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**Self-interaction corrections for f-electrons**

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These are preliminary lecture notes, intended only for distribution to participants

# **Self-interaction Corrections for f-electrons**

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**Trieste, Jan. 13, 2007**

# **Self-interaction Corrections for f-electrons**

- **Introduction**
- **4f: SmX**
- **5f: PuSe, Am**
- **Simplified SIC**
- **Summary**

# Self-Interaction Corrected Total Energy Functional

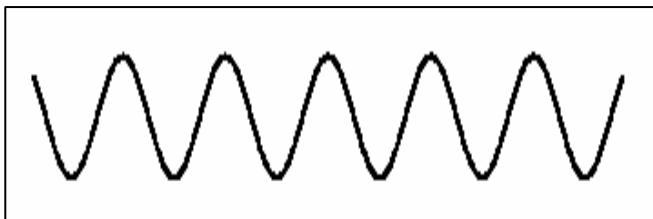
$$E[n] = E^{LSD}[n] - \sum_{\alpha}^{occ.} \delta_{\alpha}$$

$$E^{LSD}[n] = T[n] + U[n] + V_{ext}[n] + E_{xc}[n]$$

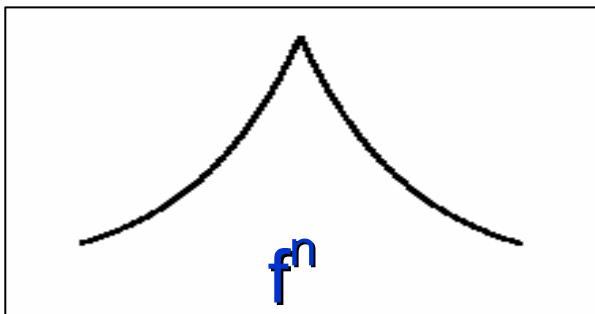
$$\delta_{\alpha} = U[n_{\alpha}] + E_{xc}[n_{\alpha}]$$

Perdew & Zunger, PRB 23, 5048 (1981)

# Introduction

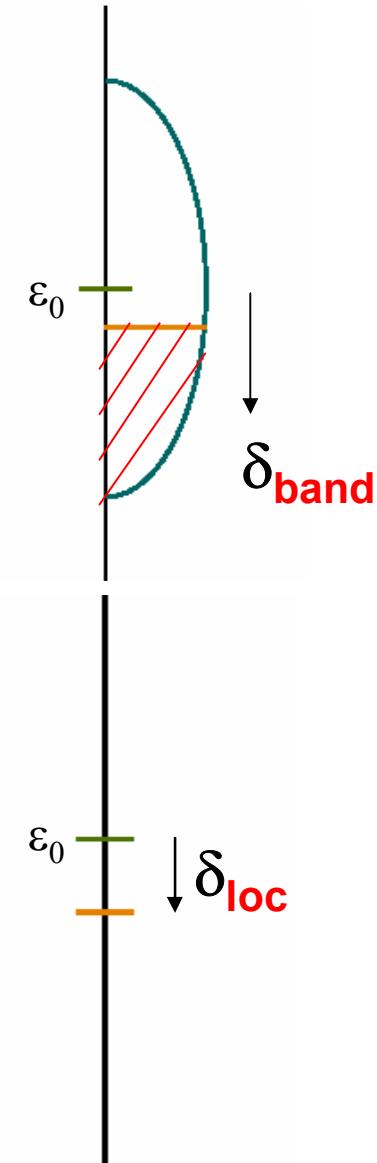


**Bloch:**  
delocalized  
band formation



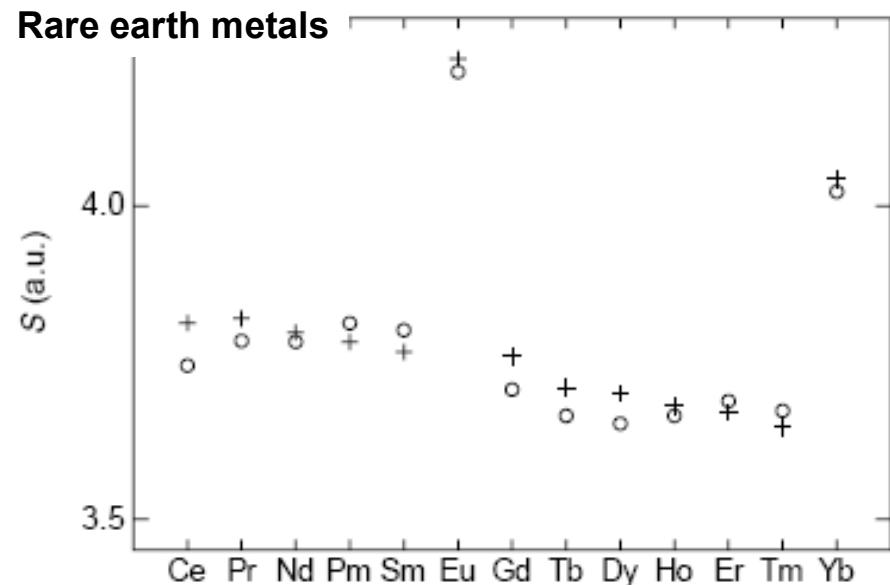
**Heitler-London:**  
localized  
multiplets

TB-LMTO(ASA)

