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SPECIAL LECTURE:
Unparticle Physics

A. S. Cornell

Institut de Physique Nucleaire de Lyon, France

Unparticle Physics

presented by

Alan S. Cornell

(Institut de Physique Nucléaire de Lyon)

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Outline of lecture

- Motivation
- Review the basic model
- Example 1: $t \to u + U$
- The Unparticle propagator
- Example 2: $e^+ + e^- \to \mu^+ + \mu^-$
- Example 3: LFV decay $\mu \to e^- + e^+ + e^-$
- Example 4: $B \to K$ + missing energy
- Example 5: Lepton anomalous magnetic moments
- Example 6: Mono-photon events in e^+e^- collisions
- Summary

Unparticle articles out there to date[4-6-2007]

- **Howard Georgi, "Unparticle Physics", arXiv: hep-ph/0703260,**
- **Howard Georgi, "Another Odd Thing About Unparticle Physics", arXiv:0704.2457,**
- K. Cheung, W-Y. Keung and T-C. Yuan, "Novel signals in unparticle physics", arXiv:0704.2588,
- M. Luo and G. Zhu, "Some Phenomenologies of Unparticle Physics", arXiv:0704.3532,
- C-H. Chen and C-Q. Geng, "Unparticle physics on CP violation", arXiv:0705.0689,
- G-J. Ding and M-L. Yan, "Unparticle Physics in DIS", arXiv:0705.0794,
- Yi Liao, "Bounds on Unparticles Couplings to Electrons: from Electron g-2 to Positronium Decays", arXiv:0705.0837,
- T. M. Aliev, A. S. Cornell and Naveen Gaur, "Lepton Flavour Violation in Unparticle Physics", arXiv:0705.1326,
- X-Q. Li and Z-T. Wei, "Unparticle Physics Effects on D-anti-D Mixing", arXiv:0705.1821,

- C-D. Lu, W. Wang, and Y-M. Wang, “Lepton flavor violating processes in unparticle physics”, arXiv:0705.2909,
- M. A. Stephanov, “Deconstruction of Unparticles”, arXiv:0705.3049,
- P. J. Fox, A. Rajaraman and Y. Shirman, “Bounds on Unparticles from the Higgs Sector”, arXiv:0705.3092,
- N. Greiner, “Constraints On Unparticle Physics In Electroweak Gauge Boson Scattering”, arXiv:0705.3518,
- H. Davoudiasl, “Constraining Unparticle Physics with Cosmology and Astrophysics”, arXiv:0705.3636,
- D. Choudhury, D. K. Ghosh and Mamta, “Unparticles and Muon Decay”, arXiv:0705.3637,
- S-L. Chen and X-G. He, “Interactions of Unparticles with Standard Model Particles”, arXiv:0705.3946,
- T. M. Aliev, A. S. Cornell and N. Gaur, “$B \to K(K^*)$ missing energy in Unparticle physics, arXiv:0705.4542,
- P. Mathews and V. Ravindran, “Unparticle physics at hadron collider via dilepton production”, arXiv:0705.4599.

Motivation for this model

- Scale invariance is a very powerful concept with wide applications. In particle physics it is very predictive in analysing the asymptotic behaviour of correlation functions at high energies
- Scale invariance at low energies is broken by the masses of particles
- Our quantum mechanical world seems well described by particles
- But why can't we have a scale invariant sector in our theory?
- An interacting scale invariant theory would have no particles, it is made of ***unparticles***
- So what would ***unparticles*** look like?

Some definitions of use

Scale invariance: a feature of objects or laws that do not change if the length of energy scales are multiplied by a common factor

An Effective theory is an approximate theory that contains the appropriate degrees of freedom to describe physical phenomena occurring at a chosen length scale, but ignores the substructure and the degrees of freedom at shorter distances.

That is, we shall use the degrees of freedom appropriate to the scale of the problem, for example, we don't use quantum gravity to calculate projectile motion!

More generally, any theory at momentum $p<<M$ can be described by an effective Lagrangian

$$L_{eff} = L_0 + \sum_i \frac{C_i}{M^{n_i}} \cdot O_i$$

where the C_i's are the short distance quantities (in QCD these are perturbatively calculable if $M>>\Lambda_{QCD}$)

and the O_i's are the long distance quantities

The Basic Set-up

- Imagine that at very high energies our theory contains the SM fields and a conformal sector due to fields of a theory with a nontrivial IR fixed point (call them BZ fields)
- These two sets interact through the exchange of particles with a mass scale M_U
- Below the scale M_U, there are nonrenormalizable couplings involving both SM and BZ fields suppressed by powers of M_U $$\frac{1}{M_U^k} O_{SM} O_{BZ} \quad \text{for } k > 1$$
- As in massless non-Abelian gauge theories, renormalization affects the scale invariant BZ sector inducing dimensional transmutation at an energy scale Λ_U

- In the effective theory below the scale Λ_U the BZ operators must match onto the new (unparticle) operators, which have the following form in the effective interaction

$$\frac{C_U \Lambda_U^{d_{BZ}-d_U}}{M_U^k} O_{SM} O_U$$

where d_U is the scaling dimension of the unparticle operator C_U is a coefficient determined by the matching condition

- As a nontrivial scale invariant IR fixed point theory is thoroughly nonlinear and complicated, the matching of the BZ physics to the unparticle will be complicated

  cf. high-energy QCD and low-energy hadron states

  As such, we shall estimate these constants only roughly.

- First note though, that this scaling dimension d_U can be a non-integer number

The reason for this is (*by analogy*):

- This is a CFT defined in terms of an OPE
- When one does an OPE and then tries to evaluate the overlap of two operators we get a *Taylor-like* series (basically the perturbative expansion of the diagrams) in terms of the coupling constants
- The first term of the expansion is like the correlator function
- Higher order terms give us the anomalous dimension (higher order diagrams changing the classical dimension, hence *anomalous*)
- This anomalous dimension need not be an integer one

Typical of scale invariant theories where one needs an OPE. Note that this also happens for *B*-decays, as we use an OPE there also.

Note that in the previous equation, the unparticle operator was a Lorentz scalar, but there are other possible Lorentz structures, some of which we shall present later

So what physics do we have below Λ_U?

Note that we shall focus on the production of the unparticle stuff

- The most important effects will be those involving only one factor of

$$\frac{C_U \Lambda_U^{d_{BZ}-d_U}}{M_U^k}$$

The result will be the production of unparticle stuff, contributing to missing energy and momentum

Density of final states

- To calculate probability distributions we need to know the density of final states

  Note that this is essentially determined by the scale invariance

$$\left\langle 0 \middle| O_U(x) O_U^\dagger(x) \middle| 0 \right\rangle = \int e^{-iPx} \left| \left\langle 0 \middle| O_U(0) \middle| P \right\rangle \right|^2 \rho(P^2) \frac{d^4 P}{(2\pi)^4}$$

From scale invariance the matrix element scales with dimension *$2d_U$*, requiring

$$\left| \left\langle 0 \middle| O_U(0) \middle| P \right\rangle \right|^2 \rho(P^2) = A_{d_U} \theta(P^0) \theta(P^2) (P^2)^{d_U - 2}$$

Lets compare this to the phase space for *n* massless particles

$$(2\pi)^4 \delta^4\left(P - \sum_{j=1}^{n} p_j\right) \prod_{j=1}^{n} \delta(p_j^2)\theta(p_j^0) \frac{d^4 p_j}{(2\pi)^3} = A_n \theta(P^0)\theta(P^2)(P^2)^{n-2}$$

where

$$A_n = \frac{4\pi^{5/2}}{(2\pi)^{2n}} \frac{\Gamma(n+1/2)}{\Gamma(n-1)\Gamma(2n)}$$

Thus we could say that

Unparticle stuff with scale dimension d_U looks like a non-integral number d_U of invisible particles

Let us then identify A_n above with the A_{dU} from earlier

Note that any other choice could always be absorbed into our choice for C_U earlier

The Operators

Re-writing our parameters into the form

$$\frac{\lambda_i}{\Lambda_U^{d_U}} O_{SM} O_U$$

we note that below the scale Λ_U our effective theory will give us four-Fermi like interactions, with powers of d_U from dimensional analysis. For example

$$\frac{c_S}{\Lambda_U^{d_U}} \bar{\ell} \gamma_\mu \ell' \partial^\mu O_U \qquad \frac{c_P}{\Lambda_U^{d_U}} \bar{\ell} \gamma_\mu \gamma_5 \ell' \partial^\mu O_U$$

$$\frac{c_V}{\Lambda_U^{d_U-1}} \bar{\ell} \gamma_\mu \ell' O_U^\mu \qquad \frac{c_A}{\Lambda_U^{d_U-1}} \bar{\ell} \gamma_\mu \gamma_5 \ell' O_U^\mu$$

As my examples will focus on lepton based processes I will only use the above interactions.

