The Abdus Salam
International Centre for Theoretical Physics

United Nations
Educational, Scientific and
Cultural Organization

IAEA
International Atomic Energy Agency

SMR/1847-14

Summer School on Particle Physics

11 - 22 June 2007

Supersymmetry:
Motivation, Algebra, Models and Signatures
(Lecture 1)

M. Carena
Fermilab, USA

Strada Costiera 11, 34014 Trieste, Italy - Tel. +39 040 2240 111; Fax +39 040 224 163 - sci_info@ictp.it, www.ictp.it

Supersymmetry:
Motivation, Algebra, Models and Signatures

Marcela Carena
Theoretical Physics Department
Fermilab

ICTP 2007 Summer School on Particle Physics
Trieste, Italy, June 15, 2007

Outline

- Lecture 1:
  Motivation
  Introduction to Supersymmetry
- Lecture 2 :
  Supersymmetric Interactions
  The Minimal Supersymmetric extension of the Standard Model
- Lecture 3 :
  Gauge Coupling Unification
  Models of Supersymmetry Breakdown
- Lecture 4 :
  SUSY signals at Colliders
  The Search for SUSY Dark Matter

Lecture 1:

Motivation
and
Introduction to Supersymmetry

The Standard Model

A quantum theory that describes how all known fundamental particles interact via the strong, weak and electromagnetic forces

A gauge field theory with a symmetry group $SU(3)_c \times SU(2)_L \times U(1)_Y$

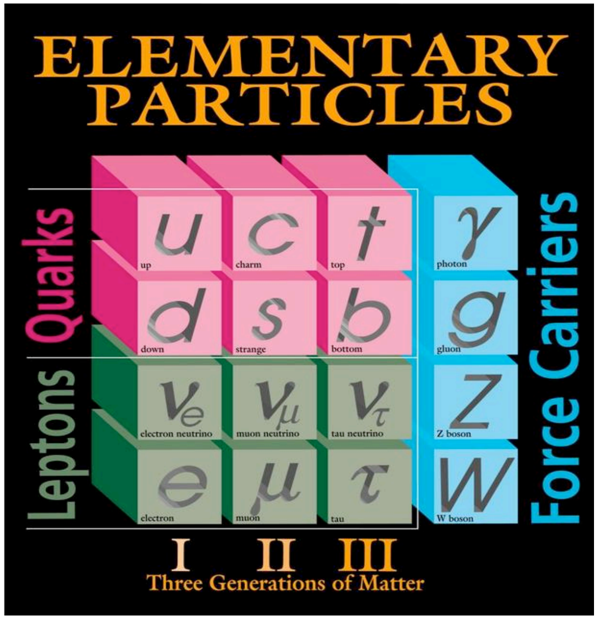
Force Carriers:

12 fundamental gauge fields:
8 gluons, 3 W_μ's and B_μ

and 3 gauge couplings: g_1, g_2, g_3

Matter fields :

3 families of quarks and leptons with the same quantum numbers under the gauge groups



SM particle masses and interactions have been tested at Collider experiments
==> incredibly successful description of nature up to energies of about 100 GeV

The Mystery of Mass

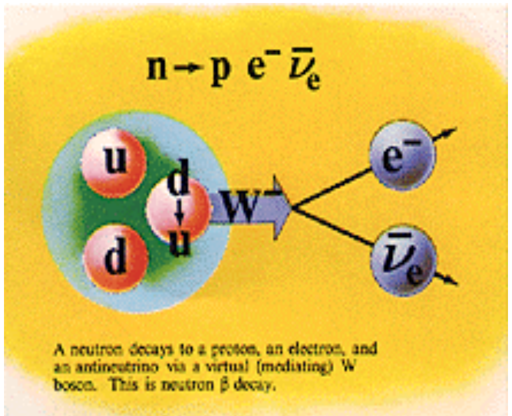
Crucial Problem in the SM: The origin of mass of all the fundamental particles

- Is not possible to give mass to the gauge bosons respecting the gauge symmetry,

  -- massless gauge bosons ==> imply long range forces --

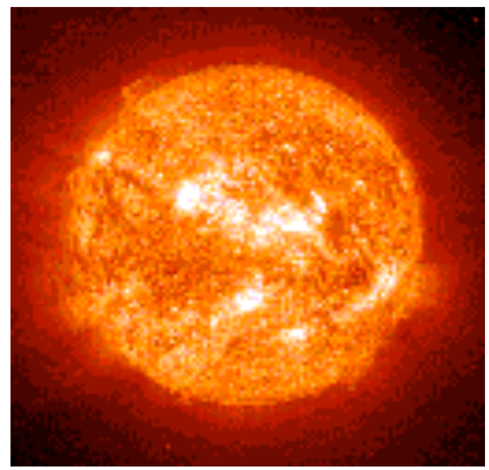
How to give mass to the Z and W gauge bosons?

