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Gauge-Higgs Unification (Extra material)

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Lecture 5: Gauge-Higgs Unification

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1. Basics of gauge-Higgs unification (GHU)

- Idea: A_5 4D scalar could be Higgs. How to find a setup where A_5 is a doublet of SU(2)xU(1) with correct hypercharge?
- Ideally, use flat space, and NO induced Scalars, just orbifold BCs

History: **1979 Manton, use 6D with monopole in sphere**

1983 Hosotani, "Wilson line" breaking: "Hosotani mechanism"

1998 Hatanaka, Inami, Lim: revive idea, no concrete model

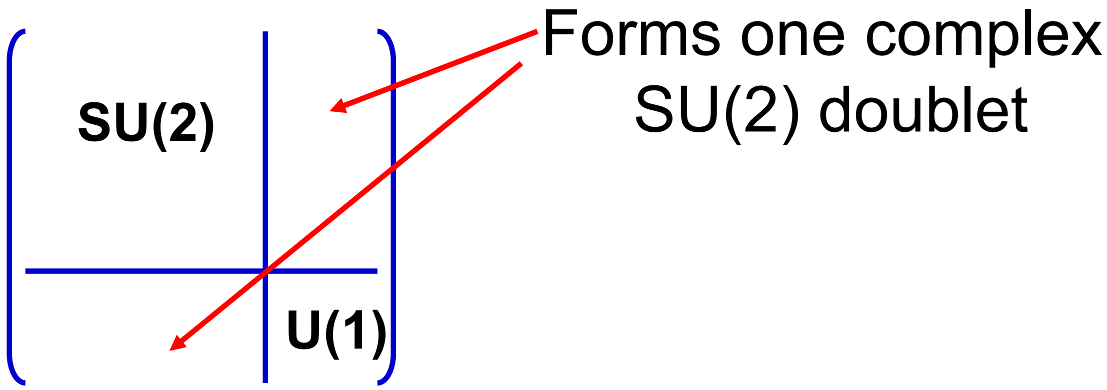
2001 Antoniadis, Benakli, Quiros: basic model, Higgs potential calculation

2002 C.C., Grojean, Murayama; von Gersdorrf, Irges, Quiros: 6D problems, basics of flavor construction

2003 Scrucca, Serone, Silvestrini (+Wulzer): basic 5D model introduced and alayzed

2005 Cacciapaglia, C.C., Park; Panico, Serone, Wulzer: close to realistic model

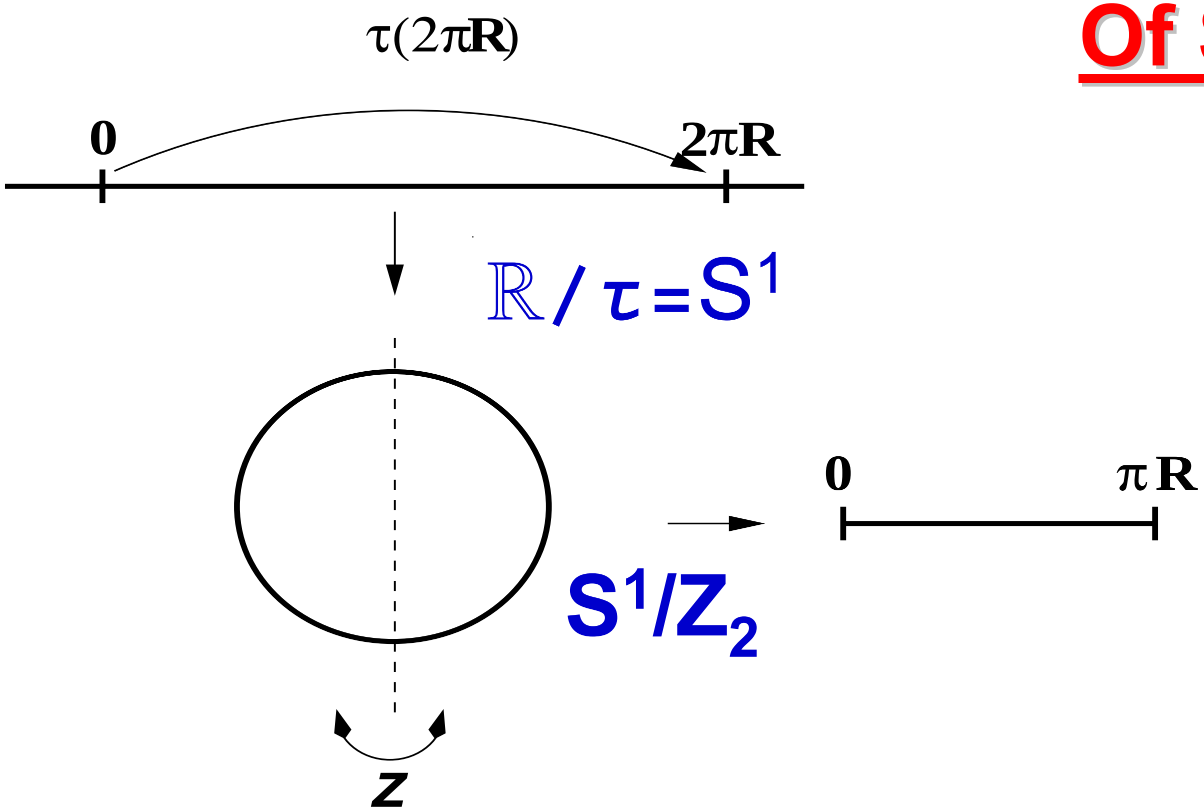
- A_5 is in adjoint of gauge group, but Higgs is doublet: need to enlarge gauge group.
- If we want to use simplest orbifold (does not reduce rank): extended gauge group would be rank 2
- Simplest rank 2 group SU(3)



2. Orbifolds

•Next simplest possibility: instead of circle comapctify on a line segment S^1/Z_2. Will look in two silghtly different approaches (orbifold vs. interval).

