The Abdus Salam
International Centre for Theoretical Physics

United Nations
Educational, Scientific
and Cultural Organization

International Atomic
Energy Agency

SMR 1829 - 1

Winter College on Fibre Optics, Fibre Lasers and Sensors

12 - 23 February 2007

Fundamentals of fiber waveguide modes

K. Thyagarajan

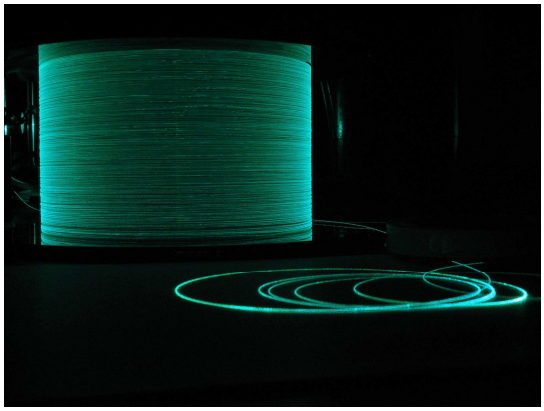
Physics Department
IIT Delhi
New Delhi, India

Fundamentals of fiber waveguide modes

K Thyagarajan
Physics Department
IIT Delhi
New Delhi, India
ktrajan@physics.iitd.ernet.in

Winter college on "*Fiber Optics, Fiber Lasers and Sensors*"
February 12-23, 2007, ICTP, Trieste

Terabits per second



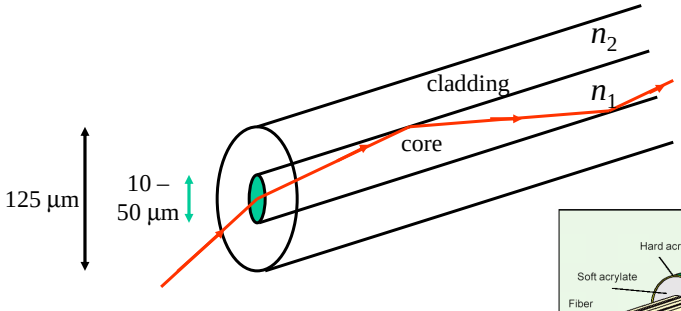
This thin glass optical fiber can transmit 30,000 copies of a 20 volume Encyclopedia in 1 second !
This corresponds to a capacity of ~ 9 x 10^{12} bits/s

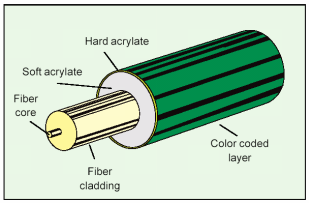
Outline

- Optical fiber
- Concept of modes
- Modes in a simple optical waveguide
- Modes in step index optical fibers
- Multimode and single mode optical fibers
- Dispersion and dispersion compensation
- Modes and fiber components
- Summary

Optical fiber

- Made of silica (SiO_2)
- Light guidance through *Total Internal Reflection*
- n_1 ~ 1.47; n_2 ~ 1.46



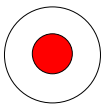


Fiber Types

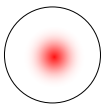
Multimode: Support a number of guided modes

Core dia.: 50 μm

Index difference ~ 0.015



Step Index

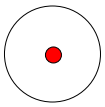


Graded Index

Singlemode: Support only a single guided mode

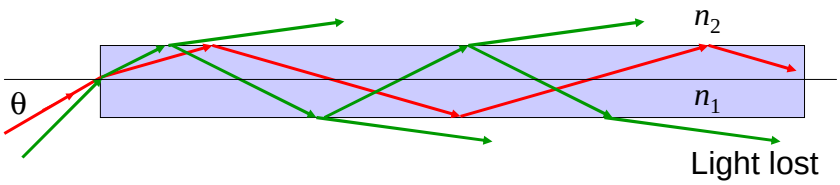
Core dia.: 9 μm

Index difference ~ 0.003



K Thyagarajan IITD

Numerical aperture



Light rays subtending an angle greater than critical angle will get guided by the fiber

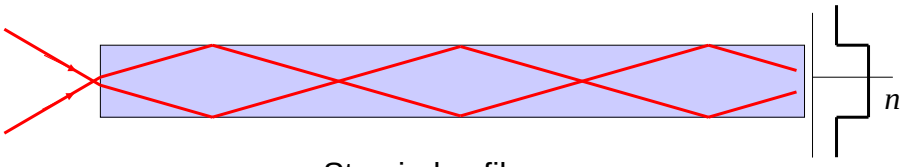
$$N.A. = \sin\theta_{\max} = \sqrt{n_1^2 - n_2^2}$$

Typical values of NA:	0.2 to 0.6
Acceptance angles:	11.5^o to 37^o

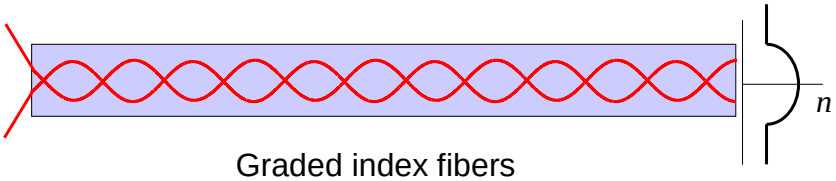
Higher NA implies higher light gathering power

K Thyagarajan IITD

Propagation through fibers



Step index fibers



Graded index fibers

Simplified picture

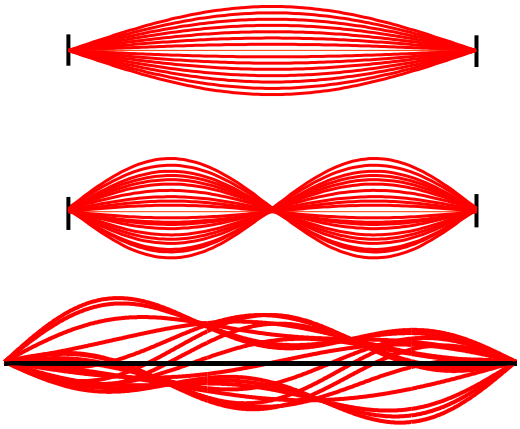
- Describing light propagation through optical fibers in terms of rays is an approximate picture
- An accurate description is in terms of *modes* which are obtained as solutions of the Maxwell's equations

Modes of propagation

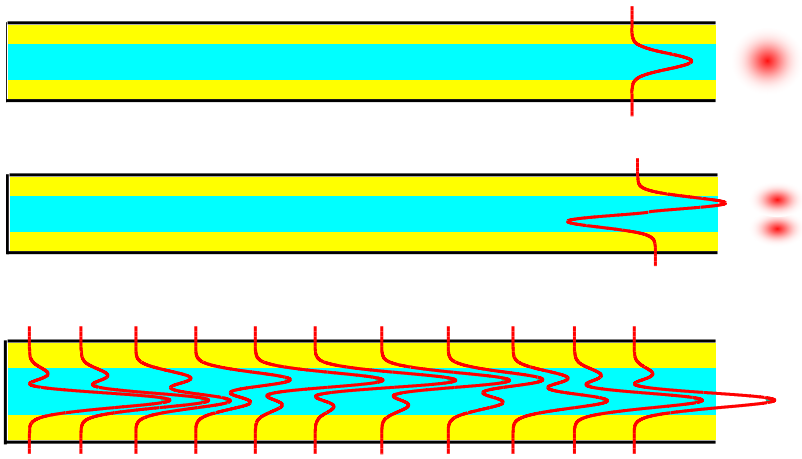
Mode:
- Certain electric field patterns that propagate unchanged
- Solution to Maxwell's equations satisfying appropriate boundary conditions
- Characteristics:
  - Definite transverse electric field pattern
  - Definite phase and group velocity
  - Discrete set of guided modes

Similar to
- Modes of oscillation of a string fixed at two ends
- Eigenstates of a potential well in quantum physics

Modes of oscillation of a string

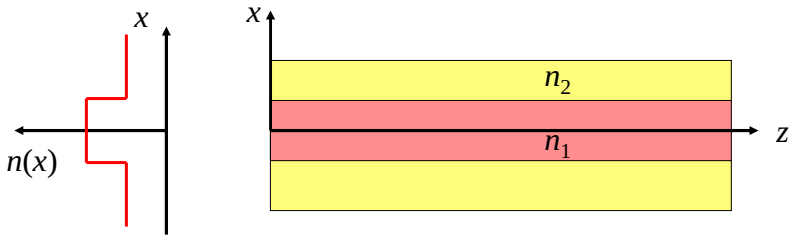


Modes of propagation in a waveguide



Modes of a simple optical waveguide

- Planar step index waveguide
- Refractive index depends only on *x*



$$n(x) = n_1; \qquad |x| < \frac{d}{2}$$

$$= n_2; \qquad |x| > \frac{d}{2}$$

What are modes?

- We look for solutions of the form

$$E(x,z,t) = \psi(x)\exp[i(\omega t - \beta z)]$$

- Here ω is the frequency and β is referred to as the propagation constant
- The trial solution is such that as the field propagates along the *z*-direction, the transverse field distribution specified by $\psi(x)$ remains same; only phase changes
- *Modes are specific field patterns which remain unchanged as they propagate through the waveguide*

Plane wave and mode

	Plane wave	Mode
Electric field:	$E = E_0 e^{i(\omega t - kz)}$	$E = E(x,y)e^{i(\omega t - \beta z)}$
Propagation constant:	$k = \frac{\omega}{c} n$	$\beta = \frac{\omega}{c} n_{eff}$
Phase velocity:	$v_p = \frac{c}{n}$	$v_p = \frac{c}{n_{eff}}$
Group velocity:	$v_g = \left(\frac{dk}{d\omega}\right)^{-1}$	$v_g = \left(\frac{d\beta}{d\omega}\right)^{-1}$

Maxwell's equations

$$\nabla\times\vec{E}=-\frac{\partial\vec{B}}{\partial t}=-\mu_0\frac{\partial\vec{H}}{\partial t}$$

$$\nabla\times\vec{H}=\frac{\partial\vec{D}}{\partial t}=\varepsilon_0 n^2(x)\frac{\partial\vec{E}}{\partial t}$$

- If we assume the *t*- and *z*-dependence given earlier, then these equations split into two independent sets:
  - Set containing only E_y, H_x and H_z: Transverse Electric (TE) modes
  - Set containing only H_y, E_x and E_z: Transverse magnetic (TM) modes