Weak Force



↔

Nuclear Fusion in the Sun



$m_W = 80.449 \pm 0.034$ GeV

Determines strength of the weak force

↓

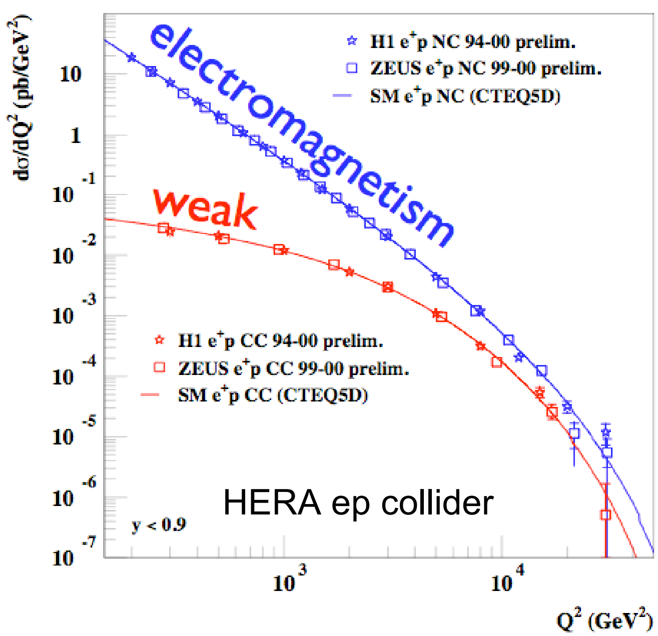
Sun still burning !

- A fermion mass term $L = m\,\bar{\psi}\psi = m\,(\bar{\psi}_L\psi_R + \bar{\psi}_R\psi_L)$ is forbidden because it would mix left- and right-handed fermions which have different quantum numbers

The gauge symmetries of the model do not allow to generate mass at all!

•What is the origin of Mass of the Fundamental Particles ?
or
the source of Electroweak Symmetry Breakdown (EWSB)

- There is a Field that fills all the Universe
  - -- it does not disturb gravity and electromagnetism but it renders the weak force short-ranged
  - -- it slows down the fundamental particles from the speed of light



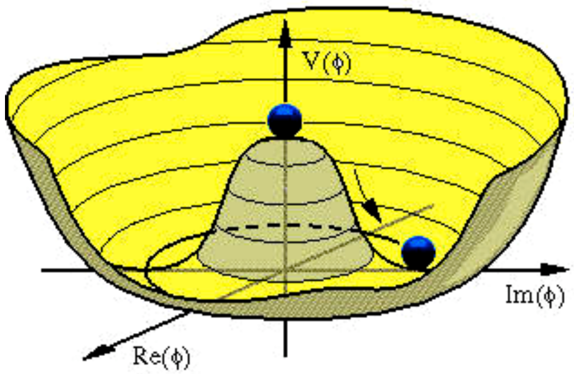
The electromagnetic and weak forces are unified
==> electroweak theory

what breaks the symmetry
==> the mysterious Field

EWSB occurs at the electroweak scale
New phenomena should lie in
the TeV range or below
within LHC/ILC reach

The Higgs Mechanism

A self interacting complex scalar doublet with no trivial quantum numbers under $SU(2)_L \times U(1)_Y$



The Higgs field acquires non-zero value to minimize its energy

$$V(\Phi) = \mu^2 \Phi^+ \Phi + \frac{\lambda}{2}\left(\Phi^+ \Phi\right)^2 \qquad \mu^2 < 0$$

$$< \phi^\dagger \phi > = \mathrm{v}^2 = -\mu^2/\lambda$$

v is the scale of EWSB

- Spontaneous breakdown of the symmetry generates 3 massless Goldstone bosons which are absorbed to give mass to W and Z gauge bosons

- Higgs neutral under strong and electromagnetic interactions exact symmetry $SU(3)_C \times SU(2)_L \times U(1)_Y \Longrightarrow SU(3)_C \times U(1)_{em}$

$$m_\gamma = 0 \quad m_g = 0$$

- Masses of fermions and gauge bosons proportional to their couplings to the Higgs

$$M_V^2 = g_{\phi VV}\, \mathrm{v}/2 \qquad m_f = \mathrm{h_f}\ \mathrm{v}$$

- One extra physical state -- Higgs Boson -- left in the spectrum

$$m^2_{H_{SM}} = 2\lambda\ \mathrm{v}^2$$

The Hierarchy Problem of the SM Higgs Sector

- SM is an effective theory $\Longrightarrow$ low energy quantities (masses, couplings) expected to be given as a function of parameters of the fundamental theory valid at Q> $\Lambda_{eff.}$

✦ **low energy dimensionless couplings: receive quantum corrections prop. to log ($\Lambda_{eff.}$)**

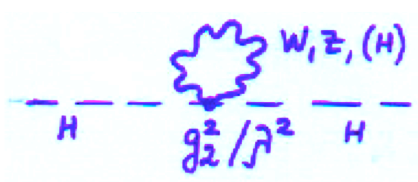
✦ **what about the Higgs potential mass parameter μ?** $\quad \mathrm{v}^2 = |\mu|^2 / 2\lambda$

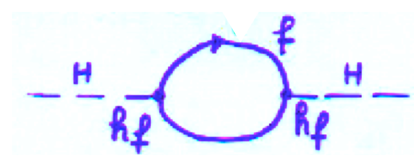
Quantum corrections to μ^2 are quadratically divergent

$$\mu^2 = \mu^2(\Lambda_{eff.}) + \Delta\mu^2 \quad \longrightarrow \quad \Delta\mu^2 \approx \frac{\mathrm{n}_W g_{hWW}^2 + \mathrm{n}_h \lambda^2 - \mathrm{n}_f g_{hf\bar{f}}^2}{16\pi^2} \Lambda^2_{eff.}$$

to explain v $\approx O(m_W)$

either $\Lambda_{eff.} \leq$ 1 TeV or extreme fine tuning to give cancellation



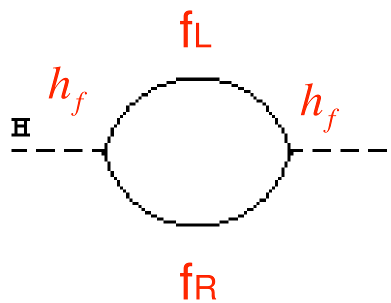


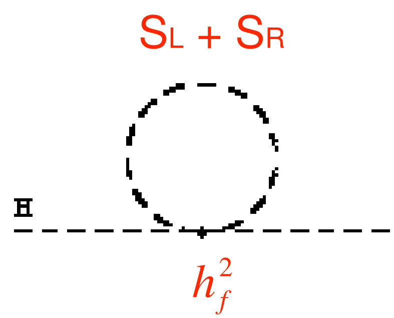
Quantum Corrections to the Higgs Mass Parameter

