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Neutrino Physics
(Lecture 3 & 4)

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Neutrino Physics

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Tentative Outline for the Next Four Lectures

1. Brief History of the Neutrino;
2. Neutrino Puzzles – The Discovery of Neutrino Masses;
3. Neutrino Oscillations;
4. What We Know We Don't Know – Next-Generation ν Oscillations;
5. What We Know We Don't Know – Majorana versus Dirac Neutrinos;
6. Neutrino Masses As Physics Beyond the Standard Model;
7. Ideas for Tiny Neutrino Masses, and Some Consequences;
8. Comments on Lepton Mixing. (see Steve King's lectures)

[note: Questions are ALWAYS welcome]

Short, Biased List of Recent References:

- A. Strumia and F. Vissani, hep-ph/0606054;
- R. Mohapatra and A. Yu. Smirnov, hep-ph/0603118;
- R. Mohapatra *et al.*, hep-ph/0510213;
- AdG, hep-ph/0503086;
- AdG, hep-ph/0411274.

Neutrino History:

"Are There Really Neutrinos? – An Evidential History," Allan Franklin, Perseus Books, 2001.

1 - Brief History of the Neutrino

1. 1896: Henri Becquerel discovers natural radioactivity while studying phosphorescent properties of uranium salts.
   - α rays: easy to absorb, hard to bend, positive charge, mono-energetic;
   - β rays: harder to absorb, easy to bend, negative charge, spectrum?;
   - γ rays: no charge, very hard to absorb.
2. 1897: (J.J. Thompson discovers the electron.)
3. 1914: Chadwick presents definitive evidence for a continuos β-ray spectrum. Origin unkown. Different options include several different energy loss mechanisms.

It took 15+ years to decide that the "real" β-ray spectrum was really continuos. Reason for continuos spectrum was a total mystery:

- QM: Spectra are discrete;
- Energy-momentum conservation: $N \to N' + e^-$ — electron energy and momentum well-defined.

Nuclear Physics before 1930: nucleus $= n_p p + n_e e^-$.

Example: ^{4}He $= 4p + 2e^-$, works well. However: ^{14}N $= 14p + 7e^-$ is expected to be a fermion. However, it was experimentally known that ^{14}N was a boson!

There was also a problem with the magnetic moment of nuclei: $\mu_N, \mu_p \ll \mu_e$ ($\mu = eh/4mc$). How can the nuclear magnetic moment be so much smaller than the electron one if the nucleus contains electrons?

SOLUTION: Bound, nuclear electrons are very weird!

This can also be used to solve the continuous β-ray spectrum: energy need not be conserved in nuclear processes! (N. Bohr)

"... This would mean that the idea of energy and its conservation fails in dealing with processes involving the emission and capture of nuclear electrons. This does not sound improbable if we remember all that has been said about peculiar properties of electrons in the nucleus." (G. Gamow, Nuclear Physics Textbook, 1931).

enter the neutrino...

1. 1930: Postulated by Pauli to (a) resolve the problem of continuous β-ray spectra, and (b) reconcile nuclear model with spin-statistics theorem. $\Rightarrow$
2. 1932: Chadwick discovers the neutron.

   neutron $\neq$ Pauli's neutron = neutrino (Fermi);
3. 1934: Fermi theory of Weak Interactions – current-current interaction

   $$\mathcal{H} \sim G_F\,(\bar{p}\Gamma n)\,(\bar{e}\Gamma\nu_e)\,, \quad \text{where} \;\; \Gamma = \{1, \gamma_5, \gamma_\mu, \gamma_\mu\gamma_5, \sigma_{\mu\nu}\}$$

   Way to "see" neutrinos: $\bar{\nu}_e + p \to e^+ + n$. Prediction for the cross-section - too small to ever be observed...
4. 1935: (Yukawa postulates the existence of mesons (pions) as mediators of the nuclear (strong) force: $m_\pi \sim 100$ MeV.)
5. 1936/37: ("Meson" discovered in cosmic rays. Another long, tortuous story. Turns out to be the muon...)
6. 1947: (Marshak, Bethe postulate the 2 meson hypothesis ($\pi \to \mu$). Pion observed in cosmic rays.)

Dear Radioactive Ladies and Gentlemen,
I have come upon a desperate way out regarding the wrong statistics of the ^{14}N and ^{6}Li nuclei, as well as the continuous β-spectrum, in order to save the "alternation law" statistics and the energy law. To wit, the possibility that there could exist in the nucleus electrically neutral particles, which I shall call "neutrons," and satisfy the exclusion principle... The mass of the neutrons should be of the same order of magnitude as the electron mass and in any case not larger than 0.01 times the proton mass. The continuous β-spectrum would then become understandable from the assumption that in β-decay a neutron is emitted along with the electron, in such a way that the sum of the energies of the neutron and the electron is constant...
For the time being I dare not publish anything about this idea and address myself to you, dear radioactive ones, with the question how it would be with experimental proof of such a neutron, if it were to have the penetrating power equal to about ten times larger than a γ-ray.
I admit that my way out may not seem very probable *a priori* since one would probably have seen the neutrons a long time ago if they exist. But only the one who dares wins, and the seriousness of the situation concerning the continuous β-spectrum is illuminated by my honored predecessor, Mr Debye who recently said to me in Brussels: "Oh, it is best not to think about this at all, as with new taxes." One must therefore discuss seriously every road to salvation. Thus, dear radioactive ones, examine and judge. Unfortunately, I cannot appear personally in Tübingen since a ball... in Zürich... makes my presence here indispensible....
Your most humble servant, W. Pauli

Adapted summary of an English Translation to Pauli's letter dated December 4, 1930, from Ref. 3.

observing the unobservable:

1. 1956: "Discovery" of the neutrino (Reines and Cowan) in the Savannah River Nuclear Reactor site. $\Rightarrow$

   $\bar{\nu}_e + p \to e^+ + n$. Measure positron ($e^+e^- \to \gamma$s) <u>and</u> neutron ($nN \to N^* \to N + \gamma$s) in delayed coincidence in order to get rid of backgrounds.

2. 1958: Neutrino Helicity Measured (Goldhaber et al.). Neutrinos are purely left-handed. Interact only weakly (Parity violated maximally).

   $e^- + ^{152}\mathrm{Eu}(J=0) \to ^{152}\mathrm{Sm}^*(J=1) + \nu \to ^{152}\mathrm{Sm}(J=1) + \nu + \gamma$

3. 1962: The second neutrino: $\nu_\mu \neq \nu_e$ (Lederman, Steinberger, Schwarts at BNL). First neutrino beam.

   $$p + Z \to \pi^+ X \to \mu^+ \nu_\mu \quad \Rightarrow \quad \begin{array}{l} \nu_\mu + Z \to \mu^- + Y \ (\text{"always"}) \\ \nu_\mu + Z \to e^- + Y \ (\text{"never"}) \end{array}$$

4. 2001: ν_τ directly observed (DONUT experiment at FNAL). Same strategy: $\nu_\tau + Z \to \tau^- + Y$. ($\tau$-leptons discovered in the 1970's). $\Rightarrow$

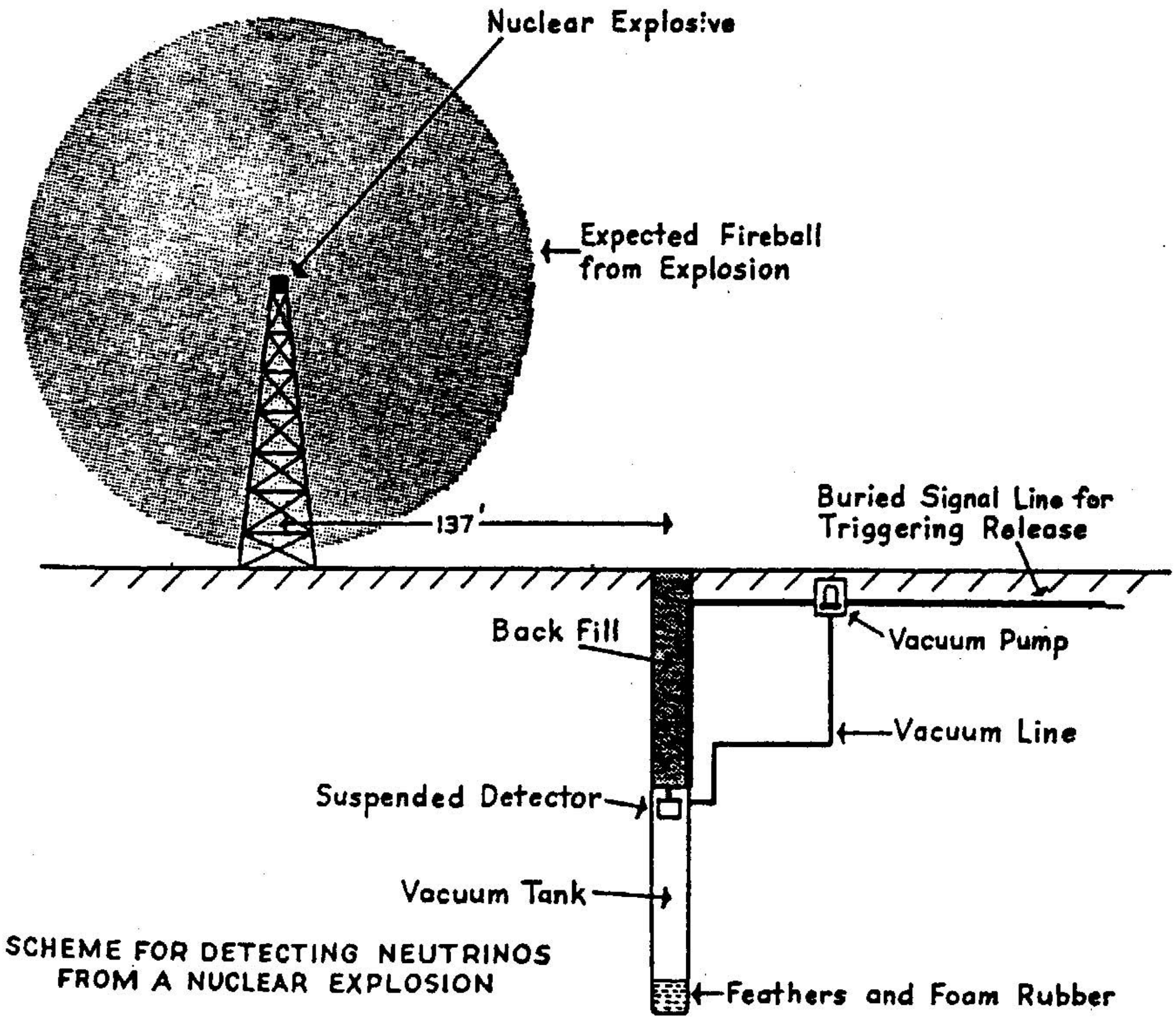


FIGURE 5.1 Scheme for detecting neutrinos from a nuclear e
(Cowan, 1964).

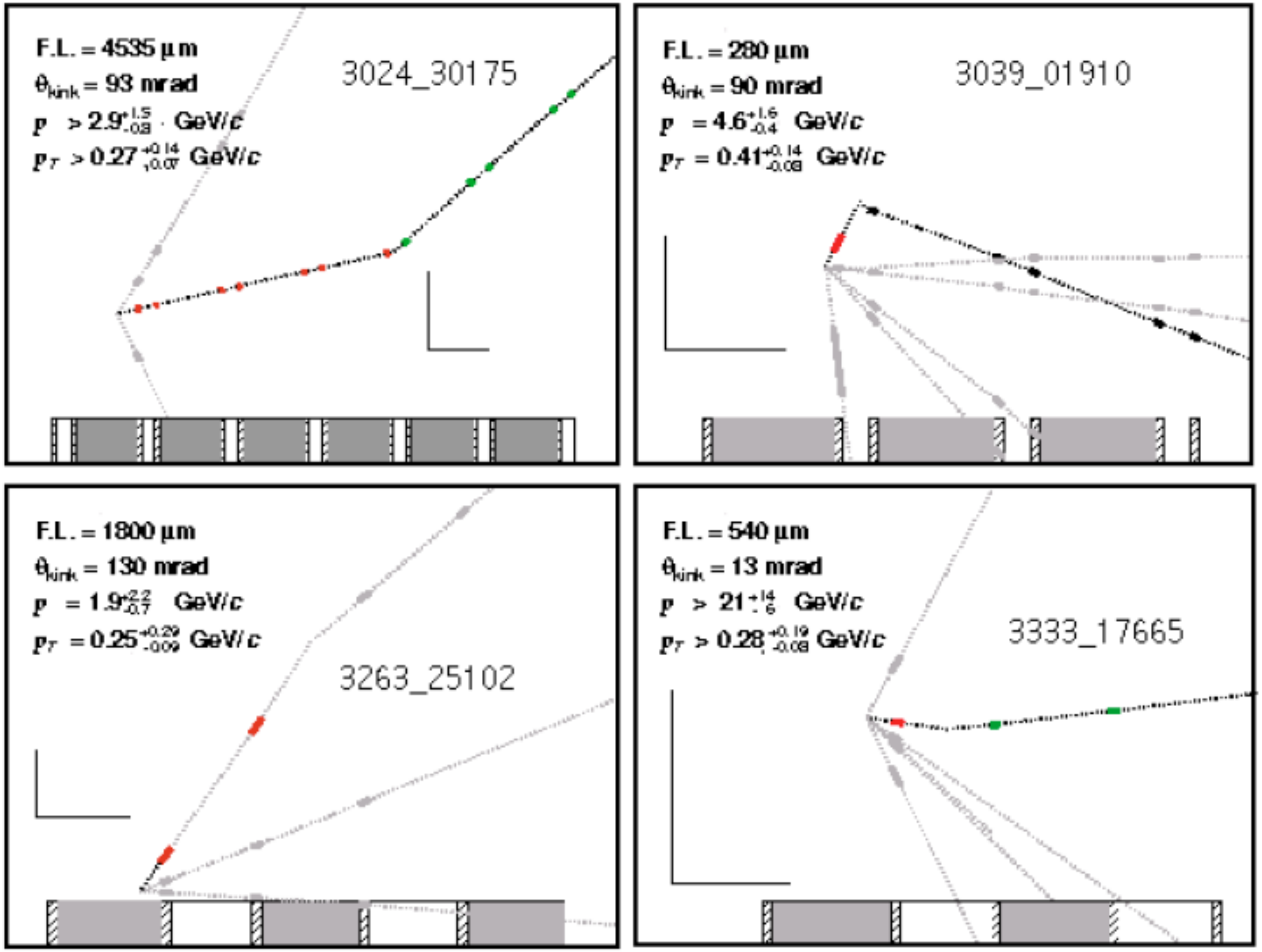


Figure 4-6: The four tau neutrino charged current events. The scale is given by the perpendicular lines (vertical: 0.1 mm, horizontal: 1 mm). The bar on the bottom shows the target material (solid: steel, hatched: emulsion, clear: plastic base).

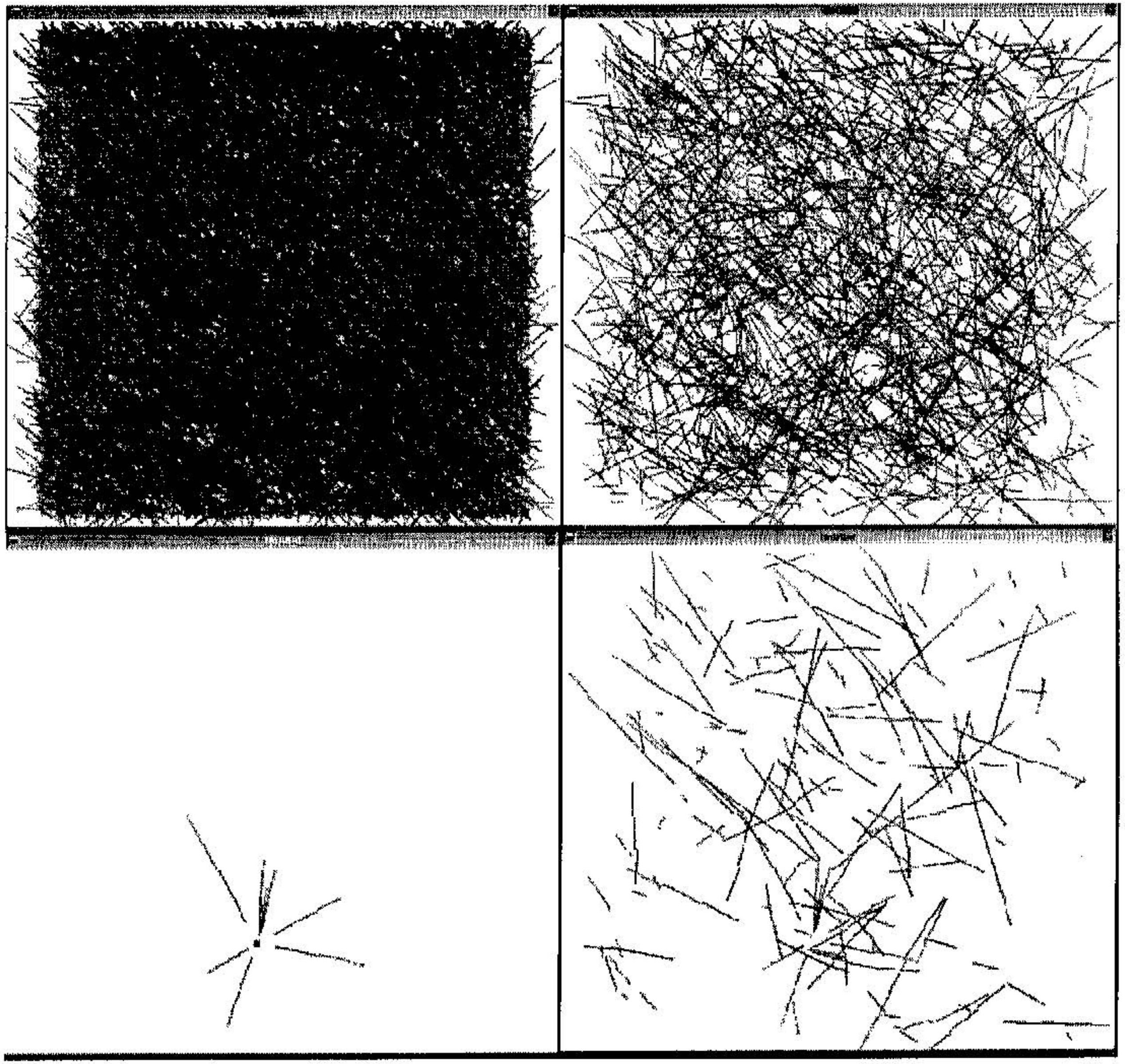
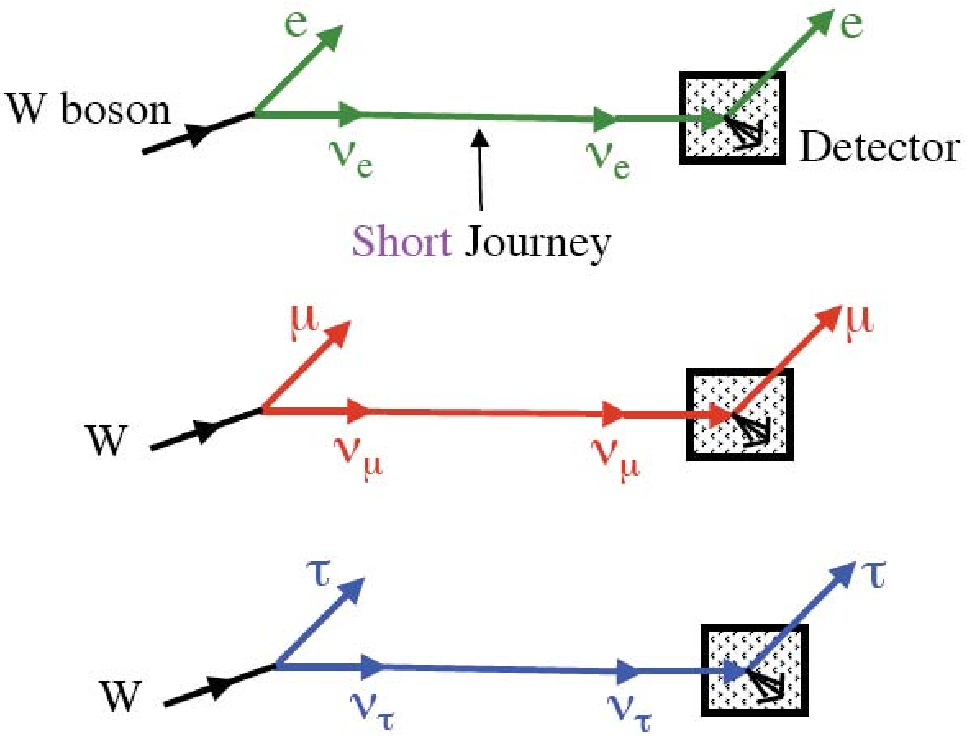


図 5.12: net scan 反応点探索の各段階 (左上から時計回り)。1) 読み込んだ全ての飛跡 ($5 \times 5\text{mm}^2$)、2) 測定領域を突き抜けている飛跡の排除、3) 低運動量の飛跡の排除、4) 一点 (4μm 以内) 収束している飛跡

HUNTING FOR

Until recently, this is how we pictured neutrinos:



- come in three flavors (see figure);
- interact only via weak interactions ($W^\pm, Z^0$);
- have ZERO mass – helicity good quantum number;
- ν_L field describes 2 degrees of freedom:
  - left-handed state ν,
  - right-handed state $\bar{\nu}$ (CPT conjugate);
- neutrinos carry lepton number:
  - $L(\nu) = +1$,
  - $L(\bar{\nu}) = -1$.

2– Neutrino Puzzles

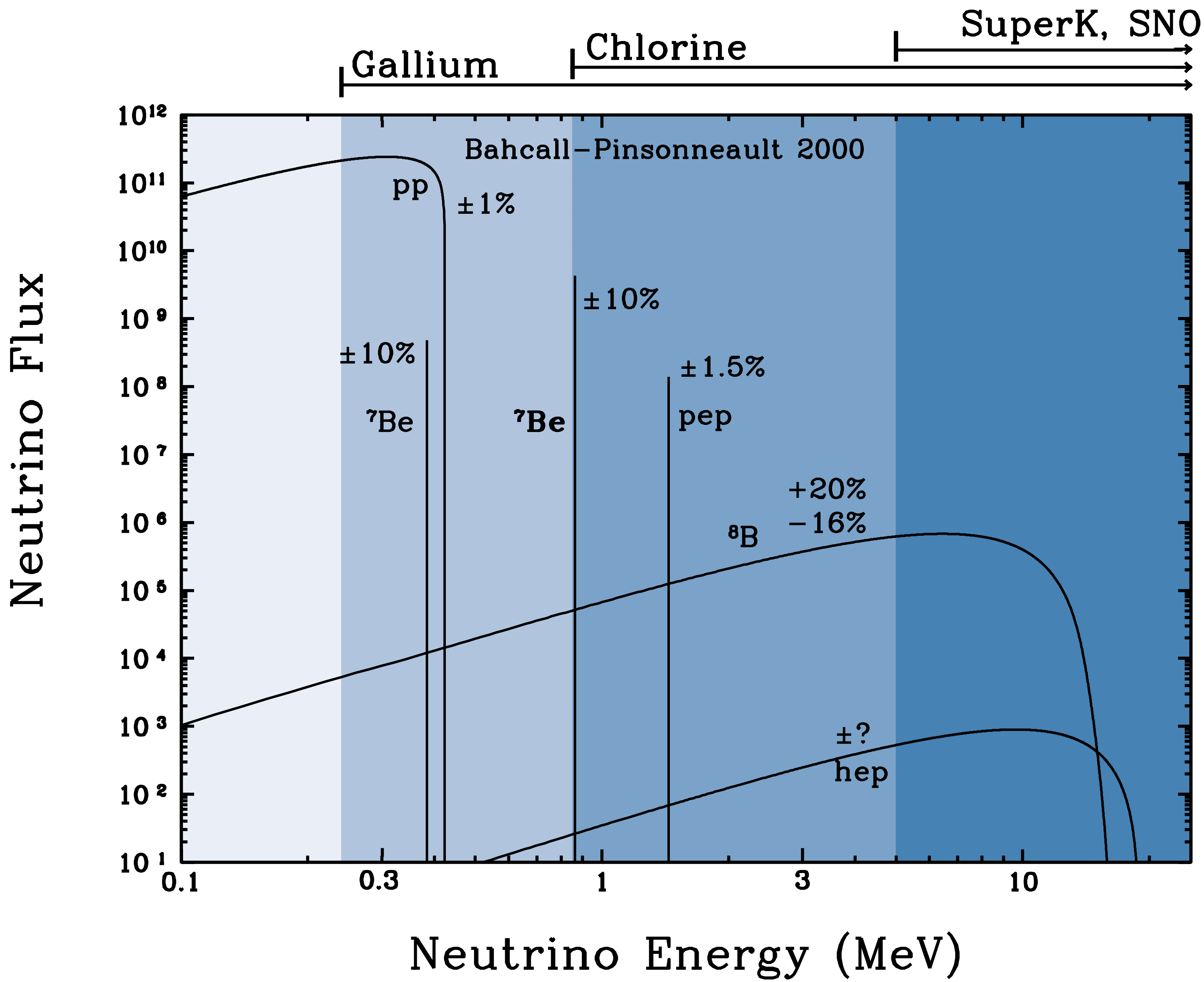
Long baseline neutrino experiments have revealed that neutrinos change flavor after propagating a finite distance, violating the definitions in the previous slide. The rate of change depends on the neutrino energy E_ν and the baseline L.

- $\nu_\mu \to \nu_\tau$ and $\bar{\nu}_\mu \to \bar{\nu}_\tau$ — atmospheric experiments [“indisputable”];
- $\nu_e \to \nu_{\mu,\tau}$ — solar experiments [“indisputable”];
- $\bar{\nu}_e \to \bar{\nu}_{\rm other}$ — reactor neutrinos [“indisputable”];
- $\nu_\mu \to \nu_{\rm other}$ from accelerator experiments [“really strong”].

Table 1. Nuclear reactions responsible for producing almost all of the Sun's energy and the different "types" of solar neutrinos (nomenclature): *pp*-neutrinos, *pep*-neutrinos, *hep*-neutrinos, ^{7}Be-neutrinos, and ^{8}B-neutrinos. 'Termination' refers to the fraction of interacting protons that participate in the process.

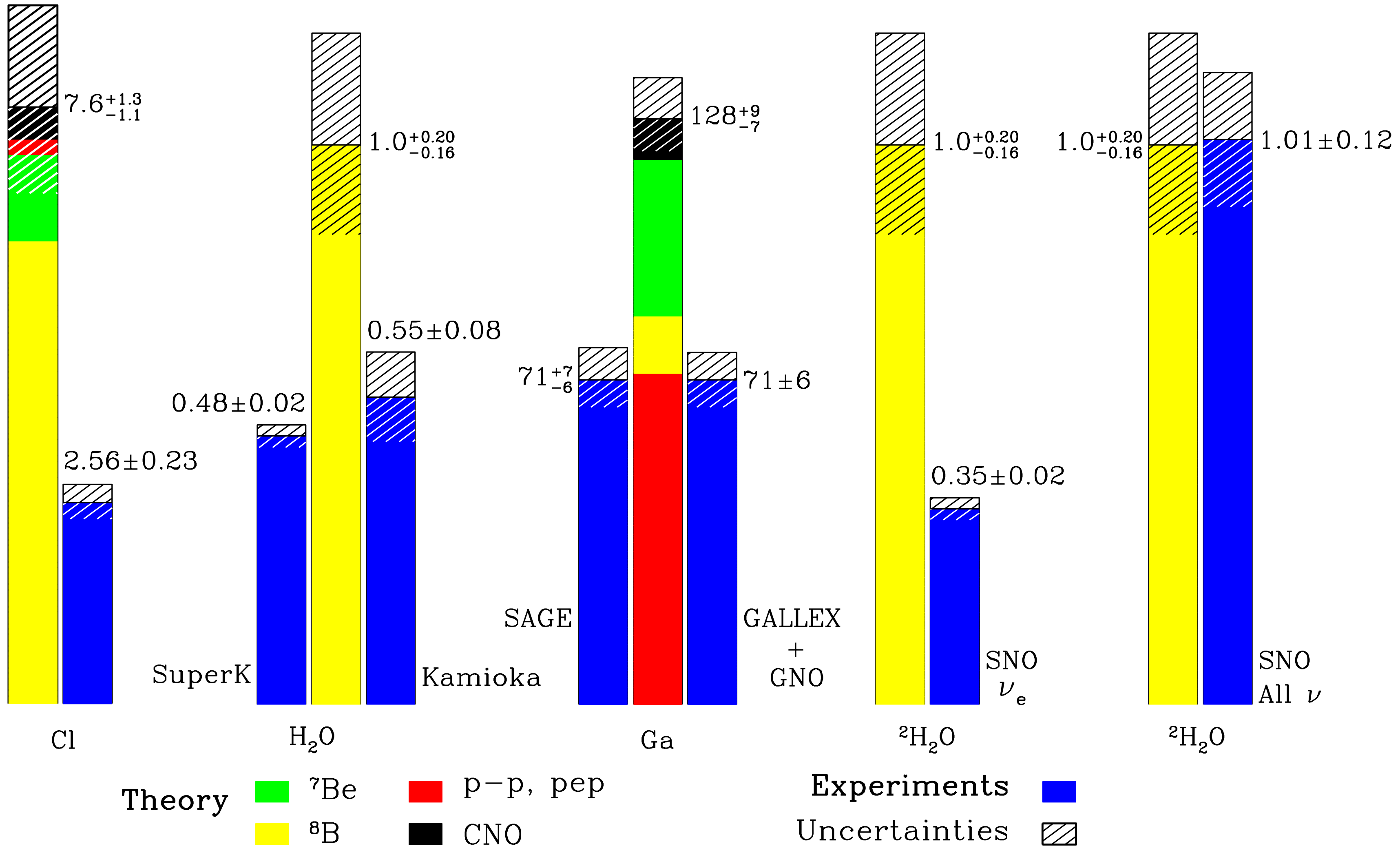
Reaction	Termination (%)	Neutrino Energy (MeV)	Nomenclature
$p+p\to{}^2\text{H}+e^+ +\nu_e$	99.96	< 0.423	*pp*-neutrinos
$p+e^- +p\to{}^2\text{H}+\nu_e$	0.044	1.445	*pep*-neutrinos
$^2\text{H}+p\to{}^3\text{He}+\gamma$	100	–	–
$^3\text{He}+{}^3\text{He}\to{}^4\text{He}+p+p$	85	–	–
$^3\text{He}+{}^4\text{He}\to{}^7\text{Be}+\gamma$	15	–	–
$^7\text{Be}+e^-\to{}^7\text{Li}+\nu_e$	15	0.863(90%) 0.386(10%)	^{7}Be-neutrinos
$^7\text{Li}+p\to{}^4\text{He}+{}^4\text{He}$		–	–
$^7\text{Be}+p\to{}^8\text{B}+\gamma$	0.02	–	–
$^8\text{B}\to{}^8\text{Be}^* +e^+ +\nu_e$		< 15	^{8}B-neutrinos
$^8\text{Be}\to{}^4\text{He}+{}^4\text{He}$		–	–
$^3\text{He}+p\to{}^4\text{He}+e^+ +\nu_e$	0.00003	< 18.8	*hep*-neutrinos

Note: Adapted from Ref. 12. Please refer to Ref. 12 for a more detailed explanation.

