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Higgs and Electroweak Symmetry Breaking
(Lectures 4 & 5)

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The little Higgs

Hierarchy problem: expect new physics to appear at $\Lambda \sim$ TeV scale!

Problem: if there are particles with mass ~ 1TeV (which are responsible for solving the hierarchy problem) would not see them till $E \sim 1$TeV beyond current reach of detectors.

BUT: indirect effects should already have been seen.

Example:

Broken symmetry	op.	Suppression scale
B, L	$\frac{QQQL}{\Lambda^2}$	$\Lambda \gtrsim 10^{13}$TeV
1^{st}, 2^{nd} family family, CP	$\frac{\bar{d}s\bar{d}s}{\Lambda^2}$	$\Lambda \gtrsim 1000$TeV
2^{nd}, 3^{rd} fam. flavor	$\frac{m_b\ \bar{s}\sigma_{\mu\nu}F^{\mu\nu}b}{\Lambda^2}$	$\Lambda \gtrsim 50$TeV
custodial SU(2)	$(h^+ D_\mu h)^2 \frac{1}{\Lambda^2}$	$\Lambda \gtrsim 5$TeV
none S param	$D^2 h^+ D^2 h \frac{1}{\Lambda^2}$	$\Lambda \gtrsim 5$TeV

Generic new physics $\Lambda \gtrsim 5$TeV at least.

Little hierarchy problem: if there is new physics at TeV, why have we not already seen some indirect effects at least?

Idea of little higgs models: find a rationale why one-loop quadratic divergences to Higgs mass cancel. Then largest corrections from 2 loops $\Delta m_h^2 \propto \frac{\Lambda^2}{(16\pi^2)^2} \propto \left(\frac{\Lambda}{16\pi^2}\right)^2$

$\Lambda \sim 10\,\text{TeV}$ would be natural!

Could postpone the appearance of solution to hierarchy problem to 10 TeV — at the price of having some new physics at 1 TeV that cancels 1-loop divergences...

Idea: higgs as pseudo-Goldstone boson.

We know: if there is a spont. broken global symmetry, $\exists$ exactly massless scalar field → Goldstone boson. Can we somehow use Goldstone's thm. to protect the Higgs scalar from 1-loop quadratic divergences?

Goldstone bosons

U(1) example: ϕ complex scalar field

$\phi \to e^{i\alpha}\phi$ global symmetry (no gauging)

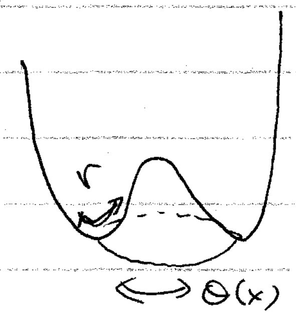
$$\mathcal{L} = \partial_\mu \phi^* \partial^\mu \phi - V(\phi^*\phi)$$

Assume potential Mexican hat:

$$V(\phi) = -\mu^2 \phi^*\phi + \lambda(\phi^*\phi)^2$$

$$\langle\phi\rangle = \frac{f}{\sqrt{2}} \qquad -\mu^2\cancel{\langle\phi\rangle} + 2\lambda\langle(\phi^*\phi)\rangle\cancel{\langle\phi\rangle} = 0$$

$$\boxed{f^2 = \frac{\mu^2}{\lambda}}$$



Parameterize field as:

$$\phi(x) = \frac{1}{\sqrt{2}}[f + r(x)] \; e^{i\theta(x)/f}$$

radial excitation

angle → no potential along this direction!

Action: $\phi^*\phi$ independent of θ → no mass term for θ field → GB

$$\partial_\mu\phi = \frac{\partial_\mu r}{\sqrt{2}} e^{i\theta/f} + \frac{1}{\sqrt{2}}(f+r)\, i\frac{\partial_\mu\theta}{f} e^{i\theta}$$

$$\mathcal{L} = \frac{1}{2} \left| \partial_\mu r + \left(1+\frac{r}{f}\right) i \partial_\mu \theta \right|^2 - V(r)$$

$$= \frac{1}{2}(\partial_\mu r)^2 + \frac{1}{2}(\partial_\mu \theta)^2 + \frac{1}{2}\frac{r^2}{f^2}(\partial_\mu \theta)^2 + \frac{r}{f}(\partial_\mu \theta)^2 - V(r)$$

no mass for θ, only <u>derivative</u> interactions.

Lagrangian invariant under <u>shift</u> symmetry

$\theta(x) \to \theta(x) + \alpha$ (remnant of global symmetry)

$\to$ forbids mass term!

<u>More complicated Goldstone bosons</u>

Assume $SU(N) \to SO(N-1)$ via VEV of fundamental field ϕ of $SO(N)$!

How many broken generators?

$SU(N) \to N^2-1$

$SU(N-1) \to (N-1)^2-1$

broken gen $= \#GB's = N^2-1 - (N-1)^2+1$

$= N^2-1-N^2+2N-1+1$

$= 2N-1$ real fields.

Which generators?

broken generators of ϕ

SU(N) $\rightarrow$ traceless hermitian $\rightarrow$

2N-1 of them.

<u>Theorem:</u> (non-linear σ-model, CWZ, ...)

parametrization of fields in terms of GB's.

$$e^{i\pi^a T^a/f}\,\phi_0$$

<u>always</u>

<u>In our case:</u> $\frac{1}{\sqrt{N(N-1)}}$ ...

$$\pi^a T^a = \begin{pmatrix} \pi^0 & & & \pi_1 \\ & \ddots & & \pi_2 \\ & & & \vdots \\ & & \pi_0 & \pi_{N-1} \\ \pi_1^* & \cdots & \pi_{N-1}^* & -\pi_0(N-1) \end{pmatrix}$$

$(N-1)^2 + N-1$
$= (N-1)N$

$$\begin{pmatrix} & \pi^1 \\ \pi_1^* & \end{pmatrix}\begin{pmatrix} & \pi_1 \\ \pi_1^* & \end{pmatrix} = 2\pi^1\pi_1^*$$

<u>Non-linear σ-model:</u> effective Lagrangian for GB's only

→ replace field ϕ with non-linear parametrization. If don't know underlying theory, just symmetry (or dynamics can generate lots of non-perturbative terms) → just write down all possible terms in terms of ϕ consistent with the symmetry!

We want to create a Higgs → PGB doublet of $SO(2)$! How to get a doublet?

Simplest possibility: $SO(3) \to SO(2)$

$2hh^* \to h_1^2 + h_2^2$

Pion matrix:

$$\Pi = \left(\begin{array}{cc|c} -\eta/2\sqrt{\frac{2}{3}} & & h \\ & -\eta/2\sqrt{\frac{2}{3}} & \\ \hline & h^+ & \eta\sqrt{\frac{2}{3}} \end{array}\right)$$

$\eta \to SO(2)$ singlet ignore for now.

What is the effective Lagrangian for the h field?
(valid below the cutoff scale Λ TBD?)