Geometric construction Of S^1/Z_2



Effects on the fields

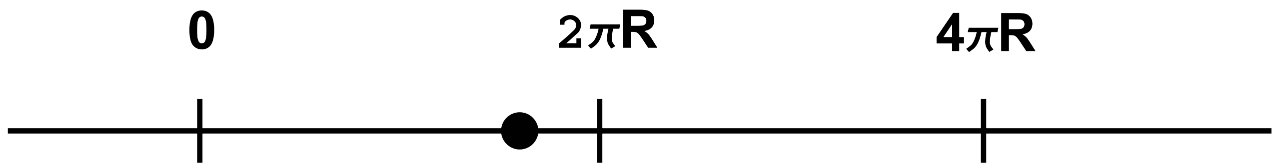
- τ, Z have to be symmetries of action
- Fields have to agree UP TO a symmetry transformation **T,Z** (**T** is SS-twist)

$$\tau(2\pi R)\varphi(y) = T^{-1}\varphi(y + 2\pi R)$$
$$\mathcal{Z}\varphi(y) = Z\varphi(-y)$$

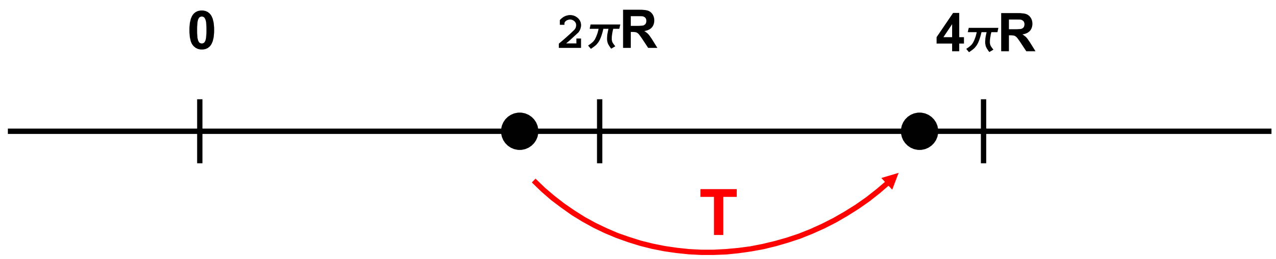
- Field identification will be