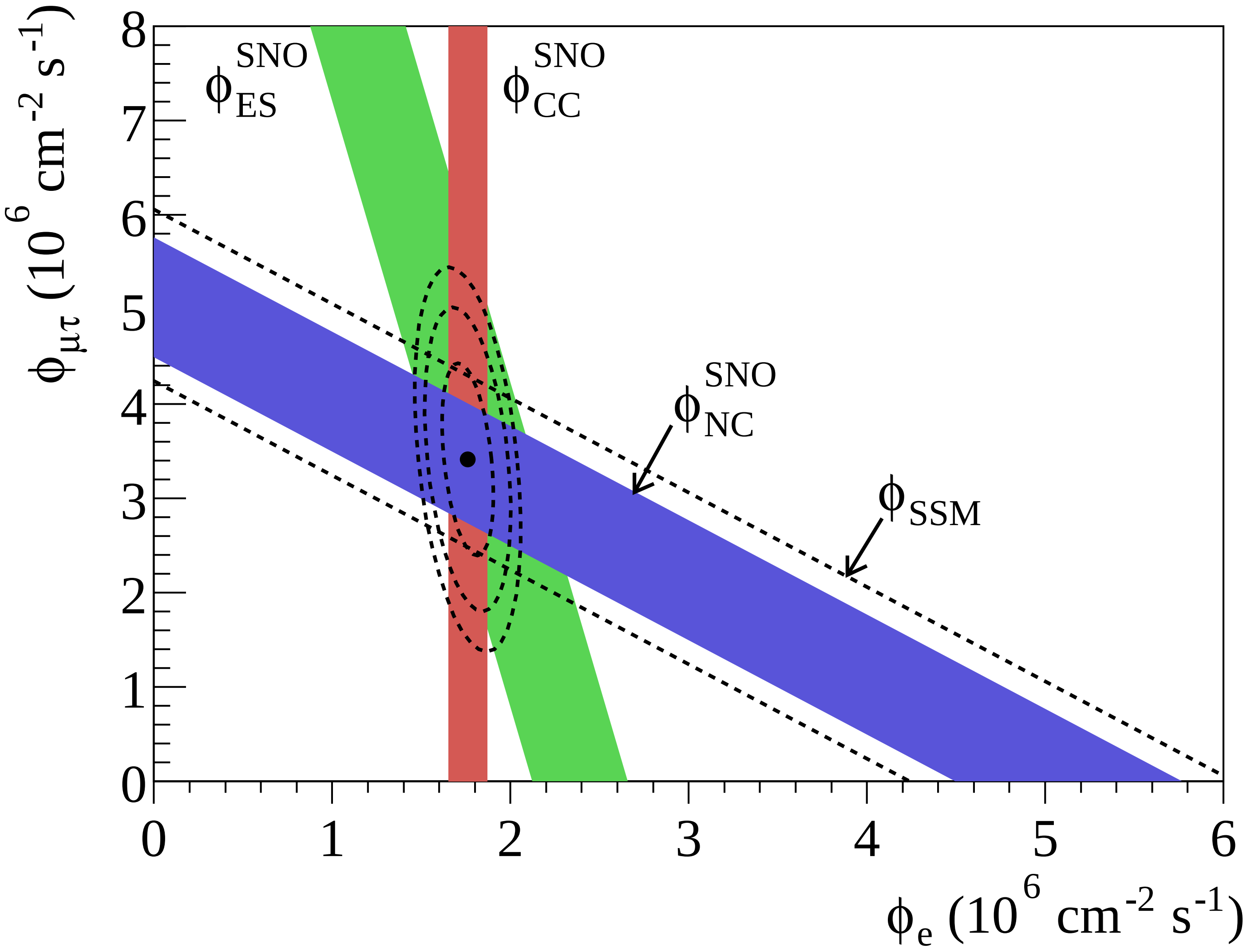

Gallium
Chlorine
SuperK, SNO
Bahcall−Pinsonneault 2000
pp
±1%
±10%
±10%
±1.5%
7Be
7Be
pep
+20%
−16%
8B
±?
hep
Neutrino Flux
Neutrino Energy (MeV)
10^12
10^11
10^10
10^9
10^8
10^7
10^6
10^5
10^4
10^3
10^2
10^1
0.1
0.3
1
3
10

Total Rates: Standard Model vs. Experiment

Bahcall–Pinsonneault 2000



The SNO Experiment: conclusive evidence for flavor change



SNO Measures:

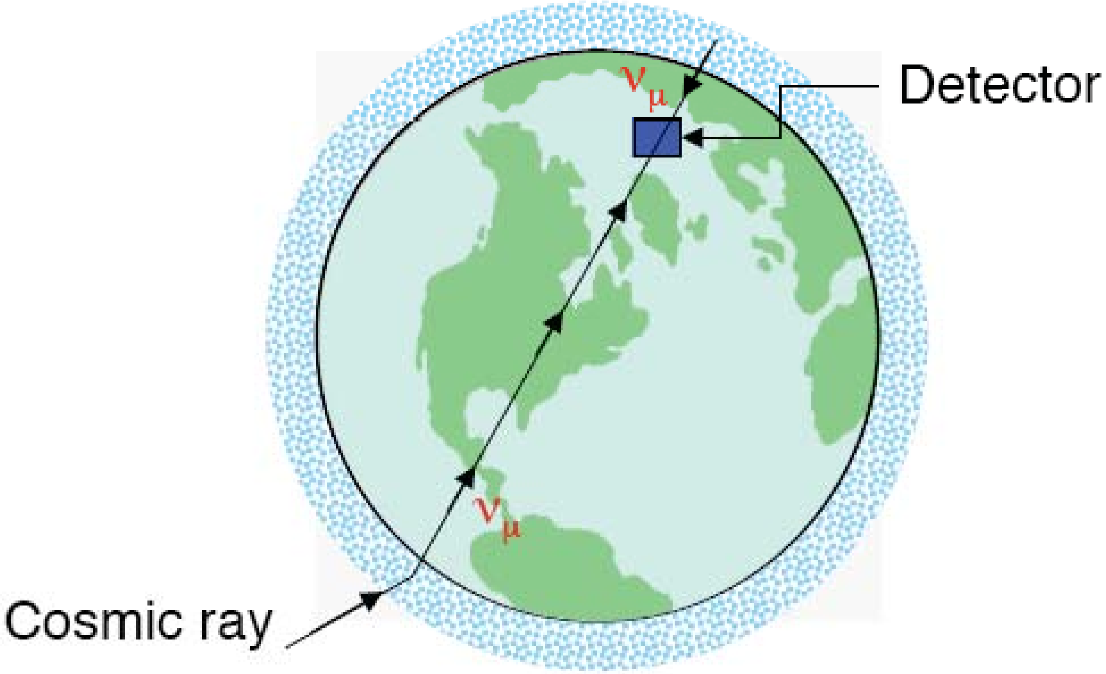
$[CC]\ \nu_e + {}^2H \to p + p + e^-$

$[ES]\ \nu + e^- \to \nu + e^-$

$[NC]\ \nu + {}^2H \to p + n + \nu$

different reactions sensitive to different neutrino flavors.

Atmospheric Neutrinos



Isotropy of the $\gtrsim 2$ GeV cosmic rays + Gauss' Law + No ν_μ disappearance

$$\Rightarrow \quad \frac{\phi_{\nu_\mu}(\text{Up})}{\phi_{\nu_\mu}(\text{Down})} = 1\ .$$

But Super-Kamiokande finds for $E_\nu > 1.3$ GeV

$$\frac{\phi_{\nu_\mu}(\text{Up})}{\phi_{\nu_\mu}(\text{Down})} = 0.54 \pm 0.04\ .$$

UP $\neq$ DOWN – neutrinos can tell time! $\rightarrow$ neutrinos have mass.

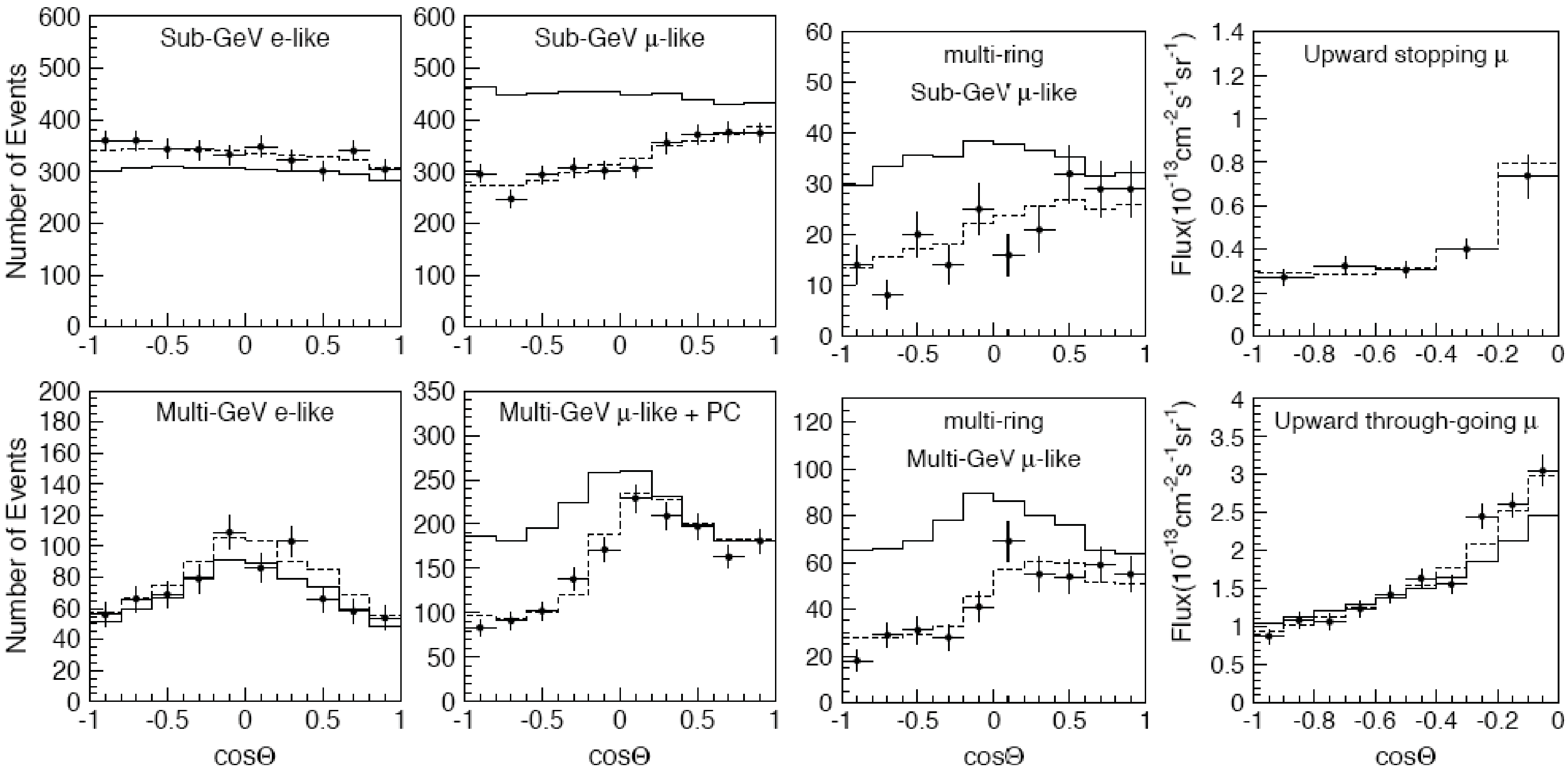


Figure 4. Zenith angle distribution for fully-contained single-ring e-like and μ-like events, multi-ring μ-like events, partially contained events and upward-going muons. The points show the data and the solid lines show the Monte Carlo events without neutrino oscillation. The dashed lines show the best-fit expectations for $\nu_\mu \leftrightarrow \nu_\tau$ oscillations. From M. Ishitsuka [Super-Kamiokande Collaboration], hep-ex/0406076.

3 - Mass-Induced Neutrino Flavor Oscillations

Neutrino Flavor change can arise out of several different mechanisms. The simplest one is to appreciate that, once **neutrinos have mass**, **leptons can mix**. This turns out to be the correct mechanism (certainly the dominant one), and **only** explanation that successfully explains **all** long-baseline data consistently.

Neutrinos with a well defined mass:

$$\nu_1, \nu_2, \nu_3, \dots \qquad \text{with masses} \;\; m_1, m_2, m_3, \dots$$

How do these states (neutrino mass eigenstates) relate to the neutrino flavor eigenstates (ν_e, ν_μ, ν_τ)?

$$\nu_\alpha = U_{\alpha i} \nu_i \qquad \alpha = e, \mu, \tau, \quad i = 1, 2, 3$$

U is a unitary mixing matrix. I'll talk more about it later.

The Propagation of Massive Neutrinos

Neutrino mass eigenstates are eigenstates of the free-particle Hamiltonian:

$$|\nu_i\rangle = e^{-iE_i t}|\nu_i\rangle, \qquad E_i^2 - |\vec{p}_i|^2 = m_i^2$$

The neutrino flavor eigenstates are linear combinations of ν_i's, say:

$$\begin{aligned} |\nu_e\rangle &= \cos\theta|\nu_1\rangle + \sin\theta|\nu_2\rangle. \\ |\nu_\mu\rangle &= -\sin\theta|\nu_1\rangle + \cos\theta|\nu_2\rangle. \end{aligned}$$

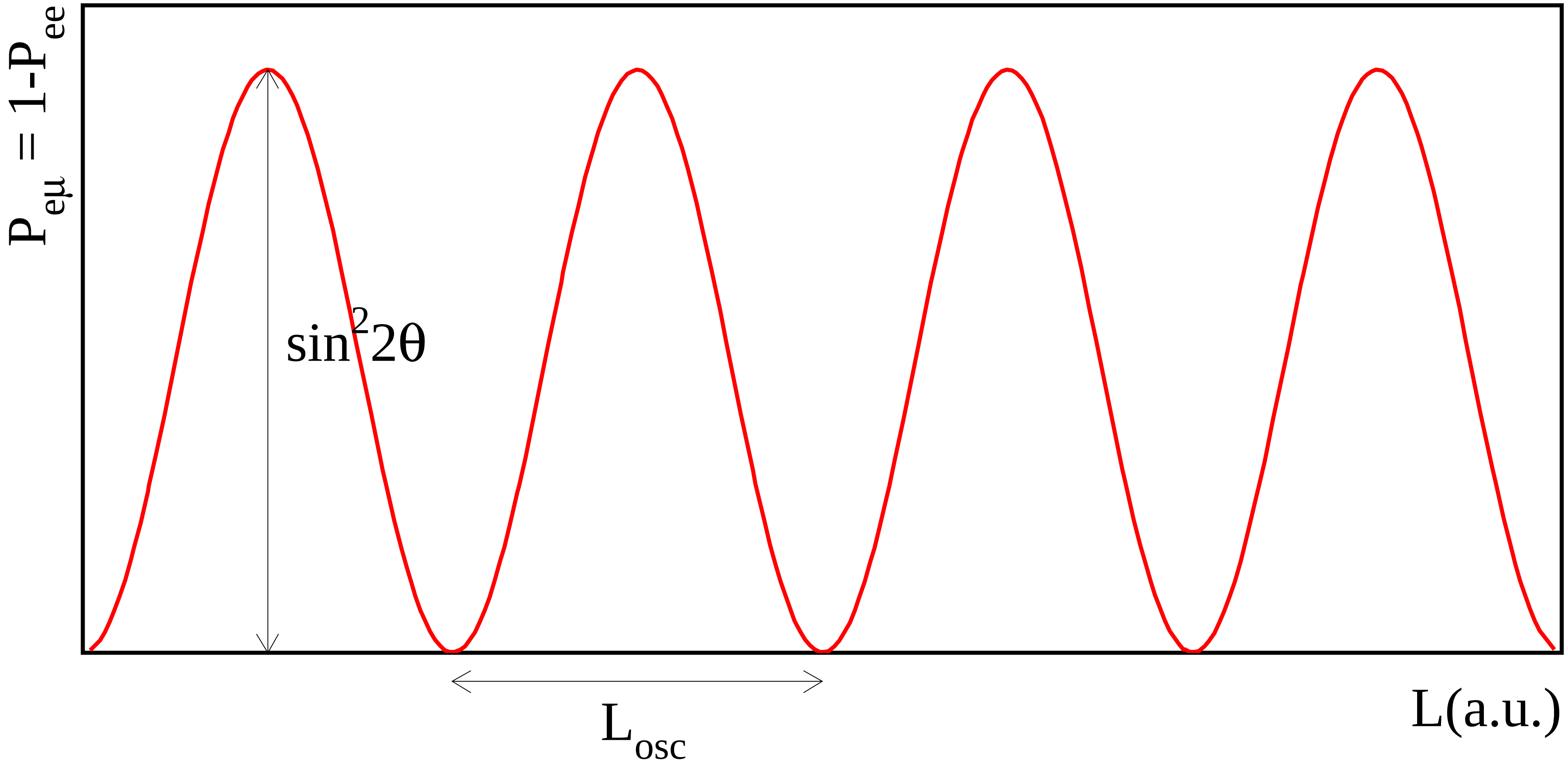
If this is the case, a state produced as a ν_e evolves in vacuum into

$$|\nu(t,\vec{x})\rangle = \cos\theta e^{-ip_1 x}|\nu_1\rangle + \sin\theta e^{-ip_2 x}|\nu_2\rangle.$$

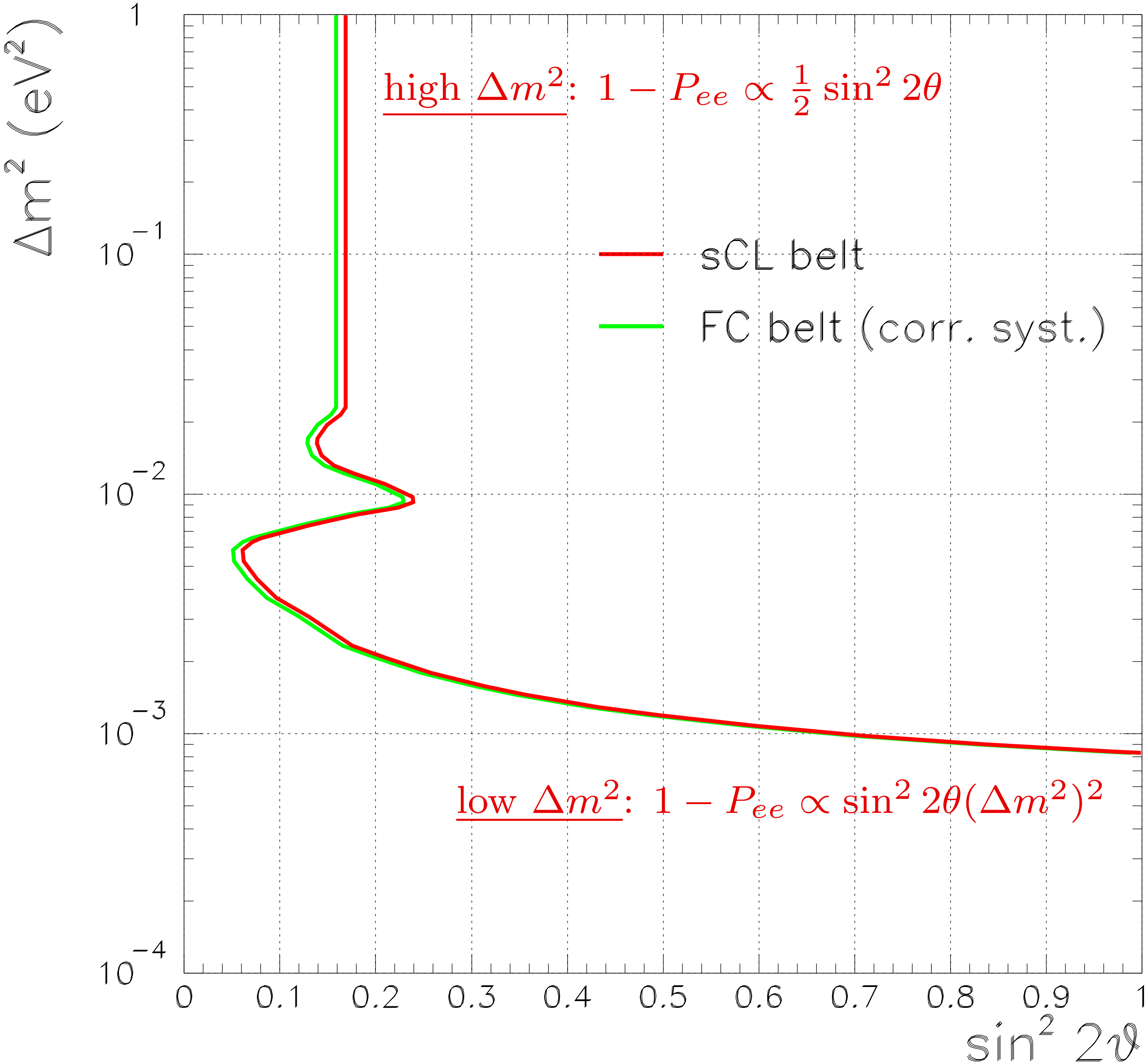
It is trivial to compute $P_{e\mu}(L) \equiv |\langle\nu_\mu|\nu(t,z=L)\rangle|^2$. It is just like a two-level system from basic undergraduate quantum mechanics! In the ultrarelativistic limit (always a good bet), $t \simeq L$, $E_i - p_{z,i} \simeq (m_i^2)/2E_i$, and

$$\boxed{P_{e\mu}(L) = \sin^2 2\theta \sin^2\left(\frac{\Delta m^2 L}{4E_\nu}\right)}$$

oscillation parameters: $\begin{cases} \pi \frac{L}{L_{\rm osc}} \equiv \frac{\Delta m^2 L}{4E} = 1.267 \left(\frac{L}{\rm km}\right) \left(\frac{\Delta m^2}{\rm eV^2}\right) \left(\frac{\rm GeV}{E}\right) \\ \text{amplitude } \sin 2\theta \end{cases}$



CHOOZ experiment



$$P_{ee}=1-\sin^2 2\theta\sin^2\left(\frac{\Delta m^2 L}{4E}\right)$$

result: $1-P_{ee}<0.05$

$P_{\mu\mu} = 1 - \sin^2 2\theta \sin^2\left(\frac{\Delta m^2 L}{4E}\right)$ Works great for $\sin^2 2\theta \sim 1$ and $\Delta m^2 \sim 10^{-3}$ eV^2

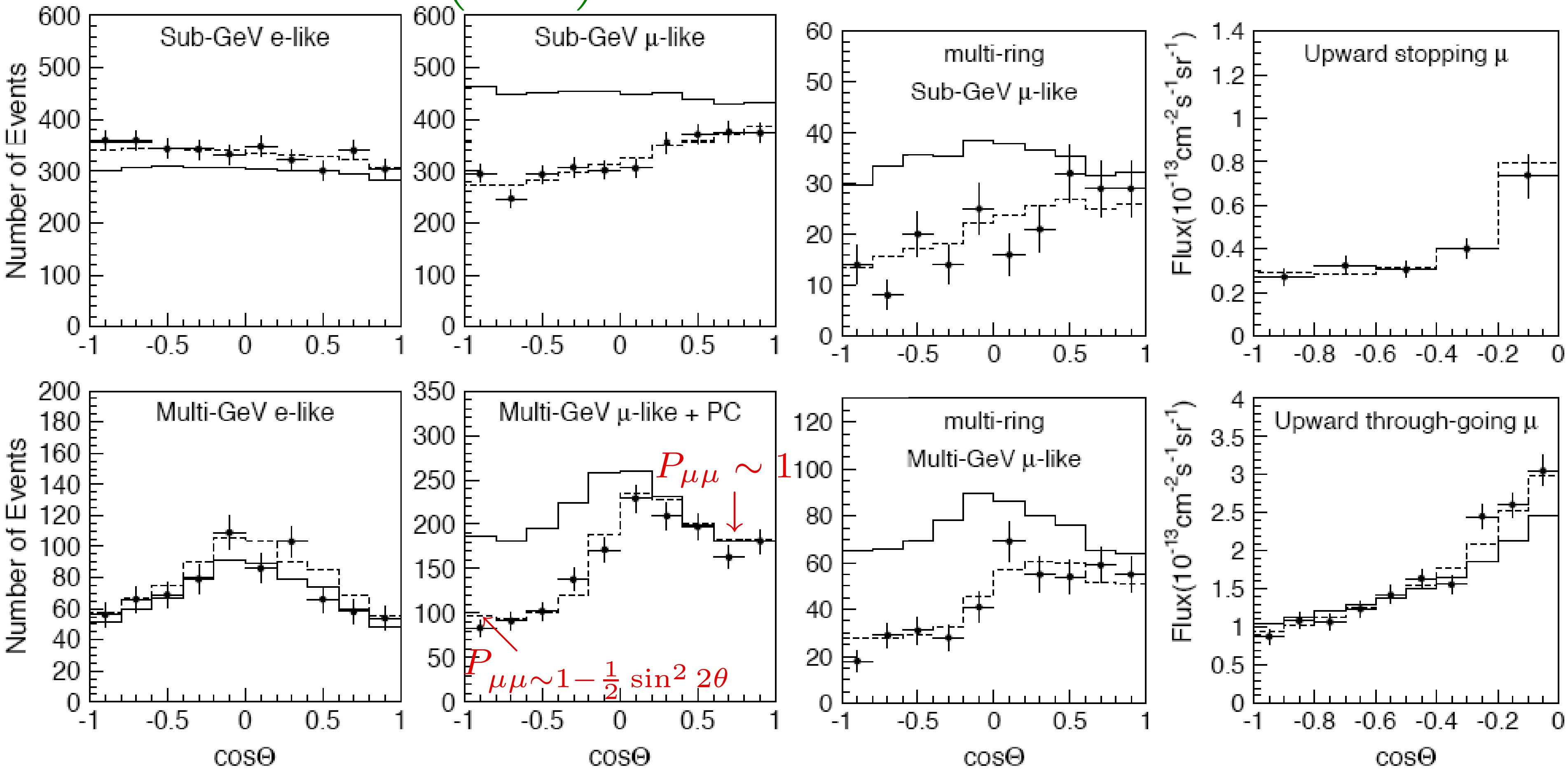
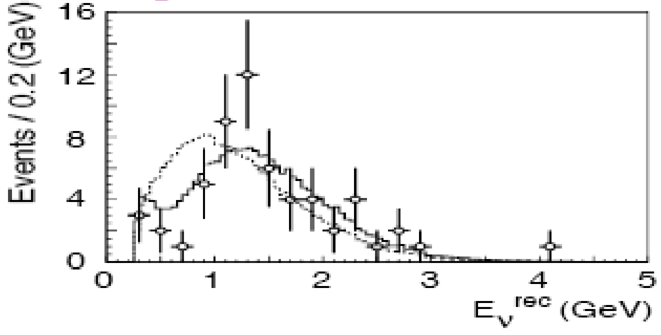


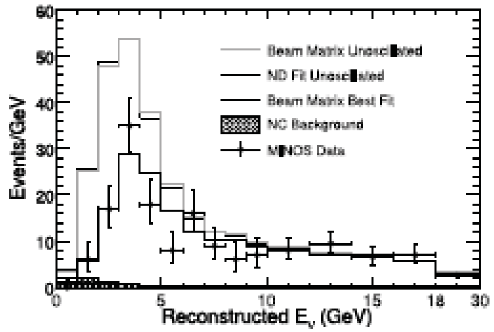
Figure 4. Zenith angle distribution for fully-contained single-ring e-like and μ-like events, multi-ring μ-like events, partially contained events and upward-going muons. The points show the data and the solid lines show the Monte Carlo events without neutrino oscillation. The dashed lines show the best-fit expectations for $\nu_\mu \leftrightarrow \nu_\tau$ oscillations. From M. Ishitsuka [Super-Kamiokande Collaboration], hep-ex/0406076.

K2K	ν_μ at KEK	SK	L=250 km
MINOS	ν_μ at Fermilab	Soundan	L=735 km
Opera/Icarus	ν_μ at CERN	Gran Sasso	L=740 km

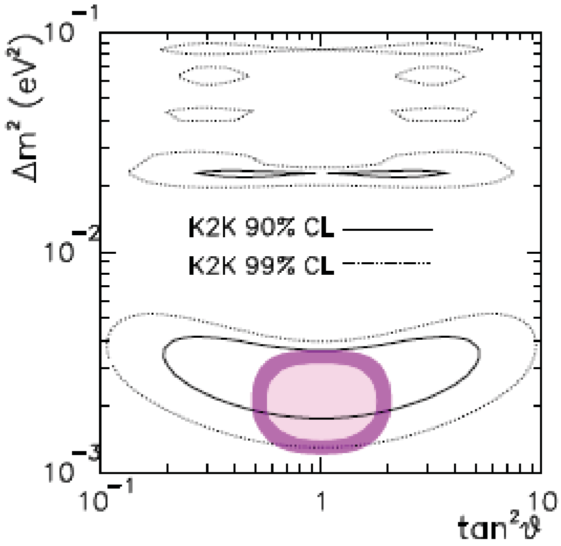
K2K 2004: spectral distortion



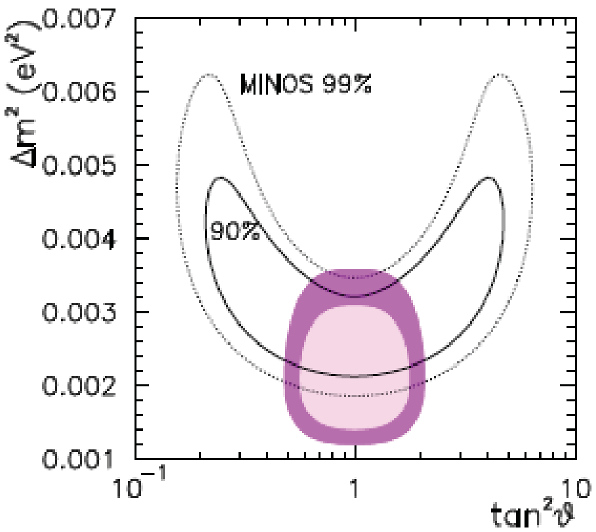
MINOS 2006: spectral distortion



Confirmation of ATM oscillations



Confirmation of ATM oscillations



[Gonzalez-Garcia, PASI 2006]

Matter Effects

The neutrino propagation equation, in the ultra-relativistic approximation, can be re-expressed in the form of a Shrödinger-like equation. In the mass basis:

$$i\frac{\mathrm{d}}{\mathrm{d}L}|\nu_i\rangle = \frac{m_i^2}{2E}|\nu_i\rangle,$$

up to a term proportional to the identity. In the weak/flavor basis

$$i\frac{\mathrm{d}}{\mathrm{d}L}|\nu_\beta\rangle = U_{\beta i}\frac{m_i^2}{2E}U^\dagger_{i\alpha}|\nu_\alpha\rangle.$$

In the 2×2 case,

$$i\frac{\mathrm{d}}{\mathrm{d}L}\left(\begin{array}{c} |\nu_e\rangle \\ |\nu_\mu\rangle \end{array}\right) = \frac{\Delta m^2}{2E}\left(\begin{array}{cc} \sin^2\theta & \cos\theta\sin\theta \\ \cos\theta\sin\theta & \cos^2\theta \end{array}\right)\left(\begin{array}{c} |\nu_e\rangle \\ |\nu_\mu\rangle \end{array}\right),$$

(again, up to additional terms proportional to the 2×2 identity matrix).