Quadratic Divergent contributions:

One loop corrections to the Higgs mass parameter cancel if the couplings of bosons and fermions are equal to each other

$$\delta m_H^2 = \frac{N_C h_f^2}{16\pi^2}\left[-2\Lambda^2 + 3m_f^2 \log\left(\frac{\Lambda^2}{m_f^2}\right) + 2\Lambda^2 - 2m_s^2 \log\left(\frac{\Lambda^2}{m_s^2}\right)\right]$$





If the mass proceed from a v.e.v of H, the cancellation of the log terms is ensured by the presence of an additional diagram induced by trilinear Higgs couplings. The fermion and scalar masses are the same in this case: $m_f = m_s = h_f v$

Supersymmetry is a symmetry between bosons and fermions that ensures the equality of couplings and masses

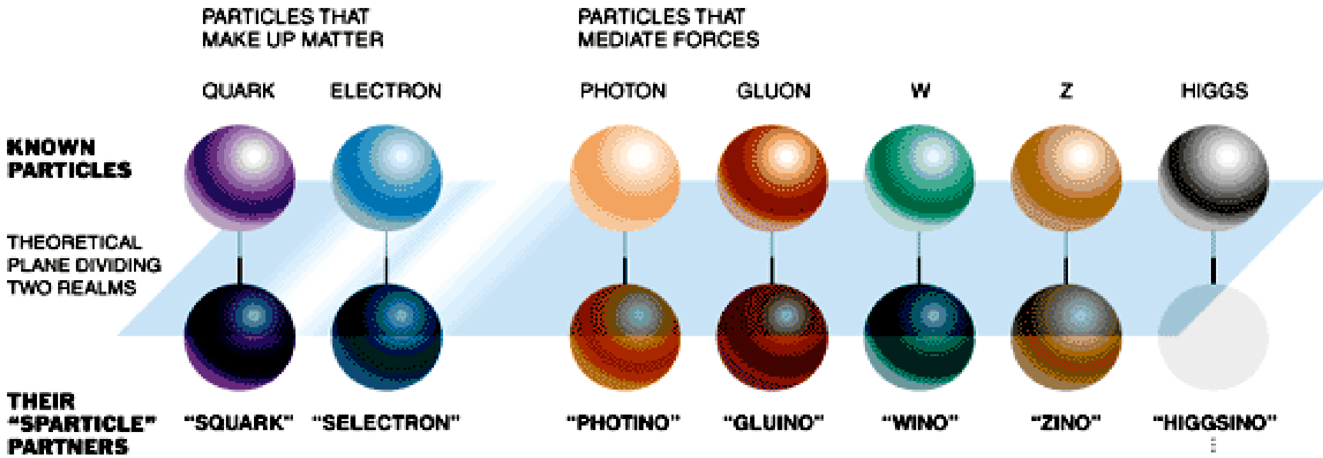
Automatic cancellation of loop corrections to the Higgs mass parameter

Supersymmetry

lesson from history: electron self energy → fluctuations of em fields generate a quadratic divergence but existence of electron antiparticle cancels it, otherwise QED will break down well below M_{Pl}

Will history repeat itself? Take SM and double particle spectrum

New Fermion-boson Symmetry: SUPERSYMMETRY (SUSY)



No new dimensionless couplings
Couplings of SUSY particles equal to couplings of SM particles

Why Supersymmetry?

- Helps stabilize the weak scale-Planck scale hierarchy

- SUSY algebra contains the generator of space translations
  $\longrightarrow$ necessary ingredient of theory of quantum gravity

- Allows for gauge coupling Unification at a scale ~ 10^{16} GeV

- Starting from positive Higgs mass parameters at high energies, induces electroweak symmetry breaking radiatively

- Provides a good Dark matter candidate : the Lightest SUSY Particle

- Provides a solution to the baryon asymmetry of the universe

Structure of Supersymmetric Theories

- The Standard Model is based on a Gauge Theory.
- A supersymmetric extension of the Standard Model has then to follow the rules of Supersymmetric Gauge Theories.
- These theories are based on two set of fields:
  - Chiral fields, that contain left handed components of the fermion fields and their superpartners.
  - Vector fields, containing the vector gauge bosons and their superpartners.
- Right-handed fermions are contained on chiral fields by means of their charge conjugate representation

$$(\psi_R)^C = (\psi^C)_L \qquad \text{with } \psi^C = i\gamma_2\psi^* \qquad \gamma_2 = \begin{pmatrix} 0 & \sigma_2 \\ \sigma_2 & 0 \end{pmatrix}$$

