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Les Houches Lectures on Warped Models and Holography

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Abstract

The theoretical tools required to construct models in warped extra dimensions are presented. This includes how to localise zero modes in the warped bulk and how to obtain the holographic interpretation using the AdS/CFT correspondence. Several models formulated in warped space are then discussed including nonsupersymmetric and supersymmetric theories as well as their dual interpretation. Finally it is shown how grand unification occurs in warped models.

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1 Introduction

Warped extra dimensions have provided a new framework for addressing the hierarchy problem in extensions of the Standard Model. The curved fifth dimension is compactified on a line segment where distance scales are measured with the nonfactorisable metric of anti-de Sitter (AdS) space. Since distance and hence energy scales are location dependent in AdS space, the hierarchy problem can be redshifted away. However this novel solution also has a more mundane explanation. By the AdS/CFT correspondence, five-dimensional (5D) AdS space has a dual four-dimensional (4D) interpretation in terms of a strongly-coupled conformal field theory (CFT). In this 4D guise the hierarchy problem is solved by having a low cutoff scale that is associated with the conformal symmetry breaking scale of the CFT. Both interpretations are equally valid and allow the infrared (IR) scale to be hierarchically smaller and stable compared to the ultraviolet (UV) scale. This is much like the photon and electron of QED compared with the baryons and mesons of QCD. The Planck scale, M_P provides the cutoff scale for QED, while in QCD the composite baryons and mesons are only valid up to the QCD scale $\Lambda_{QCD} \ll M_P$. In warped models of electroweak physics the Higgs is composite at the TeV scale, while the Standard Model fermions and gauge bosons are partly composite to varying degrees, ranging from an elementary electron to a composite top quark. The Planck scale again provides the UV cutoff for the elementary states, while the TeV scale is the compositeness or IR scale.

This hybrid framework of electroweak physics has led to a renaissance of composite Higgs models that have many desirable features. They are consistent with electroweak precision data that place strong bounds on the scales of new physics. New physics at the TeV scale generically leads to flavor problems, but these are absent in warped models, since a GIM-like mechanism operates. The warped models also explain the fermion mass hierarchy (including neutrino masses) as well as incorporate grand unification with logarithmic running and unified gauge couplings. This sophisticated level of model building is due to the complementary nature of the AdS/CFT framework. The 5D warped bulk not only provides a simple geometric picture in which to construct these models, but more importantly is also a weakly-coupled description in which calculations can be performed. In fact this theoretical tool provides a unique window into strongly-coupled 4D gauge theories and goes beyond the application to electroweak physics that will be the primary concern of these lectures. The 5D picture is complemented by the 4D description which is more intuitive, primarily from our understanding of QCD, except that the 4D gauge theory is strongly-coupled and a perturbative analysis is not possible.

Hence the plan of these lectures will be to begin in 5D warped space and describe how to localise zero modes anywhere in a slice of AdS. This will enable us to quite simply construct the Standard Model in the bulk. The more intuitive dual 4D interpretation will then be obtained by writing down the AdS/CFT dictionary. This dictionary will allow us to give either a 5D or 4D description for any type of warped

model. The AdS/CFT framework need not only apply to electroweak physics. Supersymmetry will be subsequently introduced and the model-building possibilities surveyed. Lastly the novel features of grand unification in warped space will be discussed.

Since the aim of these lectures is to emphasize the dual nature of warped models a little background knowledge of warped extra dimensions will be useful. This is nicely reviewed in Refs [1, 2, 3, 4, 5]. Some theoretical aspects of warped models presented in these lectures have also been covered in Refs [3, 5, 6]. Higgsless models are not covered in these lectures, but these type of warped models are discussed in Refs [6, 7]. Finally, while these lectures concentrate on the theoretical aspects of warped models, the phenomenological aspects are just as important, and these can be found in Ref [6, 8].

2 Bulk fields in a slice of AdS₅

2.1 A slice of AdS₅

Let us begin by considering a 5D spacetime with the AdS₅ metric

$$ds^2 = e^{-2ky} \eta_{\mu\nu} dx^\mu dx^\nu + dy^2 \equiv g_{MN} dx^M dx^N, \quad (1)$$

where k is the AdS curvature scale. The spacetime indices $M = (\mu, 5)$ where $\mu = 0, 1, 2, 3$ and $\eta_{\mu\nu} = \text{diag}(- + + +)$ is the Minkowski metric. The fifth dimension y is compactified on a Z_2 orbifold with a UV (IR) brane located at the orbifold fixed points $y^* = 0(\pi R)$. Between these two three-branes the metric (1) is a solution to Einstein's equations provided the bulk cosmological constant and the brane tensions are appropriately tuned (see, for example, the lectures by Rubakov [1]). This slice of AdS₅ is the Randall-Sundrum solution [9] (RS1) and is geometrically depicted in Fig.1.

In RS1 the Standard Model particle states are confined to the IR brane. The hierarchy problem is then solved by noting that generic mass scales M in the 5D theory are scaled down to $M e^{-\pi k R}$ on the IR brane at $y = \pi R$. In particular since the Higgs boson H is localised on the IR brane this means that the dimension two Higgs mass term gets rescaled by an amount

$$m_H^2 |H|^2 \rightarrow (m_H e^{-\pi k R})^2 |H|^2, \quad (2)$$

so that a Higgs mass parameter $m_H \sim \mathcal{O}(M_5)$ in the 5D theory will naturally be associated with a hierarchically smaller scale on the IR brane (where M_5 is the 5D fundamental mass scale). However on the IR brane higher-dimension operators with dimension greater than four, such as those associated with proton decay, flavour changing neutral currents (FCNC) and neutrino masses will now be suppressed by

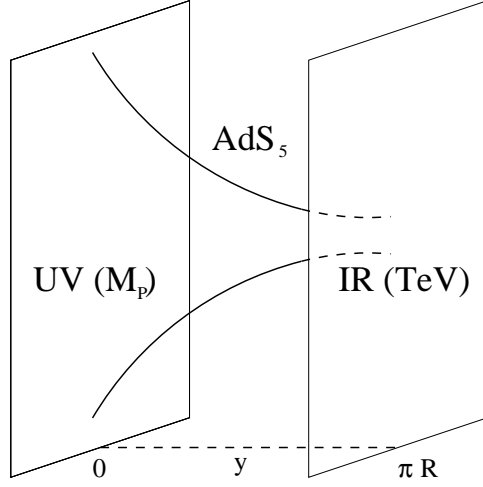


Figure 1: A slice of AdS_5 : The Randall-Sundrum scenario.

the warped down scale

$$\frac{1}{M_5^2} \bar{\Psi}_i \Psi_j \bar{\Psi}_k \Psi_l \rightarrow \frac{1}{(M_5 e^{-\pi k R})^2} \bar{\Psi}_i \Psi_j \bar{\Psi}_k \Psi_l , \quad (3)$$

$$\frac{1}{M_5} \nu \nu H H \rightarrow \frac{1}{M_5 e^{-\pi k R}} \nu \nu H H , \quad (4)$$

where Ψ_i is a Standard Model fermion and ν is the neutrino. This leads to generic problems with proton decay and FCNC effects, and also neutrino masses are no longer consistent with experiment. Thus, while the hierarchy problem has been addressed in the Higgs sector by a classical rescaling of the Higgs field, this has come at the expense of introducing proton decay and FCNC problems from higher-dimension operators that were sufficiently suppressed in the Standard Model.

- **EXERCISE:** The classical rescaling $\Phi \rightarrow e^{d_\Phi \pi k R} \Phi$ where $d_\Phi = 1(\frac{3}{2})$ for scalars (fermions), suffers from a quantum anomaly and leads to the addition of the Lagrangian term

$$\delta \mathcal{L}_{\text{anomaly}} = \pi k R \sum_i \frac{\beta(g_i)}{4g_i^3} \text{Tr} F_{\mu\nu,i}^2 , \quad (5)$$

where $\beta(g_i)$ is the β -function for the corresponding gauge couplings g_i . Show that this anomaly implies that quantum mass scales, such as the gauge coupling unification scale M_{GUT} , are also redshifted by an amount $M_{GUT} e^{-\pi k R}$.

Instead in the slice of AdS_5 with the Standard Model fields confined on the IR brane one has to resort to discrete symmetries to forbid the offending higher-dimension operators. Of course it is not adequate just to forbid the leading higher-dimension

operators. Since the cutoff scale on the IR brane is low ($\mathcal{O}(\text{TeV})$), successive higher-dimension operators must also be eliminated to very high order.

This feature of RS1 stems from the fact that all Standard Model particles are localised on the IR brane. However to address the hierarchy problem, only the Higgs field needs to be localised on the IR brane. The Standard Model fermions and gauge fields do not have a hierarchy problem and therefore can be placed anywhere in the bulk [10, 11, 12]. In this way the UV brane can be used to provide a sufficiently high scale to help suppress higher-dimension operators while still solving the hierarchy problem [11].

2.2 The bulk field Lagrangian

Let us consider fermion Ψ , scalar Φ and vector A_M bulk fields. In a slice of AdS_5 the 5D action is given by

$$\begin{aligned}
S_5 = & - \int d^4x dy \sqrt{-g} \left[\frac{1}{2} M_5^3 R + \Lambda_5 \right. \\
& + \frac{1}{4g_5^2} F_{MN}^2 + |D_M \Phi|^2 + i \bar{\Psi} \Gamma^M \nabla_M \Psi \\
& \left. + m_\phi^2 |\Phi|^2 + im_\psi \bar{\Psi} \Psi \right] , \tag{6}
\end{aligned}$$

where M_5 is the 5D fundamental scale, Λ_5 is the bulk cosmological constant and g_5 is the 5D gauge coupling. In curved space the gamma matrices are $\Gamma_M = e_M^A \gamma_A$, where e_M^A is the funfbein defined by $g_{MN} = e_M^A e_N^B \eta_{AB}$ and $\gamma_A = (\gamma_\alpha, \gamma_5)$ are the usual gamma matrices in flat space. The curved space covariant derivative $\nabla_M = D_M + \omega_M$, where ω_M is the spin connection and D_M is the gauge covariant derivative for fermion and/or scalar fields charged under some gauge symmetry. The action (6) includes all terms to quadratic order that are consistent with gauge symmetries and general coordinate invariance. In particular this only allows a mass term m_ϕ for the bulk scalar and a mass term m_ψ for the bulk fermion.

In general the equation of motion for the bulk fields is obtained by requiring that $\delta S_5 = 0$. This variation of the action (6) can be written in the form

$$\delta S_5 = \int d^5x \delta\phi (\mathcal{D}\phi) + \int d^4x \delta\phi (\mathcal{B}\phi)|_{y^*} , \tag{7}$$

where ϕ is any bulk field. Requiring the first term in (7) to vanish gives the equation of motion $\mathcal{D}\phi = 0$. However the second term in (7) is evaluated at the boundaries y^* of the fifth dimension y . The vanishing of the second term thus leads to the boundary conditions $\delta\phi|_{y^*} = 0$ or $\mathcal{B}\phi|_{y^*} = 0$. Note that there are also boundary terms arising from the orthogonal directions x^μ , but these are automatically zero because ϕ is assumed to vanish at the 4D boundary $x^\mu = \pm\infty$.

2.2.1 Scalar fields

Suppose that the bulk scalar field has a mass squared $m_\phi^2 = ak^2$ where we have defined the bulk scalar mass in units of the curvature scale k with dimensionless coefficient a . The equation of motion derived from the scalar part of the variation of the action (6) is

$$\partial^2 \Phi + e^{2ky} \partial_5 (e^{-4ky} \partial_5 \Phi) - m_\phi^2 e^{-2ky} \Phi = 0, \quad (8)$$

where $\partial^2 = \eta^{\mu\nu} \partial_\mu \partial_\nu$. We are interested in the zero mode solution of this equation. The solution is obtained by assuming a separation of variables

$$\Phi(x, y) = \frac{1}{\sqrt{\pi R}} \sum_n \Phi^{(n)}(x) \phi^{(n)}(y), \quad (9)$$

where $\Phi^{(n)}$ are the Kaluza-Klein modes satisfying $\partial^2 \Phi^{(n)}(x) = m_n^2 \Phi^{(n)}(x)$ and $\phi^{(n)}(y)$ is the profile of the Kaluza-Klein mode in the bulk. The general solution for the zero mode ($m_0 = 0$) is given by

$$\phi^{(0)}(y) = c_1 e^{(2-\alpha)ky} + c_2 e^{(2+\alpha)ky}, \quad (10)$$

where $\alpha \equiv \sqrt{4+a}$ and c_1, c_2 are arbitrary constants. These constants can be determined by imposing boundary conditions at the brane locations, which following from the second term of (7) can be either Neumann $\partial_5 \phi^{(n)}|_{y^*} = 0$ or Dirichlet $\phi^{(n)}|_{y^*} = 0$. However for $a \neq 0$ imposing Neumann conditions leads to $c_1 = c_2 = 0$. Similarly Dirichlet conditions lead to $c_1 = c_2 = 0$. This implies that there is no zero mode solution with simple Neumann or Dirichlet boundary conditions.

Instead in order to obtain a zero mode we need to modify the boundary action and include boundary mass terms [11]

$$S_{bdy} = - \int d^4x dy \sqrt{-g} 2 b k [\delta(y) - \delta(y - \pi R)] |\Phi|^2, \quad (11)$$

where b is a dimensionless constant parametrising the boundary mass in units of k . The Neumann boundary conditions are now modified to

$$(\partial_5 \phi^{(0)} - b k \phi^{(0)}) \Big|_{0, \pi R} = 0. \quad (12)$$

• **EXERCISE:** Verify that the boundary conditions (12) follow from (7) after including the boundary mass terms (11).

