The Abdus Salam
International Centre for Theoretical Physics

United Nations
Educational, Scientific and
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IAEA
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SMR/1837-5

2007 ICTP Oceanography Advanced School

30 April - 11 May, 2007

Two-Layer Exchange Flows

H. Bryden
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Southampton, UK

Two-Layer Exchange Flows

Lecture for ICTP Advanced School on Oceanography
International Centre for Theoretical Physics
Trieste, Italy April-May 2007

Prof. Harry L. Bryden

Miller (1983)

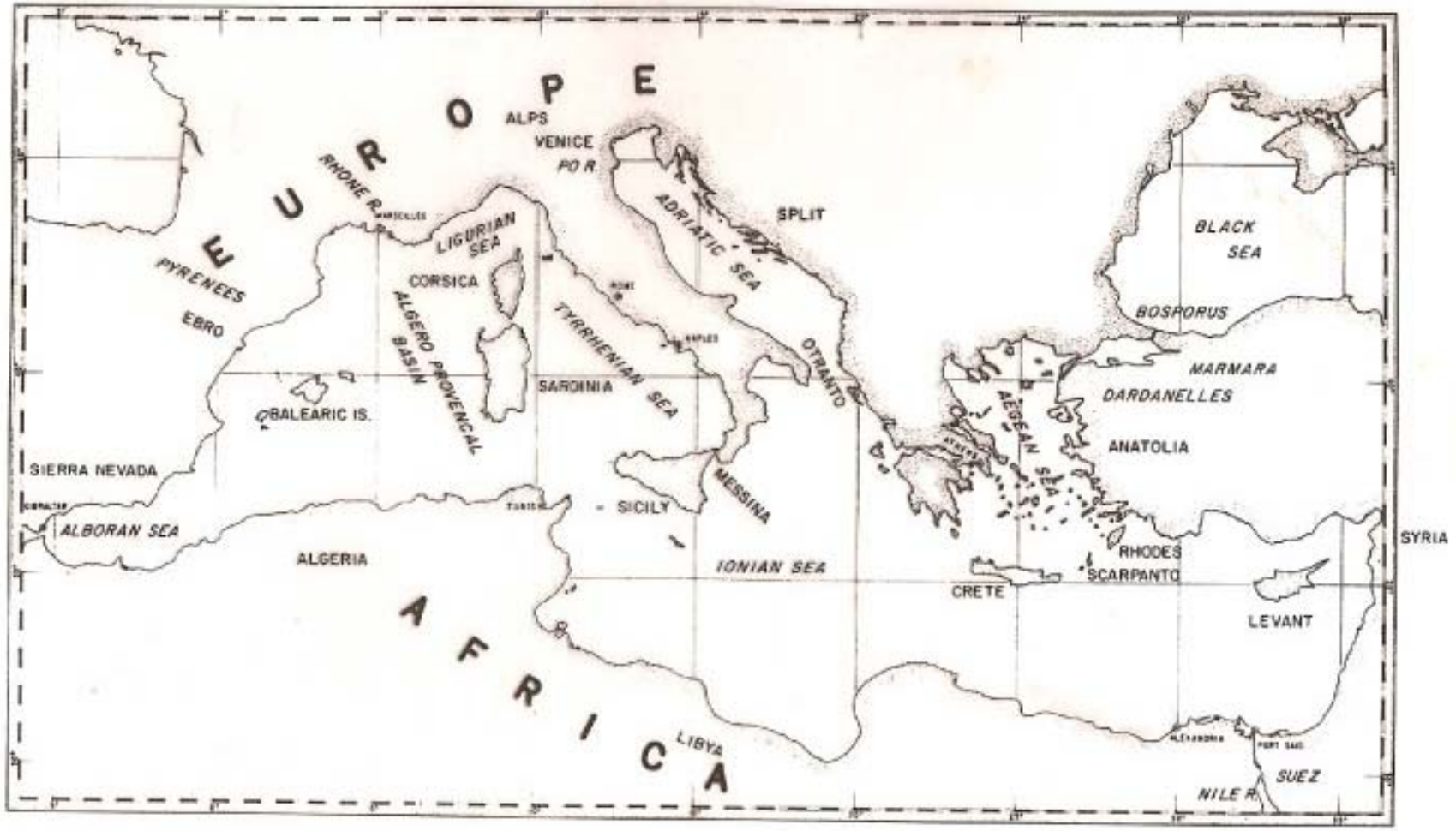
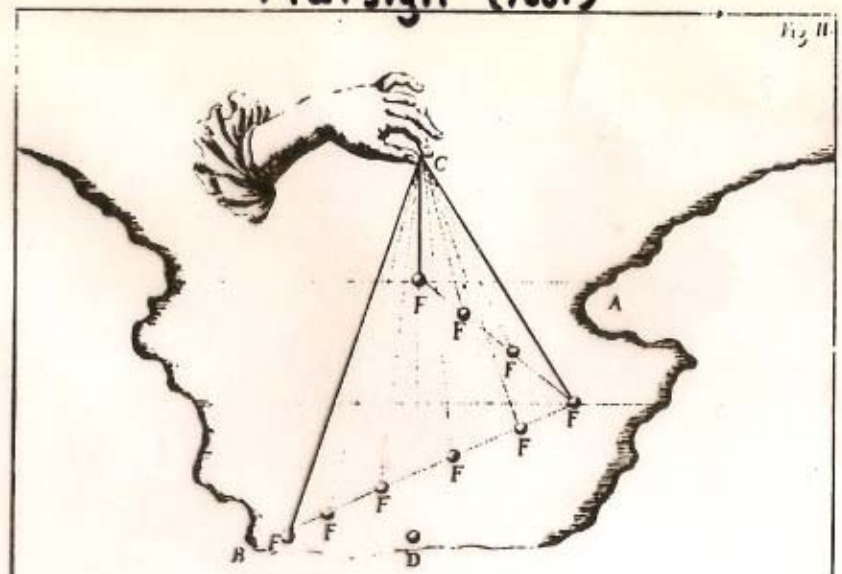
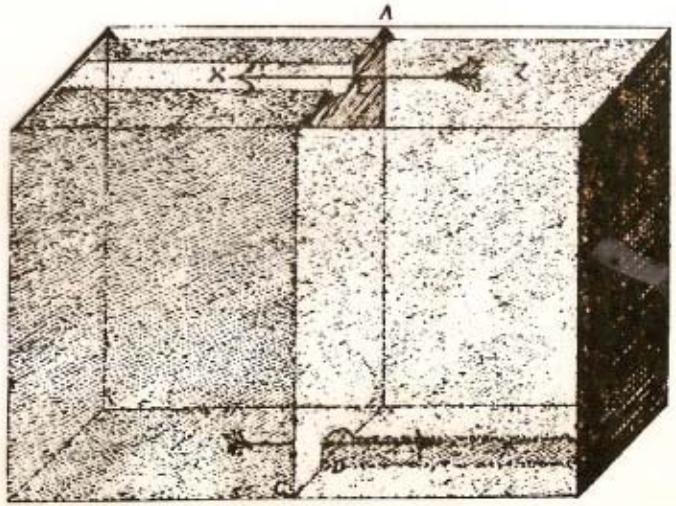