- Higgs fields are described by chiral fields, with fermion superpartners

Supersymmetry Generators

For every fermion there is a boson of equal mass and couplings

Supersymmetric transformations relate bosonic to fermionic degrees of freedom

the operator Q that generates that transformation acts, schematically

$$Q|B>=|F> \quad Q|F>=|B> \qquad Q^\dagger|B>=|F> \quad Q^\dagger|F>=|B>$$

The SUSY generators, *Q* and $Q^\dagger$

are two component anti-commuting spinors satisfying:

$$\{Q_\alpha, Q^\dagger_\alpha\} = 2\sigma^\mu_{\alpha\dot{\alpha}} P_\mu \qquad \{Q_\alpha, Q_\beta\} = \{Q^\dagger_\alpha, Q^\dagger_\beta\} = 0 \quad [Q_\alpha, P^\mu] = [Q^\dagger_{\dot{\alpha}}, P^\mu] = 0$$

where $\sigma^\mu = (I, \vec{\sigma})$, $\bar{\sigma}^\mu = (I, -\vec{\sigma})$, and σ^i are Pauli Matrices

$P^\mu = (H, \vec{p})$ is the generator of spacetime translations: part of the SUSY algebra

Two spinors may contract to form a Lorentz invariant:

$$\psi^\alpha \chi_\alpha = \psi^\alpha \epsilon_{\alpha\beta} \chi^\beta \qquad \bar{\psi}^{\dot{\alpha}} \bar{\chi}_{\dot{\alpha}} = \bar{\psi}^{\dot{\alpha}} \epsilon_{\dot{\alpha}\dot{\beta}} \bar{\chi}^{\dot{\beta}}$$

Hamiltonian of Supersymmetric Theories

- Since there is a relation between the momentum operator and the SUSY generators, one can compute the energy operator

$$H = \frac{1}{4}\left(Q_1 Q_1^\dagger + Q_1^\dagger Q_1 + Q_2 Q_2^\dagger + Q_2^\dagger Q_2\right)$$

- Two things may be concluded from here. First, the Hamiltonian operator is semidefinite positive.

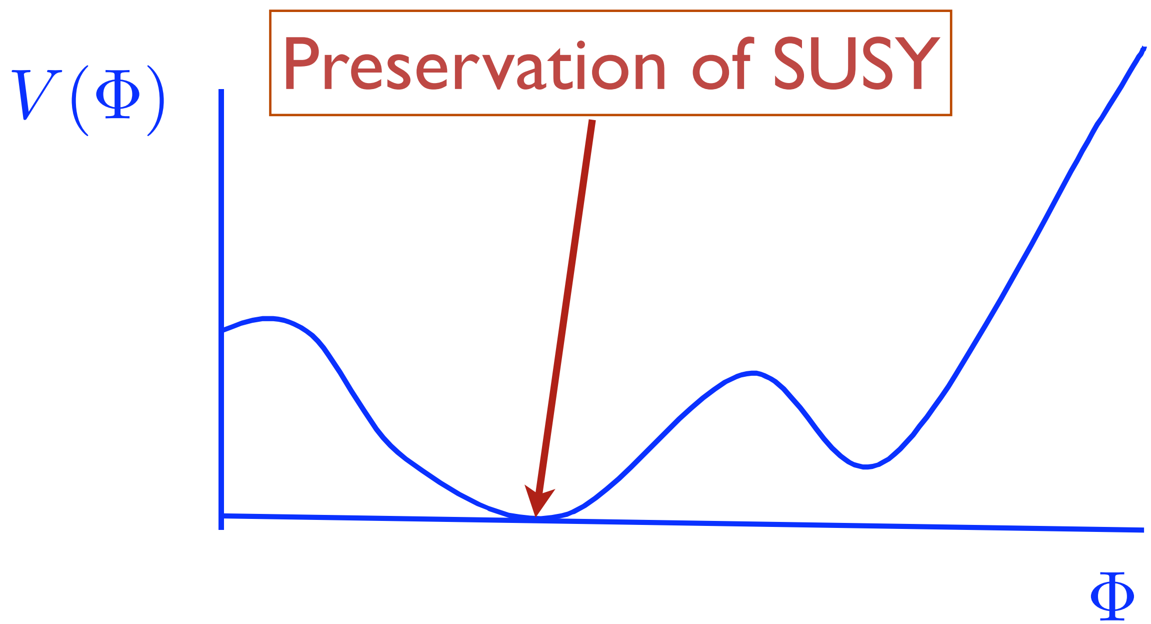
$$< \ H \ > = \ E \geq 0$$

- Second, if the theory is supersymmetric, then the vacuum state should be annihilated by supersymmetric charges

$$Q_\alpha |0> = 0, \qquad Q^\dagger_{\dot\alpha}|0> = 0 \qquad \Longrightarrow \qquad <0|H|0> = 0$$