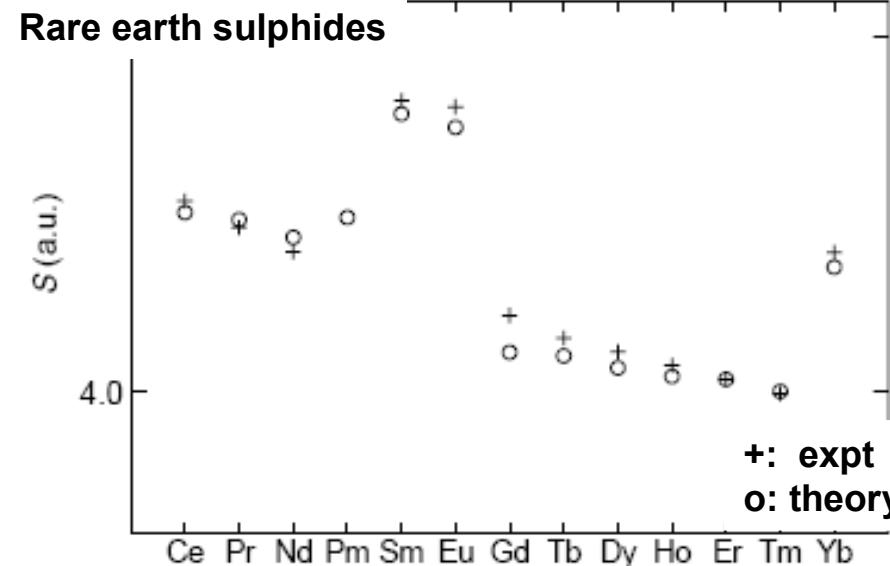


# Lattice constant

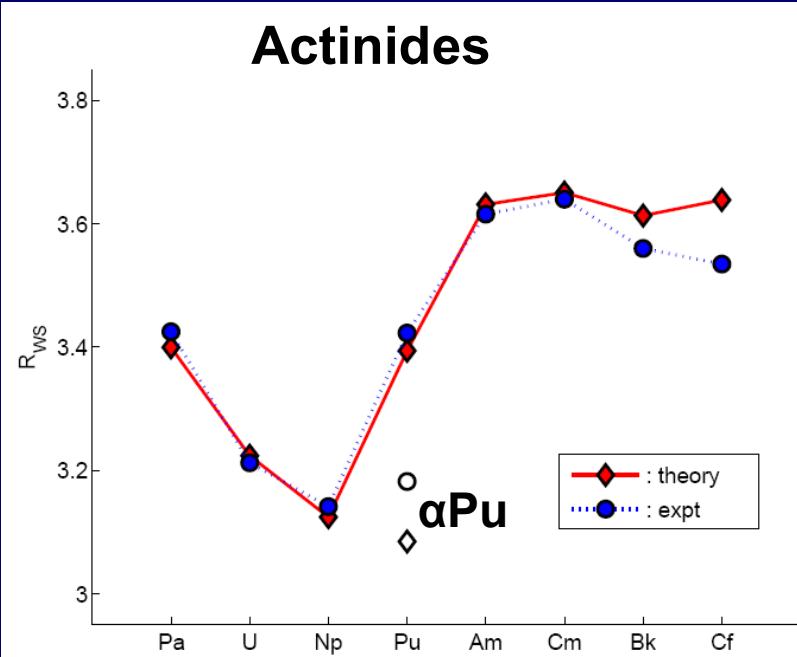
Rare earth metals



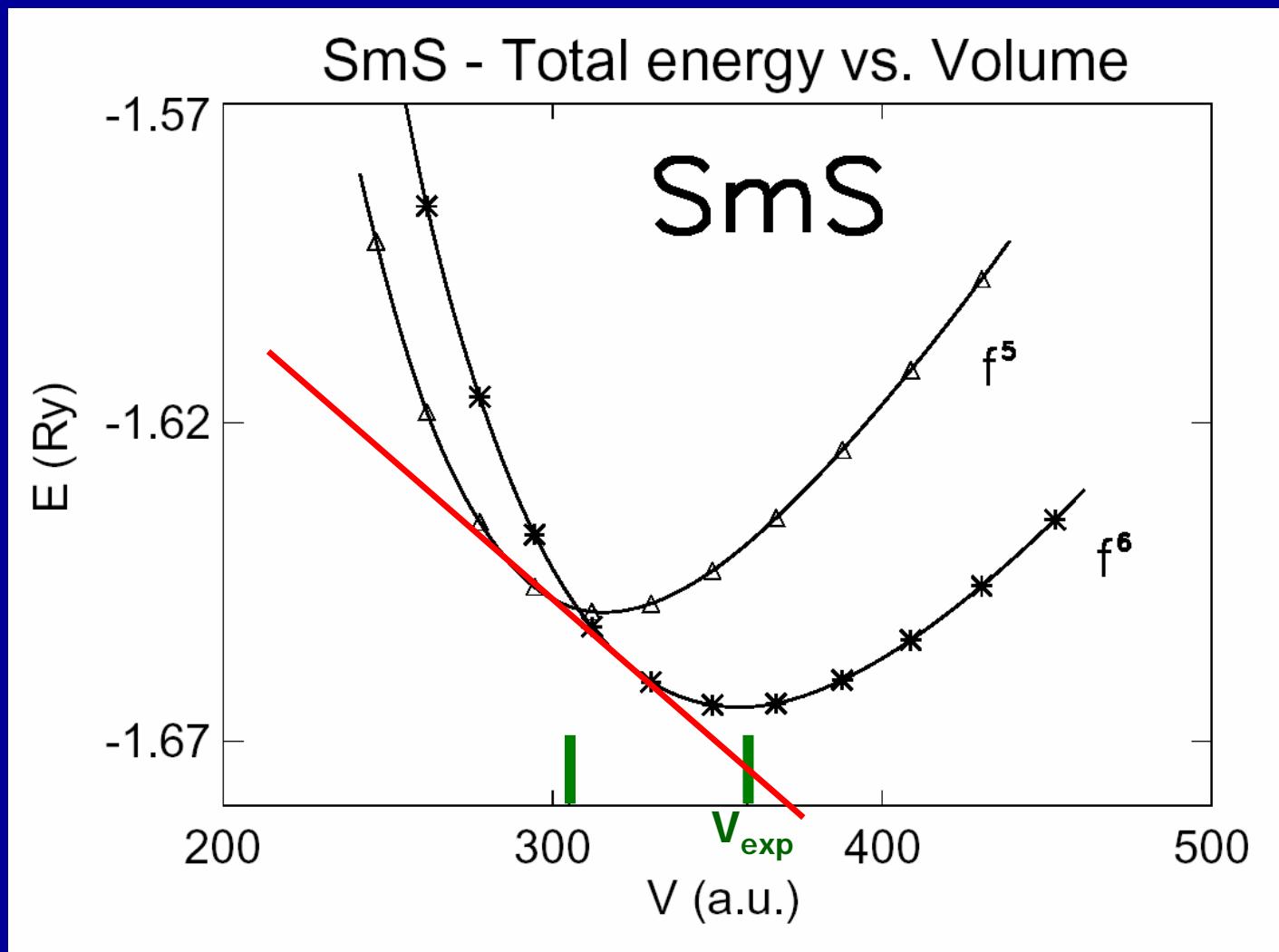
Rare earth sulphides



Actinides

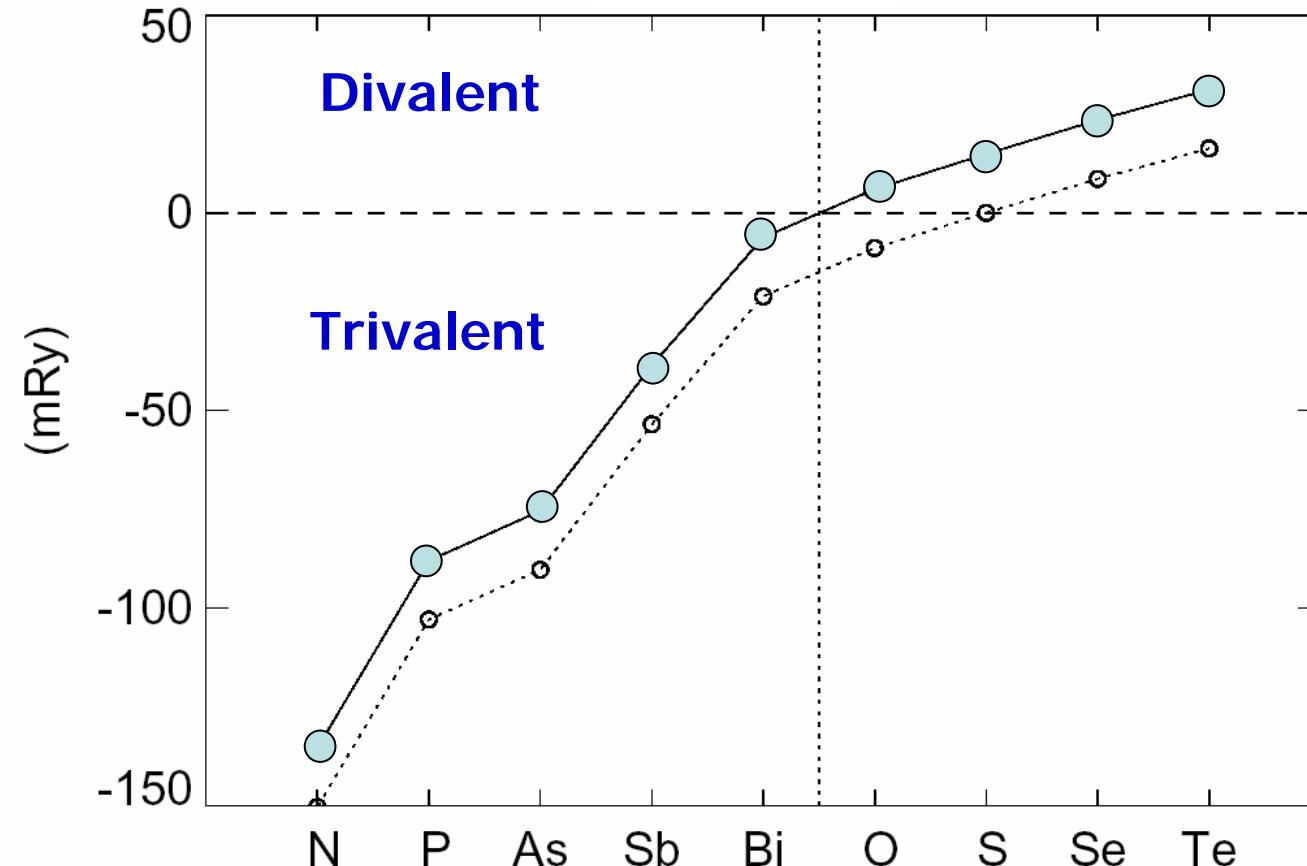


# SmS

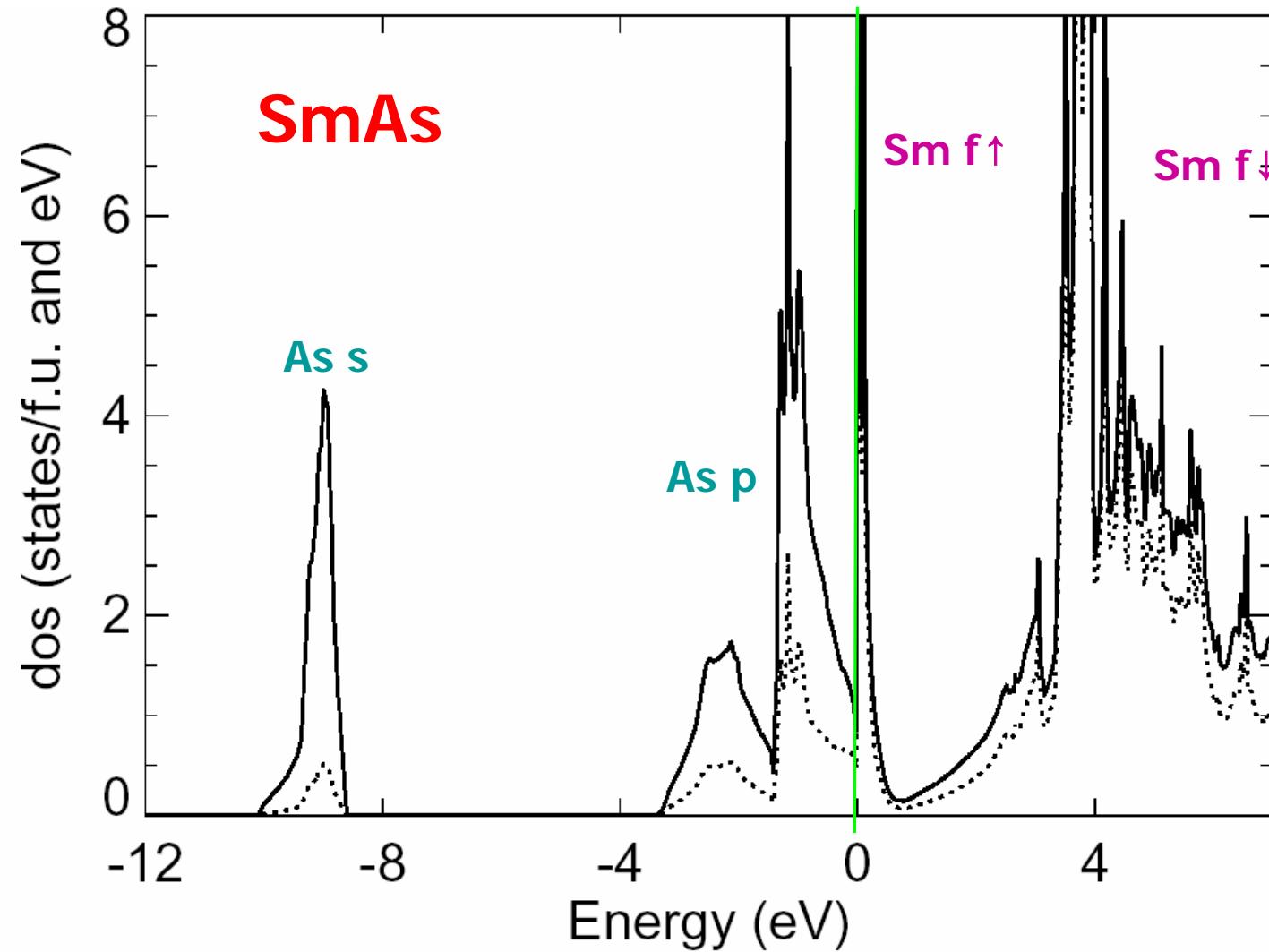


# SmX

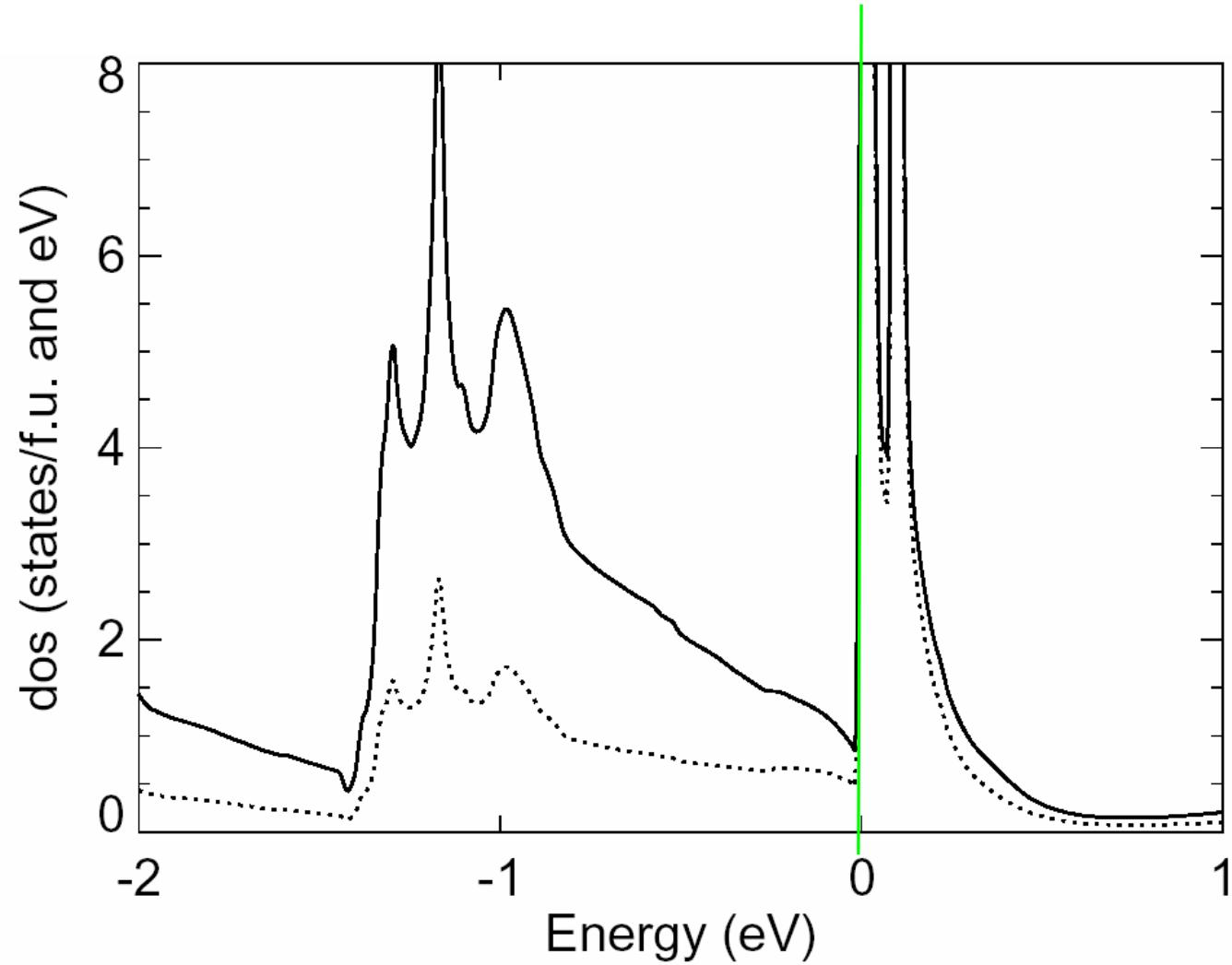
$E(f^5) - E(f^6)$



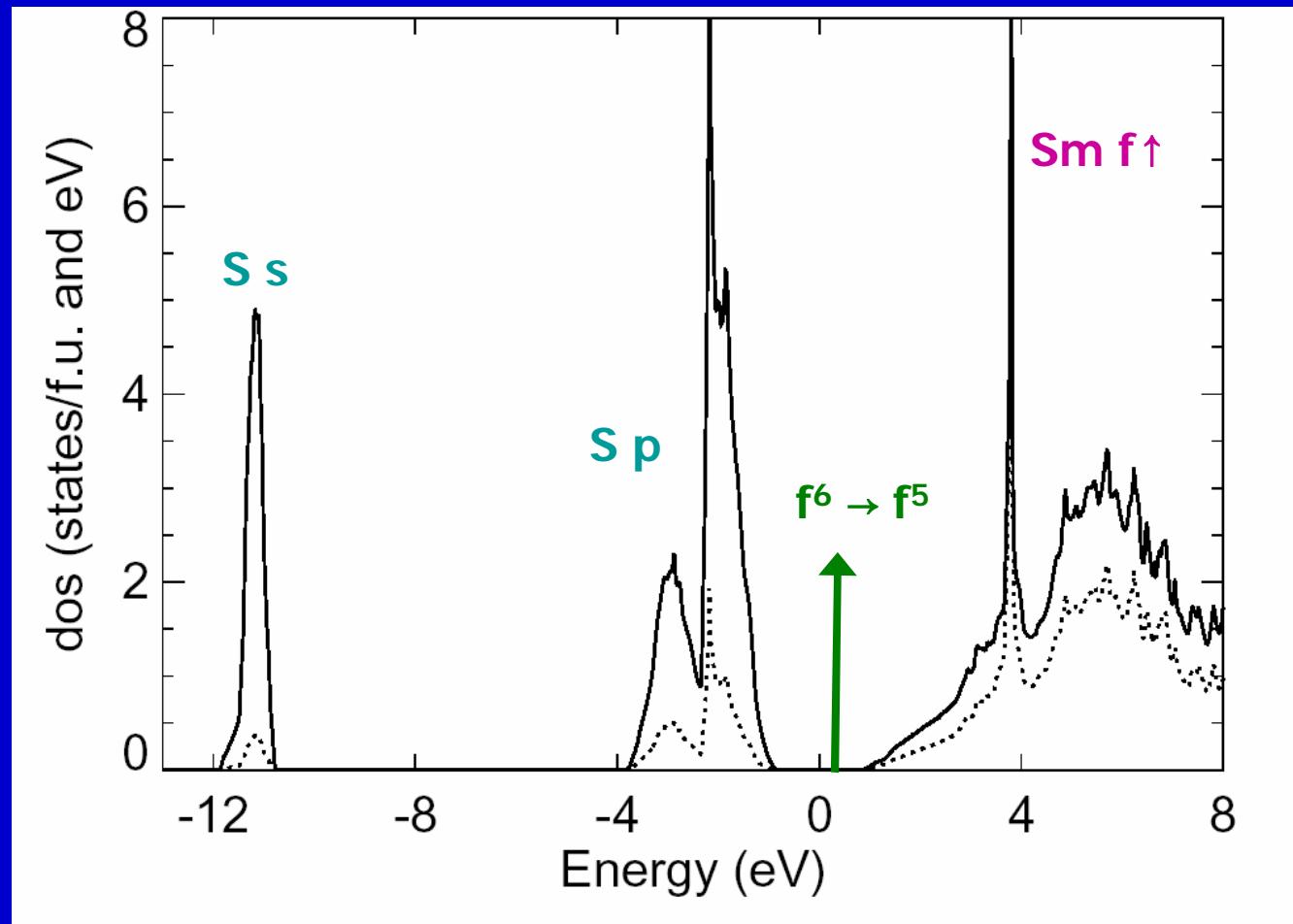
# SmAs - density of states



# SmAs - density of states

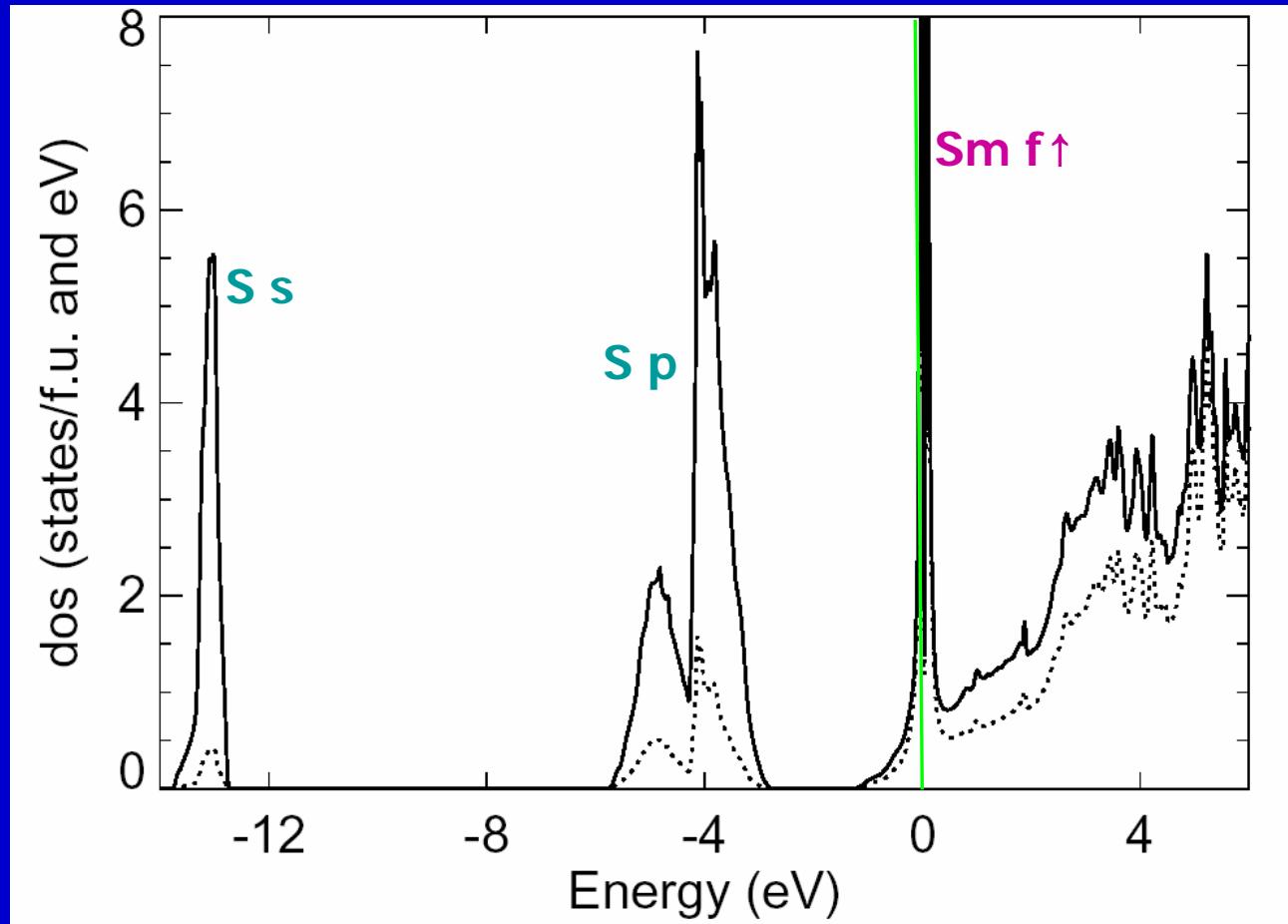


# SmS - density of states

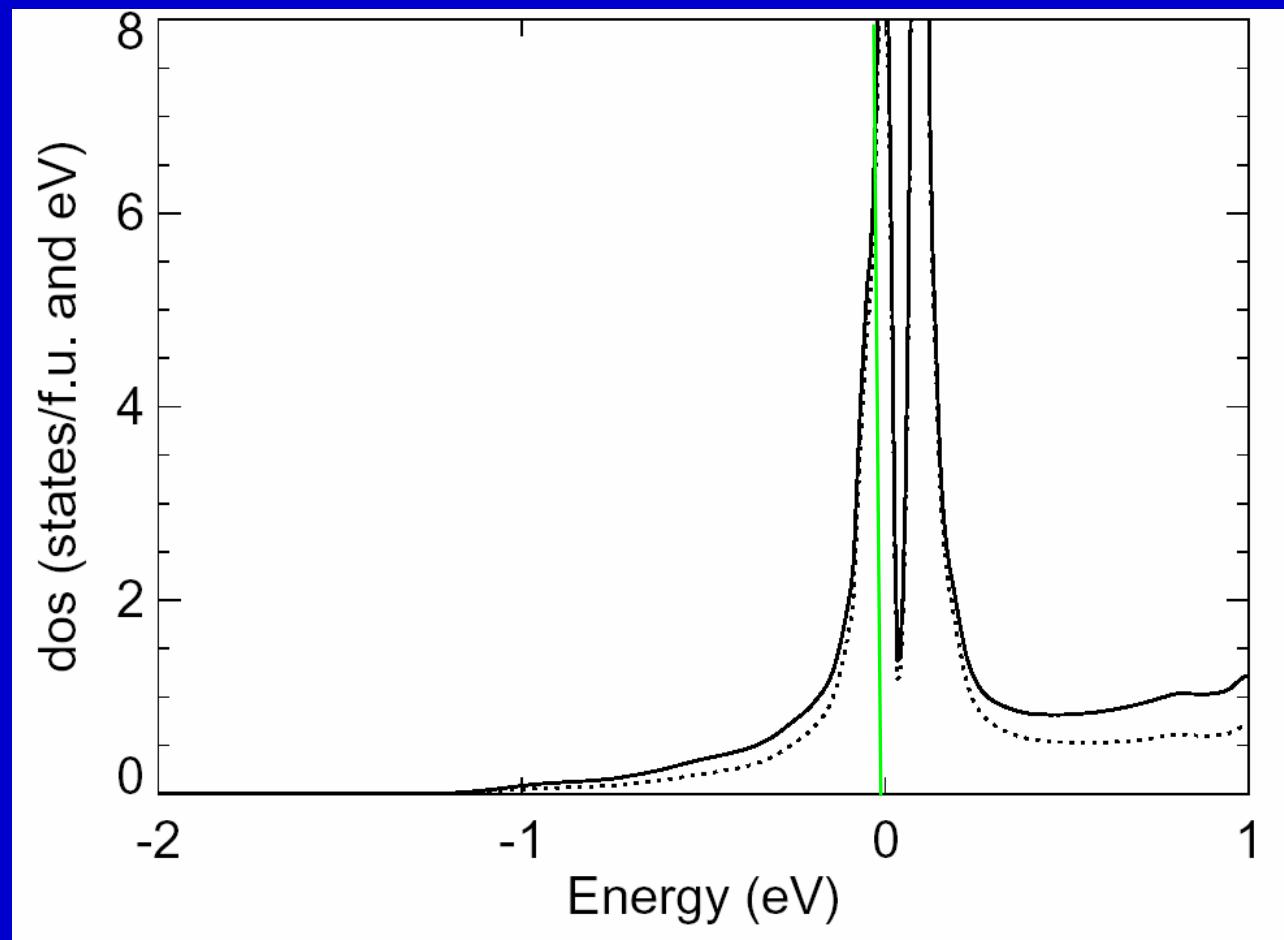


$$E_g = 0.71 \text{ eV}$$

# $\text{Sm}(3+)\text{S}$ - density of states

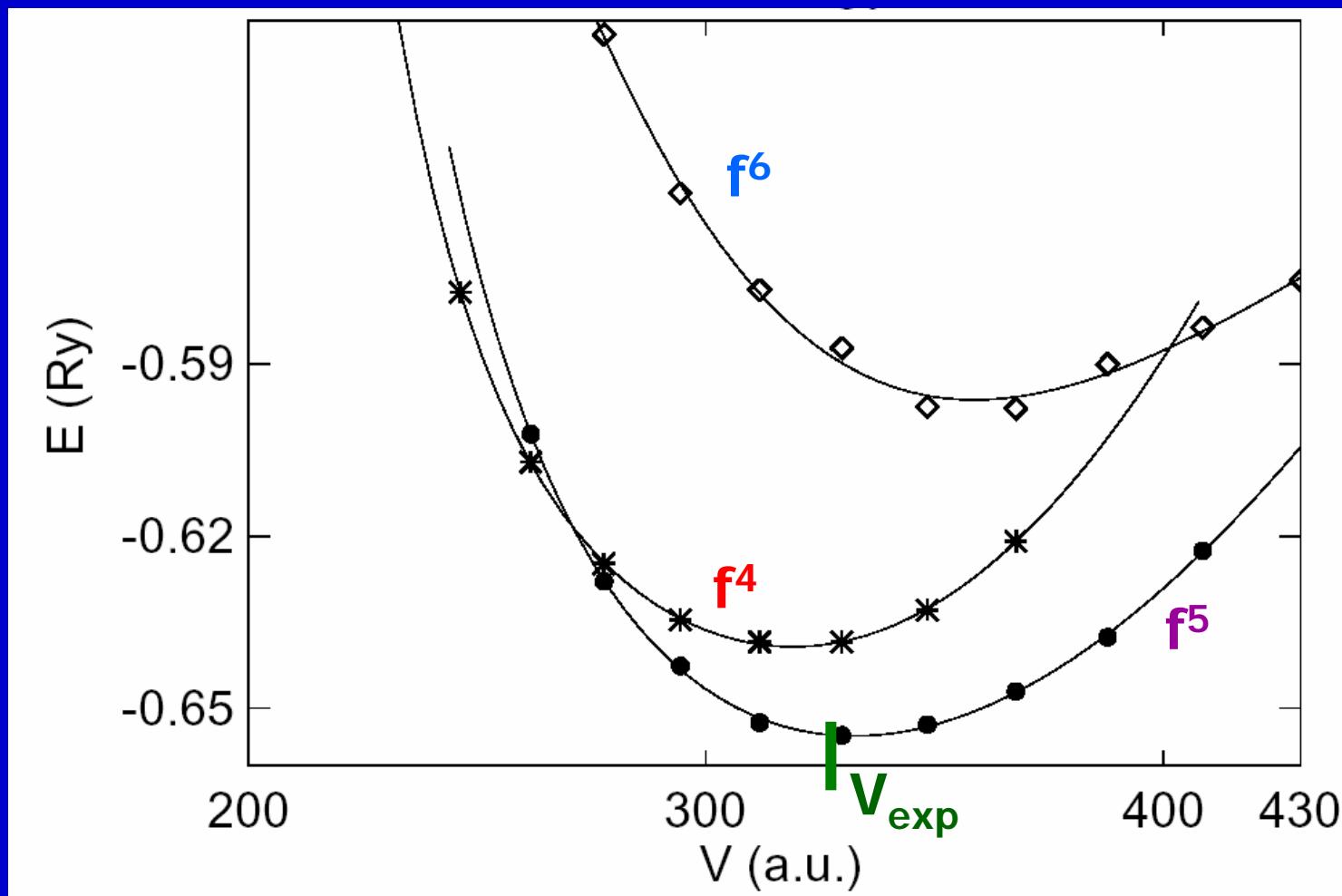


