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CP Violation in Meson Decays

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Abstract

This is a written version of a series of lectures aimed at graduate students in the field of (theoretical and experimental) high energy physics. The main topics covered are: (i) The flavor sector of the Standard Model and the Kobayashi-Maskawa mechanism of CP violation; (ii) Formalism and theoretical interpretation of CP violation in meson decays; (iii) K decays; (iv) D decays; (v) B decays: $b \to c\bar{c}s$, $b \to s\bar{s}s$, $b \to u\bar{u}d$ and $b \to c\bar{u}s, u\bar{c}s$; (vi) CP violation as a probe of new physics and, in particular, of supersymmetry.

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I. INTRODUCTION

The Standard Model predicts that the only way that CP is violated is through the Kobayashi-Maskawa mechanism [1]. Specifically, the source of CP violation is a *single* phase in the mixing matrix that describes the charged current weak interactions of quarks. In the introductory chapter, we briefly review the present evidence that supports the Kobayashi-Maskawa picture of CP violation, as well as the various arguments against this picture.

A. Why believe the Kobayashi-Maskawa mechanism?

Experiments have measured to date nine independent CP violating observables:[1]

[1] The list of measured observables in B decays is somewhat conservative. I include only observables where the combined significance of Babar and Belle measurements (taking an inflated error in case of inconsis-

1. Indirect CP violation in $K \to \pi\pi$ decays [2] and in $K \to \pi\ell\nu$ decays is given by

$$\varepsilon_K = (2.28 \pm 0.02) \times 10^{-3}\ e^{i\pi/4}. \tag{1}$$

2. Direct CP violation in $K \to \pi\pi$ decays [3, 4, 5] is given by

$$\varepsilon'/\varepsilon = (1.72 \pm 0.18) \times 10^{-3}. \tag{2}$$

3. CP violation in the interference of mixing and decay in the $B \to \psi K_S$ and other, related modes is given by [6, 7]:

$$S_{\psi K_S} = +0.69 \pm 0.03. \tag{3}$$

4. CP violation in the interference of mixing and decay in the $B \to K^+K^-K_S$ mode is given by [8, 9]

$$S_{K^+K^-K_S} = -0.45 \pm 0.13. \tag{4}$$

5. CP violation in the interference of mixing and decay in the $B \to D^{*+}D^{*-}$ mode is given by [10, 11]

$$S_{D^{*+}D^{*-}} = -0.75 \pm 0.23. \tag{5}$$

6. CP violation in the interference of mixing and decay in the $B \to \eta' K^0$ modes is given by [12, 13, 14]

$$S_{\eta' K_S} = +0.50 \pm 0.09(0.13). \tag{6}$$

7. CP violation in the interference of mixing and decay in the $B \to f_0 K_S$ mode is given by [13, 15]

$$S_{f_0 K_S} = -0.75 \pm 0.24. \tag{7}$$

8. Direct CP violation in the $\overline{B}^0 \to K^-\pi^+$ mode is given by [16, 17]

$$\mathcal{A}_{K^\mp\pi^\pm} = -0.115 \pm 0.018. \tag{8}$$

9. Direct CP violation in the $B \to \rho\pi$ mode is given by [18, 19]

$$\mathcal{A}_{\rho\pi}^{-+} = -0.48 \pm 0.14. \tag{9}$$

tencies) is above 3σ.

All nine measurements – as well as many other, where CP violation is not (yet) observed at a level higher than 3σ – are consistent with the Kobayashi-Maskawa picture of CP violation. In particular, the measurement of the phase β from the CP asymmetry $B \to \psi K$, the measurement of the phase α from CP asymmetries and decay rates in the $B \to \pi\pi, \rho\pi$ and $\rho\rho$ modes, and the measurement of the phase γ from $B \to DK$ decays, have provided the first three precision tests of CP violation in the Standard Model. Since the model has passed these tests successfully, we are able, for the first time, to make the following statement: *The Kobayashi-Maskawa phase is, very likely, the dominant source of CP violation in low-energy flavor-changing processes.*

In contrast, various alternative scenarios of CP violation that have been phenomenologically viable for many years are now unambiguously excluded. Two important examples are the following:

- The superweak framework [20], that is, the idea that CP violation is purely indirect, is excluded by the evidence that $\varepsilon'/\varepsilon \neq 0$.
- Approximate CP, that is, the idea that all CP violating phases are small (see, for example, [21]), is excluded by the evidence that $S_{\psi K_S} = \mathcal{O}(1)$.

Indeed, I am not aware of any viable, reasonably motivated, scenario which provides a complete alternative to the KM mechanism, that is of a framework where the KM phase plays no significant role in the observed CP violation.

The experimental results from the B-factories, such as those in Eqs. (3-9), and their implications for theory signify a new era in the study of CP violation. In this series of lectures we explain these recent developments and their significance.

B. Why doubt the Kobayashi-Maskawa mechanism?

1. *The baryon asymmetry of the Universe*

Baryogenesis is a consequence of CP violating processes [22]. Therefore the present baryon number, which is accurately deduced from nucleosynthesis and CMBR constraints,

$$Y_B \equiv \frac{n_B - n_{\overline{B}}}{s} \simeq 9 \times 10^{-11}, \tag{10}$$

is essentially a CP violating observable! It can be added to the list of known CP violating observables, Eqs. (1-9). Within a given model of CP violation, one can check for consistency between the data from cosmology, Eq. (10), and those from laboratory experiments.

The surprising point is that the Kobayashi-Maskawa mechanism for CP violation fails to account for (10). It predicts present baryon number density that is many orders of magnitude below the observed value [23, 24, 25]. This failure is independent of other aspects of the Standard Model: the suppression of Y_B from CP violation is much too strong, even if the departure from thermal equilibrium is induced by mechanisms beyond the Standard Model. This situation allows us to make the following statement: *There must exist sources of CP violation beyond the Kobayashi-Maskawa phase.*

Two important examples of viable models of baryogenesis are the following:

1. Leptogenesis [26]: a lepton asymmetry is induced by CP violating decays of heavy fermions that are singlets of the Standard Model gauge group (sterile neutrinos). Departure from thermal equilibrium is provided if the lifetime of the heavy neutrino is long enough that it decays when the temperature is below its mass. Processes that violate $B + L$ are fast before the electroweak phase transition and partially convert the lepton asymmetry into a baryon asymmetry. The CP violating parameters may be related to CP violation in the mixing matrix for the light neutrinos (but this is a model dependent issue [27]).

2. Electroweak baryogenesis (for a review see [28]): the source of the baryon asymmetry is the interactions of top (anti)quarks with the Higgs field during the electroweak phase transition. CP violation is induced, for example, by supersymmetric interactions. Sphaleron configurations provide baryon number violating interactions. Departure from thermal equilibrium is provided by the wall between the false vacuum ($\langle\phi\rangle = 0$) and the expanding bubble with the true vacuum, where electroweak symmetry is broken.

2. The strong CP problem

Nonperturbative QCD effects induce an additional term in the SM Lagrangian,

$$\mathcal{L}_\theta = \frac{\theta_{\rm QCD}}{32\pi^2}\epsilon_{\mu\nu\rho\sigma}F^{\mu\nu a}F^{\rho\sigma a}. \tag{11}$$

This term violates CP. In particular, it induces an electric dipole moment (EDM) to the neutron. The leading contribution in the chiral limit is given by [29]

$$d_N = \frac{g_{\pi NN}\bar{g}_{\pi NN}}{4\pi^2 M_N} \ln\frac{M_N}{m_\pi} \approx 5\times 10^{-16}\ \theta_{\rm QCD}\ e\ {\rm cm}, \tag{12}$$

where M_N is the nucleon mass, and $g_{\pi NN}$ ($\bar{g}_{\pi NN}$) is the pseudoscalar coupling (CP-violating scalar coupling) of the pion to the nucleon. (The leading contribution in the large N_c limit was calculated in the Skyrme model [30] and leads to a similar estimate.) The experimental bound on d_N is given by [31]

$$d_N \leq 6.3\times 10^{-26}\ e\ {\rm cm}. \tag{13}$$

It leads to the following bound on $\theta_{\rm QCD}$:

$$\theta_{\rm QCD} \lesssim 10^{-10}. \tag{14}$$

Since $\theta_{\rm QCD}$ arises from nonperturbative QCD effects, it is impossible to calculate it. Yet, there are good reasons to expect that these effects should yield $\theta_{\rm QCD} = \mathcal{O}(1)$ (for a review, see [32]). Within the SM, a value as small as in Eq. (14) is unnatural, since setting $\theta_{\rm QCD}$ to zero does not add symmetry to the model. [In particular, as we will see below, CP is violated by $\delta_{\rm KM} = \mathcal{O}(1)$.] Understanding why CP is so small in the strong interactions is the strong CP problem.

It seems then that the strong CP problem is a clue to new physics. Among the solutions that have been proposed are a massless u-quark (for a review, see [33]), the Peccei-Quinn mechanism [34, 35] and spontaneous CP violation.

3. New Physics

Almost any extension of the Standard Model provides new sources of CP violation. For example, in the supersymmetric extension of the Standard Model (with R-parity), there are 44 independent phases, most of them in flavor changing couplings. If there is new physics at or below the TeV scale, it is quite likely that the KM phase is not the only source of CP violation that is playing a role in meson decays.

C. Will new CP violation be observed in experiments?

The SM picture of CP violation is testable because the Kobayashi-Maskawa mechanism is unique and predictive. These features are mainly related to the fact that there is a single phase that is responsible to all CP violation. As a consequence of this situation, one finds two classes of tests:

(i) Correlations: many independent CP violating observables are correlated within the SM. For example, the SM predicts that the CP asymmetries in $B \to \psi K_S$ and in $B \to \phi K_S$, which proceed through different quark transitions, are equal to each other (to a few percent accuracy) [36, 37]. Another important example is the strong SM correlation between CP violation in $B \to \psi K_S$ and in $K \to \pi\nu\bar{\nu}$ [38, 39, 40]. It is a significant fact, in this context, that several CP violating observables can be calculated with very small hadronic uncertainties. To search for violations of the correlations, precise measurements are important.

(ii) Zeros: since the KM phase appears in flavor-changing, weak-interaction couplings of quarks, and only if all three generations are involved, many CP violating observables are predicted to be negligibly small. For example, the transverse lepton polarization in semileptonic meson decays, CP violation in $t\bar{t}$ production, tree level D decays, and (assuming $\theta_{\rm QCD} = 0$) the electric dipole moment of the neutron are all predicted to be orders of magnitude below the (present and near future) experimental sensitivity. To search for lifted zeros, measurements of CP violation in many different systems should be performed.

The strongest argument that new sources of CP violation must exist in Nature comes from baryogenesis. Whether the CP violation that is responsible for baryogenesis would be manifest in measurements of CP asymmetries in B decays depends on two issues:

(i) The scale of the new CP violation: if the relevant scale is very high, such as in leptogenesis, the effects cannot be signalled in these measurements. To estimate the limit on the scale, the following three facts are relevant: First, the Standard Model contributions to CP asymmetries in B decays are $\mathcal{O}(1)$. Second, the expected experimental accuracy would reach in some cases the few percent level. Third, the contributions from new physics are expected to be suppressed by $(\Lambda_{\rm EW}/\Lambda_{\rm NP})^2$. The conclusion is that, if the new source of CP violation is related to physics at $\Lambda_{\rm NP} \gg 1\ TeV$, it cannot be signalled in B decays. Only if the true mechanism is electroweak baryogenesis, it can potentially affect B decays.

(ii) The flavor dependence of the new CP violation: if it is flavor diagonal, its effects on

B decays would be highly suppressed. It can still manifest itself in other, flavor diagonal CP violating observables, such as electric dipole moments.