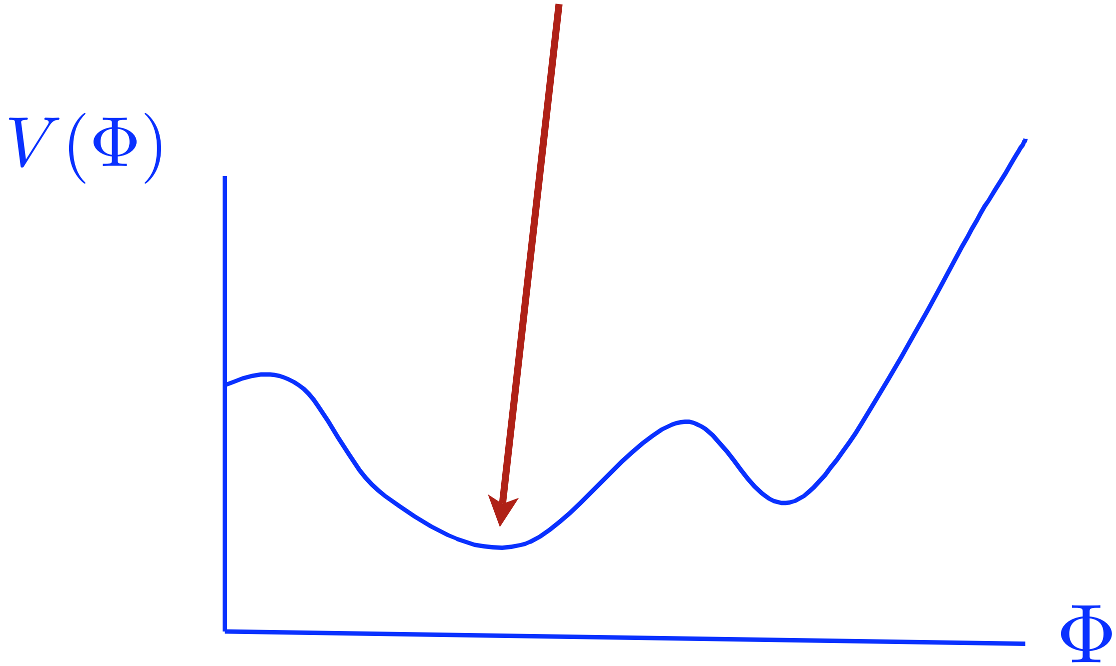
- So, the vacuum state energy is zero ! The vacuum energy is the order parameter for Supersymmetry breaking.

Effective Potential of a Supersymmetric Theory



Non-trivial Minimum could lead to the breakdown of gauge or global symmetries but SUSY is preserved, provided the value of the effective potential at the minimum is equal to zero, like in the Figure above.

Spontaneous breakdown of SUSY



If the Minimum of the Potential is at a value different from zero, then the vacuum state is not supersymmetric and SUSY has been broken spontaneously.
A massless fermion, the Goldstino, appears in the spectrum of the Theory.
In Supergravity (local supersymmetry) theories, this Goldstino appears as the longitudinal component of the Gravitino, of spin 3/2.

Four-component vs. two-component Weyl fermions

* A Dirac spinor is a four component object whose components are

$$\psi_D = \begin{pmatrix} \chi_\alpha \\ \bar{\psi}^{\dot{\alpha}} \end{pmatrix} \qquad \psi_D^C = \begin{pmatrix} \psi_\alpha \\ \bar{\chi}^{\dot{\alpha}} \end{pmatrix}$$

* A Majorana spinor is a four component object whose components are

$$\psi_M = \begin{pmatrix} \chi_\alpha \\ \bar{\chi}^{\dot{\alpha}} \end{pmatrix} \qquad \psi_M^C = \psi_M$$

* Gamma Matrices

$$\gamma^\mu = \begin{pmatrix} 0 & \sigma^\mu \\ \bar{\sigma}^\mu & 0 \end{pmatrix} \qquad \gamma^5 = \begin{pmatrix} -I & 0 \\ 0 & I \end{pmatrix}$$

* Observe that $\psi_{D,L} = \chi; \quad \psi_{D,R} = \bar{\psi} \equiv \psi\dagger$

* Usual Dirac contractions may then be expressed in terms of two components contractions

$$\bar{\psi}_D \psi_D = \psi\chi + h.c. \qquad \text{with } \bar{\psi}_D = (\psi^\alpha \quad \bar{\chi}_{\dot{\alpha}})$$

In particular:

$$\bar{\psi}_D \gamma^\mu \psi_D = \psi\sigma^\mu\bar{\psi} + \bar{\chi}\bar{\sigma}^\mu\chi = -\bar{\psi}\bar{\sigma}^\mu\psi + \bar{\chi}\bar{\sigma}^\mu\chi$$

Observe that Majorana particles lead to vanishing vector currents
Hence, they must be neutral under electromagnetic interactions

Chiral currents, instead, do not vanish

$$\bar{\psi}_D \gamma^\mu \gamma_5 \psi_D = \psi\sigma^\mu\bar{\psi} - \bar{\chi}\bar{\sigma}^\mu\chi = -\bar{\psi}\bar{\sigma}^\mu\psi - \bar{\chi}\bar{\sigma}^\mu\chi$$

They may couple to the Z boson

Other relations may be found in the literature

Superspace

- In order to describe supersymmetric theories, it proves convenient to introduce the concept of superspace.
- Apart from the ordinary coordinates x^μ, one introduces new anticommuting spinor coordinates θ^α and $\bar{\theta}_{\dot{\alpha}}$; $[\theta] = [\bar{\theta}] =$ -1/2.
- One can also define derivatives

$$\{\theta_\alpha, \theta_\beta\} = 0; \qquad \theta\theta\theta = 0; \qquad [\theta Q, \bar{\theta}\bar{Q}] = 2\theta\bar{\theta}\sigma^\mu P_\mu$$

$$\partial_\alpha = \frac{\partial}{\partial\theta^\alpha}; \qquad \partial_\alpha \theta^\beta = \delta_\alpha^\beta; \qquad \partial_\alpha(\theta^\beta\theta_\beta) = 2\theta_\alpha$$

Supersymmetry representation

- Supersymmetry is a particular translation in superspace, characterized by a Grassman parameter ξ.
- Supersymmetry generators may be given as derivative operators

$$Q_\alpha = i\left[-\partial_\theta - i\sigma^\mu\bar{\theta}\partial_\mu\right]$$

One can check that these differential generators fulfill the SUSY algebra.

- Superspace allows to represent fermion and boson fields by the same superfield, by fields in superspace
- The operator

$$\bar{D} = -\partial_{\dot{\alpha}} + i\theta\sigma^\mu\partial_\mu$$

commutes with the supersymmetry transformations.