$$\varphi(y + 2\pi R) = T\varphi(y)$$
$$\varphi(-y) = Z\varphi(y)$$

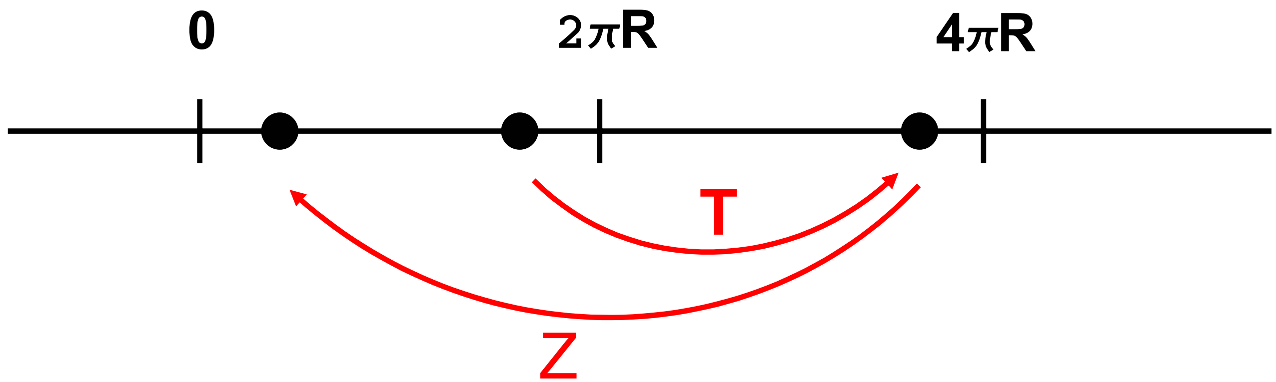
A consistency condition



A consistency condition



A consistency condition



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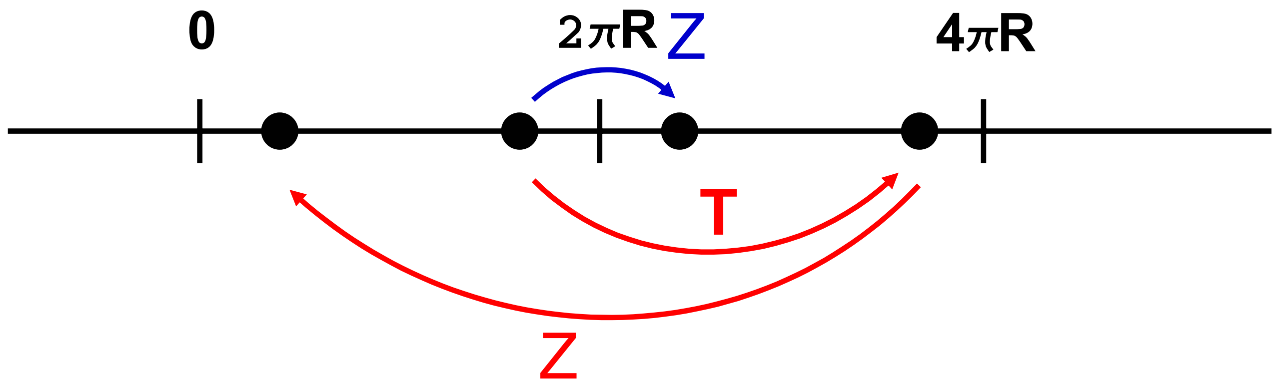

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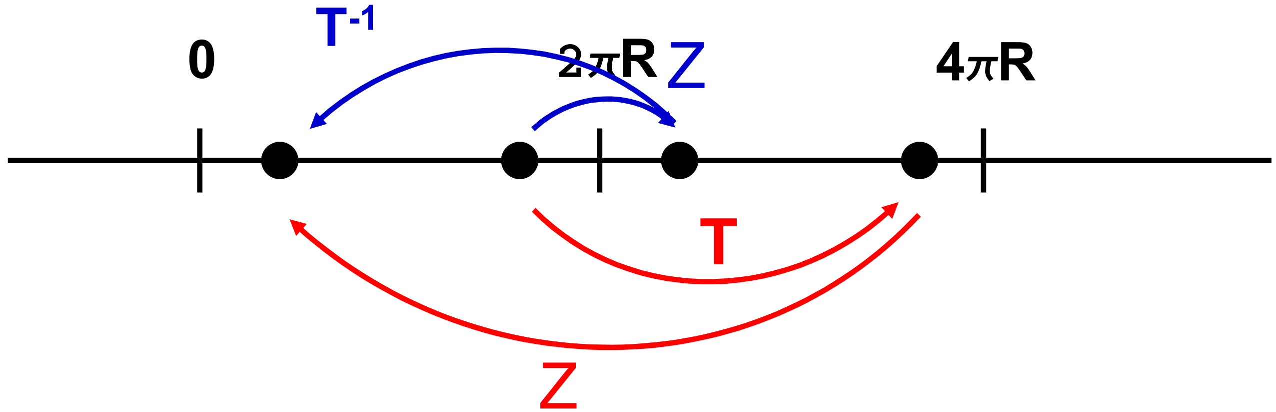
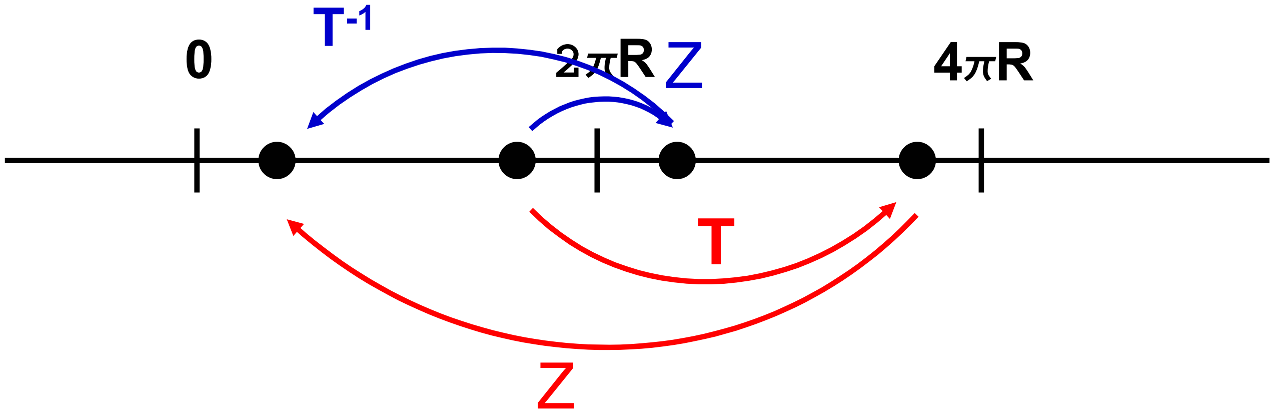


