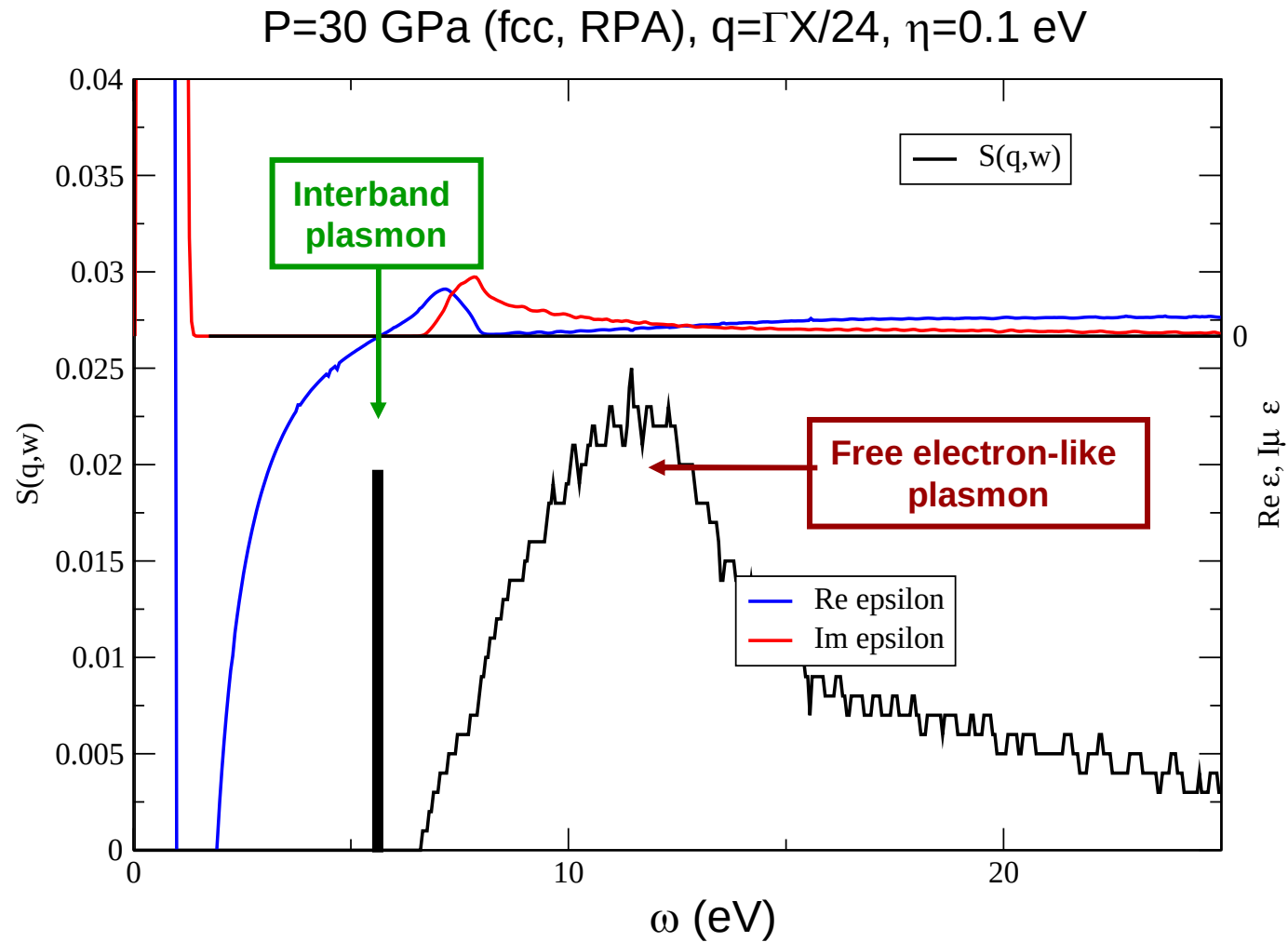
New Undamped Plasmon in Lithium under Pressure