- So, if a field depends only on the variable $y^\mu = x^\mu - i\bar{\theta}\sigma^\mu\bar{\theta}$, since $\bar{D}y^\mu = 0$

$$\text{satisfies } \bar{D}\Phi = 0$$

the supersymmetric transformation of such field depend only on y

Chiral Superfields: $(\bar{D}\Phi = 0)$

- A generic scalar, chiral field is given by

$$\Phi(x, \theta, \bar{\theta} = 0) = A(x) + \sqrt{2}\,\theta\,\psi(x) + \theta^2 F(x)$$
$$\Phi(x, \theta, \bar{\theta}) = \exp(-i\partial_\mu \theta \sigma^\mu \bar{\theta})\;\Phi(x, \theta, \bar{\theta} = 0)$$

The supersymmetric transformation of a chiral field is chiral.

- A, ψ and F are the scalar, fermion and auxiliary components.
- Under supersymmetric transformations, the components of chiral fields transform like

$$\begin{aligned} \delta A &= \sqrt{2}\xi\psi, \qquad\qquad \delta F = -i\sqrt{2}\bar{\xi}\bar{\sigma}^\mu \partial_\mu \psi \\ \delta\psi &= -i\sqrt{2}\sigma^\mu \bar{\xi}\partial_\mu A + \sqrt{2}\xi F \end{aligned}$$

- The F component transforms like a total derivative.

Properties of Chiral Superfields:

- The product of two superfields is another superfield.
- For instance, the F-component of the product of two superfields Φ_1 and Φ_2 is obtained by collecting all the terms in θ^2, and is equal to

$$[\Phi_1 \Phi_2]_F = A_1 F_2 + A_2 F_1 - \psi_1 \psi_2$$

- For a generic Polynomial function of several fields $P(\Phi_i)$, the result is

$$[P(\Phi)]_F = (\partial_{A_i} P(A)) F_i - \frac{1}{2} \left(\partial^2_{A_i A_j} P(A) \right) \psi_i \psi_j$$

- Finally, a single chiral field has dimensionality $[A] = [\Phi] = 1$, $[\psi]=$ $3/2$ and $[F] = 2$. For $P(A)$, $[P(\Phi)]_F = [P(\Phi)] + 1 \quad ([\theta] = [\bar{\theta}] =$ -1/2).

Expansion of a Chiral Superfield

* In the above, we only used the form of the chiral superfield at $\bar{\theta} = 0$

However, for many applications the full expression of the chiral superfield is necessary:

$$\Phi(x, \theta, \bar{\theta}) = \exp(-i\partial_\mu \theta \sigma^\mu \bar{\theta})\ \Phi(x, \theta, \bar{\theta} = 0)$$

$$\downarrow$$

$$\Phi(x, \theta, \bar{\theta}) = A(x) + i\partial^\mu A(x)\theta\sigma_\mu\bar{\theta} - \frac{1}{4}\partial^2 A(x)\theta^2\bar{\theta}^2$$

$$+\sqrt{2}\,\theta\psi(x) + i\frac{\theta^2}{2}\partial^\mu\psi(x)\sigma_\mu\bar{\theta} + F(x)\theta^2$$

Vector Superfields

*Vector Superfields are generic hermitian fields. The minimal irreducible representations may be obtained by:

$$V(x,\theta,\bar{\theta}) = -\left(\theta\sigma^{\mu}\bar{\theta}\right)V_{\mu} + i\theta^2\bar{\theta}\bar{\lambda} - i\bar{\theta}^2\theta\lambda + \frac{1}{2}\theta^2\bar{\theta}^2 D$$

*Vector Superfields contain a regular gauge vector field V_{μ}, its fermionic superpartner λ and an auxiliary scalar field D

Under supersymmetric transformations the components transform like:

$$\delta V_{\mu}^{a} = -\bar{\xi}\bar{\sigma}_{\mu}\lambda^{a} - \bar{\lambda}^{a}\bar{\sigma}_{\mu}\xi \qquad \delta\lambda_{\alpha}^{a} = -\frac{i}{2}(\sigma^{\mu}\bar{\sigma}^{\nu}\xi)_{\alpha}F_{\mu\nu}^{a} + \xi_{\alpha}D^{a}$$

$$\delta D^{a} = i(\bar{\xi}\bar{\sigma}^{\mu}\nabla_{\mu}\lambda^{a} - \nabla_{\mu}\bar{\lambda}^{a}\bar{\sigma}^{\mu}\xi) \qquad \text{with } \nabla_{\mu} = \partial_{\mu} + ig\,V_{\mu}^{a}\,T^{a}$$

The D component of a vector field transforms as a total derivative

- $D = [V] + 2$; $[V_{\mu}] = [V] + 1$; $[\lambda] = [V] + 3/2$. If V_{μ} describes a physical gauge field, then [V] = 0.

Superfield Strength and Gauge transformations

- Similarly to $F_{\mu\nu}$ in the regular case, there is a field that contains the field strength. It is a chiral field, derived from V ($W = -\bar{D}\bar{D}DV/4$), and it is given by

$$W^{\alpha}(x,\theta,\bar{\theta}=0) = -i\lambda^{\alpha} + (\theta\sigma_{\mu\nu})^{\alpha} F^{\mu\nu} + \theta^{\alpha} D - \theta^2 \left(\bar{\sigma}^{\mu}\mathcal{D}_{\mu}\bar{\lambda}\right)^{\alpha}$$

- Under gauge transformations, superfields transform like

$$\Phi \rightarrow \exp(-ig\Lambda)\Phi, \qquad W_{\alpha} \rightarrow \exp(-ig\Lambda) W_{\alpha} \exp(ig\Lambda)$$

$$\exp(gV) \rightarrow \exp(-ig\bar{\Lambda}) \exp(gV) \exp(ig\Lambda)$$

where Λ is a chiral field of dimension 0.