However, other interactions are just as easily constructed

Example 1: $t \rightarrow u + U$

H. Georgi, arXiv:hep-ph/0703260

To illustrate the procedure in a realistic situation, consider the decay $t \rightarrow u + U$, from the coupling

$$i\frac{\lambda}{\Lambda^{d_U}}\bar{u}\gamma_\mu\left(1-\gamma_5\right)t\partial^\mu O_U + h.c.$$

where the constant $\lambda = \dfrac{C_U \Lambda_U^{d_{BZ}}}{M_U^k}$

Ignoring the u mass, the final state densities are

$$d\Phi_u\left(p_u\right) = 2\pi\theta\left(p_u^0\right)\delta\left(p_u^2\right)$$

$$d\Phi_U\left(p_U\right) = A_{d_U}\theta\left(p_U^0\right)\theta\left(p_U^2\right)\left(p_U^2\right)^{d_U-2}$$

The phase space factor is then

$$d\Phi(P) = \int (2\pi)^4 \delta^4\left(P - \sum_j p_j\right) \prod_j d\Phi(p_j) \frac{d^4 p_j}{(2\pi)^4}$$

and the differential decay rate as

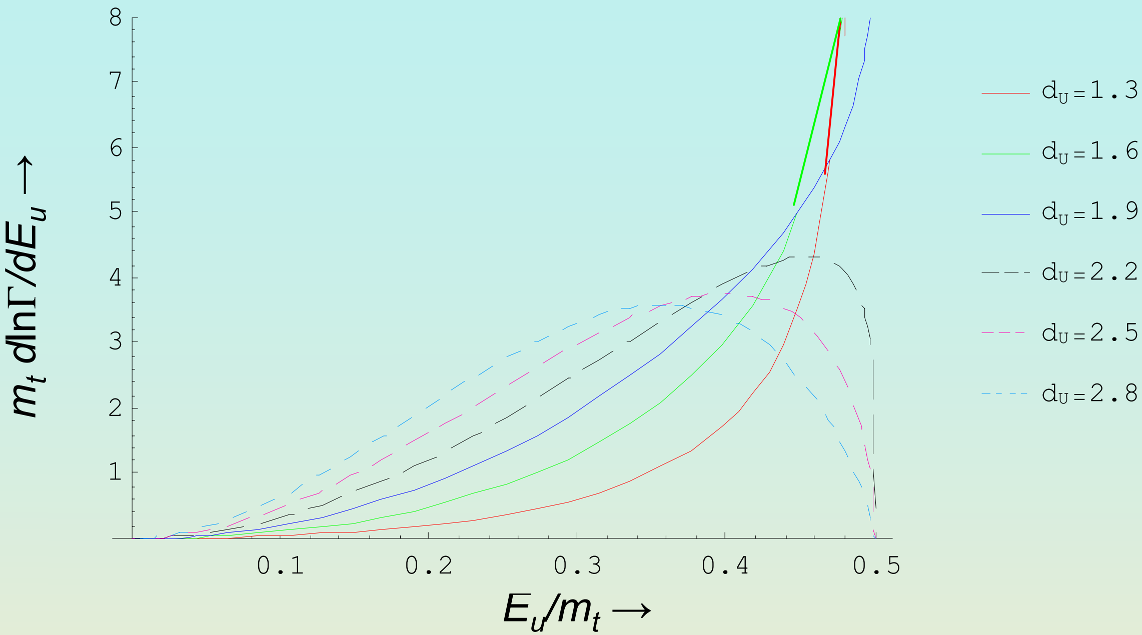
$$d\Gamma = \frac{|M|^2}{2m} d\Phi(P)$$

$$\Rightarrow \frac{d\Gamma}{dE_u} = \frac{A_{d_U} m_t^2 E_u^2 |\lambda|^2}{2\pi^2 \Lambda_U^{2d_U}} \frac{\theta(m_t - 2E_u)}{(m_t^2 - 2m_t E_u)^{2-d_U}}$$

$$or \frac{1}{\Gamma}\frac{d\Gamma}{dE_u} = 4d_U\left(d_U^2 - 1\right)\left(1 - 2E_u / m_t\right)^{d_U - 2} E_u^2 / m_t^2$$

$d\ln\Gamma/dE_u$ versus E_u in units of m_t with d_U = *1.3* to *2.8*.

The dashes get smaller as *j* increases



As $d_U \rightarrow 1$ from above, $d\ln\Gamma/dE_U$ becomes more peaked at $E_u = m_t/2$, matching smoothly onto the 2-particle decay limit, as expected.

For higher d_U the shape depends sensitively on d_U

The kind of peculiar distributions of missing energy that we see in this figure may allow us to discover unparticles experimentally!

The Unparticle Propagator

We shall now consider some virtual effects of unparticles. Note that interference between SM and virtual unparticle amplitudes can be a very sensitive probe of high-energy processes (as we shall see)

Working, as before, to lowest trivial order in the small couplings of unparticles to SM fields, we will require our unparticle operators to be Hermitian and transverse

$$\partial_\mu O_U^\mu = 0$$

Note that unparticle operators with different tensor structures can be dealt with in a similar way

The transverse 4-vector unparticle propagator is given by

$$\int e^{iPx}\left\langle 0\left|T(O_U^{\mu}(x)O_U^{\nu}(x))\right|0\right\rangle d^4x = i\frac{A_{d_U}}{2\pi}\int_0^{\infty}(M^2)^{d_U-2}\frac{-g^{\mu\nu}+P^{\mu}P^{\nu}/P^2}{P^2-M^2+i\varepsilon}dM^2$$

$$= i\frac{A_{d_U}}{2\pi}\frac{-g^{\mu\nu}+P^{\mu}P^{\nu}/P^2}{\sin(d_U\pi)}\left(-P^2-i\varepsilon\right)^{d_U-2}$$

Where the tensor structure reflects our requirement on the operator

And the powers of d_U such that we maintain scale invariance

Note that there will be an imaginary part to the propagator when $P^2 > 0$ (spacelike) and none when $P^2 < 0$

This can be checked by finding the discontinuity across the cut for $P^2 > 0$

$$i\frac{A_{d_U}}{2}\frac{1}{\sin(d_U\pi)}(P^2)^{d_U-2}\left((-1-i\varepsilon)^{d_U-2}-(-1+i\varepsilon)^{d_U-2}\right)=i\frac{A_{d_U}}{2}\frac{1}{\sin(d_U\pi)}(P^2)^{d_U-2}\left(e^{-i(d_U-2)\pi}-e^{i(d_U-2)\pi}\right)$$

$$i\frac{A_{d_U}}{2}\frac{1}{\sin(d_U\pi)}(P^2)^{d_U-2}\left(-2i\sin(d_U\pi)\right)=A_{d_U}(P^2)^{d_U-2}$$

Note that while the discontinuity is not singular for integer $d_U > 1$, the propagator is singular in the denominator.