$(\partial_\mu \phi^*)(\partial^\mu \phi)$

$$= \left| \partial_\mu e^{i\Pi/f} \begin{pmatrix} 0 \\ f \end{pmatrix} \right|^2 =$$

$$e^{i\Pi/f}\begin{pmatrix}0\\f\end{pmatrix} = \left[1+\begin{pmatrix} & ih/f\\ \frac{ih^+}{f} & 0\end{pmatrix} + \frac{1}{2}\begin{pmatrix} & ih/f\\ ih^+/f & \end{pmatrix}\begin{pmatrix} & ih/f\\ ih^+/f & \end{pmatrix} + \dots\right]\begin{pmatrix}0\\f\end{pmatrix}$$

$$= \left[1 + \begin{pmatrix} & ih/f\\ ih^+/f & \end{pmatrix} + \frac{1}{2}\begin{pmatrix} -hh^+/f^2 & \\ & -\frac{h^+h}{f^2}\end{pmatrix}\right]\begin{pmatrix}0\\f\end{pmatrix}$$

$$= \begin{pmatrix} ih \\ f - \frac{h^+h}{2f}\end{pmatrix} + \dots$$

$$\phi \sim \begin{pmatrix} ih \\ f - \frac{h^+h}{2f}\end{pmatrix} + \dots$$

$$|\partial_\mu \phi|^2 = \begin{bmatrix} i\partial_\mu h \\ -\frac{[(\partial_\mu h^+)h + h^+\partial_\mu h]}{2f}\end{bmatrix}^2$$

$$= \partial_\mu h^+ \partial^\mu h + 2\frac{1}{4f^2}2|\partial_\mu h|^2 h^+h =$$

$$[\partial_\mu(h^+h)]^2$$

$$= \partial_\mu(h^+h)\,\partial^\mu(h^+h)$$

$$= -\partial_\mu\partial^\mu(h^+h)\,(h^+h)$$

$$\frac{1}{4f^2}\left((\partial_\mu h^+)h + h^+\partial_\mu h\right)\left((\partial_\mu h^+)h + h^+\partial_\mu h\right)$$

→ Fierz transformation

$$\rightarrow [\partial_\mu h^{+}]^2 + \frac{(\partial_\mu h)^2 h^+h}{f^2} + \dots$$

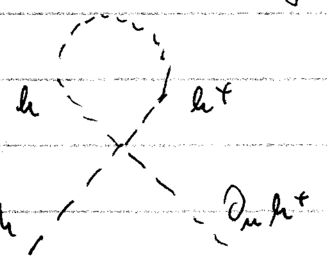
Lagrangian contains non-renormalizable terms (≡ eff. Lagrangian below scale of radial mode).

What is the cutoff scale?

when effect of loop = tree level operator.

from $\frac{|\partial_\mu h|^2 h^+ h}{f^2}$

→ get renormalization of leading kinetic term



$\propto \frac{\Lambda^2}{16\pi^2 f^2}$

$$\frac{\Lambda^2}{16\pi^2 f^2} = 1 \rightarrow \boxed{\Lambda = 4\pi f}$$

generic relation between cutoff scale

Summary: - theory produces global SU(2) Higgs doublet

- exact GB → exactly massless.
- becomes strongly coupled at $\Lambda = 4\pi f$

But: exact GB → shift symmetry, only ~~forbids~~ derivative coupling → unlike Higgs
forbids - gauge
- Yukawa
- quartic self-coupling

} need to generate them

1.) How to introduce SU(2) gauge interactions?

First attempt: just gauge SU(2)

→ couple h to SU(2) gauge bosons W_μ^a

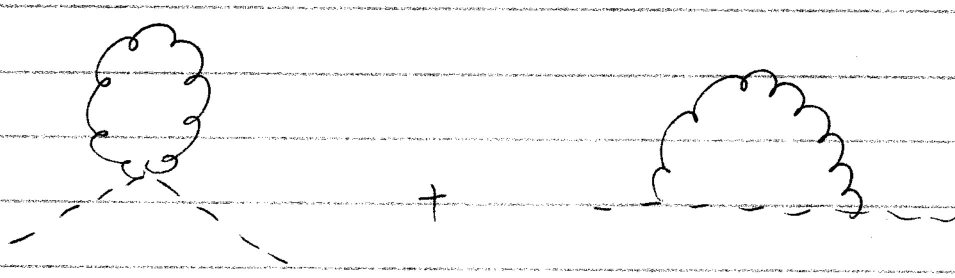
$$\mathcal{L} = |\partial_\mu h|^2 + (\partial_\mu h)^2 \frac{h^+ h}{f^2} + \dots$$

$$\partial_\mu h \to \left(\partial_\mu - i g W_\mu^a \frac{\tau^a}{2}\right) h$$

But: this term explicitly & completely breaks original SU(3) global symmetry!

Contains interaction term

$$|g W_\mu h|^2$$



$$\propto \frac{g^2}{16\pi^2} \Lambda^2 h^+ h$$

just like in SM. Should not be surprised — added SM interactions → get SM divergences!

Gained nothing: introduced SU(3), shift symmetry → broke shift symmetry → recovered quadratic divergence!

~~Method~~ <u>Write this in a more fancy way:</u>

$$\mathcal{L}_{int} = \left| g \begin{pmatrix} W_\mu & \\ & 0 \end{pmatrix} \phi \right|^2 \quad \phi = e^{i\Pi/f} \begin{pmatrix} 0 \\ 0 \\ f \end{pmatrix}$$

→ to get quadratically divergent piece:

$$\frac{g^2}{16\pi^2} \Lambda^2 \operatorname{Tr} M^{+} M$$

$$M = \begin{pmatrix} 1 & & \\ & 1 & \\ & & 0 \end{pmatrix} \phi$$

$$= \frac{g^2}{16\pi^2} \Lambda^2 \Big[\underbrace{\phi^+ \begin{pmatrix} 1 & & \\ & 1 & \\ & & 0 \end{pmatrix} \phi}_{h^+h + \dots} \Big] = \frac{g^2 \Lambda^2}{16\pi^2} h^+h \to \text{same quadratic div}$$

<u>Second attempt:</u> if there was no $\begin{pmatrix} 1 & & \\ & 1 & \\ & & 0 \end{pmatrix}$ projection matrix → would get a complete ~~matrix~~ $\phi^+\phi$ independent of Π's!

<u>Gauge</u> entire SU(3) so that we don't break the global symmetry explicitly!

$$\mathcal{L} = |D_\mu \phi|^2$$

→ quadratically divergent diagram:

$$\frac{g^2}{16\pi^2}\Lambda^2 \, \mathrm{Tr}\underbrace{\left(\phi^+ \begin{pmatrix}1&&\\&1&\\&&1\end{pmatrix}\phi\right)}_{\text{indep. of }\Pi\text{'s!}}$$

No dependence on Higgs field → no quadratic divergence! But ALSO NO Higgs.

This Goldstone is just eaten by SO(3) gauge field, not a physical field!

Need to follow this route, but such that we get a physical Goldstone at the same time!

~~Next~~ Third attempt: two copies of fields ϕ_1, ϕ_2

& add SO(3) invariant covariant derivatives for both. Expect: no quadratic divergence for both & only one linear combination eaten.