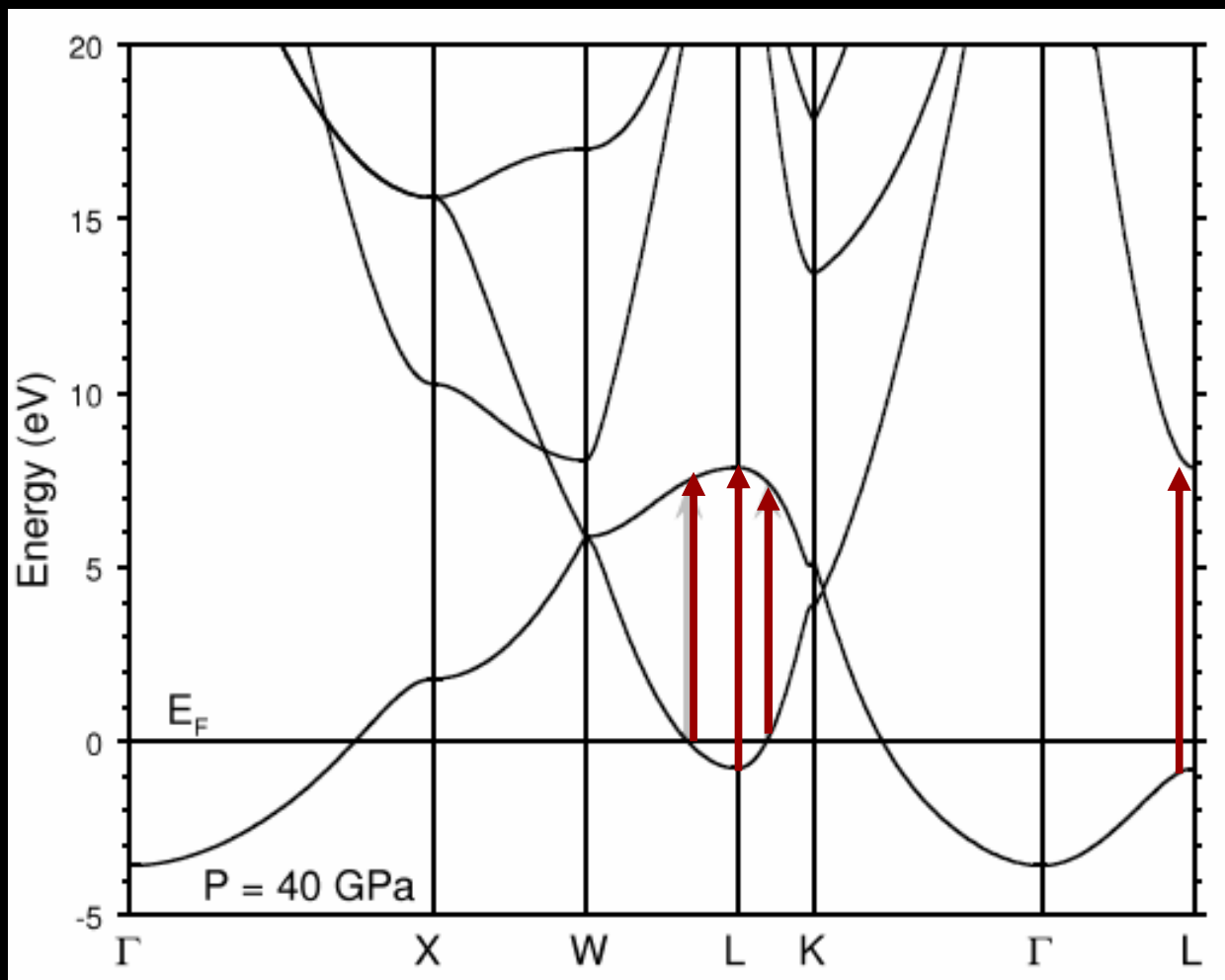
Band structure (fcc, $P=10$ GPa)