We conclude that new measurements of CP asymmetries in meson decays are particularly sensitive to new sources of CP violation that come from physics at (or below) the few TeV scale and that are related to flavor changing couplings. This is, for example, the case, in certain supersymmetric models of baryogenesis [41, 42]. The search for electric dipole moments can reveal the existence of new flavor diagonal CP violation.

Of course, there could be new flavor physics at the TeV scale that is not related to the baryon asymmetry and may give signals in B decays. The best motivated extension of the SM where this situation is likely is that of supersymmetry.

Finally, we would like to mention that, in the past, flavor physics and the physics of CP violation led indeed to the discovery of new physics or to probing it before it was directly observed in experiments:

- The smallness of $\frac{\Gamma(K_L\to\mu^+\mu^-)}{\Gamma(K^+\to\mu^+\nu)}$ led to predicting a fourth (the charm) quark;
- The size of Δm_K led to a successful prediction of the charm mass;
- The size of Δm_B led to a successful prediction of the top mass;
- The measurement of ε_K led to predicting the third generation.

II. THE KOBAYASHI-MASKAWA MECHANISM

A. Yukawa interactions are the source of CP violation

A model of elementary particles and their interactions is defined by three ingredients:

1. The symmetries of the Lagrangian;
2. The representations of fermions and scalars;
3. The pattern of spontaneous symmetry breaking.

The Standard Model (SM) is defined as follows:

1. The gauge symmetry is

$$G_{\rm SM} = SU(3)_{\rm C} \times SU(2)_{\rm L} \times U(1)_{\rm Y}. \tag{15}$$

2. There are three fermion generations, each consisting of five representations of $G_{\rm SM}$:

$$Q^I_{Li}(3,2)_{+1/6},\quad U^I_{Ri}(3,1)_{+2/3},\quad D^I_{Ri}(3,1)_{-1/3},\quad L^I_{Li}(1,2)_{-1/2},\quad E^I_{Ri}(1,1)_{-1}. \tag{16}$$

Our notations mean that, for example, left-handed quarks, Q^I_L, are triplets of $SU(3)_{\rm C}$, doublets of $SU(2)_{\rm L}$ and carry hypercharge $Y=+1/6$. The super-index I denotes interaction eigenstates. The sub-index $i=1,2,3$ is the flavor (or generation) index. There is a single scalar representation,

$$\phi(1,2)_{+1/2}. \tag{17}$$

3. The scalar ϕ assumes a VEV,

$$\langle\phi\rangle=\begin{pmatrix}0\\ \frac{v}{\sqrt{2}}\end{pmatrix}, \tag{18}$$

so that the gauge group is spontaneously broken,

$$G_{\rm SM}\to SU(3)_{\rm C}\times U(1)_{\rm EM}. \tag{19}$$

The Standard Model Lagrangian, $\mathcal{L}_{\rm SM}$, is the most general renormalizable Lagrangian that is consistent with the gauge symmetry (15), the particle content (16,17) and the pattern of spontaneous symmetry breaking (18). It can be divided to three parts:

$$\mathcal{L}_{\rm SM}=\mathcal{L}_{\rm kinetic}+\mathcal{L}_{\rm Higgs}+\mathcal{L}_{\rm Yukawa}. \tag{20}$$

As concerns the kinetic terms, to maintain gauge invariance, one has to replace the derivative with a covariant derivative:

$$D^\mu=\partial^\mu+ig_sG^\mu_aL_a+igW^\mu_bT_b+ig'B^\mu Y. \tag{21}$$

Here G^μ_a are the eight gluon fields, W^μ_b the three weak interaction bosons and B^μ the single hypercharge boson. The L_a's are $SU(3)_{\rm C}$ generators (the 3×3 Gell-Mann matrices $\frac{1}{2}\lambda_a$ for triplets, 0 for singlets), the T_b's are $SU(2)_{\rm L}$ generators (the 2×2 Pauli matrices $\frac{1}{2}\tau_b$ for doublets, 0 for singlets), and the Y's are the $U(1)_{\rm Y}$ charges. For example, for the left-handed quarks Q^I_L, we have

$$\mathcal{L}_{\rm kinetic}(Q_L)=i\overline{Q^I_{Li}}\gamma_\mu\left(\partial^\mu+\frac{i}{2}g_sG^\mu_a\lambda_a+\frac{i}{2}gW^\mu_b\tau_b+\frac{i}{6}g'B^\mu\right)Q^I_{Li}, \tag{22}$$

while for the left-handed leptons L^I_L, we have

$$\mathcal{L}_{\rm kinetic}(L_L)=i\overline{L^I_{Li}}\gamma_\mu\left(\partial^\mu+\frac{i}{2}gW^\mu_b\tau_b-ig'B^\mu\right)L^I_{Li}. \tag{23}$$

These parts of the interaction Lagrangian are always CP conserving.

The Higgs potential, which describes the scalar self interactions, is given by:

$$\mathcal{L}_{\rm Higgs} = \mu^2 \phi^\dagger \phi - \lambda (\phi^\dagger \phi)^2. \tag{24}$$

For the Standard Model scalar sector, where there is a single doublet, this part of the Lagrangian is also CP conserving. For extended scalar sectors, such as that of a two Higgs doublet model, $\mathcal{L}_{\rm Higgs}$ can be CP violating. Even in case that it is CP symmetric, it may lead to spontaneous CP violation.

The quark Yukawa interactions are given by

$$-\mathcal{L}^{\rm quarks}_{\rm Yukawa} = Y^d_{ij} \overline{Q^I_{Li}} \phi D^I_{Rj} + Y^u_{ij} \overline{Q^I_{Li}} \tilde{\phi} U^I_{Rj} + \text{h.c.}. \tag{25}$$

This part of the Lagrangian is, in general, CP violating. More precisely, CP is violated if and only if [43]

$$\mathcal{I}m(\det[Y^d Y^{d\dagger}, Y^u Y^{u\dagger}]) \neq 0. \tag{26}$$

An intuitive explanation of why CP violation is related to *complex* Yukawa couplings goes as follows. The hermiticity of the Lagrangian implies that $\mathcal{L}_{\rm Yukawa}$ has its terms in pairs of the form

$$Y_{ij} \overline{\psi_{Li}} \phi \psi_{Rj} + Y^*_{ij} \overline{\psi_{Rj}} \phi^\dagger \psi_{Li}. \tag{27}$$

A CP transformation exchanges the operators

$$\overline{\psi_{Li}} \phi \psi_{Rj} \leftrightarrow \overline{\psi_{Rj}} \phi^\dagger \psi_{Li}, \tag{28}$$

but leaves their coefficients, Y_{ij} and Y^*_{ij}, unchanged. This means that CP is a symmetry of $\mathcal{L}_{\rm Yukawa}$ if $Y_{ij} = Y^*_{ij}$.

The lepton Yukawa interactions are given by

$$-\mathcal{L}^{\rm leptons}_{\rm Yukawa} = Y^e_{ij} \overline{L^I_{Li}} \phi E^I_{Rj} + \text{h.c.}. \tag{29}$$

It leads, as we will see in the next section, to charged lepton masses but predicts massless neutrinos. Recent measurements of the fluxes of atmospheric and solar neutrinos provide evidence for neutrino masses (for a review, see [44]). That means that $\mathcal{L}_{\rm SM}$ cannot be a complete description of Nature. The simplest way to allow for neutrino masses is to add

dimension-five (and, therefore, non-renormalizable) terms, consistent with the SM symmetry and particle content:

$$-\mathcal{L}_{\rm Yukawa}^{\rm dim-5} = \frac{Y_{ij}^{\nu}}{M} L_i L_j \phi\phi + {\rm h.c.}. \tag{30}$$

The parameter M has dimension of mass. The dimensionless couplings Y_{ij}^{ν} are symmetric ($Y_{ij}^{\nu} = Y_{ji}^{\nu}$). We refer to the SM extended to include the terms $\mathcal{L}_{\rm Yukawa}^{\rm dim-5}$ of Eq. (30) as the "extended SM" (ESM):

$$\mathcal{L}_{\rm ESM} = \mathcal{L}_{\rm kinetic} + \mathcal{L}_{\rm Higgs} + \mathcal{L}_{\rm Yukawa} + \mathcal{L}_{\rm Yukawa}^{\rm dim-5}. \tag{31}$$

The inclusion of non-renormalizable terms is equivalent to postulating that the SM is only a low energy effective theory, and that new physics appears at the scale M.

How many independent CP violating parameters are there in $\mathcal{L}_{\rm Yukawa}^{\rm quarks}$? Each of the two Yukawa matrices Y^q ($q = u, d$) is 3×3 and complex. Consequently, there are 18 real and 18 imaginary parameters in these matrices. Not all of them are, however, physical. One can think of the quark Yukawa couplings as spurions that break a global symmetry,

$$U(3)_Q \times U(3)_D \times U(3)_U \;\rightarrow\; U(1)_B. \tag{32}$$

This means that there is freedom to remove 9 real and 17 imaginary parameters [the number of parameters in three 3×3 unitary matrices minus the phase related to $U(1)_B$]. We conclude that there are 10 quark flavor parameters: 9 real ones and a single phase. This single phase is the source of CP violation in the quark sector.

How many independent CP violating parameters are there in the lepton Yukawa interactions? The matrix Y^e is a general complex 3×3 matrix and depends, therefore, on 9 real and 9 imaginary parameters. The matrix Y^{ν} is symmetric and depends on 6 real and 6 imaginary parameters. Not all of these 15 real and 15 imaginary parameters are physical. One can think of the lepton Yukawa couplings as spurions that break (completely) a global symmetry,

$$U(3)_L \times U(3)_E. \tag{33}$$

This means that 6 real and 12 imaginary parameters are not physical. We conclude that there are 12 lepton flavor parameters: 9 real ones and three phases. These three phases induce CP violation in the lepton sector.

B. CKM mixing is the (only!) source of CP violation in the quark sector

Upon the replacement $\mathcal{R}e(\phi^0) \to \frac{v+H^0}{\sqrt{2}}$ [see Eq. (18)], the Yukawa interactions (25) give rise to mass terms:

$$-\mathcal{L}^q_M = (M_d)_{ij}\overline{D^I_{Li}}D^I_{Rj} + (M_u)_{ij}\overline{U^I_{Li}}U^I_{Rj} + \text{h.c.}, \tag{34}$$

where

$$M_q = \frac{v}{\sqrt{2}}Y^q, \tag{35}$$

and we decomposed the $SU(2)_{\rm L}$ quark doublets into their components:

$$Q^I_{Li} = \begin{pmatrix} U^I_{Li} \\ D^I_{Li} \end{pmatrix}. \tag{36}$$

The mass basis corresponds, by definition, to diagonal mass matrices. We can always find unitary matrices V_{qL} and V_{qR} such that

$$V_{qL}M_qV^\dagger_{qR} = M^{\rm diag}_q \quad (q = u, d), \tag{37}$$

with $M^{\rm diag}_q$ diagonal and real. The quark mass eigenstates are then identified as

$$q_{Li} = (V_{qL})_{ij}q^I_{Lj}, \quad q_{Ri} = (V_{qR})_{ij}q^I_{Rj} \quad (q = u, d). \tag{38}$$

The charged current interactions for quarks [that is the interactions of the charged $SU(2)_{\rm L}$ gauge bosons $W^\pm_\mu = \frac{1}{\sqrt{2}}(W^1_\mu \mp iW^2_\mu)$], which in the interaction basis are described by (22), have a complicated form in the mass basis:

$$-\mathcal{L}^q_{W^\pm} = \frac{g}{\sqrt{2}}\overline{u_{Li}}\gamma^\mu(V_{uL}V^\dagger_{dL})_{ij}d_{Lj}W^+_\mu + \text{h.c.}. \tag{39}$$

The unitary 3×3 matrix,

$$V = V_{uL}V^\dagger_{dL}, \quad (VV^\dagger = \mathbf{1}), \tag{40}$$

is the Cabibbo-Kobayashi-Maskawa (CKM) *mixing matrix* for quarks [1, 45]. A unitary 3×3 matrix depends on nine parameters: three real angles and six phases.