This would be a real effect

As such we shall look to see what virtual effects, from unparticles, this imaginary part will have

Example 2: $e^+ + e^- \to \mu^+ + \mu^-$

H. Georgi, arXiv:0704.2457[hep-ph]

Let us compute the cross-section for $e^+ + e^- \to \mu^+ + \mu^-$ in the presence of the interactions

$$\frac{c_V \Lambda_U^{k+1-d_U}}{M_U^k} \bar{\ell} \gamma_\mu \ell' O_U^\mu + \frac{c_A \Lambda_U^{k+1-d_U}}{M_U^k} \bar{\ell} \gamma_\mu \gamma_5 \ell' O_U^\mu$$

ignoring lepton mass where $q^2 = s$, the total CM energy, and θ is the angle between the μ^- and the e^-

$$|M|^2 = 2s^2 [\left(|\Delta_{VV}(s)|^2 + |\Delta_{AA}(s)|^2 + |\Delta_{VA}(s)|^2 + |\Delta_{AV}(s)|^2 \right)\left(1+\cos^2\theta\right)$$

$$+ \left(\mathrm{Re}\left(\Delta_{VV}^*(s)\Delta_{AA}(s)\right) + \mathrm{Re}\left(\Delta_{VA}^*(s)\Delta_{AV}(s)\right)\right) 4\cos\theta]$$

where $\Delta_{xy}(s) \equiv \sum_{j=\gamma,Z,U} d_{xj}^{e} d_{yj}^{\mu *} \Delta_j(s)$ and $x,y = V$ or A

The *d*'s and Δ's are

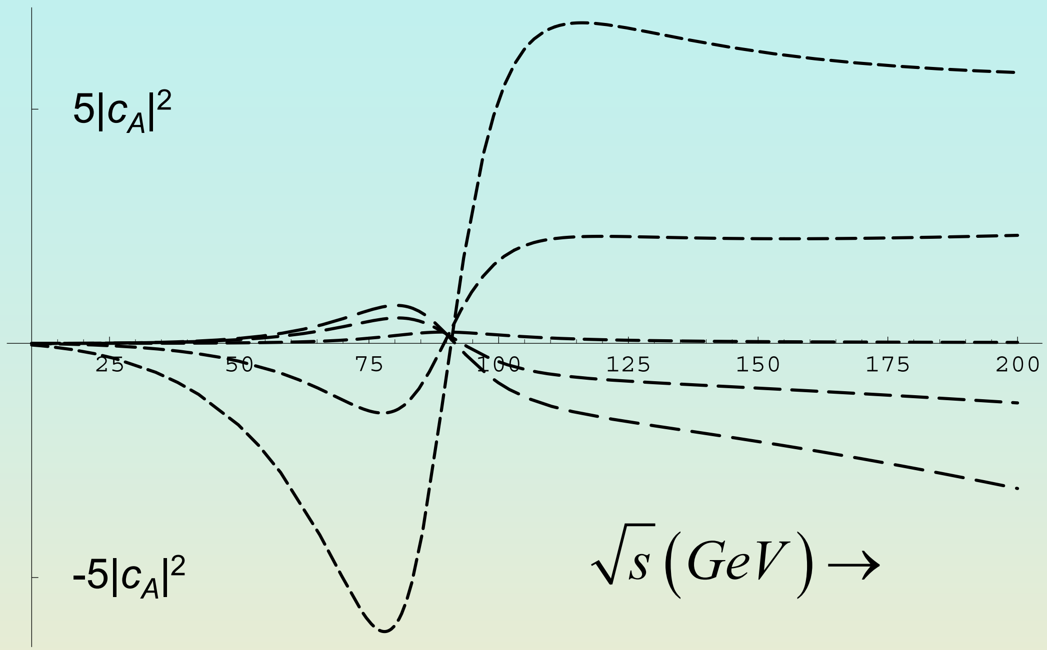
j	γ	Z	U
d_{Vj}	e	$\frac{e}{\sin\theta\cos\theta}\left(-\frac{1}{4}+\sin^2\theta\right)$	$\frac{c_V}{M_Z^{d_U-1}}$
d_{Aj}	0	$\frac{e/4}{\sin\theta\cos\theta}$	$\frac{c_A}{M_Z^{d_U-1}}$
Δ_j	$\frac{1}{s}$	$\frac{1}{s-M_Z^2+iM_Z\Gamma_Z}$	$\frac{A_{d_U}}{2\sin\left(d_U\pi\right)}s^{d_U-2}e^{-i\left(d_U-2\right)\pi}$

Note that we have assumed that unparticles are lepton-flavour blind, but our earlier expression is entirely general

Consider now the total cross-section in the LEP region, where we are used to thinking the Z pole is a poor place to look for interference effects

This prejudice is not warranted for unparticle interactions as this can interfere with both real and imaginary parts of the SM

Beginning with $c_V = 0$ photon and *U* do not directly contribute. Expect only interference with *Z*

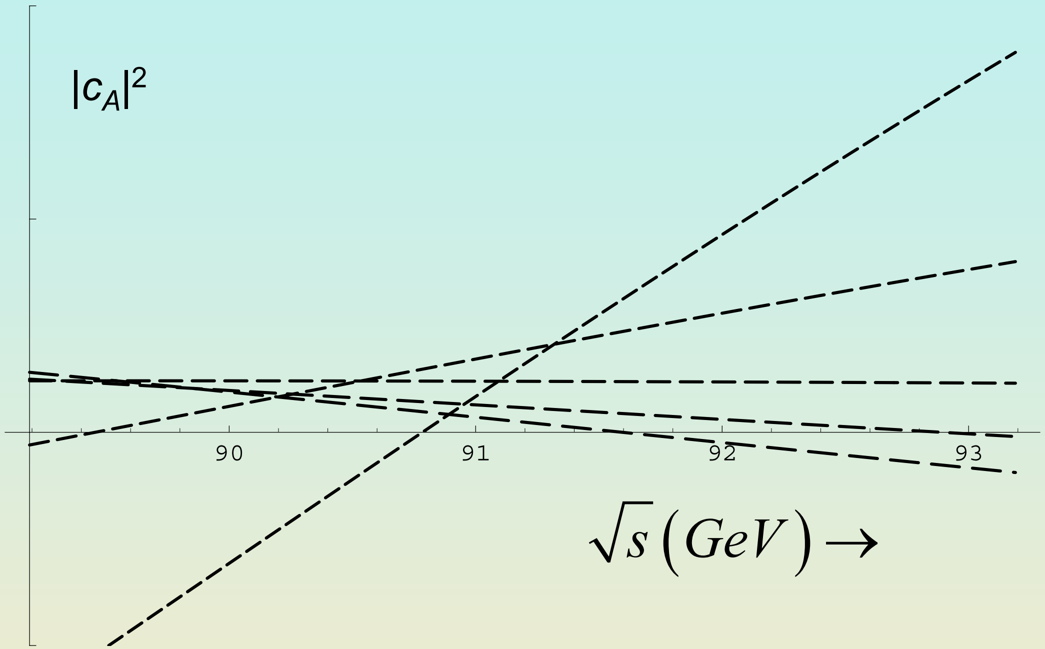


Shown is the fractional change in total Γ for small c_A for various d_U between 1 and 2.

This is extremely sensitive to d_U. We can understand this by thinking about the phase of the *U* propagator along the cut

$$\phi_{dU} = -(d_U - 1)\,\pi$$

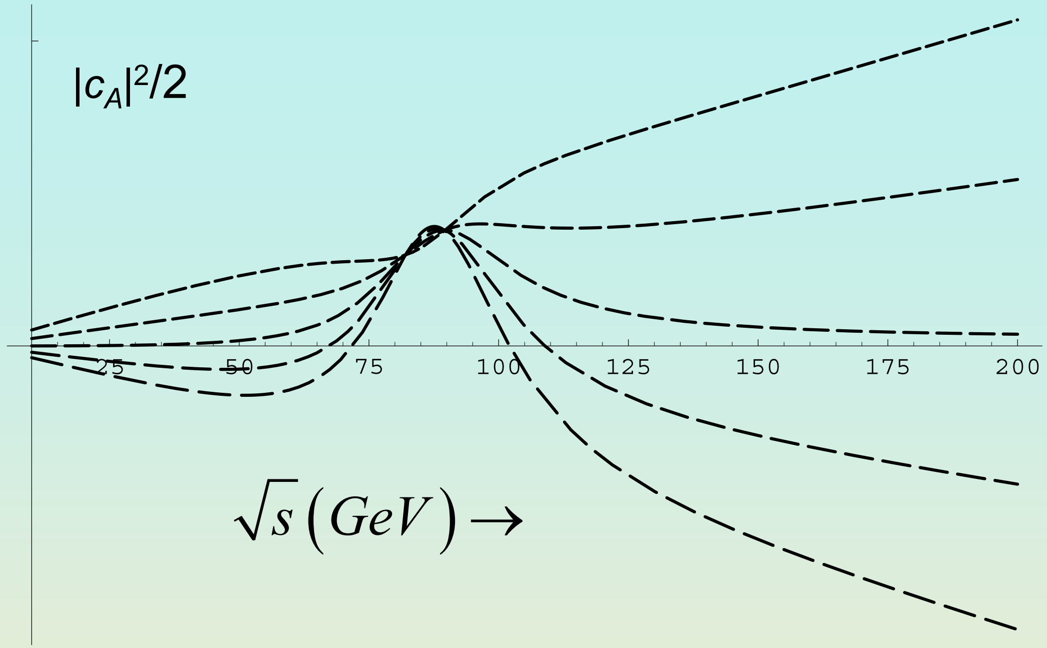
The real part is positive for $1 < d_U < 3/2$ and negative for $3/2 < d_U < 2$. Thus away from the *Z* pole we expect destructive (constructive) interference below (above) the pole for $1 < d_U < 3/2$ and vice-versa for $3/2 < d_U < 1$.