TE modes

Wave equation satisfied by $E_y(x)$

$$\frac{d^2E_y}{dx^2}+\left[k_0^2n^2(x)-\beta^2\right]E_y=0$$

Equations in core and cladding:

$$\frac{d^2E_y}{dx^2}+\left[k_0^2n_1^2-\beta^2\right]E_y=0;\qquad |x|<d/2\qquad \text{Core}$$

$$\frac{d^2E_y}{dx^2}+\left[k_0^2n_2^2-\beta^2\right]E_y=0;\qquad |x|>d/2\qquad \text{Cladding}$$

Boundary conditions: $E_y(x)$ and dE_y/dx must be continuous at $x = \pm d/2$

Solutions

- $\beta^2 > k_0^2 n_1^2$:

  No solutions are possible

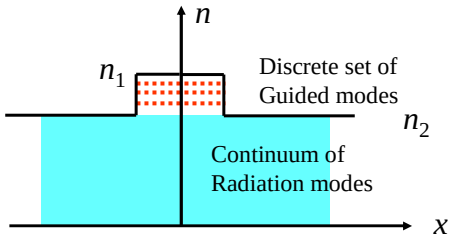
- $k_0^2 n_1^2 > \beta^2 > k_0^2 n_2^2$:

  Solutions oscillatory in core and exponentially decaying in cladding: GUIDED MODES

- $k_0^2 n_2^2 > \beta^2$

  Solutions oscillatory in core and in cladding : RADIATION MODES

Guided and radiation modes



$n_2 < n_{eff} < n_1$

$n_{eff} < n_2$

$n_{eff} = \frac{\beta}{k_0}$ Effective index of the mode

$$E(x,z,t) = \psi(x)\exp[i(\omega t - k_0 n_{eff} z)]$$

Guided TE modes

- Since $n(x)$ is symmetric, the modal fields are either symmetric or antisymmetric in x
- For the symmetric mode, solutions of the wave equation in core and cladding:

$$E_y(x) = A\cos\kappa x; \qquad |x| < d/2$$
$$= Ce^{-\gamma|x|}; \qquad |x| > d/2$$

Here

$$\kappa^2 = k_0^2 n_1^2 - \beta^2$$
$$\gamma^2 = \beta^2 - k_0^2 n_2^2$$

Eigen value equation

- Applying boundary conditions of continuity of $E_y(x)$ and dE_y/dx at $x = d/2$, we get the following eigenvalue equation:

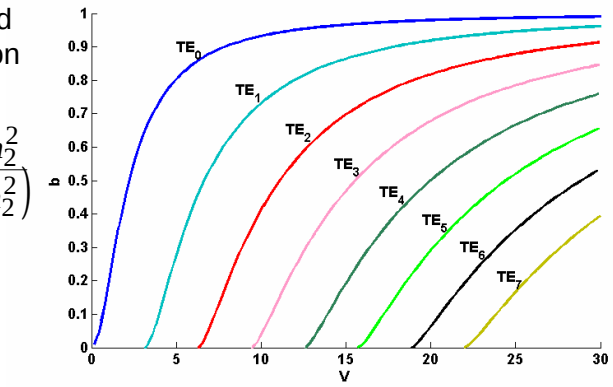
$$\tan\left(\frac{\kappa d}{2}\right) = \frac{\gamma}{\kappa}$$

- Eigenvalue equation has a discrete set of solutions
- Each solution corresponds to a guided mode
- Similar equation for antisymmetric modes
- Modes designated as TE_0, TE_1, TE_2 etc.

Normalized plot

Normalized propagation constant

$$b = \frac{\beta^2 - k_0^2 n_2^2}{k_0^2\left(n_1^2 - n_2^2\right)}$$



$$V = k_0 d\sqrt{n_1^2 - n_2^2}$$

Physical picture

- Field of a guided mode in the core:

$$E(x,z,t) = A\cos(\kappa x)e^{i(\omega t-\beta z)}$$
$$= \frac{1}{2}A\left[e^{i(\omega t-\kappa x-\beta z)} + e^{i(\omega t+\kappa x-\beta z)}\right]$$

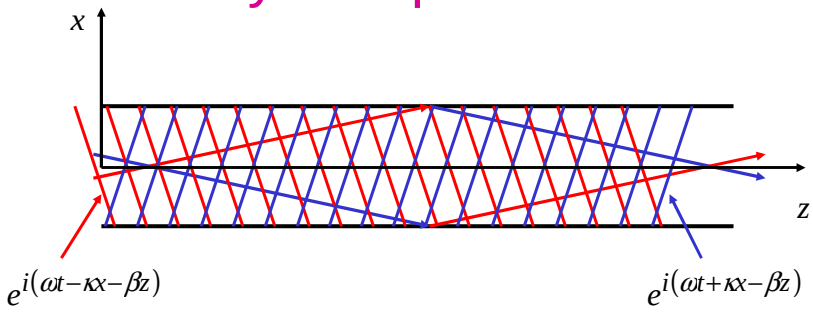
- Comparing with the expression for a plane wave

$$E(x,y,z,t) = Ae^{i\left(\omega t-\vec{k}.\vec{r}\right)} = Ae^{i\left(\omega t-k_x x-k_y y-k_z z\right)}$$

- k_x, k_y and k_z define the propagation direction of the wave
- Mode is composed of two plane waves with components

$$k_x = \pm\kappa, \quad k_y = 0, \quad k_z = \beta$$

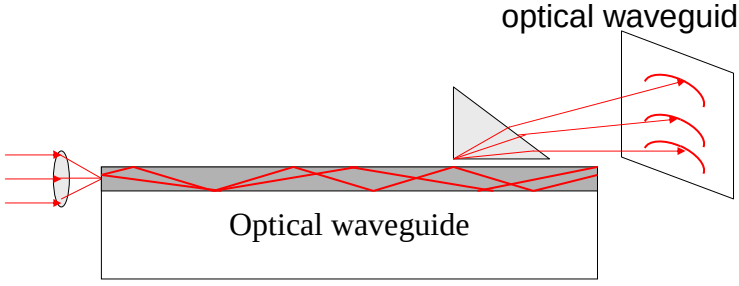
Physical picture



- Modes are standing wave patterns in the transverse direction formed by superposition of plane waves
- Discrete solutions means that only certain allowed angles of propagation are allowed

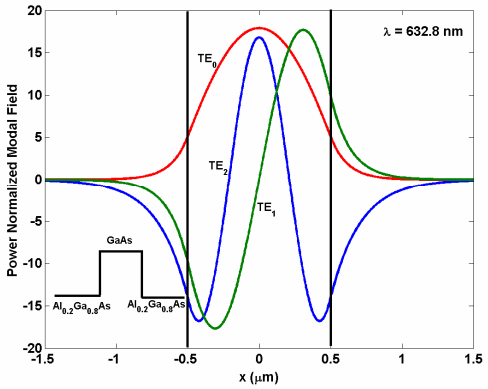
Interesting experiment

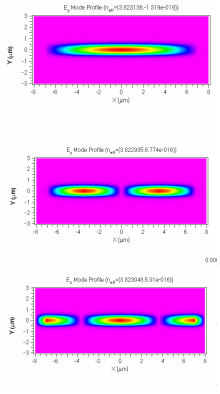
Discrete modes of an optical waveguide



- Different discrete modes of the waveguide exit at different angles from the prism
- By measuring the angles, it is possible to obtain the propagation constants of the modes
- A planar waveguide can be easily fabricated by diffusion of silver into glass

Field patterns of some modes





TE_0

TE_1

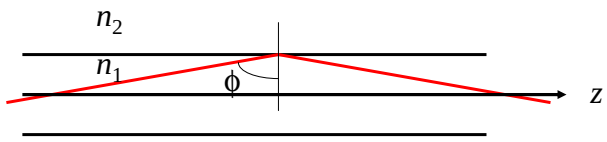
TE_2

- Note that the field extends into the cladding region
- Higher order modes have greater penetration into the cladding
- Fundamental mode (TE_0 mode) has no zero crossings

K Thyagarajan IITD

Cut off of a mode

- As the wavelength increases, the propagation constant β reduces
- When $\beta = k_0 n_2$, then the mode is referred to have reached cutoff
- Cut off is equivalent to loss of total internal reflection



At cutoff $\beta = k_0 n_1 \sin\phi = k_0 n_2$

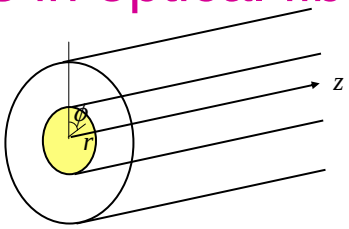
which is nothing but the condition for total internal reflection

K Thyagarajan IITD

Weakly guiding waveguides

- Similar eigen value equations can be obtained for TM modes
- For waveguides with a small value of $(n_1 - n_2)/n_1$, TE and TM modes have almost same characteristics
- Such waveguides are called *weakly guiding*
- In weakly guiding waveguides, polarization does not play any role and all polarization states have same propagation characteristics

Modes in optical fibers



- Optical fibers have a refractive index profile dependent only on the radial cylindrical coordinate r
- Aximuthal dependence is of the form $\cos l\phi$ and $\sin l\phi$
- For fibers with small difference between core and cladding indices, the modes can be assumed to be linearly polarized (LP modes)

$$E(r,\phi,z,t) = R(r)e^{i(\omega t-\beta z)}\begin{Bmatrix}\cos l\phi\\ \sin l\phi\end{Bmatrix}$$

Guided mode fields

Step index fibers LP_{lm}

$$E(r,\phi,z,t)=\begin{cases}\dfrac{A}{J_l(U)}J_l\left(\dfrac{Ur}{a}\right) & r<a\\ \dfrac{A}{K_l(W)}K_l\left(\dfrac{Wr}{a}\right) & r>a\end{cases}\begin{bmatrix}\cos l\phi\\ \sin l\phi\end{bmatrix}e^{i(\omega t-\beta z)}$$

$$U=a\left(k_0^2n_1^2-\beta^2\right)^{1/2}$$

$$W=a\left(\beta^2-k_0^2n_2^2\right)^{1/2}$$

$$V^2=U^2+W^2=\left(k_0a\sqrt{n_1^2-n_2^2}\right)^2$$

LP: Linearly Polarized

Ref: A Ghatak and K Thyagarajan, *Introduction to Fiber Optics*, Cambridge University Press, 1998