New Undamped Plasmon in Lithium under Pressure



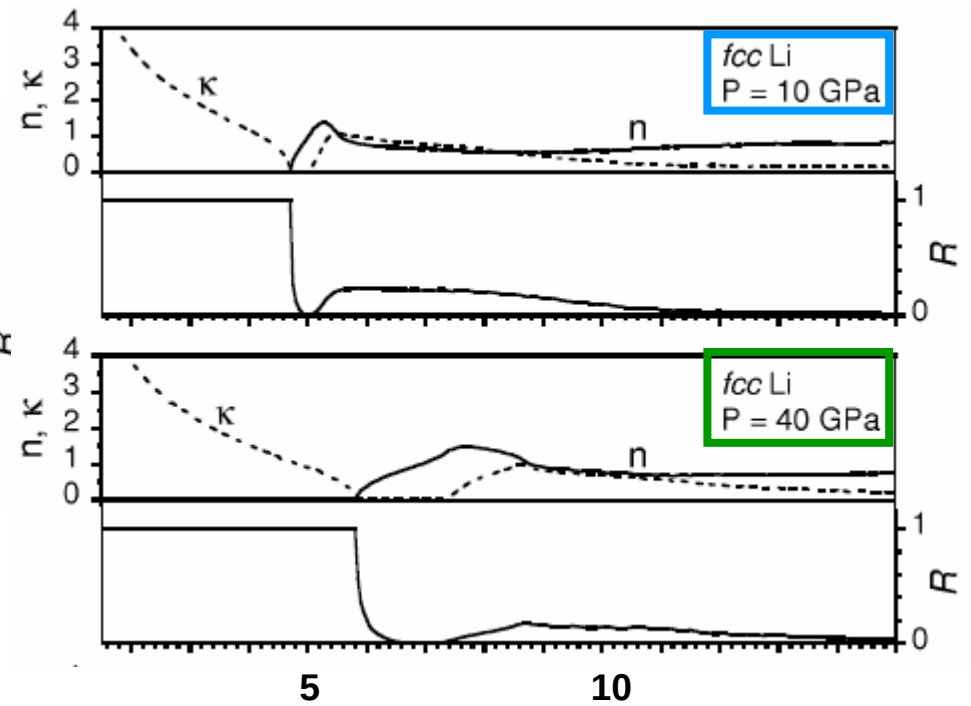
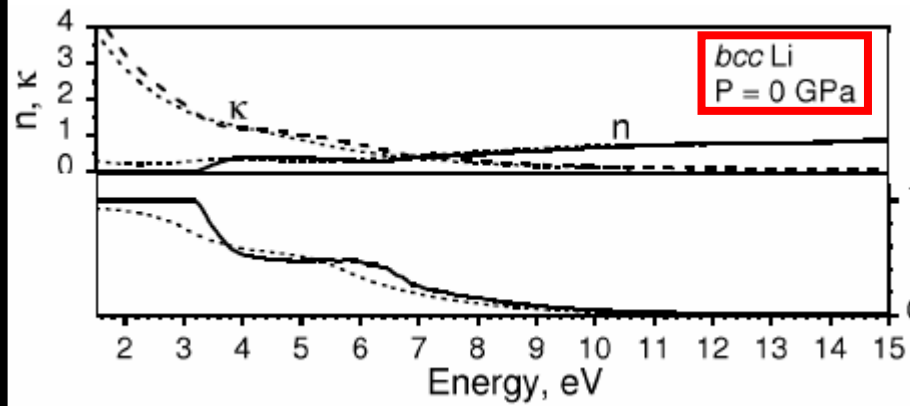
Band structure (fcc, P=30 GPa)



Plasmon energies and line-widths

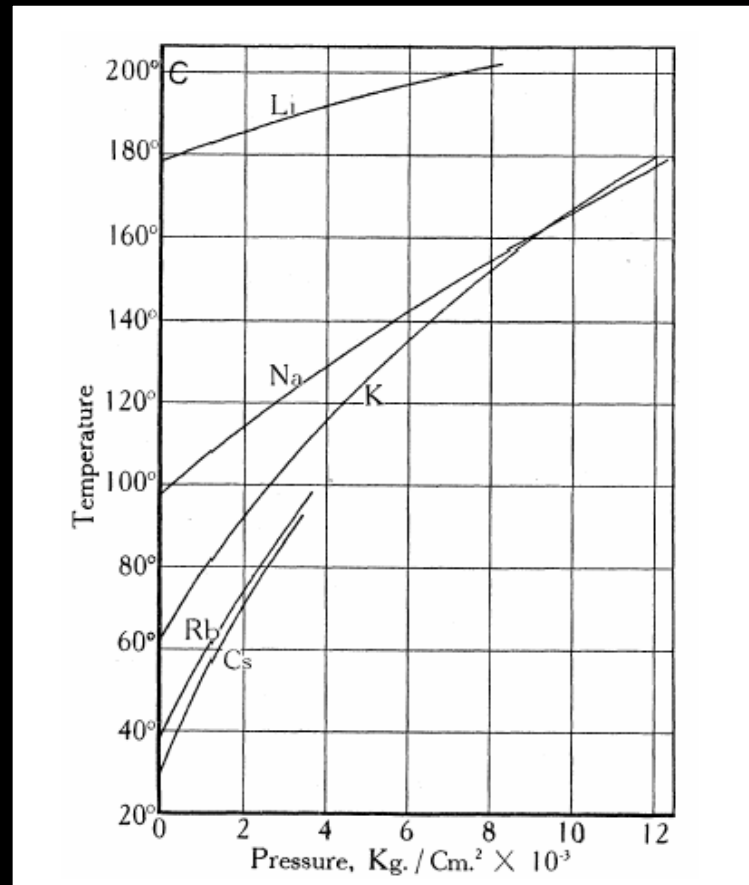
Pressure	E_p^A	Δ_p^A	E_p^B	E_o
<i>fcc</i>				
P=0	7.6	2.3	—	3.3
P=10	9.2	4.8	4.7	5.1
P=20	10.4	5.2	5.25	6.2
P=30	11.5	5.4	5.6	7.0
P=40	12.4	5.5	5.8	7.5
<i>bcc</i>				
P=0	7.28	2.6		