Fermi Lagrangian, after a Fiertz rearrangement of the charged-current terms:

$$\mathrm{L} \supset \bar{\nu}_{eL} i\partial_\mu \gamma^\mu \nu_{eL} - 2\sqrt{2}G_F \left(\bar{\nu}_{eL}\gamma^\mu \nu_{eL}\right)\left(\bar{e}_L \gamma_\mu e_L\right) + \ldots$$

Equation of motion for one electron neutrino state in the presence of a non-relativistic electron background, in the rest frame of the electrons:

$$\langle \bar{e}_L \gamma_\mu e_L \rangle = \delta_{\mu 0}\frac{N_e}{2}$$

where $N_e \equiv e^\dagger e$ is the average electron number density (at rest, hence $\delta_{\mu 0}$ term). Factor of 1/2 from the "left-handed" half.

Dirac equation for a one neutrino state inside a cold electron "gas" is (ignore mass)

$$(i\partial^\mu \gamma_\mu - \sqrt{2}G_F N_e \gamma_0)|\nu_e\rangle = 0.$$

In the ultrarelativistic limit, (plus $\sqrt{2}G_F N_e \ll E$), dispersion relation is

$$E \simeq |\vec{p}| \pm \sqrt{2}G_F N_e, \qquad + \text{ for } \nu, \quad - \text{ for } \bar{\nu}$$

$$i\frac{\mathrm{d}}{\mathrm{d}L}\begin{pmatrix} |\nu_e\rangle \\ |\nu_\mu\rangle \end{pmatrix} = \left[\frac{\Delta m^2}{2E}\begin{pmatrix} \sin^2\theta & \cos\theta\sin\theta \\ \cos\theta\sin\theta & \cos^2\theta \end{pmatrix} + \begin{pmatrix} A & 0 \\ 0 & 0 \end{pmatrix}\right]\begin{pmatrix} |\nu_e\rangle \\ |\nu_\mu\rangle \end{pmatrix},$$

$A = \pm\sqrt{2}G_F N_e$ (+ for neutrinos, − for antineutrinos).

Note: Similar effect from neutral current interactions common to all (active) neutrino species → proportional to the identity.

In general, this is hard to solve, as A is a function of L: two-level non-relativistc quantum mechanical system in the presence of time dependent potential.

In some cases, however, the solution is rather simple.

Constant A: good approximation for neutrinos propagating through matter inside the Earth [exception: neutrinos that see Earth's internal structure (the crust, the mantle, the outer core, the inner core)]

$$i\frac{\mathrm{d}}{\mathrm{d}L}\begin{pmatrix}|\nu_e\rangle \\ |\nu_\mu\rangle\end{pmatrix} = \begin{pmatrix} A & \Delta/2\sin 2\theta \\ \Delta/2\sin 2\theta & \Delta\cos 2\theta\end{pmatrix}\begin{pmatrix}|\nu_e\rangle \\ |\nu_\mu\rangle\end{pmatrix}, \quad \Delta \equiv \Delta m^2/2E.$$

$$P_{e\mu} = \sin^2 2\theta_M \sin^2\left(\frac{\Delta_M L}{2}\right),$$

where

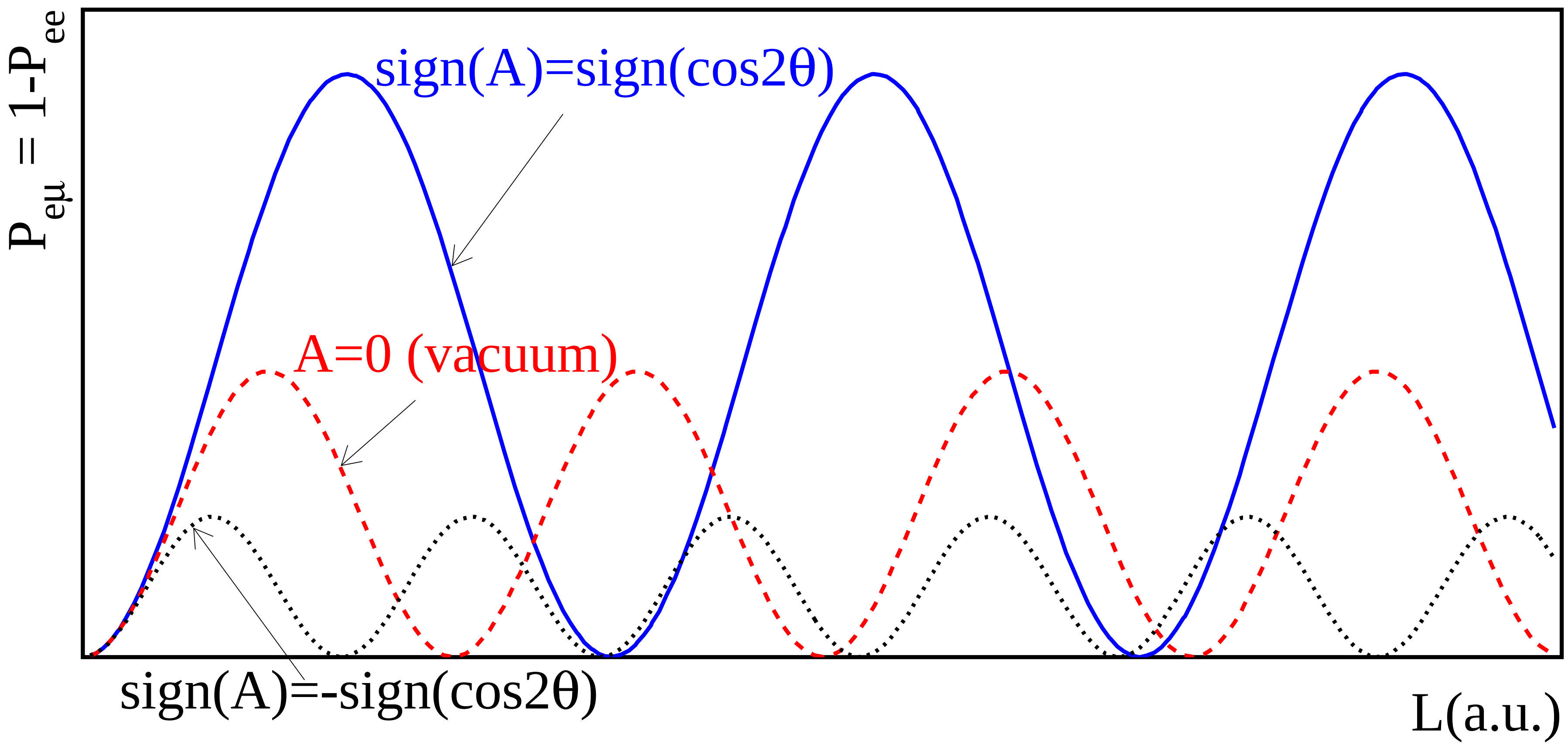
$$\begin{aligned}\Delta_M &= \sqrt{(A-\Delta\cos 2\theta)^2 + \Delta^2\sin^2 2\theta}, \\ \Delta_M \sin 2\theta_M &= \Delta\sin 2\theta, \\ \Delta_M\cos 2\theta_M &= A - \Delta\cos 2\theta.\end{aligned}$$

The presence of matter affects neutrino and antineutrino oscillation differently.

Nothing wrong with this: CPT-theorem relates the propagation of neutrinos in an electron background to the propagation of antineutrinos in a positron background.

Enlarged parameter space in the presence of matter effects.

For example, can tell whether $\cos 2\theta$ is positive or negative.



The MSW Effect

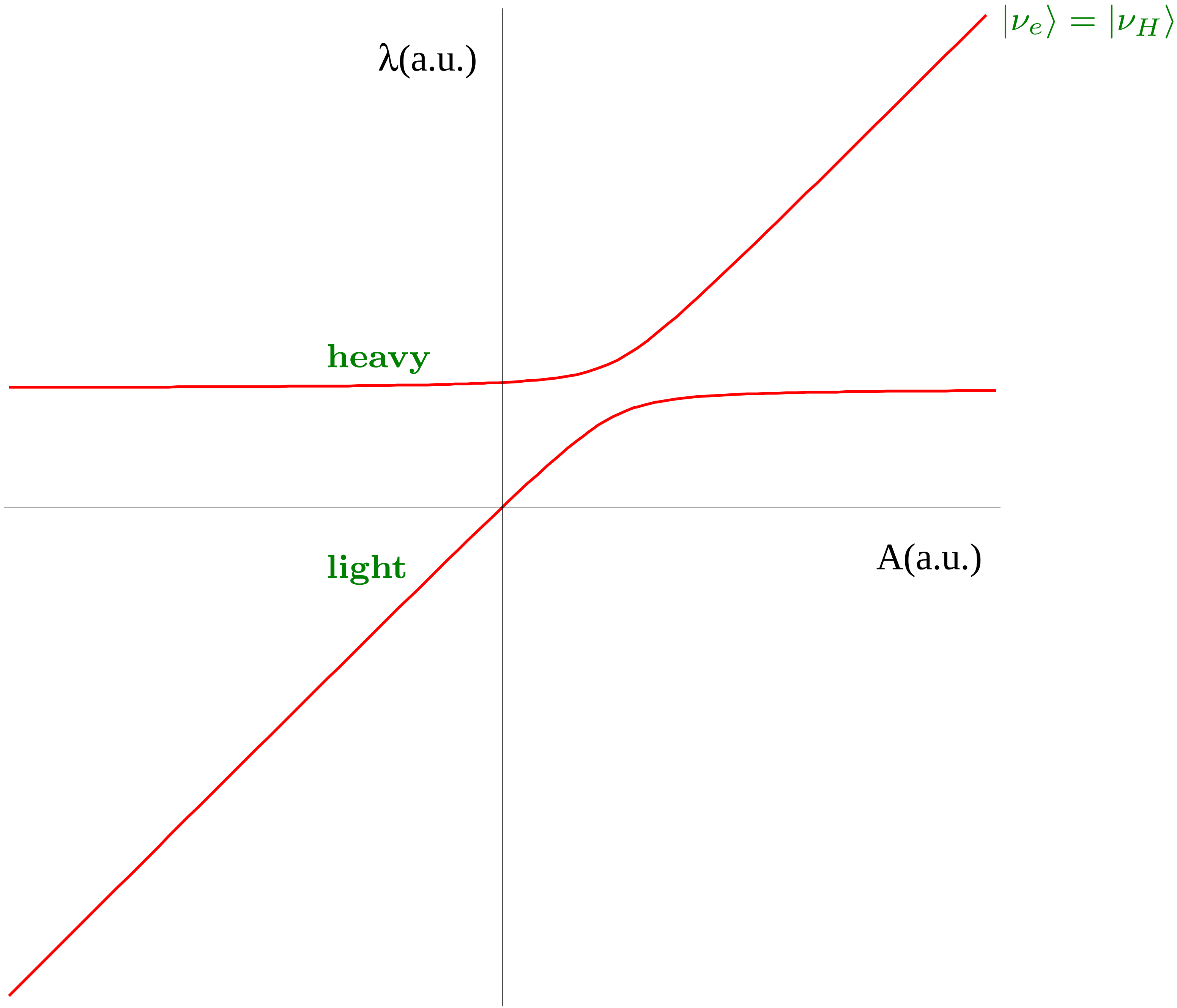
Curiously enough, the oldest neutrino puzzle is the one that is most subtle to explain. This is because solar neutrinos traverse a strongly varying matter density on their way from the center of the Sun to the surface of the Earth.

For the Hamiltonian

$$\left[\Delta\left(\begin{array}{cc}\sin^2\theta & \cos\theta\sin\theta \\ \cos\theta\sin\theta & \cos^2\theta\end{array}\right)+A\left(\begin{array}{cc}1 & 0 \\ 0 & 0\end{array}\right)\right],$$

it is easy to compute the eigenvalues as a function of A:

(remember, $\Delta = \Delta m^2/2E$)



$|\nu_e\rangle = |\nu_H\rangle$
λ(a.u.)
heavy
light
A(a.u.)

A decreases "slowly" as a function of L $\Rightarrow$ system evolves adiabatically.

$$|\nu_e\rangle = |\nu_{2M}\rangle \text{ at the core } \rightarrow |\nu_2\rangle \text{ in vacuum,}$$
$$P_{ee}^{\rm Earth} = |\langle\nu_e|\nu_2\rangle|^2 = \sin^2\theta.$$

Note that $P_{ee} \simeq \sin^2\theta$ applies in a wide range of energies and baselines, as long as the approximations mentioned above apply —ideal to explain the energy independent suppression of the ^{8}B solar neutrino flux!

Furthermore, large average suppressions of the neutrino flux are allowed if $\sin^2\theta \ll 1$. Compare with $\bar{P}_{ee}^{\rm vac} = 1 - 1/2\sin^2 2\theta > 1/2$.

One can expand on the result above by loosening some of the assumptions. $|\nu_e\rangle$ state is produced in the Sun's core as an *incoherent* mixture of $|\nu_{1M}\rangle$ and $|\nu_{2M}\rangle$. Introduce adiabaticity parameter P_c, which measures the probability that a $|\nu_{iM}\rangle$ matter Hamiltonian state will *not* exit the Sun as a $|\nu_i\rangle$ mass-eigenstate.

$$\begin{aligned} |\nu_e\rangle \quad &\rightarrow \quad |\nu_{1M}\rangle, \text{ with probability } \cos^2\theta_M, \\ &\rightarrow \quad |\nu_{2M}\rangle, \text{ with probability } \sin^2\theta_M, \end{aligned}$$

where θ_M is the matter angle at the neutrino **production point**.

$$\begin{aligned} |\nu_{1M}\rangle \quad &\rightarrow \quad |\nu_1\rangle, \text{ with probability } (1-P_c), \\ &\rightarrow \quad |\nu_2\rangle, \text{ with probability } P_c, \\ |\nu_{2M}\rangle \quad &\rightarrow \quad |\nu_1\rangle \text{ with probability } P_c, \\ &\rightarrow \quad |\nu_2\rangle \text{ with probability } (1-P_c). \end{aligned}$$

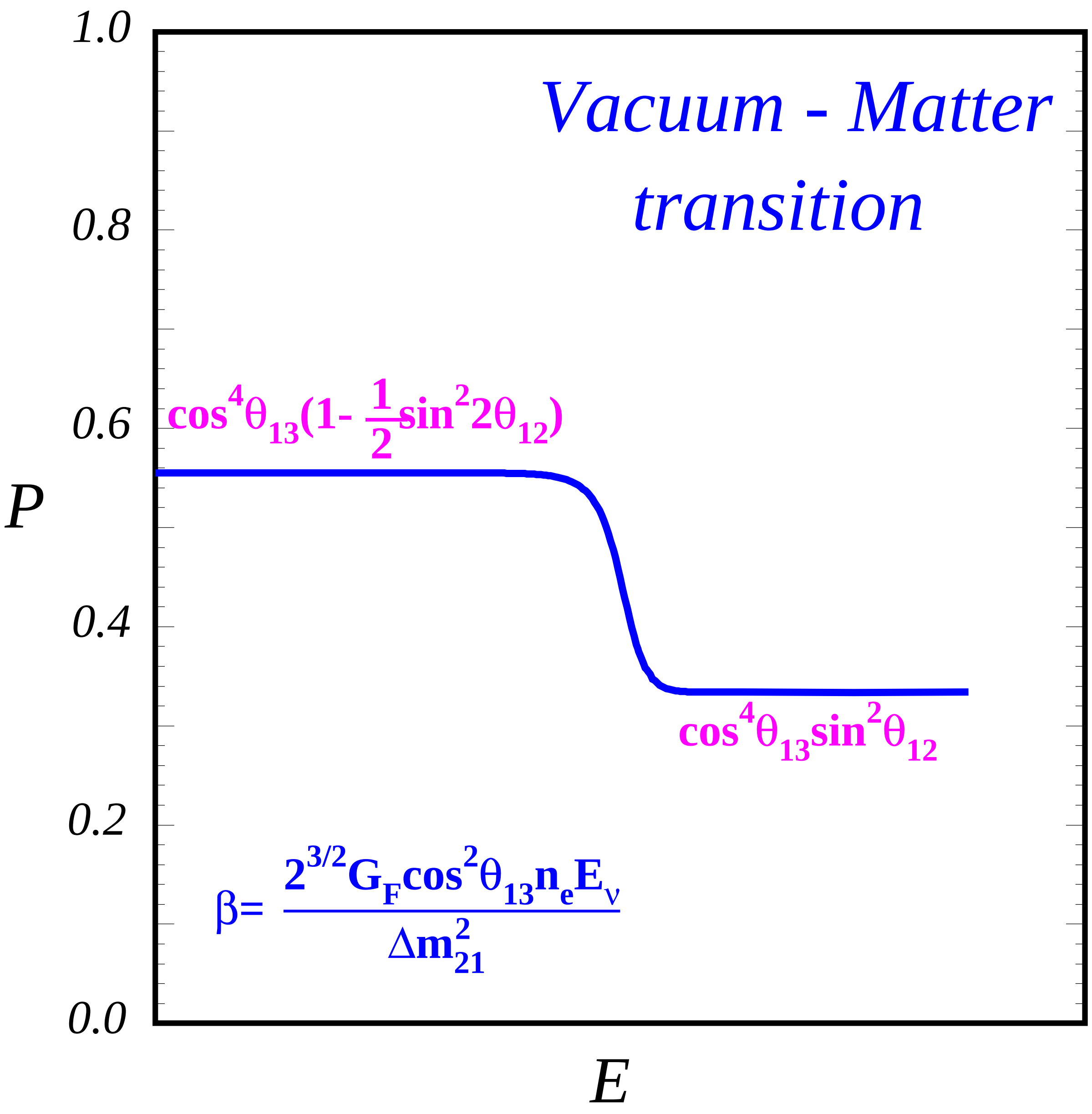
$P_{1e} = \cos^2\theta$ and $P_{2e} = \sin^2\theta$ so

$$\begin{aligned} P_{ee}^{\rm Sun} = \quad & \cos^2\theta_M\left[(1-P_c)\cos^2\theta + P_c\sin^2\theta\right] \\ & + \sin^2\theta_M\left[P_c\cos^2\theta + (1-P_c)\sin^2\theta\right]. \end{aligned}$$

For $N_e = N_{e0}e^{-L/r_0}$, P_c, (crossing probability), is exactly calculable

$$P_c = \frac{e^{-\gamma\sin^2\theta} - e^{-\gamma}}{1-e^{-\gamma}}, \quad \gamma = 2\pi r_0\Delta. \tag{1}$$

Adiabatic condition: $\gamma \gg 1$, when $P_c \rightarrow 0$.



We need:

- $P_{ee} \sim 0.3$ (^{8}B neutrinos)
- $P_{ee} \sim 0.6$ (^{7}Be, pp neutrinos)

$\Rightarrow \quad \sin^2\theta \sim 0.3$

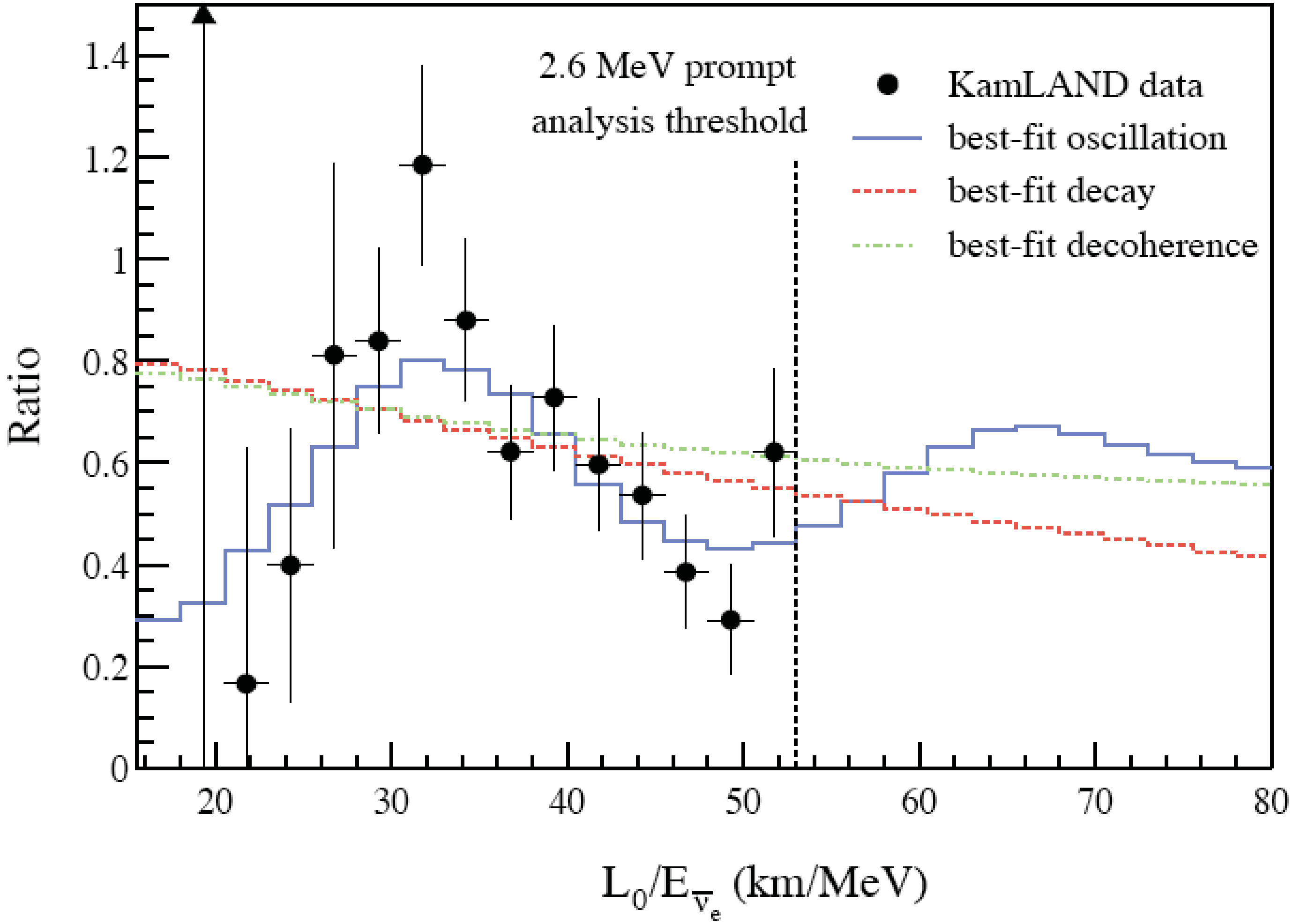
$\Rightarrow \quad \Delta m^2 \sim 10^{-(5 \text{ to } 4)}\ \text{eV}^2$

for a long time, there were many other options!

(LMA, LOW, SMA, VAC)

Solar oscillations confirmed by Reactor experiment: KamLAND!!!

$$\text{phase}= 1.27\left(\frac{\Delta m^2}{5\times 10^{-5}\ \text{eV}^2}\right)\left(\frac{5\ \text{MeV}}{E}\right)\left(\frac{L}{100\ \text{km}}\right)$$

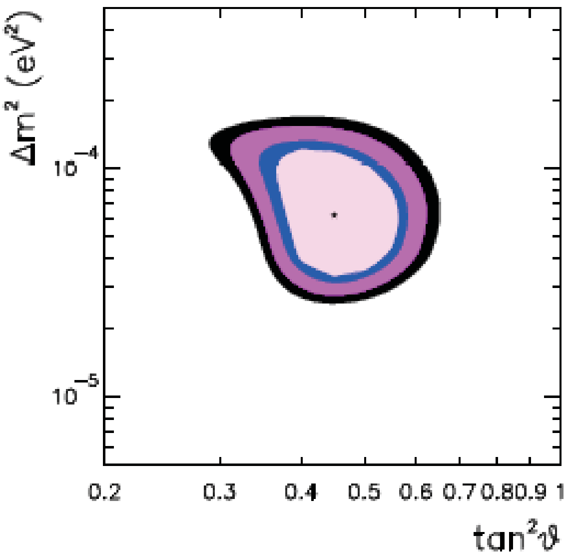


$$P_{ee} = 1 - \sin^2 2\theta \sin^2\left(\frac{\Delta m^2 L}{4E}\right)$$

hint of oscillatory behavior!

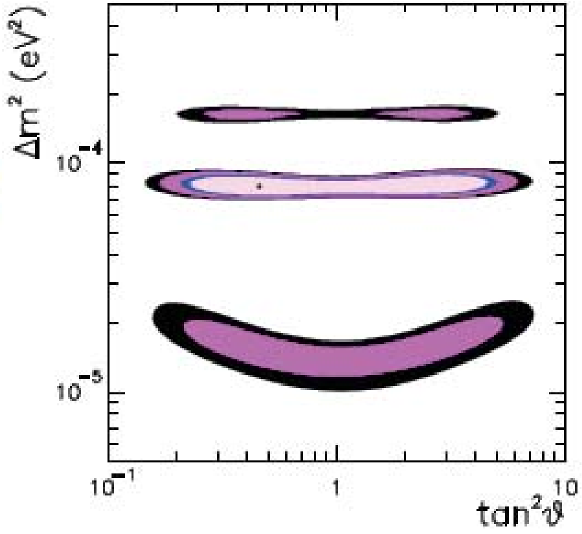
Solar

$\nu_e \rightarrow \nu_{\text{active}}$

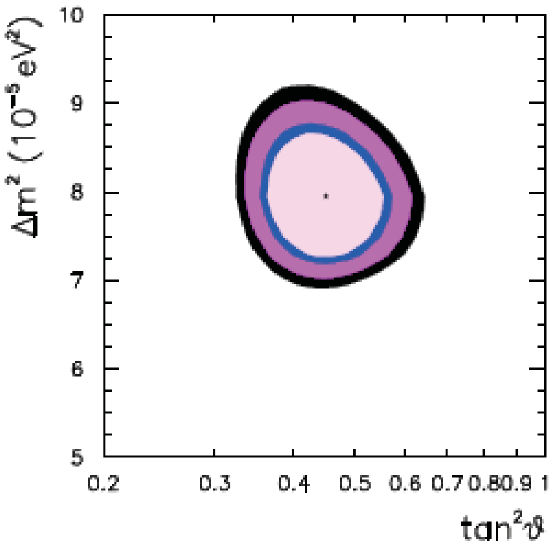


+ KamLAND

$\bar{\nu}_e \not\rightarrow \bar{\nu}_e$



ν_e oscillation parameters compatible with $\overline{\nu}_e$: *Sensible* to assume CPT: $P_{ee} = P_{\bar{e}\bar{e}}$



$$\Delta m^2_\odot = \left(8^{+0.4}_{-0.5}\right) \times 10^{-5}\ \text{eV}^2 \quad (1\sigma)$$
$$\tan^2\theta_\odot = 0.45^{+0.05}_{-0.05}$$

[Gonzalez-Garcia, PASI 2006]

Summarizing:

Both the solar and atmospheric puzzles can be properly explained in terms of two-flavor neutrino oscillations:

- **solar**: $\nu_e \leftrightarrow \nu_a$ (linear combination of ν_μ and ν_τ): $\Delta m^2 \sim 10^{-4}$ eV^2, $\sin^2\theta \sim 0.3$.
- **atmospheric**: $\nu_\mu \leftrightarrow \nu_\tau$: $\Delta m^2 \sim 10^{-3}$ eV^2, $\sin^2\theta \sim 0.5$ ("maximal mixing").