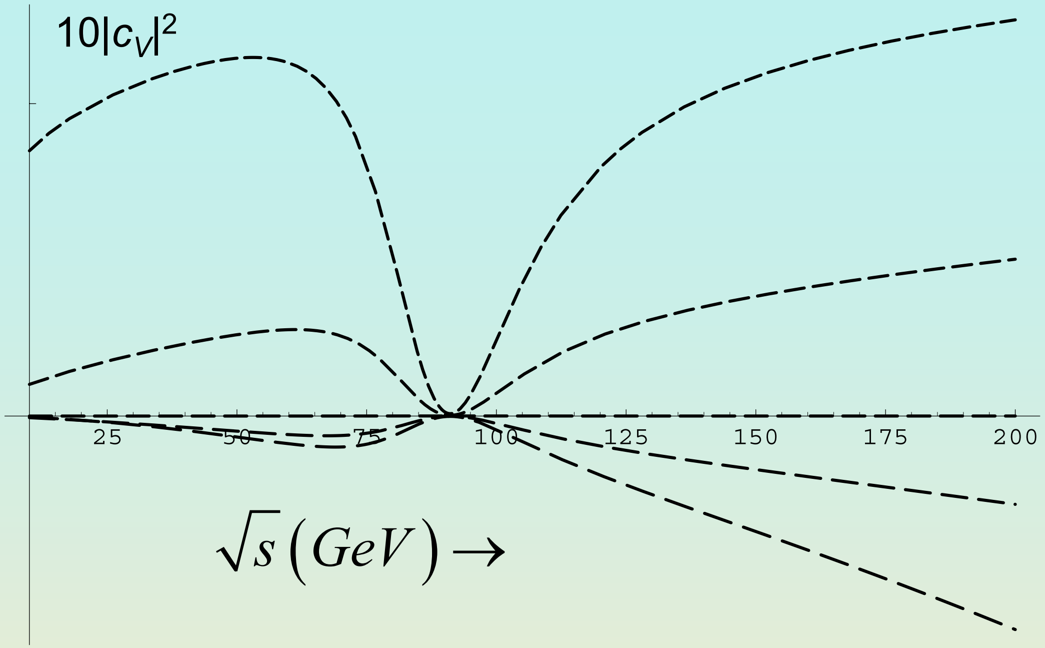
A closer view of the previous plot around the Z pole

As can be seen from this plot, things are much more complicated near the Z pole, as both real and imaginary parts contribute to the interference

Here our values of d_U are closer to *1.5* (that is, *1.48, 1.49, 1.51* and *1.52*)



The situation simplifies in an interesting way for $d_U = 3/2$.
In this case the unparticle amplitude interferes only with the imaginary part of the Z exchange. This is a smaller effect as it is proportional to the Z width, rather than $s - M_Z^2$.
It gives constructive interference that peaks on the Z pole and goes to zero far from the pole.



Now we consider the case of purely vector coupling, where we expect interference with the photon, and only weakly with the *Z*

We now expect constructive interference for $1 < d_U < 3/2$ and destructive for $3/2 < d_U < 2$.

Note that the dip at the *Z* pole arises from our plotting the fractional change, and the large contribution from the pole is in the denominator.

Note that the unparticle interference in the matrix element also gives rise to a complicated pattern of changes in the FB asymmetry, which we won't cover here.

However, this does point to some very interesting and detailed interference effects, unique to unparticle stuff, even though we lack a truly detailed picture of what it looks like!

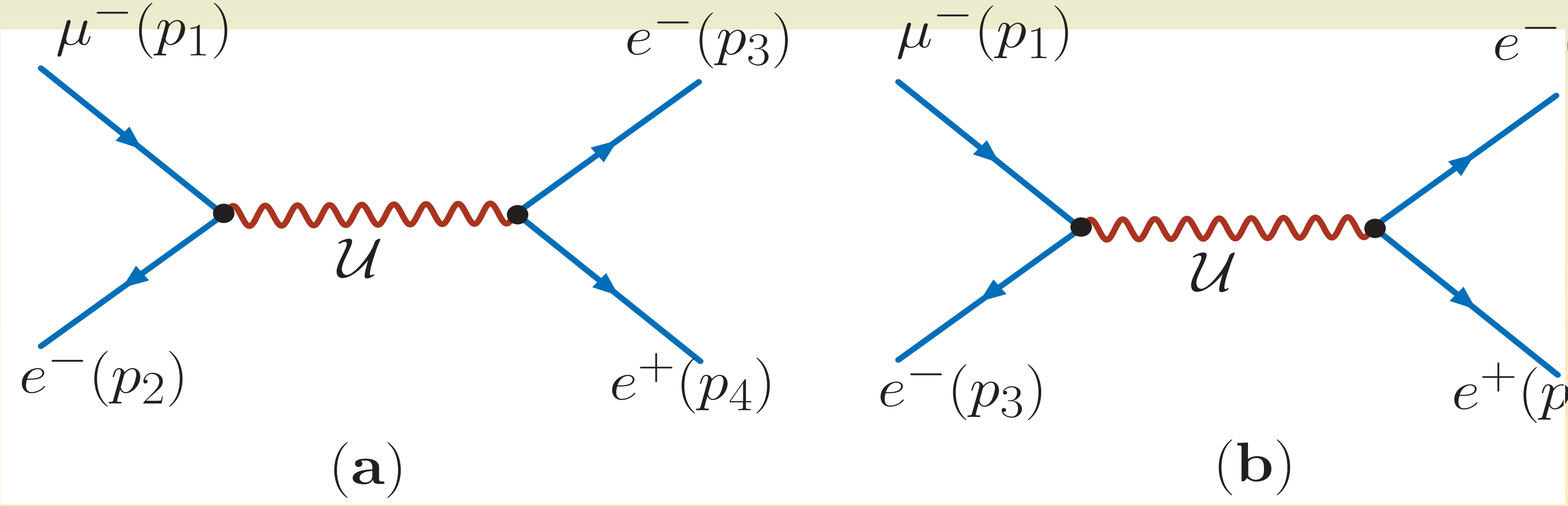
Example 3: LFV decay $\mu \to e^- + e^+ + e^-$

T. M. Aliev, A. S. Cornell and N. Gaur, arXiv:0705.1326[hep-ph]

For our study we will consider the following set of effective interactions for the unparticle operators which have couplings to leptons:

$$\frac{c_V}{\Lambda_U^{d_U-1}} \bar{\ell}\gamma_\mu \ell' O_U^\mu + \frac{c_A}{\Lambda_U^{d_U-1}} \bar{\ell}\gamma_\mu\gamma_5 \ell' O_U^\mu$$

The scalar operator, in principle, couples with SM fermions, however, their contributions are proportional to fermion mass and are suppressed here