To be explicit:

$$\left.\begin{array}{l} \phi_1 = e^{i\Pi_1/f}\begin{pmatrix}0\\f\end{pmatrix} \\[1em] \phi_2 = e^{i\Pi_2/f}\begin{pmatrix}0\\f\end{pmatrix}\end{array}\right\} \text{Assumed VEV's aligned, } f_1 = f_2 = f \text{ for simplicity.}$$

$\mathcal{L} = |D_\mu \phi_1|^2 + |D_\mu \phi_2|^2$

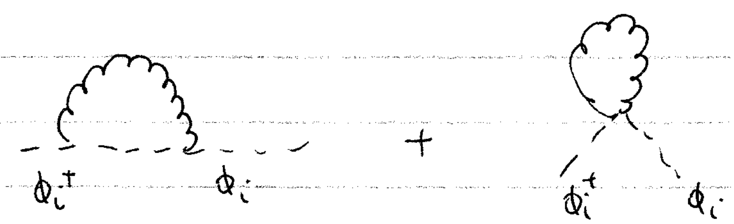
→ quadratic divergences:

$$\frac{\Lambda^2}{16\pi^2} g^2 \underbrace{\left(\phi_1^+\phi_1 + \phi_2^+\phi_2\right)}_{2f^2}$$

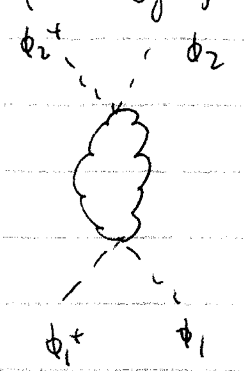
→ no potential for any GB!

$\pi_1 + \pi_2$ eaten, but $\pi_1 - \pi_2$ ~~[illegible]~~ remains as a physical GB!

Why no quadratic divergence?



A: As long as just one field appears in diagram → same as before, no possible quad. divergence.
However, if you have more than one field:



$$\propto \frac{g^4}{16\pi^2} |\phi_1^+\phi_2|^2 \log\left(\frac{\Lambda^2}{\mu^2}\right)$$

will give you log divergent mass for physical Higgs. Why?

$$\phi_1 = e^{i\begin{pmatrix} & h \\ h^+ & \end{pmatrix}/f} \begin{pmatrix} 0 \\ f \end{pmatrix}$$

<u>h: uneaten GB!</u>

$$\phi_2 = e^{-i\begin{pmatrix} & h \\ h^+ & \end{pmatrix}/f} \begin{pmatrix} 0 \\ f \end{pmatrix}$$

$$\phi_1^+ \phi_2 = (0\ f)\ e^{-2i/f\begin{pmatrix} & h \\ h^+ & \end{pmatrix}} \begin{pmatrix} 0 \\ f \end{pmatrix}$$

$$= f^2 - 2h^+h + \dots$$

→ mass² for higgs generated by loops:

$$\frac{g^4}{16\pi^2} \log\left(\frac{\Lambda^2}{\mu^2}\right) f^2 \qquad \sim (100\ \text{GeV})^2 \quad \text{if}$$

$$f \sim 1\,\text{TeV}, \quad g \sim g_{SU(2)}$$

Collective breaking

Why did we not a quadratic divergence → deeper reason

w/o gauging the SU(3) group

SU(3) × SU(3) global symmetry, each broken to SU(2)

$$\underbrace{SU(3)}_{\text{global sym.}} \times \overbrace{SU(3)}^{SU(3)\text{d gauged.}}$$

$$\begin{array}{ccc} SU(3) & \times & SU(3) \\ \downarrow & & \downarrow \\ SU(2) & & SU(2) \end{array}$$

gauging the diagonal subgroup explicitly breaks global symmetry!

$$|g A_\mu \Phi_1|^2 + |g A_\mu \Phi_2|^2$$

Imagine setting coupling of $\Phi_1 \to 0$.
Then $SU(3)$ gl. sym. on Φ_1
& $SU(3)$ sym. ~~of~~ on Φ_2, A_μ } $SU(3) \times SU(3)$ symmetry g unbroken

↓

h exact Goldstone!

Similarly, if coupling of $\Phi_2 \to 0 \to$
still $SU(3) \times SU(3)$, h still exact Goldstone.

Higgs becomes massive only via diagrams involving **BOTH** couplings of Φ_1, Φ_2 → none of these diagrams quadratically divergent!

Collective breaking: symmetry explicitly broken but in a special way: one needs more than one coupling to completely break the symmetry → any single coupling leaves some global symmetry exact & then Goldstone remains massless! Only diagrams involving 2 (or more) couplings can generate a ~~a~~ mass for Goldstone → more propagators → not quadratically divergent!

Summary little higgs

Idea: use Goldstone's thm. to cancel 1-loop quadratic divergences to Higgs

How to make Higgs a Goldstone?

$SO(3) \to SO(2)$

$$\left(\begin{array}{c|c} & h \\ \hline h^+ & \end{array}\right)$$

Non-linear realization:

$$\phi = e^{i\Pi/f}\begin{pmatrix}0\\0\\v\end{pmatrix} \approx \begin{pmatrix} ih \\ f - \frac{h^+h}{2f}\end{pmatrix}$$

To get really no quadratic divergences:

ϕ_1, ϕ_2 $\quad SO(3)\times SO(3)$ global symmetry $\to SO(3)_D$

$$\phi_1 = e^{i\Pi_1/f}\begin{pmatrix}0\\0\\f\end{pmatrix} \qquad \phi_2 = e^{i\Pi_2/f}\begin{pmatrix}0\\0\\f\end{pmatrix}$$

$\Pi_1 + \Pi_2 \to$ eaten $\to 0$

$$\phi_1 = e^{i\Pi/f}\begin{pmatrix}0\\0\\f\end{pmatrix} \qquad \phi_2 = e^{-i\Pi/f}\begin{pmatrix}0\\0\\f\end{pmatrix}$$

quadratic divergence for Π from

$SU(3)_D$ gauged:

$$\Lambda^2 \frac{g^2}{16\pi^2} \operatorname{Tr}\left(\phi_1^+\phi_1 + \phi_2^+\phi_2\right) = \text{indep. of } \Pi!$$

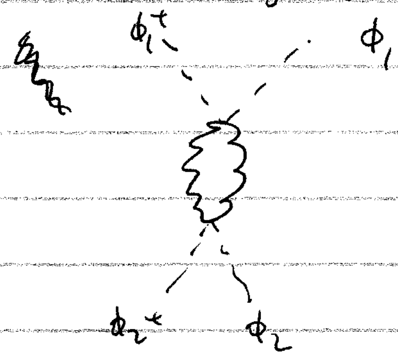
Reason for cancellation: Collective breaking:

individual term does not violate full global symmetry...
$SO(3) \times SO(3)$ w. $SO(3)_D$ gauged...

$$\mathcal{L} \supset |g A_\mu \Phi_1|^2 + |g A_\mu \Phi_2|^2$$

turn it off → still $SO(3) \times SO(3)$ symmetry intact.

~~Smallest~~ Lowest divergence



$$\propto |\phi_1^+ \phi_2|^2 \frac{g^4}{(16\pi^2)} \ln\left(\frac{\Lambda^2}{\mu^2}\right)$$

smallest possible invariant made of $\phi_1^+, \phi_1, \phi_2^+, \phi_2$ that breaks $SO(3) \times SO(3)$!

