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**Observing New Particles at High Energy Colliders
Measurement of the Supersymmetry Spectrum (Lecture 3)**

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Observing New Particles at High Energy Colliders

3. Measurement of the Supersymmetry Spectrum

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In the previous lecture, I argued that we should be able to detect signals of new physics associated with electroweak symmetry breaking and dark matter in the LHC experiments.

I also argued that this new physics is likely to include many new particles, in fact, a whole spectroscopy.

So the next question would be, **how do we measure this spectrum?**

This is a study that we will begin in pp collisions at the **LHC** and continue in e⁺e⁻ collisions at the **ILC**.

At each facility, we will take advantage of the experimental methods that I have reviewed in the previous two lectures.

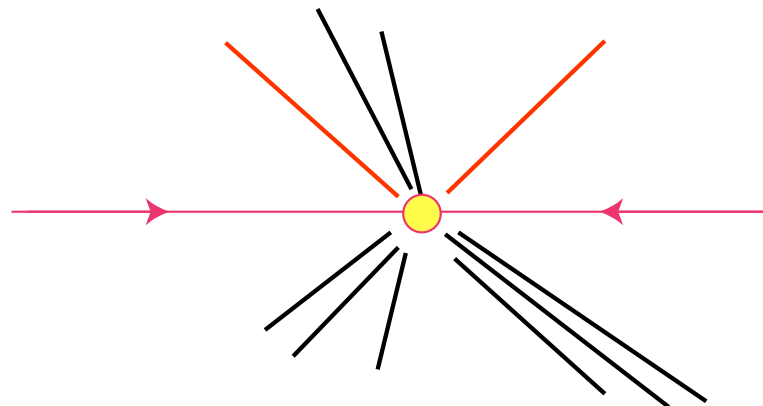
Let me begin by discussing methods for spectroscopy in hadron collisions. I will then discuss the prospects for adding to this knowledge using the special tools available in e⁺e⁻ collisions.

There are significant difficulties in trying to measure new particle masses at the LHC from resonances or features in kinematic distributions.

Any given process involves one quark or gluon colliding with another. We do not know the momenta of these individual particles. So we do not know the momentum of the initial state.



The final state contains two dark matter particles. We do not observe these particles or measure their momentum. So we have incomplete information about the final state.



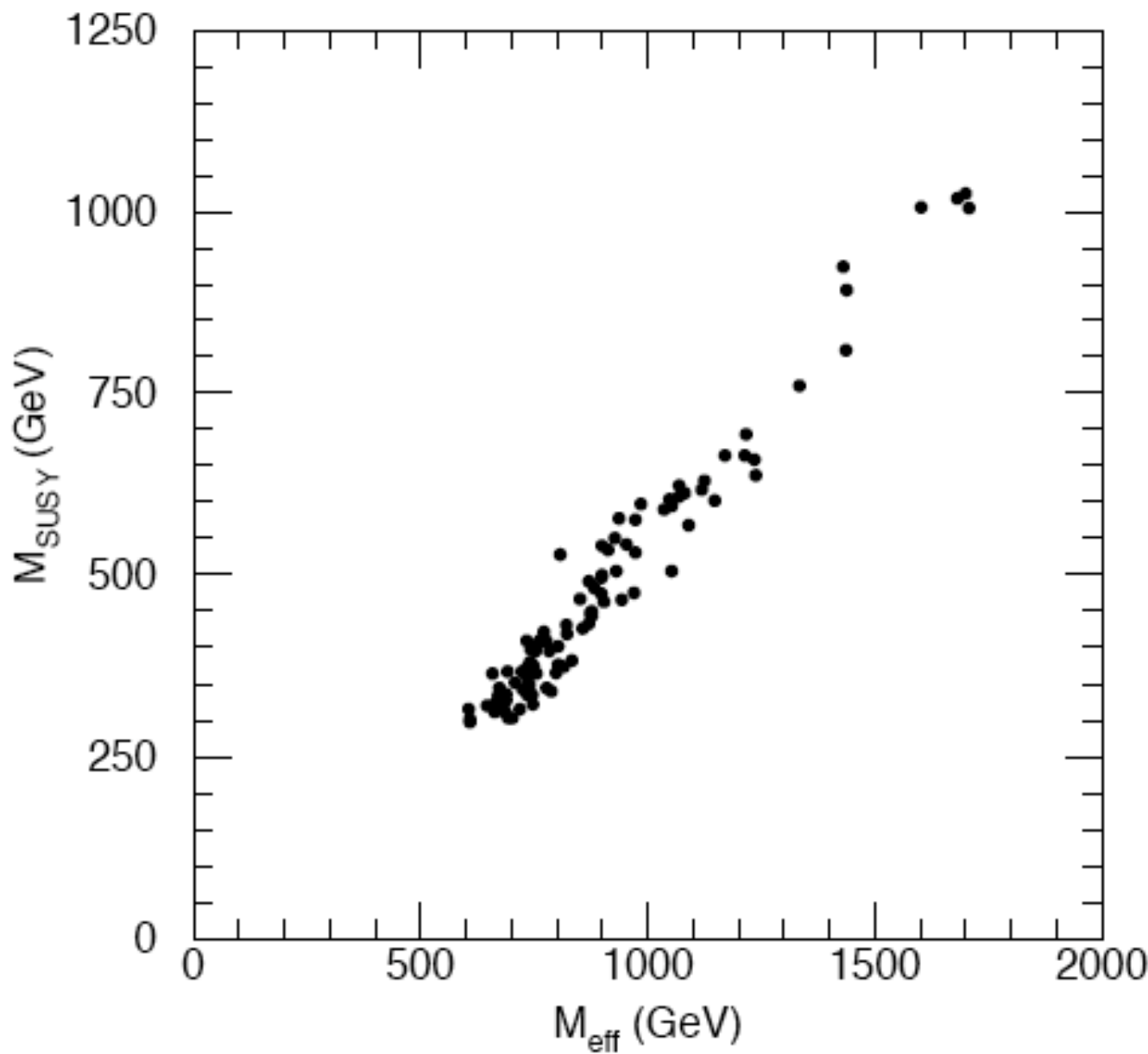
Still, gross measures of the transverse energy deposition do correlate with the mass of the colored particles that are the primary SUSY particles produced.

The variable

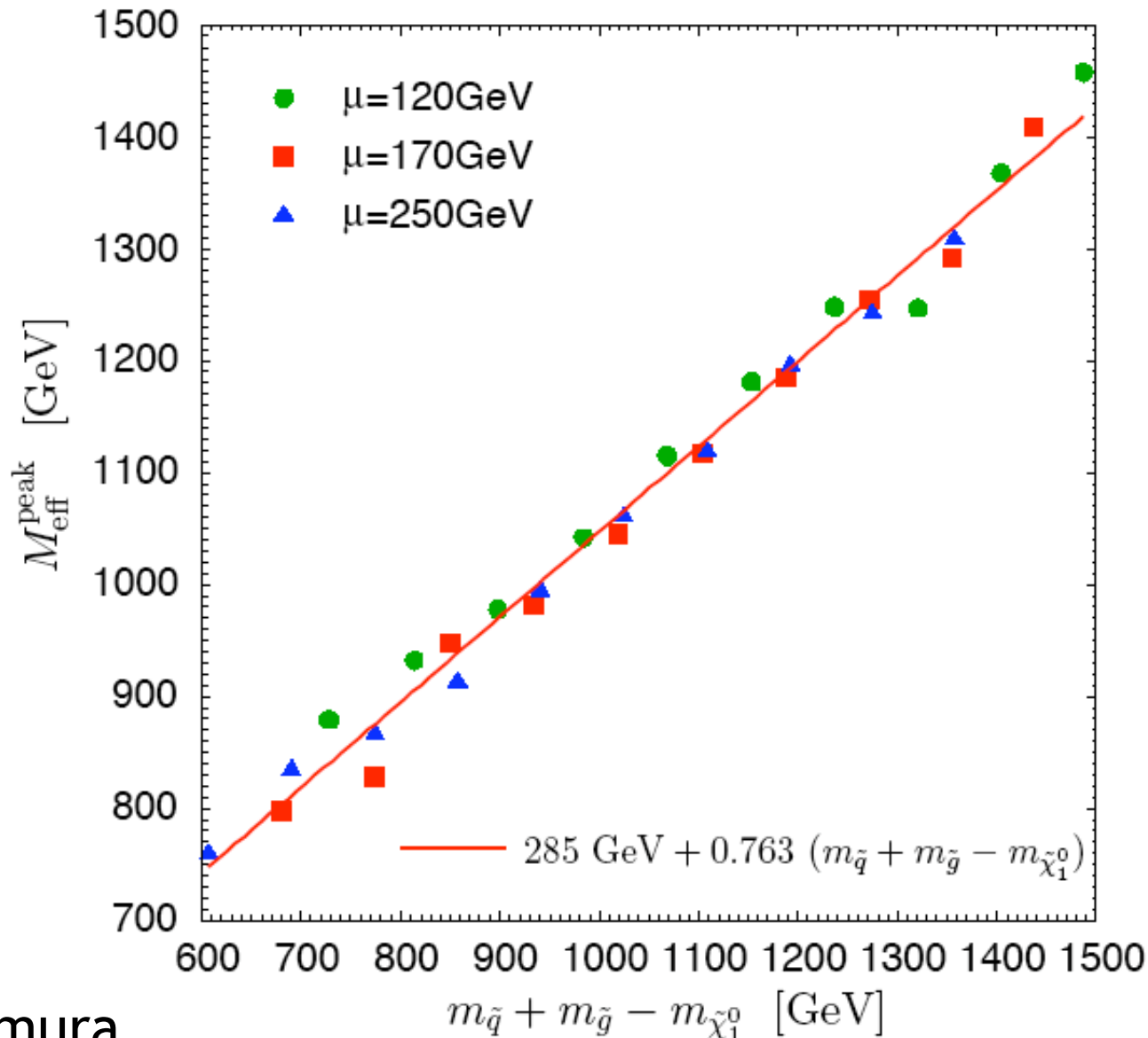
$$M_{eff} = \cancel{E}_T + \sum_{jets=1}^4 E_{Ti}$$

correlates well with the lighter of the squark and gluino masses in a class of SUSY models.

Hinchliffe, Paige,
Shapiro, Soderqvist,
Yao



When there are small mass differences among the SUSY particles, this relation breaks down, but M_{eff} remains closely proportional to the mass difference of SUSY states.



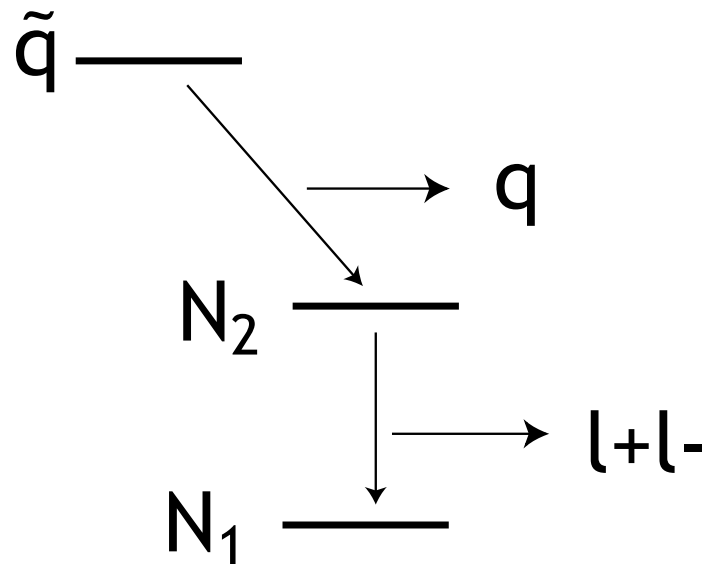
To extract more specific information, we need to perform analysis that rely on special features of the supersymmetry spectrum.

Every spectrum has special features. It is part of the art of experimentation to find and exploit them.

In the discussion to follow, I will pick out a particular feature that has been studied in a number of different analyses and use it to illustrate that level of insight that one could achieve in the hadron collider environment.

It is typical in supersymmetry models that the partners of quarks and gluons are relatively heavy states. These decay to the partners of $SU(2) \times U(1)$ gauge bosons and Higgs bosons, called **charginos** and **neutralinos**.

A feature of many supersymmetry spectra is the decay chain



The lepton momenta are measured completely, and we can construct their spectrum of invariant masses. From this point, depending on the specific model of the dilepton decay, the analysis can proceed in several different ways.

The decay of the N_2 can occur by any of the mechanisms:

$$N_2^0 \rightarrow \ell^\pm + \tilde{\ell}^\mp, \quad \tilde{\ell}^\mp \rightarrow \ell^\mp + N_1^0$$

$$N_2^0 \rightarrow N_1^0 Z^0, \quad Z^0 \rightarrow \ell^+ \ell^-$$

$$N_2^0 \rightarrow N_1^0 Z^{*0}, \quad Z^{*0} \rightarrow \ell^+ \ell^-$$

In a model with gaugino unification, $N_2 \sim \tilde{w}^0$, $N_1^0 \sim \tilde{b}^0$.
then these modes are preferred in the order listed:

2-body decays dominate over 3-body decays, and the N_2 coupling to sleptons is larger than the N_2 coupling to Z^0 .

The decay to an on-shell Z^0 is hard to work with, but the other two cases are interesting. To analyze them, consider the Dalitz plot associated with the 3-body system $N_1^0 \ell^+ \ell^-$

Let
$$x_0 = \frac{2E(N_1)}{m(N_2)} \quad x_+ = \frac{2E(\ell^+)}{m(N_2)} \quad x_- = \frac{2E(\ell^-)}{m(N_2)}$$

in the frame of the N_2 .

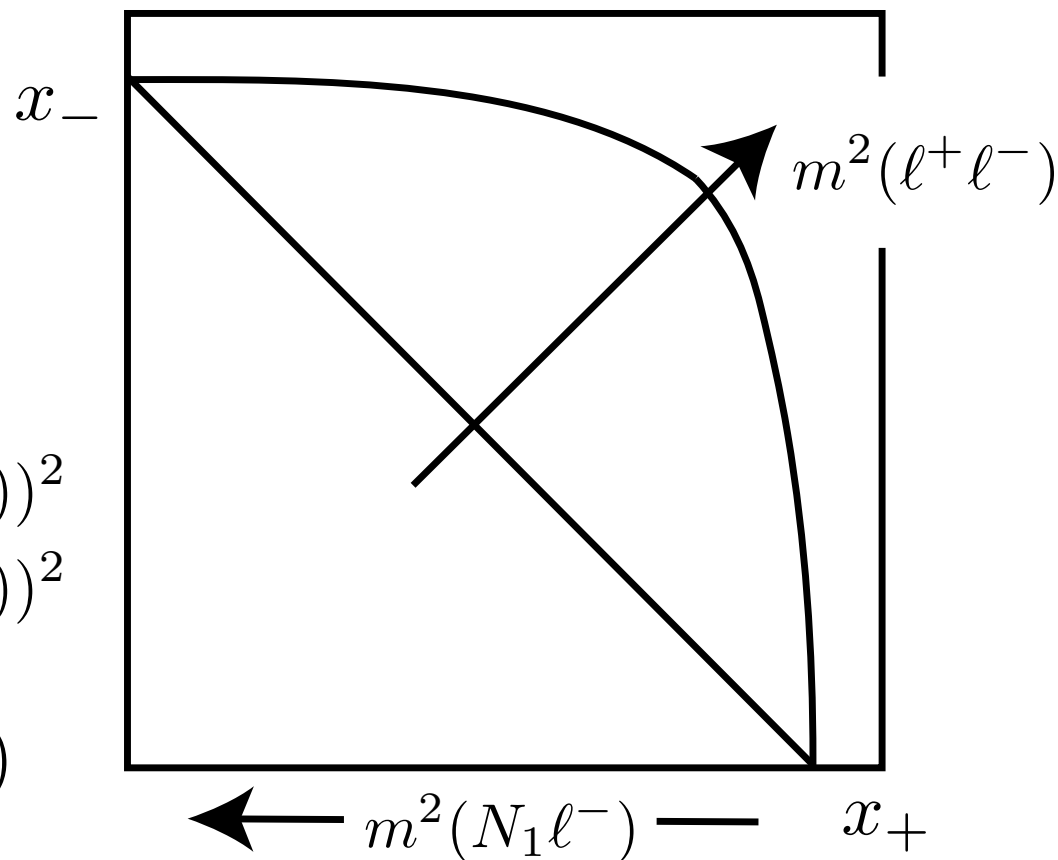
$$x_0 + x_+ + x_- = 2$$

The kinematic boundaries are located at:

$$x_+ + x_- = 1 - (m(N_1)/m(N_2))^2$$

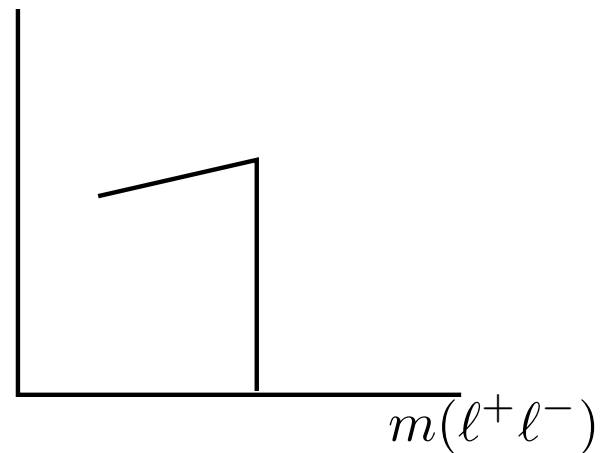
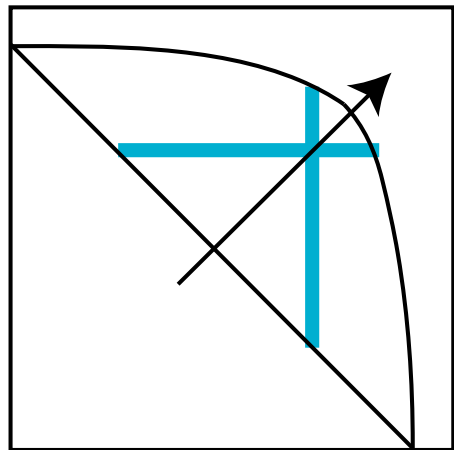
$$(1 - x_-)(1 - x_+) = (m(N_1)/m(N_2))^2$$

$$m^2(N_1 \ell^-) = m^2(N_2)(1 - x_+)$$

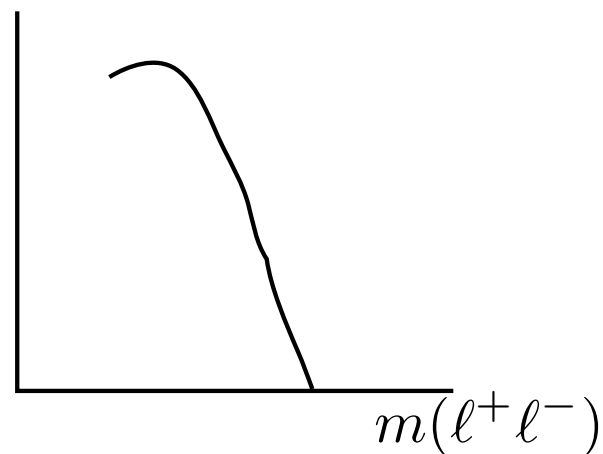
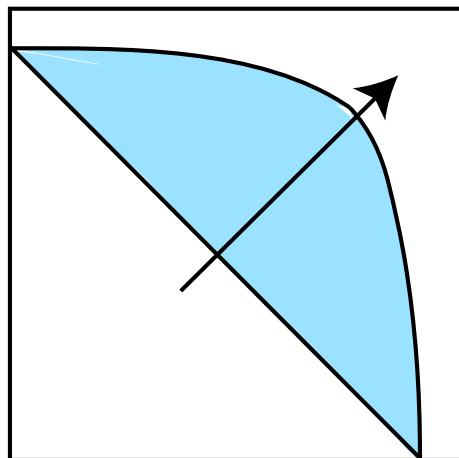


We can distinguish the cases of 2-body decay to a slepton and 3-body decay in the following way:

2-body decay populates lines on the Dalitz plot and leads to a sharp endpoint:



3-body decay populates the whole Dalitz plot and gives a slope at the endpoint:

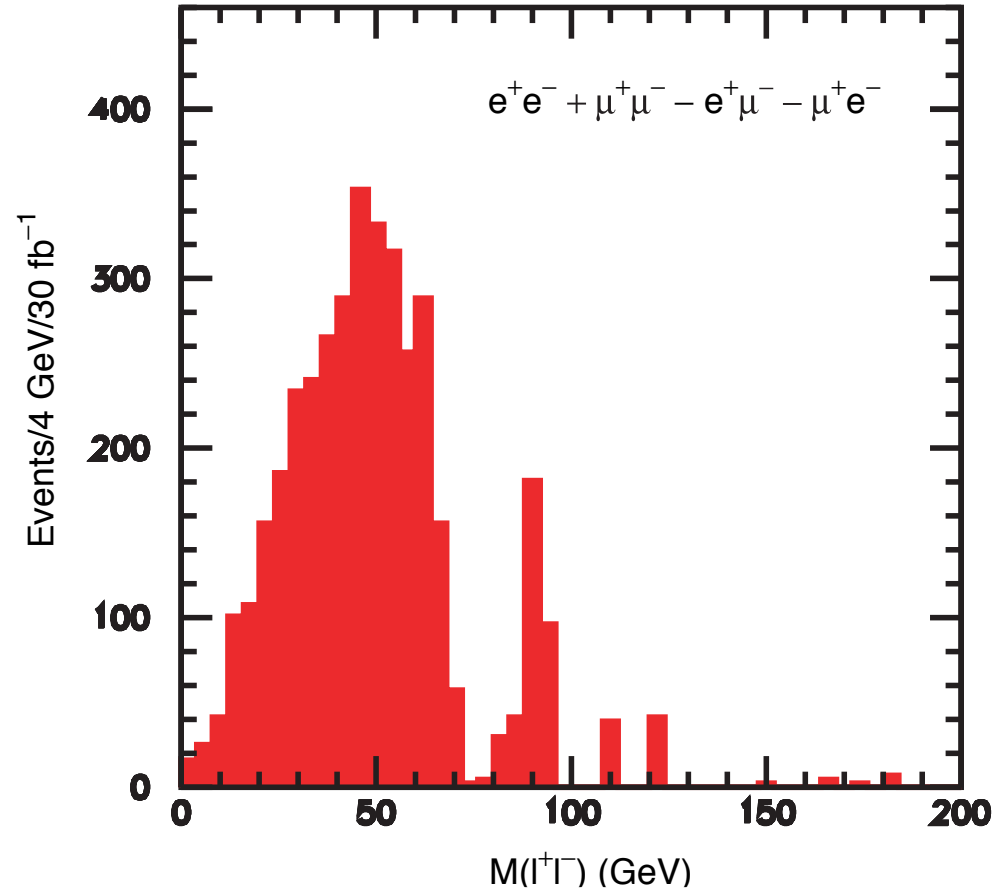
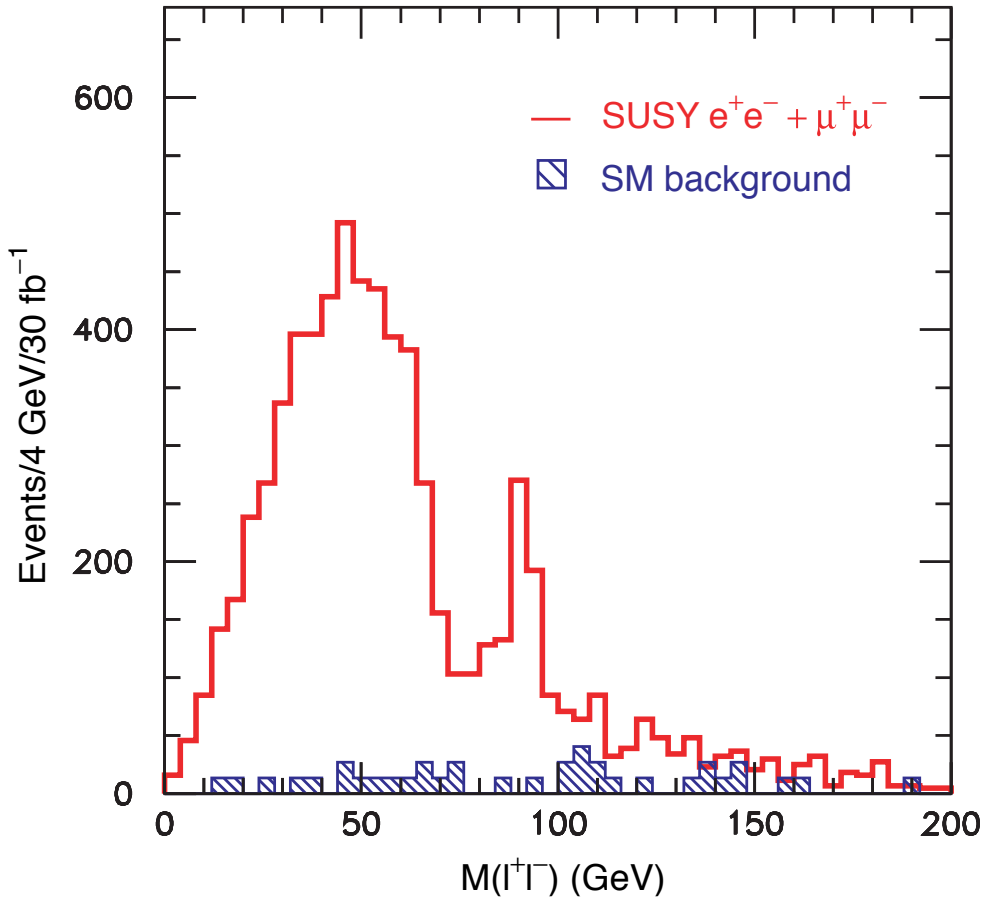


In the 3-body case, the endpoint in $m(\ell^+ \ell^-)$ is exactly

$$m(N_2) - m(N_1)$$

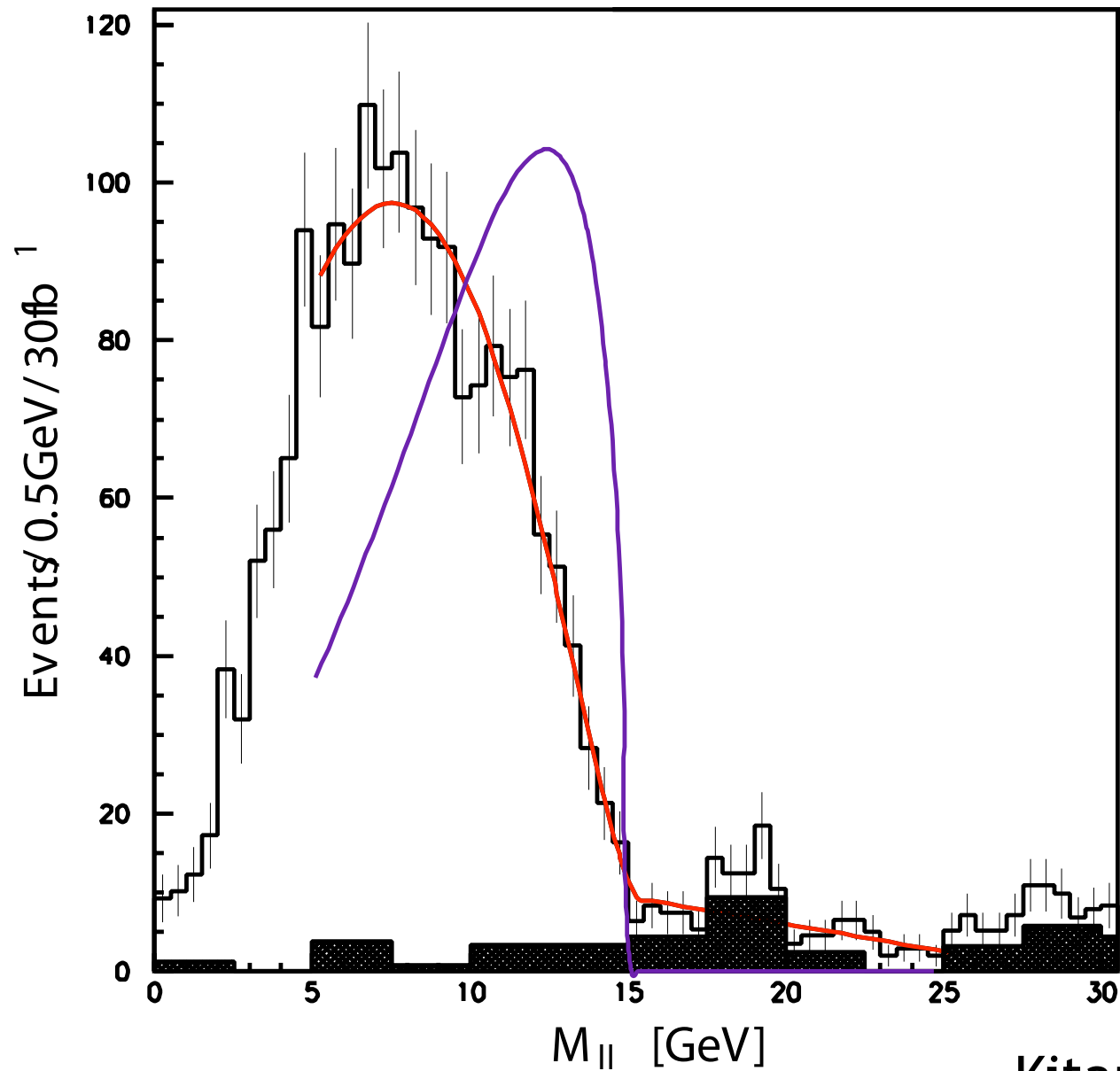
so we obtain a precise measurement of this quantity. The shape of the spectrum has more information. For example, for heavy slepton masses, this shape is different for gaugino-like or Higgsino-like lightest neutralino.

an example where the lightest neutralinos are gaugino:



Hinchliffe et al.

an example where the lightest neutralinos are Higgsino:

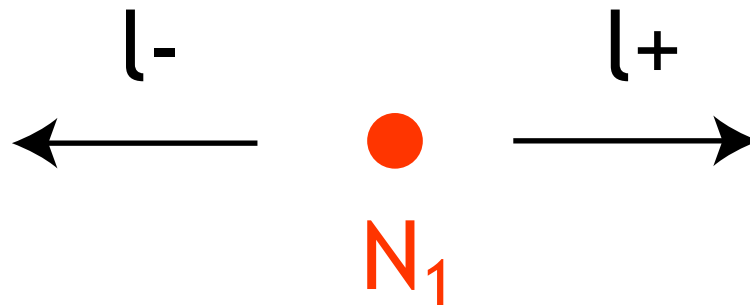


Kitano-Nomura

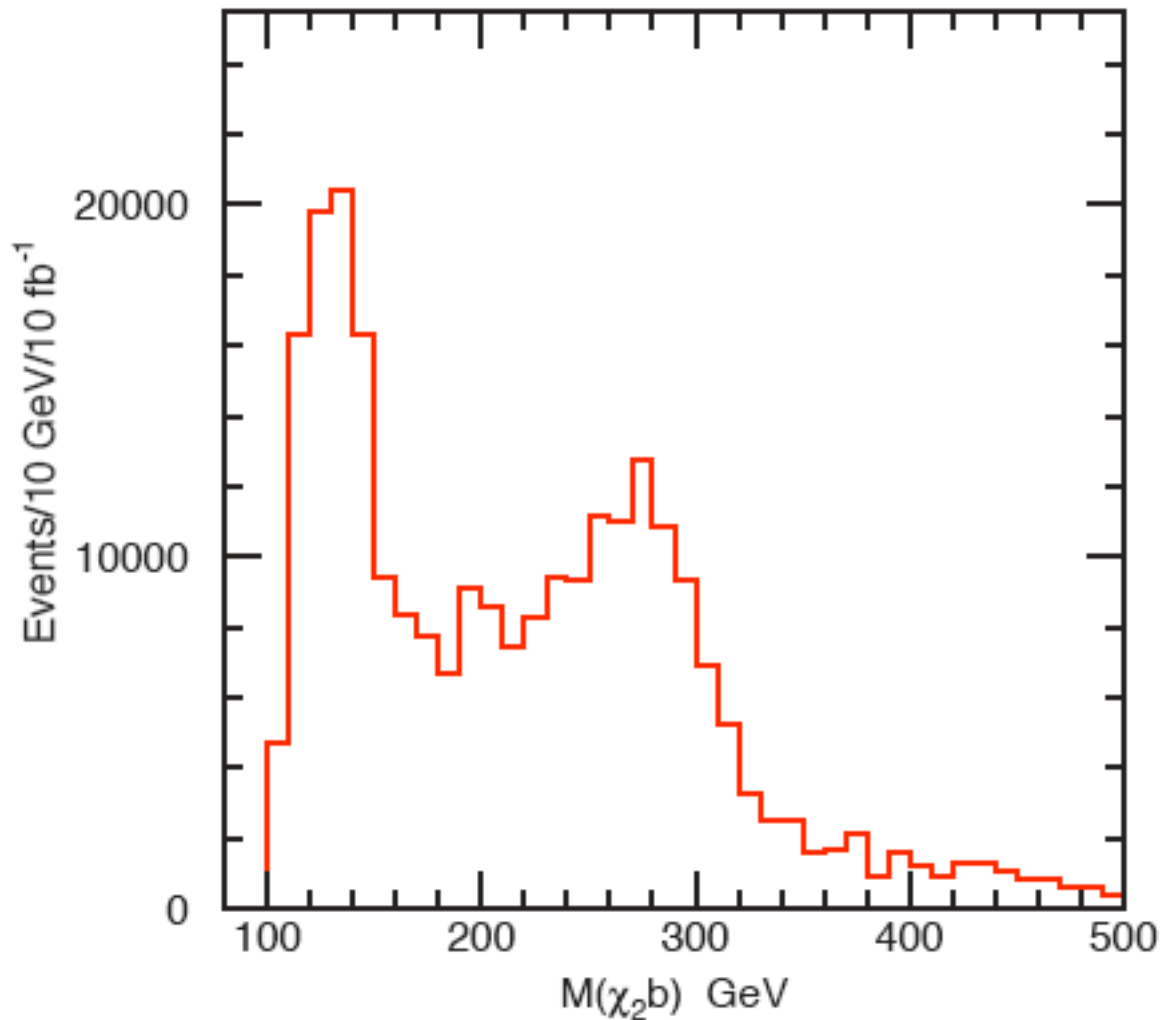
Hinchliffe et al. noticed that one could go further.

At the endpoint, the unobserved WIMP is at rest in the frame of the $l+l-$ pair. If we have an estimate of the mass of the WIMP, we can add back its 4-vector.

Now there is no more missing information. Add observed jets and reconstruct the parent squarks.



At the endpoint, the N_1 is at rest in the frame of the $\ell^+ \ell^-$. If we know (or guess) the mass of the N_1 , we know its 4-vector. **Now we have solved the problem of missing momentum**; we can add jets and try to reconstruct the parent squarks.



Hinchliffe et al.

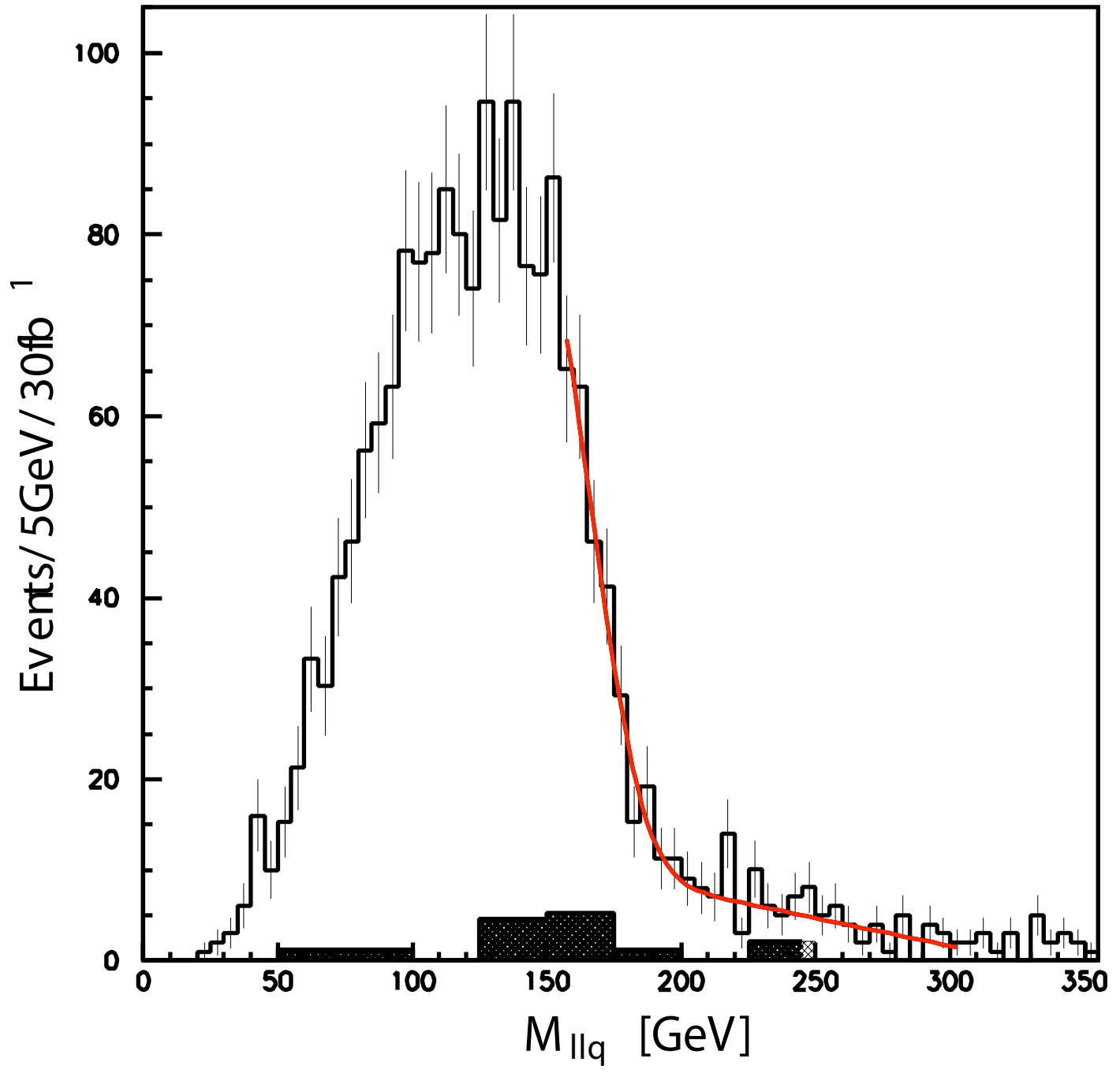
An alternative approach that does not rely so heavily on the endpoint region is to try to partially reconstruct the parent squark. Find the two hardest jets, and try to combine one with the lepton pair.

Some useful variables are:

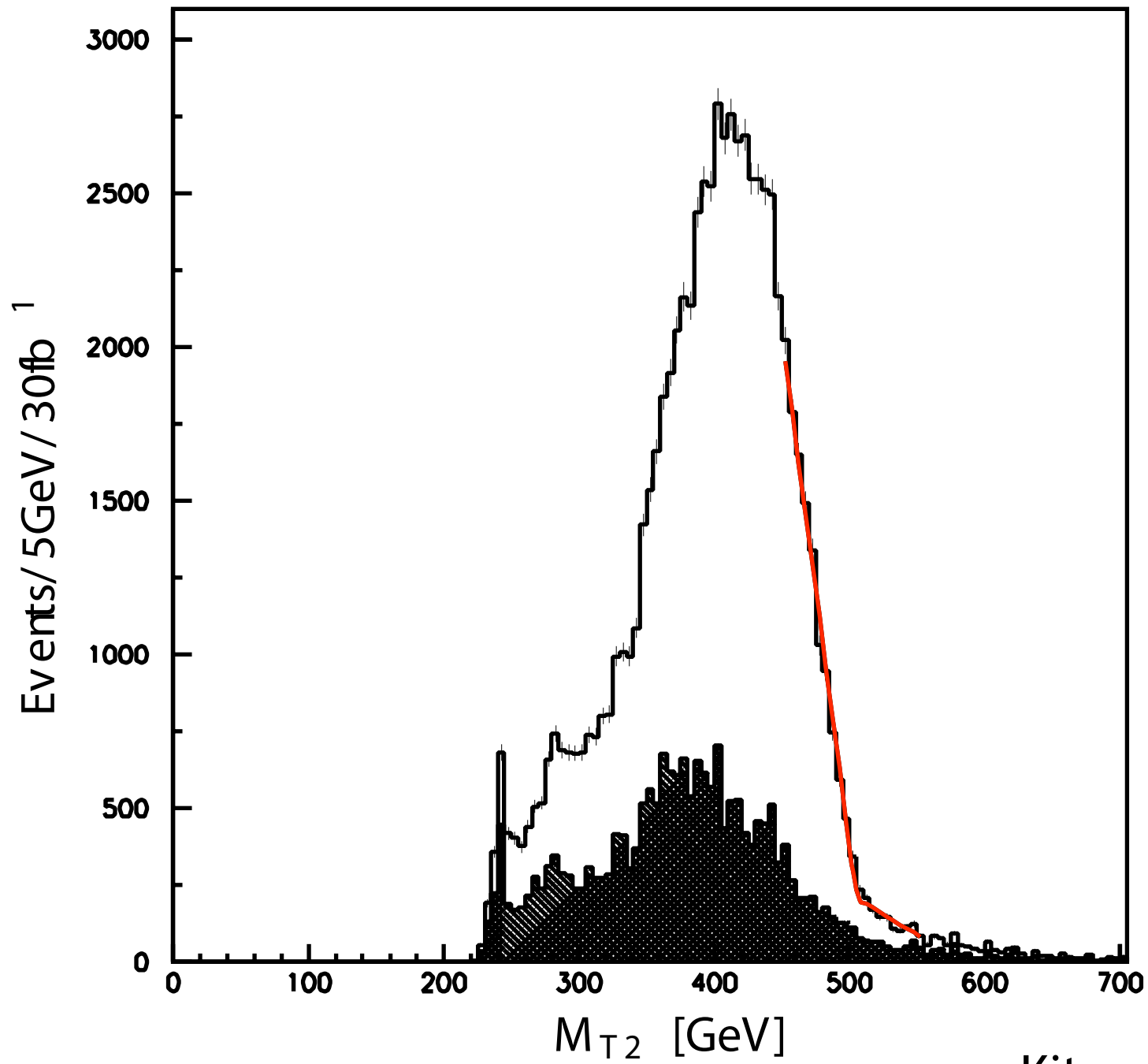
$$\min_{1,2} \{m(\ell\ell j)\}$$

$$M_T^2 = \min_{(p_{T1} + p_{T2} = p_T)} \max \{m_T^2(p_1 \cancel{p}_1), m_T^2(p_2 \cancel{p}_2)\}$$