Imposing the modified Neumann boundary conditions at $y^* = 0, \pi R$ leads to the equations

$$(2 - \alpha - b) c_1 + (2 + \alpha - b) c_2 = 0, \quad (13)$$

$$(2 - \alpha - b) c_1 e^{(2-\alpha)\pi k R} + (2 + \alpha - b) c_2 e^{(2+\alpha)\pi k R} = 0. \quad (14)$$

These equations depend on the two arbitrary mass parameters a and b . For generic values of these parameters the solution to the boundary conditions again leads to $c_1 = c_2 = 0$. However, if $b = 2 - \alpha$ then this implies that only $c_2 = 0$, while if $b = 2 + \alpha$ then we obtain $c_1 = 0$. Note that in principle there are three mass parameters if we introduce two parameters corresponding to each boundary. However in (11) we have chosen the mass parameters on the two boundaries to be equal and opposite. Thus a nonzero part of the general solution always survives and the zero mode solution becomes

$$\phi^{(0)}(y) \propto e^{bky} , \quad (15)$$

where $b = 2 \pm \alpha$. Assuming α to be real (which requires $a \geq -4$ in accord with the Breitenlohner-Freedman bound [13] for the stability of AdS space), the parameter b has a range $-\infty < b < \infty$. The localisation features of the zero mode follows from considering the kinetic term

$$\begin{aligned} & - \int d^5x \sqrt{-g} g^{\mu\nu} \partial_\mu \Phi^* \partial_\nu \Phi + \dots \\ & = - \int d^5x e^{2(b-1)ky} \eta^{\mu\nu} \partial_\mu \Phi^{(0)*}(x) \partial_\nu \Phi^{(0)}(x) + \dots \end{aligned} \quad (16)$$

Hence, with respect to the 5D flat metric the zero mode profile is given by

$$\tilde{\phi}^{(0)}(y) \propto e^{(b-1)ky} = e^{(1 \pm \sqrt{4+a})ky} . \quad (17)$$

We see that for $b < 1$ ($b > 1$) the zero mode is localised towards the UV (IR) brane and when $b = 1$ the zero mode is flat. Therefore using the one remaining free parameter b the scalar zero mode can be localised anywhere in the bulk.

The general solution of the Kaluza-Klein modes corresponding to $m_n \neq 0$ is given by

$$\phi^{(n)}(y) = e^{2ky} \left[c_1 J_\alpha \left(\frac{m_n}{ke^{-ky}} \right) + c_2 Y_\alpha \left(\frac{m_n}{ke^{-ky}} \right) \right] , \quad (18)$$

where $c_{1,2}$ are arbitrary constants. The Kaluza-Klein masses are determined by imposing the boundary conditions and in the limit $\pi kR \gg 1$ lead to the approximate values

$$m_n \approx \left(n + \frac{1}{2} \sqrt{4+a} - \frac{3}{4} \right) \pi k e^{-\pi kR} . \quad (19)$$

The fact that the Kaluza-Klein mass scale is associated with the IR scale ($ke^{-\pi kR}$) is consistent with the fact that the Kaluza-Klein modes are localised near the IR brane, and unlike the zero mode can not be arbitrarily localised in the bulk.

2.2.2 Fermions

Let us next consider bulk fermions in a slice of AdS₅ [14, 11]. In five dimensions a fundamental spinor representation has four components, so fermions are described by

Dirac spinors Ψ . Under the Z_2 symmetry $y \rightarrow -y$ a fermion transforms (up to a phase \pm) as

$$\Psi(-y) = \pm \gamma_5 \Psi(y) , \quad (20)$$

so that $\bar{\Psi}\Psi$ is odd. Since only invariant (or even) terms under the Z_2 symmetry can be added to the bulk Lagrangian the corresponding mass parameter for a fermion must necessarily be odd and given by

$$m_\psi = c k (\epsilon(y) - \epsilon(y - \pi R)) , \quad (21)$$

where c is a dimensionless mass parameter and $\epsilon(y) = y/|y|$. We have again chosen the mass parameter c to be equal and opposite on the two boundaries. The corresponding equation of motion for fermions resulting from the action (6) is

$$\begin{aligned} e^{ky} \eta^{\mu\nu} \gamma_\mu \partial_\nu \widehat{\Psi}_- + \partial_5 \widehat{\Psi}_+ + m_\psi \widehat{\Psi}_+ &= 0 , \\ e^{ky} \eta^{\mu\nu} \gamma_\mu \partial_\nu \widehat{\Psi}_+ - \partial_5 \widehat{\Psi}_- + m_\psi \widehat{\Psi}_- &= 0 , \end{aligned} \quad (22)$$

where $\widehat{\Psi} = e^{-2ky} \Psi$ and Ψ_\pm are the components of the Dirac spinor $\Psi = \Psi_+ + \Psi_-$ with $\Psi_\pm = \pm \gamma_5 \Psi_\pm$. Note that the equation of motion is now a first order coupled equation between the components of the Dirac spinor Ψ .

• **EXERCISE:** *Using the fact that the spin connection for the AdS_5 metric is given by*

$$\omega_M = \left(\frac{k}{2} \gamma_5 \gamma_\mu e^{-ky} , 0 \right) , \quad (23)$$

derive the bulk fermion equation of motion (22) from the action (6).

The solutions of the bulk fermion equation of motion (22) are again obtained by separating the variables

$$\Psi_\pm(x, y) = \frac{1}{\sqrt{\pi R}} \sum_n \Psi_\pm^{(n)}(x) \psi_\pm^{(n)}(y) , \quad (24)$$

where $\Psi_\pm^{(n)}$ are the Kaluza-Klein modes satisfying $\eta^{\mu\nu} \gamma_\mu \partial_\nu \Psi_\pm^{(n)} = m_n \Psi_\pm^{(n)}$. The zero mode solutions can be obtained for $m_n = 0$ and the general solution of (22) is given by

$$\widehat{\psi}_\pm^{(0)}(y) = d_\pm e^{\mp cky} , \quad (25)$$

where d_\pm are arbitrary constants. The Z_2 symmetry implies that one of the components ψ_\pm must always be odd. If $\gamma_5 = \text{diag}(1, -1)$, then (20) implies that ψ_\mp is odd and there is no corresponding zero mode for this component of Ψ . In fact this is how 4D chirality is recovered from the vectorlike 5D bulk and is the result of compactifying on the Z_2 orbifold. For the remaining zero mode the boundary condition obtained from (7) with the boundary mass term (21) is the modified Neumann condition

$$\left(\partial_5 \widehat{\psi}_\pm^{(0)} \pm c k \widehat{\psi}_\pm^{(0)} \right) \Big|_{0, \pi R} = 0 . \quad (26)$$

Thus there will always be a zero mode since the boundary condition is trivially the same as the equation of motion. For concreteness let us choose ψ_- to be odd, then the only nonvanishing zero mode component of Ψ is

$$\psi_+^{(0)}(y) \propto e^{(2-c)ky} . \quad (27)$$

Again the localisation features of this mode are obtained by considering the kinetic term

$$\begin{aligned} & - \int d^5x \sqrt{-g} g^{\mu\nu} i\bar{\Psi}\Gamma_\mu\partial_\nu\Psi + \dots \\ & = - \int d^5x e^{2(\frac{1}{2}-c)ky} \eta^{\mu\nu} i\bar{\Psi}_+^{(0)}(x)\gamma_\mu\partial_\nu\Psi_+^{(0)}(x) + \dots . \end{aligned} \quad (28)$$

Hence with respect to the 5D flat metric the fermion zero mode profile is

$$\tilde{\psi}_+^{(0)}(y) \propto e^{(\frac{1}{2}-c)ky} . \quad (29)$$

When $c > 1/2$ ($c < 1/2$) the fermion zero mode is localised towards the UV (IR) brane while the zero mode fermion is flat for $c = 1/2$. So, just like the scalar field zero mode the fermion zero mode can be localised anywhere in the 5D bulk.

The nonzero Kaluza-Klein fermion modes can be obtained by solving the coupled equations of motion for the Dirac components $\psi_\pm^{(n)}$. This leads to a pair of decoupled second order equations that can be easily solved. The expressions for the corresponding wave functions and Kaluza-Klein masses are summarised in the next subsection.

2.2.3 Summary

A similar analysis can also be done for the bulk graviton [15] and bulk gauge field [16, 17]. In these cases there are no bulk or boundary masses and the corresponding graviton zero mode is localised on the UV brane while the gauge field zero mode is flat and not localised in the 5D bulk¹. A summary of the bulk zero mode profiles is given in Table 1.

Similarly the Kaluza-Klein mode ($m_n \neq 0$) solutions can be obtained for all types of bulk fields and combined into one general expression [11]

$$f^{(n)}(y) = \frac{e^{\frac{s}{2}ky}}{N_n} \left[J_\alpha\left(\frac{m_n}{ke^{-ky}}\right) + b_\alpha Y_\alpha\left(\frac{m_n}{ke^{-ky}}\right) \right] , \quad (30)$$

for $f^{(n)} = (\phi^{(n)}, \widehat{\psi}_\pm^{(n)}, A_\mu^{(n)})$ where

$$b_\alpha = - \frac{(-r + \frac{s}{2})J_\alpha(\frac{m_n}{k}) + \frac{m_n}{k}J'_\alpha(\frac{m_n}{k})}{(-r + \frac{s}{2})Y_\alpha(\frac{m_n}{k}) + \frac{m_n}{k}Y'_\alpha(\frac{m_n}{k})} , \quad (31)$$

¹Note however that bulk and boundary masses can be introduced for the graviton [18] and gauge field [19], thereby changing their localization profile and corresponding operator dimensions. But this is beyond the applications that will be considered in these lectures.

Table 1: *The zero mode profiles of bulk fields and the corresponding CFT operator dimensions.*

Field	Profile	dim \mathcal{O}
scalar $\phi^{(0)}(y)$	$e^{(1\pm\sqrt{4+a})ky}$	$2 + \sqrt{4+a}$
fermion $\psi_+^{(0)}(y)$	$e^{(\frac{1}{2}-c)ky}$	$\frac{3}{2} + c + \frac{1}{2} $
vector $A_\mu^{(0)}(y)$	1	3
graviton $h_{\mu\nu}^{(0)}(y)$	e^{-ky}	4

and

$$N_n \simeq \frac{1}{\sqrt{\pi^2 R m_n} e^{-\pi k R}}, \quad (32)$$

with $s = (4, 1, 2)$, $r = (b, \mp c, 0)$ and $\alpha = (\sqrt{4+a}, |c \pm \frac{1}{2}|, 1)$. The graviton modes $h_{\mu\nu}^{(n)}$ are identical to the scalar modes $\phi^{(n)}$ except that $a = b = 0$. The Kaluza-Klein mass spectrum is approximately given by

$$m_n \simeq \left(n + \frac{1}{2}(\alpha - 1) \mp \frac{1}{4} \right) \pi k e^{-\pi k R}, \quad (33)$$

for even (odd) modes and $n = 1, 2, \dots$. Note that the Kaluza-Klein modes for all types of bulk fields are always localised near the IR brane. Unlike the zero mode there is no freedom to delocalise the Kaluza-Klein (nonzero) modes away from the IR brane.

3 The Standard Model in the Bulk

We can now use the freedom to localise scalar and fermion zero mode fields anywhere in the warped bulk to construct a bulk Standard Model. Recall that the hierarchy problem only affects the Higgs boson. Hence to solve the hierarchy problem the Higgs scalar field must be localised very near the TeV brane, and for simplicity we will assume that the Higgs is confined on the TeV brane (as in RS1). However we will now consider the possible effects of allowing fermions and gauge bosons to live in the warped bulk.

3.1 Yukawa couplings

One consequence of allowing fermions to be localised anywhere in the bulk, is that Yukawa coupling hierarchies are naturally generated by separating the fermions from the Higgs on the TeV brane. Each Standard Model fermion is identified with the zero mode of a corresponding 5D Dirac spinor Ψ . For example, the left-handed electron doublet e_L is identified with the zero mode of Ψ_{eL+} , which is the even component of

the 5D Dirac spinor $\Psi_{eL} = \Psi_{eL+} + \Psi_{eL-}$. The odd component Ψ_{eL-} does not have a zero mode, but at the massive level it pairs up with the massive modes of Ψ_{eL+} to form a vectorlike Dirac mass. This embedding of 4D fermions into 5D fermions is repeated for each Standard Model fermion. The Standard Model Yukawa interactions, such as $\bar{\Psi}_{eL}\Psi_{eR}H$, are then promoted to 5D interactions in the warped bulk. This gives

$$\begin{aligned} & \int d^4x \int dy \sqrt{-g} \lambda_{ij}^{(5)} [\bar{\Psi}_{iL}(x, y)\Psi_{jR}(x, y) + h.c.] H(x)\delta(y - \pi R) \\ & \equiv \int d^4x \lambda_{ij} (\bar{\Psi}_{iL+}^{(0)}(x)\Psi_{jR+}^{(0)}(x)H(x) + h.c. + \dots) , \end{aligned} \quad (34)$$

where i, j are flavour indices, $\lambda_{ij}^{(5)}$ is the (dimensionful) 5D Yukawa coupling and λ_{ij} is the (dimensionless) 4D Yukawa coupling. Given that the zero mode profile is

$$\tilde{\psi}_{iL+,R+}^{(0)}(y) \propto e^{(\frac{1}{2}-c_{iL,R})ky} , \quad (35)$$

this leads to an exponential hierarchy in the 4D Yukawa coupling [11]

$$\lambda_{ij} \simeq \lambda_{ij}^{(5)} k \sqrt{(c_{iL} - 1/2)(c_{iR} - 1/2)} e^{(1-c_{iL}-c_{jR})\pi k R} , \quad (36)$$

for $c_{iL,R} > 1/2$. Assuming $c_{iL} = c_{jR}$ for simplicity then the electron Yukawa coupling $\lambda_e \sim 10^{-6}$ is obtained for $c_e \simeq 0.64$. Instead when $c_{iL,R} < 1/2$, both fermions are localised near the IR brane giving

$$\lambda_{ij} \simeq \lambda_{ij}^{(5)} k \sqrt{(1/2 - c_{iL})(1/2 - c_{iR})} , \quad (37)$$

with no exponential suppression. Hence the top Yukawa coupling $\lambda_t \sim 1$ is obtained for $c_t \simeq -0.5$. The remaining fermion Yukawa couplings, c_f then range from $c_t < c_f < c_e$ [11, 20]. Thus, we see that for bulk mass parameters c of $\mathcal{O}(1)$ the fermion mass hierarchy is explained. The fermion mass problem is now reduced to determining the c parameters in the 5D theory. This requires a UV completion of the 5D warped bulk model with fermions, such as string theory.