A consistency condition



$$Z\,T = T^{-1}\,Z$$

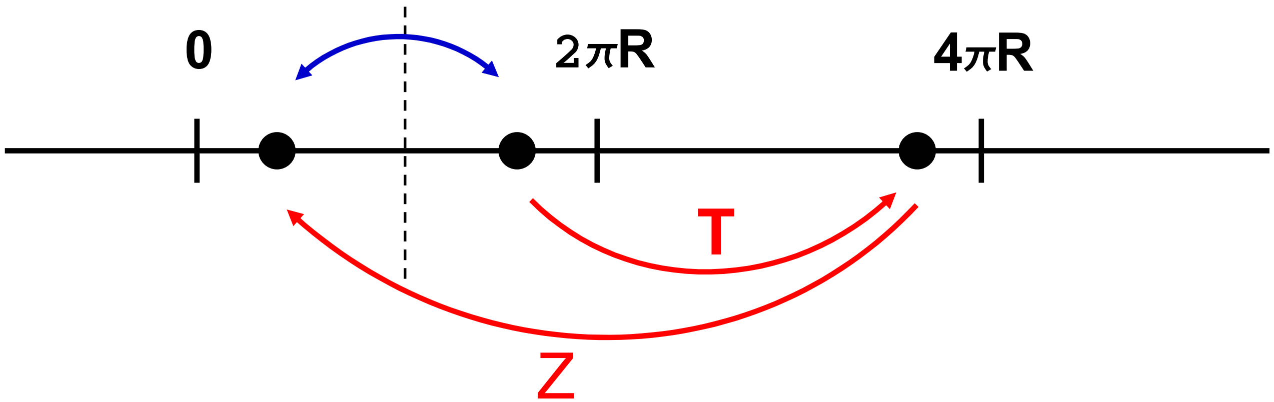
- **Z** is a projection $Z^2 = 1$

$$Z\,T\,Z = T^{-1}$$

- **ZT** is also a projection

$$(ZT)^2 = ZTZT = T^{-1}T = 1$$

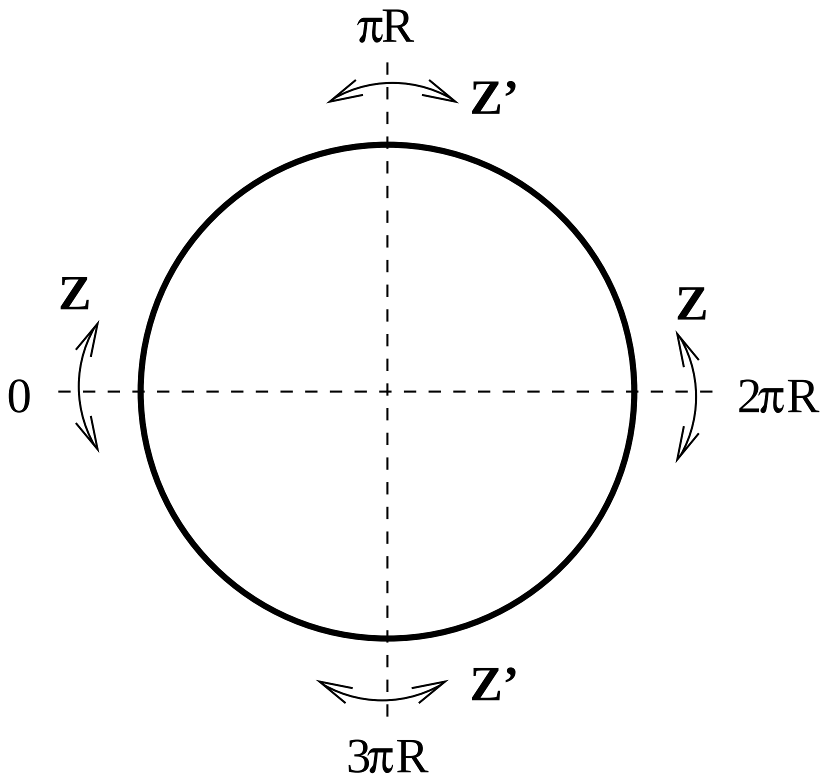
The effect of ZT



•**ZT** is a reflection around π**R**

•A generic $\mathbf{S^1/Z_2}$ is a combination of two (not necessarily commuting) parities **Z** and **ZT**

A simple way to picture the orbifold BC's



- Need to assign + or – parities to fields
- Parity assignments don't have to be in same basis (eg. there could be a Scherk-Schwarz twist if parities don't commute)

The orbifold BC's

- Assign parities under two Z_2's:
  - Scalars: $\varphi(-y)=P\varphi(y)$. $P=\pm 1$
  - Gauge fields: $A_\mu(-y)= PA_\mu(y)P^{-1}$

    $A_5(-y)=-PA_5(y)P^{-1}$
  - Fermions: $\chi(-y)=P\chi(y)$

    $\psi(-y)=-P\psi(y)$

- Reason:
  - A_5 opposite parity as A_μ (vector)
  - Term in fermion action: $\psi\partial_5\chi$

- The KK spectrum

  - Gauge bosons: If A_μ has zero mode, A_5 will NOT (and vice versa)
  - LH (χ) and RH (ψ) fermions have opposite BC's: if one has zero mode, the other doesn't $\rightarrow$ theory CHIRAL

3. Application to GH unification

- The necessary projection (at both endpoints):

$$P=\left(\begin{array}{cc|c} 1 & & \\ & 1 & \\ \hline & & -1 \end{array}\right)$$

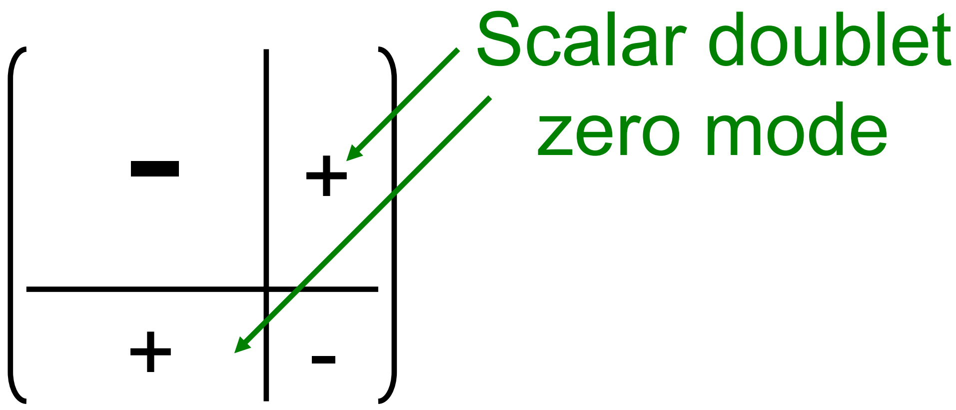
- Action on A_μ

$$PA_\mu(-y)P^{-1} \qquad \left(\begin{array}{c|c} + & - \\ \hline - & + \end{array}\right)$$