Lester and Summers

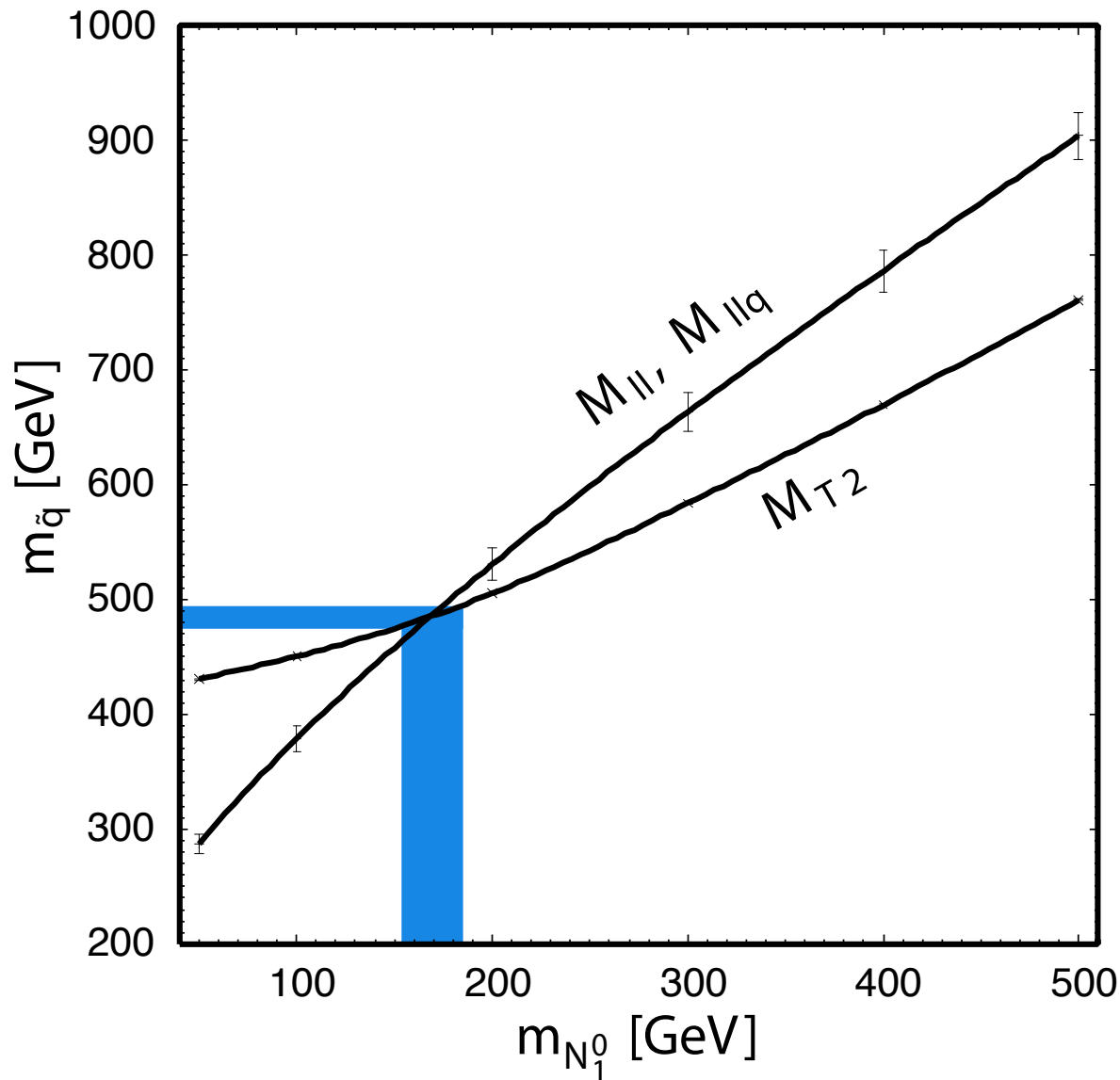


Kitano-Nomura



Kitano-Nomura

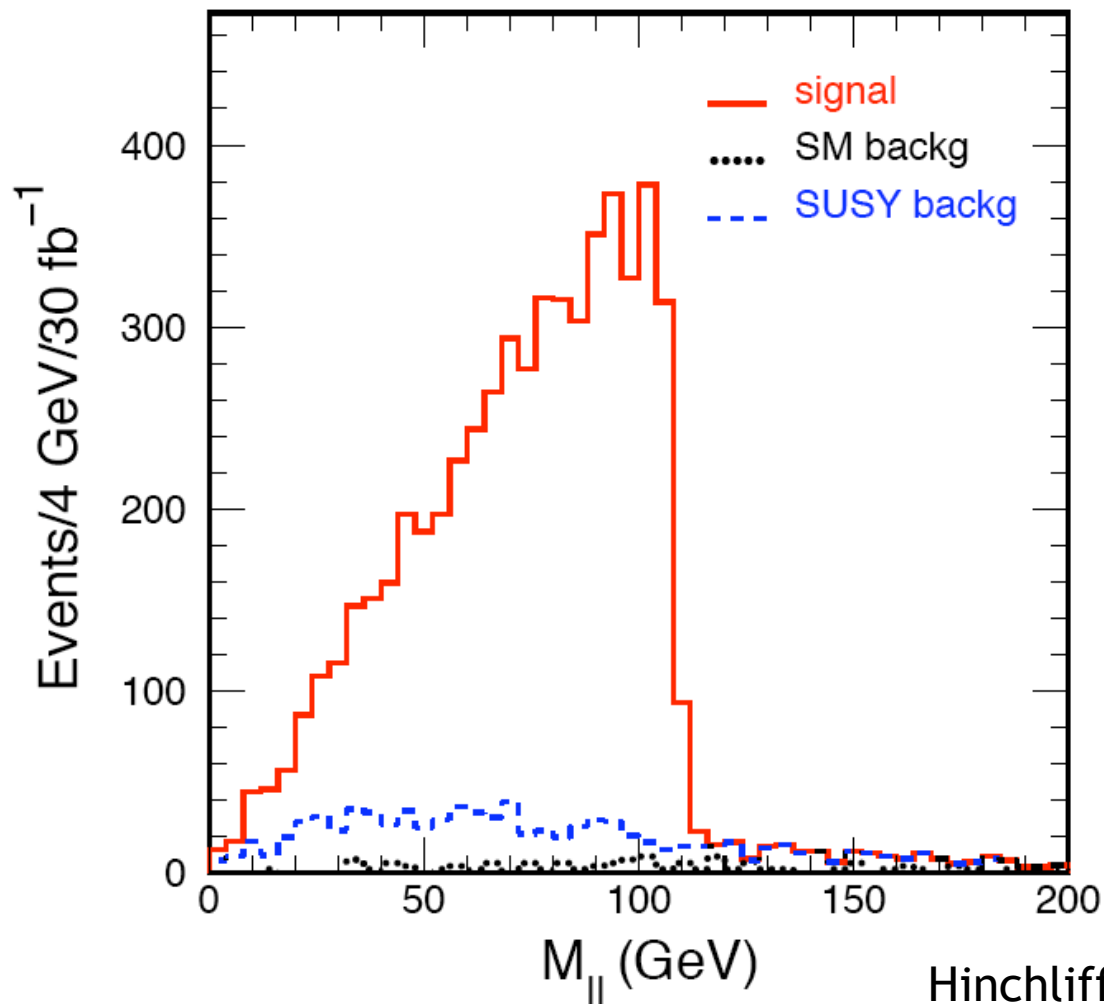
The endpoint positions have a different functional dependence on the squark and neutralino masses. Demand consistency:



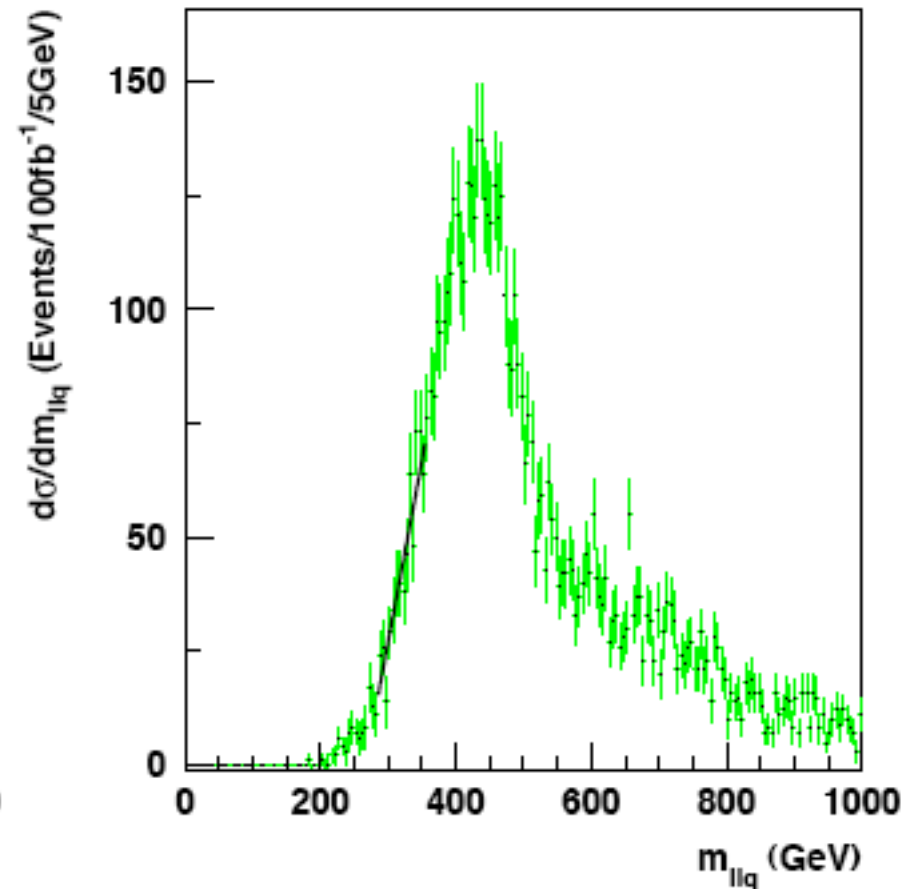
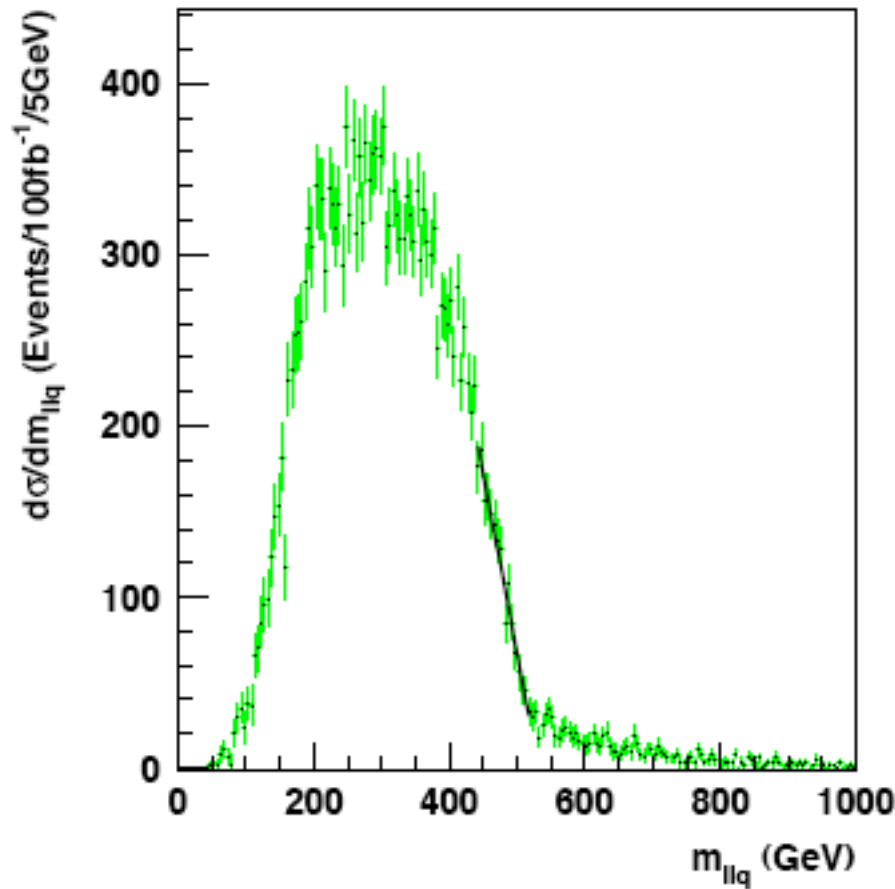
$$m_{N_1} = 169 \pm 17 \text{ GeV} \quad m_{\tilde{q}} = 486 \pm 11 \text{ GeV}$$

The case of a 2-body decay is even nicer. There is a sharp endpoint at

$$m(\ell^+ \ell^-) = m(N_2) \sqrt{1 - \left(\frac{m(\tilde{\ell})}{m(N_2)}\right)^2} \sqrt{1 - \left(\frac{m(N_1)}{m(\tilde{\ell})}\right)^2}$$



The decay $\tilde{q} \rightarrow qN_2$ is also 2-body, and so there are also upper and lower kinematic endpoints in combinations $(j\ell)$, $(j\ell\ell)$. From 4 endpoints, one can solve for the 4 unknown masses in the problem.



Hinchliffe and Paige

With these and other tricks, one can determine masses at the level of

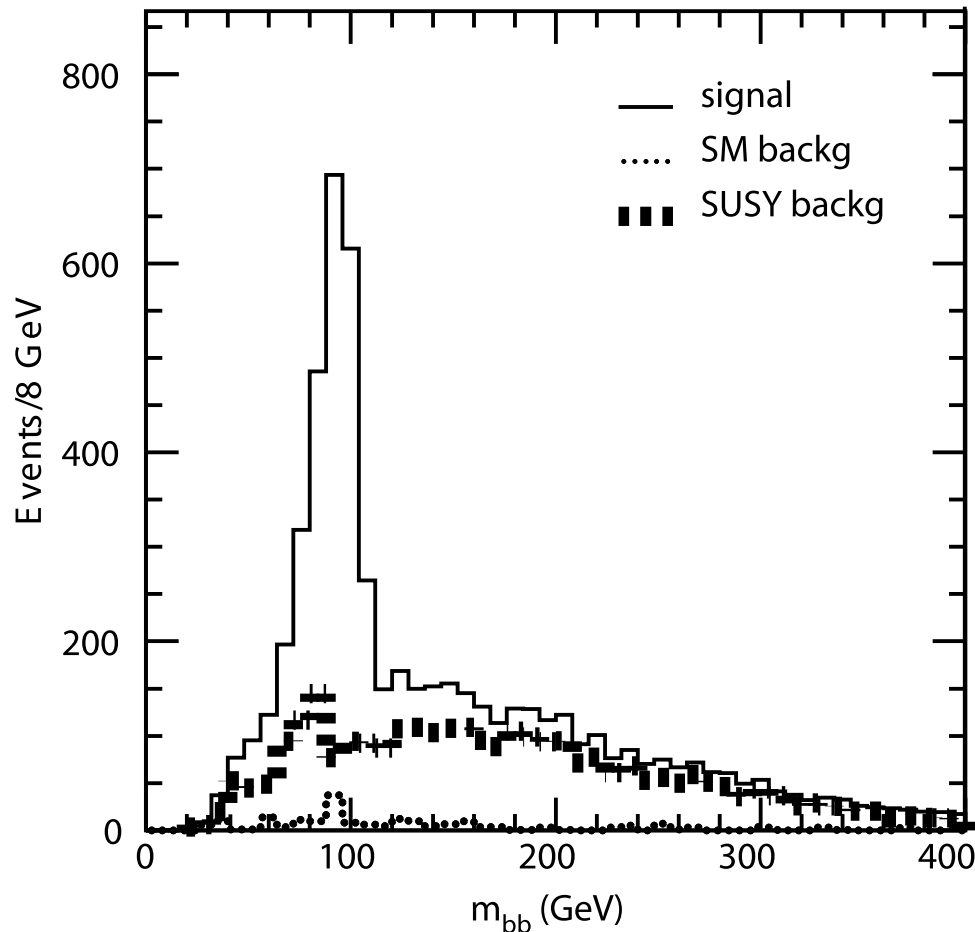
10% or below for WIMP, squark, gluino masses

1% for mass differences in $l+l^-$ cascades

One more case of an $N_2 \rightarrow N_1$ decay should be mentioned. If the 2-body decays to sleptons are not kinematically allowed, the dominant 2-body decay might be

$$N_2 \rightarrow N_1 + h^0$$

In this case, supersymmetry production can provide a copious source of Higgs bosons.



In each of these examples, the authors took advantage of special features of the spectrum to derive constraints or mass splittings. Once we identify a few states of the new particle spectrum, we can try to find the key observables. Also, we can try to identify paths that give the mass differences to the higher states in the spectrum.

However, there are several issues in supersymmetry spectroscopy that seem very difficult for hadron collider experiments:

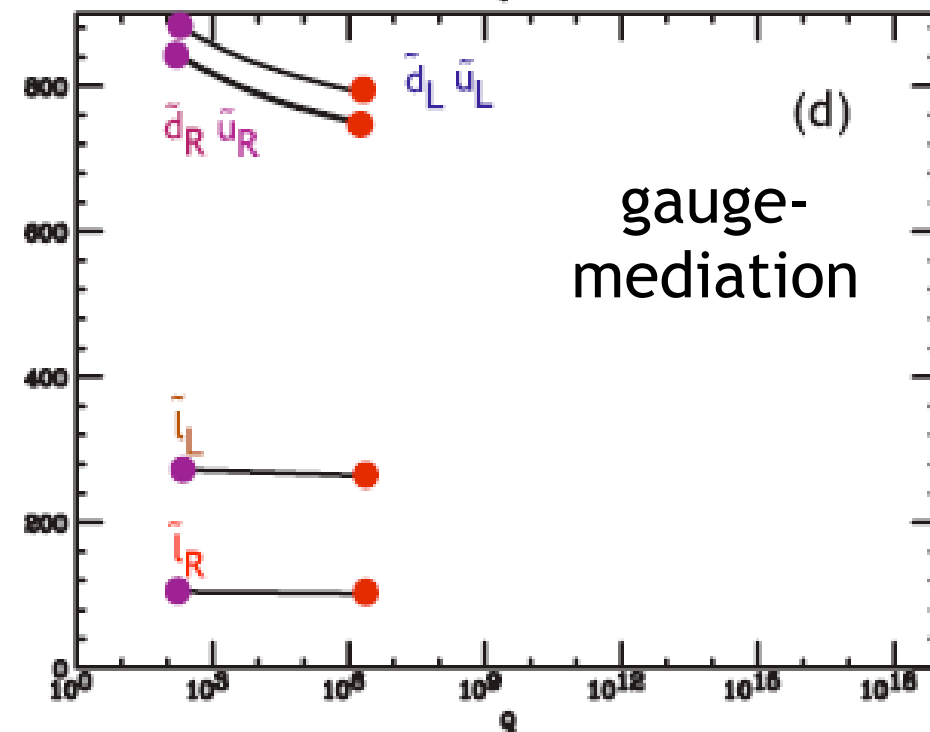
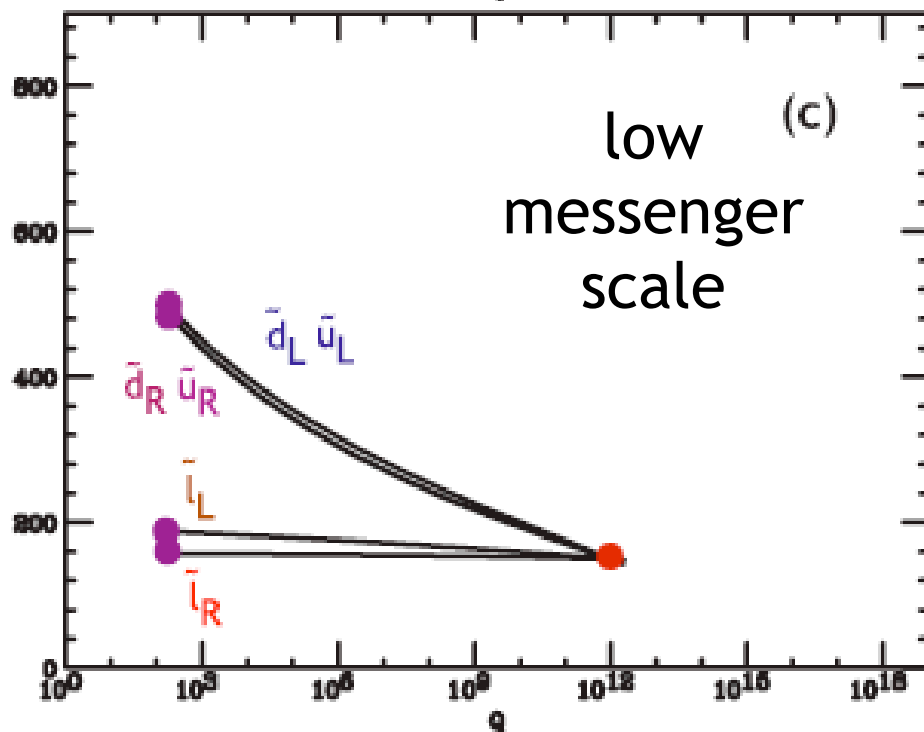
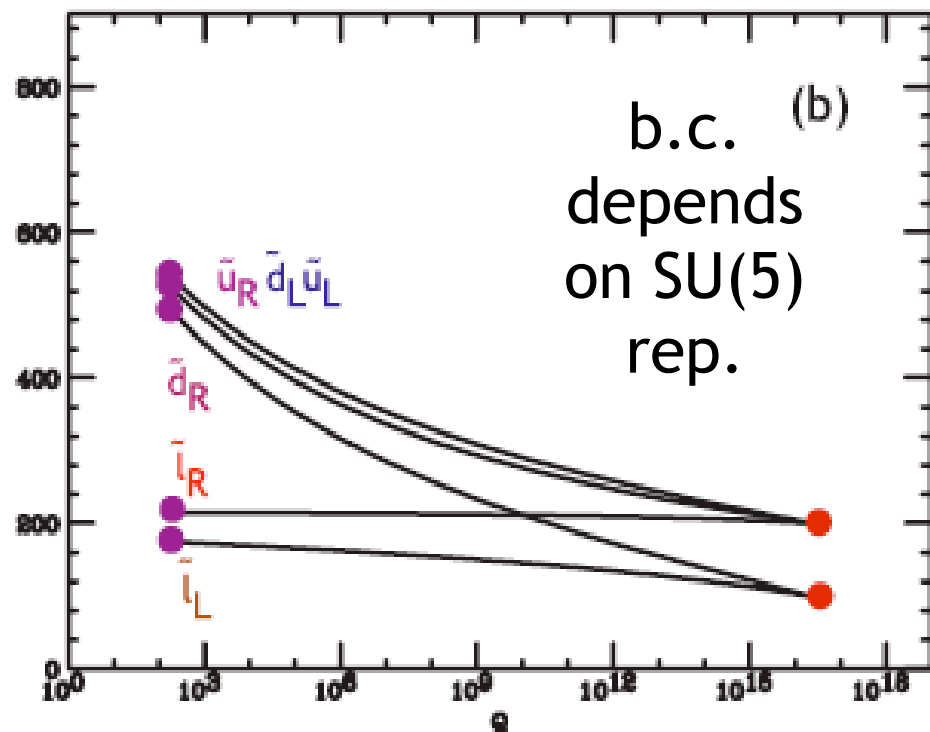
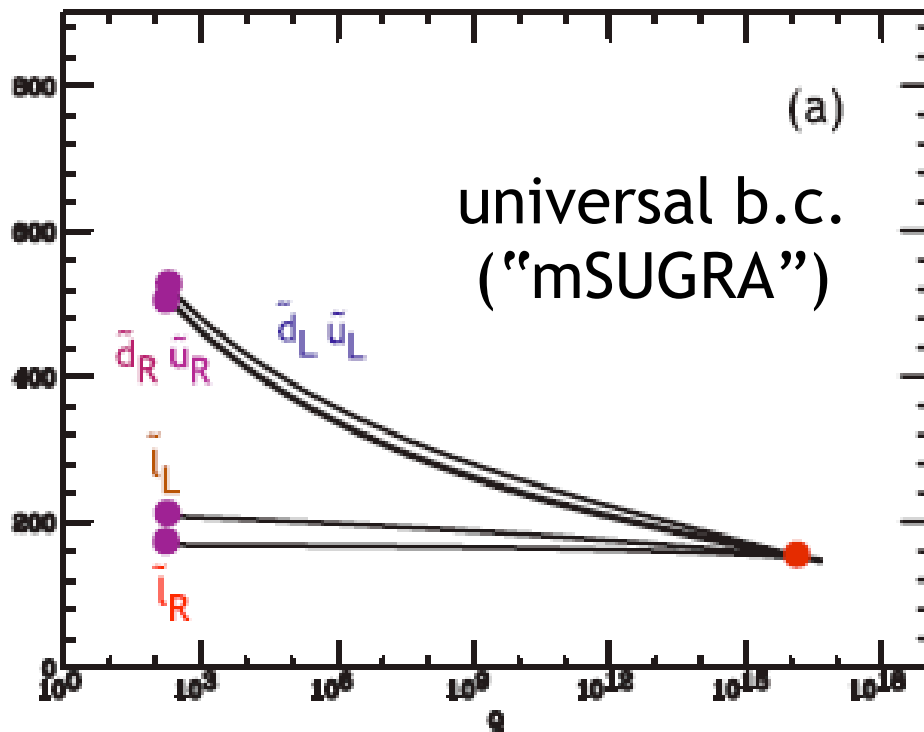
measurement of the properties of the τ **lepton** partners

separation of **chiral partners** and measurement of their mass ratios, in particular $m(\tilde{\ell}_R)/m(\tilde{\ell}_L)$

measurement of the **gaugino vs. Higgsino** nature of the charginos and neutralinos, and the associated mixing angles

measurement of $\tan \beta$, stop mixing parameters; **tests of the physics of electroweak symmetry breaking**

In addition, to test models of **unification and supersymmetry breaking**, we would like to know superparticle masses to the **1% level or better**.



To answer these questions, it will be very useful to be able to produce the superpartners in e^+e^- collisions:

electrons are elementary particles,
so **the initial CM system is known**

pair production in e^+e^- depends in a characteristic way on the Standard Model quantum numbers

so **we can determine spins and quantum numbers unambiguously**

the Standard Model annihilation cross sections are small and can be computed precisely,

so **backgrounds are small and controlled**

the CM energy can be adjusted,

so **we can concentrate on the lightest new particles with the simplest decay processes**

A major new e^+e^- collider is now under design.

the International Linear Collider (ILC)

The design CM energy is 500 GeV, with the potential for upgrade to 1000 GeV.

The ILC will be a global project. The design team is drawn from laboratories in the US, Europe, and Japan.

Argonne, Brookhaven, Cornell, DESY, Fermilab, Frascati, KEK, Novosibirsk, Orsay, and SLAC are among the labs with major involvement in this project.

Let us first discuss the pair production of **sleptons** in e^+e^- annihilation. We will start with the $\tilde{\mu}$ s, which provide an especially simple case. As we move from $\tilde{\mu}$ to $\tilde{\tau}$ to \tilde{e} , new complications and new observables will arise at each stage.

The process $e^+e^- \rightarrow \tilde{\mu}^+\tilde{\mu}^-$ is especially simple. The cross section is characteristic of scalar production

$$\frac{d\sigma}{d\cos\theta} = \frac{\pi\alpha^2}{2s} \beta^3 \sin^2\theta |f_{ab}|^2$$

for polarized initial electron and positron states, where the last factor depends on the SM quantum numbers

$$f_{ab} = 1 + \frac{(I_e^3 + \frac{1}{2}s_w^2)(I_\mu^3 + \frac{1}{2}s_w^2)}{c_w^2 s_w^2} \frac{s}{s - m_Z^2}$$

Note that there is a **strong dependence on the polarization states:**

$$|f_{ab}|^2 = \begin{array}{ll} e_R^- \rightarrow \tilde{\mu}^- & : 1.69 \quad e_R^- \rightarrow \tilde{\mu}^+ & : 0.42 \\ e_L^- \rightarrow \tilde{\mu}^- & : 0.42 \quad e_L^- \rightarrow \tilde{\mu}^+ & : 1.98 \end{array}$$

This reaction gives an elegant diagnostic of all of the SUSY partner quantum numbers: **spin, I, Y.**

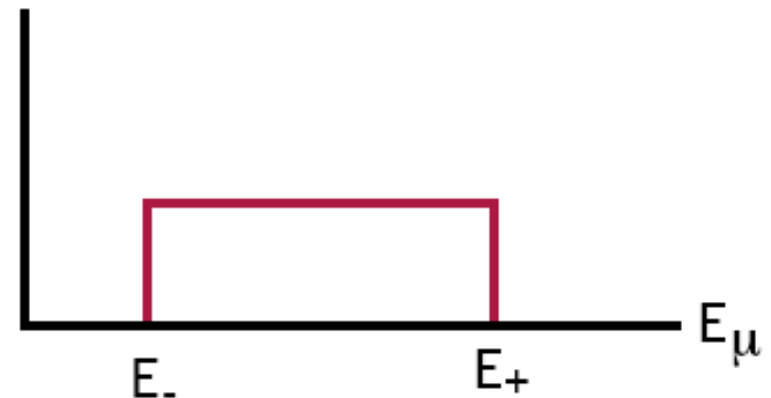
If the smuon is light, it decays to $\tilde{\mu} \rightarrow \mu N_1^0$

I continue to assume that the N_1^0 is stable and weakly interacting. Then it exits unseen from collider detectors.

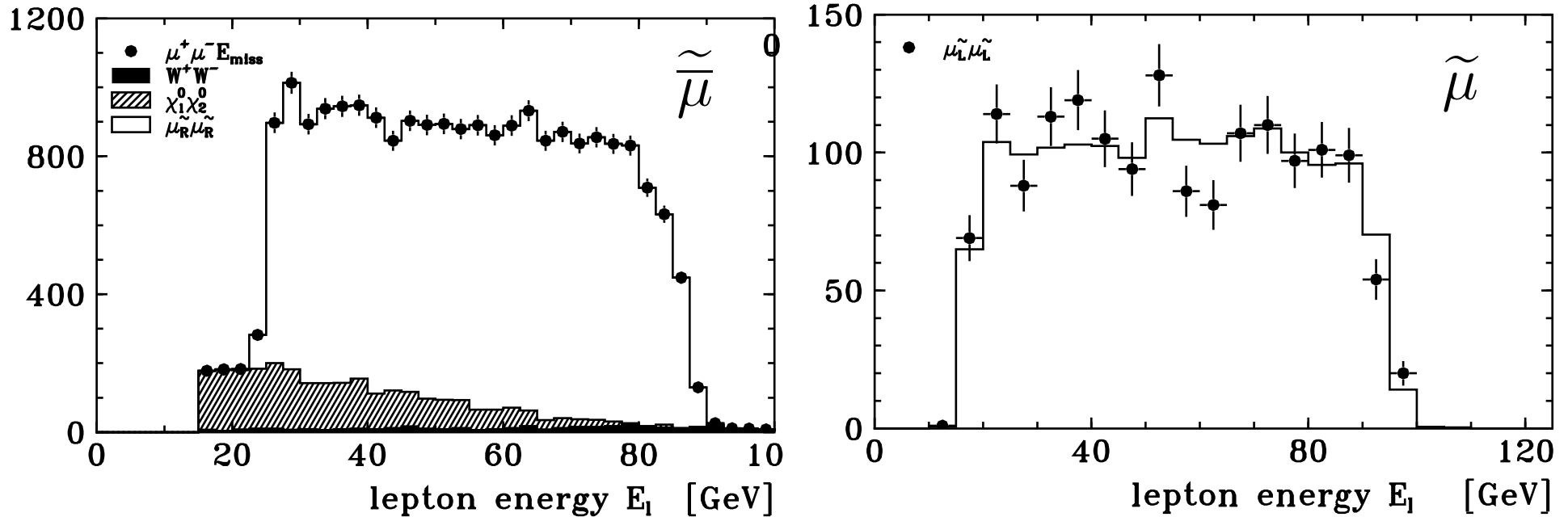
The reaction is then observed as

$$e^+e^- \rightarrow \mu^+\mu^- + (\text{missing } E \text{ and } p)$$

The spectrum of the observed muons is very simple: since $\tilde{\mu}$ is spin 0, it decays **isotropically**. In e^+e^- the $\tilde{\mu}$ is produced at a definite CM energy. **The boost of an isotropic distribution is a flat distribution in energy.** So the muon energy spectrum is flat between kinematic endpoints. These endpoints are determined by the masses of the $\tilde{\mu}$ and the N_1^0 .



Here are two examples of muon energy distributions from the TESLA simulation studies:



Blair and Martyn

It is expected that these masses could be measured at a next-generation e^+e^- collider (ILC) to a few hundred MeV (parts per mil).

For $e^+e^- \rightarrow \tilde{\tau}^+\tilde{\tau}^-$, there are a few additional complications.

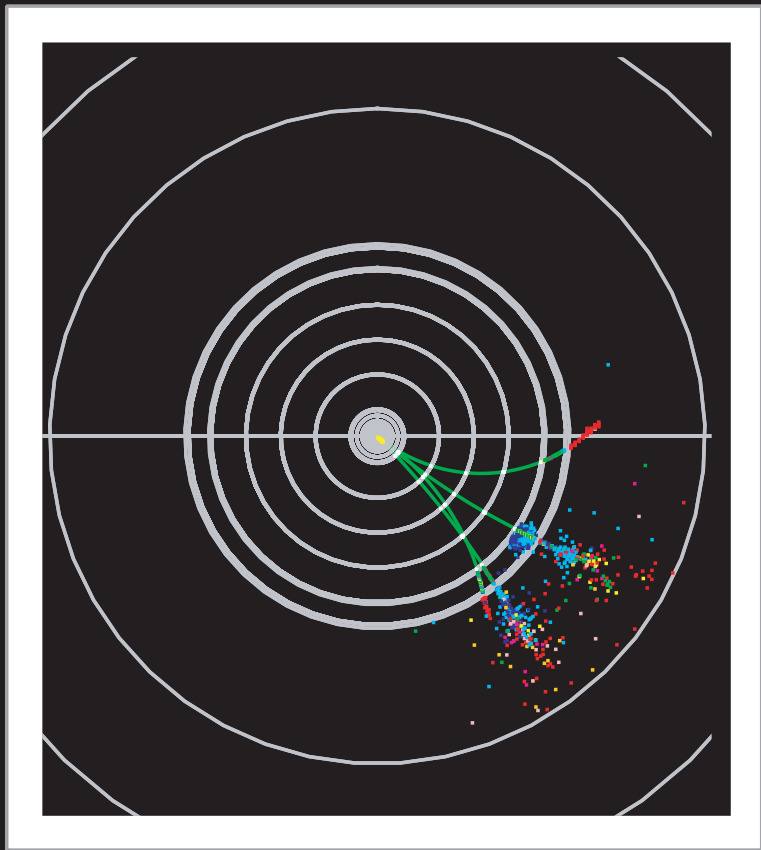
First, mixing may be important, especially if $\tan\beta$ is large. The cross section formulae reflect the mass mixing. For example, when we go from $\tilde{\tau}^-$ to $\tilde{\tau}_1^-$, the cross section formula gets

$$f_{RR} \rightarrow f_{RR} \cos^2 \theta_\tau + f_{RL} \sin^2 \theta_\tau$$

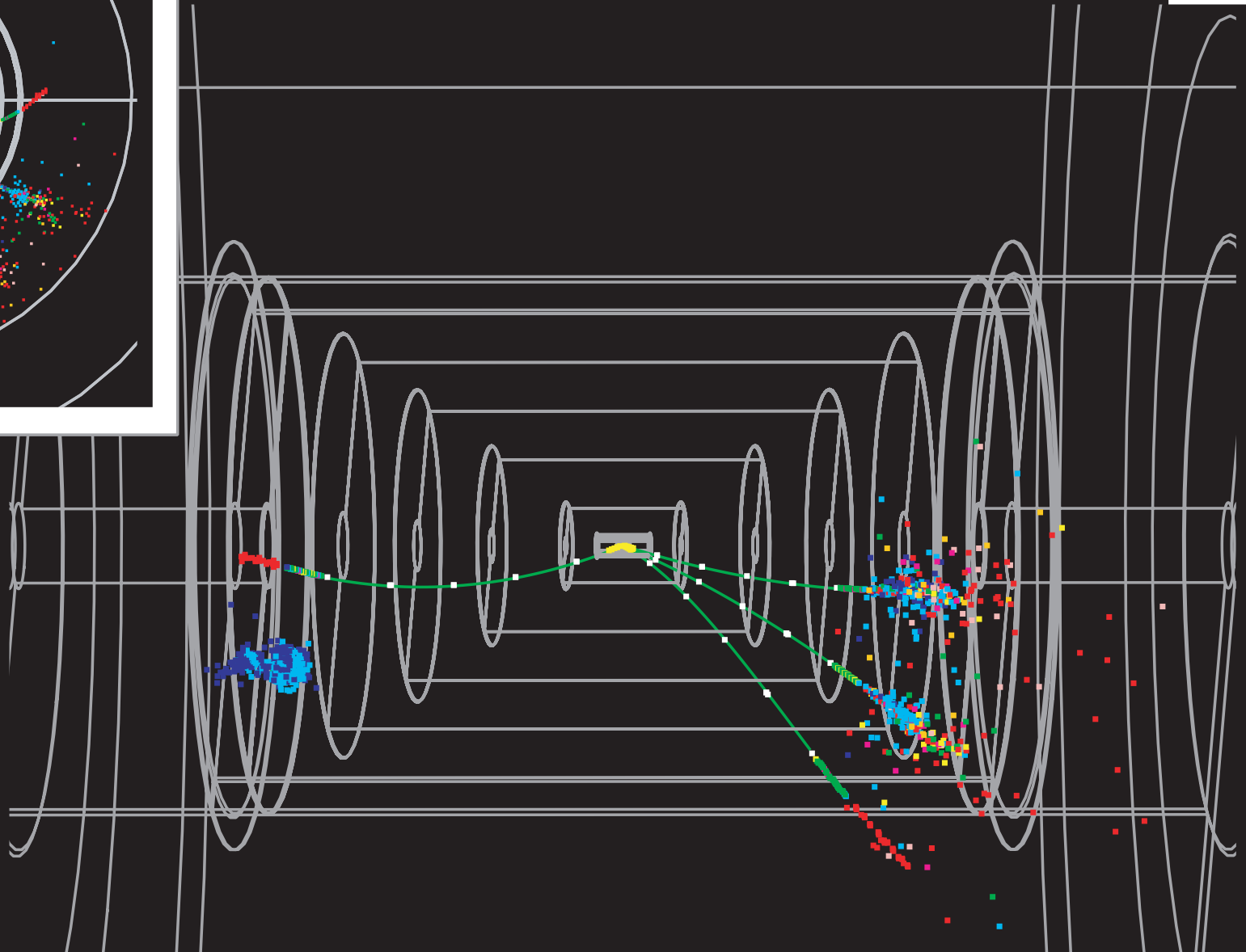
where θ_τ is the $\tilde{\tau}$ mixing angle.