The form of the matrix is not unique:

(i) There is freedom in defining V in that we can permute between the various generations. This freedom is fixed by ordering the up quarks and the down quarks by their masses, *i.e.* $(u_1, u_2, u_3) \to (u, c, t)$ and $(d_1, d_2, d_3) \to (d, s, b)$. The elements of V are written as follows:

$$V = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}. \tag{41}$$

(*ii*) There is further freedom in the phase structure of V. Let us define P_q ($q = u, d$) to be diagonal unitary (phase) matrices. Then, if instead of using V_{qL} and V_{qR} for the rotation (38) to the mass basis we use $\tilde{V}_{qL}$ and $\tilde{V}_{qR}$, defined by $\tilde{V}_{qL} = P_q V_{qL}$ and $\tilde{V}_{qR} = P_q V_{qR}$, we still maintain a legitimate mass basis since $M_q^{\rm diag}$ remains unchanged by such transformations. However, V does change:

$$V \to P_u V P_d^*. \tag{42}$$

This freedom is fixed by demanding that V has the minimal number of phases. In the three generation case V has a single phase. (There are five phase differences between the elements of P_u and P_d and, therefore, five of the six phases in the CKM matrix can be removed.) This is the Kobayashi-Maskawa phase $\delta_{\rm KM}$ which is the single source of CP violation in the quark sector of the Standard Model [1].

As a result of the fact that V is not diagonal, the $W^\pm$ gauge bosons couple to quark (mass eigenstates) of different generations. Within the Standard Model, this is the only source of *flavor changing* quark interactions.

C. The three phases in the lepton mixing matrix

The leptonic Yukawa interactions (29) and (30) give rise to mass terms:

$$-\mathcal{L}_M^\ell = (M_e)_{ij}\overline{e_{Li}^I} e_{Rj}^I + (M_\nu)_{ij}\nu_{Li}^I \nu_{Lj}^I + \text{h.c.}, \tag{43}$$

where

$$M_e = \frac{v}{\sqrt{2}} Y^e, \quad M_\nu = \frac{v^2}{2M} Y^\nu, \tag{44}$$

and we decomposed the $SU(2)_{\rm L}$ lepton doublets into their components:

$$L_{Li}^I = \begin{pmatrix} \nu_{Li}^I \\ e_{Li}^I \end{pmatrix}. \tag{45}$$

We can always find unitary matrices V_{eL} and V_ν such that

$$V_{eL} M_e M_e^\dagger V_{eL}^\dagger = \text{diag}(m_e^2, m_\mu^2, m_\tau^2), \quad V_\nu M_\nu^\dagger M_\nu V_\nu^\dagger = \text{diag}(m_1^2, m_1^2, m_3^2). \tag{46}$$

The charged current interactions for leptons, which in the interaction basis are described by (23), have the following form in the mass basis:

$$-\mathcal{L}_{W^\pm}^\ell = \frac{g}{\sqrt{2}} \overline{e_{Li}} \gamma^\mu (V_{eL} V_\nu^\dagger)_{ij} \nu_{Lj} W_\mu^- + \text{h.c.}. \tag{47}$$

The unitary 3×3 matrix,

$$U = V_{eL} V_\nu^\dagger, \tag{48}$$

is the *lepton mixing matrix* [46]. Similarly to the CKM matrix, the form of the lepton mixing matrix is not unique. But there are differences in choosing conventions:

(*i*) We can permute between the various generations. This freedom is usually fixed in the following way. We order the charged leptons by their masses, *i.e.* $(e_1, e_2, e_3) \to (e, \mu, \tau)$. As concerns the neutrinos, one takes into account that the atmospheric and solar neutrino data imply that $\Delta m^2_{\rm atm} \gg \Delta m^2_{\rm sol}$. It follows that one of the neutrino mass eigenstates is separated in its mass from the other two, which have a smaller mass difference. The convention is to denote this separated state by ν_3. For the remaining two neutrinos, ν_1 and ν_2, the convention is to call the heavier state ν_2. In other words, the three mass eigenstates are defined by the following conventions:

$$|\Delta m^2_{3i}| \gg |\Delta m^2_{21}|, \quad \Delta m^2_{21} > 0. \tag{49}$$

Note in particular that ν_3 can be either heavier ('normal hierarchy') or lighter ('inverted hierarchy') than $\nu_{1,2}$. The elements of U are written as follows:

$$U = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix}. \tag{50}$$

(*ii*) There is further freedom in the phase structure of U. One can change the charged lepton mass basis by the transformation $e_{(L,R)i} \to e'_{(L,R)i} = (P_e)_{ii} e_{(L,R)i}$, where P_e is a phase matrix. There is, however, no similar freedom to redefine the neutrino mass eigenstates: From Eq. (43) one learns that a transformation $\nu_L \to P_\nu \nu_L$ will introduce phases into the diagonal mass matrix. This is related to the Majorana nature of neutrino masses, assumed in Eq. (30). The allowed transformation modifies U:

$$U \to P_e U. \tag{51}$$

This freedom is fixed by demanding that U will have the minimal number of phases. Out of six phases of a generic unitary 3×3 matrix, the multiplication by P_e can be used to remove three. We conclude that the three generation U matrix has three phases. One of these is the analog of the Kobayashi-Maskawa phase. It is the only source of CP violation in processes that conserve lepton number, such as neutrino flavor oscillations. The other two phases can affect lepton number changing processes.

With $U \neq \mathbf{1}$, the $W^{\pm}$ gauge bosons couple to lepton (mass eigenstates) of different generations. Within the ESM, this is the only source of *flavor changing* lepton interactions.

D. The flavor parameters

Examining the quark mass basis, one can easily identify the flavor parameters. In the quark sector, we have six quark masses and four mixing parameters: three mixing angles and a single phase.

The fact that there are only three real and one imaginary physical parameters in V can be made manifest by choosing an explicit parameterization. For example, the standard parameterization [47], used by the particle data group, is given by

$$V = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}, \tag{52}$$

where $c_{ij} \equiv \cos\theta_{ij}$ and $s_{ij} \equiv \sin\theta_{ij}$. The three $\sin\theta_{ij}$ are the three real mixing parameters while δ is the Kobayashi-Maskawa phase. Another, very useful, example is the Wolfenstein parametrization, where the four mixing parameters are (λ, A, ρ, η) with $\lambda = |V_{us}| = 0.22$ playing the role of an expansion parameter and η representing the CP violating phase [48, 49]:

$$V = \begin{pmatrix} 1 - \frac{1}{2}\lambda^2 - \frac{1}{8}\lambda^4 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda + \frac{1}{2}A^2\lambda^5[1 - 2(\rho + i\eta)] & 1 - \frac{1}{2}\lambda^2 - \frac{1}{8}\lambda^4(1 + 4A^2) & A\lambda^2 \\ A\lambda^3[1 - (1 - \frac{1}{2}\lambda^2)(\rho + i\eta)] & -A\lambda^2 + \frac{1}{2}A\lambda^4[1 - 2(\rho + i\eta)] & 1 - \frac{1}{2}A^2\lambda^4 \end{pmatrix}. \tag{53}$$

Various parametrizations differ in the way that the freedom of phase rotation, Eq. (42), is used to leave a single phase in V. One can define, however, a CP violating quantity in $V_{\rm CKM}$ that is independent of the parametrization [43]. This quantity, $J_{\rm CKM}$, is defined through

$$\mathcal{I}m(V_{ij}V_{kl}V_{il}^*V_{kj}^*) = J_{\rm CKM}\sum_{m,n=1}^{3}\epsilon_{ikm}\epsilon_{jln}, \quad (i,j,k,l = 1,2,3). \tag{54}$$

In terms of the explicit parametrizations given above, we have

$$J_{\rm CKM} = c_{12}c_{23}c_{13}^2 s_{12}s_{23}s_{13}\sin\delta \simeq \lambda^6 A^2\eta. \tag{55}$$

It is interesting to translate the condition (26) to the language of the flavor parameters in the mass basis. One finds that the following is a necessary and sufficient condition for CP violation in the quark sector of the SM (we define $\Delta m_{ij}^2 \equiv m_i^2 - m_j^2$):

$$\Delta m_{tc}^2\Delta m_{tu}^2\Delta m_{cu}^2\Delta m_{bs}^2\Delta m_{bd}^2\Delta m_{sd}^2 J_{\rm CKM} \neq 0. \tag{56}$$

Equation (56) puts the following requirements on the SM in order that it violates CP:

(i) Within each quark sector, there should be no mass degeneracy;

(ii) None of the three mixing angles should be zero or $\pi/2$;

(iii) The phase should be neither 0 nor π.

As concerns the lepton sector of the ESM, the flavor parameters are the six lepton masses, and six mixing parameters: three mixing angles and three phases. One can parameterize U in a convenient way by factorizing it into $U = \hat{U}P$. Here P is a diagonal unitary matrix that depends on two phases, *e.g.* $P = \mathrm{diag}(e^{i\phi_1}, e^{i\phi_2}, 1)$, while $\hat{U}$ can be parametrized in the same way as (52). The advantage of this parametrization is that for the purpose of analyzing lepton number conserving processes and, in particular, neutrino flavor oscillations, the parameters of P are usually irrelevant and one can use the same Chau-Keung parametrization as is being used for V. (An alternative way to understand these statements is to use a single-phase mixing matrix and put the extra two phases in the neutrino mass matrix. Then it is obvious that the effects of these 'Majorana-phases' always appear in conjunction with a factor of the Majorana mass that is lepton number violating parameter.) On the other hand, the Wolfenstein parametrization [Eq. (53)] is inappropriate for the lepton sector: it assumes $|V_{23}| \ll |V_{12}| \ll 1$, which does not hold here.

In order that the CP violating phase δ in $\hat{U}$ would be physically meaningful, *i.e.* there would be CP violation that is not related to lepton number violation, a condition similar to Eq. (56) should hold:

$$\Delta m^2_{\tau\mu}\Delta m^2_{\tau e}\Delta m^2_{\mu e}\Delta m^2_{32}\Delta m^2_{31}\Delta m^2_{21} J_\ell \neq 0. \tag{57}$$

E. The unitarity triangles

A very useful concept is that of the *unitarity triangles*. We focus on the quark sector, but analogous triangles can be defined in the lepton sector. The unitarity of the CKM matrix leads to various relations among the matrix elements, *e.g.*

$$V_{ud}V^*_{us} + V_{cd}V^*_{cs} + V_{td}V^*_{ts} = 0, \tag{58}$$

$$V_{us}V^*_{ub} + V_{cs}V^*_{cb} + V_{ts}V^*_{tb} = 0, \tag{59}$$

$$V_{ud}V^*_{ub} + V_{cd}V^*_{cb} + V_{td}V^*_{tb} = 0. \tag{60}$$

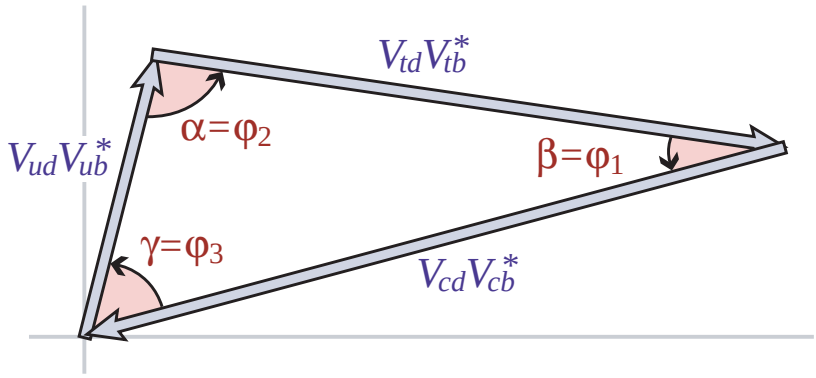


FIG. 1: Graphical representation of the unitarity constraint $V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0$ as a triangle in the complex plane.