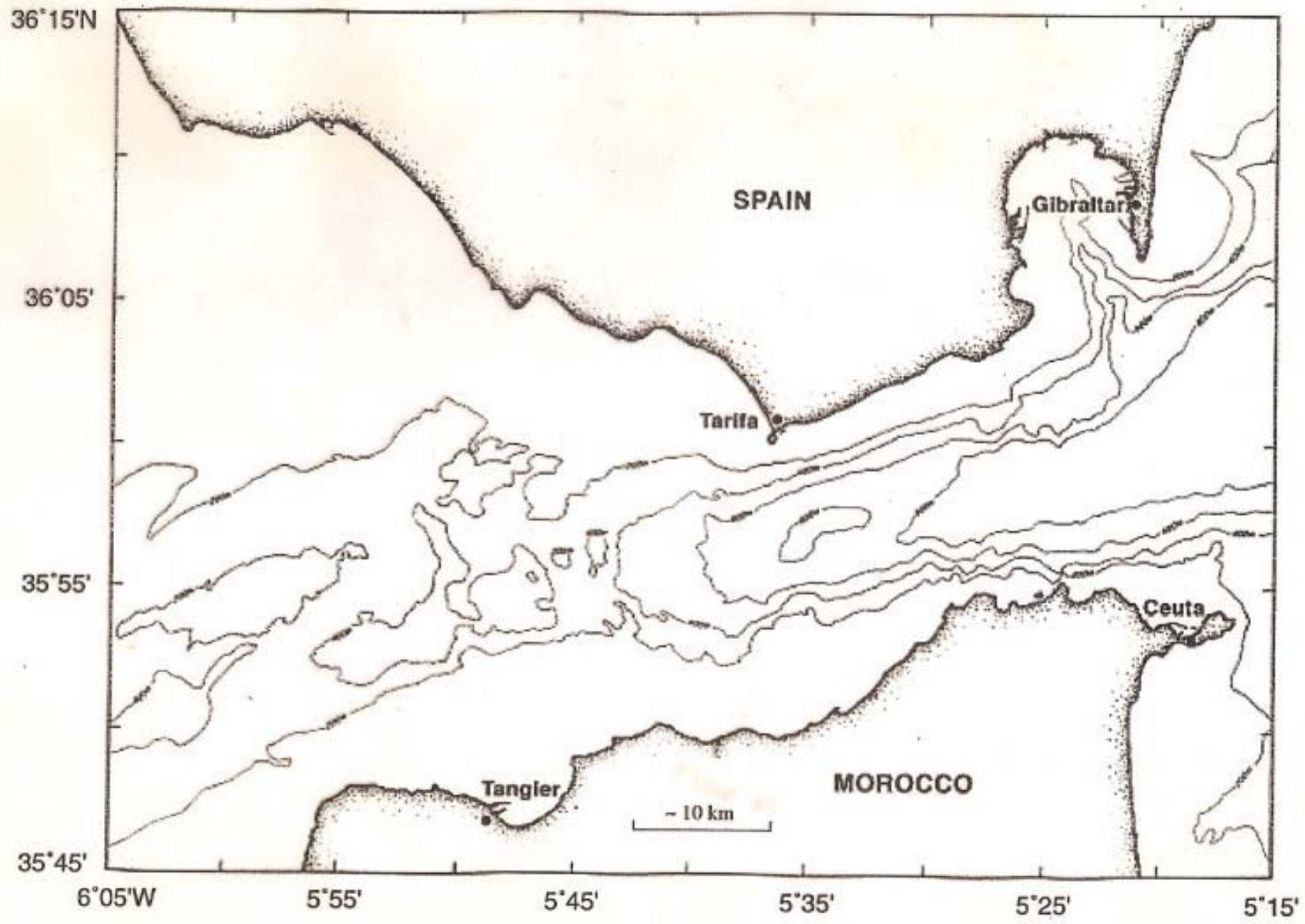
Fig. 9.1. The Mediterranean Sea.