Optical Properties



Unexplored Properties

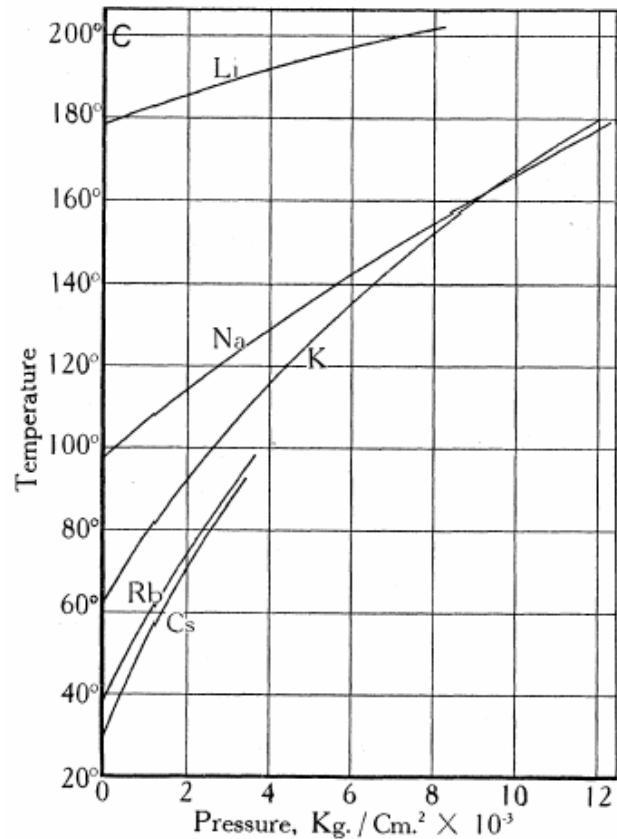
Melting Curve of the Alkalies



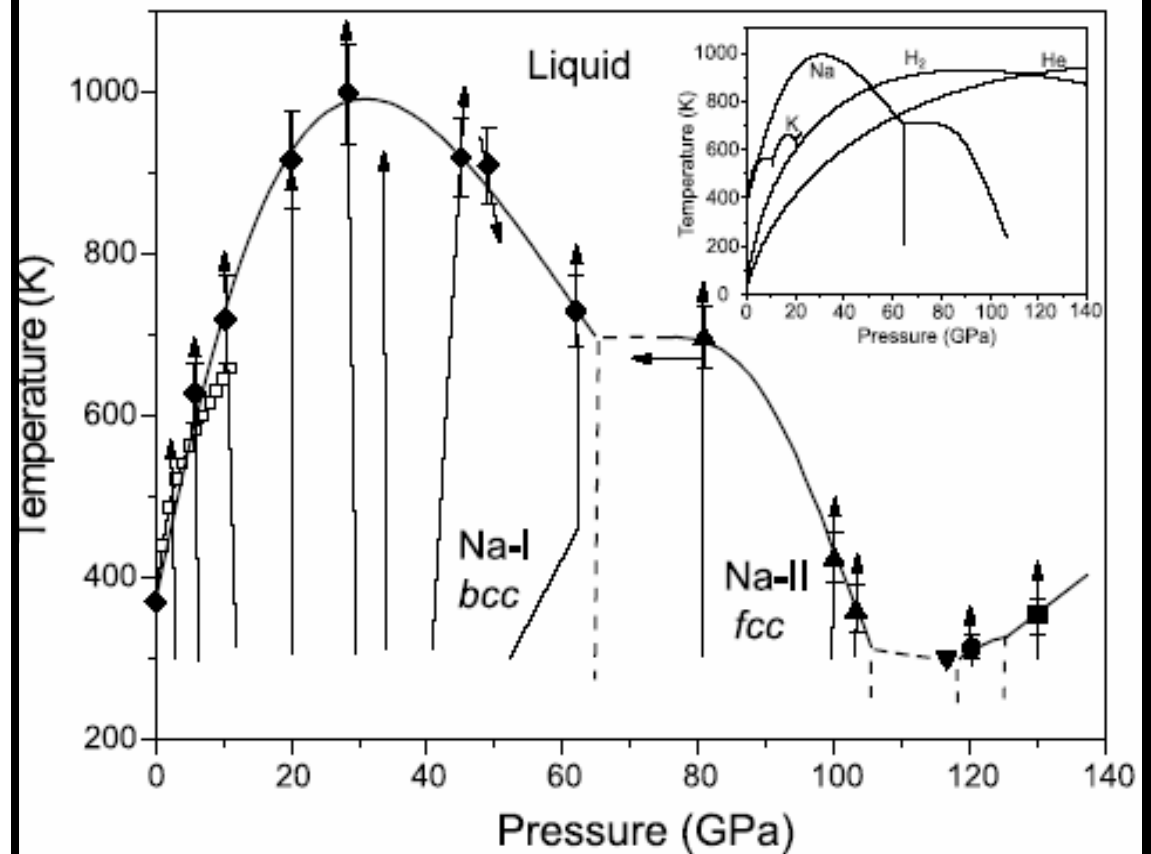
P.W. Bridgman,
Phys. Rev. **27**, 68 (1926)

Unexplored Properties

Melting Curve of the Sodium