Eigenvalue equation

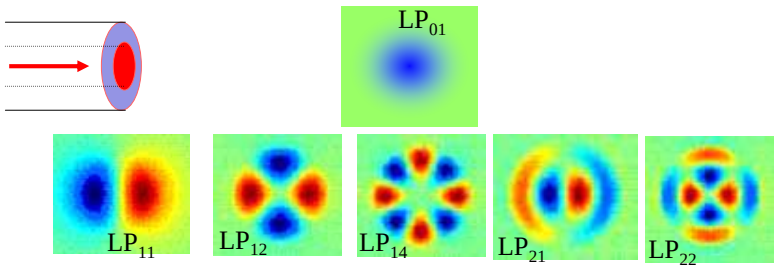
- Applying continuity conditions at r = a, we obtain the following eigenvalue equation for the guided modes:

$$U\frac{J_l'(U)}{J_l(U)}=W\frac{K_l'(W)}{K_l(W)}$$

- m^{th} solution of the above equation is referred to as the LP_{lm} mode
- For every LP_{lm} mode there is a value of V for which β = k_0n_2 and this is referred to as cut off of the mode
- The mode can exist for V values higher than the cut off V value

Modal patterns

Transverse intensity patterns of different modes



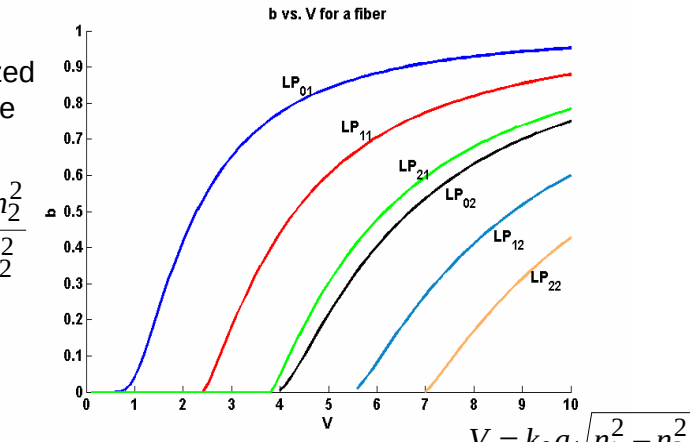
Multimode fiber: Supports many modes

Singlemode fiber: Supports a single guided mode

Normalized plot

Normalized effective index

$$b = \frac{n_{eff}^2 - n_2^2}{n_1^2 - n_2^2}$$

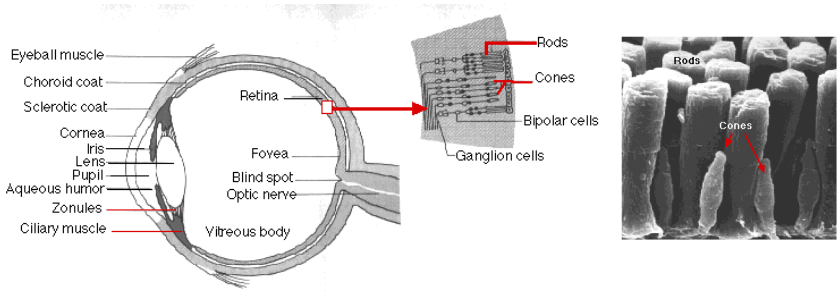


$$V = k_0 a \sqrt{n_1^2 - n_2^2}$$

- b increases as V increases
- Value of b depends only on V value
- Different fibers having same V will have same b value but different n_{eff}

Optical guides in our eyes!

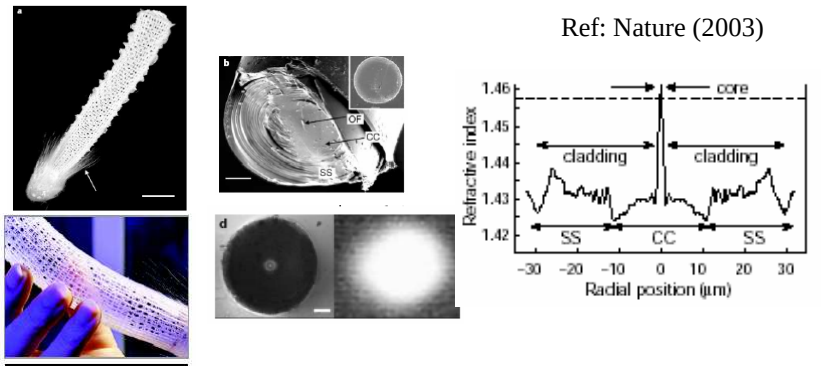
Rods and Cones



Natural optical fibers

- Natural optical fiber: Sponge named *venus flower basket*

Ref: Nature (2003)



- Nature's ability to evolve highly effective & sophisticated optical systems – even superior to man made analogs
- We can learn a lot from Nature

l = 0 modes

- Eigenvalue equation for modes with no ϕ dependence

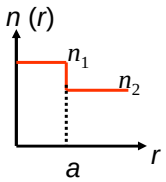
$$U\,\frac{J_1(U)}{J_0(U)} = W\,\frac{K_1(W)}{K_0(W)}$$

- Solution with highest β value is LP_{01} mode
- Referred to as the fundamental mode
- Unlike other modes, the fundamental LP_{01} mode has no cutoff; it exists at all wavelengths

Single mode fibers

- For only the LP_{01} mode to exist

$$\frac{a\sqrt{n_1^2 - n_2^2}}{\lambda_0} < 0.38$$



- Such fibers are referred to as single mode fibers (SMF)
- Typical parameters:
- Core radius ~ 5 μm
- Index difference ($n_1 - n_2$) ~ 0.004

Important Parameters of a single mode fiber

Attenuation coefficient (α)

Cut-off wavelength λ_c:

Mode Field Diameter (MFD):

Dispersion and dispersion slope (*D, S*)
Polarization mode dispersion

Nonlinear coefficient (γ)

Losses in optical fibers

- Rayleigh scattering
- Absorption due to impurities
- Losses due to bend and other fiber imperfections
- Loss measured in dB/km

decibel scale

- Variation of power with distance along the fiber

$$P(z) = P(0)e^{-\tilde{\alpha}z}$$

$\tilde{\alpha}$: units of m^{-1} or km^{-1}

- Whenever the range of variation of a quantity is very large we use the units of decibel (dB)
- This ensures that with a manageable range of numbers it is possible to describe the variation of the quantity.
- Multiplication in normal scale becomes addition in logarithmic scale; calculations get simplified

decibel scale

- Loss in decibels:

$$\alpha(\text{dB}) = 10\log\left(\frac{P_{in}}{P_{out}}\right)$$

- 3 dB loss implies

$$3 = 10\log\left(\frac{P_{in}}{P_{out}}\right)$$

or

$$P_{out} = P_{in} \times 10^{-0.3} \approx \frac{1}{2}P_{in}$$

- 3 dB loss implies 50 % transmission
- 10 dB loss implies 10 % transmission
- 20 dB loss implies 1 % transmission

Loss in dB/km

- Optical fiber loss in decibels per kilometer (dB/km):

$$\alpha(\text{dB/km}) = 10\log\left(\frac{P_{in}(0)}{P_{out}(1\text{km})}\right)$$

- Using the earlier equation

$$\alpha(\text{dB/km}) = 10\log(e^{\tilde{\alpha}(\text{km}^{-1})}) = 4.34\times\tilde{\alpha}(\text{km}^{-1})$$

- If loss through 1 km is 3 dB, i.e. output after 1 km is half of input, then loss after 3 km is 9 dB which corresponds to a transmission of 12.5 %

dBm scale

- Powers can be specified in a logarithmic scale with reference to a given power
- If the reference power is 1 mW, then we define power in dBm as

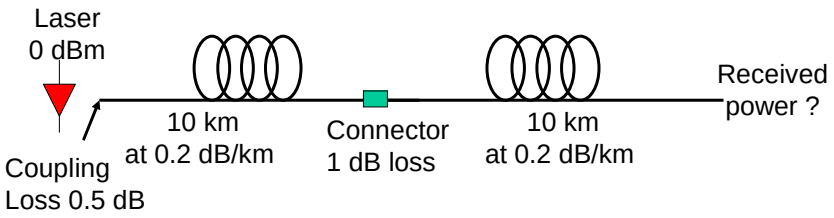
$$P(\text{dBm}) = 10\times\log(P(\text{mW}))$$

- 1 mW = 0 dBm
- 1 μW = -30 dBm
- 1 W = +30 dBm

- Calculations using dBm

$$P_{out}(\text{dBm}) = P_{in}(\text{dBm}) - Loss(\text{dB})$$

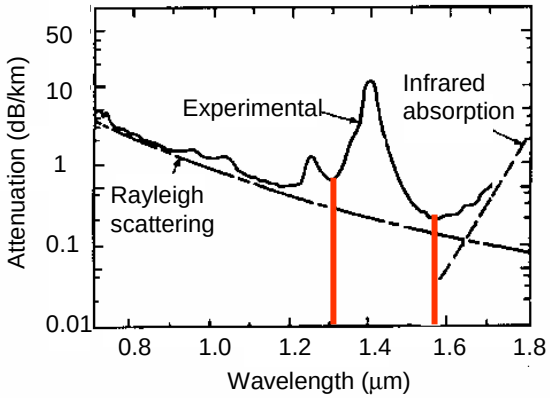
Calculating loss



$$P_{out}\,(\text{dBm}) = 0 - 0.5 - 10\times 0.2 - 1 - 10\times 0.2 = -5.5\text{dBm} \approx 0.28\ \text{mW}$$

K Thyagarajan IITD

Attenuation of a silica optical fiber

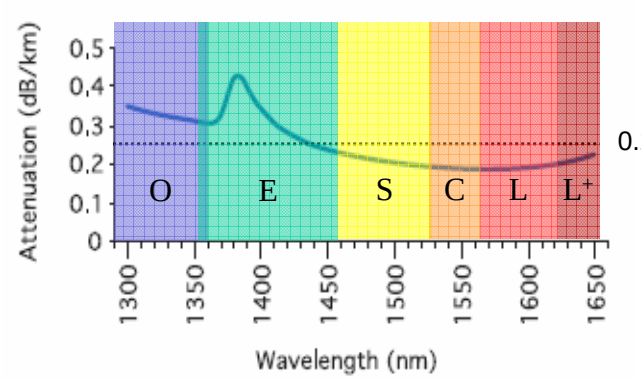


- Two low loss windows: ~ 1310 nm and ~1550 nm
- Typical loss at 1550 nm ~ 0.25 dB/km

Ref: Miya et al, Electron. Letts. 1979

K Thyagarajan IITD

Fiber attenuation spectrum



- Low loss window spanning ~ 400 nm of bandwidth
- Wavelength division multiplexing (WDM)