## Sm(3+)S - density of states

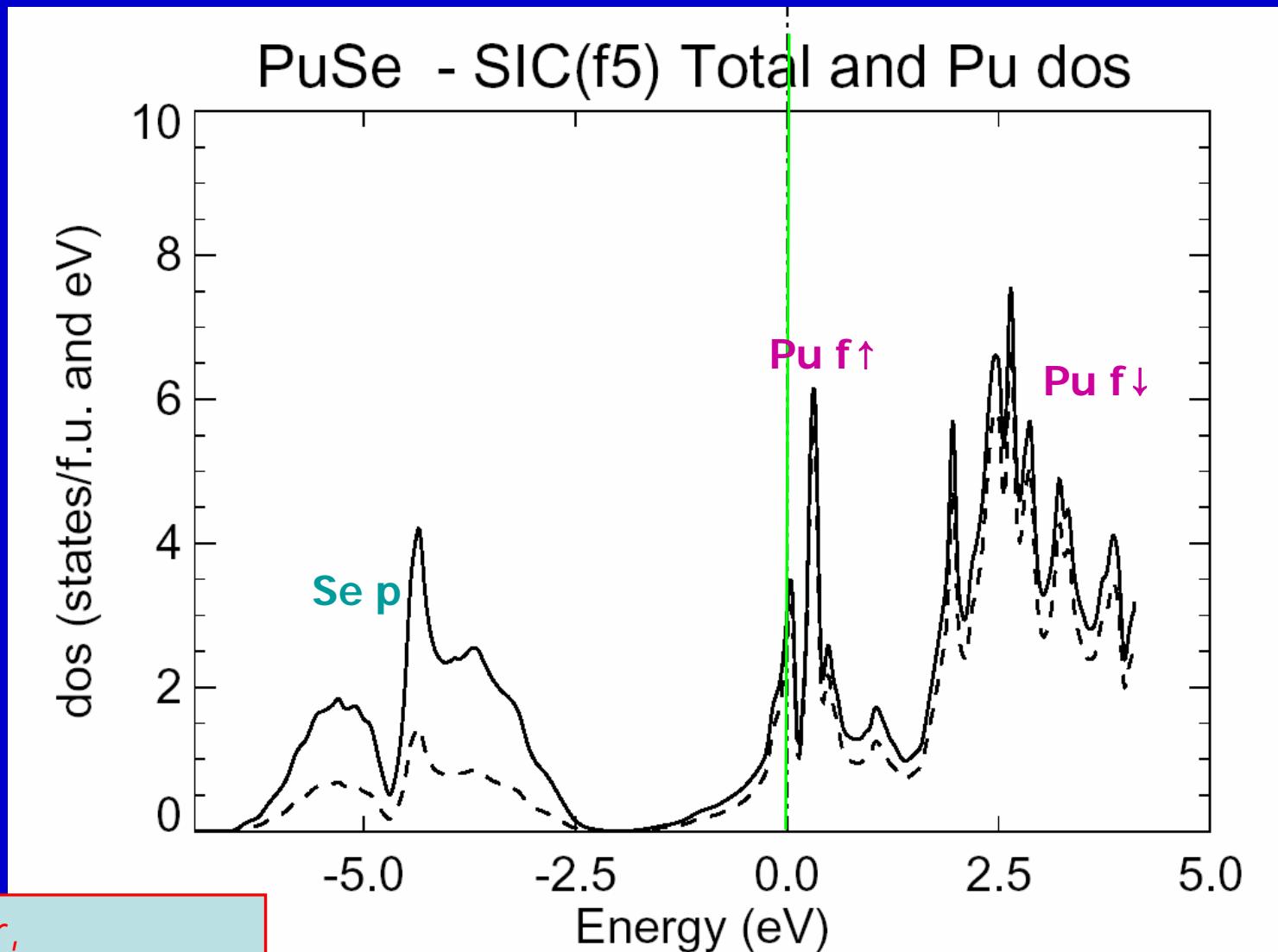


Intermediate Valence

# PuSe



# Pu(3+)Se - density of states

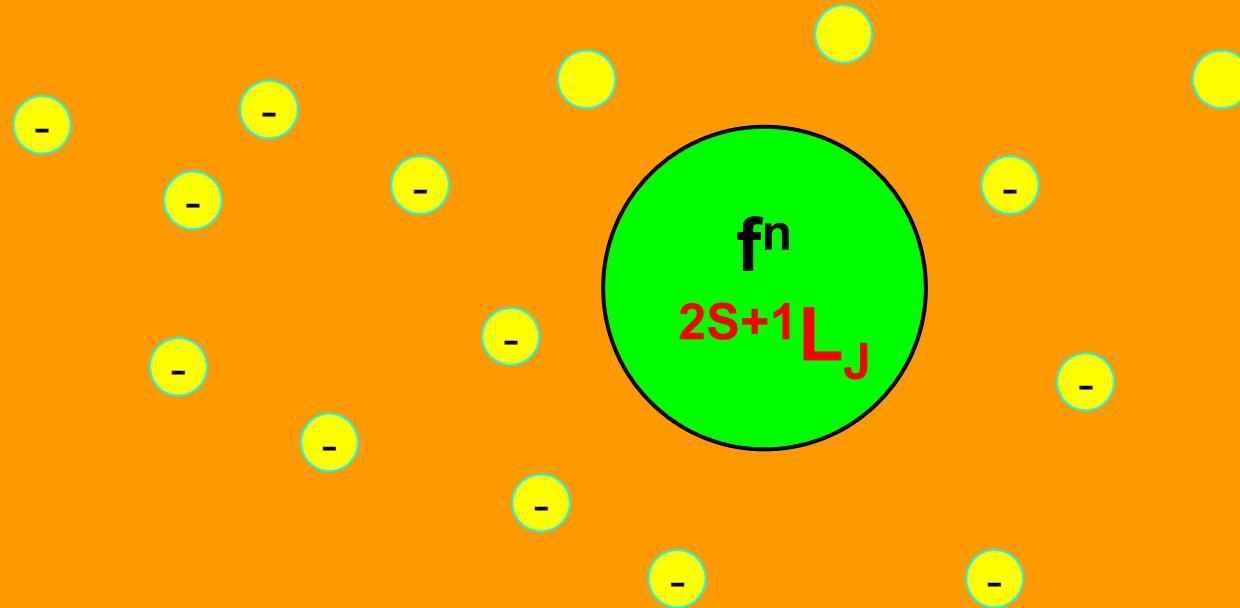


Wachter,  
SSC 127, 599 (2003)

$$\langle f^n ; i | \frac{1}{r_{12}} + \xi \vec{l} \cdot \vec{s} | f^n ; j \rangle \rightarrow |\mu\rangle, E_\mu$$

$$\sum_{ik} V_{ik} (\hat{c}_k^+ \hat{f}_i^- + \hat{f}_i^+ \hat{c}_k^-) = \int V(\varepsilon) \left( \hat{c}_k^+(\varepsilon) \hat{f}_i^- + \hat{f}_i^+ \hat{c}_k^-(\varepsilon) \right) d\varepsilon$$

$$V \sum_i (c_i f_i^+ + f_i c_i^+) \rightarrow |\tilde{\mu}\rangle, \tilde{E}_\mu$$



# DMFT implementation

$$G_{ij}^{atom}(\omega) = \sum_{\mu\nu} g_{\mu\nu} \frac{\langle \tilde{\mu} | f_i | \tilde{v} \rangle \langle \tilde{v} | f_j^+ | \tilde{\mu} \rangle}{\omega + \tilde{E}_\mu - \tilde{E}_\nu}$$