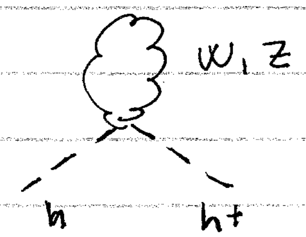
$$\phi_1^+ \phi = f^2 - 2h^+h + \dots$$

→ so largest correction to Higgs mass:

$$\sim \frac{g^4}{16\pi^2} f^2 \log\left(\frac{\Lambda^2}{\mu^2}\right)$$

for $f \sim 1$ TeV
→ $\sim [(100)\,\text{GeV}]^2$

So what cancels the divergence of the SM gauge boson to Higgs?



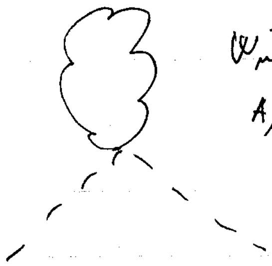
coupling of heavy gauge bosons to the higgs:

$$\left|\left(\partial_\mu - ig A^a_\mu T^a\right)\phi_{1,2}\right|^2 \longrightarrow \begin{pmatrix} & W^+ & X^+ \\ W^- & & Y^0_1 + i Y^0_2 \\ X^- & Y^0_1 - i Y^0_2 & \end{pmatrix}$$

Coupling of gauge bosons to higgs:

$$\frac{g^2}{4} h^+ h \Big[2 W^+_\mu W^{\mu -} + A^3_\mu A^{3\mu} - X^+_\mu X^{\mu -} - \frac{1}{2}\left(Y^0_{1\mu} Y^{\mu 0}_1 + Y^0_{2\mu} Y^{\mu 0}_2\right) - A^8_\mu A^{8\mu} \Big]$$

So quadratic divergences:



$W^+_\mu W^{\mu -} \to +2 \qquad \left(2 \ \frac{1}{\sqrt{2}} |W^1_\mu - i W^2_\mu|^2\right)$

$A^3_\mu \to +1$

$X^+_\mu X^{\mu -} \to -1$

$Y_{1,2} \to -1$

$A^8_\mu \to -1$

$\Sigma = 0$

Looks like a miracle in this language (just like in the case of SUSY). But we understand that there is a deep underlying reason for the cancellation!

→ upshot: quadratic divergence cancelled by the

same spin little higgs partners (vs. opposite spin partners ~~of the~~ of SUSY).

Here same spin partners related to SM particles by the ~~SU~~ global symmetry that is broken).

The fermion sector:

Numerically this is the most important source of correction to Higgs mass! Wait: introduce $SU(3)\times SU(3)$ global symmetry & maintain the collective nature of symmetry breaking!

$$SU(2)_L \qquad \begin{pmatrix} t \\ b \end{pmatrix}_L \rightarrow \begin{pmatrix} t \\ b \\ T \end{pmatrix} \equiv \Psi$$

doublet enlarged to triplet of $SU(3)$

Singlet remains $\quad t_R \rightarrow t^c$

$b_R \rightarrow b^c$

RH partner of extra top! $\rightarrow T^c$

but these will mix, call gauge eigenstates t_1^c, t_2^c

Yukawa couplings:

~~$\frac{\lambda}{\sqrt{2}}$ f T~~

$$\mathcal{L}_{Yuk} = \lambda_1 \, \phi_1^+ \Psi t_1^c + \lambda_2 \, \phi_2^+ \Psi t_2^c$$

Assume $\lambda_1 = \lambda_2 = \frac{\lambda}{\sqrt{2}}$ (reduces # of terms, since preserves parity $1 \leftrightarrow 2$)

Expand:

$$\mathcal{L} = \frac{\lambda}{\sqrt{2}} \left[\left(-ih^{+},\ f - \frac{h^{+}h}{2f}\right) \begin{pmatrix} Q \\ T \end{pmatrix} t_1^c \right.$$

$$\left. + \left(ih^{+},\ f - \frac{h^{+}h}{2f}\right) \begin{pmatrix} Q \\ T \end{pmatrix} t_2^c \right]$$

$$= \frac{\lambda}{\sqrt{2}} \left[-ih^{+} Q t_1^c + \left(f - \frac{h^{+}h}{2f}\right) T t_1^c \right.$$

$$\left. + ih^{+} Q t_2^c + \left(f - \frac{h^{+}h}{2f}\right) T t_2^c \right]$$

$$= \frac{\lambda}{\sqrt{2}} \left[ih^{+} Q (t_2^c - t_1^c) + f\left(1 - \frac{h^{+}h}{2f^2}\right) T (t_1^c + t_2^c) \right]$$

$$T^c \equiv \frac{t_1^c + t_2^c}{\sqrt{2}} \qquad t^c = \frac{i(t_2^c - t_1^c)}{\sqrt{2}}$$

$$\mathcal{L}_{Yuk} = \lambda f \left(1 - \frac{h^{+}h}{2f^2}\right) T T^c + \lambda h^{+} Q t^c + \ldots$$

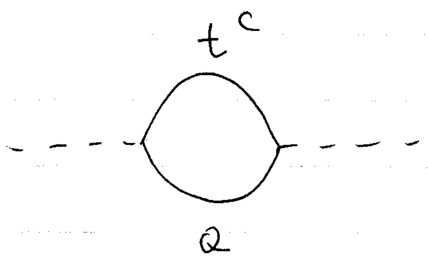
but heavy top partner will also couple to the Higgs!

normal top Yukawa coupling

heavy guy has mass

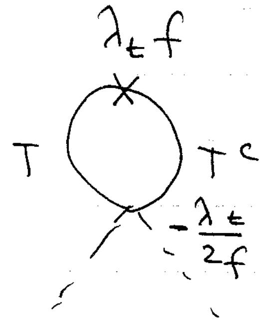
$\lambda_t f$, coupling to two higgses $-\frac{\lambda_t}{2f}$

quadratic divergences:



usual SM diagram

$+$



contribution of heavy top partners

$$\frac{\lambda_t^2}{16\pi^2}\Lambda^2 h^\dagger h$$

$$\cancel{+\frac{\lambda_t^2 f^2}{16\pi^2}}$$

$$-2\frac{\lambda_t f\cdot\lambda_t}{2f}\frac{1}{16\pi^2}\Lambda^2 h^\dagger h$$

$\Rightarrow$ again cancel due to SO(3) symmetry!

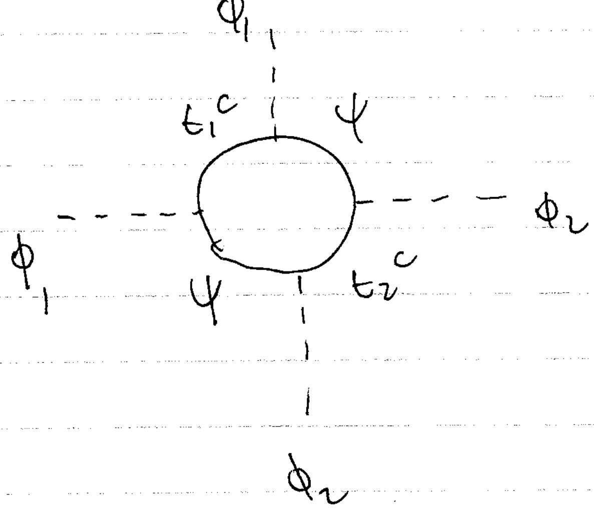
Again, can look at explanation in terms of collective breaking:

2 terms in Yukawa ~~potential~~ sector:

$$\lambda_1 \phi_1^\dagger \Psi t_1^c + \lambda_2 \phi_2^\dagger \Psi t_2^c$$

If I do an SU(3) transformation on ϕ_1 $\rightarrow$ can be compensated by transformation on ϕ_2. ~~But~~ As long as $\lambda_2 = 0$, full SU(3)$\times$SU(3) exactly unbroken!