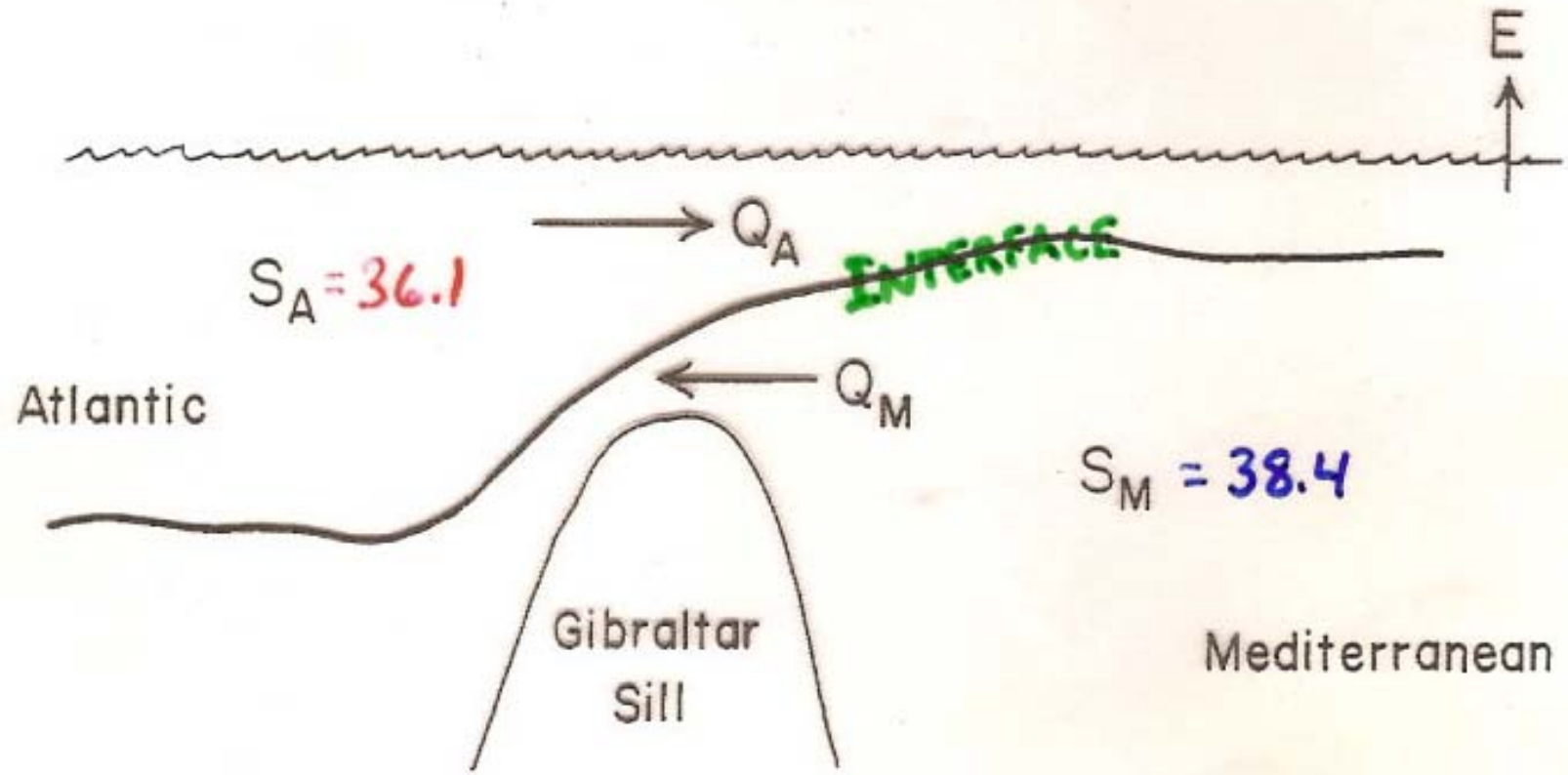
Reprinted from *Estuaries and Enclosed Seas*, edited by B.H. Ketchum