Each of these three relations requires the sum of three complex quantities to vanish and so can be geometrically represented in the complex plane as a triangle. These are "the unitarity triangles", though the term "unitarity triangle" is usually reserved for the relation (60) only. The unitarity triangle related to Eq. (60) is depicted in Fig. 1.

It is a surprising feature of the CKM matrix that all unitarity triangles are equal in area: the area of each unitarity triangle equals $|J_{\rm CKM}|/2$ while the sign of $J_{\rm CKM}$ gives the direction of the complex vectors around the triangles.

The rescaled unitarity triangle is derived from (60) by (a) choosing a phase convention such that $(V_{cd}V_{cb}^*)$ is real, and (b) dividing the lengths of all sides by $|V_{cd}V_{cb}^*|$. Step (a) aligns one side of the triangle with the real axis, and step (b) makes the length of this side 1. The form of the triangle is unchanged. Two vertices of the rescaled unitarity triangle are thus fixed at (0,0) and (1,0). The coordinates of the remaining vertex correspond to the Wolfenstein parameters (ρ, η). The area of the rescaled unitarity triangle is $|\eta|/2$.

Depicting the rescaled unitarity triangle in the (ρ, η) plane, the lengths of the two complex sides are

$$R_u \equiv \left|\frac{V_{ud}V_{ub}}{V_{cd}V_{cb}}\right| = \sqrt{\rho^2+\eta^2}, \quad R_t \equiv \left|\frac{V_{td}V_{tb}}{V_{cd}V_{cb}}\right| = \sqrt{(1-\rho)^2+\eta^2}. \tag{61}$$

The three angles of the unitarity triangle are defined as follows [50, 51]:

$$\alpha \equiv \arg\left[-\frac{V_{td}V_{tb}^*}{V_{ud}V_{ub}^*}\right], \quad \beta \equiv \arg\left[-\frac{V_{cd}V_{cb}^*}{V_{td}V_{tb}^*}\right], \quad \gamma \equiv \arg\left[-\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}\right]. \tag{62}$$

They are physical quantities and can be independently measured by CP asymmetries in B decays. It is also useful to define the two small angles of the unitarity triangles (59) and

(58):

$$\beta_s \equiv \arg\left[-\frac{V_{ts}V_{tb}^*}{V_{cs}V_{cb}^*}\right], \quad \beta_K \equiv \arg\left[-\frac{V_{cs}V_{cd}^*}{V_{us}V_{ud}^*}\right]. \tag{63}$$

To make predictions for CP violating observables, we need to find the allowed ranges for the CKM phases. There are three ways to determine the CKM parameters (see *e.g.* [52]):

(i) **Direct measurements** are related to SM tree level processes. At present, we have direct measurements of $|V_{ud}|$, $|V_{us}|$, $|V_{ub}|$, $|V_{cd}|$, $|V_{cs}|$, $|V_{cb}|$ and $|V_{tb}|$.

(ii) **CKM Unitarity** ($V^\dagger V = \mathbf{1}$) relates the various matrix elements. At present, these relations are useful to constrain $|V_{td}|$, $|V_{ts}|$, $|V_{tb}|$ and $|V_{cs}|$.

(iii) **Indirect measurements** are related to SM loop processes. At present, we constrain in this way $|V_{tb}V_{td}|$ (from Δm_B and Δm_{B_s}) and the phase structure of the matrix (for example, from ε_K and $S_{\psi K_S}$).

Direct measurements are expected to hold almost model independently. Most extensions of the SM have a special flavor structure that suppresses flavor changing couplings and, in addition, have a mass scale $\Lambda_{\rm NP}$, that is higher than the electroweak breaking scale. Consequently, new physics contributions to tree level processes are suppressed, compared to the SM ones, by at least $\mathcal{O}(m_Z^2/\Lambda_{\rm NP}^2) \ll 1$.

Unitarity holds if the only quarks (that is fermions in color triplets with electric charge $+2/3$ or $-1/3$) are those of the three generations of the SM. This is the situation in many extensions of the SM, including the supersymmetric SM (SSM).

Using tree level constraints and unitarity, the 90% confidence limits on the magnitude of the elements are [53]

$$\begin{pmatrix} 0.9739 - 0.9751 & 0.221 - 0.227 & 0.0029 - 0.0045 \\ 0.221 - 0.227 & 0.9730 - 0.9744 & 0.039 - 0.044 \\ 0.0048 - 0.014 & 0.037 - 0.043 & 0.9990 - 0.9992 \end{pmatrix}. \tag{64}$$

Note that $|V_{ub}|$ and $|V_{td}|$ are the only elements with uncertainties of order one.

Indirect measurements are sensitive to new physics. Take, for example, the $B^0 - \overline{B}^0$ mixing amplitude. Within the SM, the leading contribution comes from an electroweak box diagram and is therefore $\mathcal{O}(g^4)$ and depends on small mixing angles, $(V_{td}^* V_{tb})^2$. (It is this dependence on the CKM elements that makes the relevant indirect measurements, particularly Δm_B and $S_{\psi K_S}$, very significant in improving our knowledge of the CKM matrix.) These suppression factors do not necessarily persist in extensions of the SM. For example, in the SSM there

are (gluino-mediated) contributions of $\mathcal{O}(g_s^4)$ and the mixing angles could be comparable to, or even larger than the SM ones. The validity of indirect measurements is then model dependent. Conversely, inconsistencies among indirect measurements (or between indirect and direct measurements) can give evidence for new physics.

When all available data are taken into account, one finds [54]:

$$\lambda = 0.226 \pm 0.001, \quad A = 0.83 \pm 0.03, \tag{65}$$

$$\bar{\rho} = 0.21 \pm 0.04, \quad \bar{\eta} = 0.33 \pm 0.02, \tag{66}$$

$$\sin 2\beta = 0.720 \pm 0.025, \quad \alpha = (99 \pm 7)^o, \quad \gamma = (58 \pm 7)^o, \quad \beta_s = (1.03 \pm 0.08)^o, \tag{67}$$

$$R_u = 0.40 \pm 0.02, \quad R_t = 0.86 \pm 0.04. \tag{68}$$

Of course, there are correlations between the various parameters. The present constraints on the shape of the unitarity triangle or, equivalently, the allowed region in the $\rho - \eta$ plane, are presented in Fig. 2.

F. The uniqueness of the Standard Model picture of CP violation

In the previous subsections, we have learnt several features of CP violation as explained by the Standard Model. It is important to understand that various reasonable (and often well-motivated) extensions of the SM provide examples where some or all of these features do not hold. Furthermore, until a few years ago, none of the special features of the Kobayashi-Maskawa mechanism of CP violation has been experimentally tested. This situation has dramatically changed recently. Let us survey some of the SM features, how they can be modified with new physics, and whether experiment has shed light on these questions.

(i) $\delta_{\rm KM}$ *is the only source of CP violation in meson decays.* This is arguably the most unique feature of the SM and gives the model a strong predictive power. It is violated in almost any low-energy extension. For example, in the supersymmetric extension of the SM there are 44 physical CP violating phases, many of which affect meson decays. The measured value of $S_{\psi K_S}$ is consistent with the correlation between K and B decays that is predicted by the SM. The value of $S_{\phi K_S}$ is equal (within the present experimental accuracy) with $S_{\psi K_S}$, consistent with the SM correlation between the asymmetries in $b \to s\bar{s}s$ and $b \to c\bar{c}s$ transitions. It is therefore very likely that $\delta_{\rm KM}$ is indeed the dominant source of CP violation in meson decays.

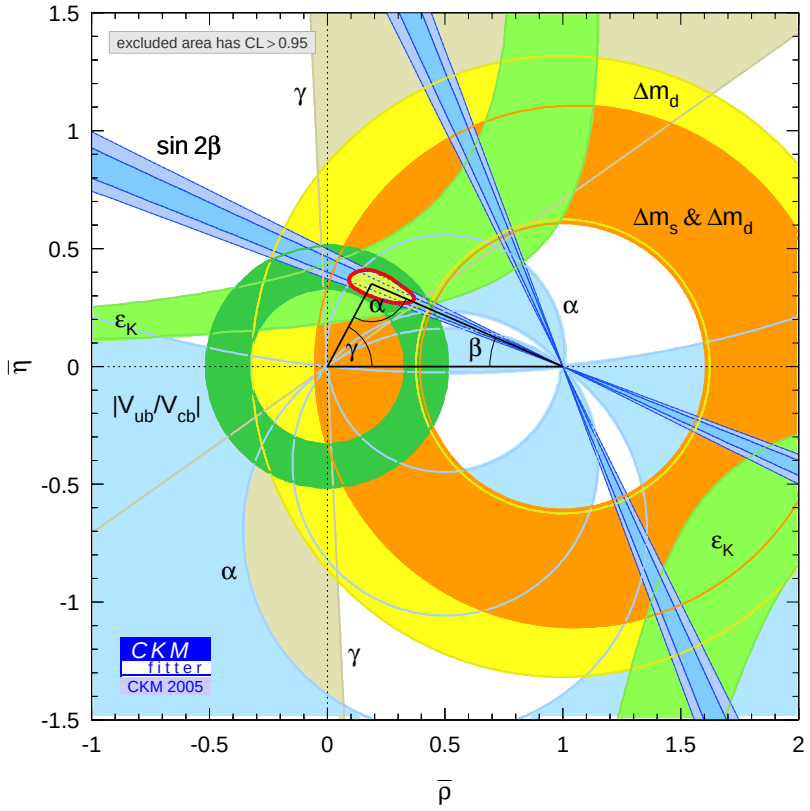


FIG. 2: Allowed region in the ρ, η plane. Superimposed are the individual constraints from charmless semileptonic B decays ($|V_{ub}/V_{cb}|$), mass differences in the B^0 (Δm_d) and B_s (Δm_s) neutral meson systems, and CP violation in $K \to \pi\pi$ (ε_K), $B \to \psi K$ ($\sin 2\beta$), $B \to \pi\pi, \rho\pi, \rho\rho$ (α), and $B \to DK$ (γ). Taken from [54].

(ii) *CP violation is small in $K \to \pi\pi$ decays because of flavor suppression and not because CP is an approximate symmetry.* In many (though certainly not all) supersymmetric models, the flavor suppression is too mild, or entirely ineffective, requiring approximate CP to hold. The measurement of $S_{\psi K_S} = \mathcal{O}(1)$ confirms that not all CP violating phases are small.

(iii) *CP violation appears in both $\Delta F = 1$ (decay) and $\Delta F = 2$ (mixing) amplitudes.* Superweak models suggest that CP is violated only in mixing amplitudes. The measurements of non-vanishing ε'/ε, $\mathcal{A}_{K^\mp\pi^\pm}$ and $\mathcal{A}_{\rho\pi}^{-+}$ confirm that there is CP violation in $\Delta S = 1$ and $\Delta B = 1$ processes.