Since bulk fermions naturally lead to Yukawa coupling hierarchies, the gauge bosons must also necessarily be in the bulk. From Table 1 the gauge field zero mode is flat and therefore couples with equal strength to both the UV and IR brane. Only the Higgs field is confined to the IR brane (or TeV brane). Thus the picture that emerges is a Standard Model in the warped bulk as depicted in Figure 2. The fermions are localised to varying degrees in the bulk with the electron zero mode, being the lightest fermion, furthest away from the Higgs on the TeV brane while the top, being the heaviest, is closest to the Higgs. This enables one to not only solve the hierarchy problem but also address the Yukawa coupling hierarchies.

The warped bulk can also be used to obtain naturally small neutrino masses. Various scenarios are possible. If the right (left) handed neutrino is localised near the UV (IR) brane then a tiny Dirac neutrino mass is obtained [14]. However this

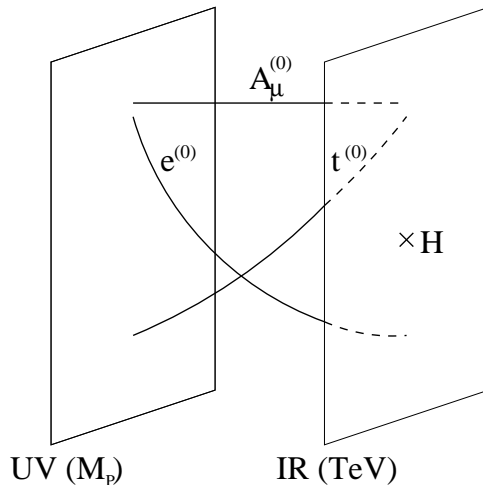


Figure 2: *The Standard Model in the warped five-dimensional bulk.*

requires that lepton number is conserved on the UV brane. Instead in the “reversed” scenario one can place the right (left) handed neutrino near the IR (UV) brane. In this case even though lepton number is violated on the UV brane, the neutrinos will still obtain naturally tiny Dirac masses [21].

3.2 Higher-dimension operators

Let us consider the following generic four-fermion operators which are relevant for proton decay and $K - \bar{K}$ mixing

$$\int d^4x \int dy \sqrt{-g} \frac{1}{M_5^3} \bar{\Psi}_i \Psi_j \bar{\Psi}_k \Psi_l \equiv \int d^4x \frac{1}{M_4^2} \bar{\Psi}_{i+}^{(0)} \Psi_{j+}^{(0)} \bar{\Psi}_{k+}^{(0)} \Psi_{l+}^{(0)} , \quad (38)$$

where the effective 4D mass scale M_4 for $1/2 \lesssim c_i \lesssim 1$ is approximately given by[11]

$$\frac{1}{M_4^2} \simeq \frac{k}{M_5^3} e^{(4-c_i-c_j-c_k-c_l)\pi k R} . \quad (39)$$

If we want the suppression scale for higher-dimension proton decay operators to be $M_4 \sim M_P$ then (39) requires $c_i \simeq 1$ assuming $k \sim M_5 \sim M_P$. Unfortunately for these values of c_i the corresponding Yukawa couplings would be too small. Nevertheless, the values of c needed to explain the Yukawa coupling hierarchies still suppresses proton decay by a mass scale larger than the TeV scale [11, 22]. Thus there is no need to impose a discrete symmetry which forbids very large higher-dimension operators.

On the other hand the suppression scale for FCNC processes only needs to be $M_4 \gtrsim 1000$ TeV. This can easily be achieved for the values of c that are needed to explain the Yukawa coupling hierarchies. In fact the FCNC constraints can be used to obtain a lower bound on the Kaluza-Klein mass scale m_{KK} . For example

Kaluza-Klein gluons can mediate $\Delta S = 2$ FCNC processes at tree level because the fermions are located at different points. In flat space with split fermions this leads to strong constraints $m_{KK} \gtrsim 25 - 300$ TeV (with the range depending on whether FCNC processes violate CP) [23]. However in warped space for $c \gtrsim 1/2$, the Kaluza-Klein gauge boson coupling to fermions is universal even though fermions are split. This is because in warped space the Kaluza-Klein gauge boson wave functions are approximately flat near the UV brane. The corresponding bound for warped dimensions is $m_{KK} \gtrsim 2 - 30$ TeV. This fact that the Kaluza-Klein gauge bosons couple universally to spatially separated fermions is akin to a GIM-like mechanism in the 5D bulk [11]. Thus, warped dimensions ameliorate the bounds on the Kaluza-Klein scale.

3.3 Higgs as a pseudo Nambu-Goldstone Boson

So far we have said very little about electroweak symmetry breaking and the Higgs mass. If the Higgs is confined to the IR (or TeV) brane then the tree-level Higgs mass parameter is naturally of order $\Lambda_{IR} = \Lambda_{UV} e^{-\pi k R}$. Since there are fermions and gauge bosons in the bulk the effects of their corresponding Kaluza-Klein modes must be sufficiently suppressed. This requires $k e^{-\pi k R} \sim \mathcal{O}(\text{TeV})$ and since in RS1 $\Lambda_{UV} \sim 10k$ we have $\Lambda_{IR} \sim \mathcal{O}(10 \text{ TeV})$. Consequently a modest amount of fine-tuning would be required to obtain a physical Higgs mass of order 100 GeV, as suggested by electroweak precision data [24]. Clearly, it is desirable to invoke a symmetry to keep the Higgs mass naturally lighter than the IR cutoff scale, such as the spontaneous breaking of a global symmetry².

Motivated by the fact that the dimensional reduction of a five-dimensional gauge boson $A_M = (A_\mu, A_5)$ contains a scalar field A_5 , one can suppose that the Higgs scalar field is part of a higher-dimensional gauge field [25]. In a slice of AdS₅ the A_5 terms in the gauge boson kinetic term of the bulk Lagrangian (6) are

$$-\frac{1}{2} \int d^4x dy e^{-2ky} ((\partial A_5)^2 - 2\eta^{\mu\nu} \partial_\mu A_5 \partial_\nu A_5) + \dots \quad (40)$$

In particular notice that the higher-dimensional gauge symmetry prevents a tree-level mass for A_5 . However the zero mode of the A_5 scalar field must be localised near the IR brane for the hierarchy problem to be solved. The solution for A_5 can be obtained by adding a gauge fixing term that cancels the mixed $A_5 A_\nu$ term [26, 27]. This gives the zero mode solution $A_5^{(0)}$ with y dependence $\propto e^{+2ky}$ and substituting back into the action leads to

$$-\frac{1}{2} \int d^4x dy e^{+2ky} (\partial_\mu A_5^{(0)}(x))^2 + \dots \quad (41)$$

²But this is not the unique possibility. As we will see later supersymmetry (instead of a global symmetry) can also be used to obtain a light Higgs mass.

Hence with respect to the flat 5D metric the massless scalar mode $A_5^{(0)}$ is indeed localised towards the IR brane and therefore can play the role of the Higgs boson.

To obtain a realistic model one assumes an $\text{SO}(5) \times \text{U}(1)_{B-L}$ gauge symmetry in the bulk for the electroweak sector [27]. On the IR brane this symmetry is spontaneously broken by boundary conditions to $\text{SO}(4) \times \text{U}(1)_{B-L}$. This leads to four Nambu-Goldstone bosons that can be identified with the Standard Model Higgs doublet. A Higgs mass is then generated because $\text{SO}(5)$ gauge symmetry is explicitly broken in the fermion sector, in particular by the top quark. At one loop this generates an effective potential and electroweak symmetry is broken dynamically via top-quark loop corrections [28]. This effect is finite and arises from the Hosotani mechanism with nonlocal operators in the bulk [29]. An unbroken $\text{SO}(3)$ custodial symmetry guarantees that the Peskin-Takeuchi parameter $T = 0$. The important point however is that radiative corrections to the Higgs mass depend on $ke^{-\pi kR}$ and not on $\Lambda_{UV}e^{-\pi kR}$. Together with the accompanying one-loop factor $\frac{1}{16\pi^2}$ this guarantees a light Higgs mass of order $m_{Higgs} \lesssim 140$ GeV. Furthermore this model can be shown to pass stringent electroweak precision tests without a significant amount of fine-tuning [30].

In summary, the hierarchy problem can be solved by placing the Standard Model in the warped bulk together with identifying the Higgs scalar field as a pseudo Nambu-Goldstone boson. This leads to a very predictive scenario for the electroweak symmetry breaking sector. Moreover the setup is radiatively stable and valid up to the Planck scale with (as we will see later) grand unification incorporated in an interesting way! This makes the 5D warped bulk a compelling alternative framework to address the hierarchy problem in a complete scenario compared to the usual 4D scenarios. But even more compelling is that this framework can be given a purely 4D holographic description in terms of a strongly coupled gauge theory as will be shown in the next section.

4 AdS/CFT and holography

Remarkably 5D warped models in a slice of AdS can be given a purely 4D description. This holographic correspondence between the 5D theory and the 4D theory originates from the AdS/CFT correspondence in string theory. In 1997 Maldacena conjectured that [31]

$$\begin{array}{ccc} \text{type IIB string theory} & \text{DUAL} & \\ \text{on AdS}_5 \times S^5 & \iff & \mathcal{N} = 4 \text{ SU(N) 4D gauge theory} \end{array} \quad (42)$$

where \mathcal{N} is the number of supersymmetry generators and

$$\frac{R_{AdS}^4}{l_s^4} = 4\pi g_{YM}^2 N, \quad (43)$$

with $R_{AdS} \equiv 1/k$, l_s is the string length and g_{YM} is the $SU(N)$ Yang-Mills gauge coupling. Qualitatively we can see that the isometry of the five-dimensional sphere S^5 is the rotation group $SO(6) \cong SU(4)$, which is the R-symmetry group of the supersymmetric gauge theory. Moreover the $\mathcal{N} = 4$ gauge theory is a conformal field theory because the isometry group of AdS_5 is precisely the conformal group in four dimensions. In particular this means that the gauge couplings do not receive quantum corrections and therefore do not run with energy.

In the warped bulk we have only considered gravity. This represents the effective low energy theory of the full string theory. In order to neglect the string corrections, so that the bulk gravity description is valid, we require that $R_{AdS} \gg l_s$. This leads to the condition that $g_{YM}^2 N \gg 1$, which means that the 4D dual CFT is strongly coupled! Thus for our purposes the correspondence takes the form of a duality in which the weakly coupled 5D gravity description is dual to a strongly coupled 4D CFT. This remarkable duality means that any geometric configuration of fields in the bulk can be given a purely 4D description in terms of a strongly coupled gauge theory. Therefore warped models provide a new way to study strongly coupled gauge theories.

While there is no rigorous mathematical proof of the AdS/CFT conjecture, it has passed many nontrivial tests and an AdS/CFT dictionary to relate the two dual descriptions can be established. For every bulk field Φ there is an associated operator \mathcal{O} of the CFT. In the AdS_5 metric (1) the boundary of AdS space is located at $y = -\infty$. The boundary value of the bulk field $\Phi(x^\mu, y = -\infty) \equiv \phi_0(x^\mu)$ acts as a source field for the CFT operator \mathcal{O} . The AdS/CFT correspondence can then be quantified in the following way [32, 33]

$$\int \mathcal{D}\phi_{CFT} e^{-S_{CFT}[\phi_{CFT}] - \int d^4x \phi_0 \mathcal{O}} = \int_{\phi_0} \mathcal{D}\phi e^{-S_{bulk}[\phi]} \equiv e^{iS_{eff}[\phi_0]} , \quad (44)$$

where S_{CFT} is the CFT action with ϕ_{CFT} generically denoting the CFT fields and S_{bulk} is the bulk 5D action. The on-shell gravity action, S_{eff} is obtained by integrating out the bulk degrees of freedom for suitably chosen IR boundary conditions. In general n -point functions can be calculated via

$$\langle \mathcal{O} \dots \mathcal{O} \rangle = \frac{\delta^n S_{eff}}{\delta \phi_0 \dots \delta \phi_0} . \quad (45)$$

In this way we see that the on-shell bulk action is the generating functional for connected Greens functions in the CFT.

So far the correspondence has been formulated purely in AdS_5 without the presence of the UV and IR branes. In particular notice from (44) that the source field ϕ_0 is a nondynamical field with no kinetic term. However since we are interested in the 4D dual of a slice of AdS_5 (and not the full AdS space) we will need the corresponding dual description in the presence of two branes.

Suppose that a UV brane is placed at $y = 0$. The $-\infty < y < 0$ part of AdS space is chopped off and the remaining $0 < y < \infty$ part is reflected about $y = 0$ with a Z_2 symmetry. The presence of the UV brane with an associated UV scale Λ_{UV} thus corresponds to an explicit breaking of the conformal invariance in the CFT at the UV scale (but only by nonrenormalisable terms) [34, 35, 36]. The fact that the CFT now has a finite UV cutoff means that the source field ϕ_0 becomes dynamical. A kinetic term for the source field will always be induced by the CFT but one can directly add an explicit kinetic term for the source field at the UV scale. Thus in the presence of a UV brane the AdS/CFT correspondence is modified to the form

$$\begin{aligned} \int \mathcal{D}\phi_0 e^{-S_{UV}[\phi_0]} \int_{\Lambda_{UV}} \mathcal{D}\phi_{CFT} e^{-S_{CFT}[\phi_{CFT}] - \int d^4x \phi_0 \mathcal{O}} \\ = \int \mathcal{D}\phi_0 e^{-S_{UV}[\phi_0]} \int_{\phi_0} \mathcal{D}\phi e^{-S_{bulk}[\phi]} , \end{aligned} \quad (46)$$

where S_{UV} is the UV Lagrangian for the source field ϕ_0 . It is understood that now the source field $\phi_0 = \Phi(x, y = 0)$. Moving away from the UV brane at $y = 0$ in the bulk corresponds in the 4D dual to running down from the UV scale to lower energy scales. Since the bulk is AdS the 4D dual gauge theory quickly becomes conformal at energies below the UV scale.