Ref: Corning LEAF fiber

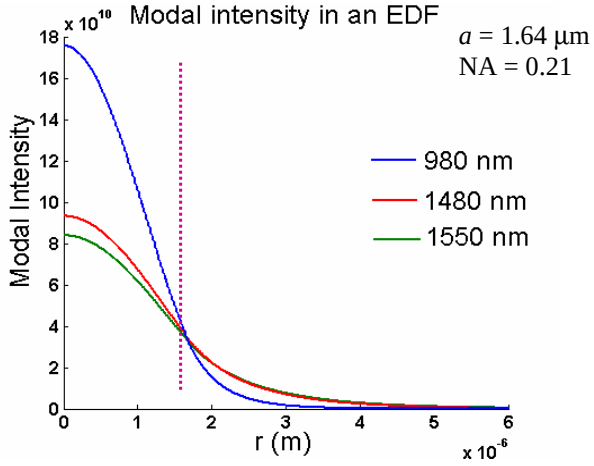
Cutoff wavelength

Cut-off wavelength (λ_c)
The wavelength above which the fiber behaves
as a single mode fiber (e.g. ~ 1260 nm for Corning SMF-28e)

$$\lambda_C = \frac{2\pi}{2.405}\, a\, \sqrt{n_1^2 - n_2^2}$$

- Cut off wavelength decided by
  - core radius,
  - NA and
  - refractive index profile of the fiber core

Modal intensity pattern



- Smaller core radius and higher NA leads to more confined modes

Spot size

- Petermann-2 spot size

$$w_P = \left[\frac{2\int_0^\infty E^2(r)\, r\, dr}{\int_0^\infty \left(\frac{dE}{dr}\right) r\, dr} \right]$$

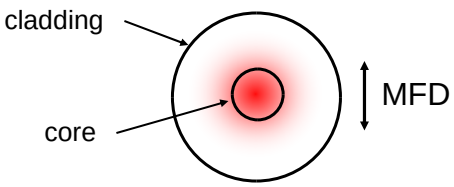
- Loss across joints is given in terms of w_P

$$\alpha(dB) = 4.34 \times \left(\frac{u}{w_P}\right)^2$$

- α is the transverse offset

Mode Field Diameter (MFD)

- MFD is defined as twice the spot size
- ~ 9.2 ±0.4 μm at 1310 nm and 10.4 ±0.8 μm at 1550 nm
- Determines splice & connector losses at joints
- Nonlinear characteristics of the fiber

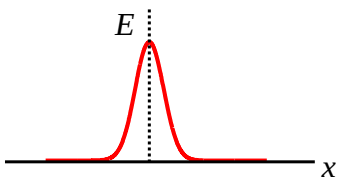


K Thyagarajan IITD

Gaussian approximation

- The mode field of the LP_{01} mode is very similar to a Gaussian distribution
- Very useful for various calculations
- Gaussian approximation

$$E(r) = Ae^{-r^2/w^2}$$



$$\frac{w}{a} \approx \left(0.65 + \frac{1.619}{V^{1.5}} + \frac{2.879}{V^6}\right); \qquad 0.8 < V < 2.5$$

Ref: Marcuse, Bell Syst. Tech. J. 56 (1977) 703

K Thyagarajan IITD

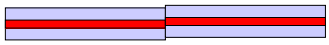
Losses across joints of two fibers

Different spot sizes

w_1 w_2

$$L(dB) = -20\log\left(\frac{2w_1w_2}{w_1^2 + w_2^2}\right)$$

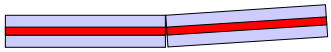
Transverse misalignment, u



w w

$$L(dB) = 4.34\times\left(\frac{u}{w}\right)^2$$

Angular misalignment, θ



w w

$$L(dB) = 4.34\times\left(\frac{\pi w\theta}{\lambda}\right)^2$$

Note: Any attempt to increase tolerance towards transverse misalignment by increasing w leads to reduced tolerance towards angular misalignment

Effective area

- Defines nonlinear interaction in fibers

$$A_{\text{eff}} = 2\pi \frac{\left[\int_0^\infty E^2(r)\, r\, dr\right]^2}{\int_0^\infty E^4(r)\, r\, dr}$$

A_{eff} at1550 nm

$\approx$ 85 μm^2 for G.652 (SMF)

$\approx$ 46 μm^2 for G.653 (DSF)

$\approx$ 60 μm^2 for G.655 (NZDSF)

$\approx$ 23 μm^2 for DCF (D < 0)

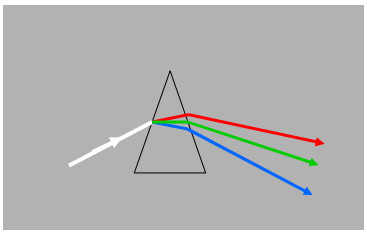
Note: In the Gaussian approximation $A_{\text{eff}} = \pi w^2$

Pulse dispersion

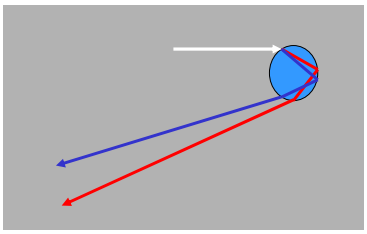
Dispersion

- Different wavelengths propagate with different velocities

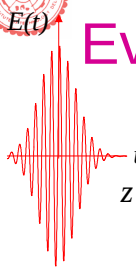
Prism



Rainbow



Evolution of a pulse of light



$z = 0$ $z = L$

Input pulse $E(t, z=0) = Ae^{-t^2/\tau_0^2} e^{i\omega_0 t}$ Gaussian pulse

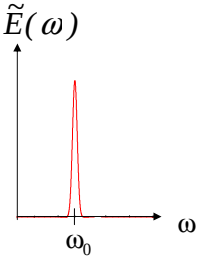
Fourier spectrum

$$E(t, z=0) = \int_{-\infty}^{\infty} \tilde{E}(\omega) e^{i\omega t} d\omega$$

$$\tilde{E}(\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} E(t, z=0) e^{-i\omega t} dt$$

Fourier spectrum

$$\tilde{E}(\omega) = \frac{A\tau_0}{2\sqrt{\pi}} \exp\left[-\frac{(\omega - \omega_0)^2 \tau_0^2}{4}\right]$$



- Fourier spectrum sharply peaked around $\omega = \omega_0$
- Width of the spectrum ~ $1/\tau_0$
- For typical pulse widths used in communication $1/\tau_0 << \omega_0$

Pulse evolution

Output pulse $E(t,z) = \int \tilde{E}(\omega) e^{i(\omega t - \beta z)} d\omega$

Expanding β around $\omega = \omega_0$

$$\beta(\omega) = \beta(\omega_0) + \left.\frac{d\beta}{d\omega}\right|_{\omega_0} (\omega - \omega_0) + \left.\frac{d^2\beta}{d\omega^2}\right|_{\omega_0} (\omega - \omega_0)^2 + \ldots$$

$$= \beta(\omega_0) + \beta_1(\omega - \omega_0) + \frac{\beta_2}{2}(\omega - \omega_0)^2 + \ldots$$

$$\beta_1 = \left.\frac{d\beta}{d\omega}\right|_{\omega_0}; \qquad \beta_2 = \left.\frac{d^2\beta}{d\omega^2}\right|_{\omega_0}$$

Solution approximating till β_1

$$E(t,z) = \int \tilde{E}(\omega)\, e^{i(\omega t - \beta_0 z - \beta_1(\omega - \omega_0)z)} d\omega$$

Substituting for the Fourier spectrum and integrating we get

$$E(t,z) = A\exp\left[-\frac{(t-\beta_1 z)^2}{\tau_0^2}\right] e^{i(\omega_0 t - \beta_0 z)}$$

- The pulse remains Gaussian
- The pulse width remains the same as it propagates
- The peak of the pulse propagates at a velocity given by

$$v_g = \frac{1}{\beta_1} = \left(\frac{d\beta}{d\omega}\right)^{-1}$$ Group velocity

Solution approximating till β_2

$$E(z,t) = \frac{A}{(1+\sigma^2)^{1/4}} \exp\left[-\frac{(t - z/v_g)^2}{\tau^2(z)}\right] \exp\left[i(\Phi(z,t) - \beta(\omega_0 z)\right]$$

$$\Phi(z,t) = \omega_0 t + \kappa(t - \frac{z}{v_g})^2 - \frac{1}{2}\tan^{-1}(\sigma)$$

$$\kappa = \frac{\sigma}{(1+\sigma^2)\tau_0^2}; \qquad \sigma = \frac{2\beta_2 z}{\tau_0^2};$$

$$\tau^2(z) = \tau_0^2(1+\sigma^2); \qquad \beta_2 = \frac{d^2\beta}{d\omega^2}$$

Pulse broadening

- Pulse remains Gaussian
- The temporal width $\tau(z)$ of the pulse increases as it propagates
  - Pulse broadening depends on β_2
- Pulse broadens irrespective of the sign of β_2
- The pulse also gets chirped
  - Instantaneous frequency of the pulse changes with time

Pulse broadening

$$\tau^2(z) = \tau_0^2\left(1 + \frac{4\beta_2^2 z^2}{\tau_0^4}\right)$$

Define pulse broadening $\Delta\tau$: $\Delta\tau = \left(\tau^2(z) - \tau_0^2\right)^{1/2} = \frac{2|\beta_2|z}{\tau_0}$

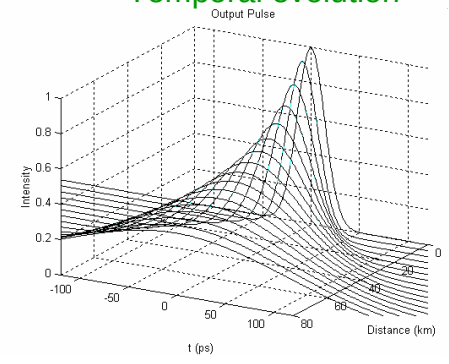
Source spectral width for a pulse width t_0 gives $\Delta\lambda = \frac{\lambda^2}{\pi c \tau_0}$

Pulse broadening: $\Delta\tau = \frac{2\pi c|\beta_2|}{\lambda^2} z\,\Delta\lambda$