Again, for large $\tan\beta$, $\tilde{\tau}^-$ can decay to $\tau_R^- \tilde{b}$ through the gauge couplings or to $\tau_L^- h_d$ through the Higgs coupling. \tilde{b} and h_d are components of \tilde{N}_1^0 . Measurement of the final τ polarizations can then determine the \tilde{N}_1^0 eigenstate.

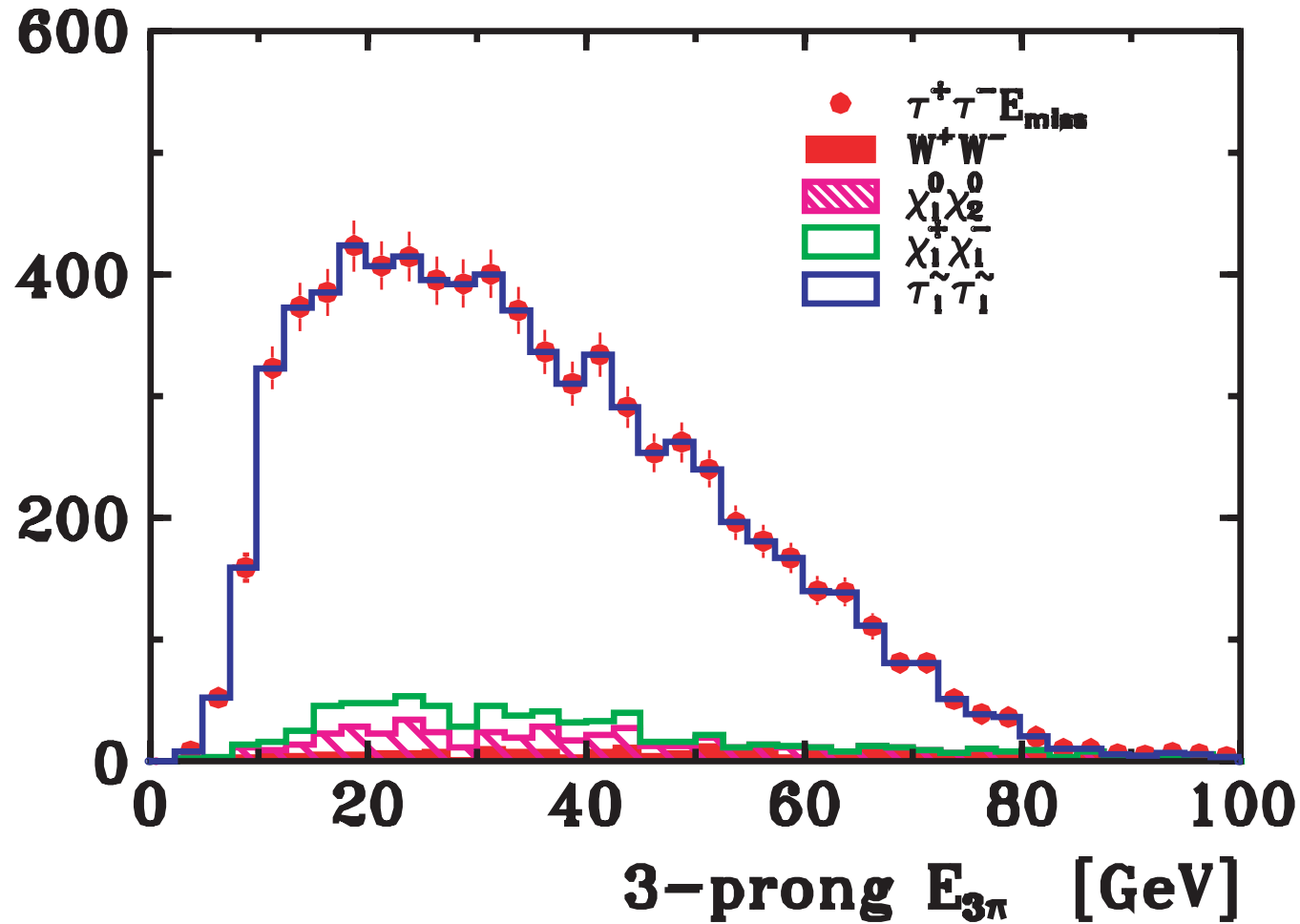
Here is a sample simulation event



$$e^+e^- \rightarrow \tilde{\tau}^+\tilde{\tau}^-$$

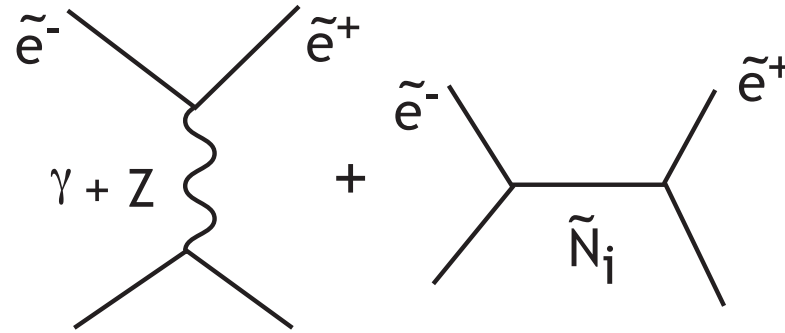


Here is the energy spectrum of visible decay products in the stau case. The kinematic endpoints are still well-defined.



Blair and Martyn

Finally, consider $e^+e^- \rightarrow \tilde{e}^-\tilde{e}^+$. Here there is a new diagram, involving t-channel neutralino exchange.



This diagram has two effects. First, it dominates annihilation through γ and Z , giving a large forward peak to the cross section. E.g., the cross section formula for $e_R^-e_L^+ \rightarrow \tilde{e}^-\tilde{e}^+$ is changed by

$$f_{RR} \rightarrow f_{RR} - \left| \frac{V_{01i}}{c_w} \right|^2 \frac{s}{m_i^2 - t}$$

where V_0 is the neutralino mixing matrix.

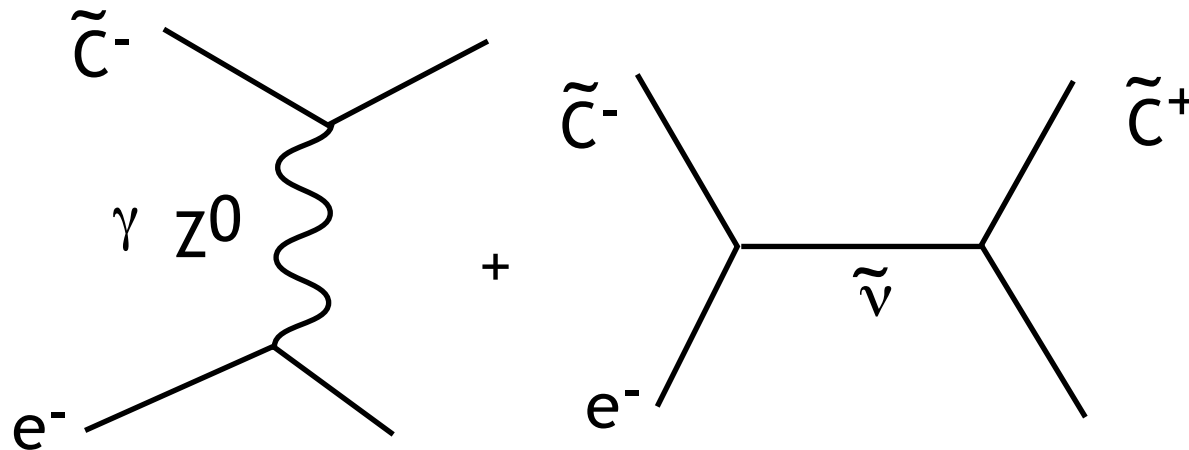
Second, it allows new processes such as $e_L^-e_L^+ \rightarrow \tilde{e}^-\tilde{e}^+$

Note the correlation of electron and positron spin with the identities of the final particles.

We can also study e^+e^- pair production of charginos and neutralinos:

$$e^+e^- \rightarrow C_k^+ C_\ell^- , N_i N_j$$

For example, for chargino production:

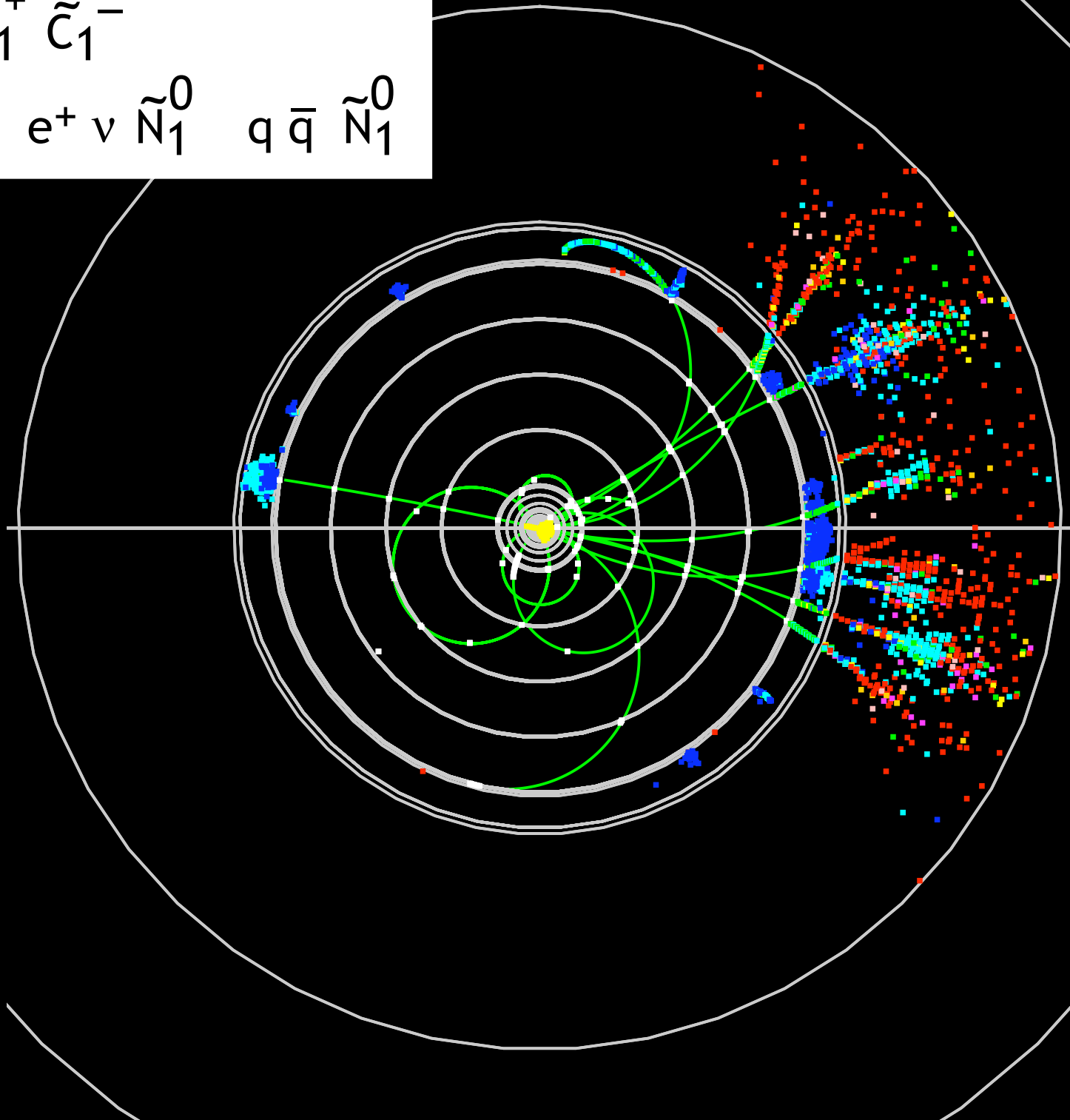


This is the supersymmetric analogue of $e^+e^- \rightarrow W^+W^-$. As in that process, the most characteristic events have hadronic decays on one side, leptonic decays on the other:

$$\tilde{C}^+ \rightarrow \ell^+ \nu \tilde{N}_1 \quad \tilde{C}^- \rightarrow q\bar{q}N_1$$