P.W. Bridgman,
Phys. Rev. **27**, 68 (1926)



E. Gregoryanz, O. Degtyareva, M. Somayazulu, R.J. Hemley, and H-k. Mao,
Phys. Rev. Letts. **94**, 185502 (2006).

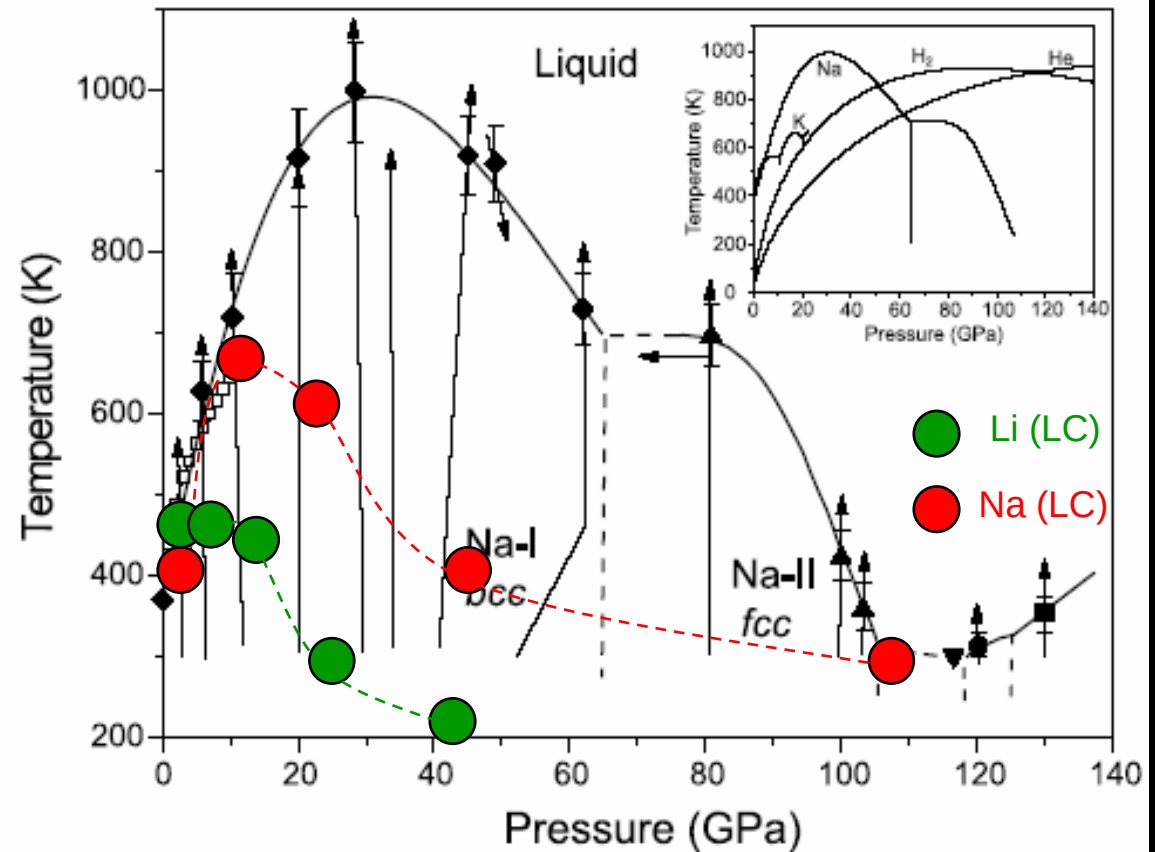
Anomalous Melting-curve of Lithium?