Towards a SUSY Lagrangian

The aim ⟶ construct a Lagrangian invariant under supersymmetric and gauge transformations

The variation $\delta\mathcal{L}$ should be a total derivative such that the action $\mathcal{S} = \int d^4x \ \mathcal{L}$ is invariant

Recall: The F-component of a chiral field (or products of chiral fields) & The D-component of a vector field transform under SUSY like a total derivative

If renormalizability is imposed, the dimension of all terms in the Lagrangian:

$$[\mathcal{L}_{int}] \leq 4$$

On the other hand the dimensions of the chiral and vector fields are:

$$[\Phi] = 1, \qquad [W_\alpha] = 3/2, \qquad [V] = 0.$$

and one should remember that $[V]_D = [V] + 2; \ [\Phi]_F = [\Phi] + 1.$

The Supersymmetric Lagrangian

- Once the above machinery is introduced, the total Lagrangian takes a particular simple form. The total Lagrangian is given by

$$\mathcal{L}_{\mathrm{SUSY}} = \frac{1}{4g^2}\left(Tr[W^\alpha W_\alpha]_F + h.c.\right) + \sum_i \left(\bar{\Phi}\exp(gV)\Phi\right)_D + \left([P(\Phi)]_F + h.c.\right)$$

where $P(\Phi)$ is the most generic dimension-three, gauge invariant, polynomial function of the chiral fields Φ, and it is called Superpotential. It has the general expression

$$P(\Phi) = c_i\Phi_i + \frac{m_{ij}}{2}\Phi_i\Phi_j + \frac{\lambda_{ijk}}{3!}\Phi_i\Phi_k\Phi_k$$

- The D-terms of V^a and the F term of Φ_i do not receive any derivative contribution: Auxiliary fields that can be integrated out by equation of motion.

SUSY Lagrangian in term of Component Fields

- The above Lagrangian has the usual kinetic terms for the boson and fermion fields. It also contain generalized Yukawa interactions and contain interactions between the gauginos, the scalar and the fermion components of the chiral superfields.

$$
\begin{aligned}
\mathcal{L}_{\mathrm{SUSY}} &= (\mathcal{D}_\mu A_i)^\dagger \mathcal{D} A_i + \left(\frac{i}{2}\bar{\psi}_i \bar{\sigma}^\mu \mathcal{D}_\mu \psi_i + \mathrm{h.c.}\right) \\
&- \frac{1}{4}\left(G^a_{\mu\nu}\right)^2 + \left(\frac{i}{2}\bar{\lambda}^a \bar{\sigma}^\mu \mathcal{D}_\mu \lambda^a + \mathrm{h.c.}\right) \\
&- \left(\frac{1}{2}\frac{\partial^2 P(A)}{\partial A_i \partial A_j}\psi_i\psi_j - i\sqrt{2} g A_i^* T_a \psi_i \lambda^a + h.c.\right) \\
&- V(F_i, F_i^*, D^a)
\end{aligned}
$$

The last term is a scalar potential term that depends only on the auxiliary fields

Notation bookkeeping

- All standard matter fermion fields are described by their left-handed components (using the charge conjugates for right-handed fields) ψ_i
- All standard matter fermion superpartners are described the scalar fields A_i. There is one complex scalar for each chiral Weyl fermion
- Gauge bosons are inside covariant derivatives and in the $G_{\mu\nu}$ terms.
- Gauginos, the superpartners of the gauge bosons are described by the fermion fields λ_a. There is one Weyl fermion for each massless gauge boson.
- Higgs bosons and their superpartners are described as regular chiral fields. Their only distinction is that their scalar components acquire a v.e.v. and, as we will see, they are the only scalars with positive R-Parity.

The Scalar Potential

$$V(F_i, F_i^*, D^a) \;=\; \sum_i F_i^* F_i \;+\; \frac{1}{2}\sum_a (D^a)^2$$

where the auxiliary fields may be obtained from their equation of motion, as a function of the scalar components of the chiral fields:

$$F_i^* = -\frac{\partial P(A)}{\partial A_i}, \qquad\qquad D^a = -g\sum_i (A_i^* T^a A_i)$$

Observe that the quartic couplings are governed by the gauge couplings and that scalar potential is positive definite ! The latter is not a surprise. From the supersymmetry algebra, one obtains,

$$H = \frac{1}{4}\sum_{\alpha=1}^{2}\left(Q_\alpha^\dagger Q_\alpha + Q_\alpha Q_\alpha^\dagger\right)$$

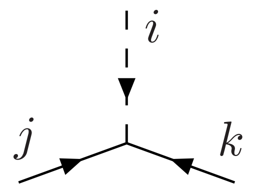
- If for a physical state the energy is zero, this is the ground state.
- Supersymmetry is broken if the vacuum energy is non-zero !