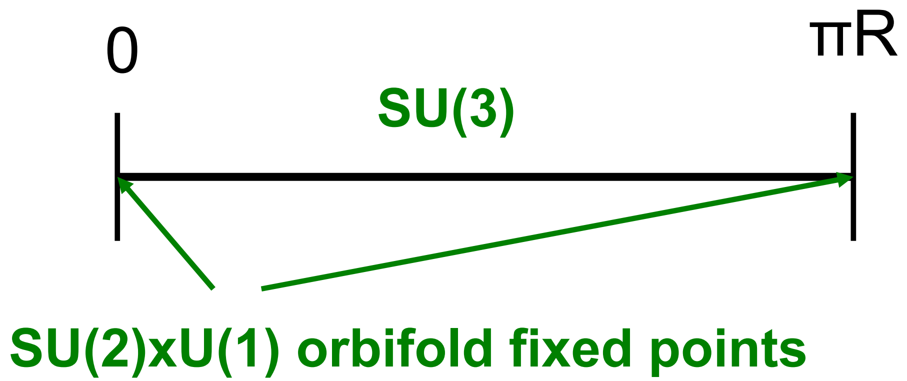
SU(2)xU(1) gauge zero modes

•Action on A_5:

$-PA_5(-y)P^{-1}$



•Picture:



•For (+,+) fields: **$\cos(ny/R)$, $m_n^2=n^2/R^2$**

•For (-,-) fields: **$\sin(ny/R)$, $m_n^2=n^2/R^2$**

•Why is this interesting? 5D gauge invariance:

$$\begin{aligned} A_\mu &\rightarrow A_\mu + \partial_\mu \epsilon(x,y) + i[\epsilon(x,y), A_\mu] \\ A_5 &\rightarrow A_5 + \partial_5 \epsilon(x,y) + i[\epsilon(x,y), A_5] \end{aligned}$$

•ε: gauge transformation param., has its own KK expansion (same as A_μ). For broken dir. $\varepsilon(0,\pi R)=0$, BUT $\partial_5 \varepsilon \neq 0$.

•Shift symmetry protects A_5 from mass even at fixed points where gauge symmetry broken

•Shift symmetry analog of broken global sym. in little Higgs models protecting Higgs.

•Shift symmetry forbids tree-level potential Also **local** radiative potential for Higgs forbidden (formulation as SS theory)
•**Non-local** loop effects could still give a **finite** Higgs potential (loop has to stretch from one fixed point to other – does not shrink to zero – result must be finite…)

•Gauge-Higgs unification protects Higgs from divergences due to higher dim. gauge invar.
•Higgs potential only generated through finite loop effects

4. The calculation of the Higgs potential

- Need Coleman-Weinberg potential for Higgs
- Assume simplest SU(3) model for now
- Higgs VEV normalization:

$$A_5 = \frac{1}{\sqrt{2}}\begin{pmatrix} - & H_5 \\ H_5^\dagger & - \end{pmatrix}$$

$$\langle H_5 \rangle = \sqrt{2}\begin{pmatrix} 0 \\ \alpha/R \end{pmatrix}$$

- **α**: VEV in units of radius. For realistic model needs to be **≪1** (to separate KK modes from) SM particles

- For Coleman-Weinberg need α-dependent Mass spectrum. For example gauge KK:

cos(ny/R) — sin(ny/R)

$$\left(\begin{array}{cc|c} \frac{1}{\sqrt{2}}A^3+\frac{1}{\sqrt{6}}A^8 & W^+ & \tilde{W}^1 \\ W^- & -\frac{1}{\sqrt{2}}A^3+\frac{1}{\sqrt{6}}A^8 & \tilde{W}^2 \\ \hline \tilde{W}^{1*} & \tilde{W}^{2*} & -\frac{2}{\sqrt{6}}A^8 \end{array}\right)$$

- Mass terms come from: $-\int_0^{\pi R}\frac{1}{2}\mathrm{Tr}\ F_{5\mu}^2$

0 in unitary gauge

$$-\frac{1}{2}\int_0^{\pi R}\mathrm{Tr}\left(\partial_5 A_\mu-\partial_\mu A_5+g_5[\langle A_5\rangle, A_\mu]\right)^2$$

- The mass matrix mixes various components In the 8x8 basis A_1-A_8 the mixing matrix is:

- TeXForm on the Mathematica output:

$$\frac{1}{R^2}\begin{pmatrix} 2\left(\alpha^2+n^2\right) & 0 & 0 & 0 & 4\alpha n & 0 & 0 & 0 \\ 0 & 2\left(\alpha^2+n^2\right) & 0 & -4\alpha n & 0 & 0 & 0 & 0 \\ 0 & 0 & 2\left(\alpha^2+n^2\right) & 0 & 0 & 0 & -4\alpha n & -2\sqrt{3}\alpha^2 \\ 0 & -4\alpha n & 0 & 2\left(\alpha^2+n^2\right) & 0 & 0 & 0 & 0 \\ 4\alpha n & 0 & 0 & 0 & 2\left(\alpha^2+n^2\right) & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2n^2 & 0 & 0 \\ 0 & 0 & -4\alpha n & 0 & 0 & 0 & 2\left(4\alpha^2+n^2\right) & 4\sqrt{3}\alpha n \\ 0 & 0 & -2\sqrt{3}\alpha^2 & 0 & 0 & 0 & 4\sqrt{3}\alpha n & 2\left(3\alpha^2+n^2\right) \end{pmatrix}$$

- Eigenvalues:

n^2/R^2	x2	$\leftarrow \gamma$
$(n\pm\alpha)^2/R^2$	x2	$\leftarrow W_\pm$
$(n\pm 2\alpha)^2/R^2$	x1	$\leftarrow Z$

- Implies most problematic part of model:

$$\mathbf{M_Z^2/M_W^2=2}$$

- Obviously due to wrong U(1) quantum number of Higgs

- Unbroken U(1) after orbifolding: T_8
- Higgs quantum number:

Usual normalization:

- $g \rightarrow 1/2 \text{ diag } (1,-1)$, etc
- $g' \rightarrow$ Higgs quantum number 1/2

- Here: for $\text{Tr } T_a T_b = 1/2$: $T_8 = 1/(2\sqrt{3}) \text{ diag}(1,1,-2)$
- Higgs quantum number $\sqrt{3}/2$. Rescale U(1):
- **$\sqrt{3}/2 g = g'/2$**

$\sin^2\theta_W = g'^2/(g^2+g'^2) = 3/(1+3) = 3/4$

- Wrong U(1) normalization, need another U(1)

The Coleman-Weinberg potential

(Antoniadis, Benakli, Quiros)

$$V_{CW}(\phi) = \frac{1}{2}\sum_I (-1)^{F_I} \int \frac{d^4p}{(2\pi)^4} \log(p^2 + M_I^2(\phi))$$

- General form of KK mass spectrum (ABQ)

$$M_{\vec{m}}^2 = \mu^2 + \sum_{i=1}^{d} \frac{(m_i + a_i(\phi))^2}{R_i^2}$$

- Expression for potential in general case in 5D:

$$V_{eff}(\beta) = \frac{\mp 1}{32\pi^2}\frac{1}{(\pi R)^4}\mathcal{F}(\beta)$$

β=k α

- Where for no bulk mass term **$m_n^2 = (n+\beta)^2/R^2$**

$$\mathcal{F}(\beta) = \frac{3}{2}\sum_{n=1}^{\infty}\frac{\cos(2\pi\beta n)}{n^5}$$

- With bulk mass term **$m_n^2 = M^2 + (n+\beta)^2/R^2$**

$$\mathcal{F}_{\kappa}(\beta) = \frac{3}{2}\sum_{n=1}^{\infty}\frac{e^{-\kappa n}\cos(2\pi\beta n)}{n^3}\left(\frac{\kappa^2}{3} + \frac{\kappa}{n} + \frac{1}{n^2}\right)$$

- Where **$\kappa = 2\pi MR$**. For large κ exponentially suppressed.

Comments

- n=1 term most important in series **$\pm\cos 2\pi\beta$**
- For fermions min. for β**=½**
- For bosons min. for β**=0**
- For twisted fermions (will see later) spectrum

$$m_n^2 = M^2 + (n+\tfrac{1}{2}+\beta)^2/R^2$$

- Effect in potential **$\beta\rightarrow\beta+\tfrac{1}{2}$**

Summary:

Can calculate finite Higgs potential for arbitrary bulk fields. Need to know, what bulk fields…