(iv) *CP is not violated in the lepton sector.* Models that allow for neutrino masses, such as the ESM framework presented above, predict CP violation in leptonic charged current interactions. Thus, while there is no measurement of leptonic CP violation, the data from

neutrino oscillation experiments, which give evidence that neutrinos are massive and mix, make it very likely that charged current weak interactions violate CP also in the lepton sector.

(v) *CP violation appears only in the charged current weak interactions and in conjunction with flavor changing processes.* Here both various extensions of the SM (such as supersymmetry) and non-perturbative effects within the SM ($\theta_{\rm QCD}$) allow for CP violation in other types of interactions and in flavor diagonal processes. In particular, it is difficult to avoid flavor-diagonal phases in the supersymmetric framework. The fact that no electric dipole moment has been measured yet poses difficulties to many models with diagonal CP violation (and, of course, is responsible to the strong CP problem within the SM).

(vi) *CP is explicitly broken.* In various extensions of the scalar sector, it is possible to achieve spontaneous CP violation. It is very difficult to test this question experimentally.

This situation, where the Standard Model has a very unique and predictive description of CP violation, is the basis for the strong interest, experimental and theoretical, in CP violation.

III. MESON DECAYS

The phenomenology of CP violation is superficially different in K, D, B, and B_s decays. This is primarily because each of these systems is governed by a different balance between decay rates, oscillations, and lifetime splitting. However, the underlying mechanisms of CP violation are identical for all pseudoscalar mesons.

In this section we present a general formalism for, and classification of, CP violation in the decay of a pseudoscalar meson P that might be a charged or neutral K, D, B, or B_s meson. Subsequent sections describe the CP-violating phenomenology, approximations, and alternate formalisms that are specific to each system. We follow here closely the discussion in [55].

A. Charged and neutral meson decays

We define decay amplitudes of a pseudoscalar meson P (which could be charged or neutral) and its CP conjugate $\overline{P}$ to a multi-particle final state f and its CP conjugate $\overline{f}$ as

$$A_f = \langle f|\mathcal{H}|P\rangle \quad , \quad \overline{A}_f = \langle f|\mathcal{H}|\overline{P}\rangle \quad , \quad A_{\overline{f}} = \langle \overline{f}|\mathcal{H}|P\rangle \quad , \quad \overline{A}_{\overline{f}} = \langle \overline{f}|\mathcal{H}|\overline{P}\rangle \; , \tag{69}$$

where $\mathcal{H}$ is the Hamiltonian governing weak interactions. The action of CP on these states introduces phases ξ_P and ξ_f that depend on their flavor content, according to

$$\begin{aligned} CP\;|P\rangle \;&=\; e^{+i\xi_P}\,|\overline{P}\rangle \quad , \quad CP\;|f\rangle = e^{+i\xi_f}\,|\overline{f}\rangle \; , \\ CP\;|\overline{P}\rangle \;&=\; e^{-i\xi_P}\,|P\rangle \quad , \quad CP\;|\overline{f}\rangle = e^{-i\xi_f}\,|f\rangle \; , \end{aligned} \tag{70}$$

so that $(CP\,)^2 = 1$. The phases ξ_P and ξ_f are arbitrary and unphysical because of the flavor symmetry of the strong interaction. If CP is conserved by the dynamics, $[CP\,,\mathcal{H}] = 0$, then A_f and $\overline{A}_{\overline{f}}$ have the same magnitude and an arbitrary unphysical relative phase

$$\overline{A}_{\overline{f}} = e^{i(\xi_f - \xi_P)}\,A_f \; . \tag{71}$$

B. Neutral meson mixing

A state that is initially a superposition of P^0 and $\overline{P}^0$, say

$$|\psi(0)\rangle = a(0)|P^0\rangle + b(0)|\overline{P}^0\rangle \; , \tag{72}$$

will evolve in time acquiring components that describe all possible decay final states $\{f_1, f_2, \ldots\}$, that is,

$$|\psi(t)\rangle = a(t)|P^0\rangle + b(t)|\overline{P}^0\rangle + c_1(t)|f_1\rangle + c_2(t)|f_2\rangle + \cdots \; . \tag{73}$$

If we are interested in computing only the values of $a(t)$ and $b(t)$ (and not the values of all $c_i(t)$), and if the times t in which we are interested are much larger than the typical strong interaction scale, then we can use a much simplified formalism [56]. The simplified time evolution is determined by a 2×2 effective Hamiltonian $\mathcal{H}$ that is not Hermitian, since otherwise the mesons would only oscillate and not decay. Any complex matrix, such as $\mathcal{H}$, can be written in terms of Hermitian matrices M and Γ as

$$\mathcal{H} = M - \frac{i}{2}\,\Gamma \; . \tag{74}$$

M and Γ are associated with $(P^0, \overline{P}^0) \leftrightarrow (P^0, \overline{P}^0)$ transitions via off-shell (dispersive) and on-shell (absorptive) intermediate states, respectively. Diagonal elements of M and Γ are associated with the flavor-conserving transitions $P^0 \to P^0$ and $\overline{P}^0 \to \overline{P}^0$ while off-diagonal elements are associated with flavor-changing transitions $P^0 \leftrightarrow \overline{P}^0$.

The eigenvectors of $\mathcal{H}$ have well defined masses and decay widths. We introduce complex parameters $p_{L,H}$ and $q_{L,H}$ to specify the components of the strong interaction eigenstates, P^0 and $\overline{P}^0$, in the light (P_L) and heavy (P_H) mass eigenstates:

$$|P_{L,H}\rangle = p_{L,H}|P^0\rangle \pm q_{L,H}|\overline{P}^0\rangle \tag{75}$$

with the normalization $|p_{L,H}|^2 + |q_{L,H}|^2 = 1$. (Another possible choice, which is in standard usage for K mesons, defines the mass eigenstates according to their lifetimes: K_S for the short-lived and K_L for the long-lived state. The K_L is experimentally found to be the heavier state.) If either CP or CPT is a symmetry of $\mathcal{H}$ (independently of whether T is conserved or violated) then $M_{11} = M_{22}$ and $\Gamma_{11} = \Gamma_{22}$, and solving the eigenvalue problem for $\mathcal{H}$ yields $p_L = p_H \equiv p$ and $q_L = q_H \equiv q$ with

$$\left(\frac{q}{p}\right)^2 = \frac{M_{12}^* - (i/2)\Gamma_{12}^*}{M_{12} - (i/2)\Gamma_{12}} \,. \tag{76}$$

If either CP or T is a symmetry of $\mathcal{H}$ (independently of whether CPT is conserved or violated), then M_{12} and Γ_{12} are relatively real, leading to

$$\left(\frac{q}{p}\right)^2 = e^{2i\xi_P} \quad \Rightarrow \quad \left|\frac{q}{p}\right| = 1 \,, \tag{77}$$

where ξ_P is the arbitrary unphysical phase introduced in Eq. (70). If, and only if, CP is a symmetry of $\mathcal{H}$ (independently of CPT and T) then both of the above conditions hold, with the result that the mass eigenstates are orthogonal

$$\langle P_H|P_L\rangle = |p|^2 - |q|^2 = 0 \,. \tag{78}$$

From now on we assume that CPT is conserved.

The real and imaginary parts of the eigenvalues of $\mathcal{H}$ corresponding to $|P_{L,H}\rangle$ represent their masses and decay-widths, respectively. The mass difference Δm and the width difference $\Delta\Gamma$ are defined as follows:

$$\Delta m \equiv M_H - M_L, \quad \Delta\Gamma \equiv \Gamma_H - \Gamma_L. \tag{79}$$

Note that here Δm is positive by definition, while the sign of $\Delta\Gamma$ is to be experimentally determined. (Alternatively, one can use the states defined by their lifetimes to have $\Delta\Gamma \equiv \Gamma_S - \Gamma_L$ positive by definition.) The average mass and width are given by

$$m \equiv \frac{M_H + M_L}{2}, \quad \Gamma \equiv \frac{\Gamma_H + \Gamma_L}{2}. \tag{80}$$

It is useful to define dimensionless ratios x and y:

$$x \equiv \frac{\Delta m}{\Gamma}, \quad y \equiv \frac{\Delta\Gamma}{2\Gamma}. \tag{81}$$

Solving the eigenvalue equation gives

$$(\Delta m)^2 - \frac{1}{4}(\Delta\Gamma)^2 = (4|M_{12}|^2 - |\Gamma_{12}|^2), \qquad \Delta m \Delta\Gamma = 4\mathcal{R}e(M_{12}\Gamma_{12}^*). \tag{82}$$

C. CP-violating observables

All CP-violating observables in P and $\overline{P}$ decays to final states f and $\overline{f}$ can be expressed in terms of phase-convention-independent combinations of A_f, $\overline{A}_f$, $A_{\overline{f}}$ and $\overline{A}_{\overline{f}}$, together with, for neutral-meson decays only, q/p. CP violation in charged-meson decays depends only on the combination $|\overline{A}_{\overline{f}}/A_f|$, while CP violation in neutral-meson decays is complicated by $P^0 \leftrightarrow \overline{P}^0$ oscillations and depends, additionally, on $|q/p|$ and on $\lambda_f \equiv (q/p)(\overline{A}_f/A_f)$.

The decay-rates of the two neutral K mass eigenstates, K_S and K_L, are different enough ($\Gamma_S/\Gamma_L \sim 500$) that one can, in most cases, actually study their decays independently. For neutral D, B, and B_s mesons, however, values of $\Delta\Gamma/\Gamma$ are relatively small and so both mass eigenstates must be considered in their evolution. We denote the state of an initially pure $|P^0\rangle$ or $|\overline{P}^0\rangle$ after an elapsed proper time t as $|P^0_{\rm phys}(t)\rangle$ or $|\overline{P}^0_{\rm phys}(t)\rangle$, respectively. Using the effective Hamiltonian approximation, we obtain

$$\begin{aligned} |P^0_{\rm phys}(t)\rangle &= g_+(t)\,|P^0\rangle - (q/p)\; g_-(t)|\overline{P}^0\rangle, \\ |\overline{P}^0_{\rm phys}(t)\rangle &= g_+(t)\,|\overline{P}^0\rangle - (p/q)\; g_-(t)|P^0\rangle\;, \end{aligned} \tag{83}$$

where

$$g_\pm(t) \equiv \frac{1}{2}\left(e^{-im_H t - \frac{1}{2}\Gamma_H t} \pm e^{-im_L t - \frac{1}{2}\Gamma_L t}\right). \tag{84}$$

One obtains the following time-dependent decay rates:

$$\frac{d\Gamma[P^0_{\rm phys}(t) \to f]/dt}{e^{-\Gamma t}\mathcal{N}_f} = \left(|A_f|^2 + |(q/p)\overline{A}_f|^2\right)\cosh(y\Gamma t) + \left(|A_f|^2 - |(q/p)\overline{A}_f|^2\right)\cos(x\Gamma t)$$

$$+\ 2\,\mathcal{R}e((q/p)A_f^*\overline{A}_f)\sinh(y\Gamma t) - 2\,\mathcal{I}m((q/p)A_f^*\overline{A}_f)\sin(x\Gamma t)\ , \qquad (85)$$

$$\frac{d\Gamma[\overline{P}^0_{\rm phys}(t)\to f]/dt}{e^{-\Gamma t}\mathcal{N}_f} = \Big(|(p/q)A_f|^2 + |\overline{A}_f|^2\Big)\cosh(y\Gamma t) - \Big(|(p/q)A_f|^2 - |\overline{A}_f|^2\Big)\cos(x\Gamma t)$$
$$+\ 2\,\mathcal{R}e((p/q)A_f\overline{A}_f^*)\sinh(y\Gamma t) - 2\,\mathcal{I}m((p/q)A_f\overline{A}_f^*)\sin(x\Gamma t)\ , \qquad (86)$$

where $\mathcal{N}_f$ is a time-independnet normalization factor. Decay rates to the CP-conjugate final state $\overline{f}$ are obtained analogously, with $\mathcal{N}_f = \mathcal{N}_{\overline{f}}$ and the substitutions $A_f \to A_{\overline{f}}$ and $\overline{A}_f \to \overline{A}_{\overline{f}}$ in Eqs. (85,86). Terms proportional to $|A_f|^2$ or $|\overline{A}_f|^2$ are associated with decays that occur without any net $P \leftrightarrow \overline{P}$ oscillation, while terms proportional to $|(q/p)\overline{A}_f|^2$ or $|(p/q)A_f|^2$ are associated with decays following a net oscillation. The $\sinh(y\Gamma t)$ and $\sin(x\Gamma t)$ terms of Eqs. (85,86) are associated with the interference between these two cases. Note that, in multi-body decays, amplitudes are functions of phase-space variables. Interference may be present in some regions but not others, and is strongly influenced by resonant substructure.