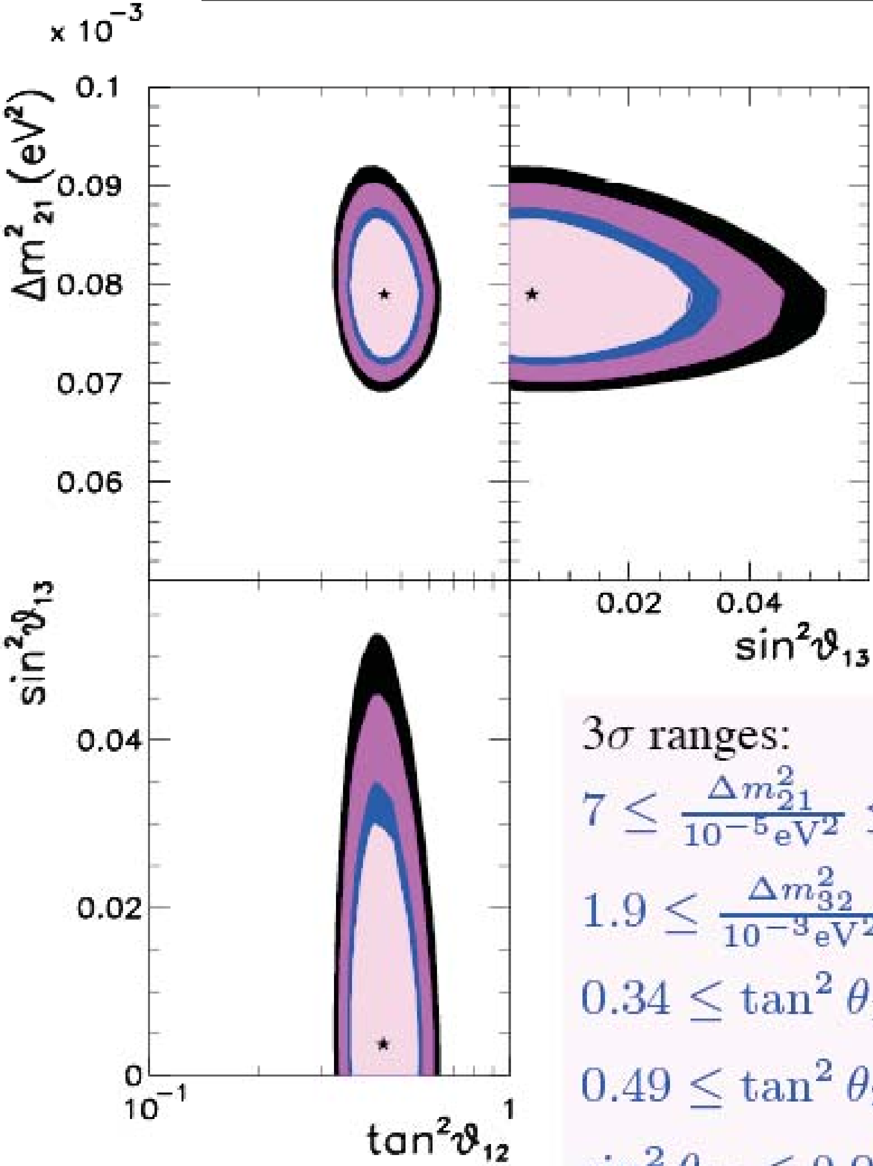
Putting it all together – 3 flavor mixing:

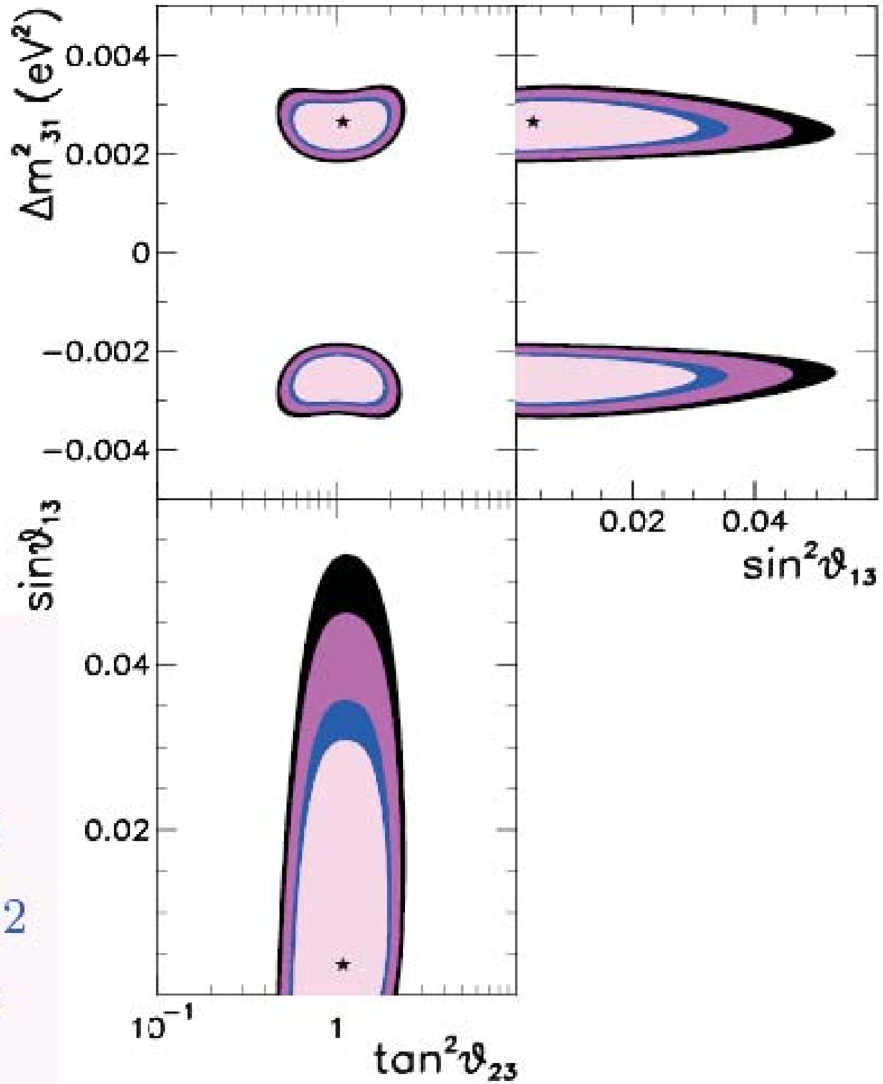
$$\left(\begin{array}{c}\nu_e\\ \nu_\mu\\ \nu_\tau\end{array}\right)=\left(\begin{array}{ccc}U_{e1} & U_{e2} & U_{e3}\\ U_{\mu1} & U_{\mu2} & U_{\mu3}\\ U_{\tau1} & U_{e\tau2} & U_{\tau3}\end{array}\right)\left(\begin{array}{c}\nu_1\\ \nu_2\\ \nu_3\end{array}\right)$$

Definition of neutrino mass eigenstates (who are ν_1, ν_2, ν_3?):

- $m_1^2 < m_2^2$ $\Delta m^2_{13} < 0$ – Inverted Mass Hierarchy
- $m_2^2 - m_1^2 \ll |m_3^2 - m_{1,2}^2|$ $\Delta m^2_{13} > 0$ – Normal Mass Hierarchy

$$\tan^2\theta_{12} \equiv \frac{|U_{e2}|^2}{|U_{e1}|^2};\quad \tan^2\theta_{23} \equiv \frac{|U_{\mu3}|^2}{|U_{\tau3}|^2};\quad U_{e3} \equiv \sin\theta_{13}e^{-i\delta}$$





3σ ranges:

$$7 \leq \frac{\Delta m^2_{21}}{10^{-5}\,\text{eV}^2} \leq 9.1$$

$$1.9 \leq \frac{\Delta m^2_{32}}{10^{-3}\,\text{eV}^2} \leq 3.25$$

$$0.34 \leq \tan^2\theta_{12} \leq 0.62$$

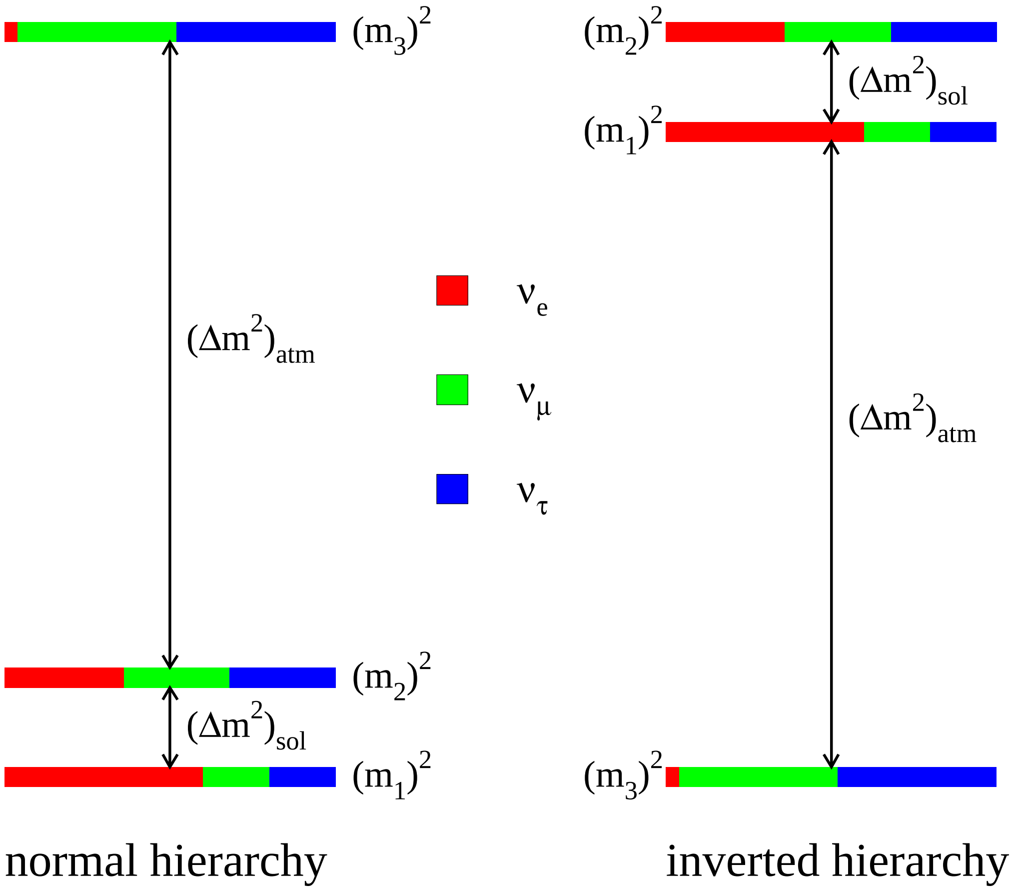
$$0.49 \leq \tan^2\theta_{23} \leq 2.2$$

$$\sin^2\theta_{13} \leq 0.045$$

$$-\pi \leq \delta \leq \pi$$

[Gonzalez-Garcia, PASI 2006]

4– What We Know We Don't Know (i)



- What is the ν_e component of ν_3? ($\theta_{13} \neq 0$?)
- Is CP-invariance violated in neutrino oscillations? ($\delta \neq 0, \pi$?)
- Is ν_3 mostly ν_μ or ν_τ? ($\theta_{23} > \pi/4$, $\theta_{23} < \pi/4$, or $\theta_{23} = \pi/4$?)
- What is the neutrino mass hierarchy? ($\Delta m^2_{13} > 0$?)

$\Rightarrow$ All of these can be addressed in neutrino oscillation experiments **if** we get lucky, that is if θ_{13} is large enough.

Hunting For θ_{13} (or U_{e3})

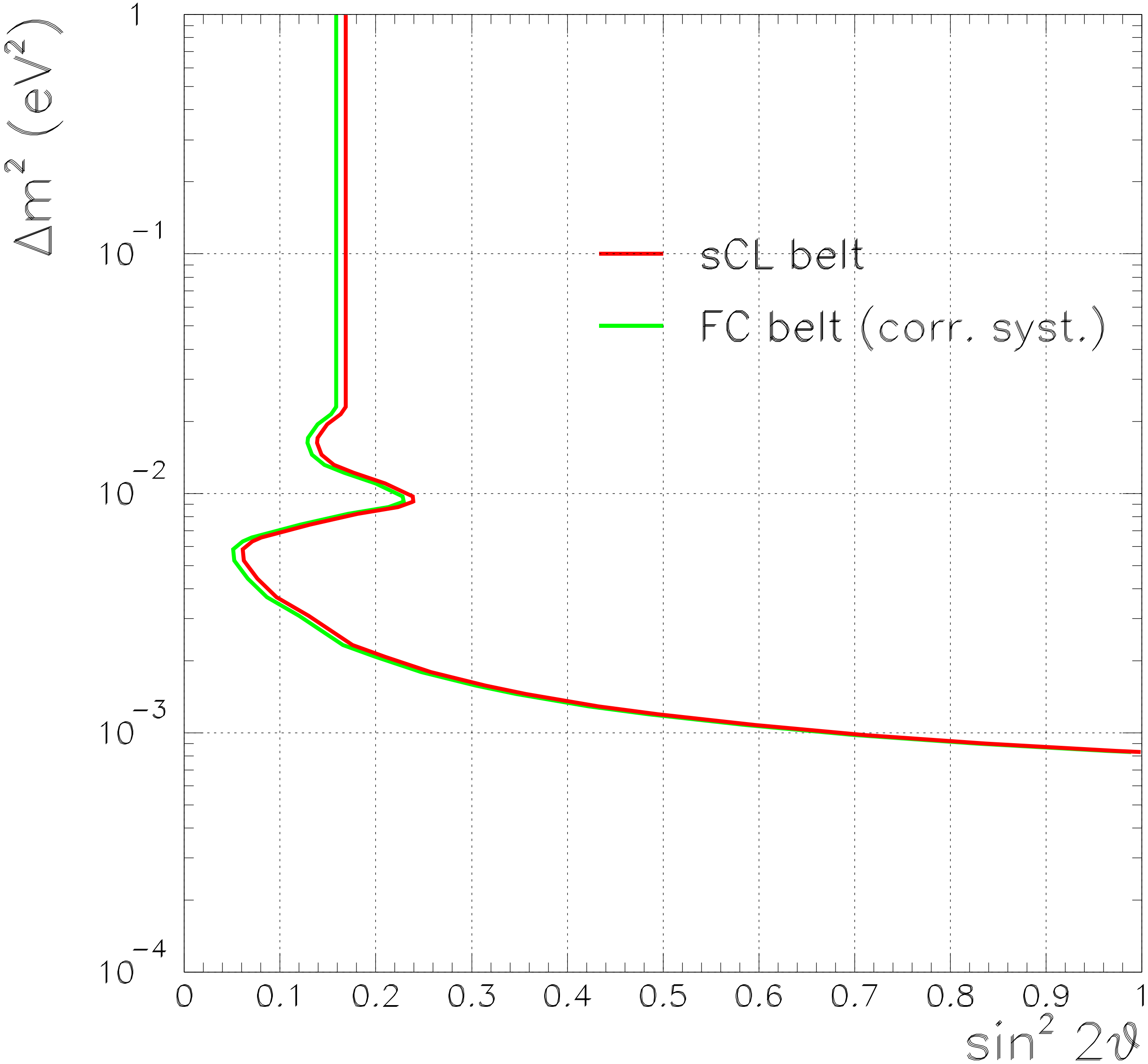
The best way to hunt for θ_{13} is to look for oscillation effects involving electron (anti)neutrinos, governed by the atmospheric oscillation frequency, Δm^2_{13} (other possibility, precision measurement of ν_μ disappearance. . .).

One way to understand this is to notice that if $\theta_{13} \equiv 0$, the ν_e state only participates in processes involving Δm^2_{12}.

Example:

$$P_{ee} \simeq 1 - \sin^2 2\theta_{13} \sin^2\left(\frac{\Delta m^2_{13} L}{4E}\right) + O\left(\frac{\Delta m^2_{12}}{\Delta m^2_{13}}\right)^2$$

Reactor Neutrino Searches for θ_{13}



- $L \sim 1$ km
- $E_\nu \sim 5$ MeV

next-generation: aim at improving CHOOZ bound by an order of magnitude.

e.g. Double CHOOZ, Daya Bay, etc

$\nu_\mu \leftrightarrow \nu_e$ at Long-Baseline Experiments

REQUIREMENTS: ν_μ beam, detector capable of seeing electron appearance. This is the case of "Superbeam Experiments" like T2K and NOνA.

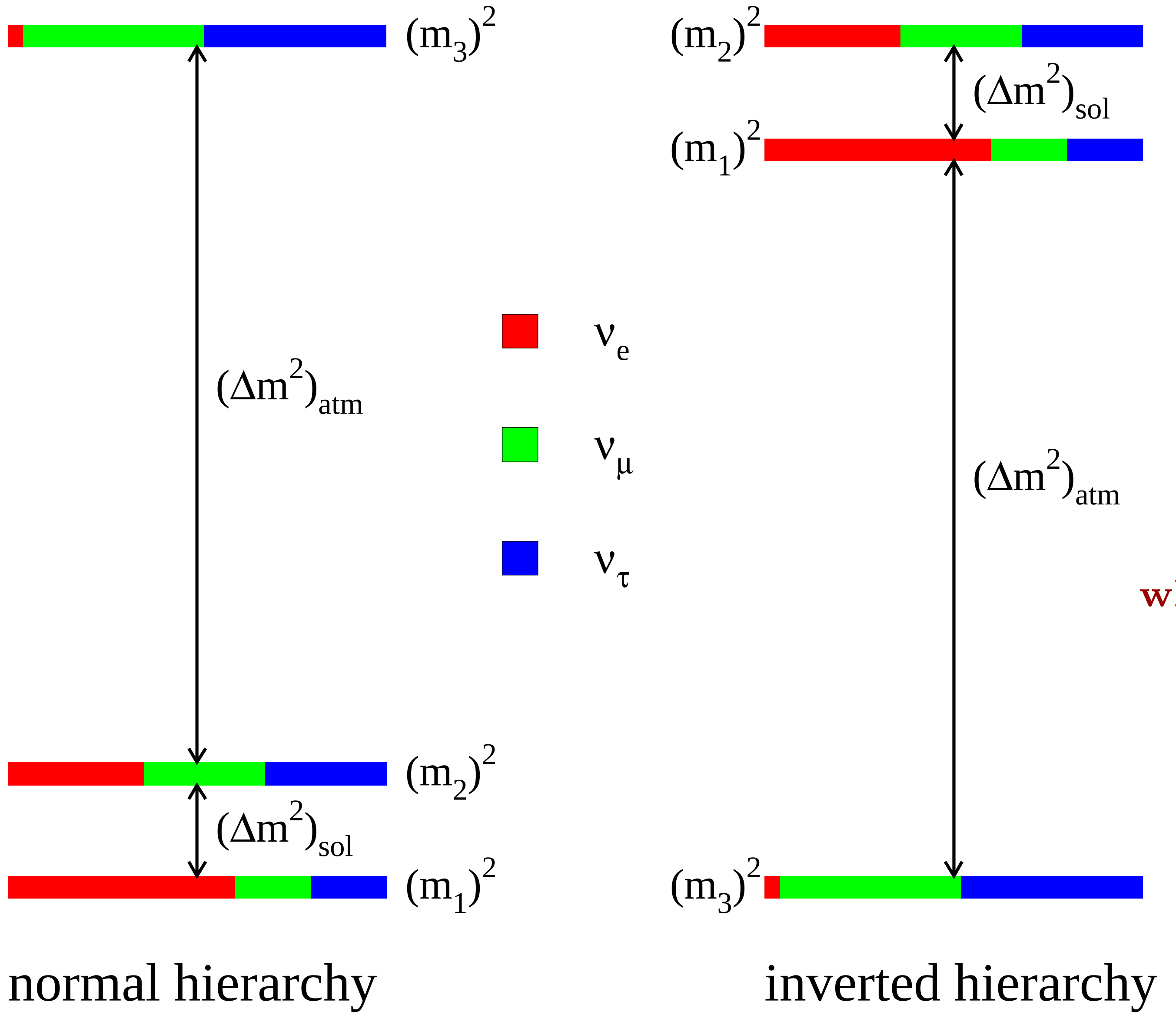
or

ν_e beam and detector capable of detecting muons (usually including sign). This would be the case of "Neutrino Factories" ($\mu^+ \to e^+\bar{\nu}_\mu\nu_e$) and "Beta Beams" ($Z \to (Z \pm 1)e^\mp \nu_e$).

In vaccum

$$P_{\mu e} = \sin^2\theta_{23}\sin^2 2\theta_{13}\sin^2\left(\frac{\Delta m^2_{13}L}{4E}\right) + \text{ “subleading”}.$$

• Sensitivity to $\sin^2\theta_{13}$. More precisely, $\sin^2\theta_{23}\sin^2 2\theta_{13}$. This leads to one potential degeneracy.



The Neutrino Mass Hierarchy

which is the right picture?

Why Don't We Know the Neutrino Mass Hierarchy?

Most of the information we have regarding θ_{23} and Δm^2_{13} comes from atmospheric neutrino experiments (SuperK). Roughly speaking, they measure

$$P_{\mu\mu} = 1 - \sin^2 2\theta_{23} \sin^2\left(\frac{\Delta m^2_{13} L}{4E}\right) + \text{ subleading.}$$

It is easy to see from the expression above that the leading term is simply not sensitive to the sign of Δm^2_{13}.

On the other hand, because $|U_{e3}|^2 < 0.05$ and $\frac{\Delta m^2_{12}}{\Delta m^2_{13}} < 0.06$ are both small, we are yet to observe the subleading effects.

Determining the Mass Hierarchy via Oscillations – the large U_{e3} route

Again, necessary to probe $\nu_\mu \to \nu_e$ oscillations (or vice-versa) governed by Δm^2_{13}. This is the oscillation channel that (almost) all next-generation, accelerator-based experiments are concentrating on, including the next generation experiments T2K and NOνA.

In vaccum

$$P_{\mu e} = \sin^2\theta_{23}\sin^2 2\theta_{13}\sin^2\left(\frac{\Delta m^2_{13}L}{4E}\right) + \text{ “subleading”},$$

so that, again, this is insensitive to the sign of Δm^2_{13} at leading order. However, in this case, matter effects may come to the rescue.

As I discussed already, neutrino oscillations get modified when these propagate in the presence of matter. Matter effects are sensitive to the neutrino mass ordering (in a way that I will describe shortly) and different for neutrinos and antineutrinos.

If $\Delta_{12} \equiv \frac{\Delta m^2_{12}}{2E}$ terms are ignored, the $\nu_\mu \to \nu_e$ oscillation probability is described, in constant matter density, by

$$P_{\mu e} \simeq P_{e\mu} \simeq \sin^2\theta_{23} \sin^2 2\theta^{\rm eff}_{13} \sin^2\left(\frac{\Delta^{\rm eff}_{13} L}{2}\right),$$

$$\sin^2 2\theta^{\rm eff}_{13} = \frac{\Delta^2_{13}\sin^2 2\theta_{13}}{(\Delta^{\rm eff}_{13})^2},$$

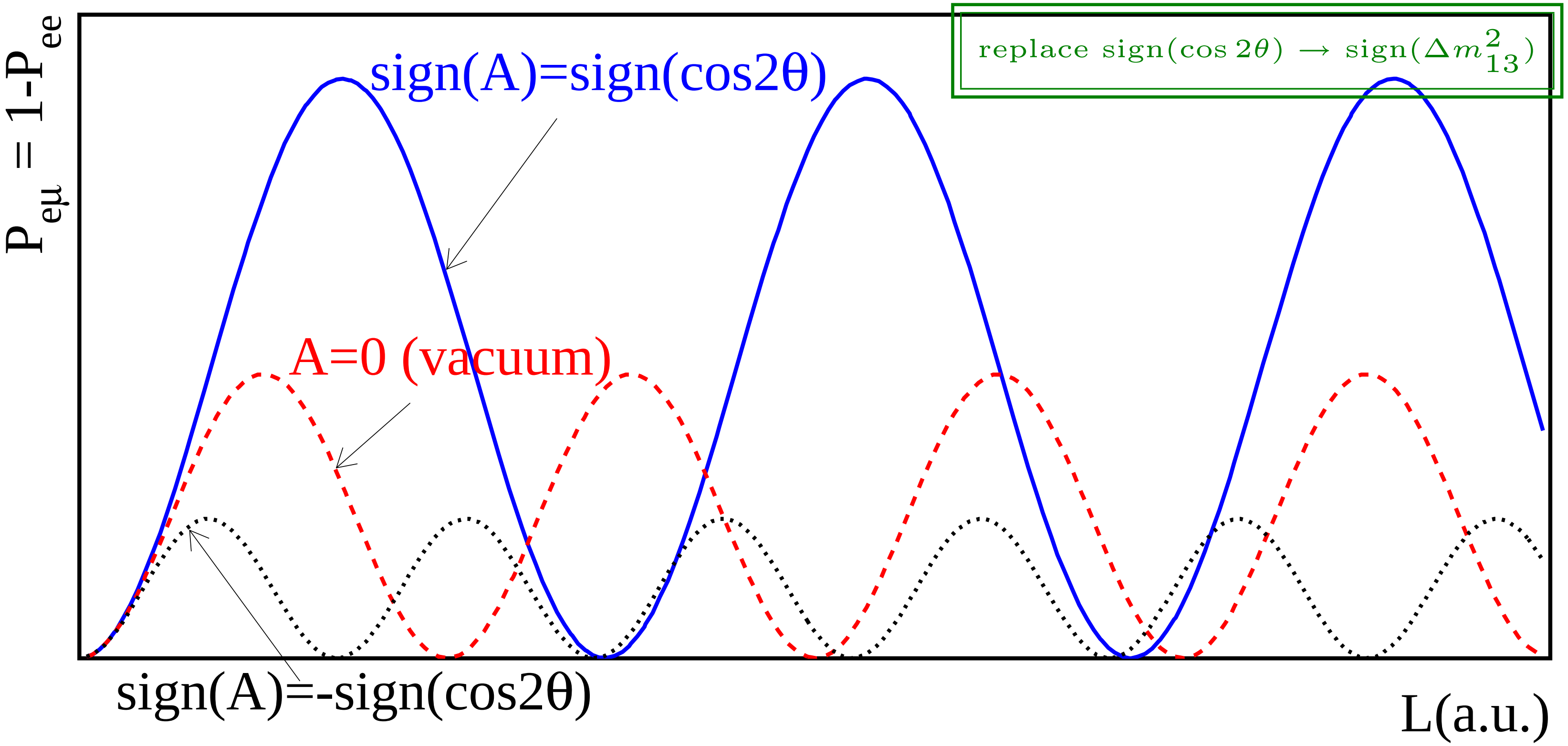
$$\Delta^{\rm eff}_{13} = \sqrt{(\Delta_{13}\cos 2\theta_{13} - A)^2 + \Delta^2_{13}\sin^2 2\theta_{13}},$$

$$\Delta_{13} = \frac{\Delta m^2_{13}}{2E},$$

$A \equiv \pm\sqrt{2} G_F N_e$ is the matter potential. It is positive for neutrinos and negative for antineutrinos.

$P_{\mu e}$ depends on the relative sign between Δ_{13} and A. It is different for the two different mass hierarchies, and different for neutrinos and antineutrinos.

replace sign$(\cos 2\theta) \rightarrow$ sign(Δm^2_{13})



Requirements:

- $\sin^2 2\theta_{13}$ large enough – otherwise there is nothing to see!
- $|\Delta_{13}| \sim |A|$ – matter potential must be significant but not overwhelming.
- $\Delta_{13}^{\rm eff} L$ large enough – matter effects are absent near the origin.

The "Holy Graill" of Neutrino Oscillations – CP Violation

In the old Standard Model, there is only one[a] source of CP-invariance violation:

$\Rightarrow$ The complex phase in V_{CKM}, the quark mixing matrix.

Indeed, as far as we have been able to test, all CP-invariance violating phenomena agree with the CKM paradigm:

- ϵ_K;
- ϵ'_K;
- $\sin 2\beta$;
- etc.

Recent experimental developments, however, provide strong reason to believe that this is not the case: neutrinos have mass, and leptons mix!

[a]modulo the QCD θ-parameter, which will be "willed away" henceforth.

CP-invariance Violation in Neutrino Oscillations

The most promising approach to studying CP-violation in the leptonic sector seems to be to compare $P(\nu_\mu \to \nu_e)$ versus $P(\bar{\nu}_\mu \to \bar{\nu}_e)$.

$$A_{\mu e} = U_{e1}^* U_{\mu 1} + U_{e2}^* U_{\mu 2} e^{i\Delta_{12}}, + U_{e3}^* U_{\mu 3} e^{i\Delta_{13}}$$

where $\Delta_{1i} = \frac{\Delta m_{1i}^2 L}{2E}$, $i = 2, 3$.

The amplitude for the CP-conjugate process is

$$\bar{A}_{\mu e} = U_{e1} U_{\mu 1}^* + U_{e2} U_{\mu 2}^* e^{i\Delta_{12}}, + U_{e3} U_{\mu 3}^* e^{i\Delta_{13}}.$$

In general, $|A|^2 \neq |\bar{A}|^2$ (CP-invariance violated) as long as:

- Nontrivial "Weak" Phases: $\arg(U_{ei}^*U_{\mu i}) \to \delta \neq 0, \pi$;
- Nontrivial "Strong" Phases: Δ_{12}, $\Delta_{13} \to L \neq 0$;
- Because of Unitarity, we need all $|U_{\alpha i}| \neq 0 \to$ three generations.

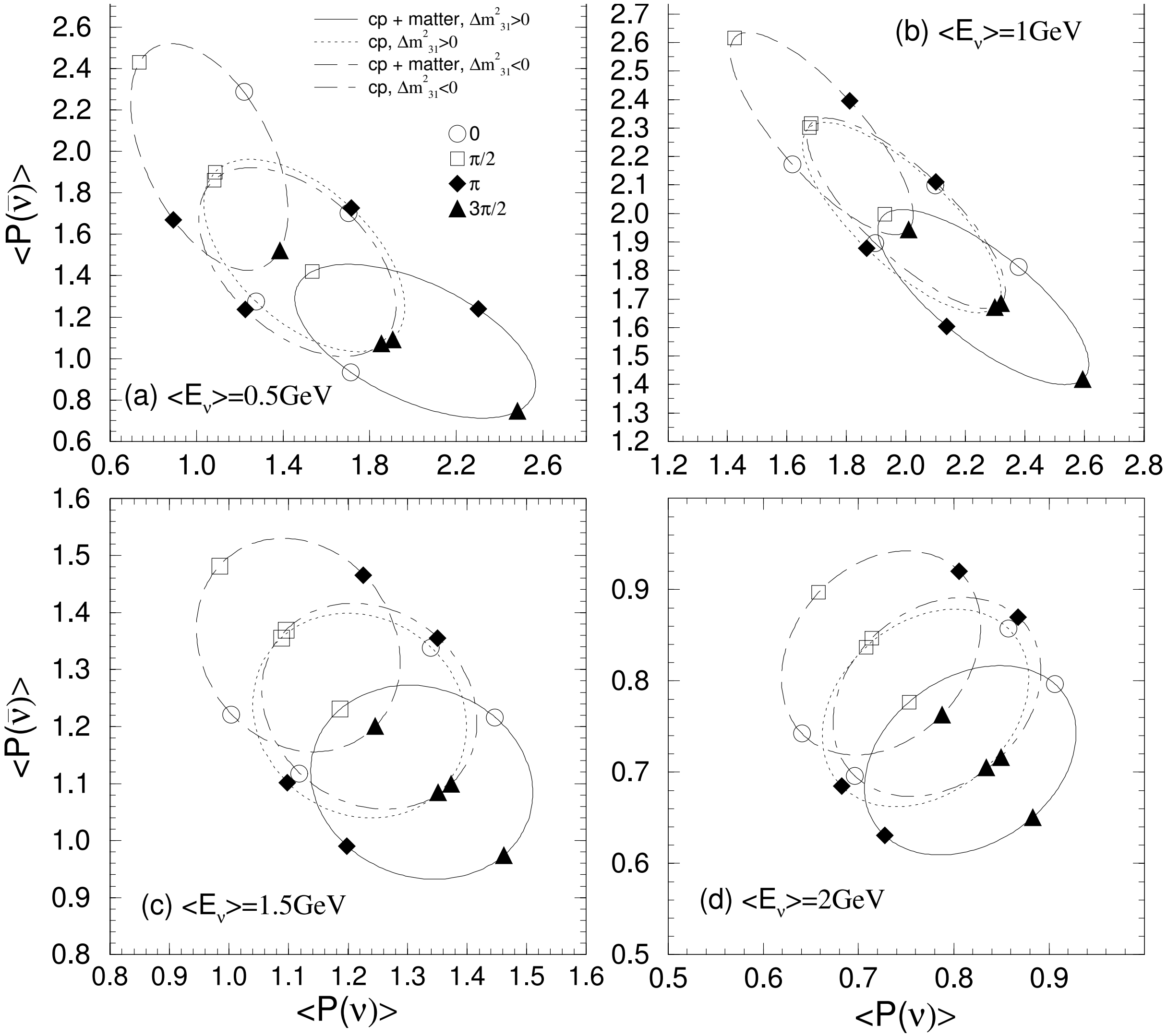
All of these can be satisfied, with a little luck: given that two of the three mixing angles are known to be large, **we need $|U_{e3}| \neq 0$.**

The goal of next-generation neutrino experiments is to determine the magnitude of $|U_{e3}|$. We need to know this in order to understand how to study CP-invariance violation in neutrino oscillations!

In the real world, life is much more complicated. The lack of knowledge concerning the mass hierarchy, θ_{13}, θ_{23} leads to several degeneracies.

Note that, in order to see CP-invariance violation, we **need** the "subleading" terms!

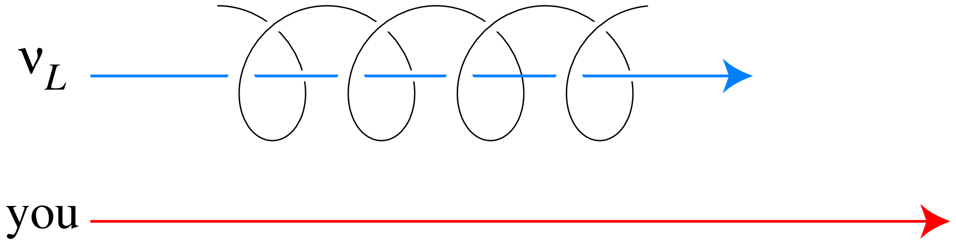
In order to ultimately measure a new source of CP-invariance violation, we will need to combine different measurements:
– oscillation of muon neutrinos and antineutrinos,
– oscillations at accelerator and reactor experiments,
– experiments with different baselines,
– etc.



Need to determine "other" oscillation parameters in order to realistically study CP-invariance violation.

[Minakata, Nunokawa, hep-ph/0108085]

5– What We Know We Don't Know (ii) – Are Neutrinos Majorana Fermions?

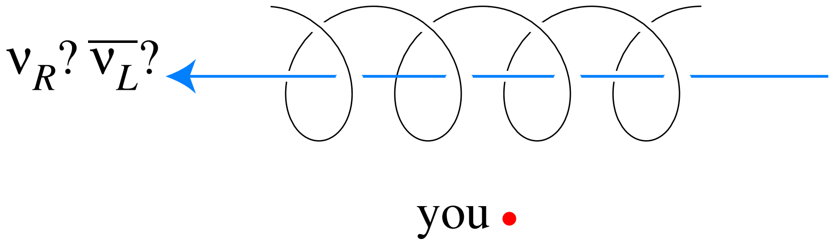


A massive charged fermion (s=1/2) is described by 4 degrees of freedom:

$$(e^-_L \leftarrow \text{CPT} \rightarrow e^+_R)$$
$$\updownarrow \text{Lorentz}$$
$$(e^-_R \leftarrow \text{CPT} \rightarrow e^+_L)$$

A massive neutral fermion (s=1/2) is described by 4 or 2 degrees of freedom:

$$(\nu_L \leftarrow \text{CPT} \rightarrow \bar{\nu}_R)$$
$$\updownarrow \text{Lorentz} \qquad \text{“DIRAC”}$$
$$(\nu_R \leftarrow \text{CPT} \rightarrow \bar{\nu}_L)$$



$$(\nu_L \leftarrow \text{CPT} \rightarrow \bar{\nu}_R)$$
$$\text{“MAJORANA”} \qquad \updownarrow \text{Lorentz}$$
$$(\bar{\nu}_R \leftarrow \text{CPT} \rightarrow \nu_L)$$

How many degrees of freedom are required to describe massive neutrinos?