The presence of the IR brane at $y = \pi R$ corresponds to a spontaneous breaking of the conformal invariance in the CFT at the IR scale $\Lambda_{IR} = \Lambda_{UV} e^{-\pi k R}$ [34, 35]. The conformal symmetry is nonlinearly realised and particle bound states of the CFT can now appear. Formally this can be understood by noting that the (massless) radion field in RS1 is localised on the IR brane, since by sending the UV brane to the AdS boundary (at $y \rightarrow -\infty$), while keeping the IR scale fixed, formally decouples the source field and keeps the radion in the spectrum. This means that at the IR scale the CFT must contain a massless particle which is interpreted as the Nambu-Goldstone boson of spontaneously broken conformal symmetry. This so called dilaton is therefore the dual interpretation of the radion. A similar phenomenon also occurs in QCD where massless pions are the Nambu-Goldstone bosons of the spontaneously broken chiral symmetry at Λ_{QCD} . Indeed the AdS/CFT correspondence suggests that QCD may be the holographic description of a bulk (string) theory.

Thus, the dual interpretation of a slice of AdS not only contains a 4D dual CFT with a UV cutoff, but also a dynamical source field ϕ_0 with UV Lagrangian $S_{UV}[\phi_0]$. In particular note that the source field is an elementary (point-like) state up to the UV scale, while particles in the CFT sector are only effectively point-like below the IR scale but are composite above the IR scale. The situation is analogous to the (elementary) photon of QED interacting with the (composite) spin-1 mesons of QCD with cutoff scale Λ_{QCD} .

4.1 Holography of scalar fields

As a simple application of the AdS/CFT correspondence in a slice of AdS₅ we shall investigate in more detail the dual theory corresponding to a bulk scalar field Φ with boundary mass terms. The qualitative features will be very similar for other spin fields. In order to obtain the correlation functions of the dual theory we first need to compute the on-shell bulk action S_{eff} . According to (30) the bulk scalar solution is given by

$$\Phi(p, z) = \Phi(p)A^{-2}(z) \left[J_\alpha(iq) - \frac{J_{\alpha\pm 1}(iq_1)}{Y_{\alpha\pm 1}(iq_1)} Y_\alpha(iq) \right], \quad (47)$$

where $z = (e^{ky} - 1)/k$, $A(z) = (1 + kz)^{-1}$, $q = p/(kA(z))$ and $\Phi(p, z)$ is the 4D Fourier transform of $\Phi(x, z)$. The \pm refers to the two branches associated with $b = b_\pm = 2 \pm \alpha$. Substituting this solution into the bulk scalar action and imposing the IR boundary condition (12) leads to the on-shell action

$$\begin{aligned} S_{eff} &= \frac{1}{2} \int \frac{d^4 p}{(2\pi)^4} \left[A^3(z) \Phi(p, z) (\Phi'(-p, z) - b k A(z) \Phi(-p, z)) \right] \Big|_{z=z_0} \\ &= \frac{k}{2} \int \frac{d^4 p}{(2\pi)^4} F(q_0, q_1) \Phi(p) \Phi(-p), \end{aligned} \quad (48)$$

where

$$\begin{aligned} F(q_0, q_1) &= \mp i q_0 \left[J_{\nu \mp 1}(i q_0) - Y_{\nu \mp 1}(i q_0) \frac{J_\nu(i q_1)}{Y_\nu(i q_1)} \right] \\ &\quad \times \left[J_\nu(i q_0) - Y_\nu(i q_0) \frac{J_\nu(i q_1)}{Y_\nu(i q_1)} \right], \end{aligned} \quad (49)$$

and $\nu \equiv \nu_\pm = \alpha \pm 1$.

- **EXERCISE:** Verify (48) is obtained by substituting the bulk scalar solution (47) into the scalar part of the action (6).

The dual theory two-point function of the operator \mathcal{O} sourced by the bulk field Φ is contained in the self-energy $\Sigma(p)$ obtained by

$$\begin{aligned} \Sigma(p) &= \int d^4 x e^{-ip \cdot x} \frac{\delta^2 S_{eff}}{\delta(A^2(z_0)\Phi(x, z_0))\delta(A^2(z_0)\Phi(0, z_0))}, \\ &= \frac{k}{g_\phi^2} \frac{q_0(I_\nu(q_0)K_\nu(q_1) - I_\nu(q_1)K_\nu(q_0))}{I_{\nu \mp 1}(q_0)K_\nu(q_1) + I_\nu(q_1)K_{\nu \mp 1}(q_0)}, \end{aligned} \quad (50)$$

where a coefficient $1/g_\phi^2$ has been factored out in front of the scalar kinetic term in (6), so that g_ϕ is a 5D expansion parameter with $\dim[1/g_\phi^2] = 1$. The behaviour of $\Sigma(p)$ can now be studied for various momentum limits in order to obtain information about

the dual 4D theory. When $A_1 \equiv A(z_1) \rightarrow 0$ the effects of the conformal symmetry breaking (from the IR brane) are completely negligible. The leading nonanalytic piece in $\Sigma(p)$ is then interpreted as the pure CFT correlator $\langle \mathcal{O}\mathcal{O} \rangle$ that would be obtained in the string AdS/CFT correspondence with $A_0 \equiv A(z_0) \rightarrow \infty$. However in a slice of AdS the poles of $\langle \mathcal{O}\mathcal{O} \rangle$ determines the pure CFT mass spectrum with a nondynamical source field ϕ_0 . These poles are identical to the poles of $\Sigma(p)$ since $\Sigma(p)$ and $\langle \mathcal{O}\mathcal{O} \rangle$ only differ by analytic terms. Hence the poles of the correlator $\Sigma(p)$ correspond to the Kaluza-Klein spectrum of the bulk scalar fields with Dirichlet boundary conditions on the UV brane.

There are also analytic terms in $\Sigma(p)$. In the string version of the AdS/CFT correspondence these terms are subtracted away by adding appropriate counterterms. However with a finite UV cutoff (corresponding to the scale of the UV brane) these terms are now interpreted as kinetic (and higher derivative terms) of the source field ϕ_0 , so that the source becomes dynamical in the holographic dual theory. The source field can now mix with the CFT bound states and therefore the self-energy $\Sigma(p)$ must be resummed and the modified mass spectrum is obtained by inverting the whole quadratic term $S_{UV} + S_{eff}$. In the case with no UV boundary action S_{UV} this means that the zeroes of (50) are identical with the Kaluza-Klein mass spectrum (19) corresponding to (modified) Neumann conditions for the source field. In both cases (either Dirichlet or Neumann) these results are consistent with the fact that the Kaluza-Klein states are identified with the CFT bound states.

- EXERCISE: *Check that the zeroes of (50) agree with the Kaluza-Klein spectrum (19).*

At first sight it is not apparent that there are an infinite number of bound states in the 4D dual theory required to match the infinite number of Kaluza-Klein modes in the 5D theory. How is this possible in the 4D gauge theory? It has been known since the early 1970's that the two-point function in large- N QCD can be written as [37, 38]

$$\langle \mathcal{O}(p)\mathcal{O}(-p) \rangle = \sum_{n=1}^{\infty} \frac{F_n^2}{p^2 + m_n^2}, \quad (51)$$

where the matrix element for \mathcal{O} to create the n th meson with mass m_n from the vacuum is $F_n = \langle 0|\mathcal{O}|n \rangle \propto \sqrt{N}/(4\pi)$. In the large N limit the intermediate states are one-meson states and the sum must be infinite because we know that the two-point function behaves logarithmically for large p^2 . Since the 4D dual theory is a strongly-coupled SU(N) gauge theory that is conformal at large scales, it will have this same behaviour. This clearly has the same qualitative features as a Kaluza-Klein tower and therefore a dual 5D interpretation could have been posited in the 1970's!

To obtain the holographic interpretation of the bulk scalar field, recall that the scalar zero mode can be localised anywhere in the bulk with $-\infty < b < \infty$ where

$b \equiv b_{\pm} = 2 \pm \alpha$ and $-\infty < b_- < 2$ and $2 < b_+ < \infty$. Since $b_{\pm} = 1 \pm \nu_{\pm}$ we have $-1 < \nu_- < \infty$ and $1 < \nu_+ < \infty$. The ν_- branch corresponds to $b_- < 2$, while the ν_+ branch corresponds to $b_+ > 2$. Hence the $\nu_-(\nu_+)$ branch contains zero modes which are localised on the UV (IR) brane.

4.1.1 ν_- branch holography

We begin first with the ν_- branch. In the limit $A_0 \rightarrow \infty$ and $A_1 \rightarrow 0$ one obtains

$$\Sigma(p) \simeq -\frac{2k}{g_{\phi}^2} \left[\frac{1}{\nu} \left(\frac{q_0}{2} \right)^2 + \left(\frac{q_0}{2} \right)^{2\nu+2} \frac{\Gamma(-\nu)}{\Gamma(\nu+1)} + \dots \right], \quad (52)$$

where the expansion is valid for noninteger ν . The expansion for integer ν will contain logarithms. Only the leading analytic term has been written in (52). The nonanalytic term is the pure CFT contribution to the correlator $\langle \mathcal{O}\mathcal{O} \rangle$. Formally it is obtained by rescaling the fields by an amount $A_0^{\nu+1}$ and taking the limit

$$\langle \mathcal{O}\mathcal{O} \rangle = \lim_{A_0 \rightarrow \infty} (\Sigma(p) + \text{counterterms}) = \frac{1}{g_{\phi}^2} \frac{\Gamma(-\nu)}{\Gamma(\nu+1)} \frac{p^{2(\nu+1)}}{(2k)^{2\nu+1}}. \quad (53)$$

Since

$$\langle \mathcal{O}(x)\mathcal{O}(0) \rangle = \int \frac{d^4p}{(2\pi)^4} e^{ipx} \langle \mathcal{O}\mathcal{O} \rangle, \quad (54)$$

the scaling dimension of the operator \mathcal{O} is

$$\dim \mathcal{O} = 3 + \nu_- = 4 - b_- = 2 + \sqrt{4 + a}, \quad (55)$$

as shown in Table 1. If A_0 is finite then the analytic term in (52) becomes the kinetic term for the source field ϕ_0 . Placing the UV brane at $z_0 = 0$ with $A_0 = 1$ leads to the dual Lagrangian below the cutoff scale $\Lambda \sim k$

$$\mathcal{L}_{4D} = -Z_0(\partial\phi_0)^2 + \frac{\omega}{\Lambda^{\nu_-}} \phi_0 \mathcal{O} + \mathcal{L}_{CFT}, \quad (56)$$

where Z_0, ω are dimensionless couplings. This Lagrangian describes a massless dynamical source field ϕ_0 interacting with the CFT via the mixing term $\phi_0 \mathcal{O}$. This means that the mass eigenstate in the dual theory will be a mixture of the source field and CFT particle states. The coupling of the mixing term is irrelevant for $\nu_- > 0$ ($b_- < 1$), marginal if $\nu_- = 0$ ($b_- = 1$) and relevant for $\nu_- < 0$ ($b_- > 1$). This suggests the following dual interpretation of the massless bulk zero mode. When the coupling is irrelevant ($\nu_- > 0$), corresponding to a UV brane localised bulk zero mode, the mixing can be neglected at low energies, and hence to a very good approximation the bulk zero mode is dual to the massless 4D source field ϕ_0 . However for relevant ($-1 < \nu_- < 0$) or marginal couplings ($\nu_- = 0$) the mixing can no longer be neglected. In this case the bulk zero mode is no longer UV-brane localised, and

the dual interpretation of the bulk zero mode is a part elementary, part composite mixture of the source field with massive CFT particle states.

The first analytic term in (52) can be matched to the wavefunction constant giving $Z_0 = 1/(2\nu g_\phi^2 k)$. However at low energies the couplings in \mathcal{L} will change. The low energy limit $q_1 \ll 1$ for $\Sigma(p)$ (and noninteger ν) leads to

$$\Sigma(p)_{IR} \simeq -\frac{2k}{g_\phi^2} \left[(1 - A_1^{2\nu_-}) \left(\frac{q_0}{2}\right)^2 \frac{1}{\nu} + \dots \right], \quad (57)$$

where $A_1 = e^{-\pi k R}$. Notice that there is no nonanalytic term because the massive CFT modes have decoupled. The analytic term has now also received a contribution from integrating out the massive CFT states. Note that when $\nu_- > 0$ the A_1 contribution to Z_0 is negligible and the kinetic term has the correct sign. On the other hand for relevant couplings the A_1 term now dominates the Z_0 term. The kinetic term still has the correct sign because $\nu_- < 0$.

• **EXERCISE:** Show that for a marginal coupling ($\nu_- = 0$) the coefficient of the kinetic term is logarithmic and has the correct sign.