Only when I turn on ~~the~~ both couplings at once do I actually explicitly break the symmetry. So I need to have a diagram that contains both λ_1 & λ_2 to get a ~~a~~ contribution to the Higgs mass



This is the smallest diagram that actually gives a Higgs ~~mass~~ potential, again of the form

$$\frac{\lambda^4}{16\pi^2}\,|\phi_1+\phi_2|^2 \log\left(\frac{\Lambda^2}{\mu^2}\right)$$

$$\rightarrow \frac{\lambda^4}{16\pi^2}\, f^2 h^\dagger h \log\left(\frac{\Lambda^2}{\mu^2}\right)$$

Formulae if you don't insist on simplest solution:

$\lambda_1 \neq \lambda_2$ $\qquad m_T = \sqrt{\lambda_1^2 f_1^2 + \lambda_2^2 f_2^2}$

$f_1 \neq f_2$ $\qquad \lambda_t = \lambda_1\lambda_2 \dfrac{\sqrt{f_1^2+f_2^2}}{m_T}$ zero, if either coupling $=0$

<u>Other Yukawas:</u> up-type = same way.

down-type: SM: $\lambda_b \; \epsilon_{ij} \; h^i Q^j b^c$

enlarge this to an SU(3) ϵ:

$$\underbrace{\frac{\lambda_b}{f} \; \epsilon_{ijk} \; \Phi_1^i \, \Phi_2^j \, \Psi^k \, b^c}$$

This involves both Φ_i' fields. But λ_b is very small, so the quadratic divergences ~~do not~~ are not large enough to cause a large hierarchy for $\Lambda \sim 10\,\text{TeV}$.

$$\frac{\lambda_b^2 \Lambda^2}{16\pi^2} \propto \left(\frac{10}{4\pi \cdot 30}\right)^2 \qquad \lambda_b \approx \frac{\lambda_t}{30} \leftarrow 1$$

$$\sim \left(\frac{10\,000}{300}\right)^2 \sim (30\,\text{GeV})^2$$

Even λ_b is OK natural!

To get complete model: still need

- color, hypercharge
- quartic self couplings of Higgs!

<u>color:</u> does not violate any global symmetry, can add it w/o any problems!

Hyperchange: $\begin{pmatrix} 0 \\ 0 \\ f \end{pmatrix}$ breaks $SO(3) \to SO(2)$

no $U(1)$ left

(or wrong $\sin^2\theta$).

→ need an additional $U(1)_X$

	$SU(3)$	$\times U(1)_X$
ϕ_i	3	$-1/3$

unbroken generator: $Y = -\frac{T_8}{\sqrt{3}} + X$

$$T^8 = \frac{1}{2\sqrt{3}} \begin{pmatrix} 1 & & \\ & 1 & \\ & & -2 \end{pmatrix}$$

$$\alpha \begin{pmatrix} 1 & & \\ & 1 & \\ & & -2 \end{pmatrix} - \frac{1}{3} \begin{pmatrix} 1 & & \\ & 1 & \\ & & 1 \end{pmatrix}$$

$-1/3 - 2\alpha = 0$

$\boxed{\alpha = -\frac{1}{6}}$ ↗ gives above result!

→ once Higgs, $SU(3)$ quantum #'s are chosen: all $U(1)_X$ quantum #'s are fixed!

$$T_8 = \frac{1}{2\sqrt{3}}\begin{pmatrix}1 & & \\ & 1 & \\ & & -2\end{pmatrix}$$

	$SU(3)_C$	$\times\, SU(3)_L$	$\times\, U(1)_X$	Y
Ψ_Q	3	3	$1/3$	$1/6$
$2\times u^c$	$\bar{3}$	1	$-2/3$	$-2/3$
d^c	$\bar{3}$	1	$1/3$	
Ψ_L	1		$-1/3$	
e^c	1	1	1	
ϕ_1	1	3	$-1/3$	
ϕ_2	1	3	$-1/3$	

$$Y = -\frac{T_8}{\sqrt{3}} + X$$

$$= -\frac{1}{6}t_8 + X$$

$$t_8 = \begin{pmatrix}1 & & \\ & 1 & \\ & & -2\end{pmatrix}$$

$-1/6 +$

$-2/3 =$

$-1/2 = -1/6 + X$

$$\mathcal{L}_{Yuk} = \lambda_1 \, \phi_1^\dagger \Psi_Q \, u^c$$

$+ \dots$ all Yukawas invariant!

Note: fermion content anomalous $\Rightarrow$ can modify it to make it anomaly free...

Quartic Higgs self-coupling

Hardest part of model:

need to write $V(\phi_1, \phi_2)$

- contains no mass term for Higgs
- Contains quartic.
- quartic generated via collective breaking

Impossible in pure SU(3) model, only SU(3) invariant:

$\phi_1^+ \phi_2$ Why?

$\phi_1^+ \phi_1 =$ const

$\epsilon^{ijk} \phi_i \phi_j \phi_k = 0$

But $\phi_1^+ \phi_2$ breaks $SU(3) \times SU(3) \to SU(3)_D$

$$\phi_1^+ \phi_2 = f^2 - h^+ h + \frac{1}{f} (h^+ h)^2 + \dots$$

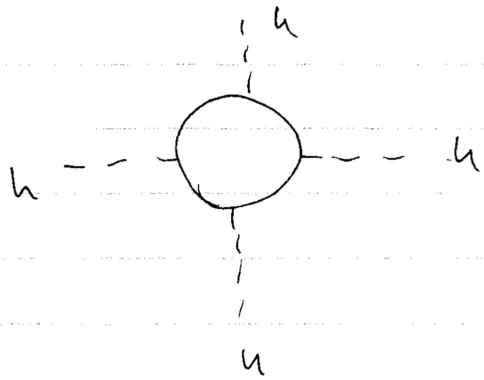
Any power: $\frac{1}{f^{2n-4}} (\phi_1^+ \phi_2)^n = \quad f^4 - f^2 h^+ h + (h^+ h)^2$

→ either large mass + quartic

→ small mass & small quartic!

Possible solutions:

- enlarge gauge group
- add op. with small coefficient → small quartic, but also top loop correction



quartic $\frac{3\lambda_t^4}{16\pi^2} \log\left(\frac{m_T^2}{m_t^2}\right) (h^+ h)^2$

+ small contribution from tree level $\phi_1^+ \phi_2$ → good EWSB!

Aspen talk 8/3/06

Overview of Higgsless EWSB

Work with C. Grojean, J. Terning
& G. Cacciapaglia, J. Hubisz, G. Marandella ~~Hitoshi~~ Murayama, L. Pilo, M. Reece
Y. Shirman

Motivation: LHC will be searching for mechanism of
EWSB. Extra dimensions can be used to
break symmetries:
~~Scherk-Schwarz~~ Wilson-line breaking (Hosotani mechanism) $\sim A_5$
Orbifold breaking

Question: can we apply them to EWSB?