Why Don't We Know the Answer (Yet)?

If neutrino masses were indeed zero, this is a nonquestion: there is no distinction between a massless Dirac and Majorana fermion.

Processes that are proportional to the Majorana nature of the neutrino vanish in the limit $m_\nu \to 0$. Since neutrinos masses are very small, the probability for these to happen is very, very small: $A \propto m_\nu/E$.

The "smoking gun" signature is the observation of LEPTON NUMBER violation. This is easy to understand: Majorana neutrinos are their own antiparticles and, therefore, cannot carry any quantum numbers — including lepton number.

Weak Interactions are Purely Left-Handed (Chirality):

For example, in the scattering process $e^- + X \to \nu_e + X$, the electron neutrino is, in a reference frame where $m \ll E$,

$$|\nu_e\rangle \sim |L\rangle + \left(\frac{m}{E}\right)|R\rangle.$$

If the neutrino is a Majorana fermion, $|R\rangle$ behaves mostly like a "$\bar{\nu}_e$," (and $|L\rangle$ mostly like a "ν_e,") such that the following process could happen:

$$e^- + X \to \nu_e + X, \text{ followed by } \nu_e + X \to e^+ + X, \qquad P \simeq \left(\frac{m}{E}\right)^2$$

Lepton number can be violated by 2 units with small probability. Typical numbers: $P \simeq (0.1 \text{ eV}/100 \text{ MeV})^2 = 10^{-18}$. VERY Challenging!

How many new CP-violating parameters in the neutrino sector?

If the neutrinos are Majorana fermions, there are more physical observables in the leptonic mixing matrix.

Remember the parameter counting in the quark sector:

9 $(3 \times 3$ unitary matrix)

$\underline{-5\quad}$ (relative phase rotation among six quark fields)

4 (3 mixing angles and 1 CP-odd phase).

If the neutrinos are Majorana fermions, the parameter counting is quite different: there are no right-handed neutrino fields to "absorb" CP-odd phases:

$$\begin{array}{rl} 9 & (3 \times 3 \text{ unitary matrix}) \\ \underline{-3\quad} & (\text{three right-handed charged lepton fields}) \\ 6 & (3 \text{ mixing angles and } 3 \text{ CP-odd phases}). \end{array}$$

There is CP-invariance violating parameters even in the 2 family case: $4 - 2 = 2$, one mixing angle, one CP-odd phase.

$$\mathcal{L} \supset \bar{e}_L U W^\mu \gamma_\mu \nu_L - \bar{e}_L (M_e) e_R - \overline{\nu_L^c} (M_\nu) \nu_L + H.c.$$

Write $U = E^{-i\xi/2} U' E^{i\alpha/2}$, where $E^{i\beta/2} \equiv \text{diag}(e^{i\beta_1/2}, e^{i\beta_2/2}, e^{i\beta_3/2})$, $\beta = \alpha, \xi$

$$\mathcal{L} \supset \bar{e}_L U' W^\mu \gamma_\mu \nu_L - \bar{e}_L E^{i\xi/2} (M_e) e_R - \overline{\nu_L^c} (M_\nu) E^{-i\alpha} \nu_L + H.c.$$

ξ phases can be "absorbed" by e_R,

α phases cannot go away!

on the other hand

Dirac Case:

$$\mathcal{L} \supset \bar{e}_L U W^\mu \gamma_\mu \nu_L - \bar{e}_L (M_e) e_R - \bar{\nu}_R (M_\nu) \nu_L + H.c.$$

$$\mathcal{L} \supset \bar{e}_L U' W^\mu \gamma_\mu \nu_L - \bar{e}_L E^{i\xi/2} (M_e) e_R - \bar{\nu}_R (M_\nu) E^{-i\alpha/2} \nu_L + H.c.$$

ξ phases can be "absorbed" by e_R, α phases can be "absorbed" by ν_R,

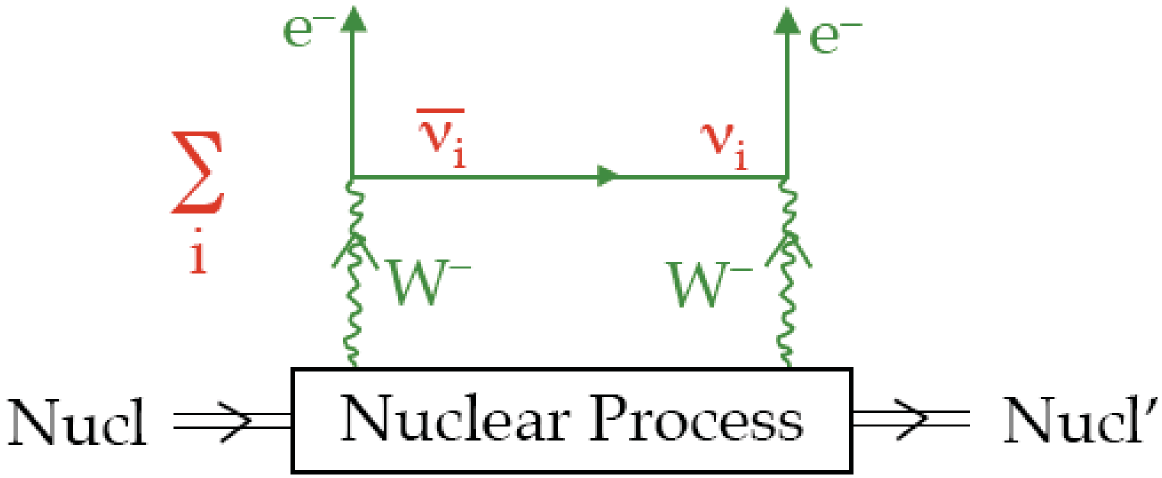
$$V_{MNS} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{e\tau 2} & U_{\tau 3} \end{pmatrix}' \begin{pmatrix} e^{i\alpha_1/2} & 0 & 0 \\ 0 & e^{i\alpha_2/2} & 0 \\ 0 & 0 & e^{i\alpha_3/2} \end{pmatrix}.$$

It is easy to see that the Majorana phases never show up in neutrino oscillations ($A \propto U_{\alpha i} U^*_{\beta_i}$).

Furthermore, they only manifest themselves in phenomena that vanish in the limit $m_i \to 0$ – after all they are only physical if we "know" that lepton number is broken.

$$A(\alpha_i) \propto m_i/E \quad \to \quad \text{tiny!}$$

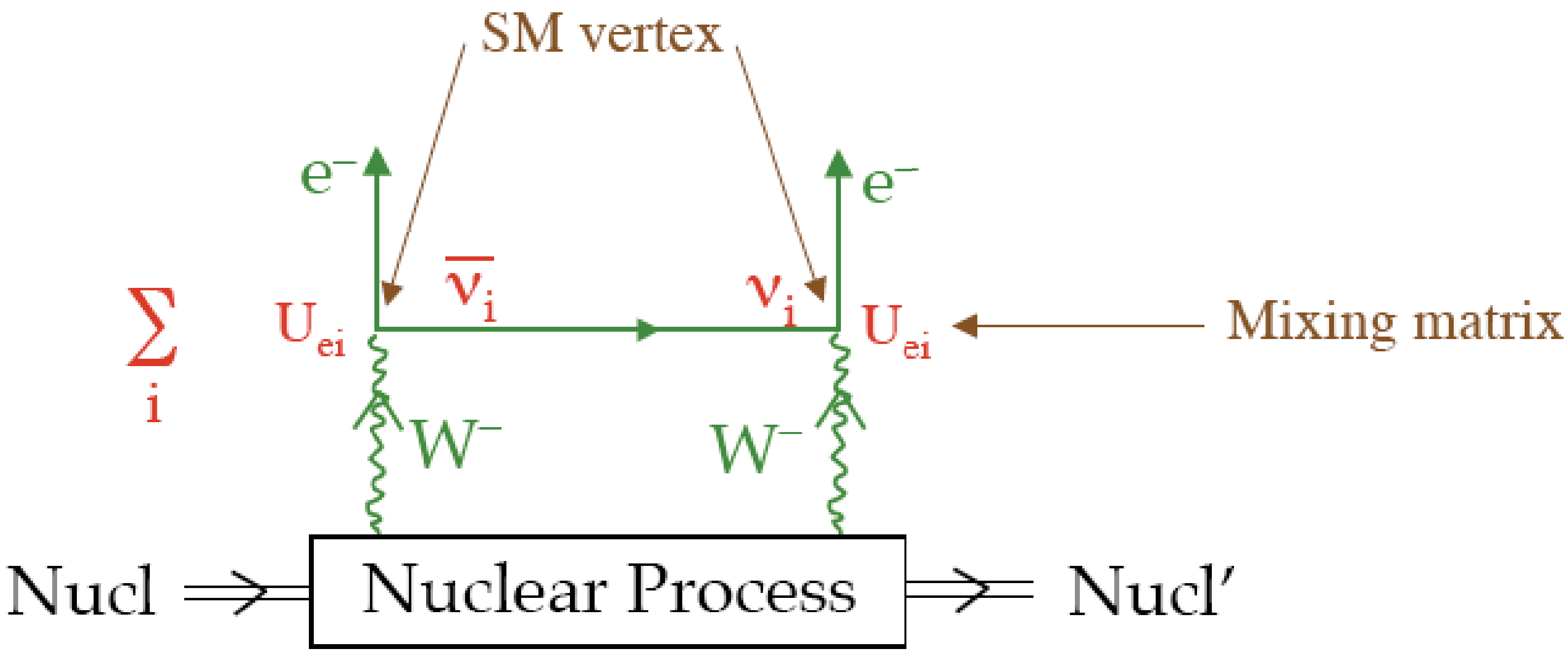
Neutrinoless Double Beta Decay [$0\nu\beta\beta$]



If we start with ***a lot*** of parent nuclei (say, one ton of them), we can cope with the small neutrino masses.

Observation would imply $\cancel{L}$ and $\overline{\nu}_i = \nu_i$.

In —

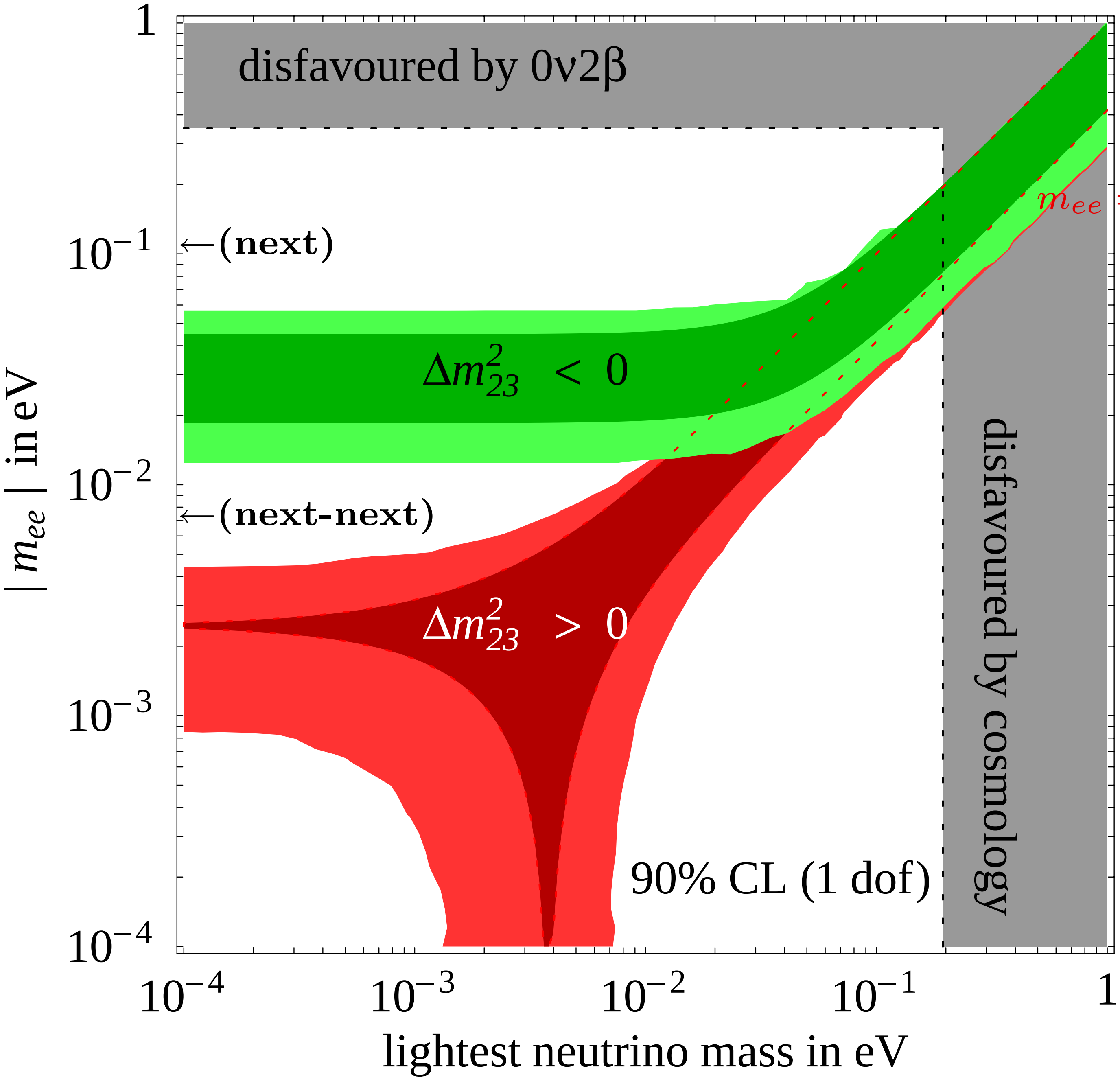


Mass (ν_i)

the $\overline{\nu}_i$ is emitted [RH + O{m_i/E}LH].

Thus, Amp [ν_i contribution] $\propto$ m_i

$$\mathrm{Amp}[0\nu\beta\beta] \propto \left| \sum_i m_i U_{ei}^{\,2} \right| \equiv m_{\beta\beta}$$



[U_{ei}^2 complex numbers]

$\downarrow$

$m_{ee} = U_{e1}^2 m_1 + U_{e2}^2 m_2 + U_{e3}^2 m_3$

Depends on Majorana Phases

e.g. inverted hierarchy:

$m_3 \ll m_1 \sim m_2 \sim \sqrt{\Delta m^2_{13}}$,

$m_{ee} \sim \sqrt{\Delta m^2_{13}} \times$
$\times \left(\cos^2\theta_{12} + e^{i\alpha}\sin^2\theta_{12}\right)$.

$m_{ee} > \sqrt{\Delta m^2_{13}} \cos 2\theta_{12}$

Exciting Prospects for the Future!

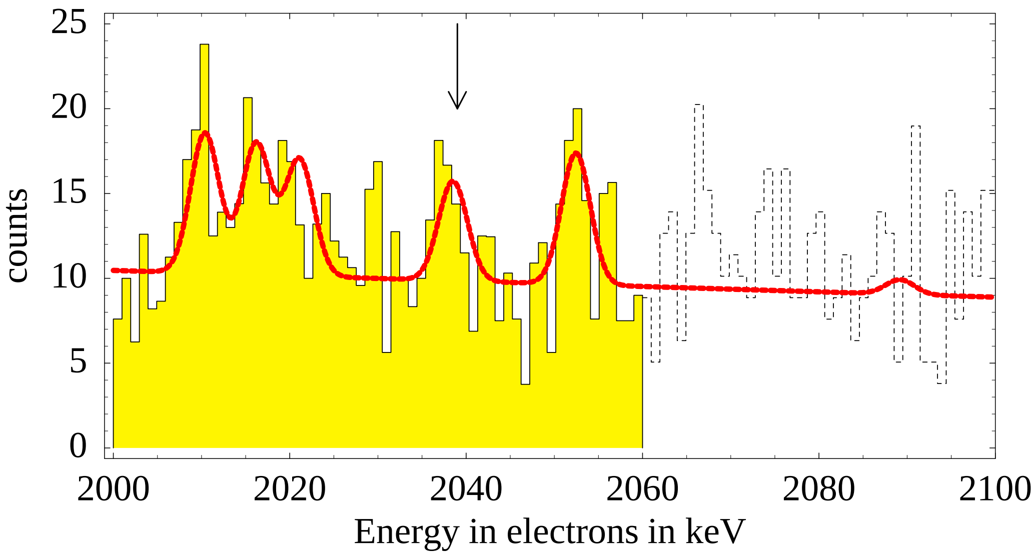
Current bound on $m_{ee} = \sum_i U_{ei}^2 m_i$ around 0.4 eV.

Next round (close, 200 kg): $m_{ee} > 0.1$ eV [degenerate masses]*,

scalable (proposed, 1 ton) to $m_{ee} > 0.01$ [inverted hierarchy]*.

Ultimate goal (R&D, $t \to \infty$, 10+ tons) $m_{ee} > 0.001$ [normal hierarchy]*.

(*beware of nuclear physics uncertainties!)



controversial claim that $0\nu\beta\beta$ has been observed consistent with $m_{ee} \sim 0.7$ eV

DETOUR:

The LSND Anomaly

The LSND experiment looks for $\bar{\nu}_e$ coming from

- $\pi^+ \to \mu^+ \nu_\mu$ decay in flight;
- $\mu^+ \to e^+ \nu_e \bar{\nu}_\mu$ decay at rest;

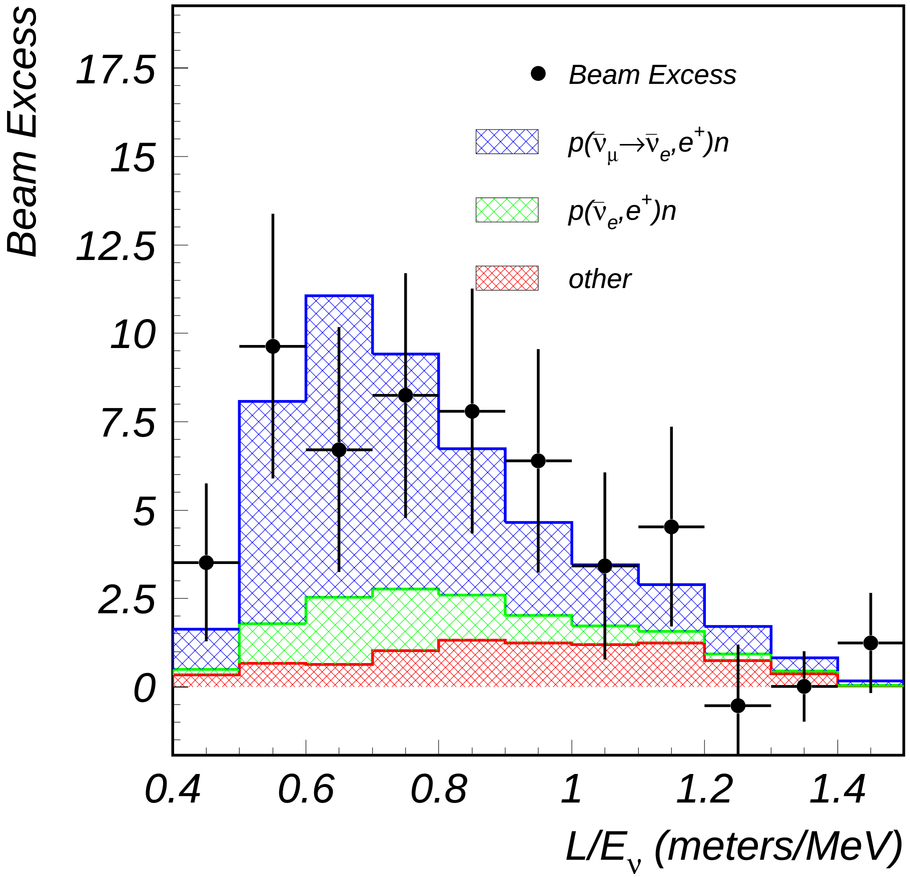
produced some 30 meters away from the detector region.

It observes a statistically significant excess of $\bar{\nu}_e$-candidates. The excess can be explained if there is a very small probability that a $\bar{\nu}_\mu$ interacts as a $\bar{\nu}_e$, $P_{\mu e} = (0.26 \pm 0.08)\%$.

However: the LSND anomaly (or any other consequence associated with its resolution) is yet to be observed in another experimental setup.

Very Unclear – Ruled out?

LSND: strong evidence for $\bar{\nu}_\mu \to \bar{\nu}_e$



If oscillations (??) $\Rightarrow \Delta m^2 \sim 1$ eV^2

× does not fit into 3 ν picture;

× $2+2$ scheme ruled out (solar, atm);

× $3+1$ scheme ruled out;

× 3 ν's CPTV ruled out (KamLAND, atm);

× $\mu \to e\nu_e\bar{\nu}_e$ ruled out (KARMEN, TWIST);

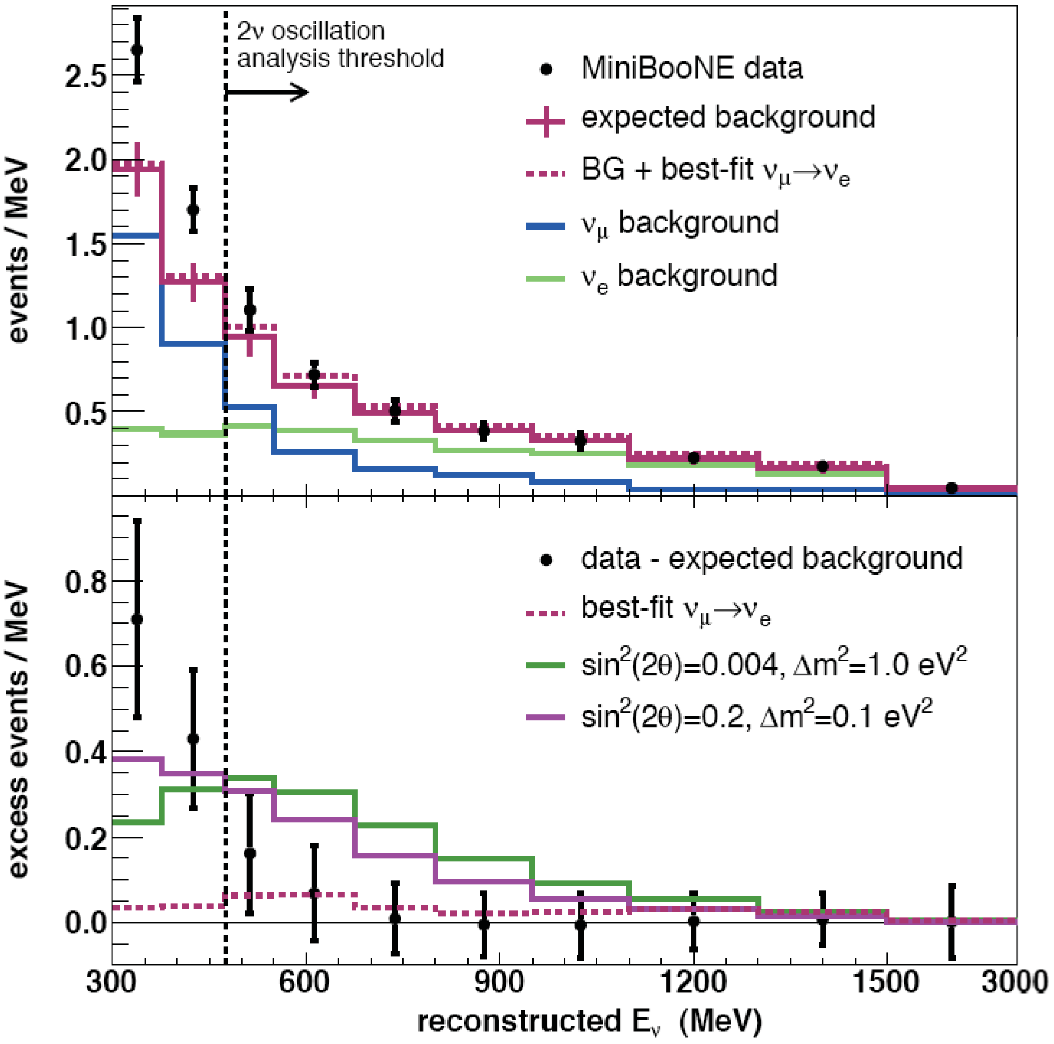
×? $3+1+1$ scheme;

○ 4 ν's CPTV

×? "heavy" decaying sterile neutrinos;

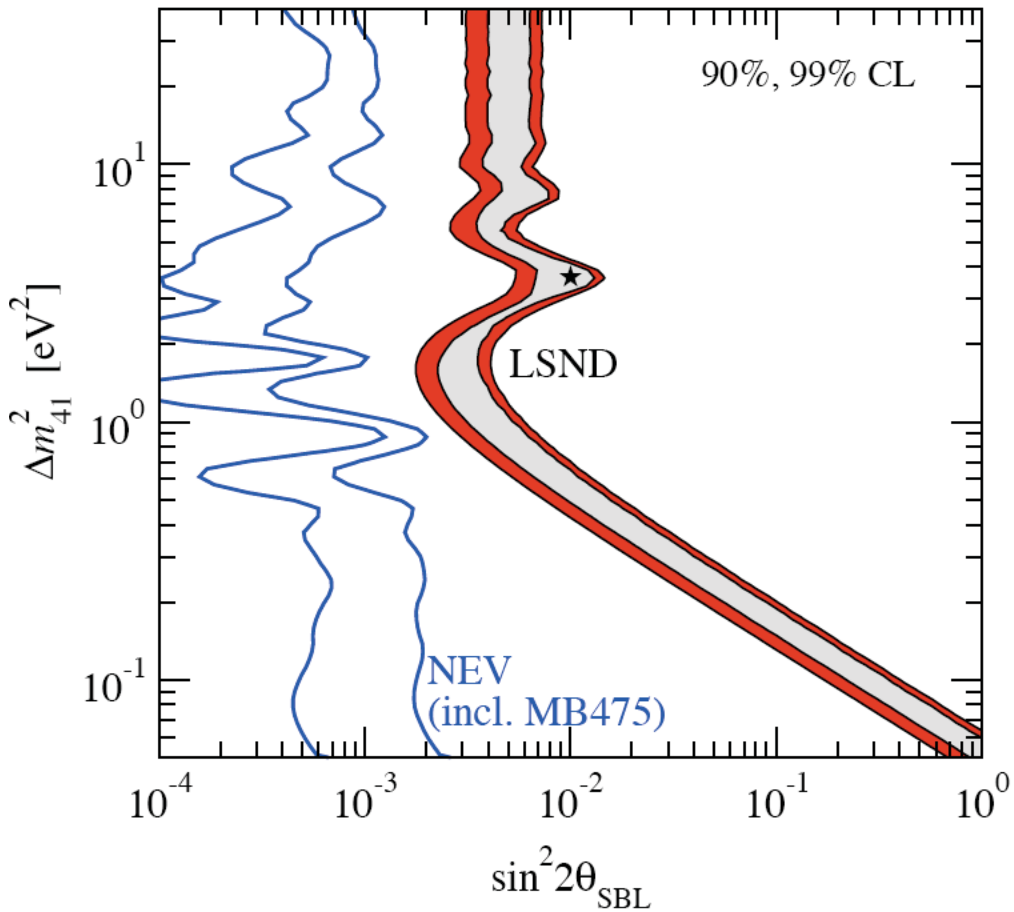
○ 3 νs and Lorentz-invariance violation;

○ something completely different.



3+1 scheme "ruled out"

[Maltoni, Schwetz, arXiv:0705.0107 [hep-ph]]



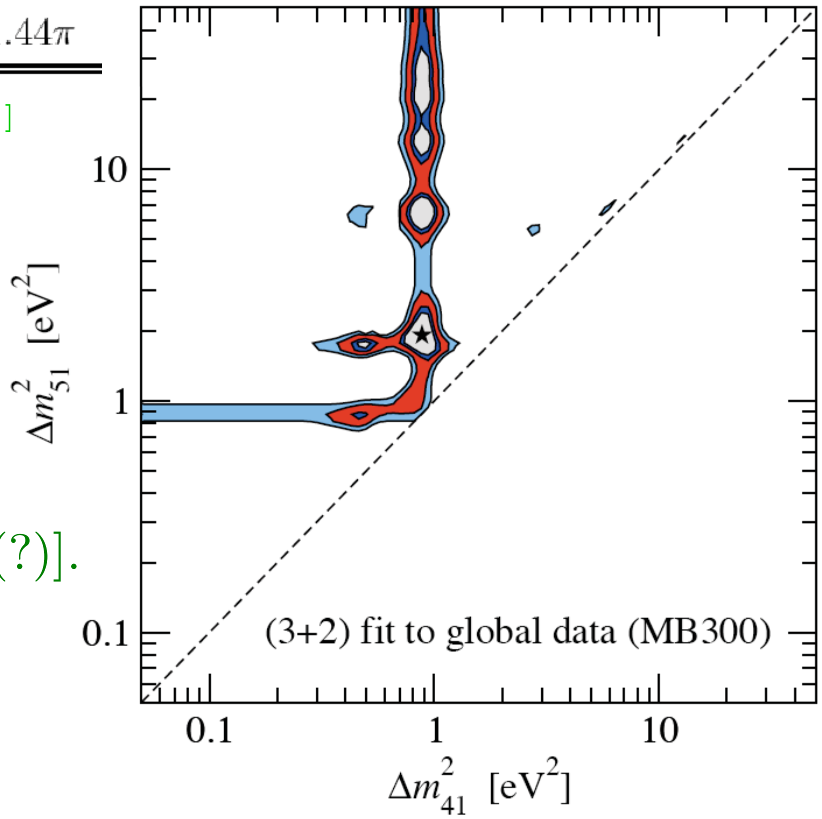
3+1+1 Fits Introduce an Extra Δm^2 and New Mixing Parameters

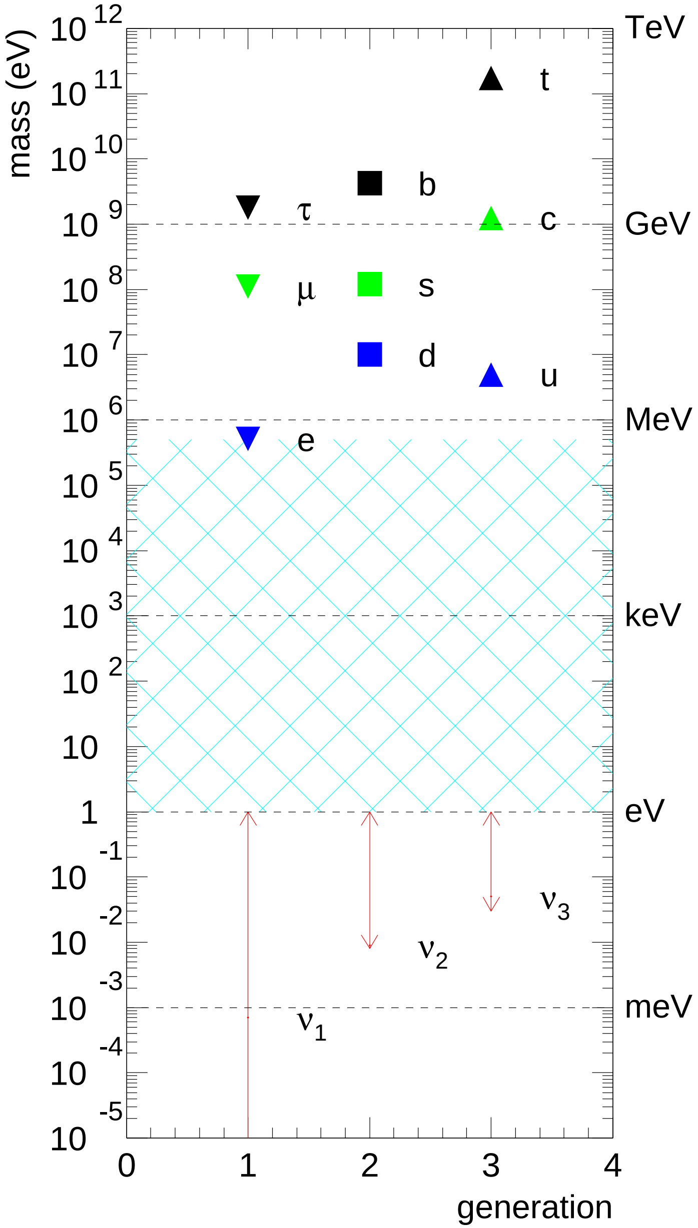
data set	$\|U_{e4}U_{\mu4}\|$		Δm^2_{41}	$\|U_{e5}U_{\mu5}\|$		Δm^2_{51}	δ – CP-violating phase
appearance (MB475)	0.044		0.66	0.022		1.44	1.12π
appearance (MB300)	0.31		0.66	0.27		0.76	1.01π
	$\|U_{e4}\|$	$\|U_{\mu4}\|$		$\|U_{e5}\|$	$\|U_{\mu5}\|$		
global data (MB475)	0.11	0.16	0.89	0.12	0.12	6.49	1.64π
global data (MB300)	0.12	0.18	0.87	0.11	0.089	1.91	1.44π

[Maltoni, Schwetz, arXiv:0705.0107 [hep-ph]]

Mini-BooNE and LSND fit "perfectly," including low-energy excess (MB300).