The decay is described by the Feynman diagrams above, with the matrix element:

$$M = M_1 + M_2 \quad \text{with}$$

$$M_1 = \bar{u}_2 \gamma_\mu \left(a_1 + a_2 \gamma_5\right) u_1 \bar{u}_3 \gamma_\nu \left(a_3 + a_4 \gamma_5\right) v_4 \frac{A_{d_U}}{\Lambda^{2d_U - 2}} \frac{\left(-g^{\mu\nu} + P^\mu P^\nu / P^2\right)}{2 \sin d_U \pi} \left(-P^2 - i\varepsilon\right)^{d_U - 2}$$

$$M_2 = -\bar{u}_3 \gamma_\mu \left(a_1 + a_2 \gamma_5\right) u_1 \bar{u}_2 \gamma_\nu \left(a_3 + a_4 \gamma_5\right) v_4 \frac{A_{d_U}}{\Lambda^{2d_U - 2}} \frac{\left(-g^{\mu\nu} + P^\mu P^\nu / P^2\right)}{2 \sin d_U \pi} \left(-P^2 - i\varepsilon\right)^{d_U - 2}$$

where $P = p_1 - p_2$ and $Q = p_1 - p_3$. If we take the particles as massless, then

$$\left|M\right|^2 = \left|M_1\right|^2 + \left|M_2\right|^2 + 2\,\mathrm{Re}\left(M_1^* M_2\right)$$

$$\left|M_1\right|^2 = 32\left|F_1\right|^2 [\left(a_1^2 + a_2^2\right)\left(a_3^2 + a_4^2\right)\left\{p_1.p_4\left(p_2.p_3\right) + p_1.p_3\left(p_2.p_4\right)\right\}$$

$$+4a_1a_2a_3a_4\left\{p_1.p_4\left(p_2.p_3\right) - p_1.p_3\left(p_2.p_4\right)\right\}]$$

$$\left|M_2\right|^2 = 32\left|F_2\right|^2 [\left(a_1^2 + a_2^2\right)\left(a_3^2 + a_4^2\right)\left\{p_1.p_4\left(p_2.p_3\right) + p_1.p_2\left(p_3.p_4\right)\right\}$$

$$+4a_1a_2a_3a_4\left\{p_1.p_4\left(p_2.p_3\right) - p_1.p_2\left(p_3.p_4\right)\right\}]$$

$$\mathrm{Re}\left(M_1^* M_2\right) = -32\,\mathrm{Re}\left(F_1^* F_2\right)\left\{\left(a_1^2 + a_2^2\right)\left(a_3^2 + a_4^2\right) + 4a_1a_2a_3a_4\right\} p_1.p_4\left(p_2.p_3\right)$$

Our calculation will be done in the CM frame of the outgoing electron and positron, as denoted by the momenta p_3 and p_4 respectively.

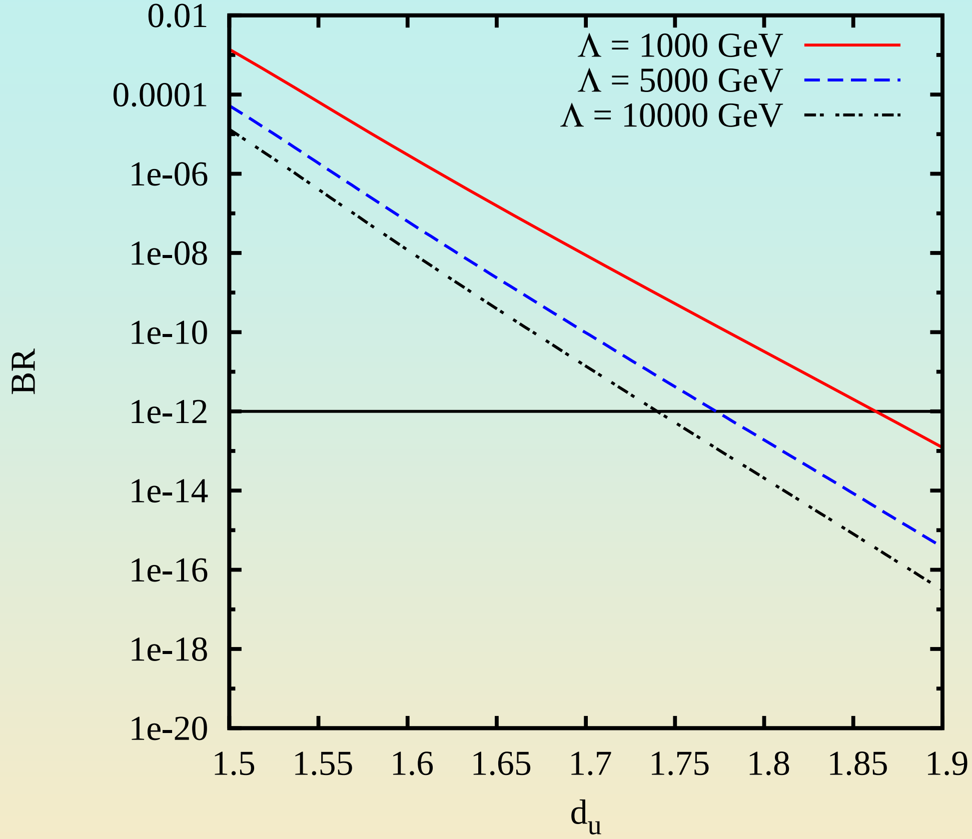
$$\Rightarrow d\Gamma = \frac{1}{2E_\mu}\frac{d^3\vec{p}_2}{(2\pi)^3 2E_2}\frac{d^3\vec{p}_3}{(2\pi)^3 2E_3}\frac{d^3\vec{p}_4}{(2\pi)^3 2E_4}(2\pi)^4\delta\left(p_1 - p_2 - p_3 - p_4\right)\frac{1}{2}\frac{1}{2}\left|M\right|^2$$

$$\frac{d\Gamma}{dsdx} = \frac{1}{2^9\pi^3}\frac{1}{\sqrt{s}}\left(\frac{m_\mu^2}{s} - 1\right)\sqrt{1 - \frac{4m_e^2}{s}}\frac{1}{2}\frac{1}{2}\left|M\right|^2$$

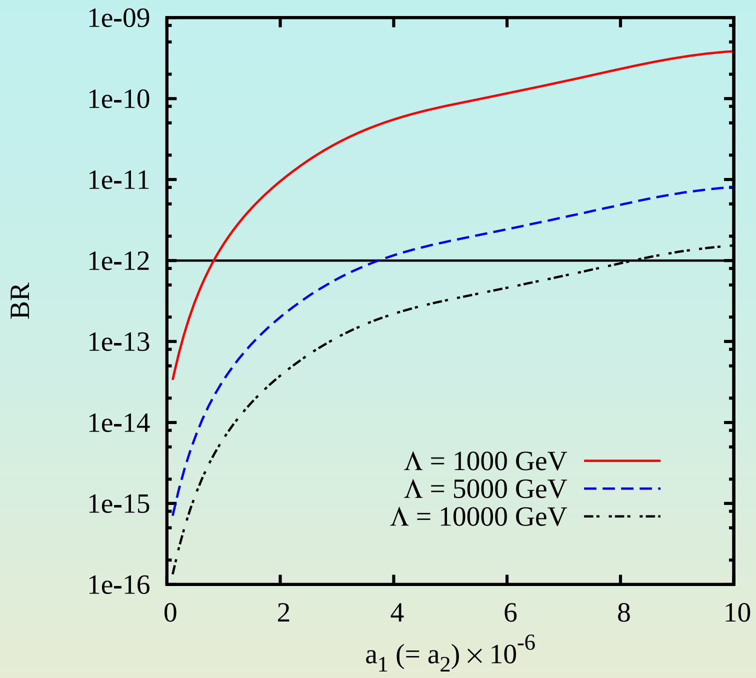
Note that the present experimental limits are