6.The fermion fields & flavor structure

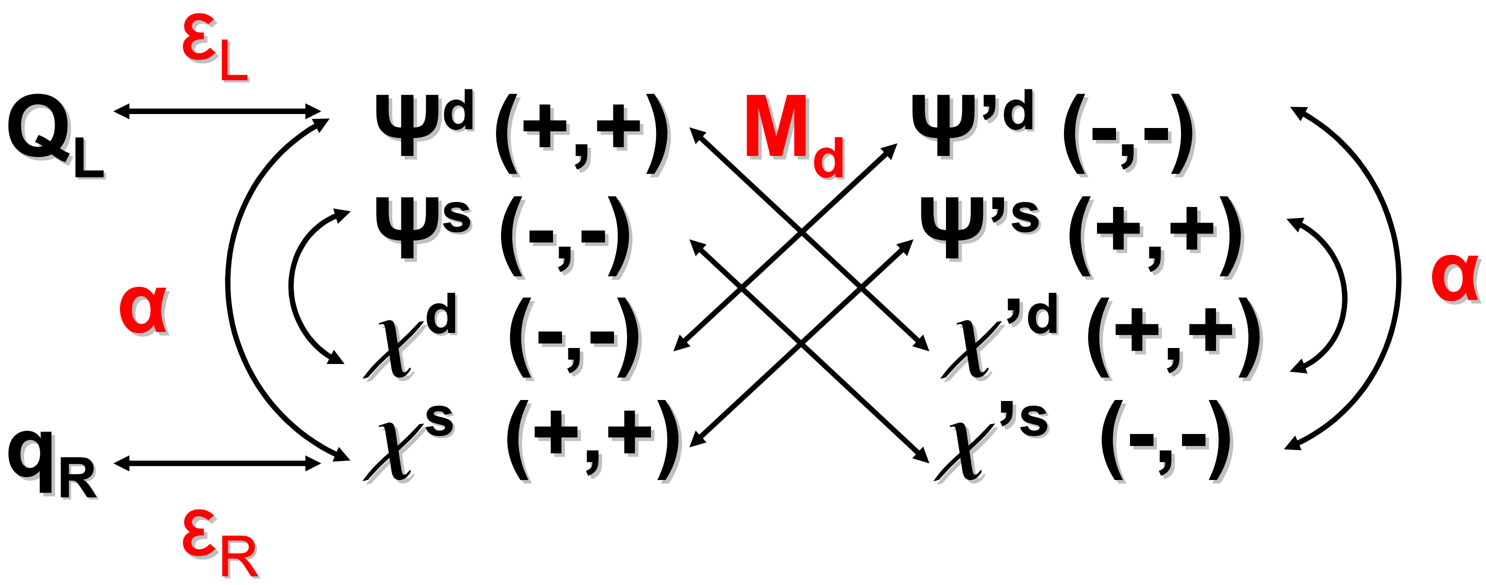
- Apparent problem: since Higgs=$\mathbf{A_5}$, Yukawa coupling=gauge coupling. How to get fermion mass hierarchy?

1. Use Arkani-Hamed **Schmaltz** idea of localizing fermions at different parts of 5D
2. Use bulk fermions mixed with localized fermions at the fixed points (an X-D version of Frogatt-Nielsen)

- Will use second approach

Example: down quark

- Use bulk triplets **3** and no twisting



- Need to write down coupled bulk equations
- Can diagonalize bulk equations
- BC's will provide equation for KK masses

7.A semi-realistic model

- To fix $\sin^2\theta_W$ we add an additional $U(1)_X$
- Gauge group $SU(3)xU(1)_X$ broken by orbifold to $SU(2)_L xU(1)_8 xU(1)_X$, and $U(1)_8 xU(1)_X \rightarrow U(1)_Y$ on the fixed point (localized Higgs or anomaly)
- This last breaking distorts wave functions, we'll have to pay the price for that…

Two main problems:

(Scrucca, Serone, Silvestrini)

- Higgs mass too small (& KK modes light)
- Top mass too small

- Reason: if assume (well motivated)
  - all mixings of same order
  - fermion hierarchy only from bulk masses

- Most bulk masses very large, contribution to CW very suppressed. Basically top dominates radiative potential, and minimum of top+gauge contribution gives

$$\alpha \sim 0.3, \quad m_h \sim 0.2 - 0.3 m_W$$
$$1/R \sim 3 - 5 m_W \sim 250 - 400 \text{GeV}$$
$$m_t \leq m_W$$

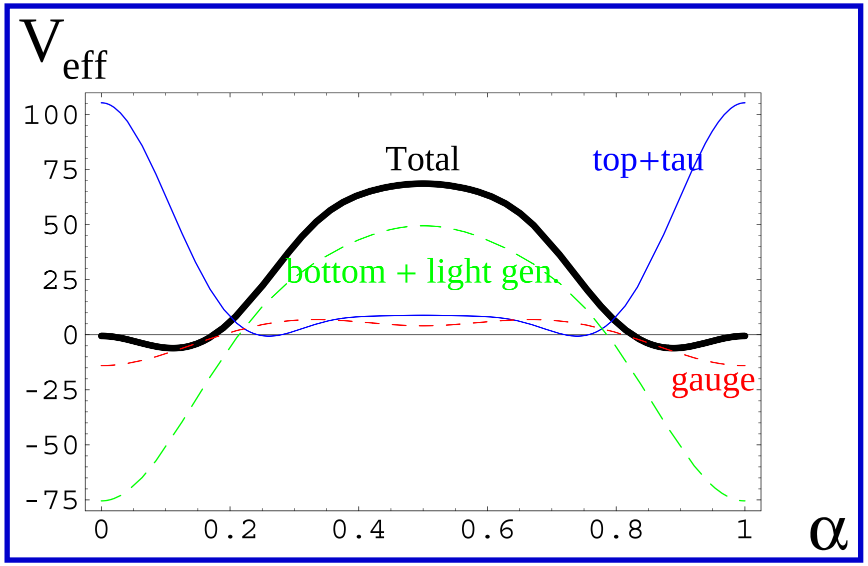
- This is obviously bad

- **Fix Higgs mass and VEV**: assume that some light fermions light **due to small mixing** rather than due to large bulk mass
- These bulk fermions will also contribute
- Take different representations and twist some of fermions→get a much more versatile Higgs potential