Yes: (1.) Gauge-higgs unification
(2.) Higgsless.

Focus on 2. boundary condition breaking of symmetries

- no elementary scalar in spectrum
- could have different phenomenology
- could solve naturalness problem...

Assumption: $\exists$ extra dim with a boundary,
simplest possibility just an interval.
BC's at ends of interval will be used
to break symmetry.

Mostly interested in gauge fields for now.

BC's:

Neumann	$\partial_5 A_\mu = 0$
Dirichlet	$A_\mu = 0$
mixed	$\partial_5 A_\mu + M A_\mu = 0$

If use Dirichlet: no massless mode in KK expansion, just massive KK tower. Want W, Z to be lowest modes of a purely massive KK tower $\Rightarrow$ gauge symmetry broken w/o physical scalar.

What BC's to impose?

Want to make sure that gauge symm. breaking "spontaneous", not hard!
Have to be careful!

- A safe limiting procedure

Think of fields as first all having Neumann but some of them have a localized mass term due to localized Higgs

Then BC:

$$\partial_5 A_\mu + g_5^2 v^2 A_\mu = 0 \quad \text{mixed BC}$$

$v \to 0 \qquad \partial_5 A_\mu = 0$ Neumann

$v \to \infty \qquad A_\mu = 0$ Dirichlet.

Crucial observation:

In $v \to \infty$ limit gauge field repelled from brane, but mass NOT infinity. Instead just becomes a massive KK mode. In this ~~case~~ limit Higgs mass $\propto \lambda v \to \infty$ totally decouples from the other modes of ~~theory~~!

Theory higgsless, even if obtained in limit from theory with localized Higgs.

How to build an actual model?

Naively very difficult, problems:

- $M_{KK} \sim \frac{n}{R}$

- How to ensure $\frac{M_W^2}{M_Z^2} = \cos^2\theta_W = \frac{g^2}{g^2+g'^2}$?

- How to ensure $\frac{M_{Z'}}{M_Z} > 2$?

Answer: warping

To gain insight, ~~we~~ note: in SM

$\rho = 1$ ensured by custodial $SU(2)$

$\underbrace{SU(2)_L \times SU(2)_R}_{\text{global sym.}} \to SU(2)_D$

Need a construction where this is implemented in extra dimension.

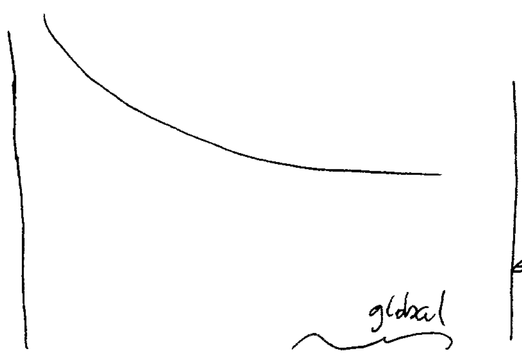
To get global sym. in XD $\to$ need AdS/CFT

bulk of AdS_5 $\Leftrightarrow$ 4D CFT

gauge field in bulk $\Leftrightarrow$ CFT has global symmetry

gauge field has normalizable zero mode $\Leftrightarrow$ global symmetry weakly gauged

So construction (similar to what H-C. mentioned)



$$ds^2 = \left(\frac{R}{z}\right)^2 \left(dx^\mu dx^\nu \eta_{\mu\nu} - dz^2\right)$$

AdS_5 metric

should have $SU(2)_L \times SU(2)_R \to SU(2)_D$ ~~bre~~ EWSB in IR

UV

$SU(2)_R \times U(1)_{B-L} \to U(1)_Y$

$\overbrace{SU(2)_L \times SU(2)_R}^{\text{global}} \times U(1)_{B-L}$

bulk

should not have zero (or light) mode

IR

BC on TeV breaks EWSB $\Longleftrightarrow$ ~~Gravity~~ 5D dual of (walking) technicolor

The actual BC's we could use:

Planck brane:

$$\partial_5 A_\mu^{La} = 0$$

$$\partial_5 \left(g_5 B_\mu + \tilde{g}_5 A_\mu^{R3} \right) = 0$$

unbroken $SU(2)_L \times U(1)_Y$

$$A_\mu^{R1,2} = \tilde{g}_5 B_\mu - g_5 A_\mu^{R3} = 0$$

TeV brane:

$$\partial_5 \left(A_\mu^{La} + A_\mu^{Ra} \right) = 0 \qquad \partial_5 B_\mu = 0$$

unbroken $SU(2)_D \times U(1)_B$

$$A_\mu^{La} - A_\mu^{Ra} = 0$$

broken $\dfrac{SU(2) \times SU(2)}{SU(2)}$

Mass spectrum: for simplest case $g_{5L} = g_{5R}$

$$R = \frac{1}{M_{Pl}} \qquad \log \frac{R'}{R} \sim 30$$

$$M_W^2 = \frac{1}{R'^2 \log (R'/R)}$$

$$M_Z^2 = \frac{g_5^2 + 2\tilde{g}_5^2}{g_5^2 + \tilde{g}_5^2} \frac{1}{R'^2 \log R'/R}$$

$$\left\{ \begin{array}{l} \dfrac{1}{g'^2} = R \log R'/R \left(\dfrac{1}{g_5^2} + \dfrac{1}{\tilde{g}_5^2} \right) \\ \dfrac{1}{g^2} = R \log (R'/R) \dfrac{1}{g_5^2} \end{array} \right.$$

$\rightarrow g \sim 1$

KK modes

$$M^n \sim \frac{\pi}{2}\,(n+1/2)\,\frac{1}{R'}$$

$$\frac{M_{W'}}{M_W} \propto \sqrt{\log R'/R}$$ → warping ensures clean separation of scales.

Explanation in terms of AdS/CFT

$$\frac{M_{W'}}{M_W} \sim \frac{m_g}{g f_\pi}$$

$$m_g \sim \frac{1}{R'} \qquad g \sim \frac{g_5}{\sqrt{R \log R'/R}}$$

$$f_\pi \sim \sqrt{\frac{R}{g_5}}\,\frac{1}{R'}$$

$$\frac{M_{W'}}{M_W} \propto \sqrt{\log R'/R}$$

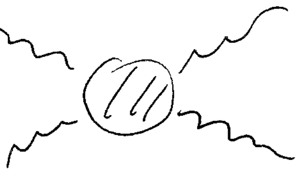
warping also gives clean separation of KK modes from lightest modes!