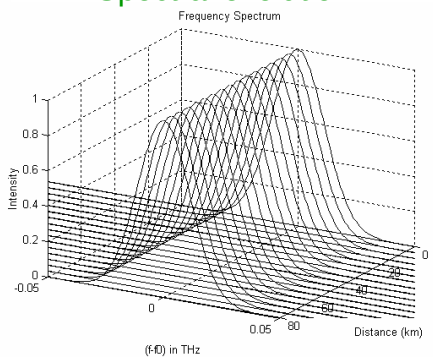
K Thyagarajan IITD

Temporal & Spectral evolution of an optical pulse

Temporal evolution



Spectral evolution



- Pulse gets broader in time domain
- Frequency spectrum of the pulse remains same

K Thyagarajan IITD

Pulse chirping

- In general we can define an instantaneous frequency
- If time variation is $e^{i\phi(t)}$, then we define the instantaneous frequency as

$$\omega(t) = \frac{d\phi}{dt}$$

- Time variation of phase of output pulse

$$\Phi(z,t) = \omega_0 t + \kappa(t - \frac{z}{v_g})^2 - \frac{1}{2}\tan^{-1}(\sigma)$$

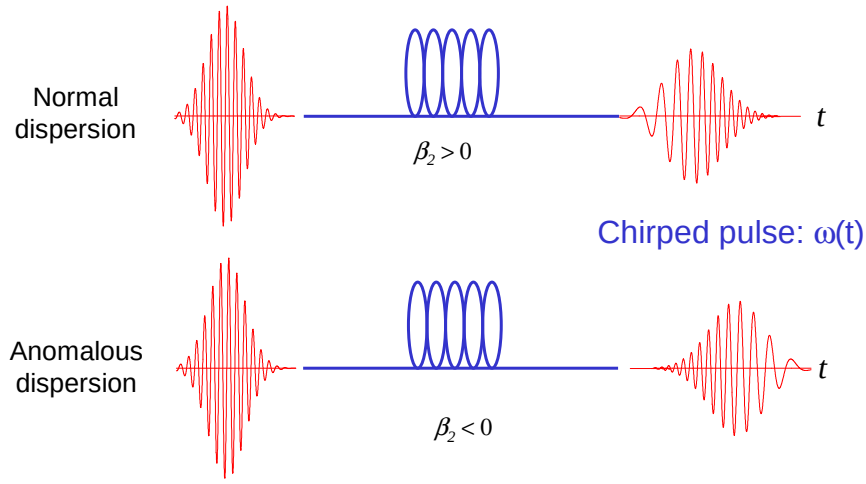
- Instantaneous frequency is given by

$$\omega(t) = \omega_0 + 2\kappa(t - \frac{z}{v_g})$$

Chirped pulse

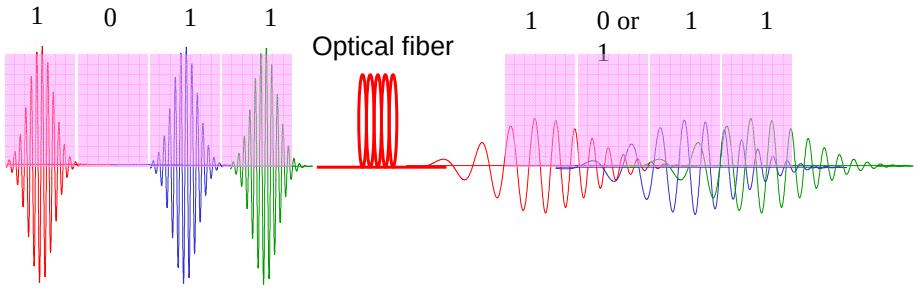
- Pulse whose instantaneous frequency changes with time is called a *chirped pulse*
- Dispersion is accompanied by chirping in the pulse
- For positive κ (positive β_2 or normal dispersion) instantaneous frequency increases with time and conversely

Dispersion & Chirping of the pulse



K Thyagarajan IITD

Impact of dispersion



Resolved pulses

Unresolved pulses

- Information lost due to overlap of pulses due to errors in identifying 0 and 1

K Thyagarajan IITD

Dispersion coefficient

$$\beta = \frac{\omega}{c} n_{eff} \qquad \beta_2 = \frac{d^2\beta}{d\omega^2} = -\frac{\lambda^2}{2\pi c} D$$

Dispersion coefficient *D* (measured in ps/km-nm)

$$D(\lambda) = -\frac{\lambda}{c} \frac{d^2 n_{eff}}{d\lambda^2}$$

Material dispersion: n_1 and n_2 depend on λ

Waveguide dispersion: explicit dependence of n_{eff} on λ

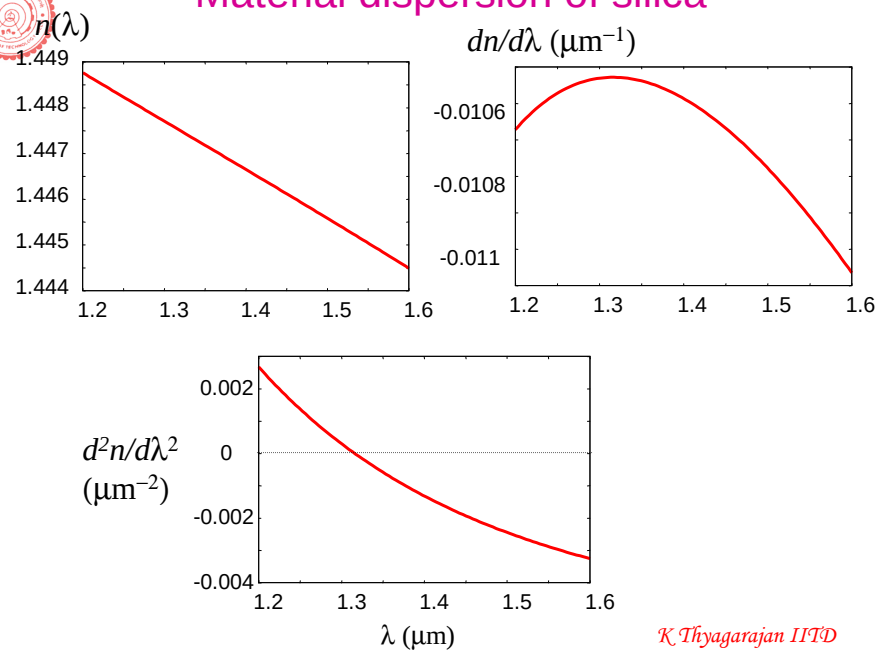
Approximate expression for $n(\lambda)$ for silica

Empirical expression for 0.5 μm < λ < 1.6 μm

$$n(\lambda) = 1.451 - 0.003\left(\lambda^2 - \frac{1}{\lambda^2}\right)$$ λ in micrometers

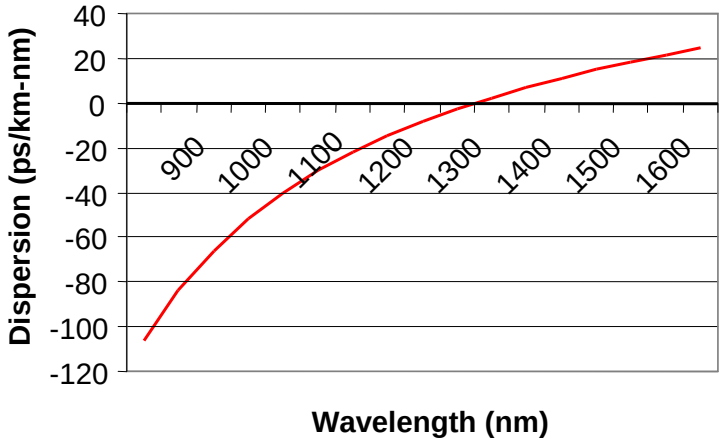
Less accurate but more convenient to use

Material dispersion of silica



K Thyagarajan IITD

Material dispersion of silica



- Note material dispersion passes through zero at ~ 1270 nm
- Zero material dispersion wavelength

K Thyagarajan IITD

Waveguide dispersion

Since $b = \dfrac{n_{eff} - n_2}{n_1 - n_2}$

$$\beta = \frac{\omega}{c}[n_2 + (n_1 - n_2)b(V)]$$

Assuming n_1 and n_2 to be independent of frequency (no material dispersion), we obtain

$$\frac{d\beta}{d\omega} = \frac{n_2}{c} + \frac{n_1 - n_2}{c}\left[\frac{d(bV)}{dV}\right]$$

and

$$\frac{d^2\beta}{d\omega^2} = \frac{n_1 - n_2}{c\omega}\left[\frac{Vd^2(bV)}{dV^2}\right]$$

Waveguide dispersion

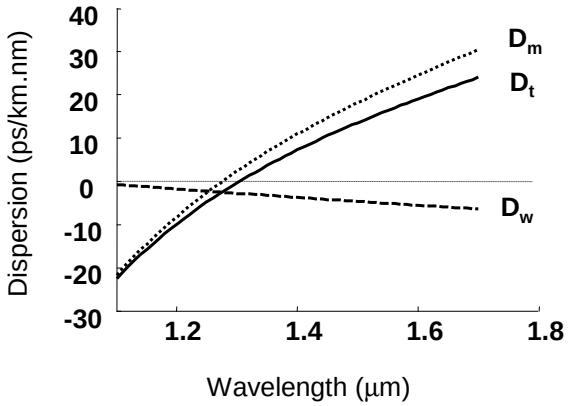
$$D_w = -\frac{2\pi c}{\lambda^2}\frac{d^2\beta}{d\omega^2} = -\frac{n_1 - n_2}{c\lambda}\left[V\frac{d^2(bV)}{dV^2}\right]$$

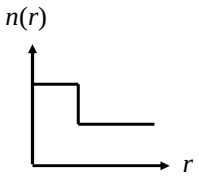
Accurate empirical formula for a step index fiber:

$$V\frac{d^2(bV)}{dV^2} \approx 0.080 + 0.549(2.834 - V)^2$$

Ref: Marcuse, Applied Optics, 18 (1979) 2930

Material, waveguide and total dispersion in SMF

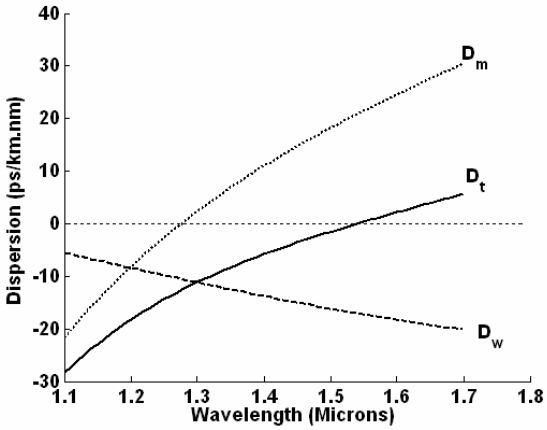




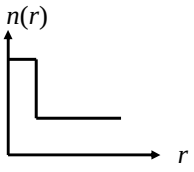
- Zero dispersion wavelength ~ 1310 nm
- A pulse at this wavelength will undergo very little dispersion