$$\Sigma^{atom}(\omega) = G^{bath}(\omega)^{-1} - G^{atom}(\omega)^{-1}$$

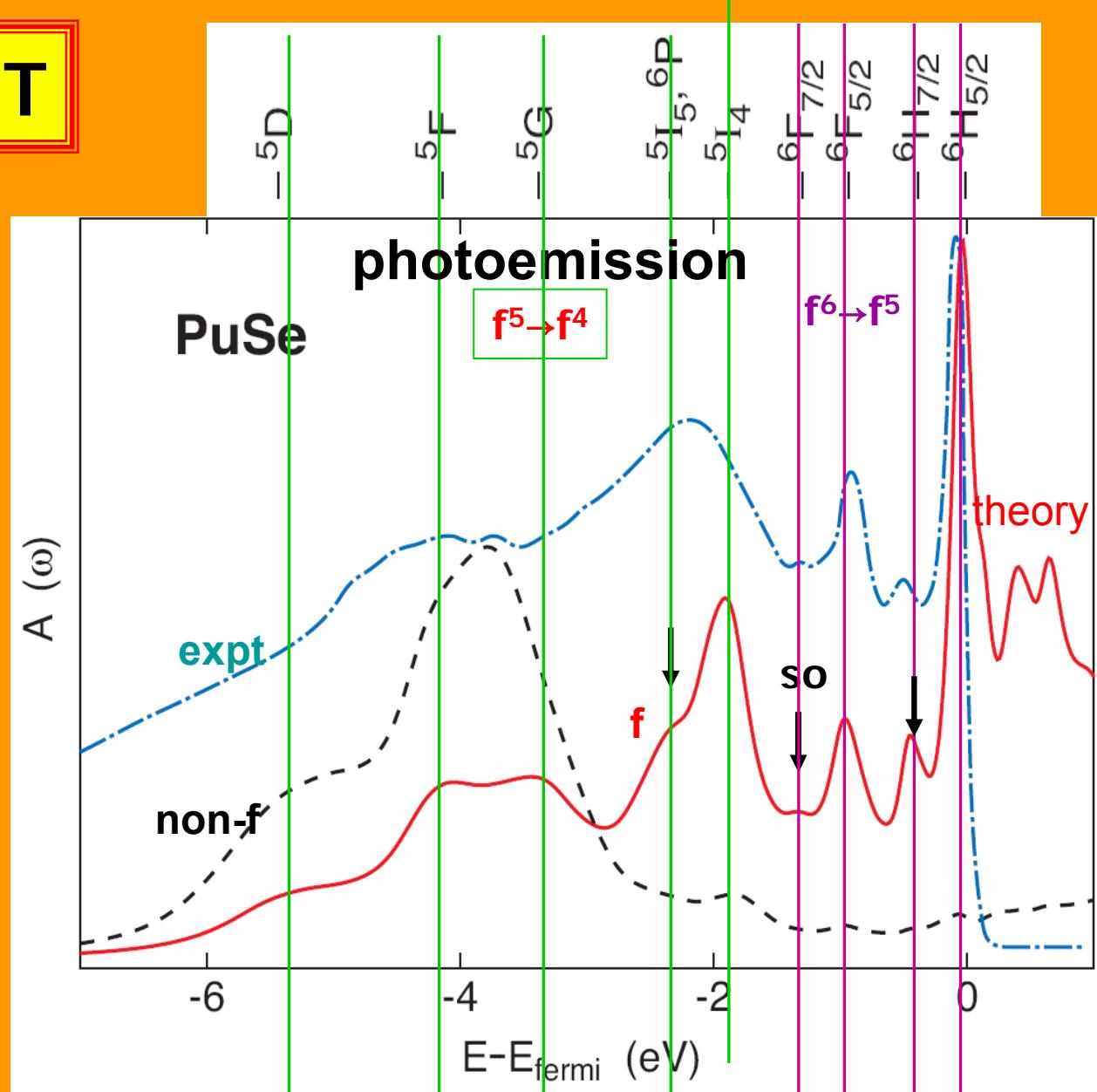
$$G^{solid}(\omega) = \frac{1}{\omega - H_{LDA} - \Sigma^{atom}(\omega)}$$

$$G^{bath}(\omega)^{-1} = \overline{G}^{solid}(\omega)^{-1} + \Sigma^{atom}(\omega)$$

Lichtenstein&Katsnelson,  
PRB **57**, 6884 (1998)

# PuSe - DMFT

$E(f^5) - E(f^6) = -44 \text{ meV}$   
 $V = 0.12 \text{ eV}$   
 $|\langle \Psi_{\text{gs}} | f^5 \rangle|^2 = 0.70$   
 $|\langle \Psi_{\text{gs}} | f^6 \rangle|^2 = 0.30$



**PES-exp.:**  
*Gouder et al.*,  
 PRL 84, 3378 (2000)

Solid State Commun. 140, 364 (2006)