$$e^+e^- \rightarrow \tilde{C}_1^+ \tilde{C}_1^-$$

$$\rightarrow e^+ \nu \tilde{N}_1^0 \quad q \bar{q} \tilde{N}_1^0$$

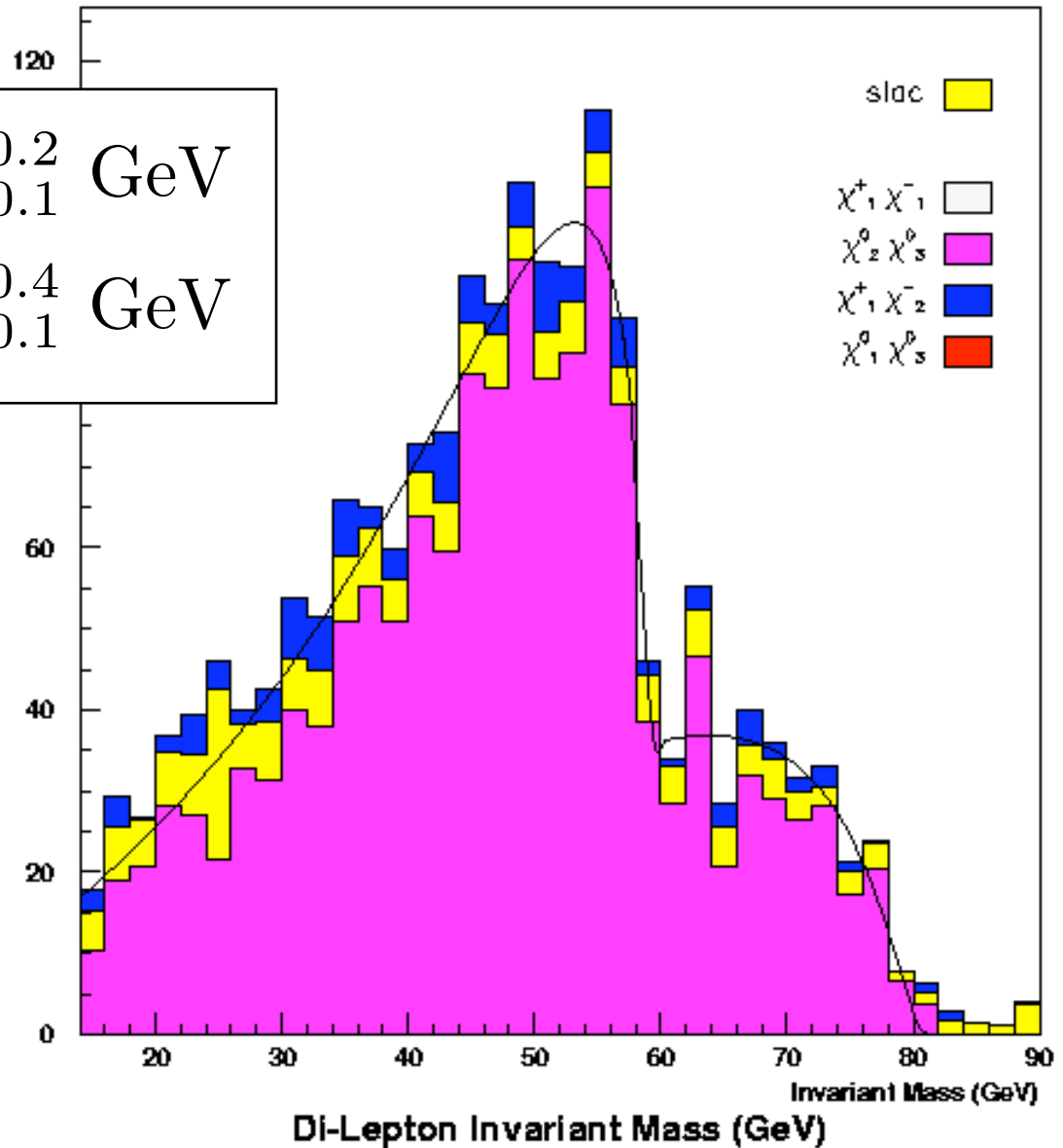


2J2L, Di-Lepton Invariant Mass, With Cuts, 500fb⁻¹

$$m(\tilde{N}_2) - m(\tilde{N}_1) = 58.7^{+0.2}_{-0.1} \text{ GeV}$$

$$m(\tilde{N}_3) - m(\tilde{N}_1) = 82.0^{+0.4}_{-0.1} \text{ GeV}$$

We have the phenomenology of neutralino dilepton decays that we have already discussed in the LHC case. Here, though, the endpoints can be determined to parts per mil.



These cross sections have a strong dependence on the chargino and neutralino mixing angles. This is especially clear if we consider polarized initial states.

For definiteness, consider $e_L^+ e_R^- \rightarrow C_1^+ C_1^-$

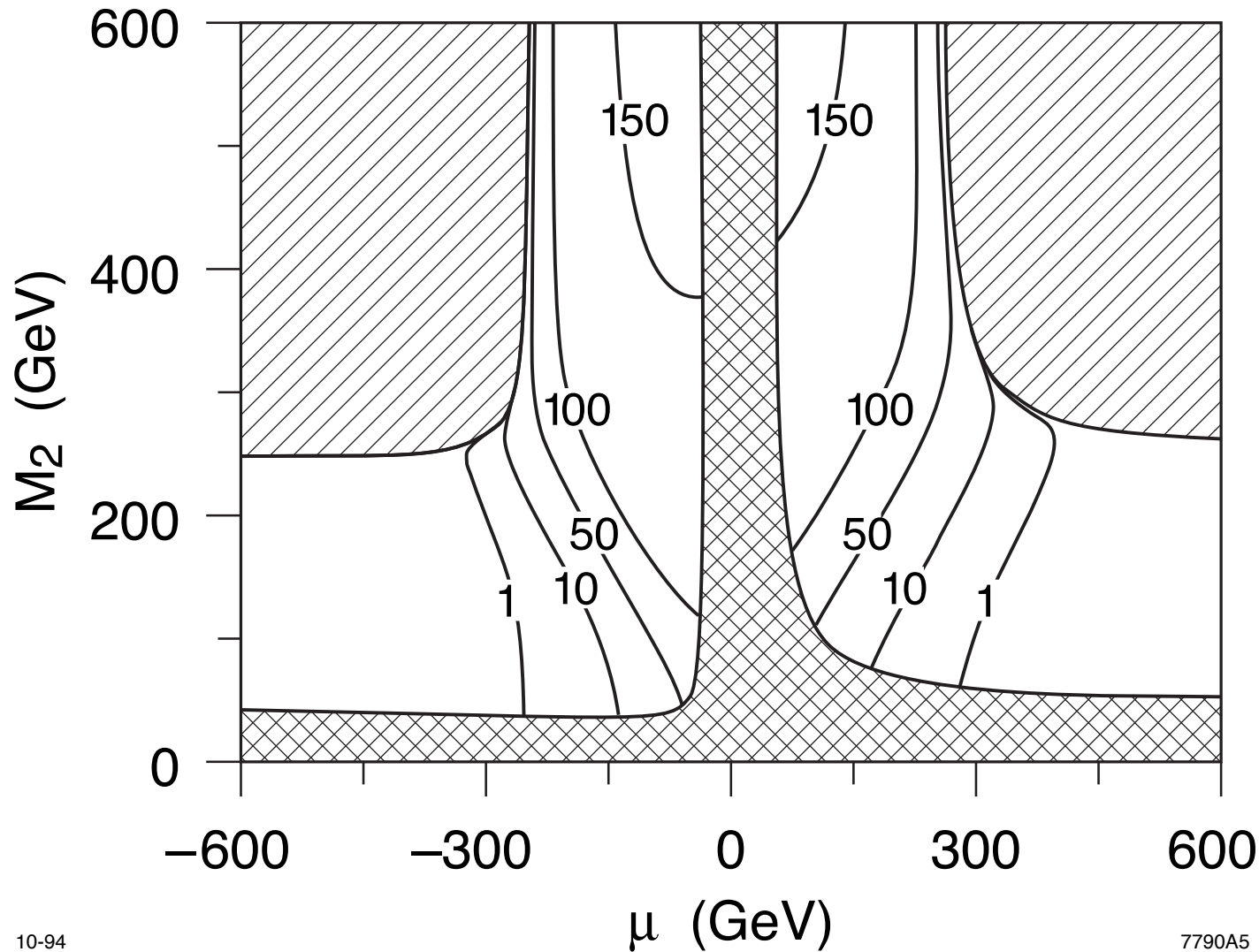
For e_R^- , the t-channel neutrino diagram does not appear, so we have only the s-channel diagrams.

We are at high energy, so it is a good approximation to consider gauge eigenstates, (B, W^0) instead of (γ, Z) . The e_R^- couples only to B . But, \tilde{w}^+, \tilde{w}^- do not couple to B . So, this cross section measures the **Higgsino content** of C_1^+ and C_1^- .

Let me make one more simplifying assumption: high energy. Then the \tilde{h}_L^+ and \tilde{h}_L^- go forward; the \tilde{h}_R^+ goes backward. Finally:

$$\frac{d\sigma}{d\cos\theta}(e_L^+ e_R^-) \sim \frac{\pi\alpha^2}{8c_w^2 s} [(|V_{+21}|^4 (1 + \cos\theta)^2 + |V_{-21}|^4 (1 - \cos\theta)^2)]$$

Here is a calculation of this polarized cross section without making high-energy assumptions. It does clearly distinguish the gaugino and Higgsino scenarios.



10-94

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$E_{cm} = 500$ GeV; σ in fb

Feng et al.

The values of the chargino and neutralino mixing angles turn out to be important for the connection between supersymmetry and dark matter.

We can discover a weakly interacting heavy neutral particle at colliders, but we still will not know whether this particle makes up all or even some of the cosmic dark matter.

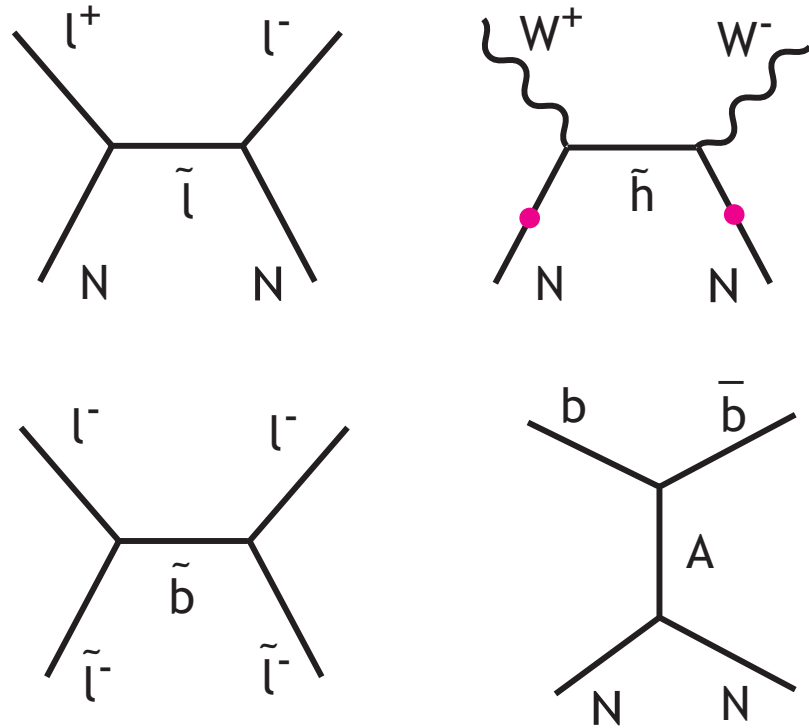
To learn this, we need

the **pair annihilation cross section** $\sigma(NN \rightarrow X)$, to evaluate the microscopic prediction for its cosmic density

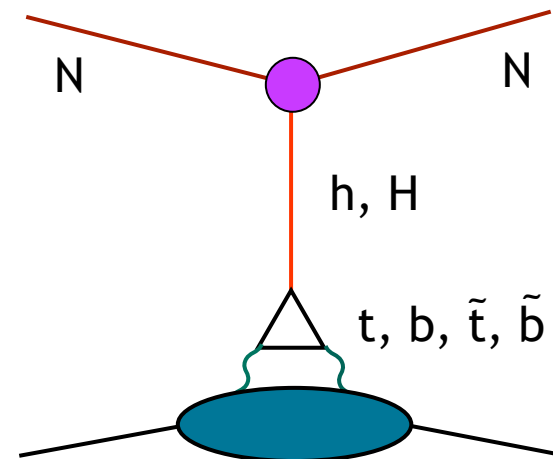
the **direct detection cross section** $\sigma(Np \rightarrow Np)$, to compare to direct detection rates and, eventually, to evaluate the density of neutralinos in the galactic halo.

Both cross sections depend strongly on the neutralino mixing angles.

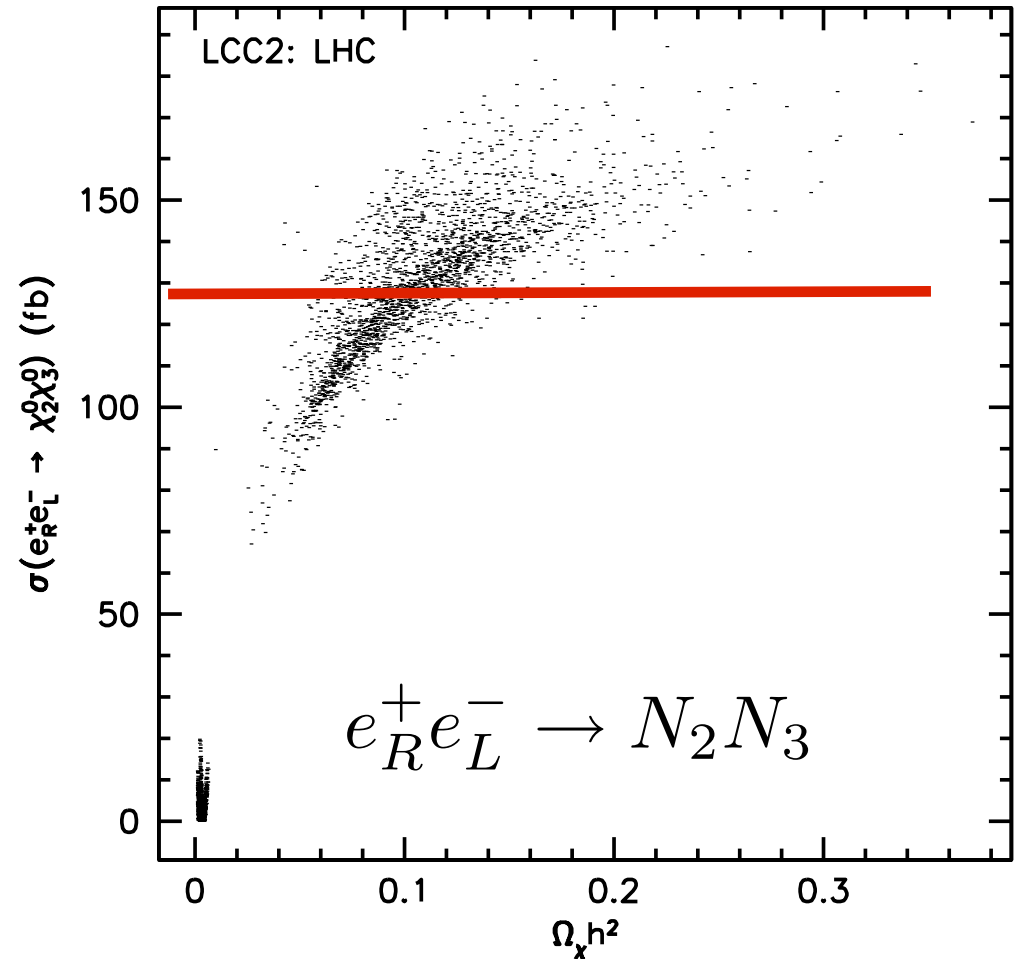
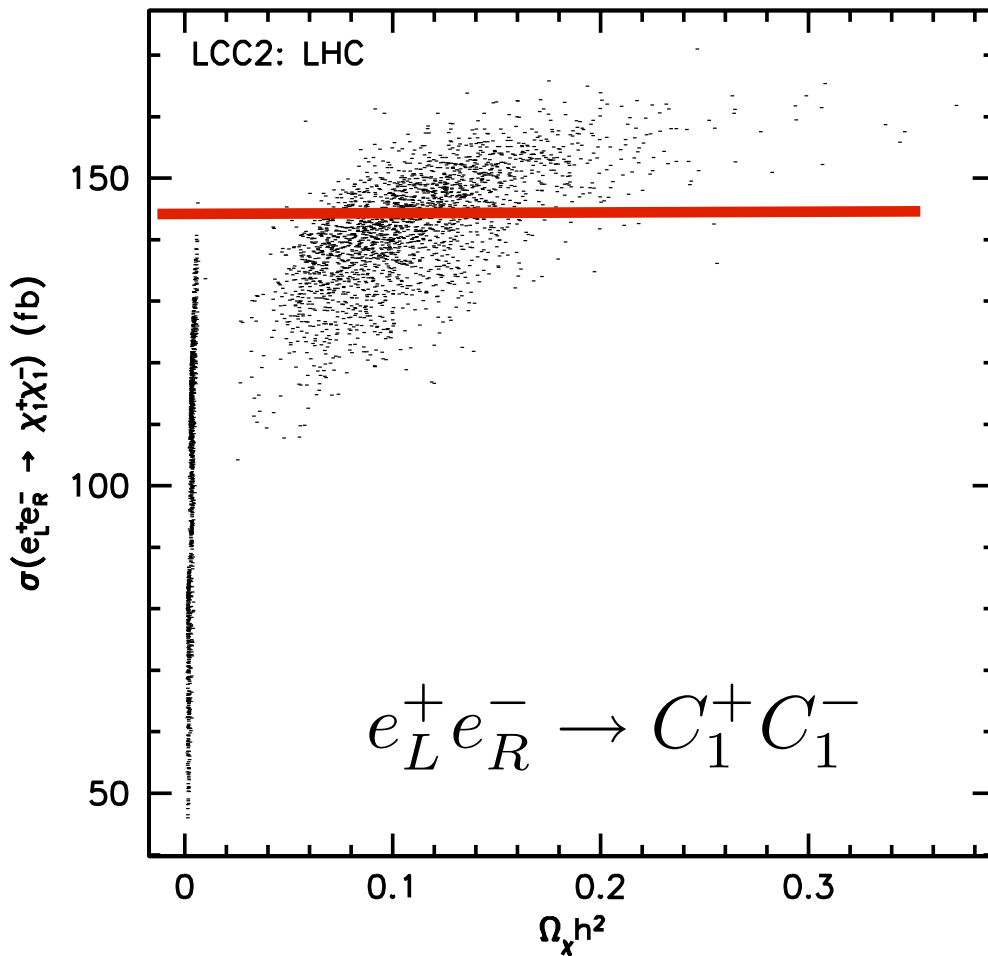
For the annihilation cross section, the mixing angles are needed not only to obtain the value of the cross section but also to predict the dominant mechanism of annihilation.