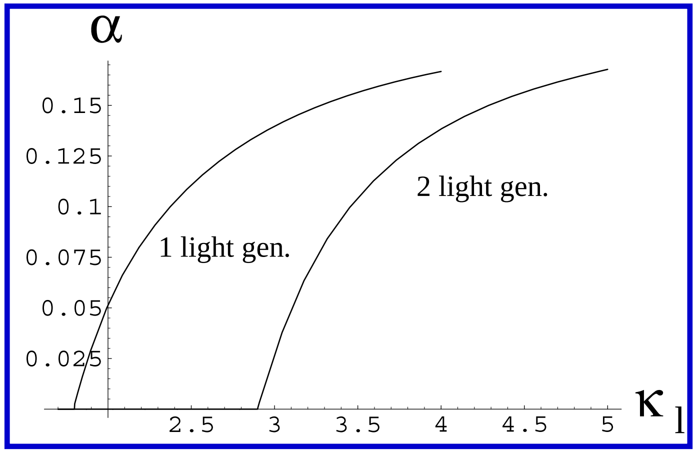
A successful example

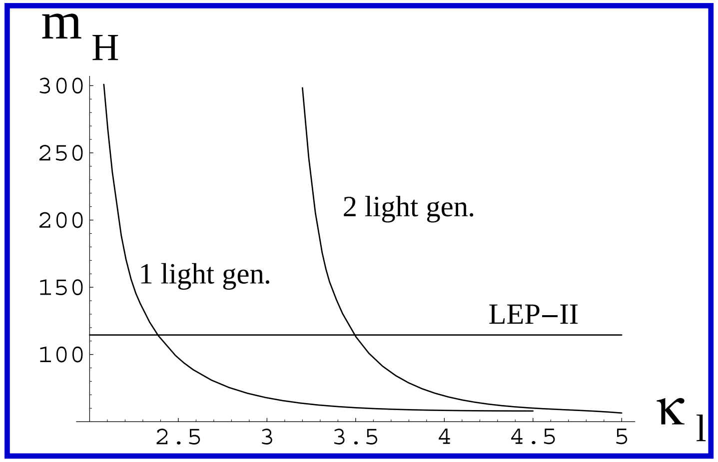
- Top: rep. $\mathbf{\overline{6}}$, large mixing $\epsilon_{L,R}\sim 3$, $\kappa_t \sim 1$
- Bottom: twisted **3**, $\kappa_b = 0$
- Tau: **10**, $\kappa_\tau = 1$
- Light gens: twisted $\mathbf{3+\overline{6}+10}$, common κ_l

• The Higgs potential:



• VEV and Higgs mass





•**Fix top mass:** upper bound on fermion mass actually depends on representation

$$m_t \leq k m_W$$

•**k^2:** number of indices of rep. top is embedded

•For $\mathbf{m_t=2m_W}$ need a 4-index irrep…

•Simplest possibility $\overline{\mathbf{15}}$ dim rep:

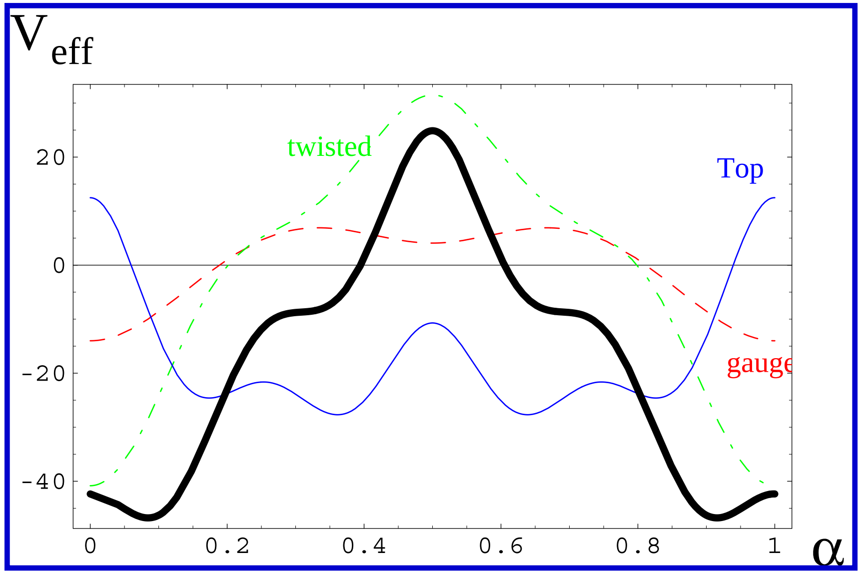
$$(\overline{15})_{-2/3} \rightarrow (1,2/3)+(2,1/6)+(3,-1/3)+(4,-5/6)+(5,-4/3)$$

•To get biggest top mass (**$2m_W$**) need top to be a bulk zero mode. So we only add a single $\overline{\mathbf{15}}$ with usual orbifold projections. Remove ad'l zero modes via mixing with localized fields

- For EWSB third generation enough (twisted fermions for **b**,τ). Possible reps (choose them as small as possible to not lower cutoff further)

	bottom	tau
model a	$(\mathbf{3},\mathbf{3})_0$	$(\mathbf{1},\mathbf{10})_0$
model b	$(\mathbf{3},\mathbf{6})_{1/3}$	$(\mathbf{1},\mathbf{3})_{-2/3}$

- The Higgs potential



8. Summary

- In extra dim's a possible solution to hierarchy problem is via gauge-Higgs unification
- Need to extand gauge group and orbifold it to **SU(2)xU(1)**
- Simplest (and most realistic) example in 5D **$SU(3)xU(1)_X$**
- Generically hard to get a large separation of Higgs VEV and KK modes, and heavy Higgs, top
- Can use many bulk fermions to generate a sufficiently generic Higgs pot.
- Top mass fixed via large bulk representation
- Constraints from **Zbb**, $\Delta\rho$: little hierarchy …