The features of the couplings in (56) at low energies can be neatly encoded into a renormalisation group equation. If we define a dimensionless running coupling $\xi(\mu) = \omega/\sqrt{Z(\mu)}(\mu/\Lambda)^\gamma$, which represents the mixing between the CFT and source sector with a canonically normalised kinetic term, then it will satisfy the renormalisation group equation [39]

$$\mu \frac{d\xi}{d\mu} = \gamma \xi + \eta \frac{N}{16\pi^2} \xi^3 + \dots, \quad (58)$$

where η is a constant and we have replaced $1/(g_\phi^2 k) = N/(16\pi^2)$. The first term arises from the scaling of the coupling of the mixing term $\phi_0 \mathcal{O}$ (i.e. $\gamma = \nu_-$), and the second term arises from the CFT contribution to the wavefunction constant Z_0 (i.e. the second term in (52)). The solution of the renormalisation group equation for an initial condition $\xi(M)$ at the scale $M \sim \Lambda$ is

$$\xi(\mu) = \left(\frac{\mu}{M}\right)^\gamma \left\{ \frac{1}{\xi^2(M)} + \eta \frac{N}{16\pi^2 \gamma} \left[1 - \left(\frac{\mu}{M}\right)^{2\gamma} \right] \right\}^{-1/2}. \quad (59)$$

When $\gamma < 0$, the constant $\eta > 0$ and the renormalisation group equation (58) has a fixed point at $\xi_* \sim 4\pi\sqrt{-\gamma/(\eta N)}$, which does not depend on the initial value $\xi(M)$. This occurs when $-1 < \nu_- < 0$ and therefore since ξ_* is nonnegligible the mixing between the source and the CFT cannot be neglected.

In the opposite limit, $\gamma > 0$, the solution (59) for $M \sim \Lambda$ becomes $\xi(\mu) \sim 4\pi\sqrt{\gamma/N}(\mu/M)^\gamma$, where the solution (59) has been matched to the low energy value $Z(ke^{-\pi k R}) = 1/(2\gamma g_\phi^2 k)(1 - e^{-2\gamma\pi k R})$ arising from (57) (with $\gamma = \nu_-$). Thus when $\nu_- > 0$ the mixing between the source and CFT sector quickly becomes irrelevant at low energies.

4.1.2 ν_+ branch holography

Consider the case $\nu = \nu_+ > 1$. In the limit $A_0 \rightarrow \infty$ and $A_1 \rightarrow 0$ we obtain for noninteger ν

$$\Sigma(p) \simeq -\frac{2k}{g_\phi^2} \left[(\nu - 1) + \left(\frac{q_0}{2}\right)^2 \frac{1}{(\nu - 2)} + \left(\frac{q_0}{2}\right)^{2\nu-2} \frac{\Gamma(2 - \nu)}{\Gamma(\nu - 1)} \right],$$

where only the leading analytic terms have been written. The nonanalytic term is again the pure CFT contribution to the correlator $\langle \mathcal{O}\mathcal{O} \rangle$ and gives rise to the scaling dimension

$$\dim \mathcal{O} = 1 + \nu_+ = b_+ = 2 + \sqrt{4 + a}. \quad (60)$$

This agrees with the result for the ν_- branch. At low energies $q_1 \ll 1$ one obtains

$$\Sigma(p)_{IR} \simeq -\frac{2k}{g_\phi^2} \left[(\nu - 1) + \left(\frac{q_0}{2}\right)^2 \frac{1}{(\nu - 2)} - \nu(\nu - 1)^2 \frac{A_1^{2\nu}}{A_0^{2\nu}} \left(\frac{2}{q_0}\right)^2 \right], \quad (61)$$

where the large- A_0 limit was taken first. We now see that at low energies the nonanalytic term has a pole at $p^2 = 0$ with the correlator

$$\langle \mathcal{O}\mathcal{O} \rangle = \frac{8k^3}{g_\phi^2} \nu_+ (\nu_+ - 1)^2 e^{-2\nu_+ \pi k R} \frac{1}{p^2}, \quad (62)$$

where $A_0 = 1$ and $A_1 = e^{-\pi k R}$. This pole indicates that the CFT has a massless scalar mode at low energies! What about the massless source field? As can be seen from (60) and (61) the leading analytic piece is a constant term which corresponds to a mass term for the source field [36]. This leads to the dual Lagrangian below the cutoff scale $\Lambda \sim k$

$$\mathcal{L}_{4D} = -\tilde{Z}_0 (\partial\phi_0)^2 + m_0^2 \phi_0^2 + \frac{\chi}{\Lambda^{\nu_+-2}} \phi_0 \mathcal{O} + \mathcal{L}_{CFT}, \quad (63)$$

where \tilde{Z}_0, χ are dimensionless parameters and m_0 is a mass parameter of order the curvature scale k . The bare parameters \tilde{Z}_0 and m_0 can be determined from (60). Thus, the holographic interpretation is perfectly consistent. There is a massless bound state in the CFT and the source field ϕ_0 receives a mass of order the curvature scale and decouples. In the bulk the zero mode is always localised towards the IR brane. Indeed for $\nu_+ > 2$ the coupling between the source field and the CFT is irrelevant and therefore the mixing from the source sector is negligible. Hence to a good approximation the mass eigenstate is predominantly the massless CFT bound state. When $1 \leq \nu_+ \leq 2$ the mixing can no longer be neglected and the mass eigenstate is again part elementary and part composite.

4.2 Dual 4D description of the Standard Model in the Bulk

The qualitative features of the scalar field holographic picture can now be used to give the 4D dual description of the Standard Model gauge fields and matter in the 5D bulk. In general for every bulk zero mode field there is a corresponding massless eigenstate in the dual 4D theory that is a mixture of (elementary) source and (composite) CFT fields. If the bulk zero mode is localised towards the UV brane, then in the dual theory the massless eigenstate is predominantly the source field. For example, as in RS1 the graviton zero mode is localised towards the UV brane so that in the dual theory the massless eigenstate is mostly composed of the graviton source field.

On the other hand the dual interpretation of a bulk zero mode localised towards the TeV brane is a state that is predominantly a CFT bound state. In this instance the source field obtains a mass of order the curvature scale and decouples from the low energy theory. Depending on the degree of localisation the bound state mixes with the massive source field. Only in the limit where the mode is completely localised on the TeV brane is the dual eigenmode a pure CFT bound state. Since the Higgs is confined to the IR brane, the Higgs field is interpreted as a pure bound state of the CFT in the dual theory. In this way we see that the RS1 solution to the hierarchy problem is holographically identical to the way 4D composite models solve the problem with a low-scale cutoff. The Higgs mass is quadratically divergent but only sensitive to the strong coupling scale Λ_{IR} which is hierarchically smaller than Λ_{UV} . To obtain a large top Yukawa coupling the top quark was localised near the IR brane, so in the dual theory the top quark is also (predominantly) a composite of the CFT. The rest of the light fermions are localised to varying degrees towards the UV brane, and therefore these states will be mostly elementary particle states in the dual theory. The detailed holographic picture of bulk fermions can be found in Ref. [39].

If zero modes are not localised in the bulk then the corresponding 4D dual massless eigenstate is partly composed of the elementary source field and the composite CFT state. In particular for bulk gauge fields whose zero modes are not localised in the bulk, the 4D dual massless eigenstate will be part composite and part elementary. Finally note that local symmetries in the bulk, such as gauge symmetries or general coordinate invariance, also have a 4D dual interpretation. The holographic dual of a local symmetry group G in the bulk is a CFT in which a subgroup G of the global symmetry group of the CFT is weakly gauged by the source gauge field [34, 40].

Thus, in summary the Standard Model in the warped 5D bulk is dual to a 4D hybrid theory with a mixture of elementary and composite states.

4.2.1 Yukawa couplings

The Yukawa coupling hierarchies can also be understood from the dual 4D theory. Consider first an electron (or light fermion) with $c > 1/2$. In the dual 4D theory the electron is predominantly an elementary field. The dual 4D Lagrangian is obtained

from analysing $\Sigma(p)$ for fermions, where the CFT induces a kinetic term for the source field $\psi_L^{(0)}$. It is given by [39]

$$\mathcal{L}_{4D} = \mathcal{L}_{CFT} + Z_0 \bar{\psi}_L^{(0)} i \gamma^\mu \partial_\mu \psi_L^{(0)} + \frac{\omega}{\Lambda^{|c+1/2|-1}} (\bar{\psi}_L^{(0)} \mathcal{O}_R + h.c.) , \quad (64)$$

where Z_0, ω are dimensionless couplings and $\dim \mathcal{O}_R = 3/2 + |c + 1/2|$. The source field $\psi_L^{(0)}$ pertains to the left-handed electron e_L and a similar Lagrangian is written for the right-handed electron e_R . At energy scales $\mu < k$ we have a renormalisation group equation like (58) for the mixing parameter ξ but with $\gamma = |c + 1/2| - 1$. Since $c > 1/2$ the first term in (58) dominates and the coupling ξ decreases in the IR. In particular at the TeV scale ($ke^{-\pi kR}$) the solution (59) gives

$$\xi(\text{TeV}) \sim \sqrt{c - \frac{1}{2}} \frac{4\pi}{\sqrt{N}} \left(\frac{ke^{-\pi kR}}{k} \right)^{c-\frac{1}{2}} = \sqrt{c - \frac{1}{2}} \frac{4\pi}{\sqrt{N}} e^{-(c-\frac{1}{2})\pi kR} . \quad (65)$$

The actual physical Yukawa coupling λ follows from the three-point vertex between the physical states. Since both e_L and e_R are predominantly elementary they can only couple to the composite Higgs via the mixing term in (64). This is depicted in Fig. 3. In a large- N gauge theory the matrix element $\langle 0 | \mathcal{O}_{L,R} | \Psi_{L,R} \rangle \sim \sqrt{N}/(4\pi)$, and the vertex between three composite states $\Gamma_3 \sim 4\pi/\sqrt{N}$ [38]. Thus if each of the elementary fields e_L and e_R mixes in the same way with the CFT so that $c_{eL} = c_{eR} \equiv c$ then

$$\lambda \propto \langle 0 | \mathcal{O}_{L,R} | \Psi_{L,R} \rangle^2 \Gamma_3 \xi^2(\text{TeV}) = \frac{4\pi}{\sqrt{N}} (c - 1/2) e^{-2(c-\frac{1}{2})\pi kR} . \quad (66)$$

This agrees precisely with the bulk calculation (36) where $\lambda_{ij}^{(5)} k \sim 4\pi/\sqrt{N}$.

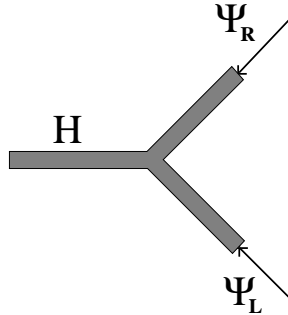


Figure 3: *The three-point Yukawa coupling vertex in the dual theory when the fermions are predominantly elementary source fields.*

- EXERCISE: *Show that when $|c| < 1/2$ the coupling of the mixing term in (64) is relevant and the renormalisation group equation (58) has a fixed point. Compare this with the bulk calculation following (36).*

Similarly we can also obtain the Yukawa coupling for the top quark with $c \lesssim -1/2$ in the dual theory. With this value of c the top quark is mostly a CFT bound state in the dual theory and we can neglect the mixing coupling with the CFT. As in the scalar field example this follows from the fact that the two point function $\langle \mathcal{O}_R \bar{\mathcal{O}}_R \rangle$ now has a massless pole. The CFT will again generate a mass term for the massless source field, so that the only massless state in the dual theory is the CFT bound state. The dual Lagrangian is given by [39]

$$\begin{aligned} \mathcal{L}_{AD} = & \mathcal{L}_{CFT} + Z_0 \bar{\psi}_L^{(0)} i \gamma^\mu \partial_\mu \psi_L^{(0)} + \tilde{Z}_0 \bar{\chi}_R i \gamma^\mu \partial_\mu \chi_R \\ & + d k (\bar{\chi}_R \psi_L^{(0)} + h.c.) + \frac{\omega}{\Lambda^{|c+\frac{1}{2}|-1}} (\bar{\psi}_L^{(0)} \mathcal{O}_R + h.c.) , \end{aligned} \quad (67)$$

where $Z_0, \tilde{Z}_0, d, \omega$ are dimensionless constants. The fermion $\psi_L^{(0)}$ pertains to t_L and a similar Lagrangian is written for t_R . Just as in the scalar case this dual Lagrangian is inferred from the behaviour of $\Sigma(p)$ for fermions. The CFT again induces a kinetic term for the source field $\psi_L^{(0)}$ but also generates a Dirac mass term of order the curvature scale k with a new elementary degree of freedom χ_R . Hence the elementary source field decouples from the low energy spectrum and the mixing term is no longer relevant for the Yukawa coupling. Instead the physical Yukawa coupling will arise from a vertex amongst three composite states so that $\lambda_t \sim \Gamma_3 \sim 4\pi/\sqrt{N} \sim \lambda^{(5)} k$, and consequently there is no exponential suppression in the Yukawa coupling. This is again consistent with the bulk calculation.

4.2.2 Minimal Composite Higgs model

Similarly the Higgs as a pseudo Nambu-Goldstone boson scenario has a 4D dual interpretation [28]. Since A_5 is localised near the TeV-brane the Higgs boson is a composite state in the dual theory. The bulk SO(5) gauge symmetry is interpreted as an SO(5) *global* symmetry of the CFT. This global symmetry is then spontaneously broken down to SO(4) at the IR scale by the (unknown) strong dynamics of the CFT. This leads to a Nambu-Goldstone boson transforming as a **4** of SO(4), which is a real bidoublet of $SU(2)_L \times SU(2)_R$.

To break electroweak symmetry, an effective Higgs potential is generated at one loop by explicitly breaking the SO(5) symmetry in the elementary (fermion) sector and transmitting it to the CFT. The top quark plays the major role in breaking this symmetry. However the top quark must also be localised near the IR brane to obtain a large overlap with the Higgs field and therefore a large top mass. Moreover to prevent large deviations from the Standard Model prediction for $Z \rightarrow \bar{b}_L b_L$ the left-handed top quark must be localised towards the UV brane ($c_{tL} \lesssim 1/2$) [41, 28]. Thus to obtain a large top mass the right-handed top quark must be localised near the IR brane. This specific localisation is compatible with electroweak symmetry breaking where the Higgs field develops a vacuum expectation value and breaks SO(4) down to

the custodial group $SO(3)$. Hence in the dual theory the physical right-handed (left-handed) top quark is mostly composite (elementary) and the custodial symmetry prevents large contributions to the T parameter.