The direct detection cross section is typically dominated by Higgs boson exchange. The NNH vertex requires both gaugino and Higgsino content.



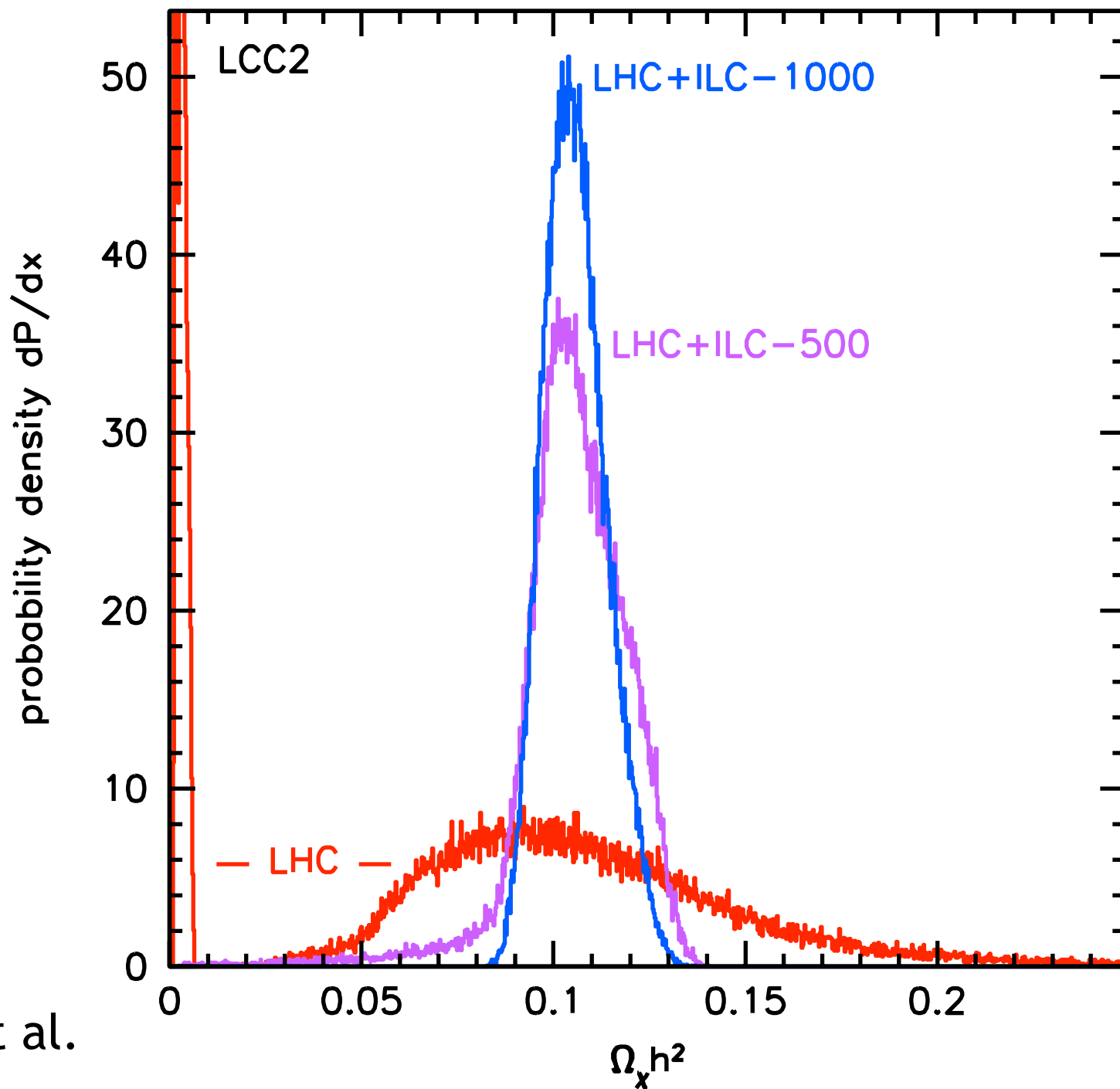
Here is an example in which the LHC data leaves an ambiguity as to whether the lightest neutralino is wino or bino. The ambiguity is resolved by measuring **polarization-dependent production cross sections** at the ILC:



Baltz et al.

In this example, the ILC measurements considerably refine the prediction of the dark matter cosmic density from microscopic information.

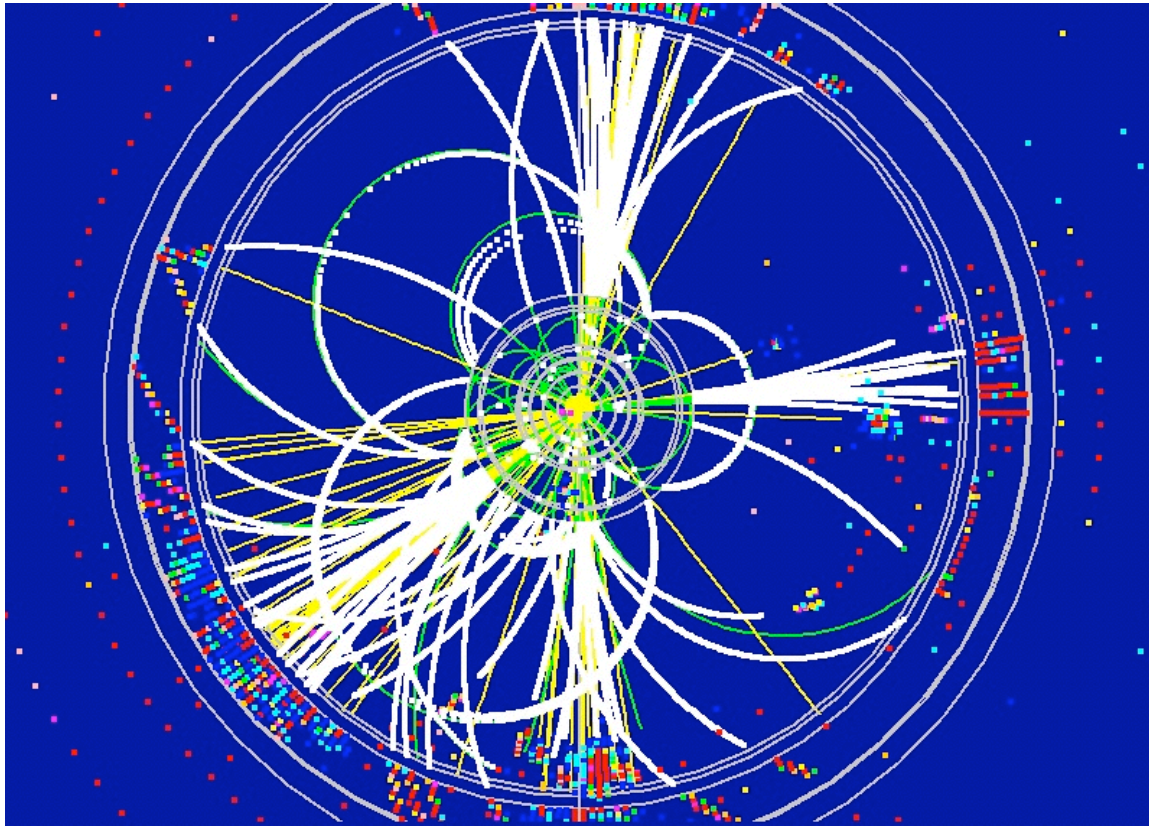
Baltz et al.



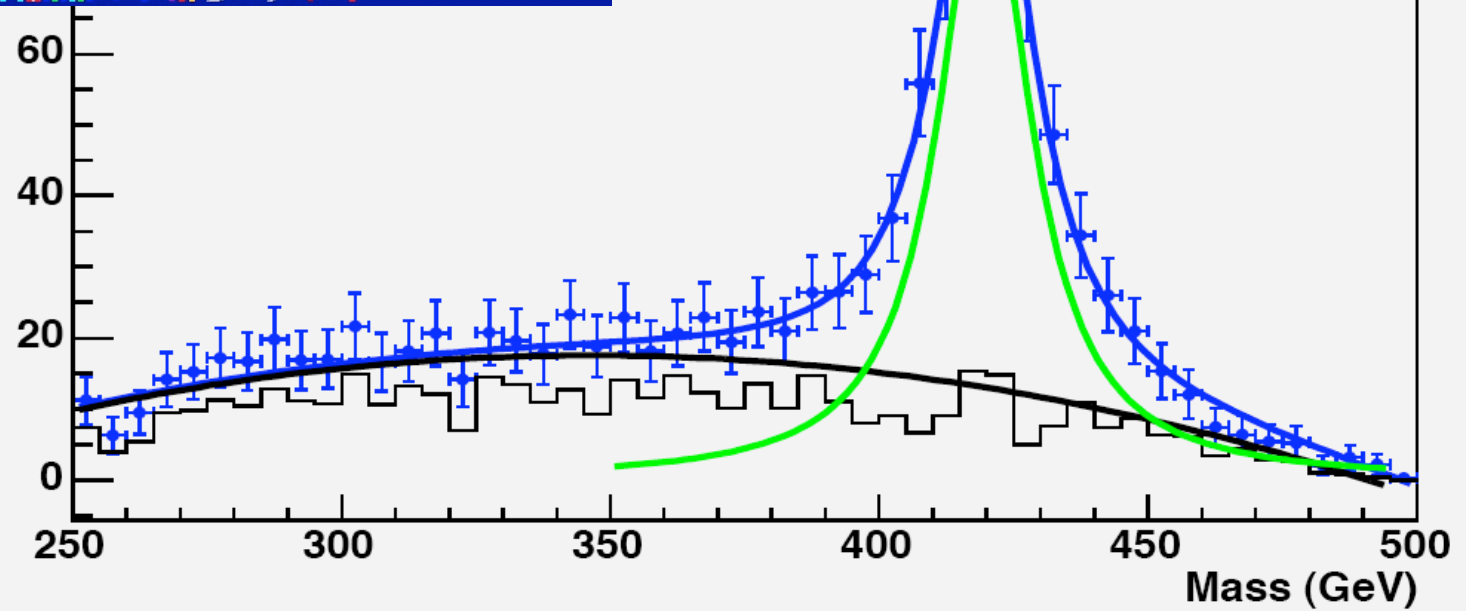
The direct detection cross section can also depend on the mass of the heavy Higgs bosons and the value of $\tan \beta$. To determine these parameters, it is usually necessary to pair-produce the Higgs bosons in

$$e^+ e^- \rightarrow H^0 A^0$$

and measure their branching ratios to $t\bar{t}$, $b\bar{b}$, $\tau^+ \tau^-$.

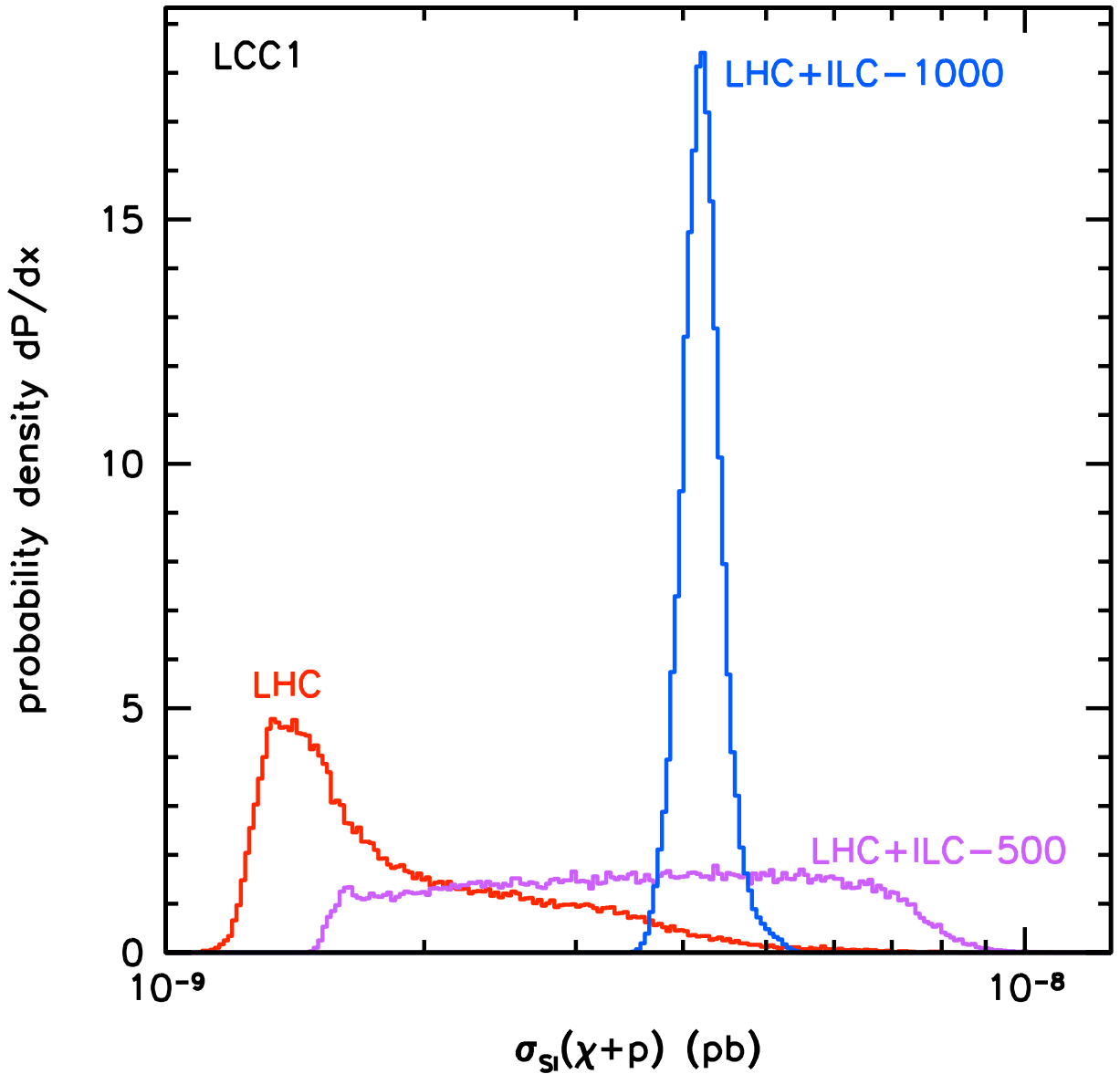


$e^+e^- \rightarrow H^0 A^0 \rightarrow 4b$
at the ILC



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This set of measurements can have a qualitative effect on our ability to predict the direct detection cross section.



In this lecture, I have given many illustrations of how we could use the data from the LHC and ILC to work out the details of the spectrum of supersymmetric particles in specific models. **The methods used build on the experimental techniques from LEP, SLC, and the Tevatron that we discussed in the earlier lectures.**

All of these methods generalize to other models of electroweak symmetry breaking.

We do not know what spectrum of new particles Nature has prepared for us. You will need to work out that puzzle. I hope that these lectures have given you a foundation from which you can attack that problem and solve it.