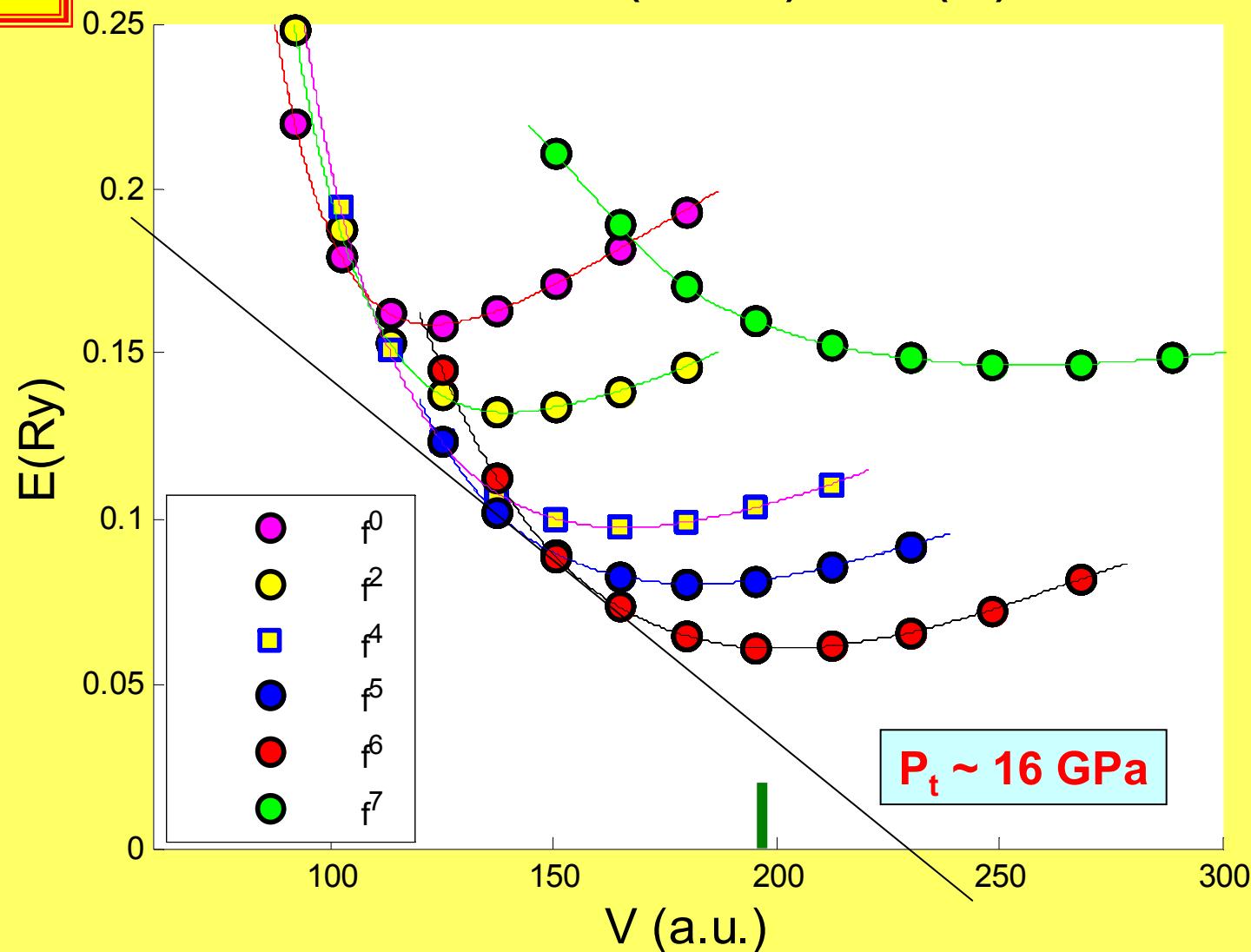
N	O
P	S
As	Se
Sb	Te
Bi	Po

# AnX valencies

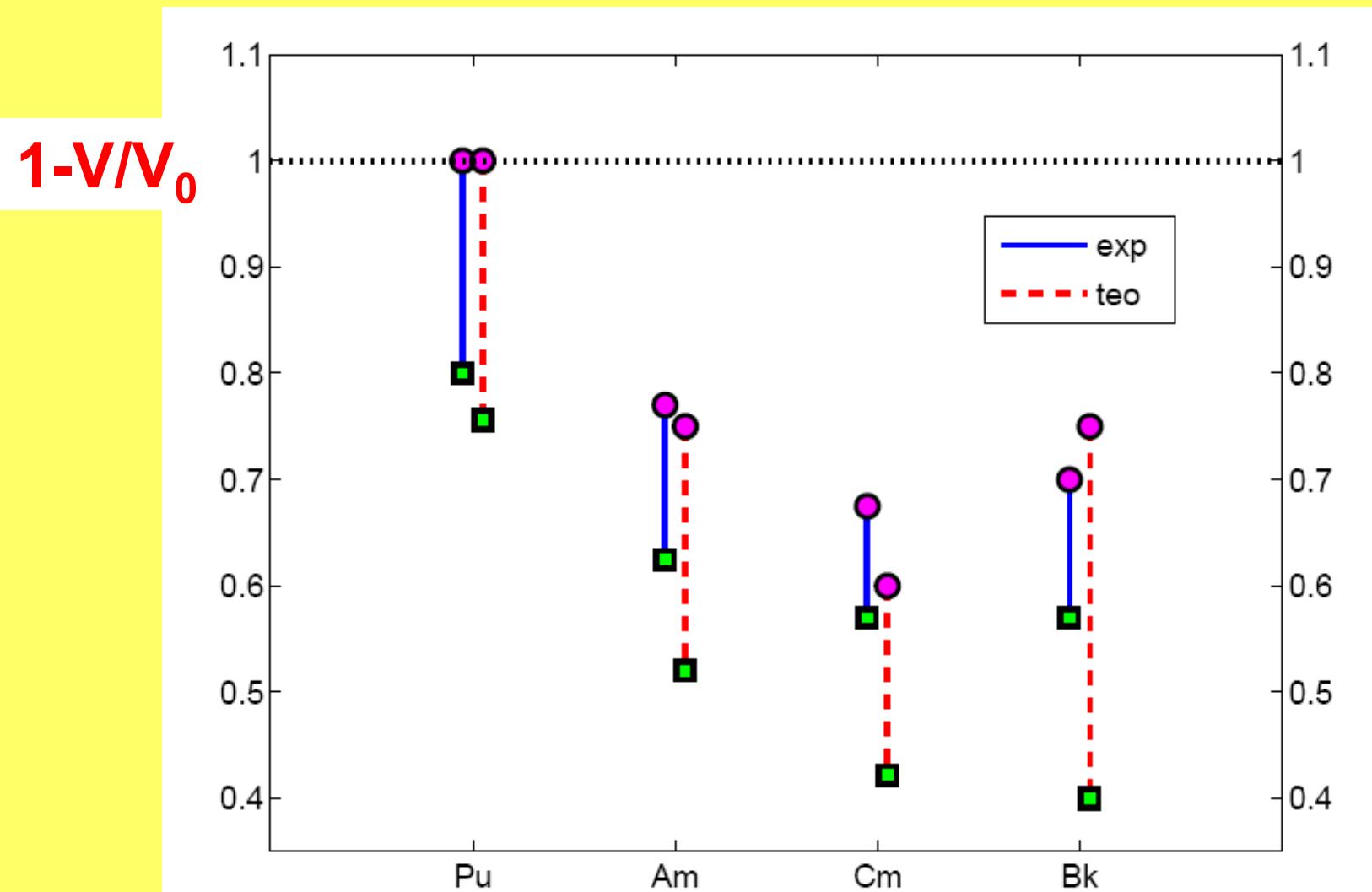
U	Np	Pu	Am	Cm
6 5	5 54	54 43	3 3	3 3
5 54	4 4	43 3	3 3	3 3
4 54	4 4	3 3	3 3	3 3
4 4	4 4	3 3	3 3	3 3
3 3	3 3	3 3	3 32	3 3
f <sup>1</sup> f <sup>2</sup>				
f <sup>3</sup> f <sup>4</sup>				
f <sup>5</sup>				
f <sup>6</sup>				
f <sup>7</sup>				

# Am

## Am SIC (FCC) : E(V)



# Actinides - delocalization



# Simplified SIC

$$E[n] = E^{LSD}[n] - \sum_{\alpha}^{\text{occ.}} \delta_{\alpha}$$

$$E^{LSD}[n] = T[n] + U[n] + V_{ext}[n] + E_{xc}[n]$$

$$\delta_{\alpha} = U[n_{\alpha}] + E_{xc}[n_{\alpha}]$$

# Simplified SIC

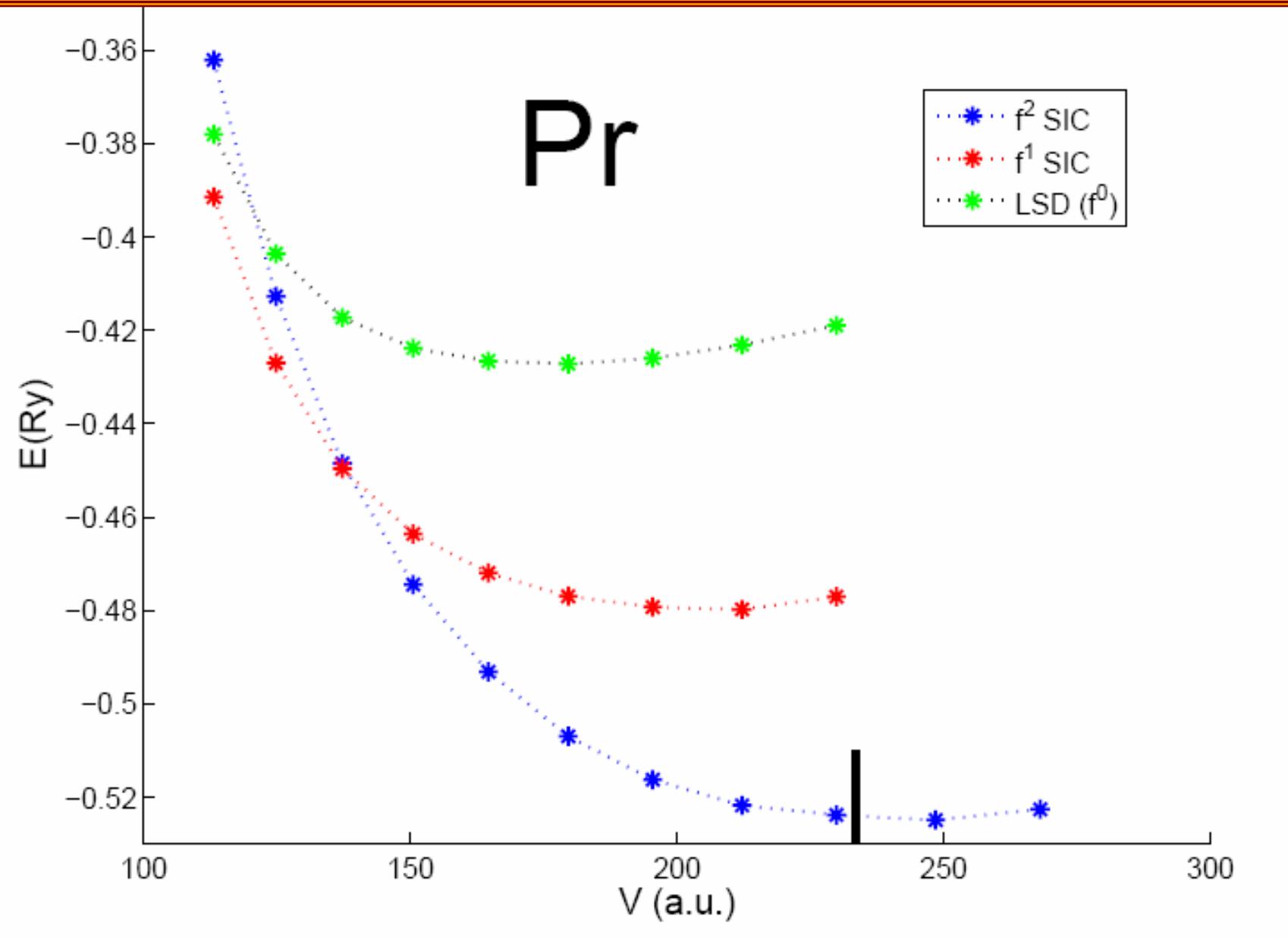
$$E[n] = E^{LSD}[n] - \sum_i^{corr.} \delta_i$$

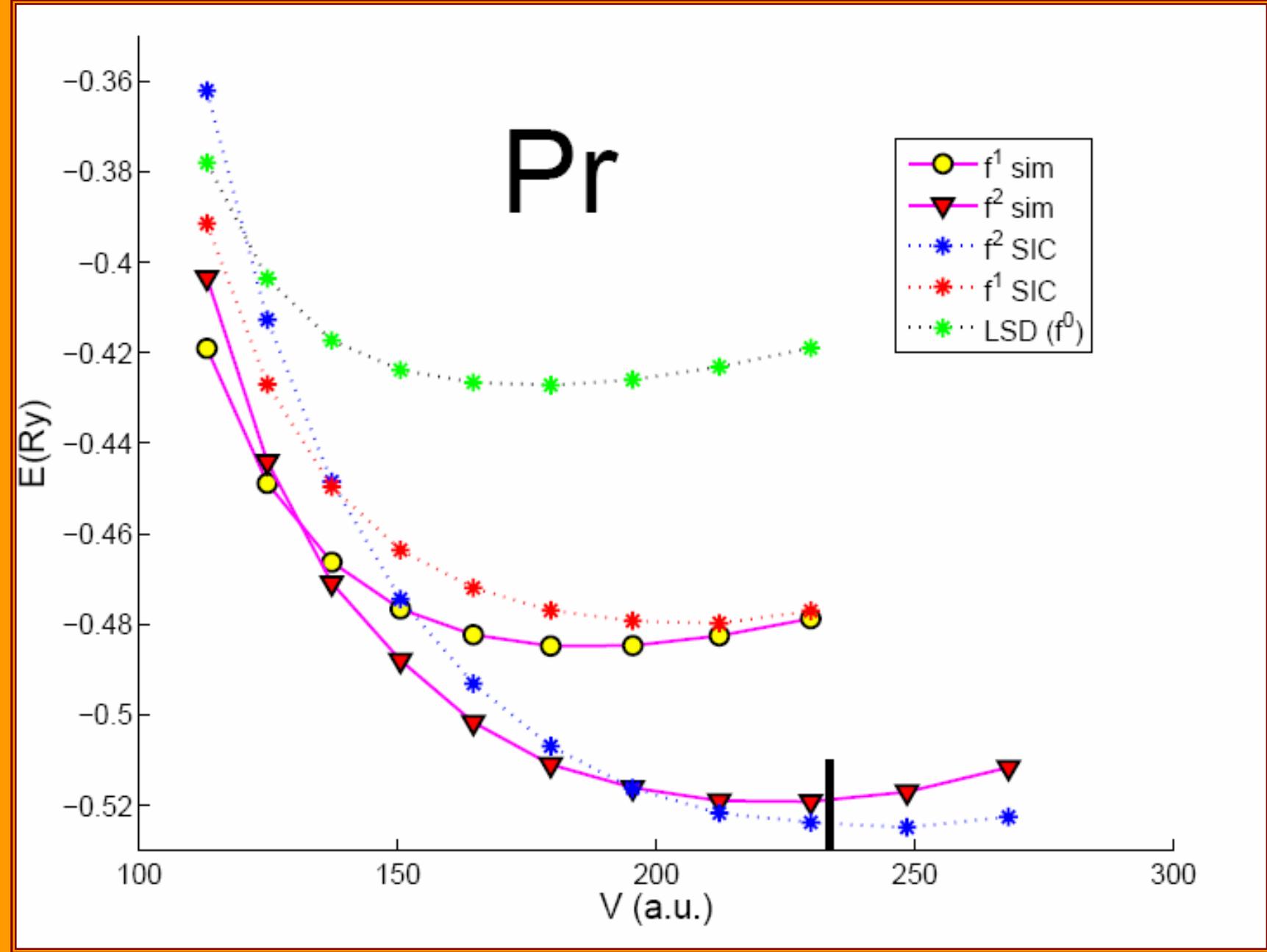
$$E^{LSD}[n] = T[n] + U[n] + V_{ext}[n] + E_{xc}[n]$$

$$\delta_i = U[n_i | \phi(r) |^2] + E_{xc}[n_i | \phi(r) |^2]$$

$$\rho_{ij} \xrightarrow{diag} n_i$$

$$\Delta V_i = -V_H[n_i | \phi(r) |^2] - V_{xc}[n_i | \phi(r) |^2]$$





# Summary

- SIC-LSD:

- $f^n$  localized states

- valency (ground state)

- Excit's.: LDA++Hubbard-I:

- multiplets

- Sm and Tm: pure atomic

- PuSe and Am: complex gr.states

- SSC 116, 399 (2000)**  
**Science 301, 498 (2003)**  
**Phase Transit. (2007)**