- Dual of technicolor
- no elementary higgs scalar

} is theory weakly coupled??

issue of unitarity

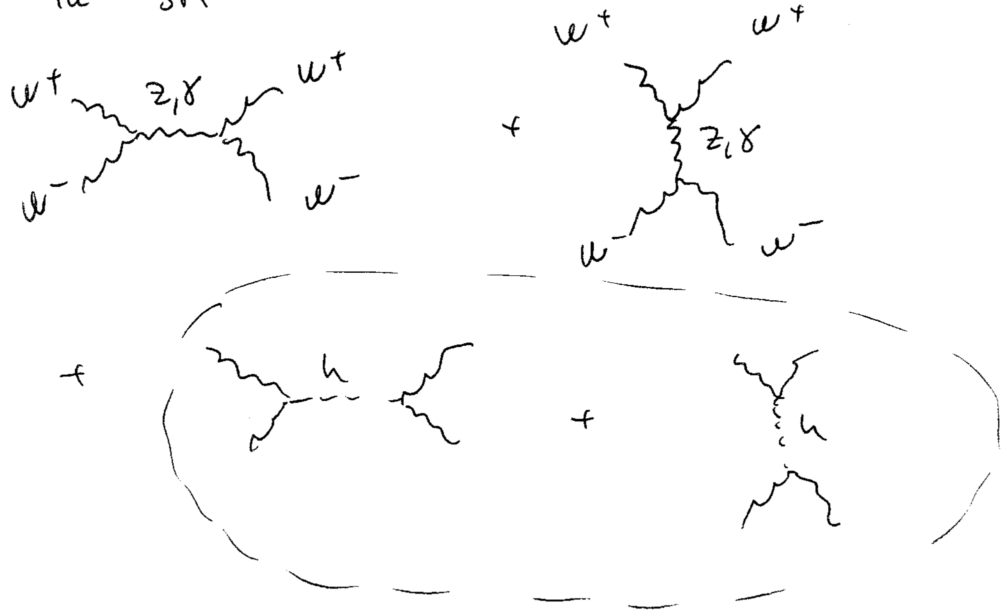
no higgs: GB scattering amplitudes $\propto E^2, E^4$



$$\mathcal{A} \sim \frac{E^2}{M_W^2}$$

for $E = \frac{4\pi M_W}{g}$ amplitude violates unitarity.

In SM

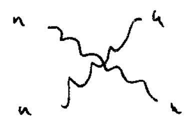


these diagrams are missing here

In 5D theory: exchange of KK modes can delay unitarity violation above $\frac{4\pi M_W}{g}$.

Cancellation of growing terms in amplitude → sum rules:

$$E^4 \to \quad g_{nnnn} = \sum_k g_{nnk}^2$$



$$E^2 \to \quad g_{nnnn} M_n^2 = \frac{3}{4} \sum_k g_{nnk}^2 M_k^2$$

Automatically satisfied in 5D gauge theory due to 5D gauge invariance.

Example easy to see:

$$g_{nnnn} = \sum_k g_{nnk}^2$$

$$g_5^2 \int f_n^4(y)\, dy = \sum_k g_5^2 \int f_n^2(y) f_k(y)\, dy \int f_n^2(y') f_k(y')\, dy'$$

$$\sum_k f_k(y) f_k(y') = \delta(y-y') \quad \text{completeness}$$

$\to$ ✓

<u>BUT</u> : not enough that asymptotically growing terms cancel.

First Z', K' need to be light enough so that cancellation happens <u>before</u> amplitude already large!

<u>Papucci's analysis:</u>

for large E unitarity still violated due to growing # of channels

$$\Lambda_{unit} \sim \Lambda_{NDA}$$

$$\Lambda_{NDA} \sim \frac{24\pi^3}{g_5^2}\left(\frac{R}{R'}\right) \sim \frac{12\pi^4}{g^2}\frac{M_W^2}{M_{W'}}$$

$M_{W'}$ should not be too heavy for $\Lambda_{NDA} > \text{TeV}$

For simplest case $R = \frac{1}{M_{Pl}}$ $g_{5L} = g_{5R}$

$M_{W',Z'}$ too heavy, but can just take

$1/R' \sim 10^8 - 10^{10}$ GeV $\rightarrow$ $M_{W'} \sim 500 - 700$ GeV

Signals at LHC

Obviously, look for W', Z'

Hewett, Rizzo, ...

For example DY

but: this is very strongly depending on GB-fermion coupling. As we will see, likely strongly suppressed.

Birkedal, Matchev, Perelstein

Vector boson fusion

$2j + 3\ell + \not{E}_T$

Almost independent of details of model. Can test essence of higgsless models ($\equiv$ sum rules).

Note also: for $WZ \to WZ$ in SM no s-channel Higgs exchange (resonance). Here there is W' exchange in s-channel

$10\, fb^{-1} \to M_{W'} \lesssim 550\, GeV$

$60\, fb^{-1} \quad M_{W'} \lesssim TeV$

Major issues

- Fermion masses & couplings
- Electroweak precision constraints

1.) How to get fermion masses?

Where to put fermions?

$SU(2)_L \times U(1)_Y$ ↑ chiral

$SU(2)_D \times U(1)_{B-L}$ ↑ not chiral

Fermions must be in bulk & feel effect of both branes.

5D fermions

4D Dirac fermion

$$\Psi = \begin{pmatrix} \chi_\alpha \\ \bar{\psi}^\alpha \end{pmatrix}$$

→ every fermion doubled, then use BC's to get a chiral theory.

For example for leptons: just usual quantum #'s

	$SU(2)_L$	$\times$ $SU(2)_R$	$\times$ $U(1)_{B-L}$
$\begin{pmatrix} \nu \\ e \end{pmatrix}_L$	$\square$	1	$-1/2$
$\begin{pmatrix} \nu \\ e \end{pmatrix}_R$	1	$\square$	$-1/2$

$$\begin{pmatrix} \chi_{\nu_L} \\ \bar\psi_{\nu_L} \\ \chi_{e_L} \\ \bar\psi_{e_L} \end{pmatrix} \begin{array}{c} ++ \\ -- \\ ++ \\ -- \end{array} \qquad \begin{pmatrix} \chi_{\nu_R} \\ \bar\psi_{\nu_R} \\ \chi_{e_R} \\ \bar\psi_{e_R} \end{pmatrix} \begin{array}{c} -- \\ ++ \\ -- \\ ++ \end{array}$$

zero mode spectrum OK.

To get actual spectrum:

need
- mass term on TeV brane giving common mass to ν, e
- Large Majorana mass for ~~it~~ ν_R ~~or~~ ~~[illegible]~~ on Planck brane

Works well for leptons & light quarks
But problem for 3rd generation

Usual BC's:

Localized mass:

$$M\,(\chi_L \psi_R + h.c.)$$

BC's get modified to

$$\psi_L = M R' \psi_R \qquad \chi_R = -M R' \chi_L$$

3rd generation:

If we use the "usual" L-R representations ~~that~~

$$\begin{pmatrix} \chi_{t_L} \\ \bar\psi_{t_L} \\ \chi_{b_L} \\ \bar\psi_{b_L} \end{pmatrix} \begin{matrix} ++ \\ -- \\ ++ \\ -- \end{matrix} \qquad \begin{pmatrix} \chi_{t_R} \\ \bar\psi_{t_R} \\ \chi_{b_R} \\ \bar\psi_{b_R} \end{pmatrix} \begin{matrix} -- \\ ++ \\ -- \\ ++ \end{matrix}$$

But if we want to get a large top mass:

$$\psi_L = MR' \psi_R \qquad \chi_R = -MR' \chi_L$$

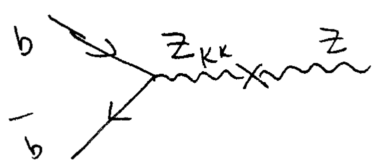
in $M \to \infty$ limit BC will be

$\psi_R = 0$, $\chi_L = 0$ so ++ field turns into +−

→ there is an upper bound set by the KK scale on how large a top mass one can get.