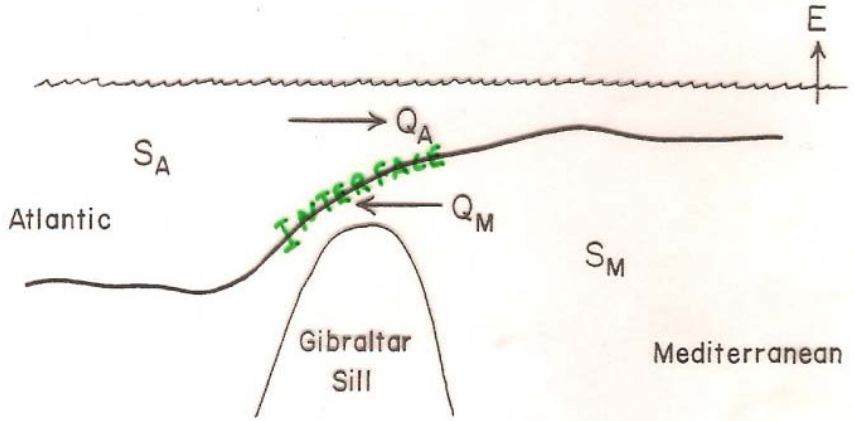
Marsigli (1681)






36°15'N
36°05'
35°55'
35°45'
6°05'W
5°55'
5°45'
5°35'
5°25'
5°15'
SPAIN
Gibraltar
Tarifa
Ceuta
Tangier
MOROCCO
~ 10 km


E
Q_A
$S_A = 36.1$
INTERFACE
Q_M
Atlantic
$S_M = 38.4$
Gibraltar
Sill
Mediterranean