In summary this 4D composite Higgs model is very predictive, with minimal particle content and is consistent with electroweak precision tests [30]. This hybrid 4D theory with elementary and composite states successfully addresses the hierarchy problem, fermion masses and flavour problems in a complete framework.

5 Supersymmetric Models in Warped Space

Supersymmetry elegantly solves the hierarchy problem because quadratic divergences to the Higgs mass are automatically cancelled thereby stabilising the electroweak sector. However this success must be tempered with the fact that supersymmetry has to be broken in nature. In order to avoid reintroducing a fine-tuning in the Higgs mass, the soft mass scale cannot be much larger than the TeV scale. Hence one needs an explanation for why the supersymmetry breaking scale is low. Since in warped space hierarchies are easily generated, the warp factor can be used to explain the scale of supersymmetry breaking, instead of the scale of electroweak breaking. This is one motivation for studying supersymmetric models in warped space. Thus, new possibilities open up for supersymmetric model building, and in particular for the supersymmetry-breaking sector. Moreover by the AdS/CFT correspondence these new scenarios have an interesting blend of supersymmetry and compositeness that lead to phenomenological consequences at the LHC.

A second motivation arises from the fact that electroweak precision data favours a light (compared to the TeV scale) Higgs boson mass [24]. As noted earlier the Higgs boson mass in a generic warped model without any symmetry is near the IR cutoff (or from the 4D dual perspective the Higgs mass is near the compositeness scale). Introducing supersymmetry provides a simple reason for why the Higgs boson mass is light and below the IR cutoff of the theory.

5.1 Supersymmetry in a Slice of AdS

It is straightforward to incorporate supersymmetry in a slice of AdS [11, 42]. The amount of supersymmetry allowed in five dimensions is determined by the dimension of the spinor representations. In five dimensions only Dirac fermions are allowed by the Lorentz algebra, so that there are eight supercharges which corresponds from the 4D point of view to an $\mathcal{N} = 2$ supersymmetry. This means that all bulk fields are in $\mathcal{N} = 2$ representations. At the massless level only half of the supercharges remain and the orbifold breaks the bulk supersymmetry to an $\mathcal{N} = 1$ supersymmetry.

Consider an $\mathcal{N} = 1$ (massless) chiral multiplet $(\phi^{(0)}, \psi^{(0)})$ in the bulk. We have seen that the zero mode bulk profiles of $\phi^{(0)}$ and $\psi^{(0)}$ are parametrised by their bulk

mass parameters a and c , respectively. Since supersymmetry treats the scalar and fermion components equally, the bulk profiles of the component fields must be the same. It is clear that in general this is not the case except when $1 \pm \sqrt{4+a} = 1/2 - c$, as follows from the exponent of the zero mode profiles in Table 1. This leads to the condition that

$$a = c^2 + c - 15/4 , \quad (68)$$

and the one remaining mass parameter c determines the profile of the chiral multiplet to be

$$\begin{pmatrix} \phi^{(0)} \\ \psi^{(0)} \end{pmatrix} \propto e^{(\frac{1}{2}-c)ky} . \quad (69)$$

Thus for $c > 1/2$ ($c < 1/2$) the chiral supermultiplet is localised towards the UV (IR) brane. It can be shown that the scalar boundary mass, that was tuned to be $b = 2 \pm \alpha$, follows from the invariance under a supersymmetry transformation [11] when (68) is satisfied.

Similarly a gauge boson with bulk profile $A_\mu^{(0)}(y) \propto 1$ and a gaugino with bulk profile $\lambda^{(0)}(y) \propto e^{(\frac{1}{2}-c_\lambda)ky}$ can be combined into an $\mathcal{N} = 1$ vector multiplet only for $c_\lambda = 1/2$. Of course this means that the gaugino zero-mode profile is flat like the gauge boson. At the massive level the on-shell field content of an $\mathcal{N} = 2$ vector multiplet is (A_M, λ_i, Σ) where λ_i is a symplectic-Majorana spinor (with $i = 1, 2$) and Σ is a real scalar in the adjoint representation of the gauge group. Invariance under supersymmetry transformations requires that Σ have bulk and boundary mass terms with $a = -4$ and $b = 2$, respectively. So, if Σ is even under the orbifold symmetry then these values will ensure a scalar zero mode.

Finally a graviton with bulk profile $h_{\mu\nu}^{(0)}(y) \propto e^{-ky}$ and a gravitino with bulk profile $\psi_\mu^{(0)}(y) \propto e^{(\frac{1}{2}-c_\psi)ky}$ can be combined into an $\mathcal{N} = 1$ gravity multiplet only for $c_\psi = 3/2$. In this case the gravitino zero-mode profile is localised on the UV brane.

5.2 The Warped MSSM

In the warped MSSM the warp factor is used to naturally generate TeV scale soft masses [43]. The UV (IR) scale is identified with the Planck (TeV) scale. The IR brane is associated with the scale of supersymmetry breaking, while the bulk and UV brane are supersymmetric. At the massless level the particle content is identical to the MSSM. The matter and Higgs superfields are assumed to be confined on the UV brane. This naturally ensures that all higher-dimension operators associated with proton decay and FCNC processes are sufficiently suppressed. In the bulk there is only gravity and the Standard Model gauge fields. These are contained in an $\mathcal{N} = 1$ gravity multiplet and vector multiplet, respectively.

Supersymmetry is broken by choosing different IR brane boundary conditions between the bosonic and fermionic components of the bulk superfields. On the boundaries the condition (20) defines a chirality since $(1 \mp \gamma_5)\Psi(y^*) = 0$ where $y^* = 0$

or πR . If opposite chiralities are chosen on the two boundaries then this leads to antiperiodic conditions for the fermions, namely

$$\Psi(y + 2\pi R) = -\Psi(y) . \quad (70)$$

If the gauginos in the bulk are assumed to have opposite chiralities on the two boundaries then supersymmetry will be broken because the gauge bosons obey periodic boundary conditions. The gaugino zero mode is no longer massless and receives a mass

$$m_\lambda \simeq \sqrt{\frac{2}{\pi k R}} k e^{-\pi k R} . \quad (71)$$

Since the theory has a $U(1)_R$ symmetry this is actually a Dirac mass where the gaugino zero mode pairs up with a Kaluza-Klein mode [43]. The Kaluza-Klein mass spectrum of the gauginos also shifts relative to that of the gauge bosons by an amount $-\frac{1}{4}\pi k e^{-\pi k R}$. Similarly for the gravity multiplet the gravitino is assumed to have antiperiodic boundary conditions, while the graviton has periodic boundary conditions. The gravitino zero mode then receives a mass

$$m_{3/2} \simeq \sqrt{8} k e^{-2\pi k R} , \quad (72)$$

while the Kaluza-Klein modes are again shifted by an amount similar to that of the vector multiplet.

• **EXERCISE:** *Using the expressions (30) and (31), check that imposing opposite chiralities on the two boundaries for the gaugino and gravitino leads to the zero mode masses (71) and (72).*

If $ke^{-\pi k R} = \text{TeV}$ then the gaugino mass (71) is $m_\lambda \simeq 0.24 \text{ TeV}$ while the gravitino mass (72) is $m_{3/2} \simeq 3 \times 10^{-3} \text{ eV}$. Even though both the gaugino and gravitino are bulk fields the difference in their supersymmetry breaking masses follows from their coupling to the IR brane, which is where supersymmetry is broken. The gaugino is not localised in the bulk and couples to the IR brane with an $\mathcal{O}(1)$ coupling. Hence it receives a TeV scale mass. On the other hand the gravitino is localised on the UV brane and its coupling to the TeV brane is exponentially suppressed. This explains why the gravitino mass is much smaller than the gaugino mass.

The scalars on the UV brane will obtain a supersymmetry breaking mass at one loop via gauge interactions with the bulk vector multiplets. The gravity interactions with the gravity multiplet are negligible. A one loop calculation leads to the soft mass spectrum

$$\tilde{m}_j^2 \propto \frac{\alpha_i}{4\pi} (\text{TeV})^2 , \quad (73)$$

where $\alpha_i = g_i^2/(4\pi)$ are individual gauge contributions corresponding to the particular gauge quantum numbers of the particle state. The exact expressions are given in

Ref. [43]. Unlike loop corrections to the usual 4D supersymmetric soft masses, the masses in (73) are finite. Normally UV divergences in a two-point function arise when the two spacetime points coincide. But the spacetime points in the 5D loop diagram can never coincide, because the two branes are assumed to be a fixed distance apart, and therefore the 5D one-loop calculation leads to a finite result (see Figure 4). This

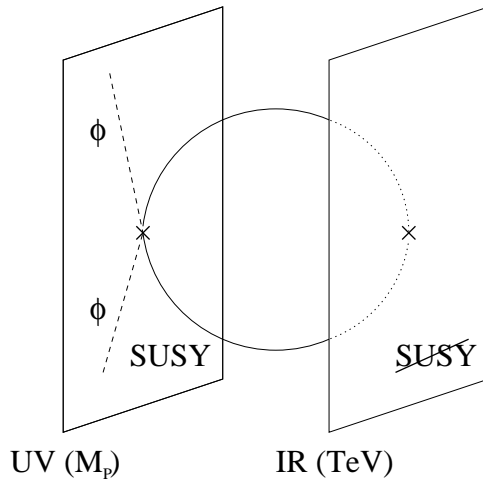


Figure 4: *The transmission of supersymmetry breaking in the warped MSSM to UV-brane localised matter fields via bulk gauge interactions which couple directly to the IR brane.*

is similar to the cancellation of divergences in the Casimir effect [44]. Since the contribution to the scalar masses is due to gauge interactions the scalar masses are naturally flavour diagonal. This means that the right-handed slepton is the lightest scalar particle since it has the smallest gauge coupling dependence. The lightest supersymmetric particle will be the superlight gravitino.

5.2.1 The dual 4D interpretation

We can use the AdS/CFT dictionary to obtain the dual 4D interpretation of the warped MSSM. Clearly the matter and Higgs fields confined to the UV brane are elementary fields external to the CFT. This is also true for the zero modes of the gravity multiplet since it is localised towards the UV brane. However, the bulk gauge field zero modes are partly composite since they are not localised. The Kaluza-Klein states, which are bound states of the CFT and localised near the IR brane, do not respect supersymmetry. Therefore at the TeV scale not only is conformal symmetry broken by the CFT but also supersymmetry. This requires some (unknown) nontrivial IR dynamics of the CFT, but the point is that supersymmetry is dynamically broken. Since the CFT is charged under the Standard Model gauge group, the gauginos (and gravitinos) will receive a tree-level supersymmetry breaking mass, while the squarks and sleptons will receive their soft mass at one loop. In some sense this model is very

similar to 4D gauge-mediated models except that there is no messenger sector since the CFT, responsible for supersymmetry breaking, is charged under the Standard Model gauge group.

In particular the bulk gaugino mass formula (71) can be understood in the dual theory. Since the gaugino mass is of the Dirac type the gaugino (source) field must marry a fermion bound state to become massive. This occurs from the mixing term $\mathcal{L} = \omega \lambda \mathcal{O}_\psi$. Since $c_\lambda = 1/2$, we have from Table 1 that $\dim \mathcal{O}_\psi = 5/2$ and therefore ω is dimensionless. This means that the mixing term coupling runs logarithmically so that at low energies the solution of (58) is

$$\xi^2(\mu) \sim \frac{16\pi^2}{N \log \frac{k}{\mu}}. \quad (74)$$

Thus at $\mu = ke^{-\pi k R}$ we obtain the correct factor in (71) since the Dirac mass $m_\lambda \propto \xi \langle 0 | \mathcal{O}_\psi | \Psi \rangle$, where in the large- N limit the matrix element for \mathcal{O}_ψ to create a bound state fermion is $\langle 0 | \mathcal{O}_\psi | \Psi \rangle \sim \sqrt{N}/(4\pi)$ [38].

Thus, in summary we have the dual picture

$$\begin{array}{ccc} \text{5D warped} & \text{DUAL} & \text{4D MSSM} \oplus \text{gravity} \\ \text{MSSM} & \longleftrightarrow & \oplus \text{strongly coupled 4D CFT} \end{array} \quad (75)$$

The warped MSSM is a very economical model of dynamical supersymmetry breaking in which the soft mass spectrum is calculable and finite, and unlike the usual 4D gauge-mediated models does not require a messenger sector. The soft mass TeV scale is naturally explained and the scalar masses are flavour diagonal. In addition, as we will show later, gauge coupling unification occurs with logarithmic running [45] arising primarily from the elementary (supersymmetric) sector as in the usual 4D MSSM[46].

5.3 The Partly Supersymmetric Standard Model

Besides solving the hierarchy problem the supersymmetric standard model has two added bonuses. First, it successfully predicts gauge coupling unification and second, it provides a suitable dark matter candidate. Generically, however, there are FCNC and CP violation problems arising from the soft mass Lagrangian, as well as the gravitino and moduli problems in cosmology [47]. These problems stem from the fact that the soft masses are of order the TeV scale, as required for a natural solution to the hierarchy problem. Of course clever mechanisms exist that avoid these problems but perhaps the simplest solution would be to have all scalar masses at the Planck scale while still naturally solving the hierarchy problem. In the partly supersymmetric standard model [48] this is precisely what happens while still preserving the successes of the MSSM.