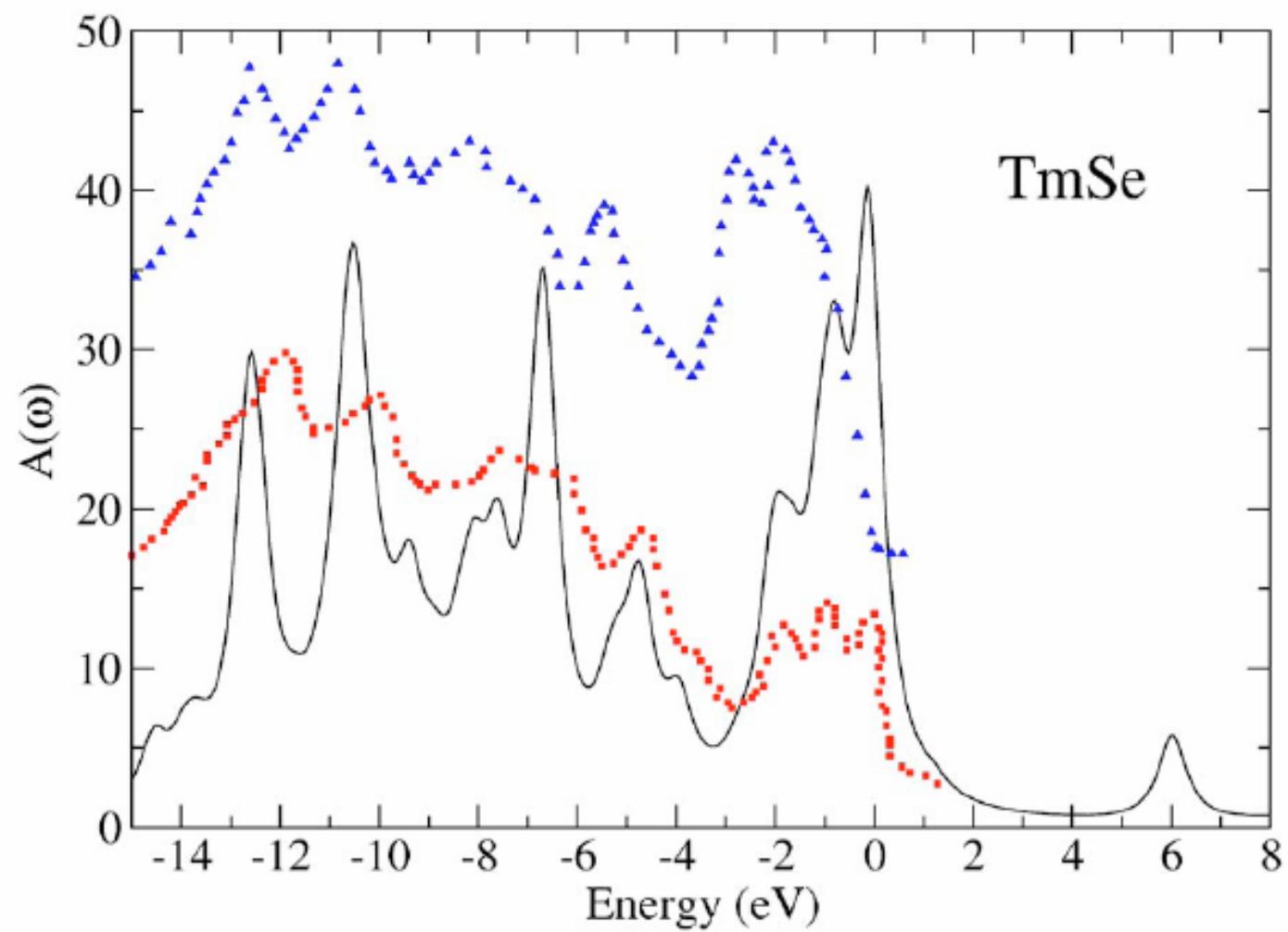
- PRB 71, 45119 (2005)**  
**PRB 72, 245102 (2005)**

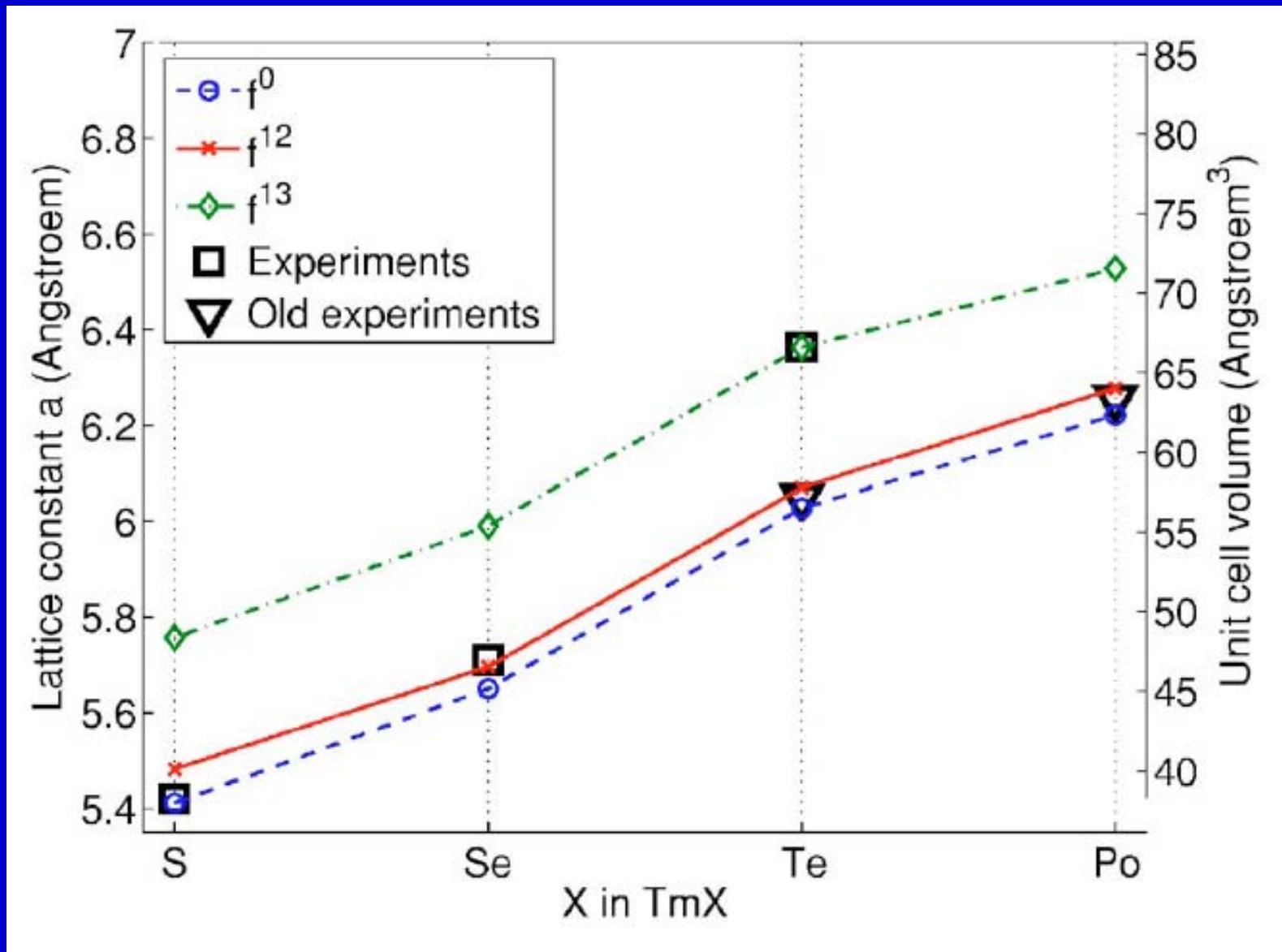
- SSC 140, 364 (2006)**

- Simplified SIC:

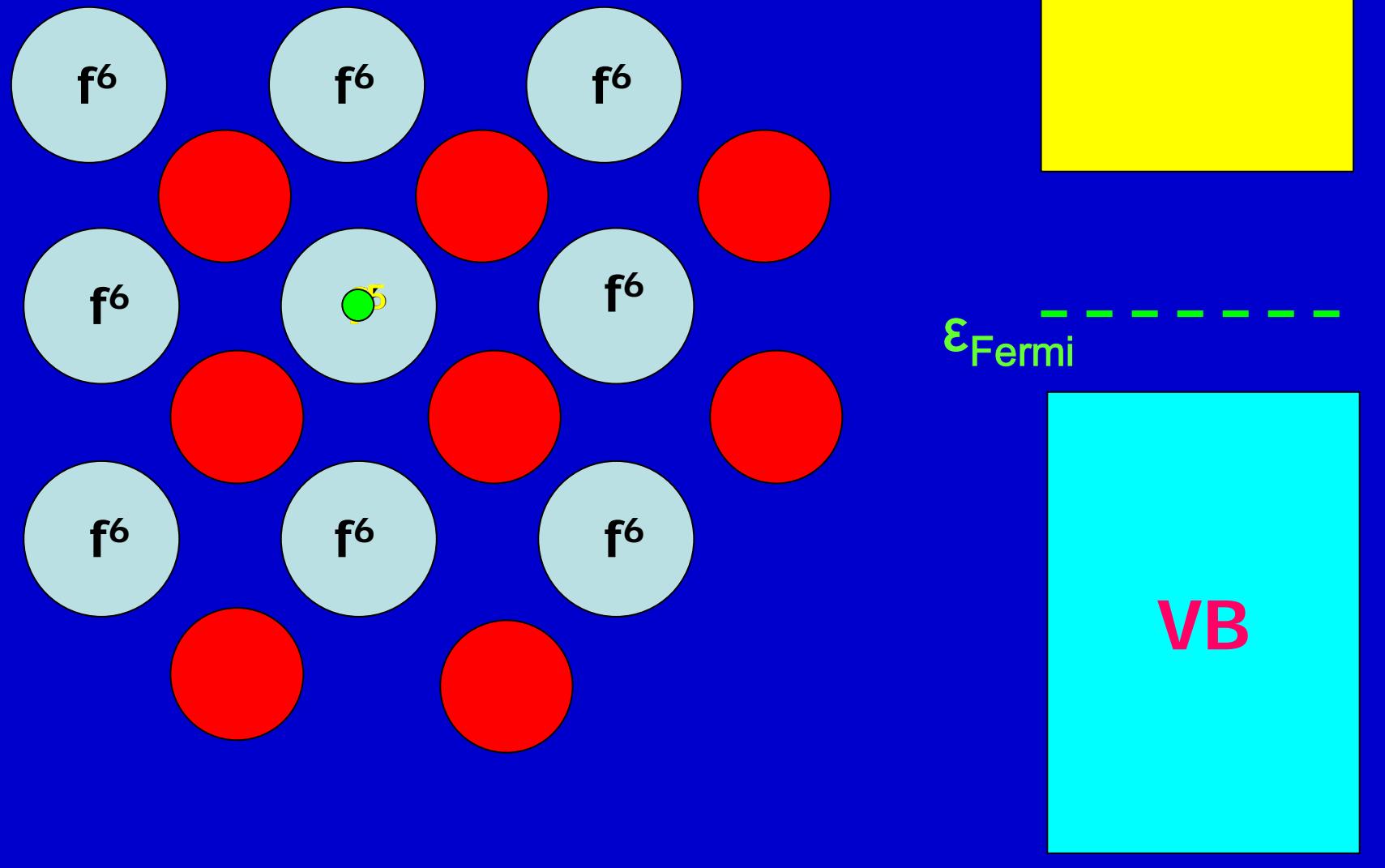
- LDA+U like ...maybe...