D. Classification of CP-violating effects

We distinguish three types of CP-violating effects in meson decays [57]:

[I] CP violation in decay is defined by

$$|\overline{A}_{\overline{f}}/A_f| \neq 1\ . \qquad (87)$$

In charged meson decays, where mixing effects are absent, this is the only possible source of CP asymmetries:

$$\mathcal{A}_{f^\pm} \equiv \frac{\Gamma(P^-\to f^-) - \Gamma(P^+\to f^+)}{\Gamma(P^-\to f^-) + \Gamma(P^+\to f^+)} = \frac{|\overline{A}_{f^-}/A_{f^+}|^2 - 1}{|\overline{A}_{f^-}/A_{f^+}|^2 + 1}\ . \qquad (88)$$

[II] CP violation in mixing is defined by

$$|q/p| \neq 1\ . \qquad (89)$$

In charged-current semileptonic neutral meson decays $P, \overline{P} \to \ell^\pm X$ (taking $|A_{\ell^+X}| = |\overline{A}_{\ell^-X}|$ and $A_{\ell^-X} = \overline{A}_{\ell^+X} = 0$, as is the case in the Standard Model, to lowest order in G_F, and in most of its reasonable extensions), this is the only source of CP violation, and can be measured via the asymmetry of "wrong-sign" decays induced by oscillations:

$$\mathcal{A}_{\rm SL}(t) \equiv \frac{d\Gamma/dt[\overline{P}^0_{\rm phys}(t)\to \ell^+X] - d\Gamma/dt[P^0_{\rm phys}(t)\to \ell^-X]}{d\Gamma/dt[\overline{P}^0_{\rm phys}(t)\to \ell^+X] + d\Gamma/dt[P^0_{\rm phys}(t)\to \ell^-X]} = \frac{1-|q/p|^4}{1+|q/p|^4}. \qquad (90)$$

Note that this asymmetry of time-dependent decay rates is actually time independent.

[III] CP violation in interference between a decay without mixing, $P^0 \to f$, and a decay with mixing, $P^0 \to \overline{P}^0 \to f$ (such an effect occurs only in decays to final states that are common to P^0 and $\overline{P}^0$, including all CP eigenstates), is defined by

$$\mathcal{I}m(\lambda_f) \neq 0 \,, \tag{91}$$

with

$$\lambda_f \equiv \frac{q}{p}\frac{\overline{A}_f}{A_f} \,. \tag{92}$$

This form of CP violation can be observed, for example, using the asymmetry of neutral meson decays into final CP eigenstates f_{CP}

$$\mathcal{A}_{f_{CP}}\,(t) \equiv \frac{d\Gamma/dt[\overline{P}^0_{\rm phys}(t) \to f_{CP}\,] - d\Gamma/dt[P^0_{\rm phys}(t) \to f_{CP}\,]}{d\Gamma/dt[\overline{P}^0_{\rm phys}(t) \to f_{CP}\,] + d\Gamma/dt[P^0_{\rm phys}(t) \to f_{CP}\,]} \,. \tag{93}$$

If $\Delta\Gamma = 0$ and $|q/p| = 1$, as expected to a good approximation for B mesons but not for K mesons, then $\mathcal{A}_{f_{CP}}$ has a particularly simple form [58, 59, 60]:

$$\begin{aligned} \mathcal{A}_f(t) &= S_f \sin(\Delta m t) - C_f \cos(\Delta m t), \\ S_f &\equiv \frac{2\,\mathcal{I}m(\lambda_f)}{1+|\lambda_f|^2}, \quad C_f \equiv \frac{1-|\lambda_f|^2}{1+|\lambda_f|^2} \,, \end{aligned} \tag{94}$$

If, in addition, the decay amplitudes fulfill $|\overline{A}_{f_{CP}}\,| = |A_{f_{CP}}\,|$, the interference between decays with and without mixing is the only source of the asymmetry and

$$\mathcal{A}_{f_{CP}}\,(t) = \mathcal{I}m(\lambda_{f_{CP}}\,)\sin(x\Gamma t). \tag{95}$$

IV. THEORETICAL INTERPRETATION: GENERAL CONSIDERATIONS

Consider the $P \to f$ decay amplitude A_f, and the CP conjugate process, $\overline{P} \to \overline{f}$, with decay amplitude $\overline{A}_{\overline{f}}$. There are two types of phases that may appear in these decay amplitudes. Complex parameters in any Lagrangian term that contributes to the amplitude will appear in complex conjugate form in the CP-conjugate amplitude. Thus their phases appear in A_f and $\overline{A}_{\overline{f}}$ with opposite signs. In the Standard Model, these phases occur only in the couplings of the $W^\pm$ bosons and hence are often called "weak phases". The weak phase of any single term is convention dependent. However, the difference between the weak phases in two different terms in A_f is convention independent. A second type of phase can

appear in scattering or decay amplitudes even when the Lagrangian is real. Their origin is the possible contribution from intermediate on-shell states in the decay process. Since these phases are generated by CP-invariant interactions, they are the same in A_f and $\overline{A}_{\overline{f}}$. Usually the dominant rescattering is due to strong interactions and hence the designation "strong phases" for the phase shifts so induced. Again, only the relative strong phases between different terms in the amplitude are physically meaningful.

The 'weak' and 'strong' phases discussed here appear in addition to the 'spurious' CP-transformation phases of Eq. (71). Those spurious phases are due to an arbitrary choice of phase convention, and do not originate from any dynamics or induce any CP violation. For simplicity, we set them to zero from here on.

It is useful to write each contribution a_i to A_f in three parts: its magnitude $|a_i|$, its weak phase ϕ_i, and its strong phase δ_i. If, for example, there are two such contributions, $A_f = a_1 + a_2$, we have

$$\begin{aligned} A_f &= |a_1| e^{i(\delta_1+\phi_1)} + |a_2| e^{i(\delta_2+\phi_2)}, \\ \overline{A}_{\overline{f}} &= |a_1| e^{i(\delta_1-\phi_1)} + |a_2| e^{i(\delta_2-\phi_2)}. \end{aligned} \tag{96}$$

Similarly, for neutral meson decays, it is useful to write

$$M_{12} = |M_{12}| e^{i\phi_M} \quad , \quad \Gamma_{12} = |\Gamma_{12}| e^{i\phi_\Gamma} \ . \tag{97}$$

Each of the phases appearing in Eqs. (96,97) is convention dependent, but combinations such as $\delta_1 - \delta_2$, $\phi_1 - \phi_2$, $\phi_M - \phi_\Gamma$ and $\phi_M + \phi_1 - \overline{\phi}_1$ (where $\overline{\phi}_1$ is a weak phase contributing to $\overline{A}_f$) are physical.

It is now straightforward to evaluate the various asymmetries in terms of the theoretical parameters introduced here. We will do so with approximations that are often relevant to the most interesting measured asymmetries.

1. The CP asymmetry in charged meson decays [Eq. (88)] is given by

$$\mathcal{A}_{f^\pm} = -\frac{2|a_1 a_2| \sin(\delta_2 - \delta_1) \sin(\phi_2 - \phi_1)}{|a_1|^2 + |a_2|^2 + 2|a_1 a_2| \cos(\delta_2 - \delta_1) \cos(\phi_2 - \phi_1)}. \tag{98}$$

The quantity of most interest to theory is the weak phase difference $\phi_2 - \phi_1$. Its extraction from the asymmetry requires, however, that the amplitude ratio and the strong phase are known. Both quantities depend on non-perturbative hadronic parameters that are difficult to calculate.

2. In the approximation that $|\Gamma_{12}/M_{12}| \ll 1$ (valid for B and B_s mesons), the CP asymmetry in semileptonic neutral-meson decays [Eq. (90)] is given by

$$\mathcal{A}_{\rm SL} = -\left|\frac{\Gamma_{12}}{M_{12}}\right| \sin(\phi_M - \phi_\Gamma). \tag{99}$$

The quantity of most interest to theory is the weak phase $\phi_M - \phi_\Gamma$. Its extraction from the asymmetry requires, however, that $|\Gamma_{12}/M_{12}|$ is known. This quantity depends on long distance physics that is difficult to calculate.

3. In the approximations that only a single weak phase contributes to decay, $A_f = |a_f| e^{i(\delta_f + \phi_f)}$, and that $|\Gamma_{12}/M_{12}| = 0$, we obtain $|\lambda_f| = 1$ and the CP asymmetries in decays to a final CP eigenstate f [Eq. (93)] with eigenvalue $\eta_f = \pm 1$ are given by

$$\mathcal{A}_{f_{CP}}(t) = \mathcal{I}m(\lambda_f)\ \sin(\Delta m t) \ \text{ with } \ \mathcal{I}m(\lambda_f) = \eta_f \sin(\phi_M + 2\phi_f). \tag{100}$$

Note that the phase so measured is purely a weak phase, and no hadronic parameters are involved in the extraction of its value from $\mathcal{I}m(\lambda_f)$.

The discussion above allows us to introduce another classification:

1. **Direct CP violation** is one that cannot be accounted for by just $\phi_M \neq 0$. CP violation in decay (type I) belongs to this class.

2. **Indirect CP violation** is consistent with taking $\phi_M \neq 0$ and setting all other CP violating phases to zero. CP violation in mixing (type II) belongs to this class.

As concerns type III CP violation, observing $\eta_{f_1}\mathcal{I}m(\lambda_{f_1}) \neq \eta_{f_2}\mathcal{I}m(\lambda_{f_2})$ (for the same decaying meson and two different final CP eigenstates f_1 and f_2) would establish direct CP violation. The significance of this classification is related to theory. In superweak models [20], CP violation appears only in diagrams that contribute to M_{12}, hence they predict that there is no direct CP violation. In most models and, in particular, in the Standard Model, CP violation is both direct and indirect. The experimental observation of $\epsilon' \neq 0$ (see Section V) excluded the superweak scenario.

V. K DECAYS

CP violation was discovered in $K \to \pi\pi$ decays in 1964 [2]. The same mode provided the first evidence for direct CP violation [3, 4, 5].