Couplings

Recall the scalar part of the superpotential $P(A) = \frac{m_{ij}}{2} A_i A_j + \frac{\lambda_{ijk}}{6} A_i A_j A_k$

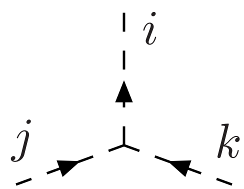
- The Yukawa couplings between scalar and fermion fields

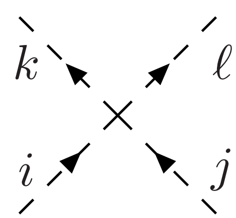
$$\frac{1}{2} \frac{\partial^2 P(A)}{\partial A_i \partial A_j} \psi_i \psi_j + h.c. \quad \rightarrow \quad \lambda_{ijk}\ \psi_i\ \psi_j\ A_k$$



are governed by the same couplings as the scalar interactions coming from

$$\left(\frac{\partial P(A)}{\partial A_i} \right)^2 \quad \rightarrow m^*_{ml} \lambda_{mjk}\ A^*_i\ A_j A_k \text{ and } \lambda_{mjk}\ \lambda^*_{mil} A_j A_k A^*_i A^*_l$$





The superpotential parameters determine all non-gauge interactions

- Similarly, the gaugino-scalar fermion interactions coming from

$$- i\sqrt{2} g A_i^* T_a \psi_i \lambda^a + h.c.$$

are governed by the gauge couplings

No new Couplings!

same couplings are obtained by replacing particles by their superpartners and changing the spinorial structure

Masses

The superpotential parameters determine also the matter field masses and give equal masses to fermions and scalars when the Higgs acquires a v.e.v

$$m_f^2 = m_s^2 = \lambda_{ffh}^2 \, \mathrm{v}^2$$

SUSY corrections to the Higgs mass parameter:

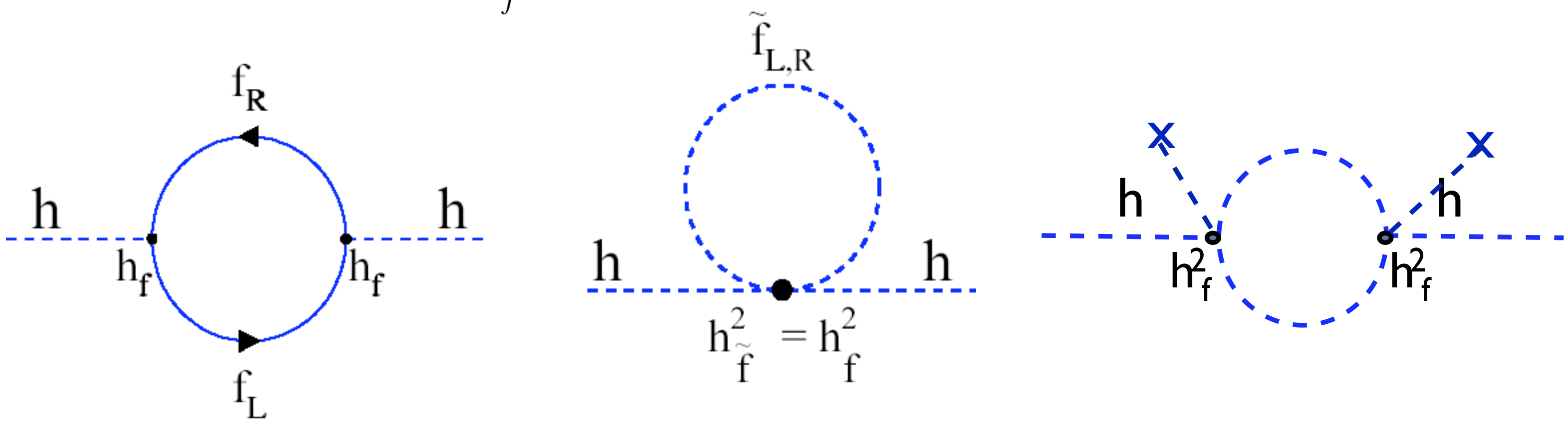
For every fermion there is a boson with equal mass and couplings

Self energy of an elementary scalar related by SUSY to the self energy of a fermion

Cancellation of quadratic divergences in Higgs mass quantum corrections has to do with SUSY relation between couplings and bosonic and fermionic degrees of freedom

$$\Delta\mu^2 \approx g^2_{hf\bar{f}}[m_f^2 - m_{\tilde{f}}^2]\ln(\Lambda^2_{eff}/m_h^2)$$

SUSY must be broken in nature



In low energy SUSY: quadratic sensitivity to Λ_{eff} replaced by quadratic sensitivity to SUSY breaking scale

The scale of SUSY breakdown must be of order 1 TeV, if SUSY is associated with scale of electroweak symmetry breakdown

Properties of Supersymmetric theories

- To each complex scalar A_i (two degrees of freedom) there is a Weyl fermion ψ_i (two degrees of freedom)
- To each gauge boson V_μ^a, there is a gauge fermion (gaugino) λ^a.
- The mass eigenvalues of fermions and bosons are the same !
- Theory has only logarithmic divergences in the ultraviolet associated with wave-function and gauge-coupling constant renormalizations.
- Couplings in superpotential $P[\Phi]$ have no counterterms associated with them.
- The equality of fermion and boson couplings are essential for the cancellation of all quadratic divergences, at all oders in perturbation theory.

Types of supermultiplets

Chiral (or "Scalar" or "Matter" or "Wess-Zumino") supermultiplet:

1 two-component Weyl fermion, helicity $\pm\frac{1}{2}$. ($n_F = 2$)

2 real spin-0 scalars = 1 complex scalar. ($n_B = 2$)

The Standard Model quarks, leptons and Higgs bosons must fit into these.

Gauge (or "Vector") supermultiplet:

1 two-component Weyl fermion gaugino, helicity $\pm\frac{1}{2}$. ($n_F = 2$)

1 real spin-1 massless gauge vector boson. ($n_B = 2$)

The Standard Model $\gamma, Z, W^{\pm}, g$ must fit into these.