However, severely disfavored by disappearance data, especially if MB300 is included [$3\sigma - 4\sigma$ (?)].





NEUTRINOS HAVE MASS

albeit very tiny ones...

SO WHAT?

Only* "Palpable" Evidence of Physics Beyond the Standard Model

The SM we all learned in school predicts that neutrinos are strictly massless. Hence, massive neutrinos imply that the the SM is incomplete and needs to be replaced/modified.

Furthermore, the SM has to be replaced by something qualitatively different.

---

* There is only a handful of questions our model for fundamental physics cannot explain properly. These are in order of palpabiloity (these are personal. Feel free to complain)

- What is the physics behind electroweak symmetry breaking? (Higgs *or* not in SM).
- What is the dark matter? (not in SM).
- Why does the Universe appear to be accelerating? Why does it appear that the Universe underwent rapid acceleration in the past? (not in SM – Is this "particle physics?").

Standard Model in One Slide, No Equations

The SM is a quantum field theory with the following defining characteristics:

- Gauge Group ($SU(3)_c \times SU(2)_{\rm L} \times U(1)_{\rm Y}$);
- Particle Content (fermions: Q, u, d, L, e, scalars: H).

Once this is specified, the SM is unambiguously determined:

- Most General Renormalizable Lagrangian;
- Measure All Free Parameters, and You Are Done! (after several decades of hard experimental work. . .)

If you follow these rules, neutrinos have no mass. Something has to give.

What is the New Standard Model? [νSM]

The short answer is – WE DON'T KNOW. Not enough available info!

$$\Updownarrow$$

Equivalently, there are several completely different ways of addressing neutrino masses. The key issue is to understand what else the νSM candidates can do. [are they falsifiable?, are they "simple"?, do they address other outstanding problems in physics?, etc]

We need more experimental input, and it looks like it may be coming in the near/intermediate future!

The νSM – Take 1

SM as an effective field theory – non-renormalizable operators

$$\mathcal{L}_{\nu\mathrm{SM}} \supset -\lambda_{ij}\frac{L^iHL^jH}{2M} + \mathcal{O}\left(\frac{1}{M^2}\right) + H.c.$$

There is only one dimension five operator [Weinberg, 1979]. If $M \gg 1$ TeV, it leads to only one observable consequence...

$$\text{after EWSB } \mathcal{L}_{\nu\mathrm{SM}} \supset \frac{m_{ij}}{2}\nu^i\nu^j; \quad m_{ij} = \lambda_{ij}\frac{v^2}{M}.$$

- Neutrino masses are small: $M \gg v \to m_\nu \ll m_f$ ($f = e, \mu, u, d$, etc)
- Neutrinos are Majorana fermions – Lepton number is violated!
- νSM effective theory – not valid for energies above at most M.
- What is M? First naive guess is that M is the Planck scale – does not work. Data require $M < 10^{15}$ GeV (anything to do with the GUT scale?)

What else is this "good for"? Depends on the ultraviolet completion!

Note that this VERY similar to the "discovery" weak interactions.
Imagine the following model:

$U(1)_{E\&M} \ + \ e(q=-1),\ \mu(q=-1),\ \nu_e(q=0),\ \nu_\mu(q=0).$

The most general renormalizable Lagrangian explains *all* QED phenomena once all couplings are known (α, m_f).

New physics: the muon decays! $\mu^- \to e^-\bar{\nu}_e\nu_\mu$. This can be interpreted as evidence of effective four fermion theory (nonrenormalizable operators):

$$-\frac{4G_F}{\sqrt{2}}\sum_\gamma g_\gamma \left(\bar{e}\Gamma^\gamma\nu\right)\left(\bar{\nu}\Gamma_\gamma\mu\right), \qquad \Gamma_\gamma = 1, \gamma_5, \gamma_\mu, \ldots$$

Prediction: will discover new physics at an energy scale **below** $\sqrt{1/G_F} \simeq 250$ GeV. We know how this turned out $\Rightarrow$ $W^\pm, Z^0$ discovered slightly below 100 GeV!

Full disclosure:

All higher dimensional operators are completely negligible, **except** those that mediate proton decay, like:

$$\frac{\lambda_B}{M^2}QQQL$$

The fact that the proton does not decay forces M/λ_B to be much larger than the energy scale required to explain neutrino masses.

Why is that? We don't know...

νSM – Take 2

The Higgs sector could be more complicated.

We add to the SM a complex Higgs triplet $\xi = (\xi^{++}\ \xi^{+}\ \xi^{0})^T$, which can couple to lepton doublets

$$-\frac{\sqrt{2}}{2}\bar{\kappa}_{\alpha\beta}\overline{L^{c\alpha}}\cdot\xi L^{\beta} + \text{h.c.}.$$

If the neutral component of ξ develops a vacuum expectation value u, the neutrinos acquire a Majorana mass

$$\bar{m}_{\alpha\beta} \equiv u\bar{\kappa}_{\alpha\beta}.$$

Lots of questions

- Why is u so small?
- Where are these ξ (note that they also couple to charged leptons) $\rightarrow$ heavy fields.
- Other technical issues render proper realizations of this ugly.

The νSM – Take 3

Why don't we just enhance the fermion sector of the theory?

One may argue that it is trivial and simpler to just add

$$\mathcal{L}_{\rm Yukawa} = -y_{i\alpha} L^i H N^\alpha + H.c.,$$

and neutrinos get a mass like all other fermions: $m_{i\alpha} = y_{i\alpha} v$

- Data requires $y < 10^{-12}$. Why so small?
- Neutrinos are Dirac fermions. $B - L$ exactly conserved.
- νSM is a renormalizable theory.

This proposal, however, violates the rules of the SM (as I defined them)! The operator $\frac{M_N}{2} NN$, allowed by all gauge symmetries, is absent. In order to explain this, we are forced to add a symmetry to the νSM. The simplest candidate is a global $U(1)_{B-L}$.

$U(1)_{B-L}$ is upgraded from accidental to fundamental (global) symmetry.

Standard Model in One Slide, No Equations, Encore

The SM is a quantum field theory with the following defining characteristics:

- Gauge Group ($SU(3)_c \times SU(2)_{\rm L} \times U(1)_{\rm Y}$);
- Particle Content (fermions: Q, u, d, L, e, scalars: H).

Once this is specified, the SM is unambiguously determined:

- Most General Renormalizable Lagrangian;
- Measure All Free Parameters, and You Are Done.

This model has *accidental global symmetries*. In particular, the anomaly free global symmetry is preserved: $U(1)_{B-L}$.

New Standard Model, Dirac Neutrinos

The SM is a quantum field theory with the following defining characteristics:

- Gauge Group ($SU(3)_c \times SU(2)_{\rm L} \times U(1)_{\rm Y}$);
- Particle Content (fermions: Q, u, d, L, e, N, scalars: H);
- Global Symmetry $U(1)_{B-L}$.

Once this is specified, the SM is unambiguously determined:

- Most General Renormalizable Lagrangian;
- Measure All Free Parameters, and You Are Done.

Naively not too different, but nonetheless qualitatively different $\rightarrow$ enhanced symmetry sector!

On very small Yukawa couplings

We would like to believe that Yukawa couplings should naturally be of order one.

Nature, on the other hand, seems to have a funny way of showing this. Of all known fermions, only one (1) has a "natural" Yukawa coupling – the top quark!

Regardless there are several very different ways of obtaining "naturally" very small Yukawa couplings. They require the more new physics.

Example 1: Non-Anomalous, Gauged $U(1)_\nu$

Add to the SM a new, non-anomalous $U(1)_\nu$ under which both SM fermions and the right-handed neutrinos transform. Charges are heavily constrained by anomaly cancellations and the fact that quarks and charged leptons have relatively large masses.

One can choose $U(1)_\nu$ charges so that all neutrino masses are forbidden by gauge invariance. This way, neutrino masses are only generated after $U(1)_\nu$ is spontaneously broken,[a] and only through higher dimensional operators, suppressed by a new ultraviolet scale Λ.

Neutrino masses are small either because Λ is very large (this is the "usual" high energy seesaw) or because it is a consequence of a very high dimensional operator: $m_\nu \propto \left(\frac{\varphi}{\Lambda}\right)^{|p|}$, where p is a large integer exponent.

[a] Assume $U(1)_\nu$ is spontaneous broken when SM singlet scalar Φ gets a vev, $\langle\Phi\rangle \equiv \varphi$.

After $U(1)_\nu$ breaking → see-saw Lagrangian plus "left–left" neutrino mass:

$$\mathcal{L} \supset \sum_{ik} \epsilon^{|p_{ik}|} \bar{L}_i (\lambda^\nu)^{ik} n_k \tilde{H} + \sum_{ij} \epsilon^{|q_{ij}|} \bar{L}^c_i \frac{(h^L)^{ij}}{\Lambda} L_j H H + \sum_{kk'} \epsilon^{|r_{kk'}|} \Lambda \bar{n}^c_k (h^R)^{kk'} n_{k'},$$

λ^ν – neutrino Yukawa coupling, h^L – "left–left" coupling), and h^R – "right–right" Majorana mass term). $i, j = 1, 2, 3$, $k, k' = 1 \ldots N$. Only allowed for integer values of p, q, and r.

After electroweak symmetry breaking, $(3+N) \times (3+N)$ neutrino mass matrix M_ν:

$$M_\nu \sim \left(\begin{array}{c|c} \frac{v^2}{\Lambda} h^L \epsilon^{|q|} & v \lambda^\nu \epsilon^{|p|} \\ \hline v(\lambda^\nu \epsilon^{|p|})^\top & \Lambda h^R \epsilon^r \end{array} \right).$$

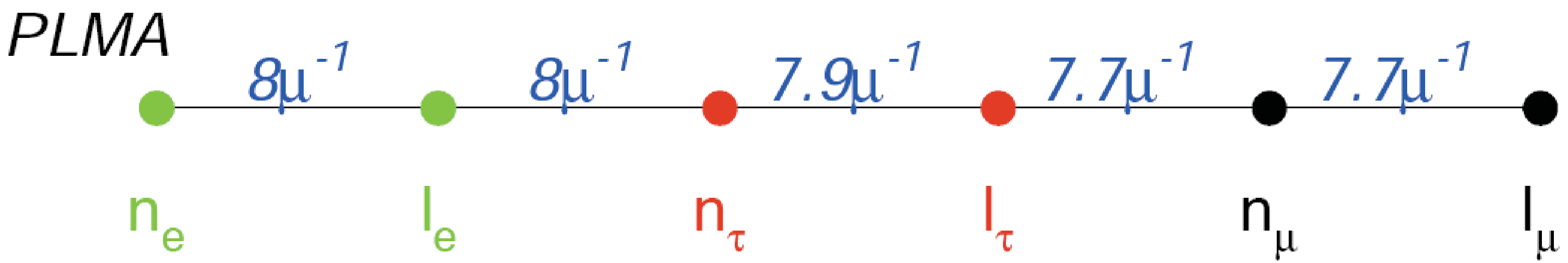
Lots of possibilities. If there are no integer q and r → Dirac Neutrinos, with suppressed masses ($m_\nu \propto v \epsilon^{|p|}$) [M.C. Chen, B. Dobrescu, AdG, work in progress]

Example 2: Extra-Dimensional Theories

- Large Extra Dimensions, with right-handed neutrinos in the bulk:

$$m_\nu = \lambda v \left(\frac{M_D}{M_{\rm Pl}}\right)^{\delta/d}$$

- Randall-Sundrum Models: left-handed, right-handed neutrinos live very close to the ultra-violet brane, the Higgs lives in the infra-red brane → small Yukawa couplings from very small small overlap of the 5-dimensional wave-functions (Grossman, Neubert).



Fat Branes: $\lambda \propto e^{-\frac{1}{2}\mu^2(f_i-f_j)^2}$, where f_i are the "positions" of the different fermion fields in the extra-dimension, all of width $1/\mu$.

easy to get very small, very hierarchical masses.
Tricky bit is the large mixing angles.

Barenboim, Branco, AdG, Rebelo, hep-ph/0104312

Massive Neutrinos and the Seesaw Mechanism

A simple[a], renormalizable Lagrangian that allows for neutrino masses is

$$\mathcal{L}_\nu = \mathcal{L}_{\rm old} - \lambda_{\alpha i} L^\alpha H N^i - \sum_{i=1}^{3} \frac{M_i}{2} N^i N^i + H.c.,$$

where N_i ($i = 1, 2, 3$, for concreteness) are SM gauge singlet fermions. $\mathcal{L}_\nu$ is the most general, renormalizable Lagrangian consistent with the SM gauge group and particle content, plus the addition of the N_i fields.

After electroweak symmetry breaking, $\mathcal{L}_\nu$ describes, besides all other SM degrees of freedom, six Majorana fermions: **six neutrinos.**

[a]Only requires the introduction of three fermionic degrees of freedom, no new interactions or symmetries.

To be determined from data: λ and M.

The data can be summarized as follows: there is evidence for three neutrinos, mostly "active" (linear combinations of ν_e, ν_μ, and ν_τ). At least two of them are massive and, if there are other neutrinos, they have to be "sterile."

This provides very little information concerning the magnitude of M_i (assume $M_1 \sim M_2 \sim M_3$)

Theoretically, there is prejudice in favor of very large M: $M \gg v$. Popular examples include $M \sim M_{\rm GUT}$ (GUT scale), or $M \sim 1$ TeV (EWSB scale).

Furthermore, $\lambda \sim 1$ translates into $M \sim 10^{14}$ GeV, while thermal leptogenesis requires the lightest M_i to be around 10^{10} GeV.

we can impose very, very few experimental constraints on M

What We Know About M:

- $M = 0$: the six neutrinos "fuse" into three Dirac states. Neutrino mass matrix given by $\mu_{\alpha i} \equiv \lambda_{\alpha i} v$.

  The symmetry of $\mathcal{L}_\nu$ is enhanced: $U(1)_{B-L}$ is an exact global symmetry of the Lagrangian if all M_i vanish. Small M_i values are 'tHooft natural.

- $M \gg \mu$: the six neutrinos split up into three mostly active, light ones, and three, mostly sterile, heavy ones. The light neutrino mass matrix is given by $m_{\alpha\beta} = \sum_i \lambda_{\alpha i} M_i^{-1} \lambda_{\beta i}$.

  This the **seesaw mechanism.** Neutrinos are Majorana fermions. Lepton number is not a good symmetry of $\mathcal{L}_\nu$, even though L-violating effects are hard to come by.

- $M \sim \mu$: six states have similar masses. Active–sterile mixing is very large. This scenario is (generically) ruled out by active neutrino data (atmospheric, solar, KamLAND, K2K, etc).

High-energy seesaw has no other observable consequences, except, perhaps, ...

Baryogenesis via Leptogenesis

One of the most basic questions we are allowed to ask (with any real hope of getting an answer) is whether the observed baryon asymmetry of the Universe can be obtained from a baryon–antibaryon symmetric initial condition plus well understood dynamics. [Baryogenesis]

This isn't just for aesthetic reasons. If the early Universe undergoes a period of inflation, baryogenesis is required, as inflation would wipe out any pre-existing baryon asymmetry.

It turns out that massive neutrinos can help solve this puzzle!

In the old SM, (electroweak) baryogenesis does not work – not enough CP-invariance violation, Higgs boson too light.

Neutrinos help by providing all the necessary ingredients for successful baryogenesis via leptogenesis.

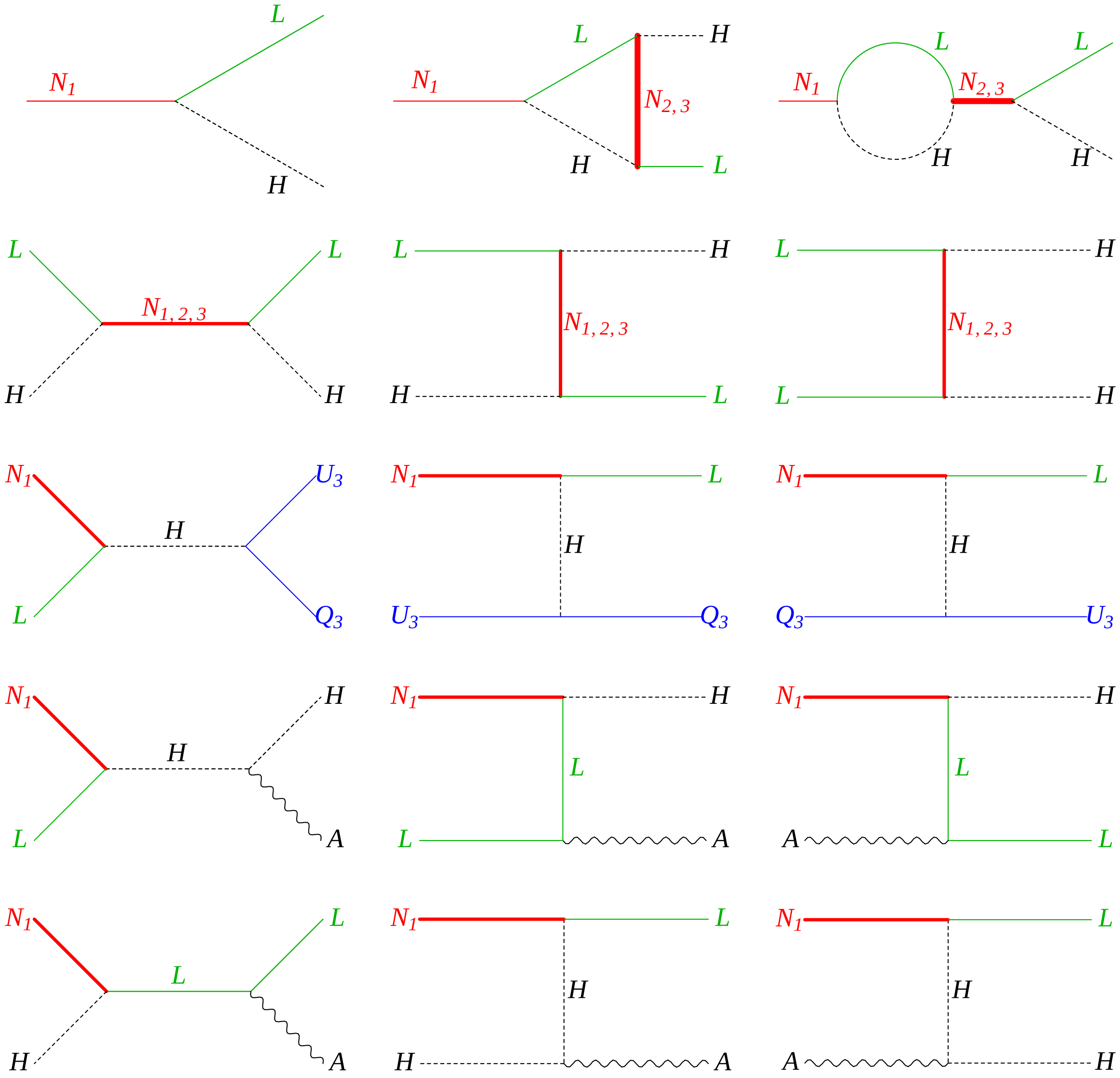
- Violation of lepton number, which later on is transformed into baryon number by nonperturbative, finite temperature electroweak effects (in one version of the νSM, lepton number is broken at a high energy scale M).

- Violation of C-invariance and CP-invariance (weak interactions, plus new CP-odd phases).

- Deviation from thermal equilibrium (depending on the strength of the relevant interactions).

E.g. – thermal, seesaw leptogenesis, $\boxed{\mathcal{L} \supset -y_{i\alpha} L^i H N^\alpha - \frac{M_N^{\alpha\beta}}{2} N_\alpha N_\beta + H.c.}$

[Fukugita, Yanagida]

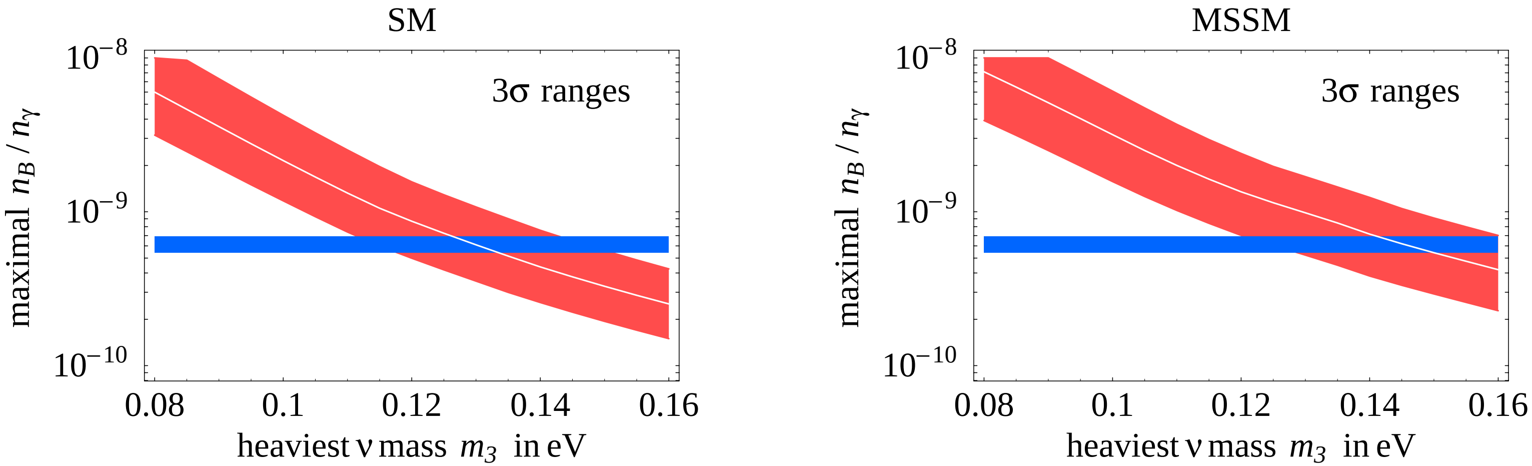


- L-violating processes
- $y \Rightarrow$ CP-violation
- deviation from thermal eq. constrains combinations of M_N and y.
- need to yield correct m_ν

not trivial!

[G. Giudice *et al*, hep-ph/0310123]

E.g. – thermal, seesaw leptogenesis, $\boxed{\mathcal{L} \supset -y_{i\alpha} L^i H N^\alpha - \frac{M_N^{\alpha\beta}}{2} N_\alpha N_\beta + H.c.}$



[G. Giudice *et al*, hep-ph/0310123]

It did not have to work – but it does

MSSM picture does not quite work – gravitino problem

(there are ways around it, of course...)

Relationship to Low Energy Observables?

In general ... no. This is very easy to understand. The baryon asymmetry depends on the (high energy) physics responsible for lepton-number violation. Neutrino masses are a (small) consequence of this physics, albeit the only observable one at the low-energy experiments we can perform nowadays.

see-saw: y, M_N have more physical parameters than $m_\nu = y^{\rm t} M_N^{-1} y$.

There could be a relationship, but it requires that we know more about the high energy Lagrangian (model depent). The day will come when we have enough evidence to refute leptogenesis (or strongly suspect that it is correct) - but more information of the kind I mentioned earlier is really necessary (charged-lepton flavor violation, collider data on EWSB, lepton-number violation, etc).

(There are other "kinds" of leptogenesis, of which I'll say nothing

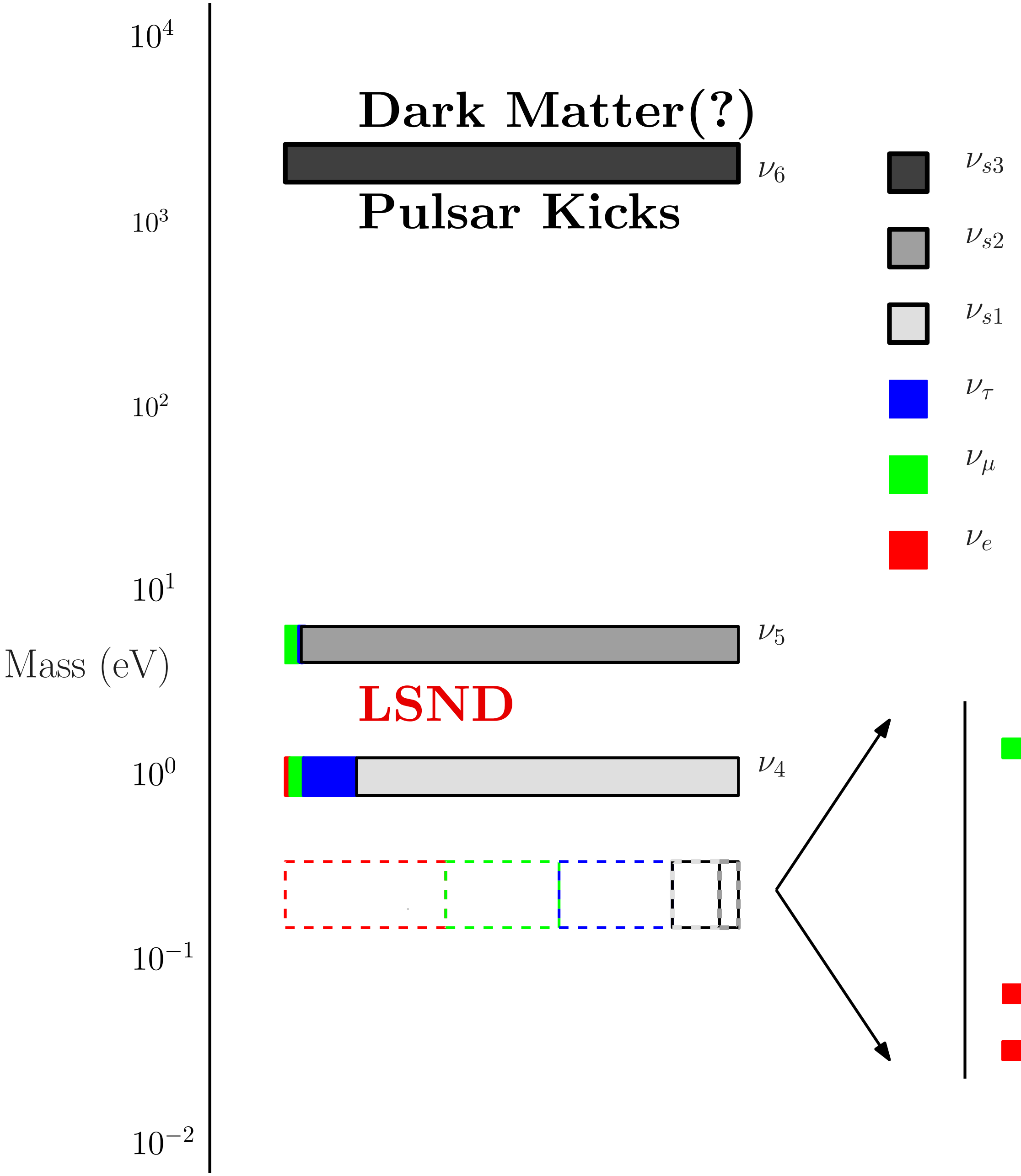
- Nonthermal leptogenesis
- Type-II see-saw leptogenesis
- Dirac leptogenesis — Lindner *et al*; Murayama and Pierce
- Soft leptogenesis — Grossman *et al*; Giudice *et al.*
- …

)

Low-Energy Seesaw [AdG PRD72,033005)]

Lets peek in the other end of the M spectrum. What do we get?

- Neutrino masses are small because the Yukawa couplings are very small $\lambda \in [10^{-6}, 10^{-11}]$;
- No standard thermal leptogenesis – right-handed neutrino way too light;
- No obvious connection with other energy scales (EWSB, GUTs, etc);
- Right-handed neutrinos are propagating degrees of freedom. They look like sterile neutrinos;
- Sterile–active mixing can be predicted – hypothesis is falsifyable.
- Sterile neutrinos could be Nature's answer to "all" our puzzles!
- Small values of M are natural (in the 'tHooft sense). In fact, theoretically, no value of M should be discriminated against!



Also effects in $0\nu\beta\beta$,
tritium beta-decay,
supernova neutrino oscillations,
NEEDS non-standard cosmology.

Understanding Fermion Mixing

The other puzzling phenomenon uncovered by the neutrino data is the fact that Neutrino Mixing is Strange. What does this mean?

It means that lepton mixing is very different from quark mixing:

$$V_{MNS} \sim \begin{pmatrix} 0.8 & 0.5 & \mathbf{0.2} \\ 0.4 & 0.6 & 0.7 \\ 0.4 & 0.6 & 0.7 \end{pmatrix} \qquad V_{CKM} \sim \begin{pmatrix} 1 & 0.2 & 0.001 \\ 0.2 & 1 & 0.01 \\ 0.001 & 0.01 & 1 \end{pmatrix}$$

$[|(V_{MNS})_{e3}| < 0.2]$

$\boxed{\text{WHY?}}$

They certainly look VERY different, but which one would you label as "strange"?

In the quark sector, the small mixing angles are interpreted, together with the hierarchical quark masses, as evidence for extra structure in the SM, i.e., there is some underlying dynamical principle (symmetry) capable of telling one quark flavor from another.

The same "must be true" in the leptonic sector. After all, charged lepton masses are also hierarchical (we don't know whether the same is true for the neutrinos yet...) and, if GUTs have anything to do with Nature, quarks and leptons may well be different low-energy manifestations of a more fundamental unified fermion.

Hence, there should also be a dynamical principle which naturally explains the form of the MNS matrix. (or should there?...)