$$Q_A + Q_M = E$$ Mass Conservation

$$Q_A S_A + Q_M S_M = 0$$ Salt Conservation

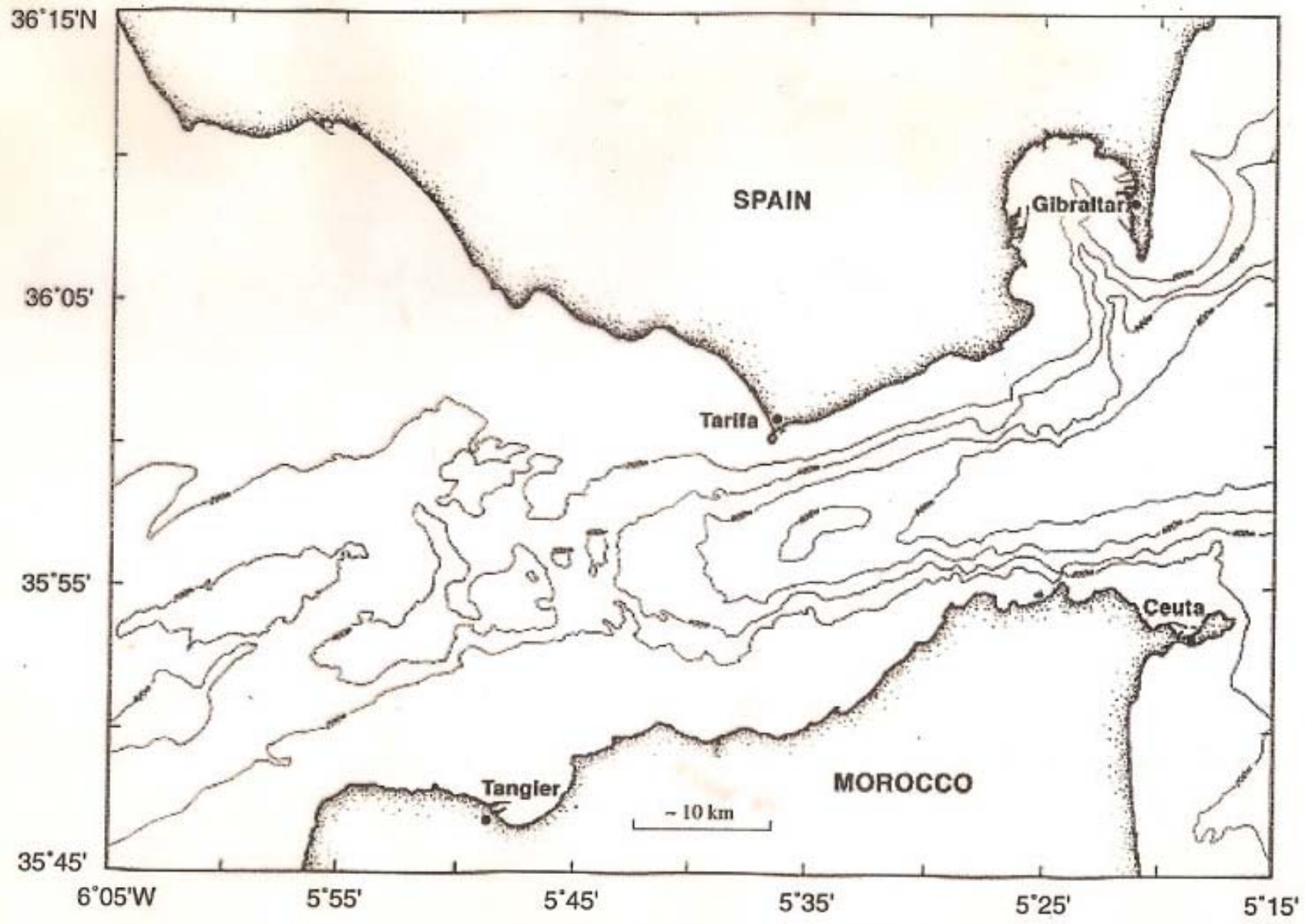
$$Q_A = \frac{E * S_M}{S_M - S_A}$$

$$Q_M = \frac{-E * S_A}{S_M - S_A}$$

Knudsen Relations