# TmSe

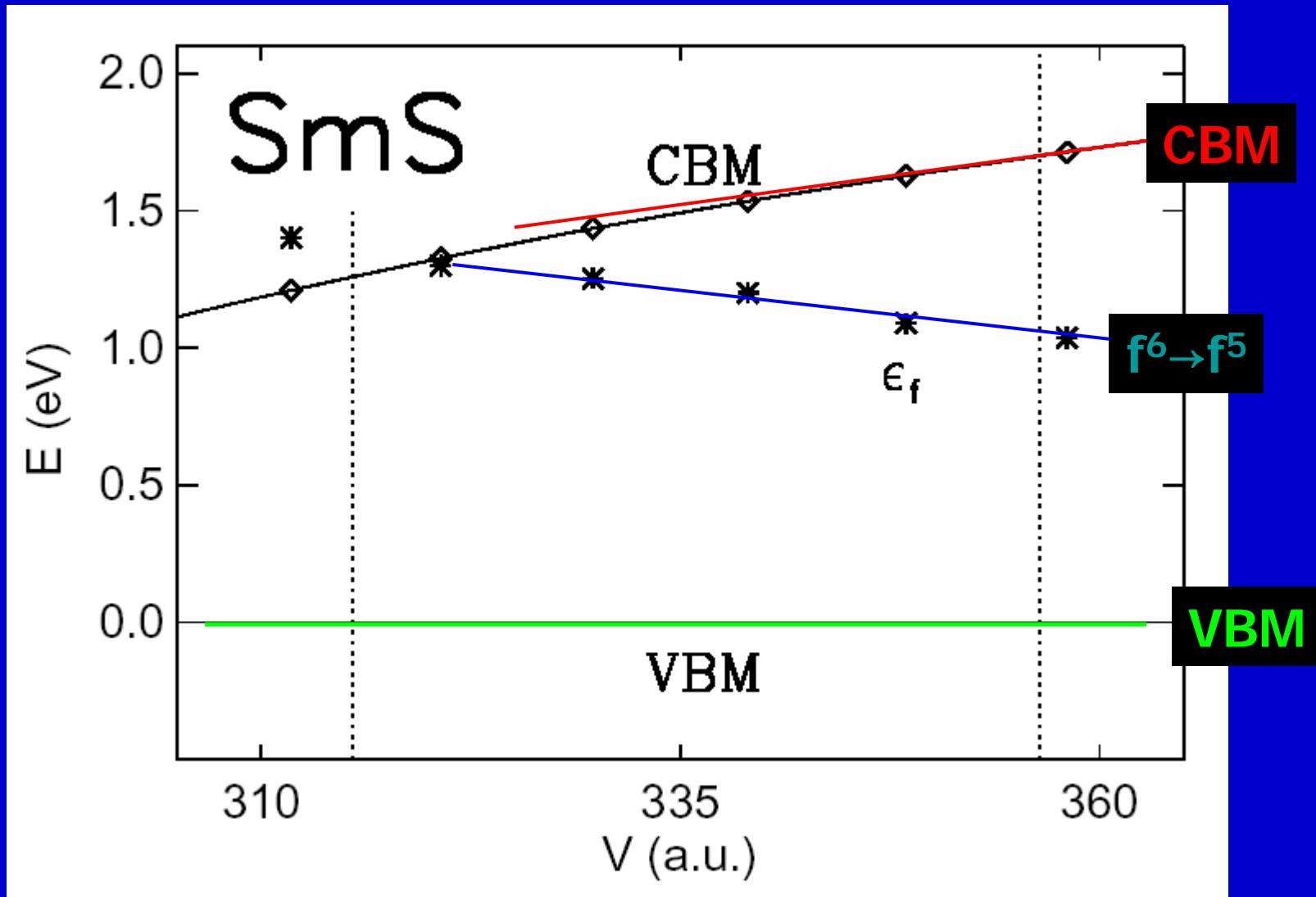




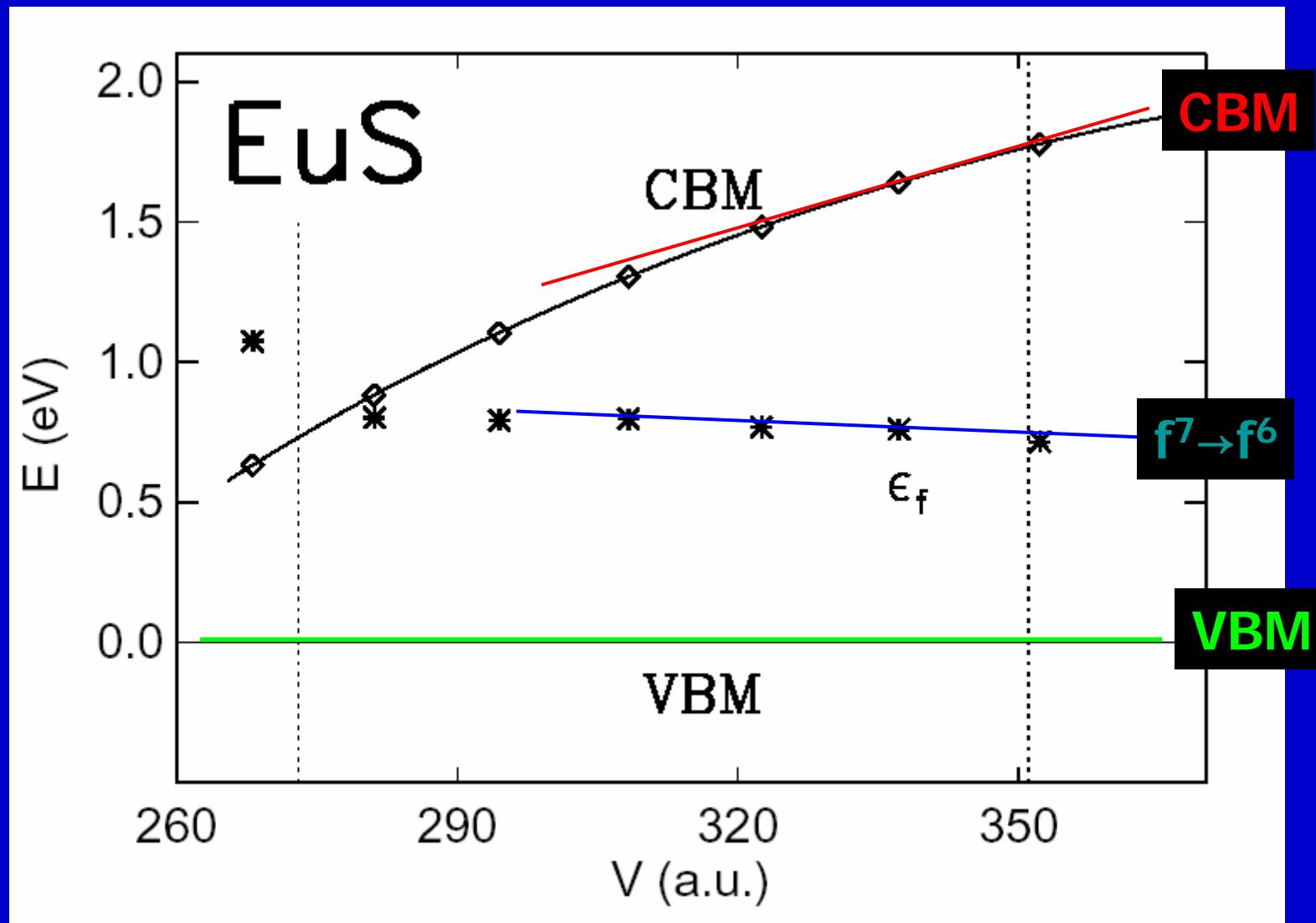
$$E(f^6) \leftrightarrow E(f^5) + \epsilon_{Fermi}$$



# SmS - band gap



# EuS - band gap



gap	$E_{\text{gap}}$		$-\frac{dE_{\text{gap}}}{dp}$	
	theory	expt.	theory	expt.
EuS	1.10	1.7	7.8	7.9, 11
SmS	0.71	0.15	11.5	10
SmSe	0.63	0.45	10.0	11

Syassen (1986);  
Jayaraman *et al.*, (1974)

eV

meV/kbar

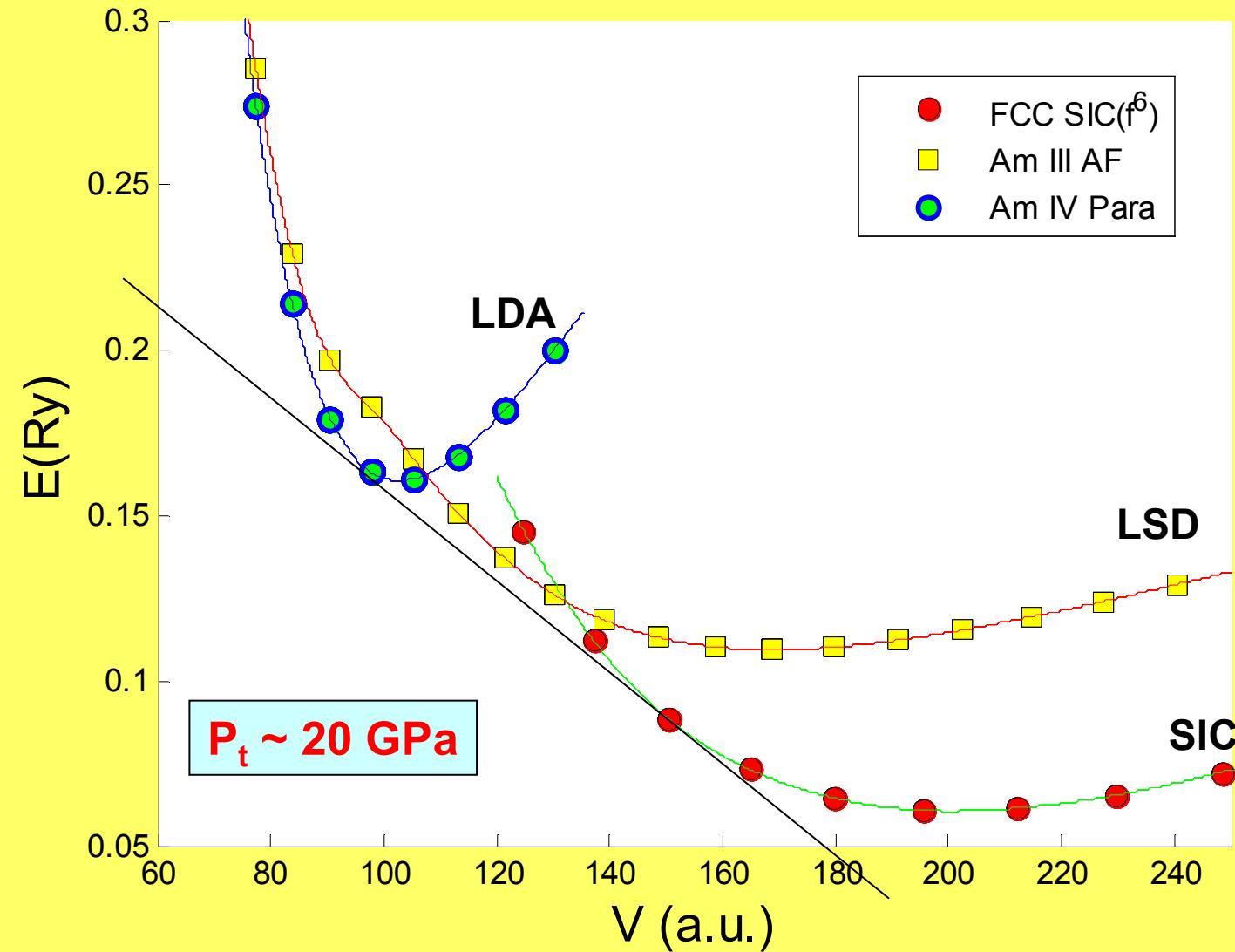
## isostructural transition pressures

GPa	theory	Expt.
SmS	0.1	0.65
SmSe	3.3	3-8
SmTe	6.2	6-8
EuS	11.6 (B1→B1)	16-20 (I→M) 20 (B1→B2)

Syassen (1986)

# Am

## Americium E(V)



# Bk

## Bk E(V)

E(Ry)

0.5  
0.4  
0.3  
0.2  
0.1  
0  
-0.1  
-0.2

60

80

100

120

140

160

180

200

220

240

260

V (a.u.)

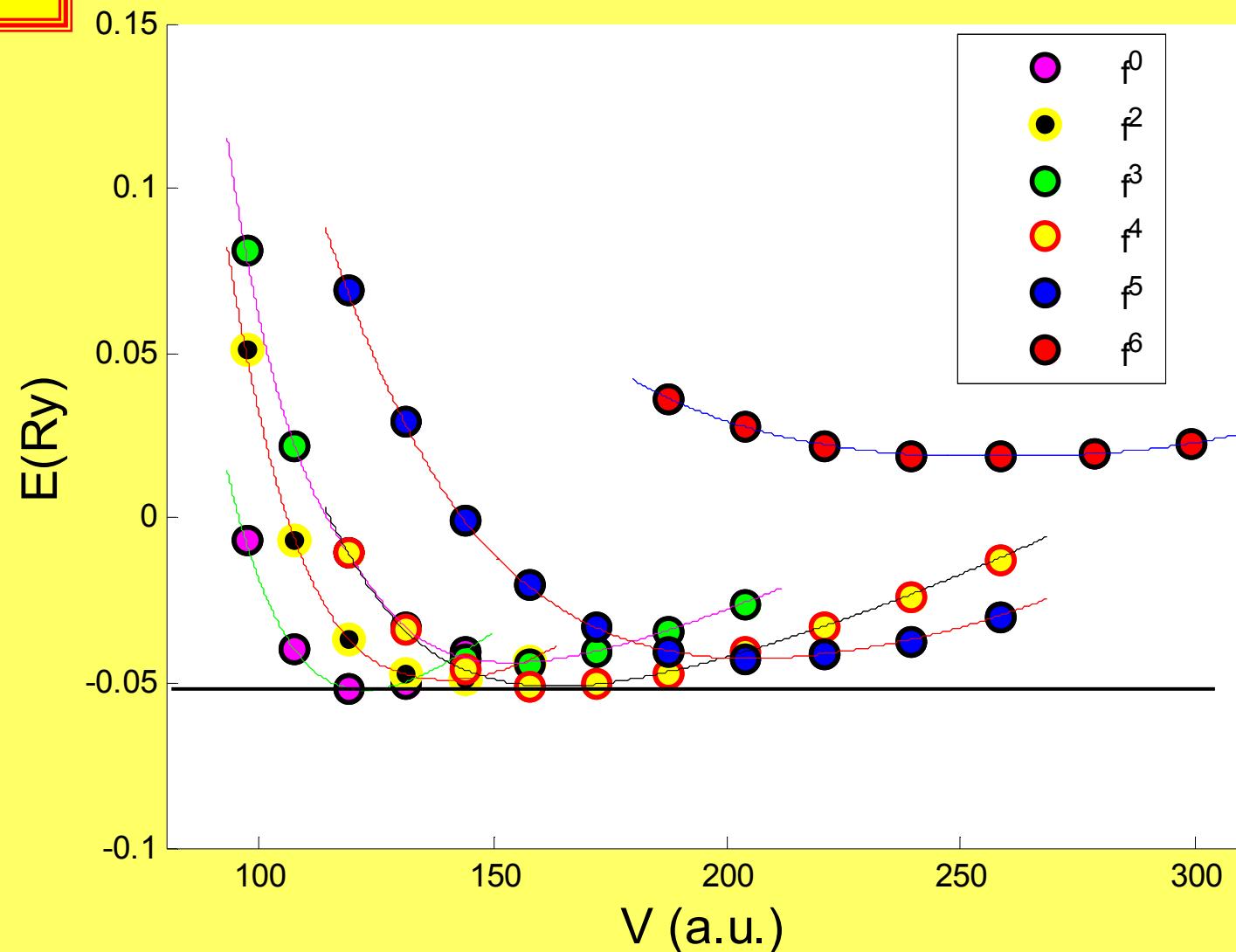
$P_t \sim 15 \text{ GPa}$

- $f^0$  (purple circle)
- $f^2$  (red square)
- $f^4$  (blue square)
- $f^5$  (black circle)
- $f^6$  (red circle)
- $f^7$  (green circle)
- $f^8$  (yellow circle)