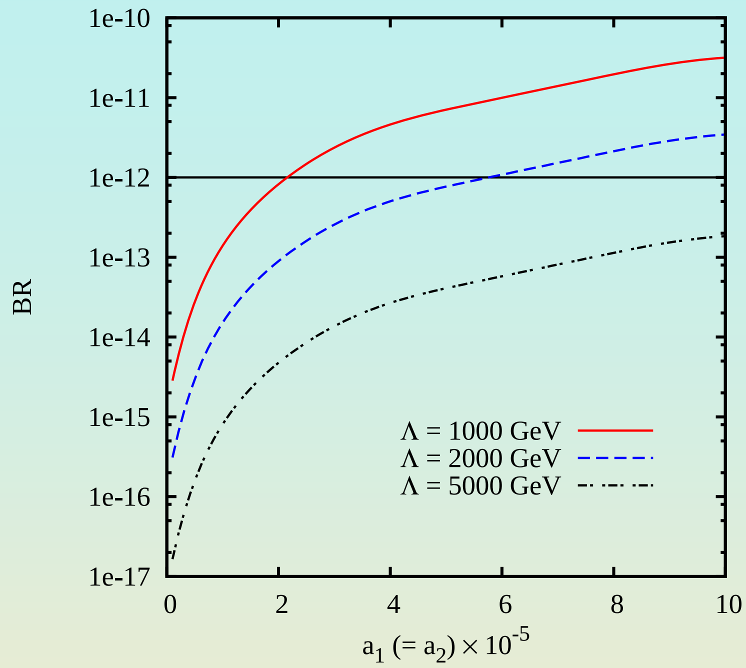
$$BR < 1 \times 10^{-12}$$

Here we have presented variation of BR against d_U for various values of Λ_U



As can be seen, the branching ratio is very sensitive to the scaling dimension





Here we show the dependence of *BR* against the various a_i's, where a_1 and a_2 correspond to the LFV interactions

Again this is done for different values of Λ_U

This shows how sensitive the *BR* is d_U and LFV couplings

Note that the same LFV couplings will be involved in other LFV processes, such as $\mu \to e\,\gamma$.

As such, an exploration of the phenomenology of LFV unparticle operators on radiative LFV processes and their possible correlation with this decay, would be interesting.

Compare this with the well known strong correlation of these LFV processes in SUSY, and how these correlations tend to change substantially in LHT[1]

[1] M. Blanke, A. J. Buras, B. Duling, A. Poschenrieder and C. Tarantino, arXiv:hep-ph/0702136

Example 4: $B \to K$ + missing energy

T. M. Aliev, A. S. Cornell and N. Gaur, arXiv:0705.4542[hep-ph]

Decays of the form $b \to s$ + *missing energy* have been the focus of much investigation at B factories, with measured results of

$$Br\left(B \to K\bar{\nu}\nu\right) < 1.4 \times 10^{-5}$$

In the SM the decay $B \to K\ \nu\nu$ is described at quark level by the Hamiltonian

$$H = \frac{\alpha G_F}{2\sqrt{2}\pi} V_{tb} V_{ts}^* C_{10} \bar{s} \gamma_\mu \left(1 - \gamma_5\right) b \bar{\nu} \gamma_\mu \left(1 - \gamma_5\right) \nu$$

After taking into account the three SM neutrino species, the differential decay width is

$$\frac{d\Gamma^{SM}}{dE_K} = \frac{\alpha^2 G_F^2}{2^7 \pi^5 m_B^2} \left|V_{tb} V_{ts}^*\right|^2 \left|C_{10}\right|^2 f_+^2 \lambda^{3/2}$$

where $\lambda = m_B^4 + m_K^4 + q^4 - 2m_B^2 q^2 - 2m_K^2 q^2 - 2m_B^2 m_K^2$

Where we have made use of the form factors

$$\left\langle K(p')\left|\bar{s}\gamma_\mu b\right|B(p)\right\rangle = \left(p+p'\right)_\mu f_+ + q_\mu f_-$$

$$\left\langle K(p')\left|\bar{s}\gamma_\mu\gamma_5 b\right|B(p)\right\rangle = 0$$

Note also that we shall use the propagator for the scalar unparticle field as

$$\int d^4x e^{iP.x}\left\langle 0\left|TO_U(x)O_U(0)\right|0\right\rangle = \frac{iA_{d_U}}{2\sin\left(d_U\pi\right)}\left(-P^2\right)^{d_U-2}$$

In the case of *B* → *K U* the following scalar operators can contribute

$$\frac{1}{\Lambda_U^{d_U}}\bar{s}\gamma_\mu\left(C_S - C_P\gamma_5\right)b\partial_\mu O_U$$

In which case the matrix element is

$$M^S = \frac{C_S}{\Lambda_U^{d_U}}\left[f_+\left(m_B^2 - m_K^2\right) + f_- q^2\right]O_U$$

In which case the decay rate is

$$\frac{d\Gamma^{SU}}{dE_K} = \frac{A_{d_U}}{8\pi^2 m_B \Lambda_U^{2d_U}} \left|C_S\right|^2 \sqrt{E_K^2 - m_K^2}\left(m_B^2 + m_K^2 - 2E_K m_B\right)^{d_U - 2}$$

$$\left[f_+\left(m_B^2 - m_K^2\right) + f_-\left(m_B^2 + 2m_K^2 - 2m_B E_K\right)\right]^2$$

Similarly for the vector unparticle operators

$$\frac{1}{\Lambda_U^{d_U - 1}} \overline{s}\gamma_\mu\left(C_V - C_A\gamma_5\right) b O_U^\mu$$

We get the matrix element

$$M^V = \frac{C_V}{\Lambda_U^{d_U - 1}}\left[f_+\left(p + p'\right)_\mu + f_- q_\mu\right] O_U^\mu$$

And the differential decay rate

$$\frac{d\Gamma^{VU}}{dE_K} = \frac{A_{d_U}}{8\pi^2 m_B \Lambda_U^{2d_U - 2}} \left|C_V\right|^2 \left|f_+\right|^2 \sqrt{E_K^2 - m_K^2}\left(m_B^2 + m_K^2 - 2E_K m_B\right)^{d_U - 2}$$

$$\left[-\left(m_B^2 + m_K^2 + 2m_B E_K\right) + \frac{\left(m_B^2 - m_K\right)^2}{m_B^2 + m_K^2 - 2m_B E_K}\right]$$

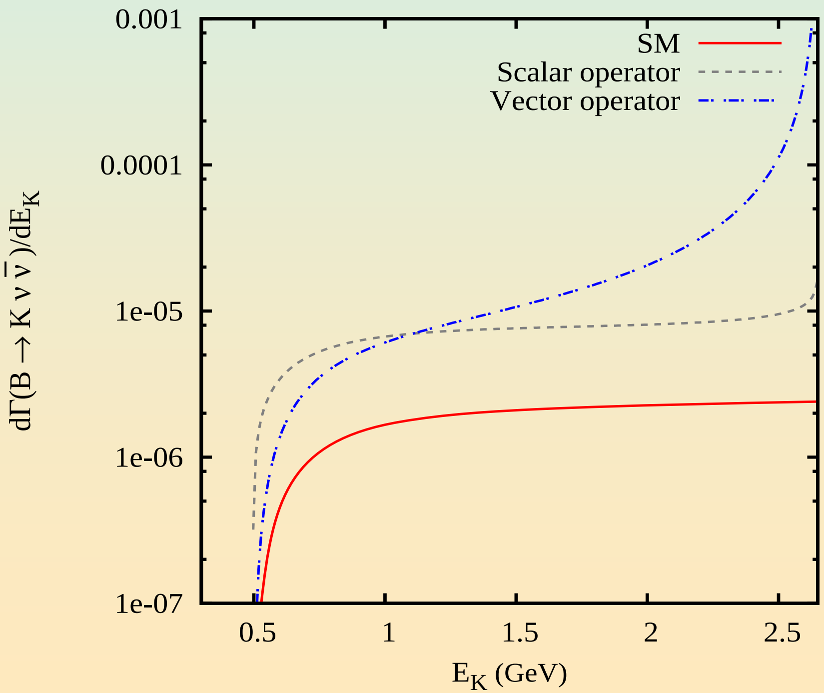
To obtain the total decay width we integrate from