Lindemann Melting Criterion

$$T_m = CMv^{2/3}\Theta_D^2$$

$$\Theta_D = 1.4 \frac{\hbar}{k_B} \sqrt{\langle \omega^2 \rangle}$$

$$\langle \omega^2 \rangle = \frac{2}{\lambda} \int_0^\infty d\omega \omega \alpha^2 F(\omega)$$



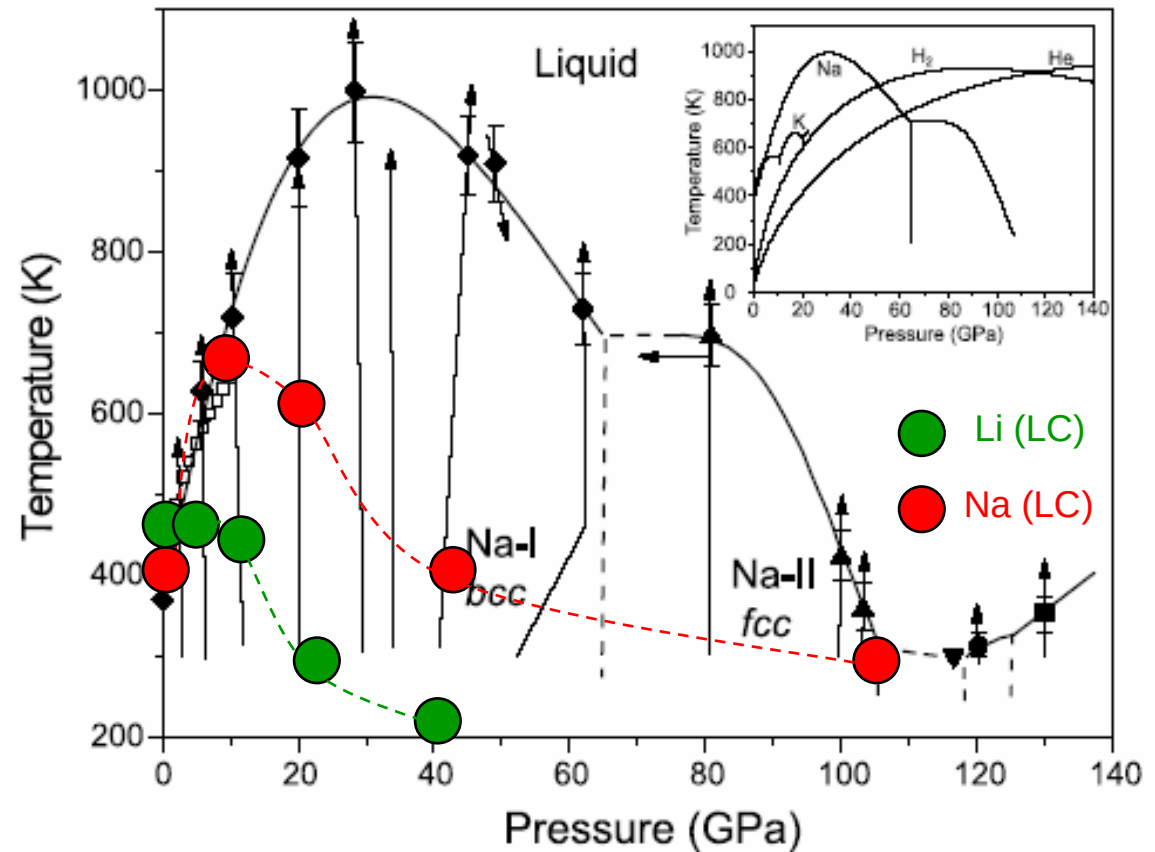
Anomalous Melting-curve of Lithium?