## Bk E(V)

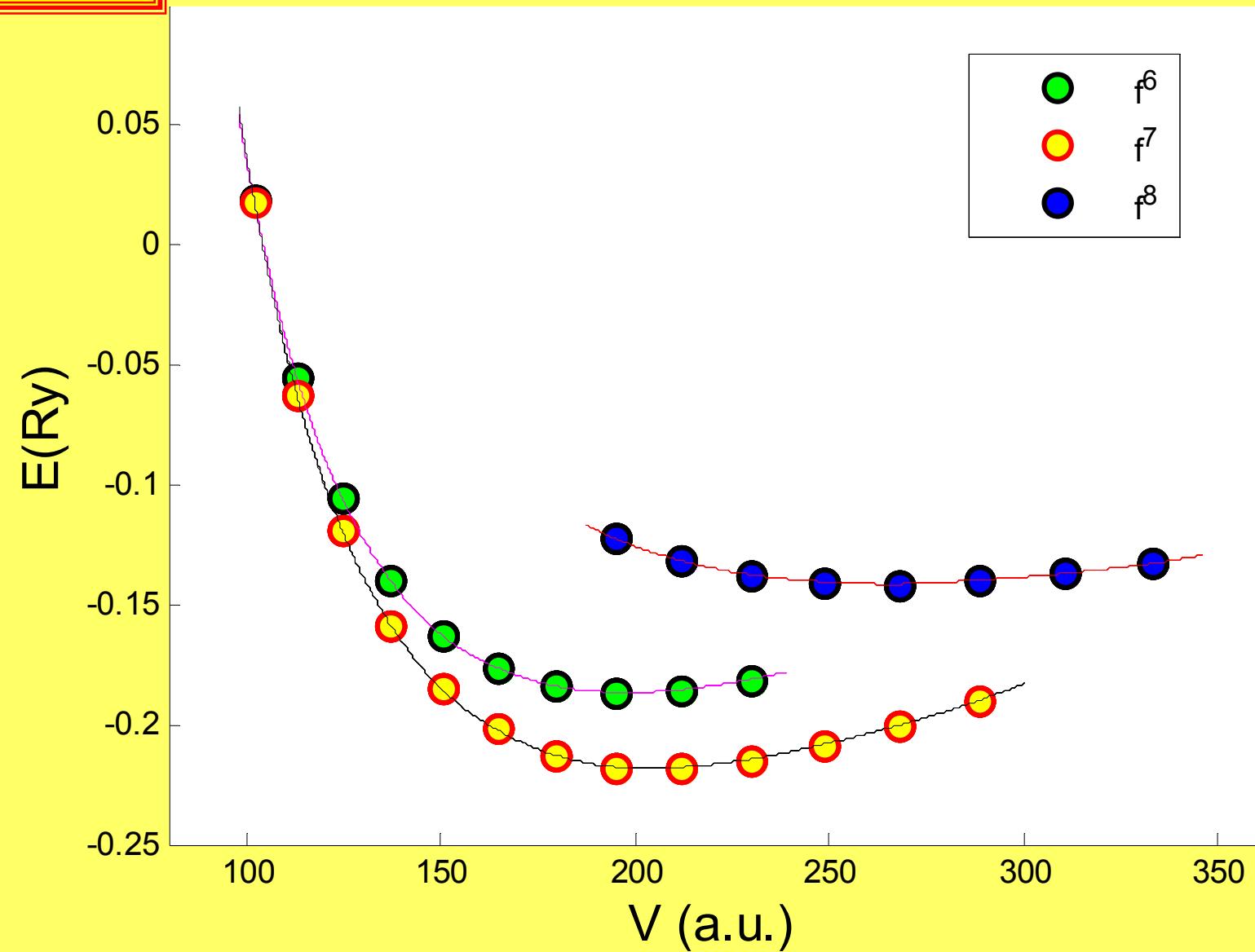
**Pu**

## $\delta$ -Pu jj coupling: $E(V)$



**Cm**

## Cm SIC-LSD: E(V)



# Actinides: equilibrium volume

a.u.	$V_0$ (theo)	$V_0$ (expt)
$\delta\text{Pu}$	<b>163.8</b>	<b>168</b>
Am	<b>196.7</b>	<b>198</b>
Cm	<b>203.5</b>	<b>202</b>
Bk	<b>201.4</b>	<b>189</b>

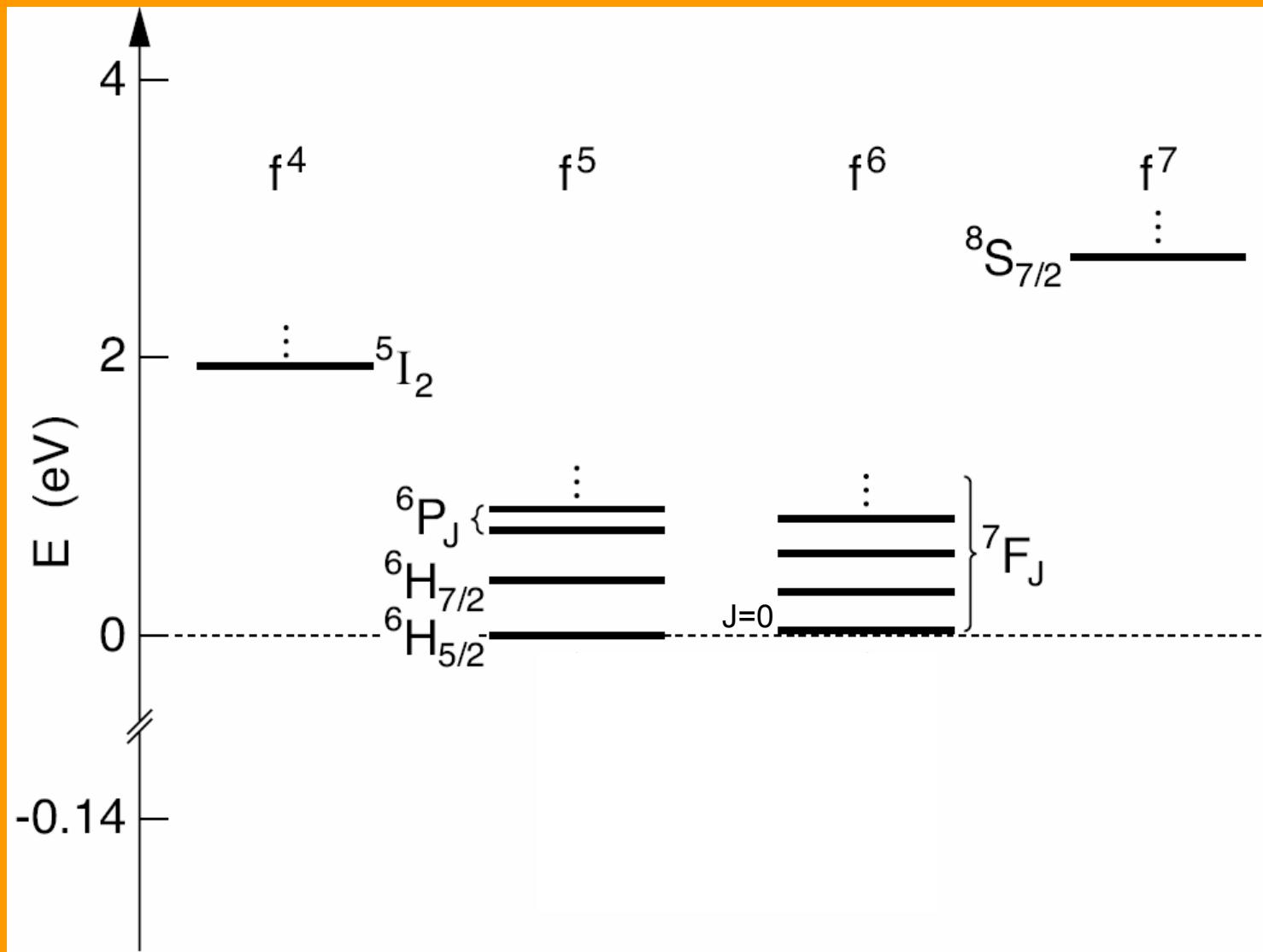
jj

jj

LS

LS

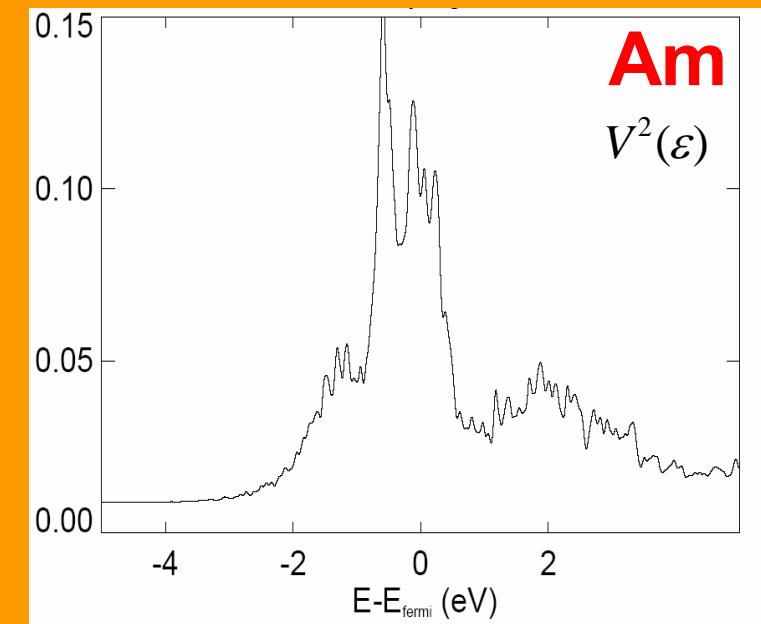
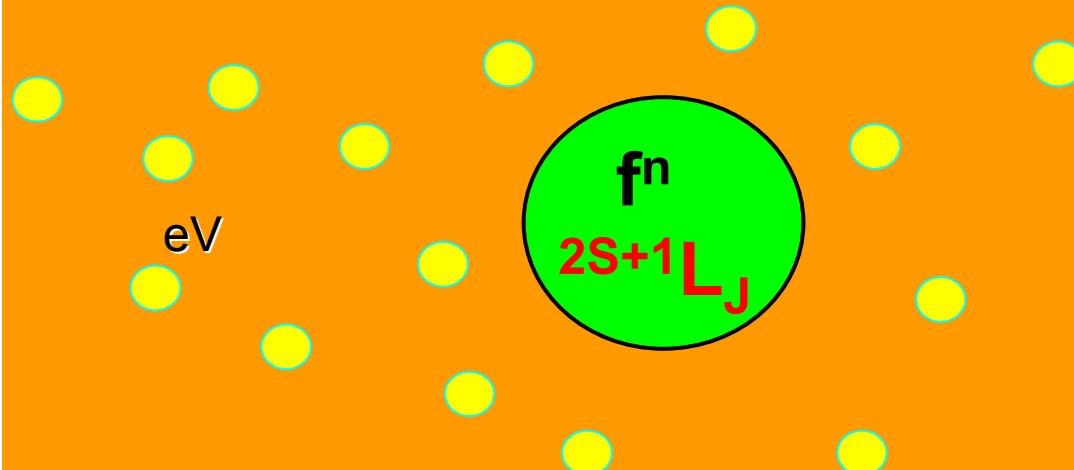
# PuSe



$$\langle fn ; i | \frac{1}{r} + \xi \vec{l} \cdot \vec{s} | fn ; j \rangle \rightarrow |\mu\rangle, E_\mu$$

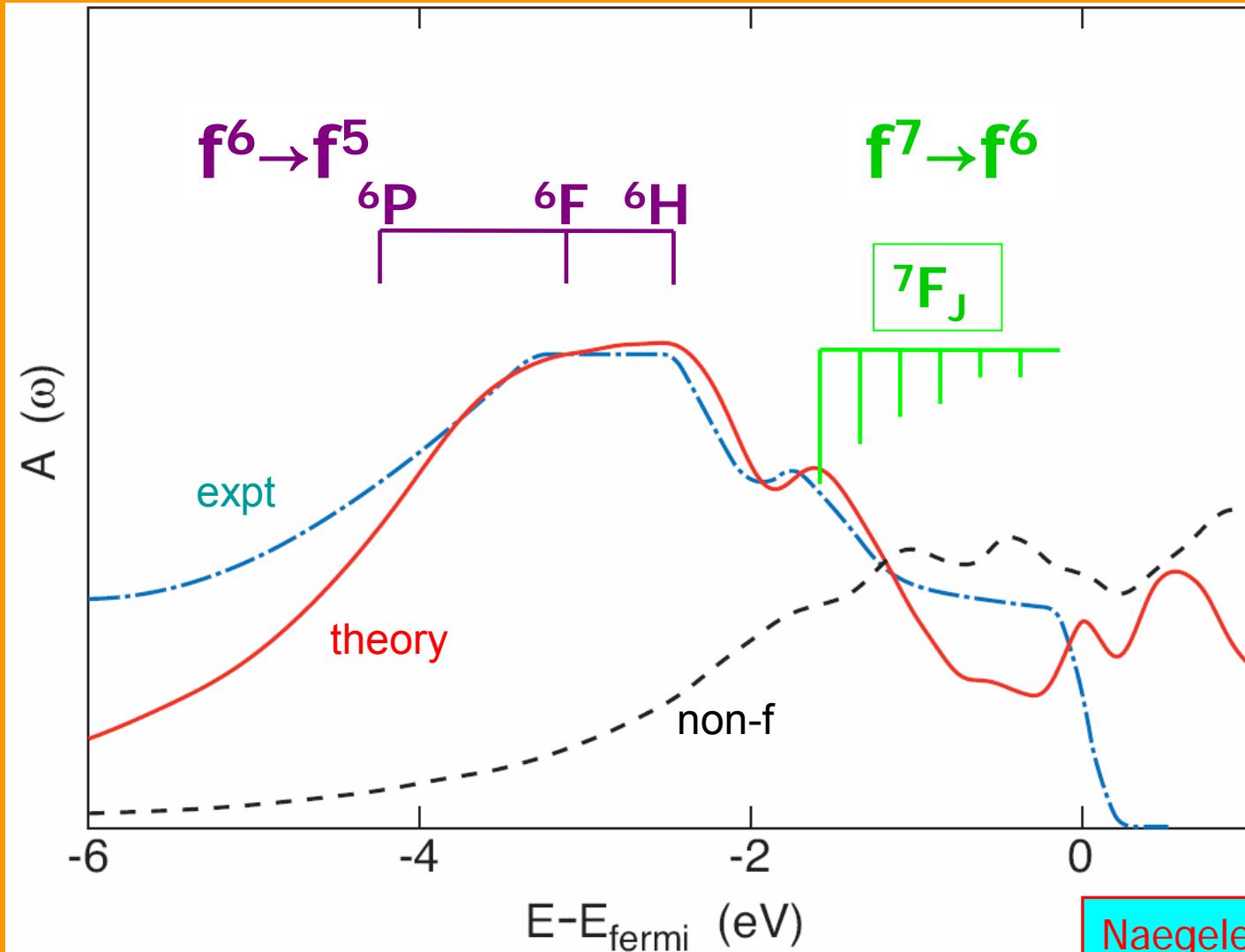
$$\sum_{ik}^{12} V_{ik} (\hat{c}_k^+ \hat{f}_i^- + \hat{f}_i^+ \hat{c}_k^-) = \int V(\varepsilon) \left( \hat{c}^+(\varepsilon) \hat{f}_i^- + \hat{f}_i^+ \hat{c}(\varepsilon) \right) d\varepsilon$$

$$V \sum_i (c_i f_i^+ + f_i c_i^+) \rightarrow |\tilde{\mu}\rangle, \tilde{E}_\mu$$



# Am

$$E(f^7) - E(f^6) = +0.7 \text{ eV}$$
$$V = 0.33 \text{ eV}$$



Naegle *et al.*,  
PRL 52, 1834 (1984)

# Am