In 5D warped space the setup of the model is as follows. Supersymmetry is assumed to be broken on the UV brane while it is preserved in the bulk and the IR

brane. The vector, matter, and gravity superfields are in the bulk while the Higgs superfield is confined to the IR brane. On the UV brane the supersymmetry breaking can be parametrised by a spurion field $\eta = \theta^2 F$, where $F \sim M_P^2$. In the gauge sector we can add the following UV brane term

$$\int d^2\theta \frac{\eta}{M_P^2} \frac{1}{g_5^2} W^\alpha W_\alpha \delta(y) + h.c. \quad (76)$$

This term leads to a gaugino mass for the zero mode $m_\lambda \sim M_P$, so that the gaugino decouples from the low energy spectrum. The gravitino also receives a Planck scale mass via a UV brane coupling and decouples from the low energy spectrum [49]. Similarly a supersymmetry breaking mass term for the squarks and sleptons can be added to the UV brane

$$\int d^4\theta \frac{\eta^\dagger \eta}{M_P^4} k S^\dagger S \delta(y) , \quad (77)$$

where S denotes a squark or slepton superfield. This leads to a soft scalar mass $\tilde{m} \sim M_P$, so that the squark and slepton zero modes also decouple from the low energy spectrum.

The Higgs sector is different because the Higgs lives on the IR brane and there is no direct coupling to the UV brane. Hence, at tree-level the Higgs mass is zero, but a (finite) soft Higgs mass will be induced at one loop via the gauge interactions in the bulk of order

$$m_H^2 \sim \frac{\alpha}{4\pi} (k e^{-\pi k R})^2 \ll M_P^2 . \quad (78)$$

As noted earlier the finiteness is due to the fact that the two 5D spacetime points on the UV and IR branes can never coincide (see Figure 5). Thus, we see that because of the warp factor the induced Higgs soft mass is much smaller than the scale of supersymmetry breaking at the Planck scale. So while at the massless level the gauginos, squarks and sleptons have received Planck scale masses, the Higgs sector remains (approximately) supersymmetric. In summary at the massless level the particle spectrum consists of the Standard Model gauge fields and matter (quarks and leptons) plus a Higgs scalar and Higgsino. This is why the model is referred to as *partly* supersymmetric.

At the massive level the Kaluza-Klein modes are also approximately supersymmetric. This is because they are localised towards the IR brane and have a small coupling to the UV brane. So the Planck scale supersymmetry breaking translates into an order TeV scale splitting between the fermionic and bosonic components of the Kaluza-Klein superfields.

Given that there are no gauginos, squarks or sleptons in the low energy spectrum it may seem puzzling how the quadratic divergences cancel in this model. Normally in the supersymmetric standard model the quadratic divergences in the Higgs mass are cancelled by a superpartner contribution of the opposite sign. However in the partly supersymmetric standard model there are no superpartners at the massless level.

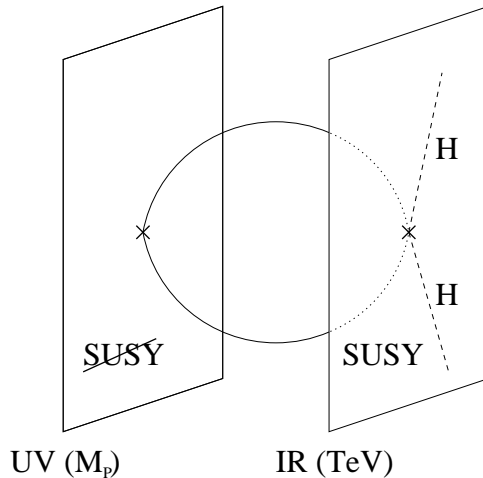


Figure 5: *The transmission of supersymmetry breaking in the partly supersymmetric standard model to the supersymmetric Higgs sector via bulk gauge interactions which couple directly to the UV brane.*

Instead what happens is that the difference between the Kaluza-Klein fermions and bosons sums up to cancel the zero mode quadratic divergence. Thus the Kaluza-Klein tower is responsible for keeping the Higgs mass natural even though supersymmetry is broken at the Planck scale.

5.3.1 Higgs sector possibilities

The motivation for making the Higgs sector supersymmetric is that the Higgs mass is induced at loop level and therefore the Higgs mass is naturally suppressed below the IR cutoff. In addition the supersymmetric partner of the Higgs, the Higgsino, provides a suitable dark matter candidate [48, 50]. However since the Higgsino is a fermion, gauge anomalies could be generated and these must be cancelled. This leads us to consider the following three possibilities:

(i) Two Higgs doublets: As in the MSSM we can introduce two Higgs doublet superfields H_1 and H_2 , so that the gauge anomaly from the two Higgsinos cancel amongst themselves. In this scenario we can add the following superpotential on the IR brane

$$\int d^2\theta (y_d H_1 Q d + y_u H_2 Q u + y_e H_1 L e + \mu H_1 H_2) . \quad (79)$$

Thus the quarks and leptons receive their masses in the usual way. In addition the μ term in (79) is naturally of order the TeV scale so that there is no μ problem. The IR brane is approximately supersymmetric and the supersymmetric mass μ has a natural TeV value. This is unlike the MSSM where the natural scale of μ is M_P and consequently a problem for phenomenology.

(ii) One Higgs doublet: At first this possibility seems to be ruled since one massless Higgsino gives rise to a gauge anomaly. However starting with a bulk Higgs $\mathcal{N} = 2$ hypermultiplet $H = (H_1, H_2)$ with bulk mass parameter $c_H = 1/2$ that consists of two $\mathcal{N} = 1$ chiral multiplets $H_{1,2}$ we can generate a Higgsino Dirac mass and only one Higgs scalar doublet in the low energy spectrum. The trick is to use mixed boundary conditions where H_1 has Neumann (Dirichlet) boundary conditions on the UV (IR) brane and vice versa for H_2 . This leads to a μ term

$$\mu \simeq \sqrt{\frac{2}{\pi k R}} k e^{-\pi k R} , \quad (80)$$

which is similar to the gaugino mass term (71) obtained in the warped MSSM. In this case the μ -term is naturally suppressed below TeV scale by the factor $1/\sqrt{\pi k R}$. Only one Higgs scalar remains in the low energy spectrum because the twisted boundary conditions localise one Higgs scalar doublet towards the UV brane where it obtains a Planck scale mass, and the other Higgs scalar is localised towards the IR brane where it obtains a mass squared μ^2 .

(iii) Zero Higgs doublet–Higgs as a Slepton: No anomalies will occur if the Higgs is considered to be the superpartner of the tau (or other lepton). This idea is not new and dates back to the early days of supersymmetry [51]. The major obstacle in implementing this possibility in the MSSM is that the gauginos induce an effective operator $\frac{g^2}{m_\lambda} \nu \nu h h$ that leads to neutrino masses of order 10 GeV which are experimentally ruled out. However in the partly supersymmetric model $m_\lambda \sim M_P$ and neutrinos masses are typically of order 10^{-5} eV. This is phenomenologically acceptable and at least makes this a viable possibility. However the stumbling block is to generate a realistic spectrum of fermion masses without introducing abnormally large coefficients [48].

5.3.2 Electroweak symmetry breaking

In this model electroweak symmetry breaking can be studied and calculated using the 5D bulk propagators. Consider, for simplicity, a one Higgs doublet version of the model. The scalar potential is

$$V(h) = \mu^2 |h|^2 + \frac{1}{8} (g^2 + g'^2) |h|^4 + V_{gauge}(h) + V_{top}(h) , \quad (81)$$

where $V_{gauge}(h)$ and $V_{top}(h)$ are one-loop contributions to the effective potential arising from gauge boson and top quark loops, respectively. The first two terms in (81), which arise at tree-level, are monotonically increasing giving rise to a minimum at $\langle |h| \rangle = 0$ and therefore do not break electroweak symmetry. This is why we need to calculate the one-loop contributions. The one-loop gauge contribution is given by

$$V_{gauge}(h) = 6 \int_0^\infty \frac{dp}{8\pi^2} p^3 \log \left[\frac{1 + g^2 |h|^2 G_B(p)}{1 + g^2 |h|^2 G_F(p)} \right] , \quad (82)$$

where $G_{B,F}(p)$ are the boson (fermion) gauge propagators in the bulk whose expressions can be found in Ref. [48]. The contribution to the effective potential from V_{gauge} is again monotonically increasing. However there is also a sizeable contribution from top quark loops (due to the large top Yukawa coupling) given by

$$V_{top}(h) = 6 \int_0^\infty \frac{dp}{8\pi^2} p^3 \log \left[\frac{1 + p^2 y_t^2 |h|^2 G_B^2(p)}{1 + p^2 y_t^2 |h|^2 G_F(p)} \right]. \quad (83)$$

This contribution generates a potential that monotonically decreases with $|h|$, destabilising the vacuum and thus triggering electroweak symmetry breaking. In order for this to occur the top quark needs to be localised near the IR brane with a bulk mass parameter $c_t \simeq -0.5$. Since the top quark $\mathcal{N} = 1$ chiral multiplet is localised near the IR brane, the top squark will only receive a TeV scale soft mass and consequently will remain in the low energy spectrum. In fact this radiative breaking of electroweak symmetry due to a large top Yukawa coupling is similar to that occurring in the usual MSSM. As in the MSSM the value of the Higgs mass is very model dependent but if no large tuning of parameters is imposed one obtains a light Higgs boson with mass $m_{Higgs} \lesssim 120$ GeV.

5.3.3 Dual 4D interpretation

The dual 4D interpretation of the partly supersymmetric model follows from applying the rules of the AdS/CFT dictionary. Supersymmetry is broken at the Planck scale in the dual 4D theory and is approximately supersymmetric at the IR scale. Thus from a 4D point of view supersymmetry is really just an accidental symmetry at low energies. At the massless level the Higgs is confined on the IR brane and the top quark is localised towards the TeV brane so both of these states are CFT composites and supersymmetric at tree level. The compositeness of the Higgs and stop explains why these states are not sensitive to the UV breaking of supersymmetry. These states are “fat” with a size of order TeV^{-1} , and are transparent to high momenta or short wavelength probes that transmit the breaking of supersymmetry. At one loop TeV-scale supersymmetry breaking effects arise from the small mixing with the elementary source fields, which directly feel the Planck scale supersymmetry breaking. The bulk gauge fields are partly composite and the light fermions which are localised to varying degrees near the UV brane are predominantly elementary fields. Since the light fermion superpartners are predominantly source fields they obtain Planck scale soft masses.

Thus the dual picture can be summarised as follows

$$\begin{array}{ccc} \text{5D partly} & & \text{4D SM} \oplus \text{Higgsino} \oplus \text{stop} \\ \text{supersymmetric SM} & \xleftrightarrow{\text{DUAL}} & \oplus \text{gravity} \\ & & \oplus \text{strongly coupled 4D CFT} \end{array} \quad (84)$$

The partly supersymmetric standard model is a natural model of high-scale supersymmetry breaking. Supersymmetry is realised in the most economical way. Only

the Higgs sector and top quark are supersymmetric and composite, while all other squarks and sleptons have Planck scale masses. The Higgsino is the dark matter candidate and even gauge coupling unification is achieved as we show in the next section.

6 Grand Unification

The unification of the gauge couplings strongly suggests that there is an underlying simplifying structure based on grand unified theories with gauge groups such as SU(5) and SO(10). We would like to preserve this structure in warped extra dimensions as well. At the quantum level gauge couplings are sensitive to particle states that can be excited in the vacuum and this leads to an energy dependence or running. At one loop the general expression for this energy dependence is

$$\frac{1}{g_a^2(p)} = \frac{1}{g_U^2} + \frac{b_a}{8\pi^2} \log \frac{M_U}{p} , \quad (85)$$

where g_U is the unified gauge coupling, M_U is the unification scale and $a = 1, 2, 3$ represents the gauge couplings of U(1)_Y, SU(2)_L and SU(3), respectively. The coefficients b_a depend on the charged particle states that can be excited in the quantum vacuum. At low energies the three gauge couplings are different and if these couplings are to unify at a high energy scale then b_a must be different. From the measurement of the three gauge couplings at the scale M_Z we can use the three equations (85) to obtain the experimental prediction for the ratio

$$B = \frac{b_3 - b_2}{b_2 - b_1} = 0.717 \pm 0.008 , \quad (86)$$

where the error is due to the experimental values of the gauge couplings [52]. This ratio can also be calculated theoretically for the particle content of any model. Note that any constant or universal contribution to b_a does not affect the ratio B , since it is a ratio of differences.

In the Standard Model the quarks and leptons contribute universally to the running because they form complete SU(5) multiplets. Therefore they do not affect the relative running of the couplings and hence the value of B . Only the Higgs and Standard Model gauge bosons, which do not form complete SU(5) multiplets at low energies, affect the relative running. This leads to a Standard Model prediction $B = 0.528$, which does not agree with the experimental value (86), even allowing for a 10% theoretical uncertainty due to threshold corrections and higher-loop effects.

On the other hand the MSSM doubles the particle spectrum with the addition of gauginos, Higgsinos, squarks and sleptons. Again the squarks and sleptons form complete SU(5) multiplets and do not contribute to the relative running. However the extra contributions from the gauginos and Higgsinos combined with the gauge bosons

and Higgs scalar field leads to the MSSM prediction $B = 0.714$. This is remarkably close to the experimental value (86), even if one accounts for theoretical uncertainties, and is one reason why supersymmetry is a leading solution to the hierarchy problem. The question we would like to now address is how does gauge coupling unification work for models in warped space?

6.1 Logarithmic Running in 5D Warped Space

Given that models in 5D warped space can be given a 4D dual description the gauge couplings should run logarithmically. But how does this happen given that the 5D model has Kaluza-Klein modes? To answer this question let us consider the one-loop corrections to the U(1) gauge coupling of the zero mode generated by an even bulk 5D scalar ϕ with charge +1 [45]. The Kaluza-Klein spectrum of the scalar is given by the expressions (33) with $\alpha_\phi = 2$. To regulate the model we introduce a Pauli-Villars (PV) regulator scalar field Φ with bulk mass $\Lambda \lesssim k$ and no boundary mass terms. This means that the PV field has no massless mode and the zero mode obtains a mass $m_0 \simeq \Lambda/\sqrt{2}$. On the other hand the Kaluza-Klein spectrum remains relatively unaffected by the bulk mass and is given by $m_{\Phi_n} = (n + \frac{\alpha_\Phi}{2} - \frac{3}{4})\pi k e^{-\pi k R}$, with $\alpha_\Phi = \sqrt{4 + \frac{\Lambda^2}{k^2}} \simeq 2 + \frac{\Lambda^2}{4k^2}$. This can also be understood as follows: Adding a bulk mass term Λ has the effect of locally adding a mass term Λe^{-ky} at any point y in the bulk. Since the zero mode of a massless bulk scalar field is localised towards $y \simeq 0$ the affect of adding a bulk mass shifts the mass of the zero mode by $\sim \Lambda$. On the other hand the Kaluza-Klein modes are localised near $y = \pi R$, so adding the bulk mass term affects them by an amount $\sim \Lambda e^{-\pi k R}$. Thus the Kaluza-Klein spectrum of the ϕ and the PV field is approximately the same while zero modes are separated by a mass scale Λ .