Lindemann Melting Criterion

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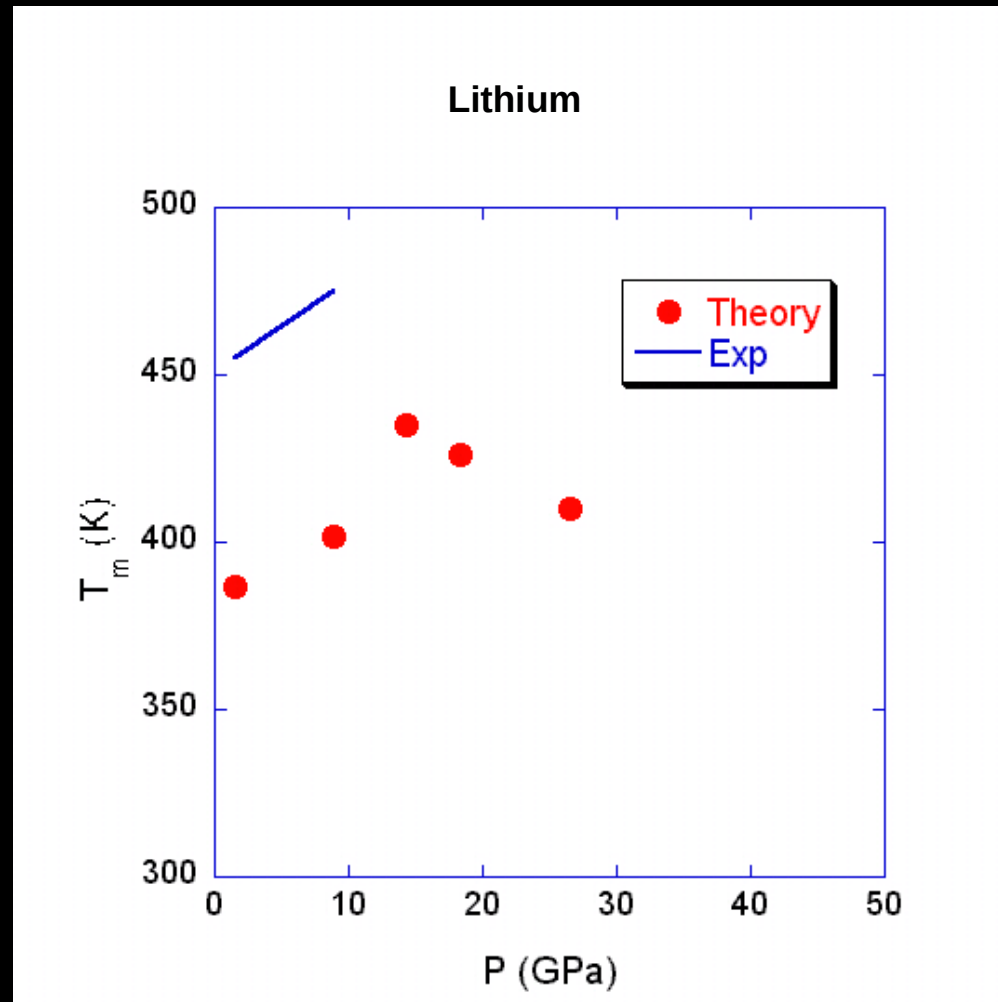
$$\langle \omega^2 \rangle = \frac{2}{\lambda} \int_0^\infty d\omega \omega \alpha^2 F(\omega)$$



$$\lambda_B = \frac{h}{p_{20\text{meV}}} \cong 0.8 \text{ \AA} \cong \frac{d_{\text{Li-Li}}^{40\text{GPa}}}{4}$$

Anomalous Melting-curve of Lithium?

A. Rodriguez-Prieto, D. Alfè, and A. Bergara;
to be published.



Conclusions

- Fermi surface deformation, nesting and increasing crystal local-field effects originates the observed complexity:
 - **Soft-modes** and structural transition to **complex phases**.
 - Enhanced Tc **Superconductivity**.
 - New Undamped **Plasmon**.
 - Possible Anomalous **Melting Curve**.
- Not unique of Lithium nor the alkalies.

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