TRADITIONAL METHOD

1. ESTIMATE NET EVAPORATION OVER MEDITERRANEAN
2. MEASURE S_A, S_M IN STRAIT OF GIBRALTAR
3. CALCULATE Q_A, Q_M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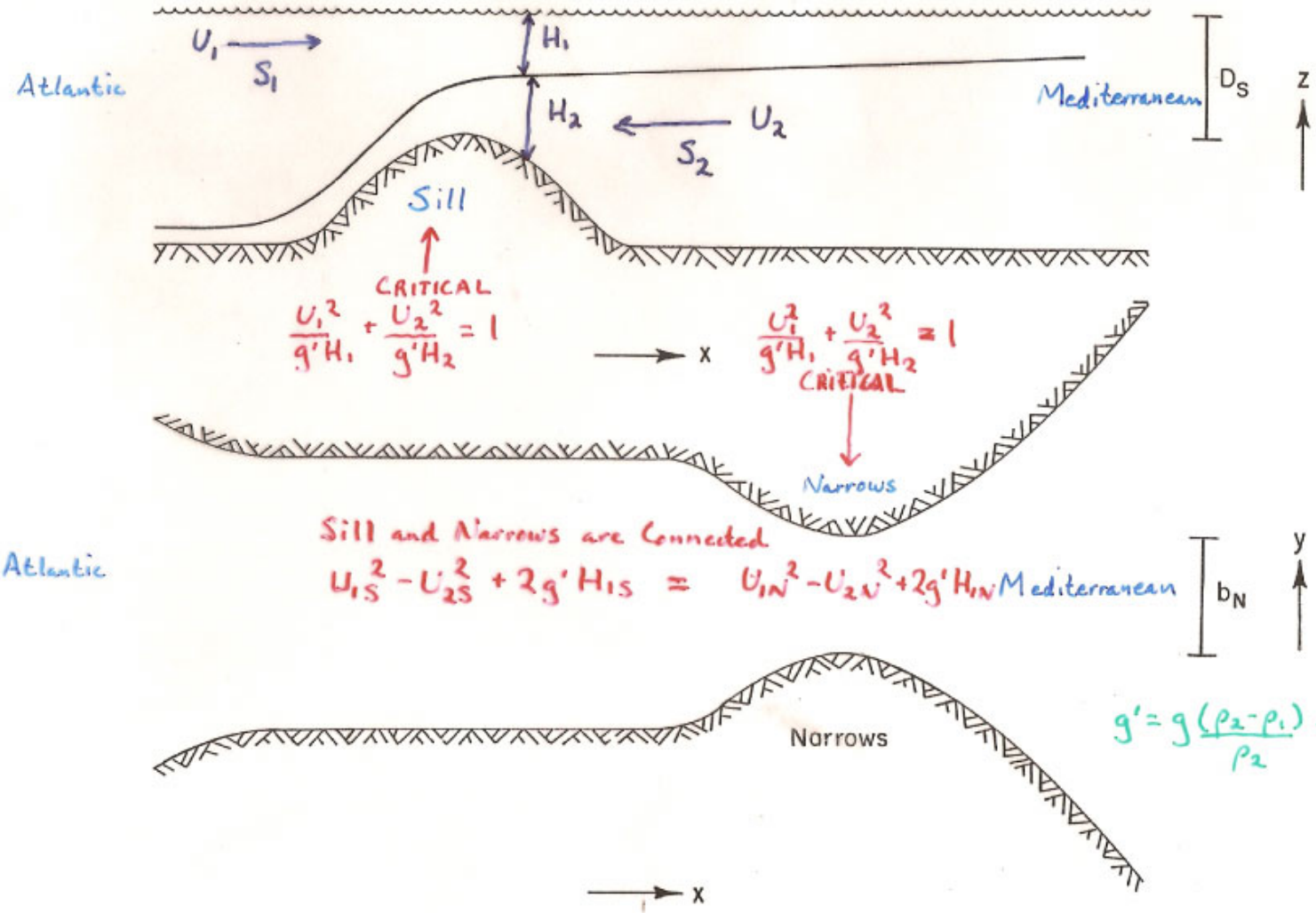
~ 10 km