First Prediction: $V_{CKM} \simeq V_{MNS}$

$\rightarrow$ "driving force" before 1998 SK results, turned out to be completely wrong.

Reference	$\sin\theta_{13}$	$\sin^2 2\theta_{13}$
SO(10)		
Goh, Mohapatra, Ng [40]	0.18	0.13
Orbifold SO(10)		
Asaka, Buchmüller, Covi [41]	0.1	0.04
SO(10) + flavor symmetry		
Babu, Pati, Wilczek [42]	$5.5 \cdot 10^{-4}$	$1.2 \cdot 10^{-6}$
Blazek, Raby, Tobe [43]	0.05	0.01
Kitano, Mimura [44]	0.22	0.18
Albright, Barr [45]	0.014	$7.8 \cdot 10^{-4}$
Maekawa [46]	0.22	0.18
Ross, Velasco-Sevilla [47]	0.07	0.02
Chen, Mahanthappa [48]	0.15	0.09
Raby [49]	0.1	0.04
SO(10) + texture		
Buchmüller, Wyler [50]	0.1	0.04
Bando, Obara [51]	0.01 .. 0.06	$4 \cdot 10^{-4}$ .. 0.01
Flavor symmetries		
Grimus, Lavoura [52, 53]	0	0
Grimus, Lavoura [52]	0.3	0.3
Babu, Ma, Valle [54]	0.14	0.08
Kuchimanchi, Mohapatra [55]	0.08 .. 0.4	0.03 .. 0.5
Ohlsson, Seidl [56]	0.07 .. 0.14	0.02 .. 0.08
King, Ross [57]	0.2	0.15
Textures		
Honda, Kaneko, Tanimoto [58]	0.08 .. 0.20	0.03 .. 0.15
Lebed, Martin [59]	0.1	0.04
Bando, Kaneko, Obara, Tanimoto [60]	0.01 .. 0.05	$4 \cdot 10^{-4}$ .. 0.01
Ibarra, Ross [61]	0.2	0.15
3×2 *see-saw*		
Appelquist, Piai, Shrock [62, 63]	0.05	0.01
Frampton, Glashow, Yanagida [64]	0.1	0.04
Mei, Xing [65] (normal hierarchy)	0.07	0.02
(inverted hierarchy)	> 0.006	$> 1.6 \cdot 10^{-4}$
Anarchy		
de Gouvêa, Murayama [66]	> 0.1	> 0.04
Renormalization group enhancement		
Mohapatra, Parida, Rajasekaran [67]	0.08 .. 0.1	0.03 .. 0.04

Table 1: Incomplete selection of predictions for θ_{13}. The numbers should be considered as order of magnitude statements.

$\Delta m^2_{13} > 0$ "typical" prediction of all* Type-I see-saw GUT models

inverted hierarchy requires* "more flavor structure"

* Albright, hep-ph/0407155

[from reactor white paper]

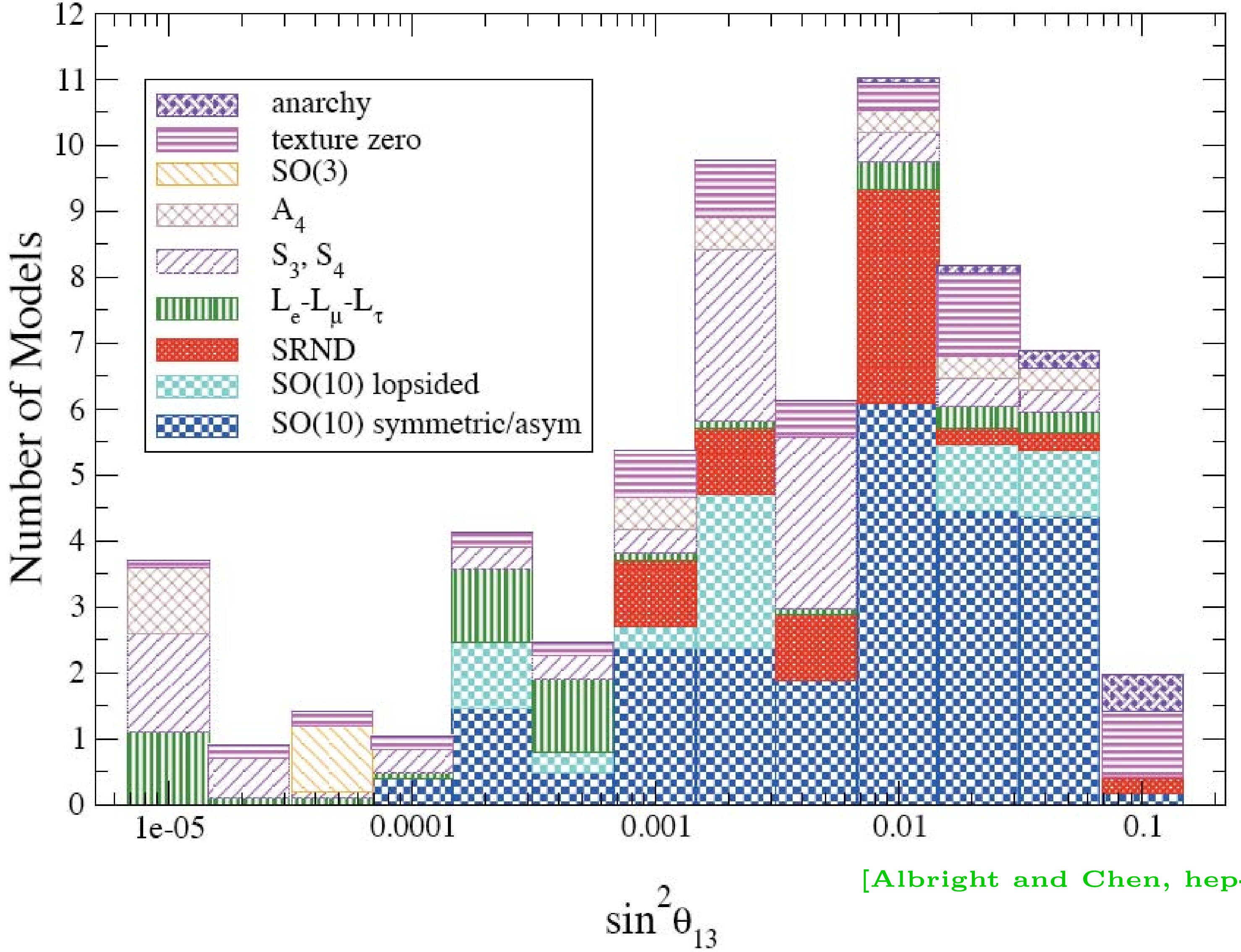
Theoretical predictions:

The literature on this subject is very large. The most exciting driving force (my opinion) is the fact that one can make *bona fide* predictions:

$\Rightarrow U_{e3}$, CP-violation, mass-hierarchy unknown!

Unfortunately, theorists have done too good a job, and people have successfully predicted everything. . .

More data needed to "sort things out."



[Albright and Chen, hep-ph/0608137]

pessimist – "We can't compute what $|U_{e3}|$ is – must measure it!"

(same goes for the mass hierarchy, δ)

Something Completely Different (?) –

maybe we are asking the wrong question! Notice that quark mixing is the one that fits the "strange" label → this is why we are convinced that there is some "hint" of more fundamental physics hidden in the CKM matrix!

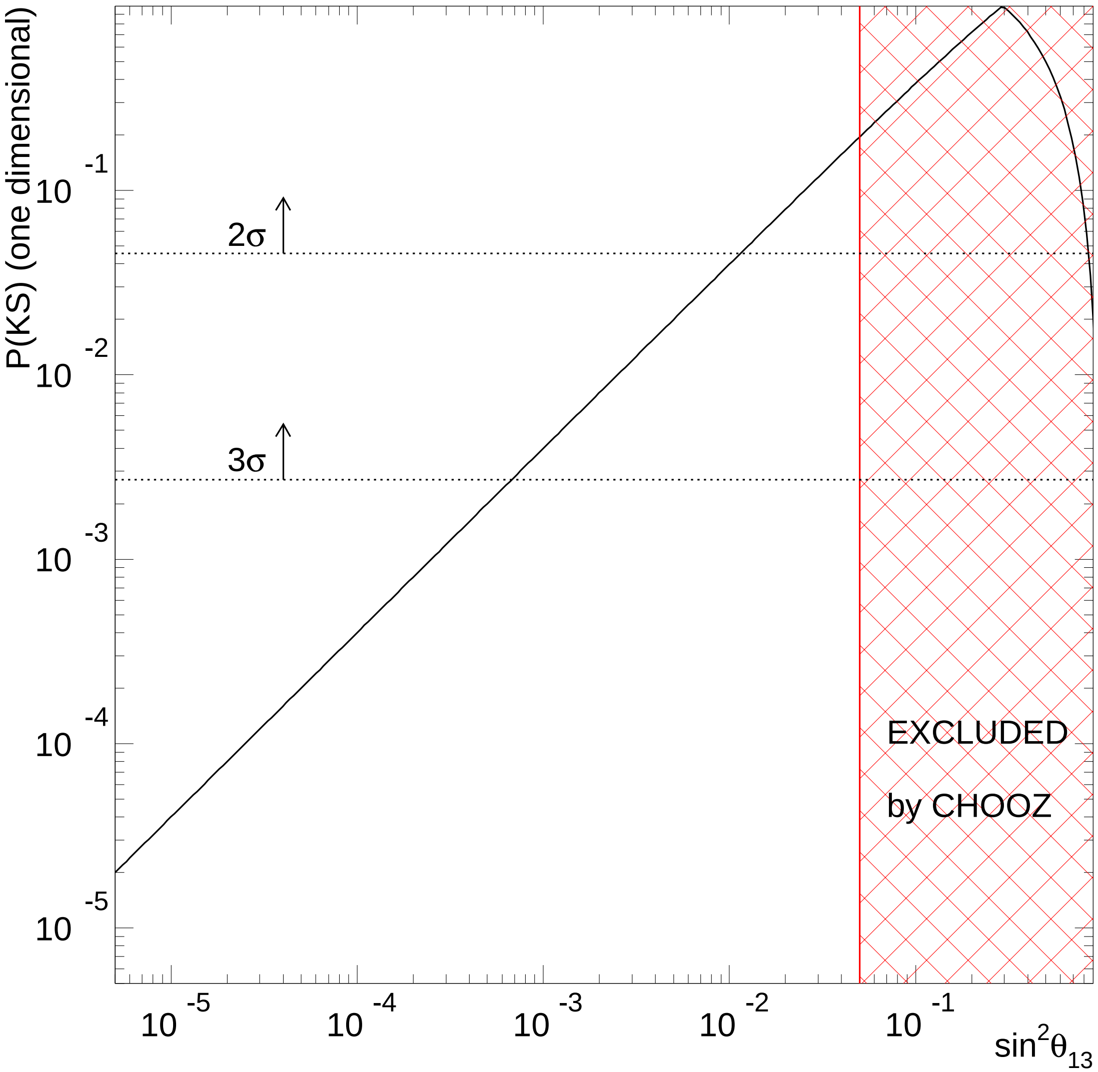
Lepton mixing, on the other hand, seems quite "ordinary." Maybe the MNS matrix is what one should expect if there was no fundamental principle "hidden" behind neutrino mixing. → *Neutrino Mass Anarchy*

Anarchy is resistant against hierarchical charged lepton masses, GUT constraints. The relevant questions are 1-can we test whether the idea is plausible and 2-can we learn anything from it? (yes, and yes!)

My only complaint is the fact that θ_{23} is maximal. But "when" should we start worrying about this? (to be discussed in the next-to-next slide)

Lower Bound for $|U_{e3}|^2$

[AdG, Murayama, PLB573, 94 (2003)]



according to the anarchical hypothesis, the probability density distribution for θ_{13} is given by $P(\cos^4\theta_{13}) \propto 1$

[Haba, Murayama, PRD63,053010 (2001)]

The probability that $|U_{e3}|^2$ is larger than 0.01 is around 95%, and if $|U_{e3}|^2$ turns out to be smaller, the anarchical hypothesis is "ruled out"!

(Prob. distribution for CP-phase: $P(\delta) \propto 1$)

generic predictions for subleading parameters. Note correlations between $|U_{e3}|$ and $\cos 2\theta_{23}$, plus dependency on mass-hierarchy.

Case	Texture	Hierarchy	$\lvert U_{e3}\rvert$	$\lvert\cos 2\theta_{23}\rvert$ (n.s.)	$\lvert\cos 2\theta_{23}\rvert$	Solar Angle
A	$\frac{\sqrt{\Delta m^2_{13}}}{2}\begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix}$	Normal	$\sqrt{\frac{\Delta m^2_{12}}{\Delta m^2_{13}}}$	O(1)	$\sqrt{\frac{\Delta m^2_{12}}{\Delta m^2_{13}}}$	O(1)
B	$\sqrt{\Delta m^2_{13}}\begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{2} & -\frac{1}{2} \\ 0 & -\frac{1}{2} & \frac{1}{2} \end{pmatrix}$	Inverted	$\frac{\Delta m^2_{12}}{\lvert\Delta m^2_{13}\rvert}$	–	$\frac{\Delta m^2_{12}}{\lvert\Delta m^2_{13}\rvert}$	O(1)
C	$\frac{\sqrt{\Delta m^2_{13}}}{\sqrt{2}}\begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}$	Inverted	$\frac{\Delta m^2_{12}}{\lvert\Delta m^2_{13}\rvert}$	O(1)	$\frac{\Delta m^2_{12}}{\lvert\Delta m^2_{13}\rvert}$	$\lvert\cos 2\theta_{12}\rvert \sim \frac{\Delta m^2_{12}}{\lvert\Delta m^2_{13}\rvert}$
Anarchy	$\sqrt{\Delta m^2_{13}}\begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$	Normal[a]	> 0.1	O(1)	–	O(1)

[a]One may argue that the anarchical texture prefers but does not require a normal mass hierarchy.

[enlarged from AdG, PRD69, 093007 (2004)]

What About Maximal Atmospheric Mixing?

"Textures" are another way to parametrize neutrino mixing and to try and understand salient features: $|U_{e3}| \ll 1$, $\cos 2\theta_{23} \ll 1$, $\Delta m^2_{12} \ll \Delta m^2_{13}$, etc. Usually "quark independent."

How Do We Learn More?

In order to learn more, we need more information. Any new data and/or idea is welcome, including

- searches for charged lepton flavor violation ($\mu \to e\gamma$, etc);
- searches for lepton number violation (neutrinoless double beta decay, etc);
- precision measurements of the neutrino oscillation parameters;
- searches for fermion electric/magnetic dipole moments (electron edm, muon $g-2$, etc);
- searches for new physics at the TeV scale – we need to understand the physics at the TeV scale before we can really claim to understand the physics behind neutrino masses (is there low-energy SUSY?, etc).

CONCLUSIONS

The venerable Standard Model has finally sprung a leak – neutrinos are not massless!

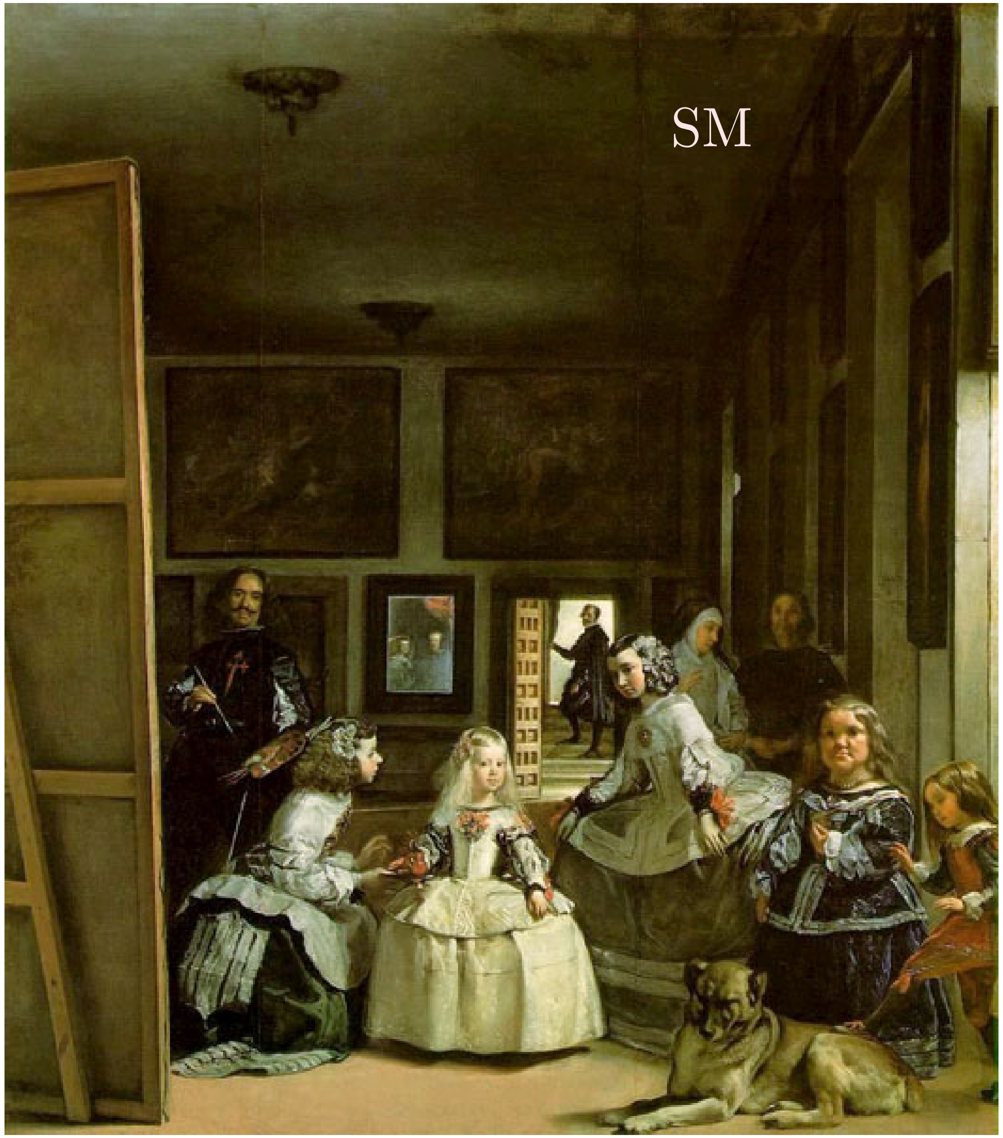
1. we have a very successful parametrization of the neutrino sector, and we have identified what we know we don't know.
2. neutrino masses are very small – we don't know why, but we think it means something important.
3. lepton mixing is very different from quark mixing – we don't know why, but we think it means something important.
4. we need a minimal νSM Lagrangian. In order to decide which one is "correct" (required in order to attack 2. and 3. above) we must uncover the faith of baryon number minus lepton number ($0\nu\beta\beta$ is the best [only?] bet).

5. We need more experimental input – and more seems to be on the way (this is a truly data driven field right now). We only started to figure out what is going on.

6. The fact that neutrinos have mass may be intimately connected to the fact that there are more baryons than antibaryons in the Universe. How do we test whether this is correct?

7. There is plenty of room for surprises, as neutrinos are very narrow but deep probes of all sorts of physical phenomena. Remember that neutrino oscillations are "quantum interference devices" – potentially very sensitive to whatever else may be out there (e.g., $M_{\rm seesaw} \simeq 10^{14}$ GeV).

neutrinos have modified our picture of the fundamental physics world



$\Rightarrow$
$\Rightarrow$
$\Rightarrow$
$\Rightarrow$
$\Rightarrow$
$\Rightarrow$

(with apologies to D. Velásquez)

(with apologies to P. Picasso)

BACK-UP MATERIAL:

- Measuring the deviation of the atmospheric mixing from maximal (is $\theta_{23} \neq \pi/4?$);
- How do you determine the neutrino mass hierarchy if θ_{13} turns out to be too small?;
- On the LSND anomaly, before the Mini-BooNE results were announced.

On measuring $\sin^2\theta_{23}$ (the atmospheric mixing angle)

More specifically, we would like to ask whether it is possible to determine:

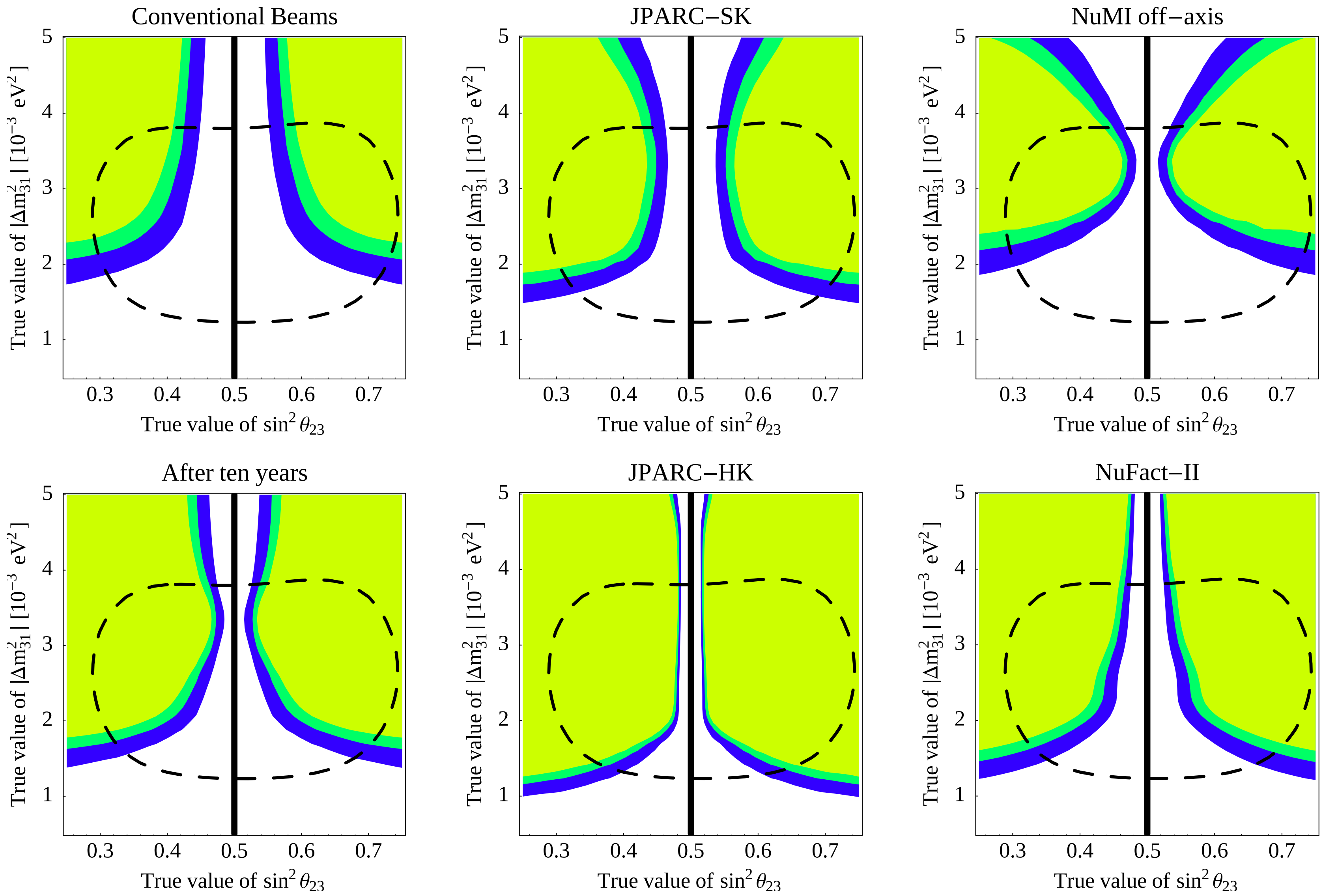
1. Is it maximal ($\sin^2\theta_{23} = 1/2$)?
2. Is $\sin^2\theta_{23} > 1/2$ or $\sin^2\theta_{23} < 1/2$?

Limited information regarding (2) from disappearance channel. — $P_{\mu\mu} \propto \sin^2 2\theta_{23}$. Simply adding $P_{\mu e} \propto \sin^2\theta_{23}\sin^2 2\theta_{13}$ does not help!

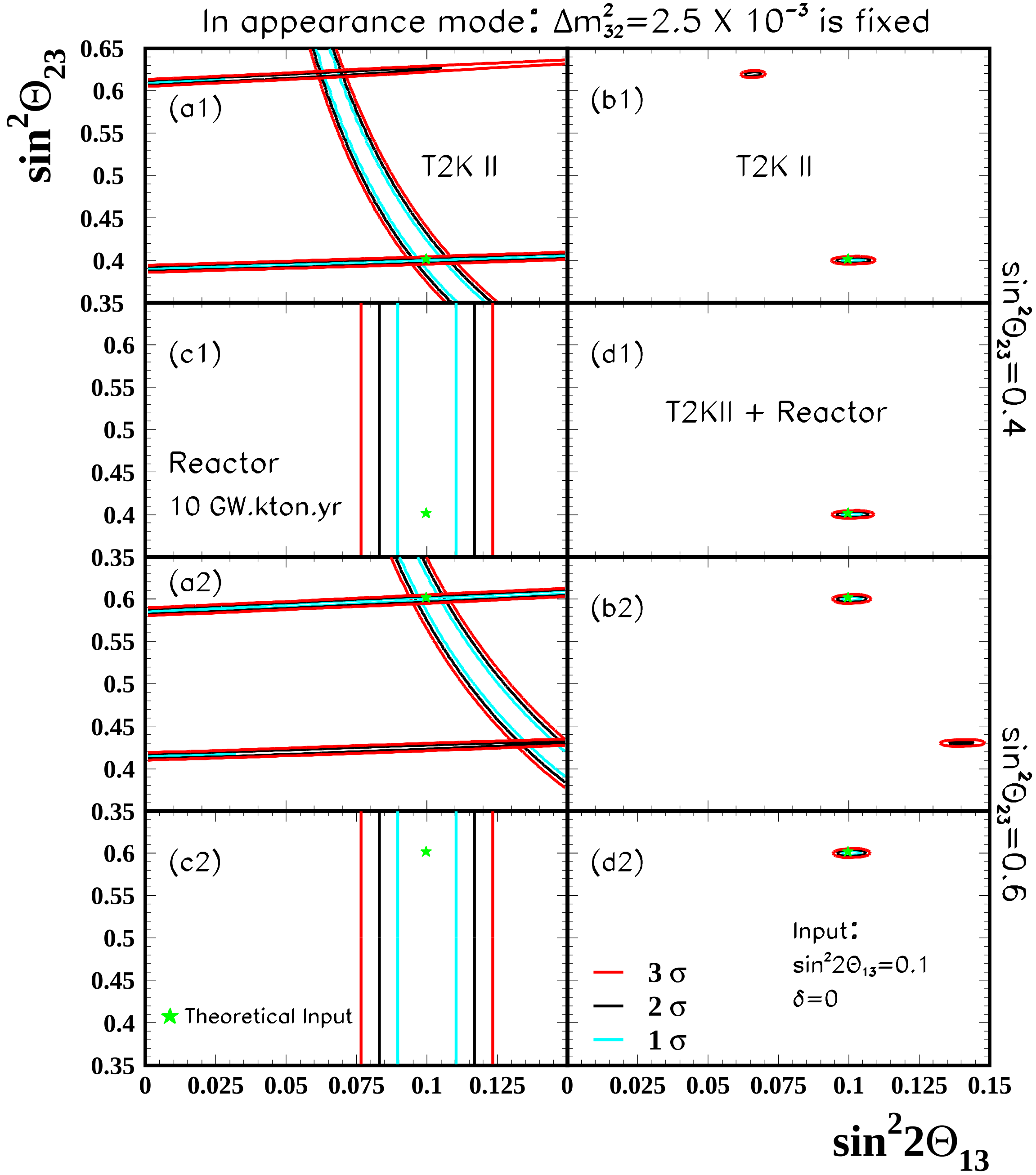
$$P_{\mu\mu} = 1 - \sin^2 2\theta_{23}\sin^2\left(\frac{\Delta m^2_{13}L}{4E}\right) + \text{subleading}.$$

In order to resolve this issue, need more information from reactors, atmospheric neutrinos, $P_{e\tau} \propto \cos^2\theta_{23}$ (which required τ appearance and is beyond the reach of "standard" next-generation LBL experiments – usually requires Neutrino Factory).

Deciding that θ_{23} is not maximal with LBL experiments



Antusch *et al.*, PRD70, 097302 (2004).



⇐ Appearance + Disappearance

not Enough

⇐ Reactors Can Resolve Degeneracy

Hiraide *et al.*, hep-ph/0601258

Determining the Mass Hierarchy via Oscillations – vanishing U_{e3} route

hep-ph/0503079, hep-ph/0507021, hep-ph/0509359

In the case of two-flavors, the "mass-hierarchy" can only be determined in the presence of matter effects: vacuum neutrino oscillations are not sensitive to the mass hierarchy.

In the case of three-flavors, this is not the case: vacuum neutrino oscillation probabilities are sensitive to the neutrino mass hierarchy. This does not depend on whether U_{e3} vanishes or not.

How does one compare the two mass hierarchies and determines which one is correct?

The question I address is the following:

For a positive choice of $\Delta m^2_{13} = \Delta m^{2+}_{13}$, is there a negative choice for $\Delta m^2_{13} = \Delta m^{2-}_{13}$ that yields identical oscillation probabilities?

If the answer is 'yes,' then one cannot tell one mass hierarchy from the other. If the answer is 'no,' then one can, in principle, distinguish the two possibilities.

More concretely: fix Δm^{2+}_{13} (which I'll often refer to as Δm^2_{13}) and define x so that

$$\Delta m^{2-}_{13} = -\Delta m^{2+}_{13} + x.$$

Question: Is there a value of x that renders $P(\Delta m^{2+}_{13}) = P(\Delta m^{2-}_{13})$?

Note: x is such that Δm^2_{13} is negative. It turns out that x's that almost do the job are of order Δm^2_{12}.

I will concentrate on survival probabilities (which will be the only relevant ones in the $U_{e3} \to 0$ limit):

$$\begin{aligned} P_{\alpha\alpha} = 1 &- 4|U_{\alpha 1}|^2|U_{\alpha 2}|^2 \sin^2\left(\frac{\Delta_{12}L}{2}\right) \\ &- 4|U_{\alpha 1}|^2|U_{\alpha 3}|^2 \sin^2\left(\frac{\Delta_{13}L}{2}\right) \\ &- 4|U_{\alpha 2}|^2|U_{\alpha 3}|^2 \sin^2\left(\frac{\Delta_{23}L}{2}\right), \end{aligned}$$

$\Delta_{ij} \equiv \Delta m^2_{ij}/2E$. Note that $\Delta_{23} = \Delta_{13} - \Delta_{12}$.

It is easy to see how the different hierarchies lead to different results. In the normal case, $|\Delta_{13}| > |\Delta_{23}|$, while in the inverted case $|\Delta_{13}| < |\Delta_{23}|$. Hence, "all" one needs to do is establish which frequency is associated to which amplitude (governed by the $U_{\alpha i}$'s).

More detail:

$$P^+_{\alpha\alpha} - P^-_{\alpha\alpha} = -4|U_{\alpha 3}|^2 \left\{ |U_{\alpha 1}|^2 \left[\sin^2\left(\frac{\Delta_{13}L}{2}\right) - \sin^2\left(\frac{(\Delta_{13}-X)L}{2}\right)\right] + |U_{\alpha 2}|^2 \left[\sin^2\left(\frac{(\Delta_{13}-\Delta_{12})L}{2}\right) - \sin^2\left(\frac{(\Delta_{13}+\Delta_{12}-X)L}{2}\right)\right]\right\},$$

$X = x/2E$.