The decay amplitudes actually measured in neutral K decays refer to the mass eigenstates K_L and K_S rather than to the K and $\overline{K}$ states referred to in Eq. (69). We define CP-violating amplitude ratios for two-pion final states,

$$\eta_{00} \equiv \frac{\langle \pi^0\pi^0|\mathcal{H}|K_L\rangle}{\langle \pi^0\pi^0|\mathcal{H}|K_S\rangle} \quad , \quad \eta_{+-} \equiv \frac{\langle \pi^+\pi^-|\mathcal{H}|K_L\rangle}{\langle \pi^+\pi^-|\mathcal{H}|K_S\rangle} \; . \tag{101}$$

Another important observable is the asymmetry of time-integrated semileptonic decay rates:

$$\delta_L \equiv \frac{\Gamma(K_L \to \ell^+\nu_\ell\pi^-) - \Gamma(K_L \to \ell^-\bar{\nu}_\ell\pi^+)}{\Gamma(K_L \to \ell^+\nu_\ell\pi^-) + \Gamma(K_L \to \ell^-\bar{\nu}_\ell\pi^+)} \; . \tag{102}$$

CP violation has been observed as an appearance of K_L decays to two-pion final states [53],

$$\begin{aligned} |\eta_{00}| &= (2.275 \pm 0.017) \times 10^{-3}, \\ |\eta_{+-}| &= (2.286 \pm 0.017) \times 10^{-3}, \\ |\eta_{00}/\eta_{+-}| &= 0.9950 \pm 0.0008, \end{aligned} \tag{103}$$

and, assuming CPT, $\phi_{+-} = \phi_{00} = 43.49^\circ \pm 0.07^\circ$ (ϕ_{ij} is the phase of the amplitude ratio η_{ij}). CP violation has also been observed in semileptonic K_L decays [53]

$$\delta_L = (3.27 \pm 0.12) \times 10^{-3} \; , \tag{104}$$

where δ_L is a weighted average of muon and electron measurements, as well as in K_L decays to $\pi^+\pi^-\gamma$ and $\pi^+\pi^-e^+e^-$ [53].

Historically, CP violation in neutral K decays has been described in terms of parameters ϵ and ϵ'. The observables η_{00}, η_{+-}, and δ_L are related to these parameters, and to those of Section III, by

$$\begin{aligned} \eta_{00} &= \tfrac{1-\lambda_{\pi^0\pi^0}}{1+\lambda_{\pi^0\pi^0}} &&= \epsilon - 2\epsilon' \; , \\ \eta_{+-} &= \tfrac{1-\lambda_{\pi^+\pi^-}}{1+\lambda_{\pi^+\pi^-}} &&= \epsilon + \epsilon' \; , \\ \delta_L &= \tfrac{1-|q/p|^2}{1+|q/p|^2} &&= \frac{2\mathcal{R}e(\epsilon)}{1+|\epsilon|^2} \; , \end{aligned} \tag{105}$$

where, in the last line, we have assumed that $|A_{\ell^+\nu_\ell\pi^-}| = |\overline{A}_{\ell^-\bar{\nu}_\ell\pi^+}|$ and $|A_{\ell^-\bar{\nu}_\ell\pi^+}| = |\overline{A}_{\ell^+\nu_\ell\pi^-}| = 0$. A fit to the $K \to \pi\pi$ data yields [53]

$$\begin{aligned} |\epsilon| &= (2.283 \pm 0.017) \times 10^{-3} \; , \\ \mathcal{R}e(\epsilon'/\epsilon) &= (1.67 \pm 0.26) \times 10^{-3} \; . \end{aligned} \tag{106}$$

In discussing two-pion final states, it is useful to express the amplitudes $A_{\pi^0\pi^0}$ and $A_{\pi^+\pi^-}$ in terms of their isospin components via

$$\begin{aligned} A_{\pi^0\pi^0} &= \sqrt{\frac{1}{3}}|A_0|e^{i(\delta_0+\phi_0)} - \sqrt{\frac{2}{3}}|A_2|e^{i(\delta_2+\phi_2)}, \\ A_{\pi^+\pi^-} &= \sqrt{\frac{2}{3}}|A_0|e^{i(\delta_0+\phi_0)} + \sqrt{\frac{1}{3}}|A_2|e^{i(\delta_2+\phi_2)} , \end{aligned} \tag{107}$$

where we parameterize the amplitude $A_I(\overline{A}_I)$ for $K^0(\overline{K}^0)$ decay into two pions with total isospin $I = 0$ or 2 as

$$A_I \equiv \langle(\pi\pi)_I|\mathcal{H}|K^0\rangle = |A_I|e^{i(\delta_I+\phi_I)} , \quad \overline{A}_I \equiv \langle(\pi\pi)_I|\mathcal{H}|\overline{K}^0\rangle = |A_I|e^{i(\delta_I-\phi_I)} . \tag{108}$$

The smallness of $|\eta_{00}|$ and $|\eta_{+-}|$ allows us to approximate

$$\epsilon \simeq \frac{1}{2}(1 - \lambda_{(\pi\pi)_{I=0}}), \qquad \epsilon' \simeq \frac{1}{6}\left(\lambda_{\pi^0\pi^0} - \lambda_{\pi^+\pi^-}\right) . \tag{109}$$

The parameter ϵ represents indirect CP violation, while ϵ' parameterizes direct CP violation: $\mathcal{R}e(\epsilon')$ measures CP violation in decay (type I), $\mathcal{R}e(\epsilon)$ measures CP violation in mixing (type II), and $\mathcal{I}m(\epsilon)$ and $\mathcal{I}m(\epsilon')$ measure the interference between decays with and without mixing (type III).

The following expressions for ϵ and ϵ' are useful for theoretical evaluations:

$$\epsilon \simeq \frac{e^{i\pi/4}}{\sqrt{2}}\frac{\mathcal{I}m(M_{12})}{\Delta m}, \qquad \epsilon' = \frac{i}{\sqrt{2}}\left|\frac{A_2}{A_0}\right| e^{i(\delta_2-\delta_0)}\sin(\phi_2 - \phi_0). \tag{110}$$

The expression for ϵ is only valid in a phase convention where $\phi_2 = 0$, corresponding to a real $V_{ud}V_{us}^*$, and in the approximation that also $\phi_0 = 0$. The phase of $\pi/4$ is approximate, and determined by hadronic parameters, $\arg\epsilon \approx \arctan(-2\Delta m/\Delta\Gamma)$, independently of the electroweak model. The calculation of ϵ benefits from the fact that $\mathcal{I}m(M_{12})$ is dominated by short distance physics. Consequently, the main source of uncertainty in theoretical interpretations of ϵ are the values of matrix elements such as $\langle K^0|(\bar{s}d)_{V-A}(\bar{s}d)_{V-A}|\overline{K}^0\rangle$. The expression for ϵ' is valid to first order in $|A_2/A_0| \sim 1/20$. The phase of ϵ' is experimentally determined, $\pi/2 + \delta_2 - \delta_0 \approx \pi/4$ and is independent of the electroweak model. Note that, accidentally, ϵ'/ϵ is real to a good approximation.

A future measurement of much interest is that of CP violation in the rare $K \to \pi\nu\bar{\nu}$ decays. The signal for CP violation is simply observing the $K_L \to \pi^0\nu\bar{\nu}$ decay. The effect here is that of interference between decays with and without mixing (type III) [61]:

$$\frac{\Gamma(K_L \to \pi^0\nu\bar{\nu})}{\Gamma(K^+ \to \pi^+\nu\bar{\nu})} = \frac{1}{2}\left[1 + |\lambda_{\pi\nu\bar{\nu}}|^2 - 2\,\mathcal{R}e(\lambda_{\pi\nu\bar{\nu}})\right] \simeq 1 - \mathcal{R}e(\lambda_{\pi\nu\bar{\nu}}), \tag{111}$$

where in the last equation we neglect CP violation in decay and in mixing (expected, model independently, to be of order 10^{-5} and 10^{-3}, respectively). Such a measurement would be experimentally very challenging and theoretically very rewarding [62]. Similar to the CP asymmetry in $B \to J/\psi K_S$, the CP violation in $K \to \pi\nu\bar{\nu}$ decay is predicted to be large (the ratio in Eq. (111) is not CKM suppressed) and can be very cleanly interpreted.

Within the Standard Model, the $K_L \to \pi^0\nu\bar{\nu}$ decay is dominated by an intermediate top quark contribution and, consequently, can be cleanly interpreted in terms of CKM parameters [63]. (For the charged mode, $K^+ \to \pi^+\nu\bar{\nu}$, the contribution from an intermediate charm quark is not negligible and constitutes a source of hadronic uncertainty.) In particular, $\mathcal{B}(K_L \to \pi^0\nu\bar{\nu})$ provides a theoretically clean way to determine the Wolfenstein parameter η [64]:

$$\mathcal{B}(K_L \to \pi^0\nu\bar{\nu}) = \kappa_L X^2 (m_t^2/m_W^2) A^4 \eta^2, \tag{112}$$

where $\kappa_L = 1.80 \times 10^{-10}$ incorporates the value of the four-fermion matrix element which is deduced, using isospin relations, from $\mathcal{B}(K^+ \to \pi^0 e^+ \nu)$, and $X(m_t^2/m_W^2)$ is a known function of the top mass.

A. Implications of ε_K

The measurement of ε_K has had (and still has) important implications. Two implications of historical importance are the following:

(i) CP violation was discovered through the measurement of ε_K. Hence this measurement played a significant role in the history of particle physics.

(ii) The observation of $\varepsilon_K \neq 0$ led to the prediction that a third generation must exist, so that CP is violated in the Standard Model. This provides an excellent example of how precision measurements at low energy can lead to the discovery of new physics (even if, at present, this new physics is old...)

Within the Standard Model, $\mathcal{I}m(M_{12})$ is accounted for by box diagrams:

$$\varepsilon_K = e^{i\pi/4} C_\varepsilon B_K \mathcal{I}m(V_{ts}^* V_{td}) \left\{ \mathcal{R}e(V_{cs}^* V_{cd})[\eta_1 S_0(x_c) - \eta_3 S_0(x_c, x_t)] - \mathcal{R}e(V_{ts}^* V_{td}) \eta_2 S_0(x_t) \right\}, \tag{113}$$

where $C_\varepsilon \equiv \frac{G_F^2 f_K^2 m_K m_W^2}{6\sqrt{2}\pi^2 \Delta m_K}$ is a well known parameter, the η_i are QCD correction factors, S_0 is a kinematic factor, and B_K is the ratio between the matrix element of the four quark

operator and its value in the vacuum insertion approximation. The measurement of ε_K has the following implications within the SM:

- This measurement allowed one to set the value of $\delta_{\rm KM}$. Furthermore, by implying that $\delta_{\rm KM} = \mathcal{O}(1)$, it made the KM mechanism plausible. Having been the single measured CP violating parameter it could not, however, serve as a test of the KM mechanism. More precisely, a value of $|\varepsilon_K| \gg 10^{-3}$ would have invalidated the KM mechanism, but any value $|\varepsilon_K| \lesssim 10^{-3}$ was acceptable. It is only the combination of the new measurements in B decays (particularly $S_{\psi K_S}$) with ε_K that provides the first precision test of the KM mechanism.

- Within the SM, the smallness of ε_K is not related to suppression of CP violation but rather to suppression of flavor violation. Specifically, it is the smallness of the ratio $|(V_{td}V_{ts})/(V_{ud}V_{us})| \sim \lambda^4$ that explains $|\varepsilon_K| \sim 10^{-3}$.

- Until recently, the measured value of ε_K provided a unique type of information on the KM phase. For example, the measurement of $\mathcal{R}e(\varepsilon_K) > 0$ tells us that $\eta > 0$ and excludes the lower half of the $\rho - \eta$ plane. Such information cannot be obtained from any CP conserving observable.

- The ε_K constraint in Eq. (113) gives hyperbole in the $\rho - \eta$ plane. It is shown in Fig. 2. The measured value is consistent with all other CKM-related measurements and further narrows the allowed region.

Beyond the SM, ε_K is an extremely powerful probe of new physics. This aspect will be discussed later.