K Thyagarajan IITD

Shifting zero dispersion wavelength



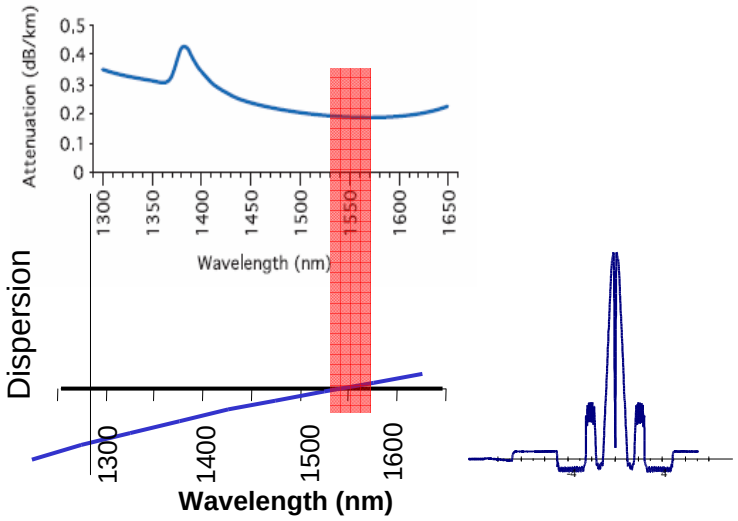
Increased index difference and smaller core radius



- Increase in waveguide dispersion by modifying refractive index profile
- Zero dispersion wavelength ~ 1550 nm

K Thyagarajan IITD

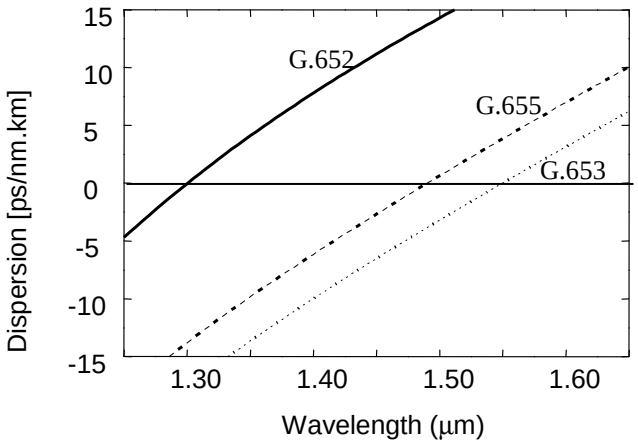
Refractive index profile modification



Minimum loss and zero dispersion at same wavelength

K Thyagarajan IITD

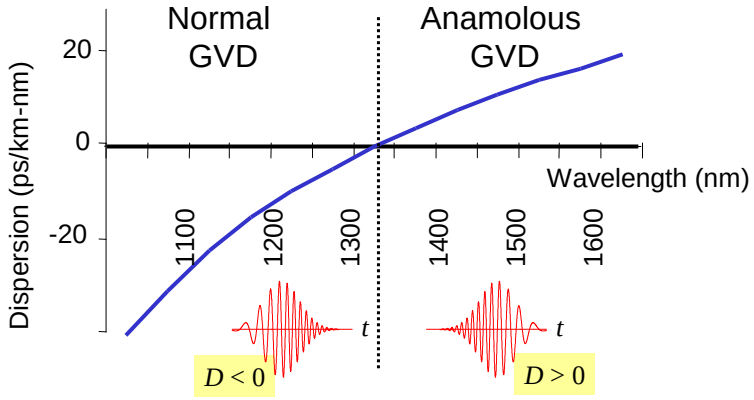
Standard fiber types



G.652 fiber: zero dispersion ~ 1310 nm
G.653 fiber: zero dispersion ~ 1550 nm
G.655 fiber: small non-zero dispersion around 1550 nm

K Thyagarajan IITD

Positive and negative *D*



Pulse broadens for both D > 0 and D < 0
Chirping is of opposite sign

K Thyagarajan IITD

Dispersion and dispersion slope

Dispersion coefficient at any λ:

$$D(\lambda)=\frac{S_0}{4}\left(\lambda-\frac{\lambda_z^4}{\lambda^3}\right)$$

$D(\lambda)$: Dispersion coefficient (ps/km-nm)
λ : Operating wavelength
λ_z: Zero dispersion wavelength
S_0: Dispersion slope (ps/km-nm^2)

Dispersion in ps: $\tau(\lambda)=D(\lambda)L\Delta\lambda$

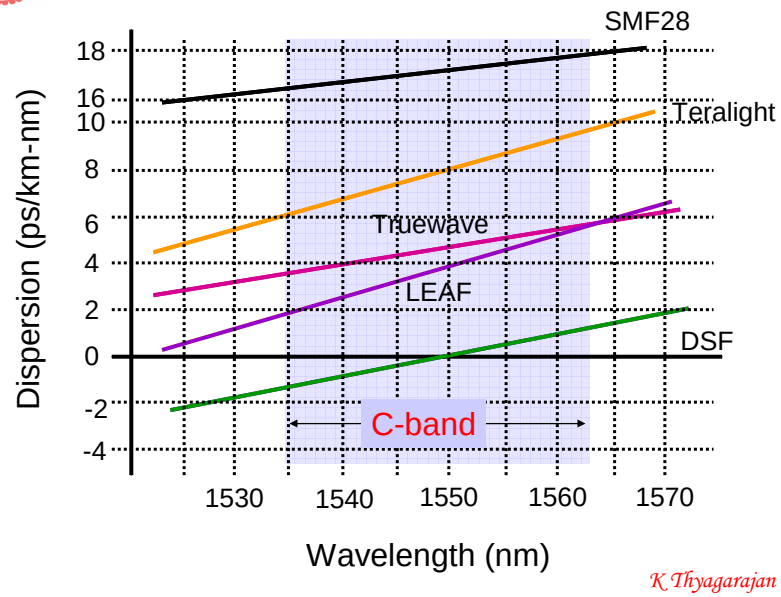
L: Fiber length; $\Delta\lambda$: spectral width of source

K Thyagarajan IITD

Typical values at 1550 nm

Fiber type	*D* (ps/km-nm)	*S* (ps/km. nm²)
Standard SMF (G.652)	17	0.058
LEAF (Corning)	4.2	0.085
Truewave-Reduced slope (OFS)	4.5	0.045
TeraLight (Alcatel)	8.0	0.057

High fiber diet



Maximum allowable dispersion

Typical acceptable dispersion value for a bit rate of B (Gb/s)

$$B^2 D L \leq 104{,}000 \text{ Gb}^2\text{ps/nm}$$

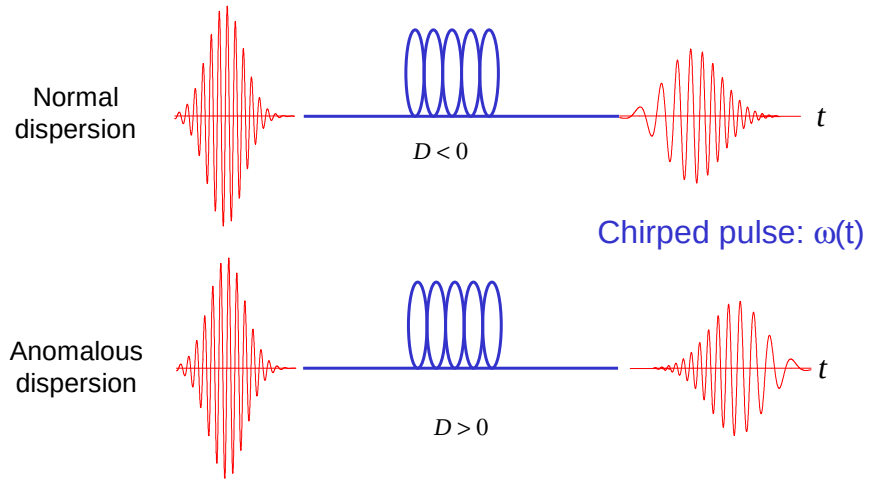
1 dB power penalty
Chirp free narrow linewidth laser
(externally modulated DFB laser - EML)

At 2.5 Gbit/s $(D\,L)_{max}$ ~ 16000 ps/nm
At 10 Gbit/s $(D\,L)_{max}$ ~ 1040 ps/nm
At 40 Gbit/s $(D\,L)_{max}$ ~ 65 ps/nm

Dispersion compensation

- Need for dispersion compensation
  - Many links using standard SMF have D ~ 17 ps/km-nm at 1550 nm
  - To avoid nonlinear effects fibers have a finite value of *D* (G.655 fibers)
  - In a wavelength division multiplexed link, different λs have different dispersions
    - Need to compensate for dispersion for different λs
    - Dispersion slope compensation

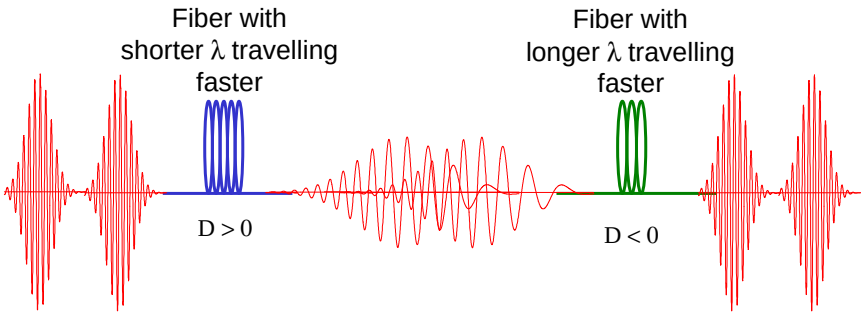
Dispersion & Chirping of the pulse



K Thyagarajan IITD

Dispersion Compensation

Optically compensate for accumulated dispersion



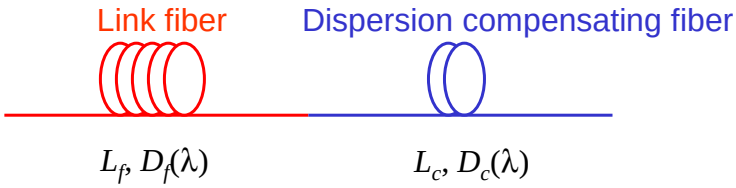
Resolved pulses

Broadened unresolved pulses

Resolved pulses

K Thyagarajan IITD

Dispersion compensation



For perfect compensation at a chosen wavelength λ_0:

$$D_f(\lambda_0)L_f + D_C(\lambda_0)L_C = 0$$

$$L_C = -\frac{D_f(\lambda_0)}{D_C(\lambda_0)}L_f$$

Dispersion slope compensation

Residual dispersion at other wavelengths:

$$\text{Dispersion} = D_f(\lambda)L_f + D_C(\lambda)L_C$$

For dispersion compensation of all channels in a DWDM system we must have

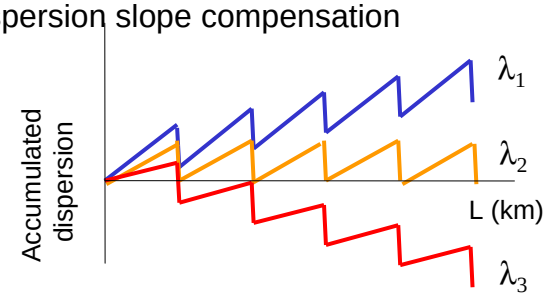
$$\kappa = \frac{D_f}{S_f} = \frac{D_c}{S_c}$$