There is no choice of x that renders this zero for all L and E,

unless (i) $|U_{\alpha 2}|^2 = |U_{\alpha 1}|^2$ (known not to happen) or (ii) $\Delta_{12} = 0$ (also does not happen) or (iii) one of the $U_{\alpha i}$'s vanishes (could happen in the case of P_{ee}).

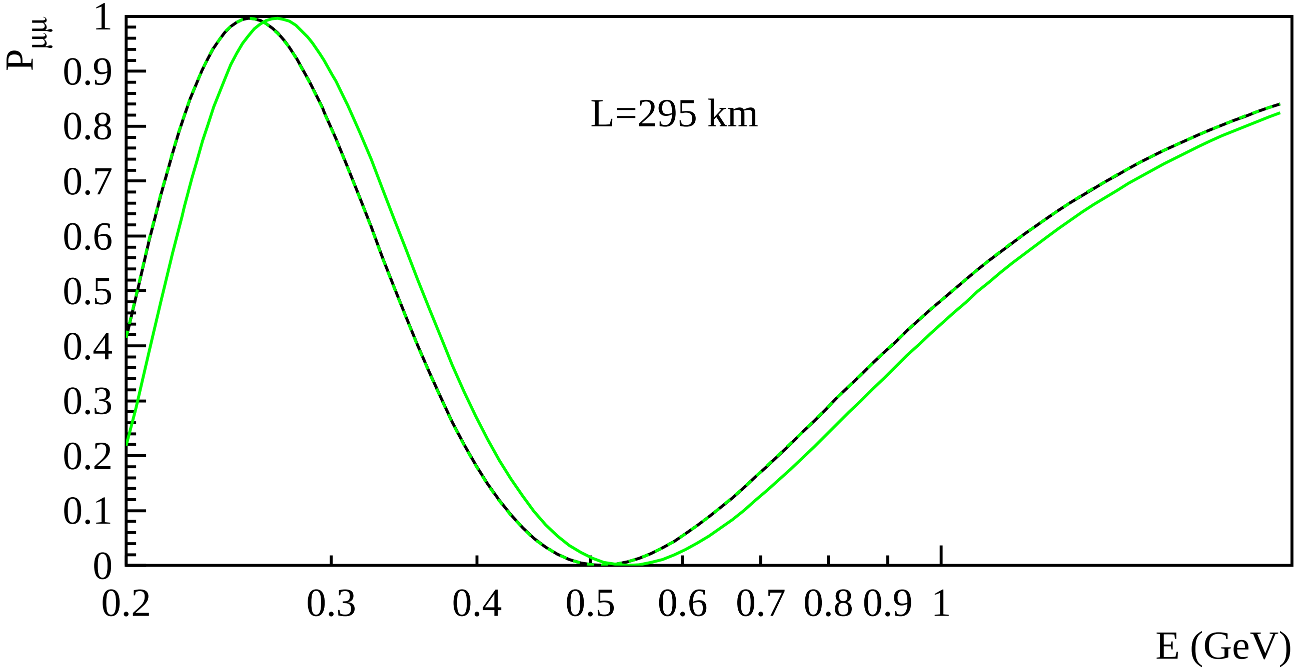
Life is not this simple. Most experimental set-ups looking for U_{e3} effects concentrate on L and E so that $\Delta_{13}L \sim 1$. This means that $\Delta_{12}L \ll 1$.

It turns out that

$$x = \frac{2|U_{\alpha 2}|^2}{|U_{\alpha 1}|^2 + |U_{\alpha 2}|^2}\Delta m^2_{12},$$

renders $P^+_{\alpha\alpha} - P^-_{\alpha\alpha} = \mathcal{O}(\Delta_{12}L)^2$.

There are two ways around this problem. One is to make sure you consider large $\Delta_{12}L$ values. The other is to note that different α's yeild different values of x. Both are very, very challenging...



$\Delta m^2_{13} = +2.20 \times 10^{-3}$ eV^2

$\Delta m^2_{13} = -2.20 \times 10^{-3}$ eV^2 (s)

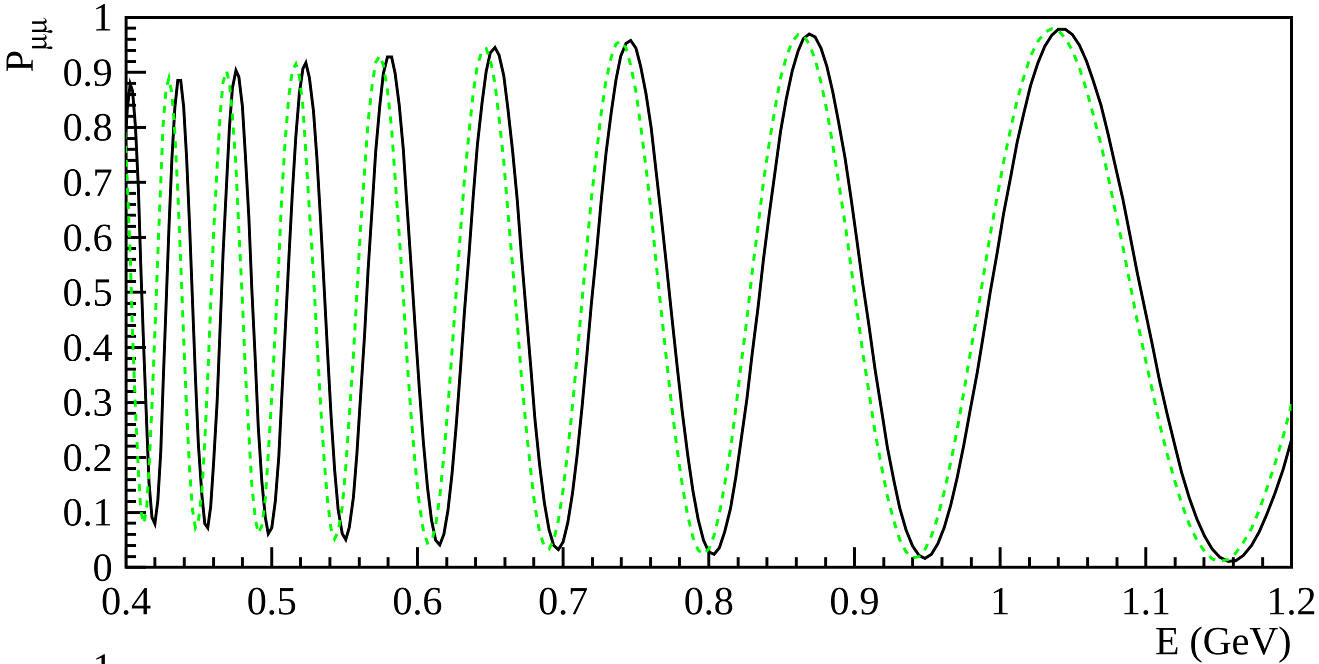
$\Delta m^2_{13} = -2.08 \times 10^{-3}$ eV^2 (d)

$\Delta m^2_{12} = 8.2 \times 10^{-5}$ eV^2,

$\sin^2 2\theta_{12} = 0.83$, $\sin^2 2\theta_{23} = 1$

The small $\Delta_{12}L$ problem: in this case $x = 2\Delta m^2_{12} \cos^2\theta_{12}$ ($= 1.16 \times 10^{-4}$ eV^2).

This would be the situation at a "short" baseline experiment: even with quasi-infinite statistics one would still end up with two different values of Δm^2_{13}, one for each hierarchy hypothesis.



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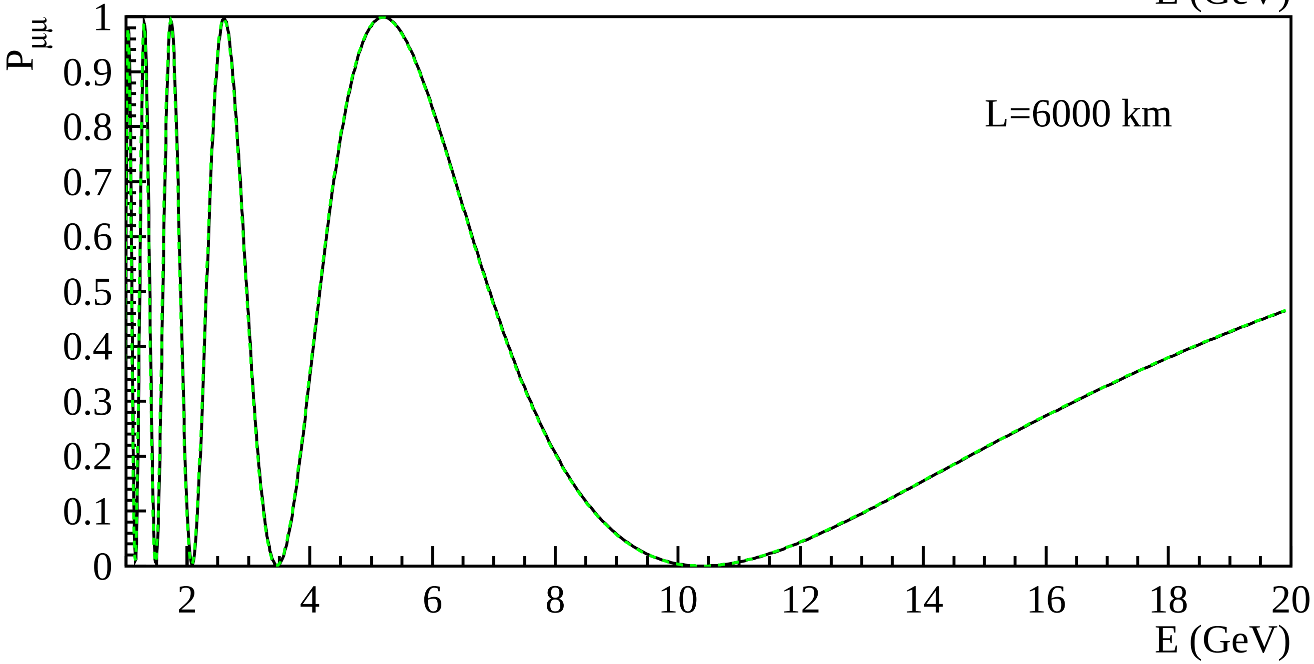
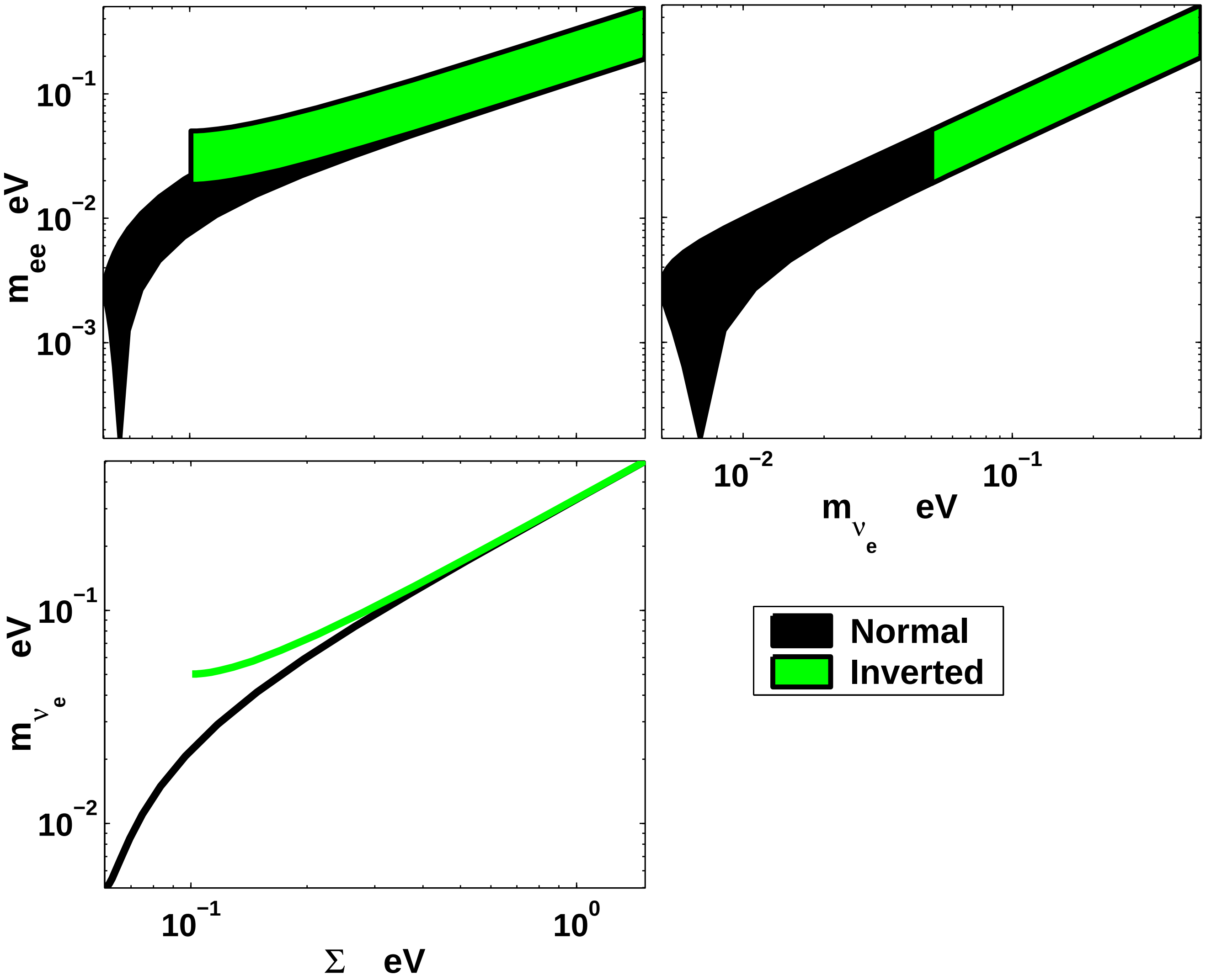


There is hope! But can we "see" the fast oscillations at low energies?

Other Tool? – Non-Oscillation Experiments

$(U_{e3} = 0,\ \Delta m^{2+}_{13} = +2.50 \times 10^{-3}\ \text{eV}^2,\ \Delta m^{2-}_{13} = -2.44 \times 10^{-3}\ \text{eV}^2)$



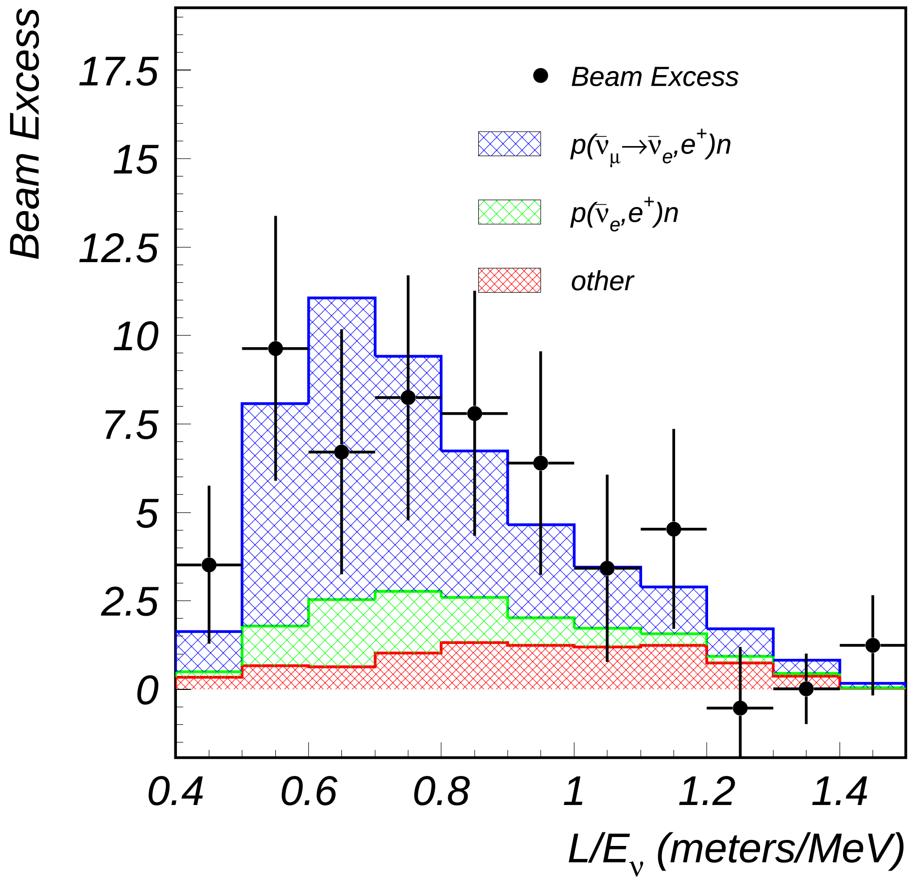
$$\Sigma = m_1 + m_2 + m_3$$

$$m^2_{\nu_e} \equiv \sum_i |U_{ei}|^2 m_i^2$$

$$m_{ee} \equiv \sum_i U_{ei}^2 m_i \equiv m_1 |U_{e1}|^2 e^{i\alpha_1} + m_2 |U_{e2}|^2 e^{i\alpha_2} + m_3 |U_{e3}|^2 e^{-2i\delta}$$

[LSND Anomaly Before Mini-BooNE]

strong evidence for $\bar{\nu}_\mu \to \bar{\nu}_e$



If oscillations $\Rightarrow \Delta m^2 \sim 1$ eV^2

× does not fit into 3 ν picture;

× $2+2$ scheme ruled out (solar, atm);

○? $3+1$ scheme disfavored (sbl searches);

× 3 ν's CPTV ruled out (KamLAND, atm);

× $\mu \to e\nu_e\bar{\nu}_e$ ruled out (KARMEN, TWIST);

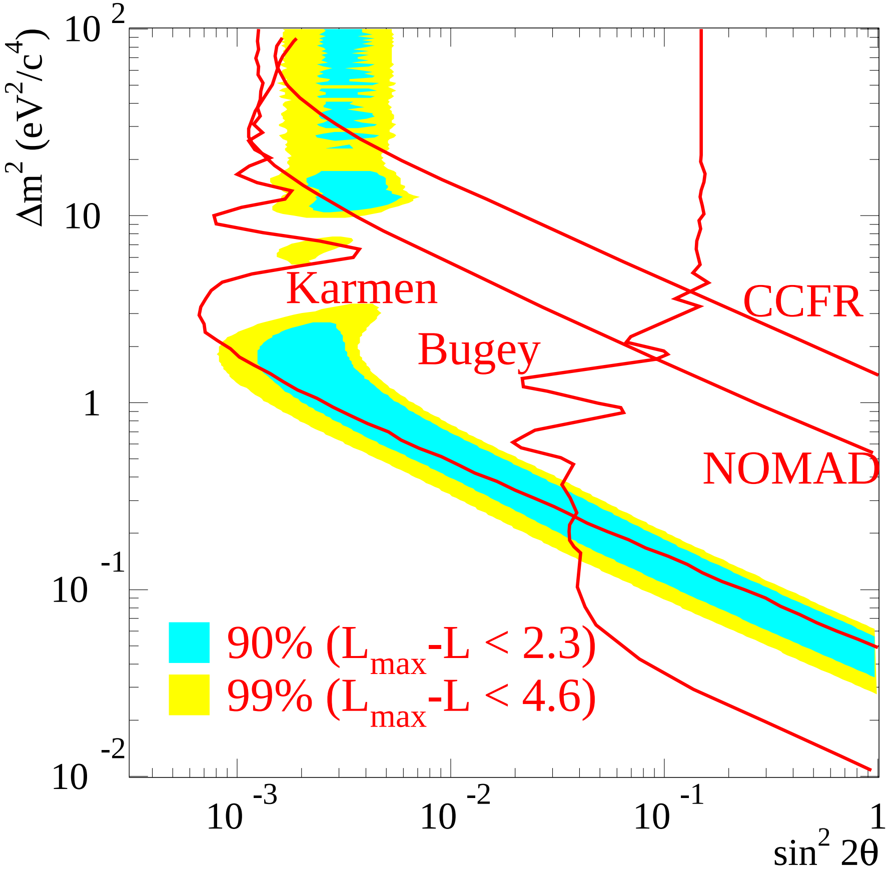
○ $3+1+1$ scheme works (finely tuned?);

○ 4 ν's CPTV

○ "heavy" decaying sterile neutrinos;

○? 3 νs and Lorentz-invariance violation;

○ something completely different.



Karmen has a similar sensitivity to

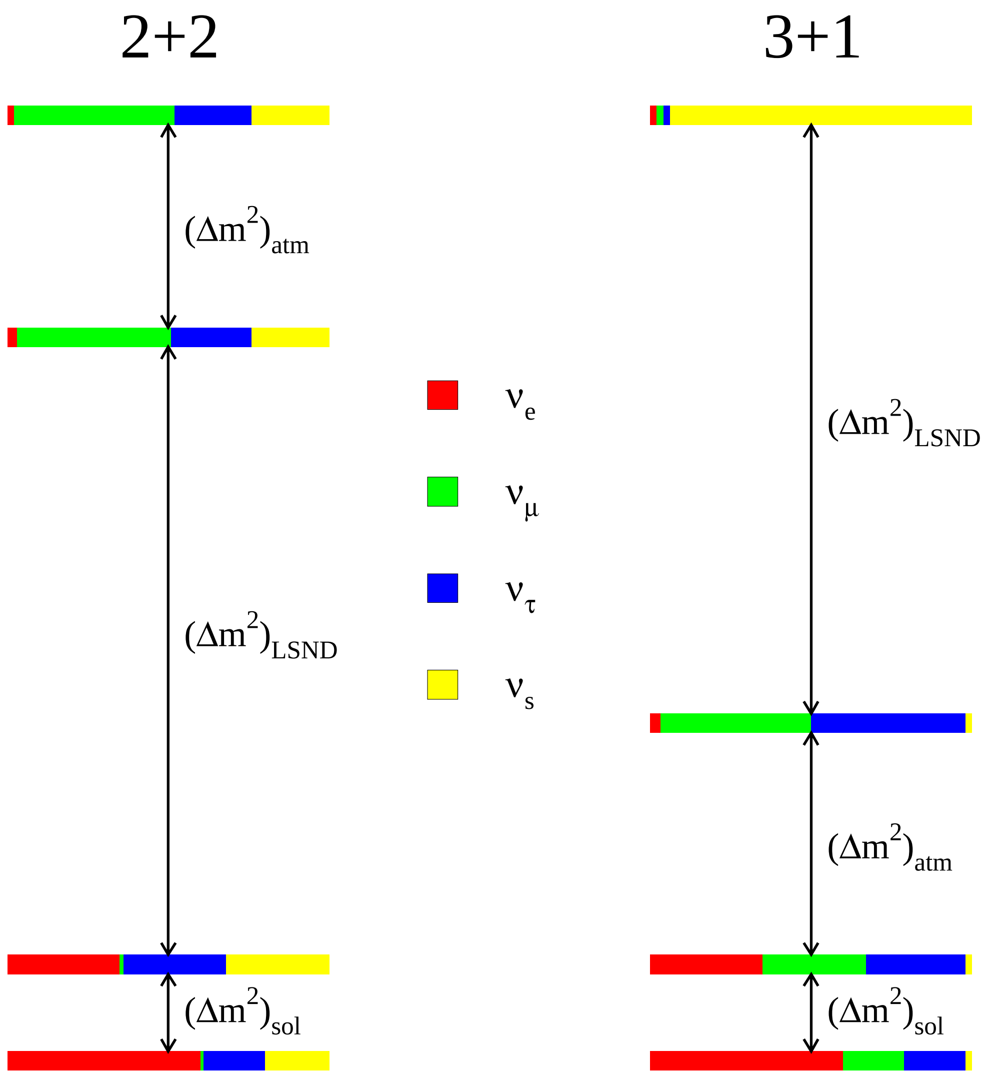
$\bar{\nu}_\mu \to \bar{\nu}_e$, but a shorter baseline ($L = 18$ m)

Other curves are failed searches for

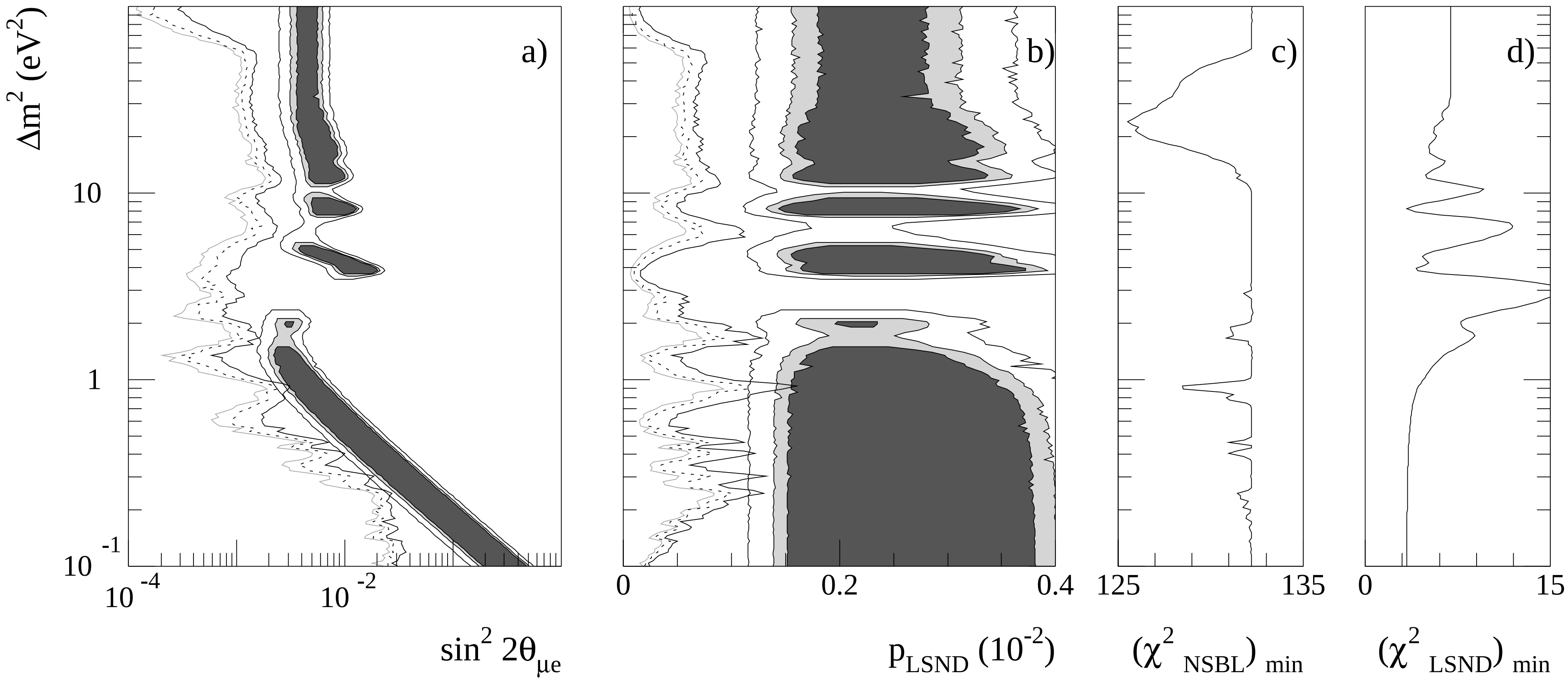
ν_μ disappearance (CCFR),

$\bar{\nu}_e$ disapperance (Bugey), etc

Remember: $P_{\mu e} = \sin^2 2\theta \sin^2 \left[1.27 \left(\frac{\Delta m^2}{\mathrm{eV}^2}\right) \left(\frac{L}{\mathrm{m}}\right) \left(\frac{\mathrm{MeV}}{E}\right)\right]$



$\Rightarrow$ 2+2 requires large sterile effects in either solar or atmospheric oscillations, not observed

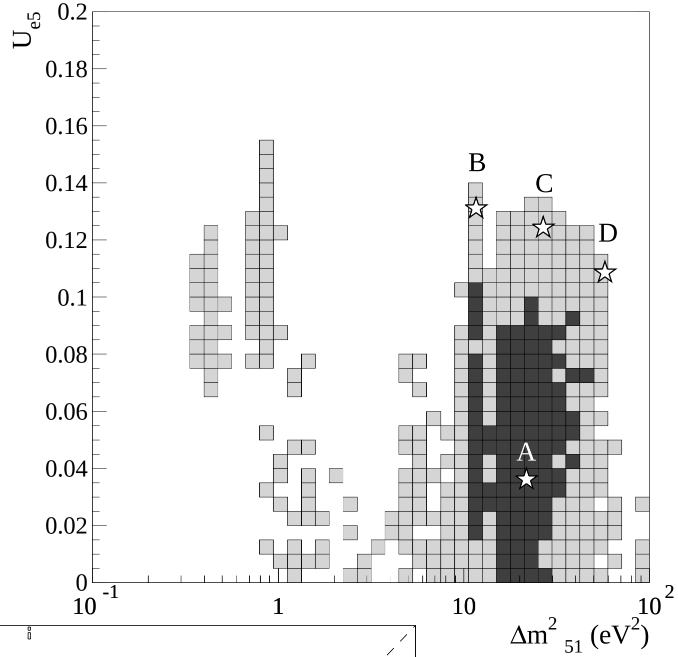


In 3+1, $\sin^2 2\theta_{\rm LSND} = \sin^2 2\theta_{\mu e} \simeq 4|U_{e4}|^2|U_{\mu 4}|^2$, while for

disappearance searches $\sin^2 2\theta_{\alpha\alpha} \simeq 4|U_{\alpha 4}|^2(1-|U_{\alpha 4}|^2)$.

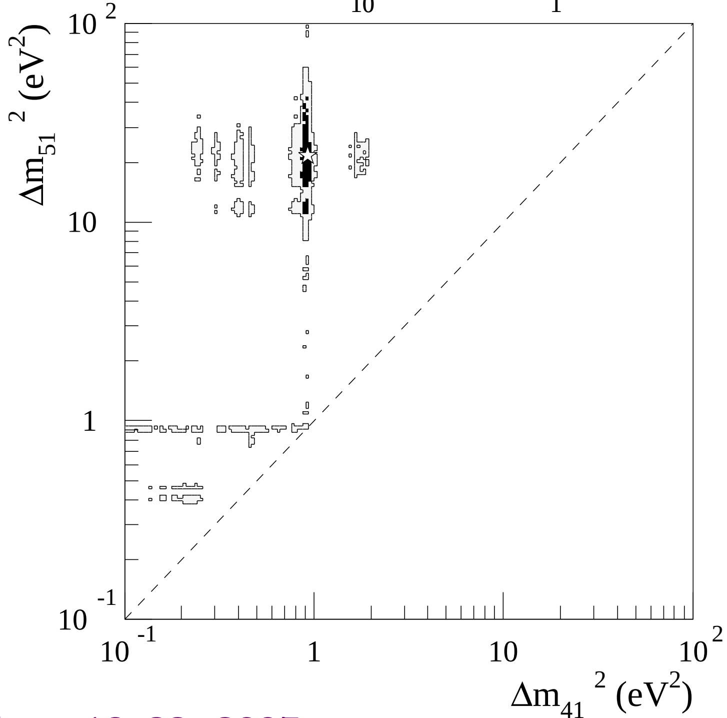
nontrivial constraints from short-baseline disappearance searches!...

3+1+1 Fits Introduce an Extra Δm^2 and Effective Mixing Angle.



Can only be better than 3+1 fit (decoupling)

The fit works by "splitting" the constraints imposed by short baseline data between the two frequencies, whose effect add up at LSND.



Is this "finely tuned"? In what sense?