The corrections to the photon self energy $\Pi_{\mu\nu}(q^2) = (q^2\eta_{\mu\nu} - q_\mu q_\nu)\Pi(q^2)$ are given by [45]

$$\begin{aligned} \Pi(0) &\propto \sum_n \int d^4p \left[\frac{1}{(p^2 + m_{\phi_n}^2)^2} - \frac{1}{(p^2 + m_{\Phi_n}^2)^2} \right], \\ &\simeq \frac{b_\phi}{8\pi^2} \log \frac{\mu}{\Lambda} - \frac{b_\phi}{64\pi^2} \frac{\Lambda^2}{k^2} (\pi k R), \end{aligned} \quad (87)$$

where we have introduced an infrared cutoff μ and $b_\phi = 1/3$ is the β -function coefficient. For $\mu \ll \Lambda < k$ the Kaluza-Klein contribution is negligible since the contributions from the Kaluza-Klein modes effectively cancels out. Instead the dominant contribution arises purely from the zero modes and is given by

$$\Pi(0) \simeq \frac{b_\phi}{8\pi^2} \log \frac{\mu}{\Lambda}. \quad (88)$$

This logarithmic dependence on the cutoff Λ is exactly what we would obtain in four dimensions and consistently agrees with the AdS/CFT correspondence.

This result can be contrasted with what one obtains in 5D flat space. If we add a bulk mass Λ to a bulk scalar field the whole Kaluza-Klein tower shifts by an amount $\sim \Lambda$. Only the Kaluza-Klein states above this scale cancel with the PV field, leaving behind the contribution from the Kaluza-Klein states below Λ . This leads to the contribution

$$\Pi(0) \simeq \frac{b_\phi}{8\pi^2} \Lambda R, \quad (89)$$

which are the power-law corrections [53].

It is also important to note that (88) can only be understood as the running of gauge couplings if matter is localised towards the UV brane. In this case the effective (field theory) cutoff is the Planck scale and the couplings can be interpreted as a running up to that scale. However for matter localised on the IR brane the effective (field theory) cutoff is the TeV scale. Above this scale the effective field theory description breaks down and there will be contributions from the fundamental (string) theory [45, 54].

6.2 Partly Supersymmetric Grand Unification

Let us now consider gauge coupling unification in the partly supersymmetric standard model. The dominant contribution to the running will be logarithmic and it is just a question of determining the β -function coefficients. Consider a 5D SU(5) gauge theory that is broken on the Planck brane, but preserved on the IR brane. We can implement this setup by imposing boundary conditions on the bulk fields. These can be either Neumann (+) or Dirichlet (-) conditions corresponding to even or odd reflections, respectively, about the orbifold fixed points.

The SU(5) gauge bosons form an $\mathcal{N} = 2$ vector multiplet, $\mathcal{V} = (V, S)$, where $V(S)$ is an $\mathcal{N} = 1$ vector (chiral) multiplet. These fields are assumed to have the boundary conditions

$$V = \begin{bmatrix} V_\mu^a(+, +) \\ V_\mu^A(-, +) \end{bmatrix}, \quad S = \begin{bmatrix} S^a(-, -) \\ S^A(+, -) \end{bmatrix}, \quad (90)$$

where the indices $a(A)$ run over the unbroken (broken) generators, and the first (second) argument refers to the UV (IR) boundary condition. These boundary conditions break the SU(5) symmetry on the UV brane and the only zero modes are the Standard Model gauge bosons $V_\mu^a(+, +)$. At the nonzero mode level, the X, Y gauge bosons (contained in $V_\mu^A(-, +)$), and the SU(5) adjoint scalar states all obtain TeV-scale masses.

Similarly, the Higgs sector is supersymmetric and contains two Higgs doublets which are embedded into two $\mathcal{N} = 2$ bulk hypermultiplets, $\mathcal{H} = (H_5, H_5^c)$ and $\bar{\mathcal{H}} = (\bar{H}_5, \bar{H}_5^c)$, each transforming in the $\mathbf{5}$ of SU(5). The boundary conditions are

$$H_5 = \begin{bmatrix} H_2(+, +) \\ H_3(-, +) \end{bmatrix}, \quad H_5^c = \begin{bmatrix} H_2^c(-, -) \\ H_3^c(+, -) \end{bmatrix}, \quad (91)$$

and similarly for $\bar{\mathcal{H}}$. The only zero modes are the two Higgs doublets, $H_2(+, +)$ and $\bar{H}_2(+, +)$, as in the MSSM. This choice of boundary conditions neatly solves the doublet-triplet splitting problem [55].

The Standard Model matter fields are also embedded into bulk $\mathcal{N} = 2$ hypermultiplets. However, while it would seem natural to put all the fermions of one generation into a single $\mathbf{5}$ and $\mathbf{10}$, parity assignments actually require the quarks and leptons to arise from different SU(5) bulk hypermultiplets [56, 57, 58]. Thus, for each generation we will suppose that there are bulk hypermultiplets $(\mathbf{5}_1, \mathbf{5}_1^c) + (\mathbf{5}_2, \mathbf{5}_2^c)$ and $(\mathbf{10}_1, \mathbf{10}_1^c) + (\mathbf{10}_2, \mathbf{10}_2^c)$ with boundary conditions

$$\mathbf{5}_1 = L_1(+, +) + d_1^c(-, +), \quad \mathbf{5}_1^c = L_1^c(-, -) + d_1(+, -), \quad (92)$$

$$\mathbf{5}_2 = L_2(-, +) + d_2^c(+, +), \quad \mathbf{5}_2^c = L_2^c(+, -) + d_2(-, -), \quad (93)$$

$$\mathbf{10}_1 = Q_1(+, +) + u_1^c(-, +) + e_1^c(-, +), \quad (94)$$

$$\mathbf{10}_1^c = Q_1^c(-, -) + u_1(+, -) + e_1(+, -), \quad (95)$$

$$\mathbf{10}_2 = Q_2(-, +) + u_2^c(+, +) + e_2^c(+, +), \quad (96)$$

$$\mathbf{10}_2^c = Q_2^c(+, -) + u_2(-, -) + e_2(-, -), \quad (97)$$

where the Standard Model fermions are identified with the zero modes of the fields with $(+, +)$ boundary conditions. Notice that this embedding elegantly explains why the fermions need not satisfy the SU(5) mass relations and although each Standard Model generation arises from different $\mathbf{5} + \mathbf{10}$ fields, the usual charge quantization and hypercharge assignments are still satisfied [56, 57]. This feature also explains why tree-level proton decay is not a problem in these models. There is simply no allowed coupling between $X, Y(-, +)$ gauge bosons and Standard Model fields $L_1(+, +)$ and $d_2^c(+, +)$ that is even under the orbifold symmetry. This is also true for couplings between Standard Model particles and the coloured Higgs triplets. However, a bulk U(1) symmetry must be introduced in order to prevent proton decay from higher-dimensional operators [59, 60].

The specific contributions to the gauge couplings are given by

$$\frac{1}{g_a^2(p)} = \frac{\pi R}{g_5^2} + \frac{1}{g_{Ba}^2(\Lambda)} + \frac{1}{8\pi^2} \Delta_a(p, \Lambda), \quad (98)$$

where g_{Ba} are boundary couplings, and Δ_a are the one-loop corrections. The first term in (98) is the universal contribution from the tree-level gauge coupling g_5 in the bulk. The second term in (98) is an SU(5) violating term that follows from breaking the SU(5) symmetry on the Planck brane. It can be neglected because $g_{Ba}(\Lambda) \simeq 4\pi$, since the theory is effectively strongly coupled at the scale $\Lambda \gtrsim k$ [61, 46, 62]. Thus, we see that the dominant contributions to the gauge couplings will arise from the logarithmically enhanced terms of Δ_a . These terms cannot be obtained in the strongly coupled 4D dual theory, but instead can be calculated using the bulk zero-mode Green functions [63].

The exact Greens function expression can be expanded at low energies $p \lesssim \text{TeV}$ to obtain the dominant logarithmic contributions. For vector bosons the dominant term is

$$\Delta^a(\mathcal{V}) = b_{\mathcal{V}}^a \ln \frac{k}{p} + \dots , \quad (99)$$

where $b_{\mathcal{V}}^a = (0, -\frac{22}{3}, -11)$. Recall that $k \simeq M_P$ is the AdS curvature scale so that (99) is the usual logarithmic contribution from the Standard Model gauge bosons. There are no corresponding gaugino zero mode contributions because these modes have received a large supersymmetry breaking mass and decouple at low energy.

Instead for the Higgs sector, the leading contribution for the Higgs doublet is

$$\Delta^a(\mathcal{H}_{++}) = T_a(\mathcal{H}_{++}) \ln \frac{T}{p} + \dots , \quad (100)$$

where $T_a(\mathcal{H}_{++}) = (\frac{3}{10}, \frac{1}{2}, 0)$ and T is shorthand for the TeV scale. In this case the leading contribution is again a logarithm but it is small. This can be interpreted as being due to the fact that the Higgs doublet is a composite particle. On the other hand the Higgs triplet contributions for $m_{-+} \lesssim p \lesssim T$, with m_{-+} the lowest lying massive state, are

$$\Delta^a(\mathcal{H}_{-+}) = T_a(\mathcal{H}_{-+}) \left[\frac{2}{3} \ln \frac{k}{p} + \ln \frac{T}{p} + \dots \right] , \quad (101)$$

where $T_a(\mathcal{H}_{-+}) = (\frac{1}{5}, 0, \frac{1}{2})$. There is now both a large and small logarithmic contribution. The small contribution is from the composite states, while the large contribution is due to elementary degrees of freedom which are required to form a Dirac state. These extra elementary states can also be inferred by directly studying the dual 4D theory [39]. Thus, the total Higgs contribution from both Higgs hypermultiplets \mathcal{H} and $\bar{\mathcal{H}}$ is

$$\Delta^a(\mathcal{H} + \bar{\mathcal{H}}) = b_{\mathcal{H}+\bar{\mathcal{H}}}^a \ln \frac{k}{p} + \ln \frac{T}{p} + \dots , \quad (102)$$

where $b_{\mathcal{H}+\bar{\mathcal{H}}}^a = (\frac{4}{15}, 0, \frac{2}{3})$.

Finally the first two matter generations are (predominantly) elementary and give rise to the contribution

$$\Delta^a(\mathbf{5}_i^{(I)} + \mathbf{10}_i^{(I)}) = \frac{4}{3} \ln \frac{k}{p} + \dots . \quad (103)$$

This is the usual universal contribution from one generation of Standard Model fermions which form a complete SU(5) multiplet. On the other hand the third generation is partly composite with composite states t_R, b_R, τ_R and elementary states $t_L, b_L, \tau_L, \nu_{\tau L}$. This gives the nonuniversal contribution

$$\Delta^a(\mathbf{5}_i^{(3)} + \mathbf{10}_i^{(3)}) = b_{(3)}^a \ln \frac{k}{p} + \frac{4}{3} \ln \frac{T}{p} + \dots , \quad (104)$$

where $b_{(3)}^a = (\frac{8}{15}, \frac{8}{3}, \frac{4}{3})$. Clearly we see that the large logarithmic contribution that arises from the elementary states introduces a differential running in the gauge couplings.

If we now add up all the Δ^a contributions (99), (102), (103) and (104) arising from the elementary states in the model, then at the leading log level we obtain the total contribution [64]

$$\Delta^a = b_{\text{total}}^a \ln \frac{k}{p} + \dots , \quad (105)$$

where $b_{\text{total}}^a = (\frac{52}{15}, -2, -\frac{19}{3})$. These β -function coefficient values give $B = 0.793$, which allowing for an approximately 10% theoretical uncertainty, agrees with the experimental value (86). Interestingly the partly composite third generation has restored the gauge coupling unification without the gauginos and Higgsinos.

- **EXERCISE:** Use the one-loop gauge coupling expressions in Ref [63] for $p \lesssim TeV$, to verify the individual logarithmic contributions in Δ^a .

Note that even though the model is partly supersymmetric, supersymmetry plays no role in obtaining gauge coupling unification because all the differential running contributions come from the UV-brane localised elementary sector which is nonsupersymmetric. This means that a similar mechanism will also work for an inherently nonsupersymmetric model such as the minimal composite Higgs model [65].

7 Conclusion

Warped models in a slice of AdS_5 provide a new framework to study solutions of the hierarchy problem at the TeV scale. The warp factor naturally generates hierarchies and can be used to either stabilise the electroweak scale or explain why the scale of supersymmetry breaking is low. Remarkably by the AdS/CFT correspondence these 5D warped models are dual to strongly coupled 4D theories. The Higgs localised on the IR brane is dual to a composite Higgs. The corresponding Higgs boson mass can be light compared to the IR cutoff by using either a global symmetry and treating the Higgs as a pseudo Nambu-Goldstone boson or using supersymmetry to make only the Higgs sector supersymmetric. The good news is that these models are testable at the LHC (and an eventual linear collider), so it will be an exciting time to discover whether Nature makes use of the fifth dimension in this novel way. If not, there is no bad news, because the warped fifth dimension literally provides a new theoretical framework for studying the dynamics of strongly coupled 4D gauge theories and this will be an invaluable tool for many years to come.

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