D: Dispersion
S: Dispersion slope

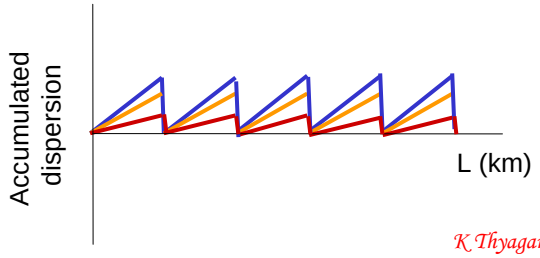
Without dispersion slope compensation:
Dispersion at extreme channels will grow and will limit the link length

Dispersion slope compensation (cont.)

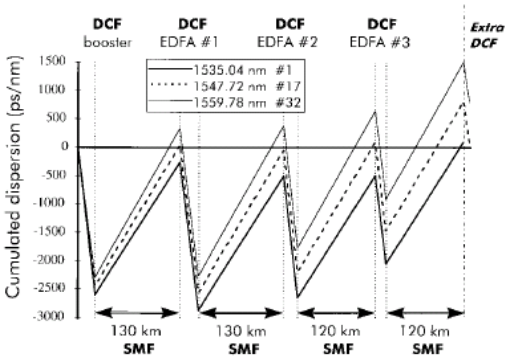
Without dispersion slope compensation



With dispersion slope compensation



Dispersion map



Ref: Bigo et al, IEEE Photonics Tech. Lett. **10** (1998) 1045

DCF for G.652 fiber

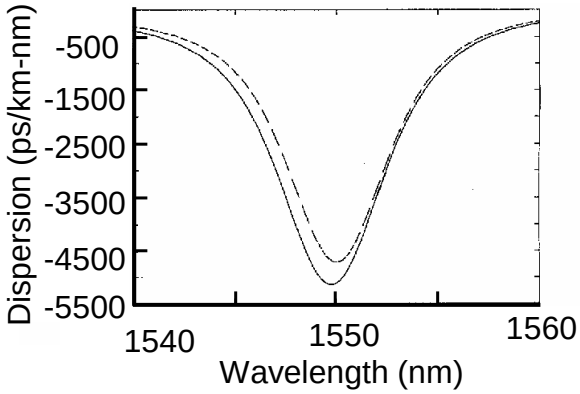
For G.652 fiber $\frac{D}{S} = 250\,\text{nm}$

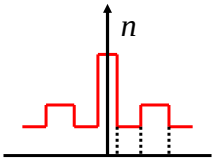
For DCF (Corning) $\frac{D}{S} = 275 \pm 25\,\text{nm}$

Insertion loss of DCF for 80 km of G.652	~ 9 dB
Polarization mode dispersion	~ 0.7 ps
Effective area	~ 20 μm^2
Nonlinear coefficient γ	~ 5.7 W^{-1} km^{-1}

K Thyagarajan IITD

Novel DCF design



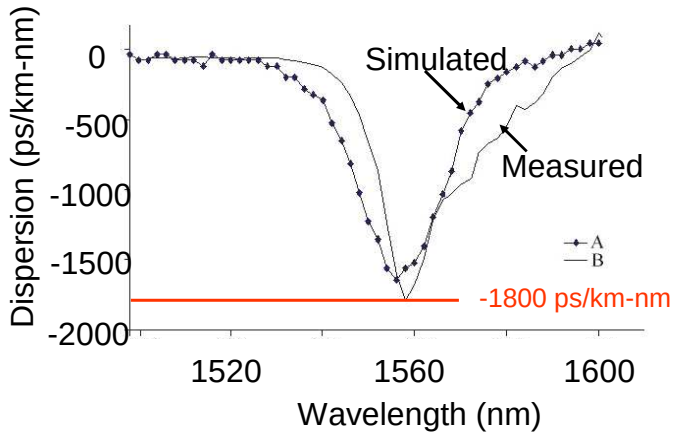


Very large negative dispersion at the phase matching λ

Ref: Thyagarajan, Varshney, Palai, Ghatak & Goyal, Photonics Tech. Letts. (1996)

K Thyagarajan IITD

Experimental realization



Ref: Auguste et al. *Electron. Lett.*, 2000.

K Thyagarajan IITD

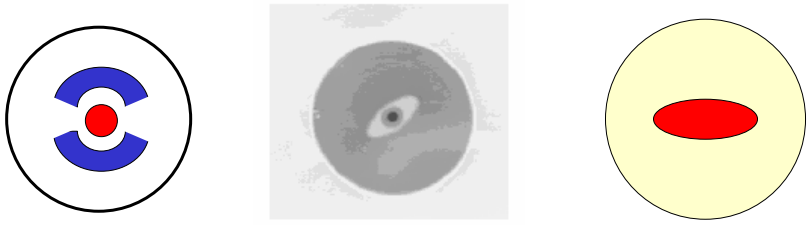
Dispersion compensating module

- Should have
  - Matched opposite dispersion to link fiber
  - Matched dispersion slope for DWDM applications
  - Small loss
  - Matching mode with single mode fiber
  - Large effective area for reduced nonlinear effects

K Thyagarajan IITD

Polarization maintaining fibers

Stress birefringent fibers



Bow-tie fibers Elliptic cladding Elliptic core

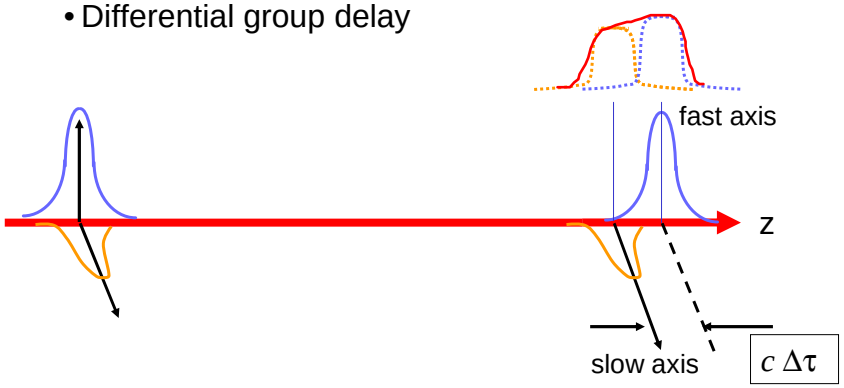
- Normal fibers do not maintain the polarization state
- PM fibers maintain the polarization state of the input light
- Applications in fiber optic sensors, integrated optics

K Thyagarajan IITD

Polarization mode dispersion

- Caused by slight fiber asymmetry and random stresses in the cable

- Different polarization states travel at different velocities
  - Differential group delay



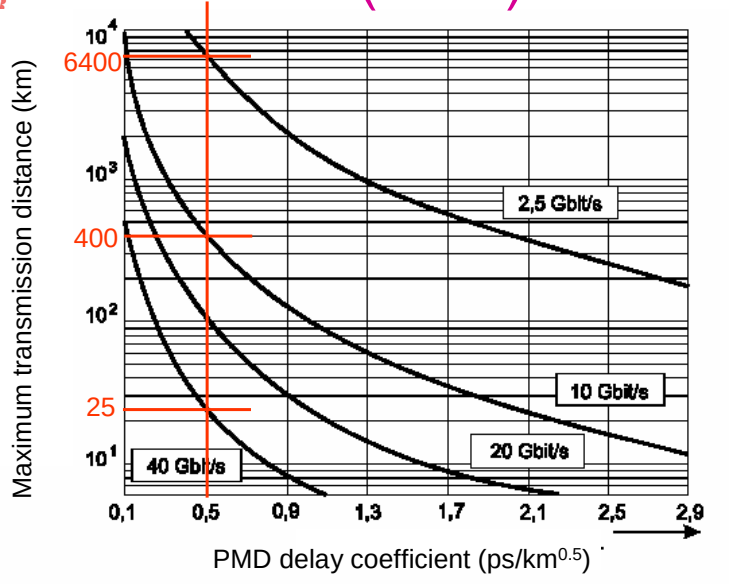
K Thyagarajan IITD

PMD

- Critical limitation for high bit rate systems
- Fluctuates slowly with time due to
  - Temperature variations, Vibrations
- Because of random coupling PMD increases as $L^{1/2}$
- PMD in current fibers < 0.05 ps/km$^{1/2}$
- PMD in many installed fibers ~ 10 ps/km$^{1/2}$
- PMD requires adaptive compensation

K Thyagarajan IITD

PMD (cont.)