HYDRAULIC CONTROL MODEL

NONLINEAR, STEADY, NO FRICTION, NO MIXING, NO ROTATION



$$g' = g\frac{(\rho_2 - \rho_1)}{\rho_2} = g\beta(S_2 - S_1)$$

Dalziel (1988)

HYDRAULIC CONTROL AND CONSERVATION OF MASS+SALT

$$\underbrace{.277\sqrt{g' D_s}\,\frac{D_s b_s}{2} = Q_1 - Q_2}_{\text{Hydraulic Control}} = \underbrace{\frac{S_M + S_A}{S_M - S_A}\,E}_{\text{Mass + Salt Conservation}}$$

$$g' = g\beta\frac{(S_M - S_A)}{\rho_o} = \frac{g\beta}{\rho_o}\Delta S \qquad S_M = S_A + \Delta S$$

$g, \beta, \rho_o, D_s, b_s, S_A$ are known constants

Solve for ΔS as function of E:

$$\frac{\Delta S^3}{\left(1 + \frac{\Delta S}{2S_A}\right)^2} = \frac{\rho_o}{g\beta D_s}\left(\frac{2S_A E}{.277\,\frac{D_s b_s}{2}}\right)^2$$

Mass + Salt Conservation suggest

$$|Q_A| + |Q_M| \propto \frac{E}{S_M - S_A}$$

Hydraulic Control Theory suggest

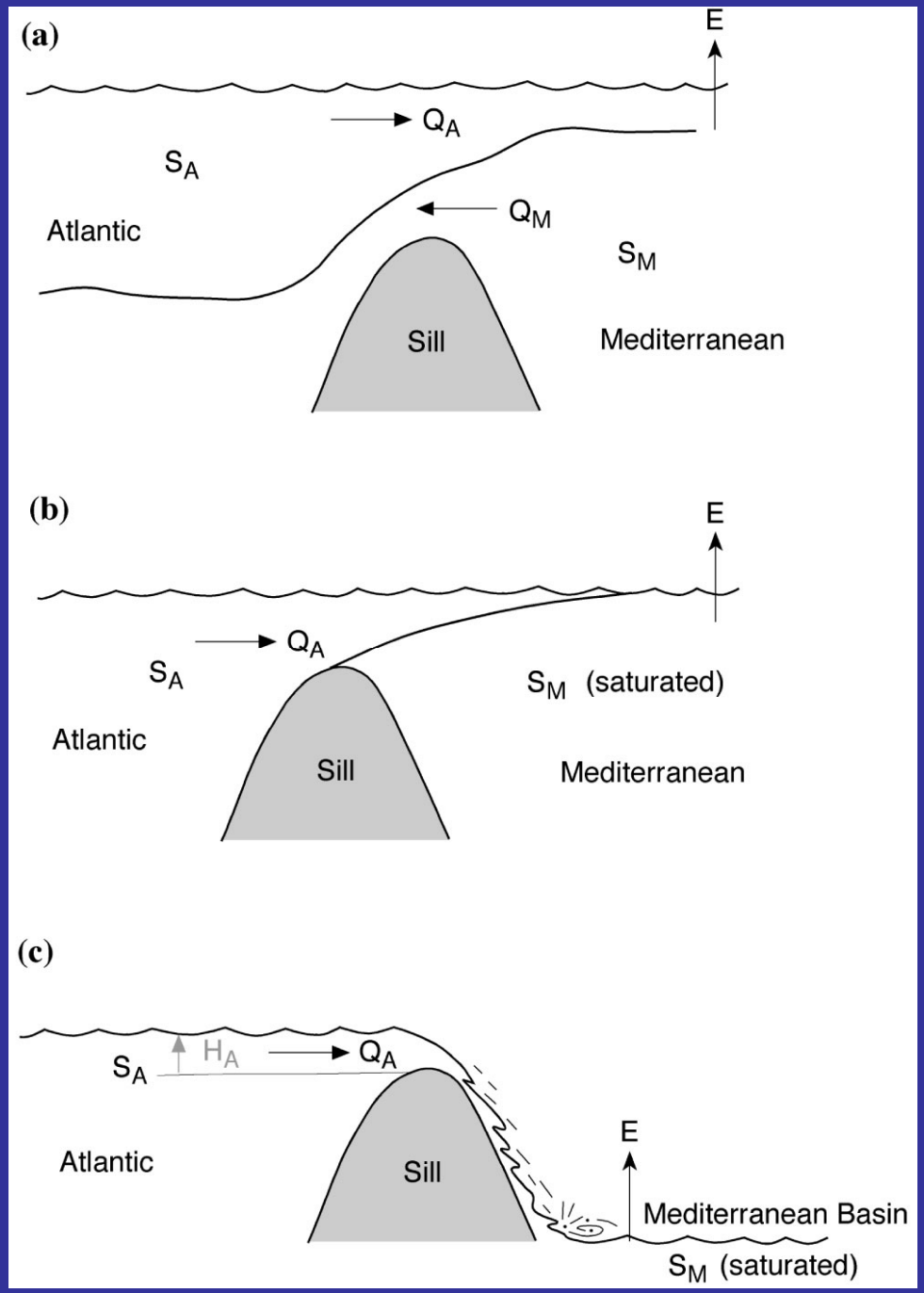
$$|Q_A| + |Q_M| \propto \sqrt{S_M - S_A}$$

For both to be satisfied

$$S_M - S_A \propto E^{2/3}$$

Table 1. *Prediction of the salinity difference, inflow and outflow transports for the steady, two-layer maximal exchange through the Strait of Gibraltar as a function of the net evaporation over the Mediterranean Basin. The net evaporation over the Mediterranean Basin,* E, *equals the average local net evaporation,* e, *times the surface area of the Mediterranean,* 2.52×10^{12} m^2

Net evaporation e (cm y^{-1})	Salinity difference, $\Delta S = S_2 - S_1$ (‰)	Inflow transport, Q_1 (10^6 m^3 s^{-1})	Outflow transport, $-Q_2$ (10^6 m^3 s^{-1})
50	1.75	0.87	0.83
60	1.98	0.92	0.88
70	2.20	0.98	0.92
80	2.40	1.03	0.96
90	2.61	1.07	1.00
100	2.80	1.11	1.03


(a)
E
Q_A
S_A
Atlantic
Q_M
S_M
Sill
Mediterranean
(b)
E
Q_A
S_A
S_M (saturated)
Atlantic
Sill
Mediterranean
(c)
S_A
H_A
Q_A
Atlantic
Sill
E
Mediterranean Basin
S_M (saturated)

THE FILLING OF THE MEDITERRANEAN SEA

BY GUY BILLOUT