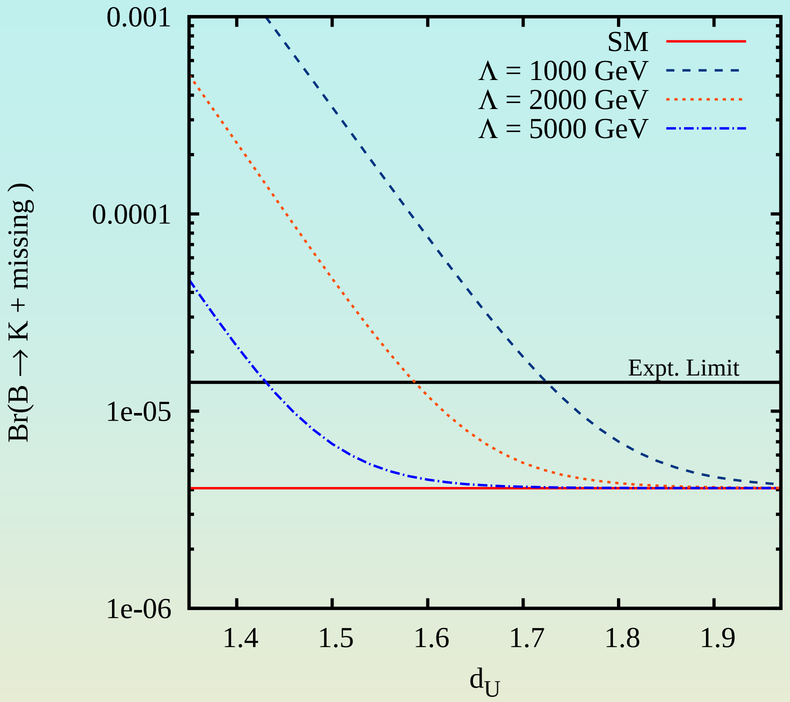
$$m_K < E_K < (m_B^2 + m_K^2)/2m_B$$

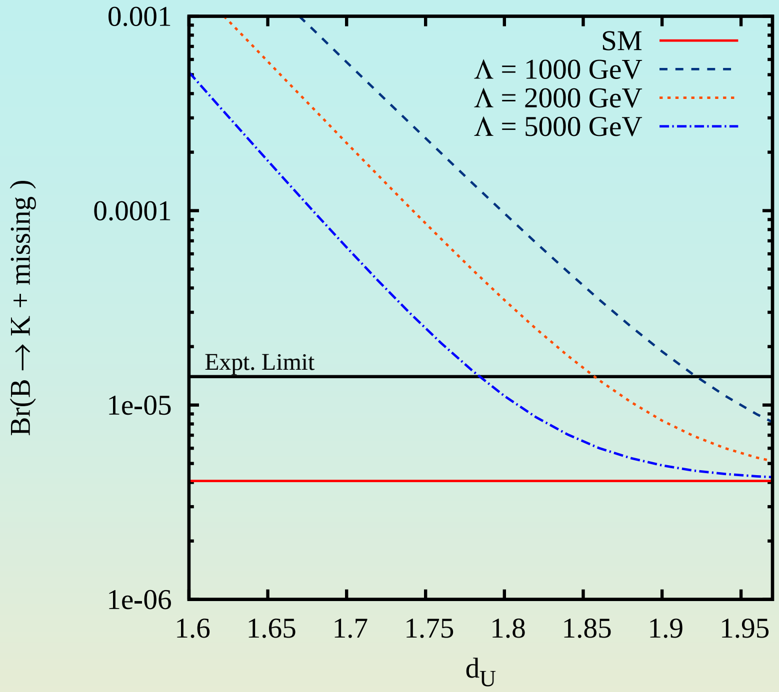
Note also that the energy distribution of final hadrons is very different from the SM compared to unparticles, such that, though the present limits are one order of magnitude above the SM expectation (SuperB will fix this) we can still place constraints now.

Here we plot the decay rate against E_K for the different decay modes.

Note the striking difference in the high energy regime for the vector operators



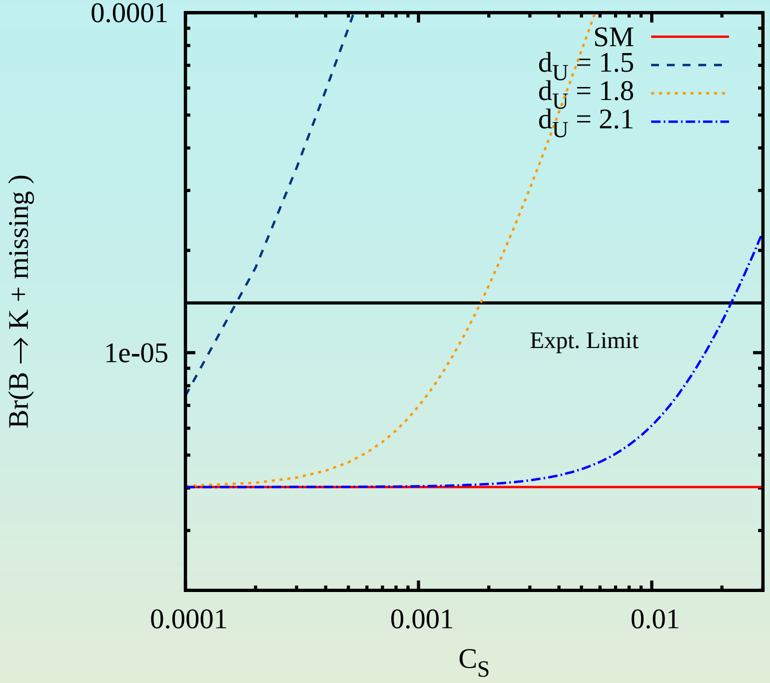


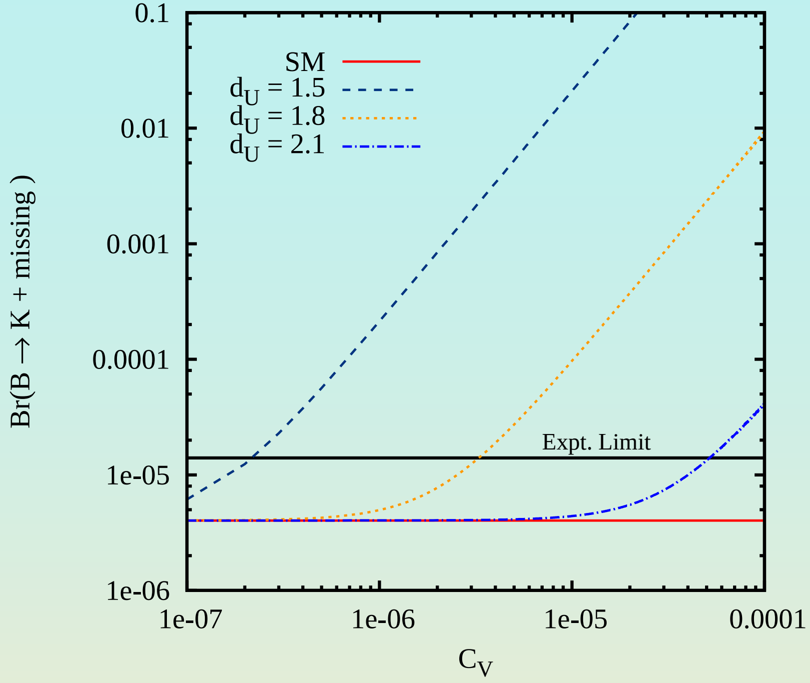


Here we look at the branching ratio against d_U for different values of Λ_U

As can be seen, it is very sensitive to both d_U and Λ_U

And that the vector operator is more strongly constrained than the scalar





Finally we plot the branching ratio against C_S and C_V for various values of d_U

These plots show how the scalar process constrains C_S whilst the vector process constrains C_V

To conclude, these operators also contribute to other processes, such as meson anti-meson mixing etc, however, our constraints here are much stronger