Ref: PMD & Polarization, Profile

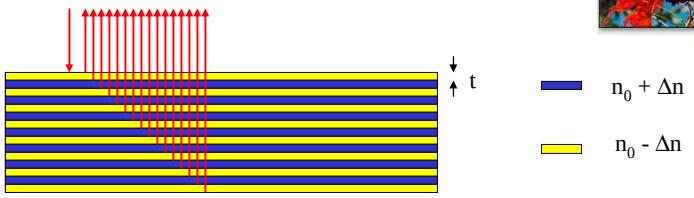
K Thyagarajan IITD

Modes and fiber components

Modes in fiber components

- In a fiber having no variation along z, the light coupled into one mode remains in the same mode
- Any perturbation along the length can be used to couple light among different modes leading to applications
- Light can also be coupled from guided to radiation modes
- Placing two fiber cores close by leads to interaction among the individual modes of the fiber

Periodic stack of films



If $\frac{2\pi}{\lambda} n_0 2t = \pi$

or $\Lambda = 2t = \frac{\lambda}{2n_0}$ BRAGG Condition

All reflections add in phase =>

STRONG REFLECTION

K Thyagarajan IITD

Optical Fibre Gratings

Periodic variation in refractive index of core used to couple Light from one mode to another

Short Period (FBG)

- Period ~ 0.5 μm
- Reflective

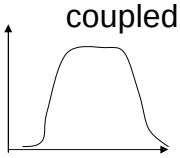
Long Period (LPG)

- Period > 100 μm
- Transmissive

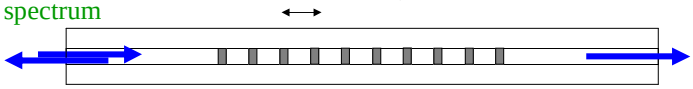
K Thyagarajan IITD

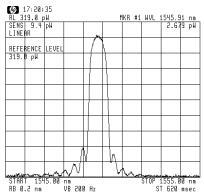
Fiber Bragg gratings

- Periodic refractive index variation along fiber
- Wavelength λ_c is strongly reflected if $\lambda_c = 2n_{eff}\Lambda$
- Forward propagating guided mode gets coupled to backward propagating guided mode



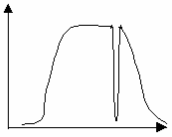
Incident spectrum



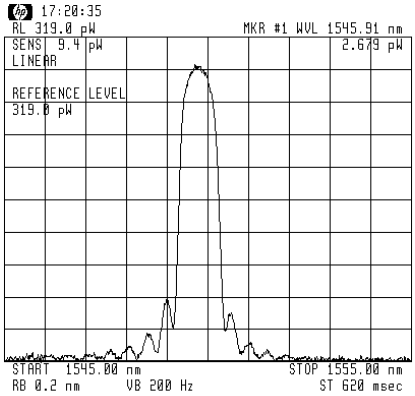


Reflected spectrum

Transmitted spectrum



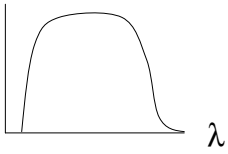
Typical measured reflectivity spectrum

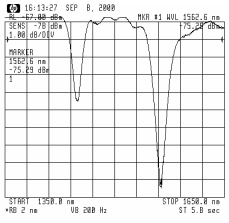


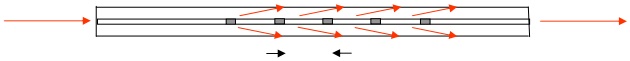
Peak reflectivity and bandwidth determined by grating length, grating strength and period of the grating

Long Period Gratings (LPG)

- Coupling of power from core mode to another core mode or a cladding mode (light guided by cladding)
- Period ~ 100 – 600 μm



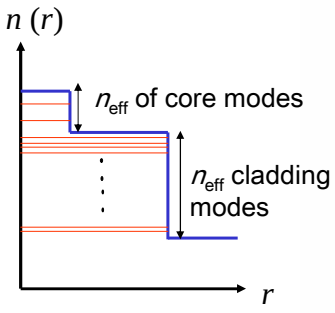


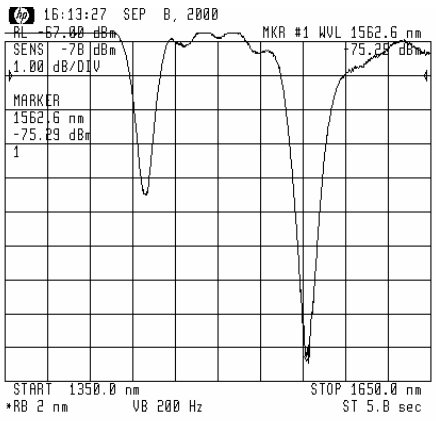


Incident spectrum

Transmitted spectrum

Typical measured transmission spectrum



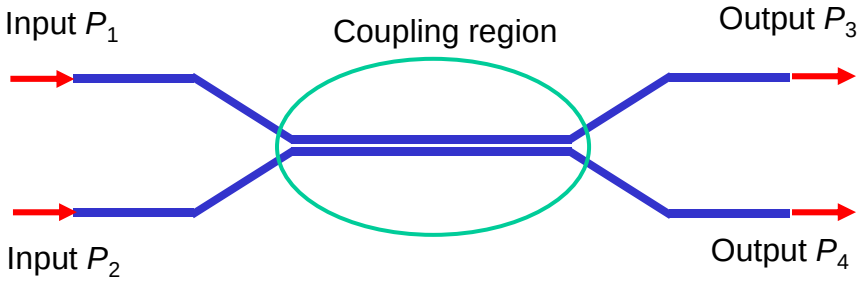


Applications

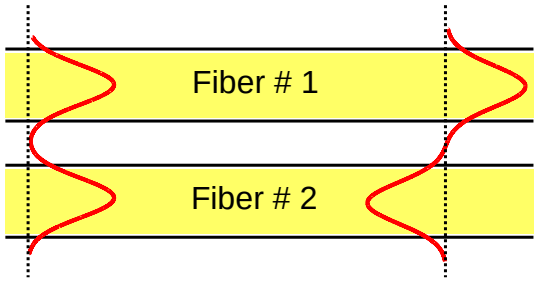
- Fiber gratings are important components in WDM systems
- Many applications demonstrated
  - Tunable dispersion compensation
  - Add/drop multiplexers
  - Wavelength lockers
- Applications in sensors
- Nonlinear switching possibilities

Fiber directional coupler

Used to split or combine optical signals



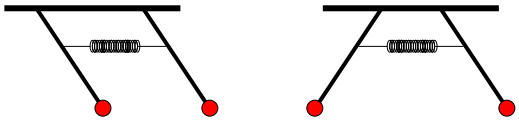
Modes of coupled waveguides



Modes have different velocities

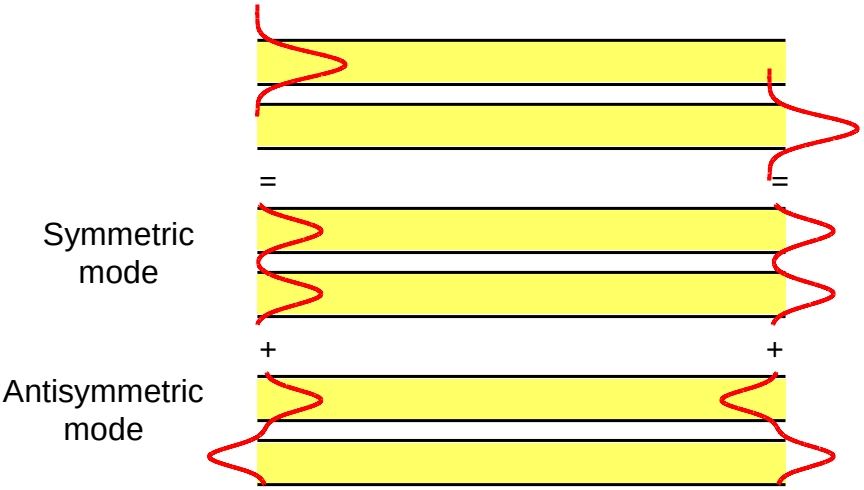
Symmetric mode

Anti-symmetric mode

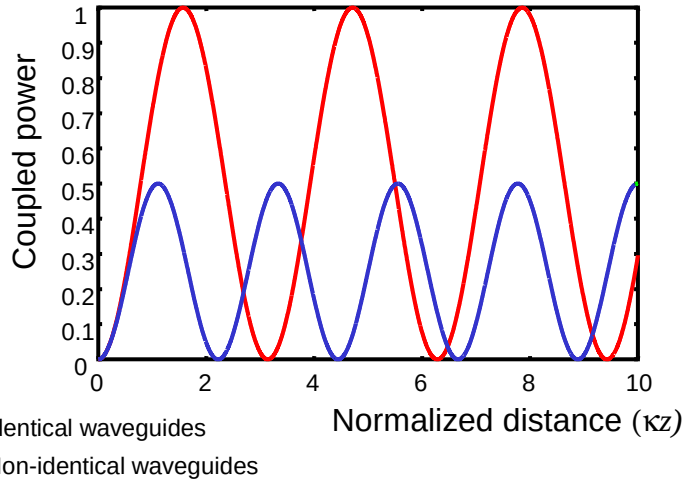


Modes have different oscillation frequencies

Principle of directional coupler



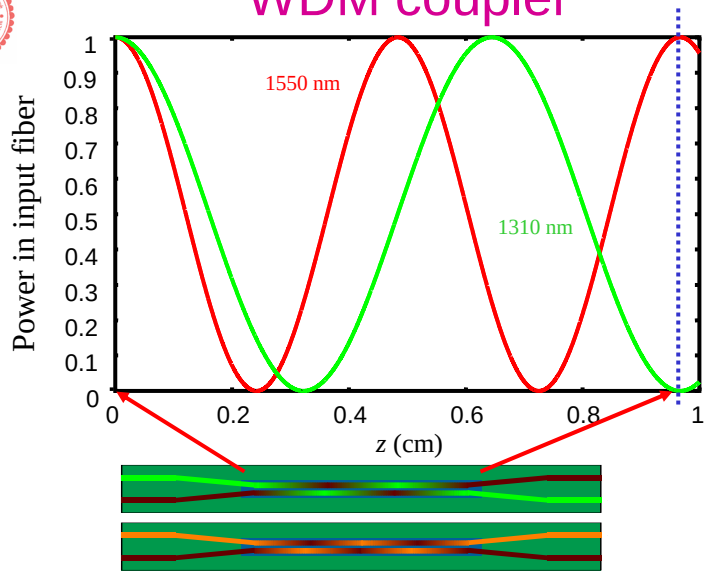
Power coupling in a directional coupler



- 3 dB coupler, tap coupler can be realized by appropriate choice of length of interaction and the fibers

K Thyagarajan IITD

WDM coupler



Ref: *Introduction to fiber optics*, A Ghatak and K Thyagarajan, Cambridge Univ Press, 1998

K Thyagarajan IITD

Summary

- Optical waveguides support guided and radiation modes
- Guided modes form a discrete spectrum and can carry power over long distances
  - form the basis of modern fiber optic communication systems
- Single mode waveguides support only one guided mode and find applications in communication, sensing and signal processing
- Understanding modal characteristics of optical waveguides can help us optimize and design waveguides and waveguide components with desired characteristics