Problem: to get large top mass,

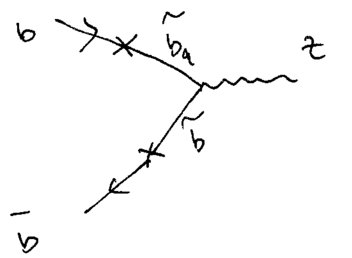
- need to localize 3rd gen close to TeV brane but this is where W, Z wave functions distorted



large deviation from $Zb\bar b$

- if not so close to TeV brane, MR' needs to be large $\chi_R = -MR' \chi_L$

left handed b mixes with LH b in $SU(2)_R$



→ together, large (> 10%) deviation for all of parameter space (while exp'l bound $\lesssim$ 0.5–1%)

→ solution later.

- Electroweak precision observables

Dual to technicolor → expect large S parameter

Indeed $$S \simeq \frac{6\pi}{g^2 \log \frac{R'}{R}}$$ for simplest case, with fermions on Planck brane

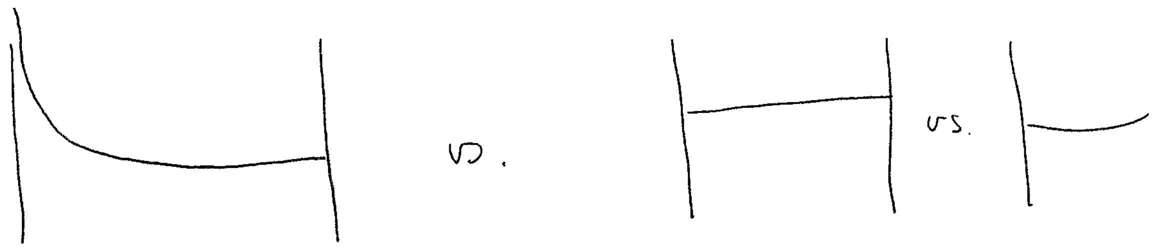
for $R \sim \frac{1}{M_{Pl}}$,

$S \simeq 1.15$, too large by $\sim$ 4–5 factor.

If increase warping: $\log\left(\frac{R'}{R}\right)$ grows, but mass scale of first KK modes as well, leave perturbative regime before $S \lesssim 0.3$

Possible way out:

S on its own meaningless, as long as the coupling of fermions $Z f\bar{f}$ not fixed to SM value.



But $Zf\bar{f}$ coupling depends on shape of fermion wave functions. By changing shape of ~~..~~ fermion S parameter can be dialed to be

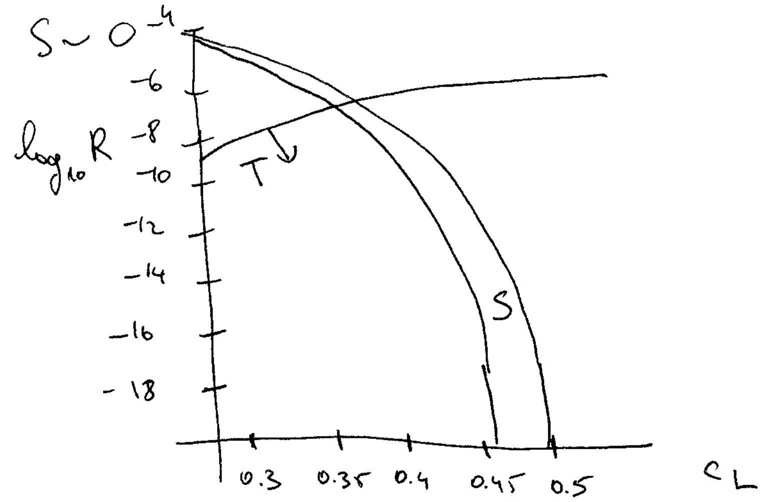
Another way to see it:

$$S = 16\pi \left(\Pi'_{33} - \Pi'_{3Q} \right)$$

$$\to Z_Z = \int_R^{R'} \left[(\psi^{L3})^2 + (\psi^{R3})^2 + (\psi^{B})^2 \right] \left(\frac{R}{z} \right) dz = 1 - (g^2 + g'^2)\Pi'_{33}$$

normalization of Z

→ can rescale Z wavefunction → S will disappear, but will pick up a shift in $Zf\bar{f}$ coupling. This can be compensated by shape of ~~....~~ fermion wave function, For $c \simeq 1/2$ (fermion wave function flat)



In allowed region fermion ~~[illegible]~~ coupling to Z, W KK modes very small.

Large contribution to S-parameter can be cancelled

Back to $Zb\bar{b}$

Recent suggestion by Agashe, Contino, da Rold, Pomarol

Can eliminate large correction to $Zb\bar{b}$ coupling by picking different representations for fermions.

Argument: (Agashe et al.)

$\frac{g}{\cos\theta} \; (T_{3L} - Q\sin^2\theta)$

$\uparrow$ not renormalized

assume $\underbrace{SU(2)_L \times SU(2)_R \times P_{LR}}_{SU(2)_D \times P_{LR}}$ — Z_2 interchange symmetry

If ψ is $+1$ eigenstate of P_{LR}

$$T_L = T_R \qquad T_{3L} = T_{3R}$$

Q_{L+R} protected by $SU(2)_D$, can not be shifted $\delta Q_L + \delta Q_R = 0$ but $\delta Q_L = \delta Q_R \Rightarrow$

$$\delta Q_L = 0$$
$$\delta Q_R = 0$$

We can choose representation like this, but b_L needs to be in a representation with $P_{LR} = +1$

	$SU(2)_L$	$\times SU(2)_R$	$\times U(1)_X$	
ψ_L	2	2	2/3	$\supset (t_L, b_L)$
t_R	1	1	2/3	$\supset t_R$
ψ_R	1	3	2/3	$\supset b_R$

$\rightarrow$ reduces immediately Zbb $40\% \rightarrow 4\%$

If an top b_r localized on UV brane $\rightarrow$

remaining 4% can be cancelled.

A representative spectrum:

$1/R = 10^8$ GeV

$1/R' = 282$ GeV

$g_5 = 0.66 \, (R \log R'/R)^{1/2}$

$\tilde{g}_5 = 0.42 \, (R \log R'/R)^{1/2}$

$c_L = 0.46$ light fermions

$c_L^{(2,2)} = 0.1$

$c_R^t = 0$

$c_R^b = -0.73$

$M_{W'}$	695 GeV
$M_{Z'}$	690 GeV
$M_{Z''}$	714 GeV
$M_{G'}$	714 GeV
$M_{t'}$	450 GeV
$M_{b'}$	664 GeV

$g_{W' ud}$	$0.07 g$
$g_{Z' q\bar{q}}$	$0.14 g_{Zq\bar{q}}$
$g_{G' q\bar{q}}$	$0.22 g_c$

⋮

<u>Conclusion</u>: at the moment have a model, which at <u>TREE</u>-level reproduces SM results, no higgs.
S, Zbb adjustable
loop effects → most should be OK
T parameter?