Example 5: Lepton Anomalous Magentic Moments

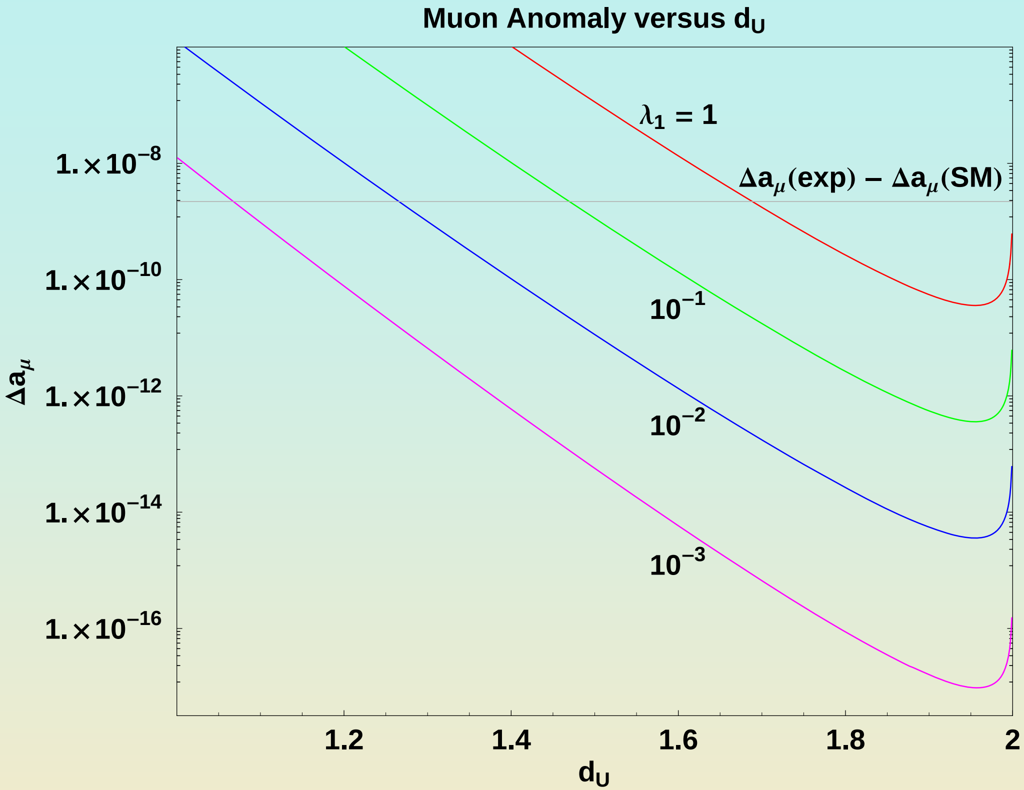
K. Cheung, W-Y. Keung and T-C. Yuan, arXiv:0704.2588[hep-ph]

If we replace one photon exchange in QED by the unparticle associated with the vector operator O_U^μ, one can derive the unparticle contribution to the lepton anomaly $\Delta a_\ell = (g_\ell - 2)/2$

$$\Delta a_\ell = -\frac{c_V^2 Z_{d_U}}{4\pi^2}\left(\frac{m_\ell^2}{\Lambda_U^2}\right)^{d_U-1}\frac{\Gamma(3-d_U)\Gamma(2d_U-1)}{\Gamma(2+d_U)}$$

where m_ℓ is the charged lepton mass and that here we assume c_V is lepton flavour blind.

As $d_U \to 1$ we get $\Delta a_\ell \to c_V^2/8\pi^2$, and if we set c_V to e we reproduce the well known QED result



Here we plot the muon anomalous magnetic moment contribution from the unparticle versus d_U for various c_V's

The horizontal line is the experimental value less the SM contribution

It is amusing to see that current experimental data of the muon anomaly can give bounds to the effective coupling c_V and the scale dimension d_U already

Example 6: Mono-photon events in e^+e^- collisions

K. Cheung, W-Y. Keung and T-C. Yuan, arXiv:0704.2588[hep-ph]

The energy spectrum of the mono-photon from the process $e^- e^+ \to \gamma U$ can also be used to probe the unparticle

Its cross-section is given by

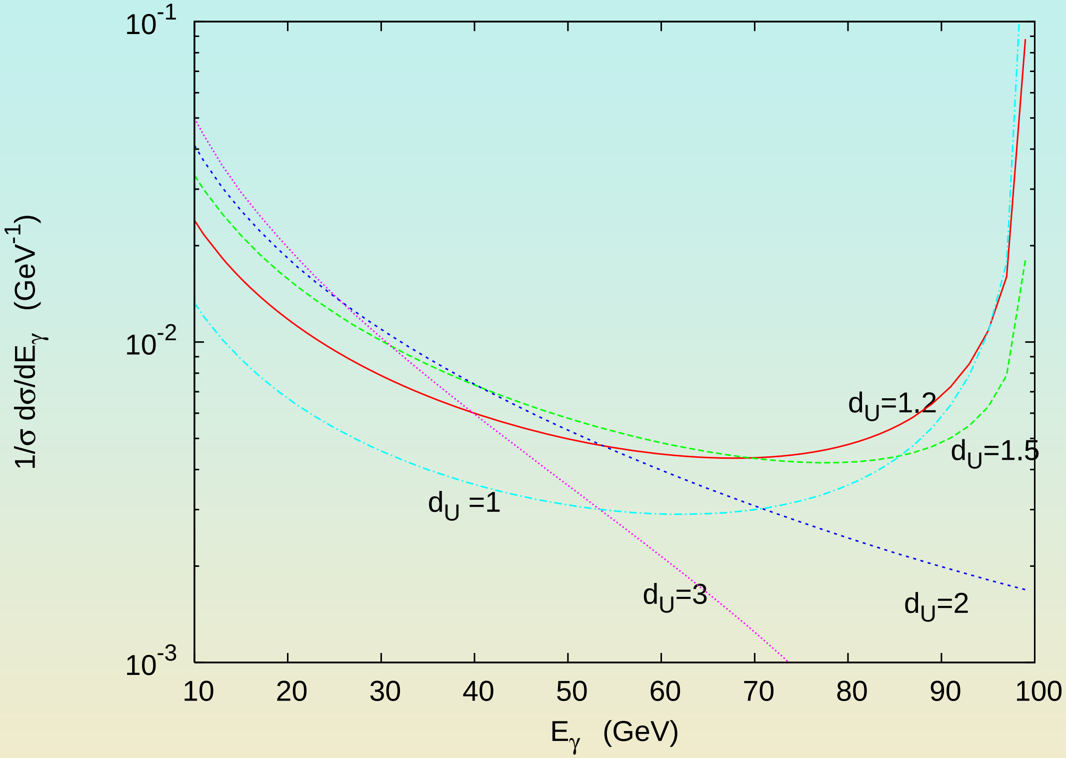
$$d\sigma = \frac{1}{2s} |M|^2 \frac{E_\gamma A_{d_U}}{16\pi^3 \Lambda_U^2} \left(\frac{P_U^2}{\Lambda_U^2} \right)^{d_U - 2} dE_\gamma d\Omega$$

with the matrix element squared

$$|M|^2 = 2e^2 Q_e^2 c_V^2 \frac{u^2 + t^2 + 2sP_U^2}{ut}$$

where P_U^2 is related to the photon energy by the recoil mass relation $P_U^2 = s - 2\, s^{1/2} E_\gamma$

Here we have plotted the mono-photon distribution for various choices of d_U



The sensitivity of the scale dimension can be easily discerned

Note that mono-photon events have been searched quite extensively at LEP experiments in other contexts and a more detailed study by K. Cheung *et al.* is expected soon

Summary

- Unparticle physics, due to conformal invariance, might appear at the TeV scale
- An effective field theory can be used to explore the unparticle effects
- It can lead to interesting phenomenological consequences, due to the scale dimension being able to take non-integer values, which can be checked at low energy experiments
- Such as particular missing energy distributions
- The unparticle propagator in the time-like region has interesting properties that force us to re-examine preconceived ideas about interference.

- In the LFV decay $e^+ + e^- \rightarrow \mu^+ + \mu^-$, we demonstrated the sensitivity of these processes to the scaling dimension and other parameters. Such that a study of other LFV processes will place strong constraints on this model.
- In our study of unparticle physics in the LFV $\mu \rightarrow e^- e^+ e^-$ we determined the decay width <u>is</u> sensitive to the virtual effects of these unparticles
- In our study of $B \rightarrow K$ + *missing energy*, we were able to constrain the scalar and vector operators from experimental data (future results will further constrain this)
- Current experimental bounds can be used to constrain some of our parameters from $(g - 2)_\mu$
- Furthermore, these operators will appear elsewhere, placing additional constraints on them.