VI. D DECAYS

Unlike the case of neutral K, B, and B_s mixing, $D^0 - \overline{D}^0$ mixing has not yet been observed. Long-distance contributions make it difficult to calculate the Standard Model prediction for the $D^0 - \overline{D}^0$ mixing parameters. Therefore, the goal of the search for $D^0 - \overline{D}^0$ mixing is not to constrain the CKM parameters but rather to probe new physics. Here CP violation plays an important role [65]. Within the Standard Model, the CP-violating effects are predicted to be negligibly small since the mixing and the relevant decays are described, to an excellent

approximation, by physics of the first two generations. Observation of CP violation in $D^0 - \overline{D}^0$ mixing (at a level much higher than $\mathcal{O}(10^{-3})$) will constitute an unambiguous signal of new physics.[2] At present, the most sensitive searches involve the $D \to K^+K^-$ and $D \to K^\pm\pi^\mp$ modes.

The neutral D mesons decay via a singly-Cabibbo-suppressed transition to the CP eigenstate K^+K^-. Since the decay proceeds via a Standard-Model tree diagram, it is very likely unaffected by new physics and, furthermore, dominated by a single weak phase. It is safe then to assume that direct CP violation plays no role here [68, 69]. In addition, given the experimental bounds [53], $x \equiv \Delta m/\Gamma \lesssim 0.03$ and $y \equiv \Delta\Gamma/(2\Gamma) = 0.008 \pm 0.005$, we can expand the decay rates to first order in these parameters. Using Eq. (85) with these assumptions and approximations yields, for $xt, yt \lesssim \Gamma^{-1}$,

$$\begin{aligned}
\Gamma[D^0_{\rm phys}(t) \to K^+K^-] &= e^{-\Gamma t}|A_{KK}|^2[1 - |q/p|(y\cos\phi_D - x\sin\phi_D)\Gamma t], \\
\Gamma[\overline{D}^0_{\rm phys}(t) \to K^+K^-] &= e^{-\Gamma t}|A_{KK}|^2[1 - |p/q|(y\cos\phi_D + x\sin\phi_D)\Gamma t],
\end{aligned} \tag{114}$$

where ϕ_D is defined via $\lambda_{K^+K^-} = -|q/p|e^{i\phi_D}$. (In the limit of CP conservation, choosing $\phi_D = 0$ is equivalent to defining the mass eigenstates by their CP eigenvalue: $|D_\mp\rangle = p|D^0\rangle \pm q|\overline{D}^0\rangle$, with $D_-(D_+)$ being the CP -odd (CP-even) state; that is, the state that does not (does) decay into K^+K^-.) Given the small values of x and y, the time dependences of the rates in Eq. (114) can be recast into purely exponential forms, but with modified decay-rate parameters [70]:

$$\begin{aligned}
\Gamma_{D^0\to K^+K^-} &= \Gamma \times [1 + |q/p|(y\cos\phi_D - x\sin\phi_D)]\,, \\
\Gamma_{\overline{D}^0\to K^+K^-} &= \Gamma \times [1 + |p/q|(y\cos\phi_D + x\sin\phi_D)]\,.
\end{aligned} \tag{115}$$

One can define CP-conserving and CP-violating combinations of these two observables (normalized to the true width Γ):

$$\begin{aligned}
Y &\equiv \frac{\Gamma_{\overline{D}^0\to K^+K^-} + \Gamma_{D^0\to K^+K^-}}{2\Gamma} - 1 \\
&= \frac{|q/p| + |p/q|}{2}\, y\cos\phi_D - \frac{|q/p| - |p/q|}{2}\, x\sin\phi_D, \\
\Delta Y &\equiv \frac{\Gamma_{\overline{D}^0\to K^+K^-} - \Gamma_{D^0\to K^+K^-}}{2\Gamma} \\
&= \frac{|q/p| + |p/q|}{2}\, x\sin\phi_D - \frac{|q/p| - |p/q|}{2}\, y\cos\phi_D.
\end{aligned} \tag{116}$$

[2] In contrast, neither $y_D \sim 10^{-2}$ [66], nor $x_D \sim 10^{-2}$ [67] can be considered as evidence for new physics.

In the limit of CP conservation (and, in particular, within the Standard Model), $Y = y$ and $\Delta Y = 0$.

The $K^\pm\pi^\mp$ states are not CP eigenstates but they are still common final states for D^0 and $\overline{D}^0$ decays. Since $D^0(\overline{D}^0) \to K^-\pi^+$ is a Cabibbo-favored (doubly-Cabibbo-suppressed) process, these processes are particularly sensitive to x and/or $y = \mathcal{O}(\lambda^2)$. Taking into account that $|\lambda_{K^-\pi^+}|, |\lambda^{-1}_{K^+\pi^-}| \ll 1$ and $x, y \ll 1$, assuming that there is no direct CP violation (again, these are Standard Model tree level decays dominated by a single weak phase) and expanding the time dependent rates for $xt, yt \lesssim \Gamma^{-1}$, one obtains

$$\begin{aligned}
\frac{\Gamma[D^0_{\rm phys}(t) \to K^+\pi^-]}{\Gamma[\overline{D}^0_{\rm phys}(t) \to K^+\pi^-]} &= r_d^2 + r_d \left|\frac{q}{p}\right| (y'\cos\phi_D - x'\sin\phi_D)\Gamma t + \left|\frac{q}{p}\right|^2 \frac{y^2+x^2}{4}(\Gamma t)^2, \\
\frac{\Gamma[\overline{D}^0_{\rm phys}(t) \to K^-\pi^+]}{\Gamma[D^0_{\rm phys}(t) \to K^-\pi^+]} &= r_d^2 + r_d \left|\frac{p}{q}\right| (y'\cos\phi_D + x'\sin\phi_D)\Gamma t + \left|\frac{p}{q}\right|^2 \frac{y^2+x^2}{4}(\Gamma t)^2, \quad (117)
\end{aligned}$$

where

$$\begin{aligned}
y' &\equiv y\cos\delta - x\sin\delta, \\
x' &\equiv x\cos\delta + y\sin\delta. \quad (118)
\end{aligned}$$

The weak phase ϕ_D is the same as that of Eq. (114) (a consequence of the absence of direct CP violation), δ is a strong phase difference for these processes, and $r_d = \mathcal{O}(\tan^2\theta_c)$ is the amplitude ratio, $r_d = |\overline{A}_{K^-\pi^+}/A_{K^-\pi^+}| = |A_{K^+\pi^-}/\overline{A}_{K^+\pi^-}|$, that is, $\lambda_{K^-\pi^+} = r_d(q/p)e^{-i(\delta-\phi_D)}$ and $\lambda^{-1}_{K^+\pi^-} = r_d(p/q)e^{-i(\delta+\phi_D)}$. By fitting to the six coefficients of the various times dependences, one can extract r_d, $|q/p|$, (x^2+y^2), $y'\cos\phi_D$, and $x'\sin\phi_D$. In particular, finding CP violation, that is, $|q/p| \neq 1$ and/or $\sin\phi_D \neq 0$, would constitute evidence for new physics.

VII. B DECAYS

The upper bound on the CP asymmetry in semileptonic B decays [53] implies that CP violation in $B^0 - \overline{B}^0$ mixing is a small effect [we use $\mathcal{A}_{\rm SL}/2 \approx 1 - |q/p|$, see Eq. (90)]:

$$\mathcal{A}_{\rm SL} = (0.3 \pm 1.3) \times 10^{-2} \quad \Longrightarrow \quad |q/p| = 0.998 \pm 0.007. \quad (119)$$

The Standard Model prediction is

$$\mathcal{A}_{\rm SL} = \mathcal{O}\left(\frac{m_c^2}{m_t^2}\sin\beta\right) \lesssim 0.001. \quad (120)$$

In models where Γ_{12}/M_{12} is approximately real, such as the Standard Model, an upper bound on $\Delta\Gamma/\Delta m \approx \mathcal{R}e(\Gamma_{12}/M_{12})$ provides yet another upper bound on the deviation of $|q/p|$ from one. This constraint does not hold if Γ_{12}/M_{12} is approximately imaginary.

The small deviation (less than one percent) of $|q/p|$ from 1 implies that, at the present level of experimental precision, CP violation in B mixing is a negligible effect. Thus, for the purpose of analyzing CP asymmetries in hadronic B decays, we can use

$$\lambda_f = e^{-i\phi_B}(\overline{A}_f/A_f) \ , \tag{121}$$

where ϕ_B refers to the phase of M_{12} [see Eq. (97)]. Within the Standard Model, the corresponding phase factor is given by

$$e^{-i\phi_B} = (V_{tb}^* V_{td})/(V_{tb} V_{td}^*) \ . \tag{122}$$

Some of the most interesting decays involve final states that are common to B^0 and $\overline{B}^0$ [71, 72, 73]. Here Eq. (94) applies [58, 59, 60]. The processes of interest proceed via quark transitions of the form $\bar{b} \to \bar{q}q\bar{q}'$ with $q' = s$ or d. For $q = c$ or u, there are contributions from both tree (t) and penguin (p^{q_u}, where $q_u = u, c, t$ is the quark in the loop) diagrams (see Fig. 3) which carry different weak phases:

$$A_f = \left(V_{qb}^* V_{qq'}\right) t_f + \sum_{q_u=u,c,t} \left(V_{q_u b}^* V_{q_u q'}\right) p_f^{q_u} \ . \tag{123}$$

(The distinction between tree and penguin contributions is a heuristic one, the separation by the operator that enters is more precise. For a detailed discussion of the more complete operator product approach, which also includes higher order QCD corrections, see, for example, ref. [74].) Using CKM unitarity, these decay amplitudes can always be written in terms of just two CKM combinations. For example, for $f = \pi\pi$, which proceeds via $\bar{b} \to \bar{u}u\bar{d}$ transition, we can write

$$A_{\pi\pi} = (V_{ub}^* V_{ud})\, T_{\pi\pi} + (V_{tb}^* V_{td})\, P_{\pi\pi}^t, \tag{124}$$

where $T_{\pi\pi} = t_{\pi\pi} + p_{\pi\pi}^u - p_{\pi\pi}^c$ and $P_{\pi\pi}^t = p_{\pi\pi}^t - p_{\pi\pi}^c$. CP violating phases in Eq. (124) appear only in the CKM elements, so that

$$\frac{\overline{A}_{\pi\pi}}{A_{\pi\pi}} = \frac{(V_{ub} V_{ud}^*)\, T_{\pi\pi} + (V_{tb} V_{td}^*)\, P_{\pi\pi}^t}{(V_{ub}^* V_{ud})\, T_{\pi\pi} + (V_{tb}^* V_{td})\, P_{\pi\pi}^t}. \tag{125}$$

For $f = J/\psi K$, which proceeds via $\bar{b} \to \bar{c}c\bar{s}$ transition, we can write

$$A_{\psi K} = (V_{cb}^* V_{cs})\, T_{\psi K} + (V_{ub}^* V_{us})\, P_{\psi K}^u, \tag{126}$$

where $T_{\psi K} = t_{\psi K} + p^c_{\psi K} - p^t_{\psi K}$ and $P^u_{\psi K} = p^u_{\psi K} - p^t_{\psi K}$. A subtlety arises in this decay that is related to the fact that B^0 decays into $J/\psi K^0$ while $\overline{B}^0$ decays into $J/\psi \overline{K}^0$. A common final state, e.g. $J/\psi K_S$, is reached only via $K^0 - \overline{K}^0$ mixing. Consequently, the phase factor corresponding to neutral K mixing, $e^{-i\phi_K} = (V^*_{cd}V_{cs})/(V_{cd}V^*_{cs})$, plays a role:

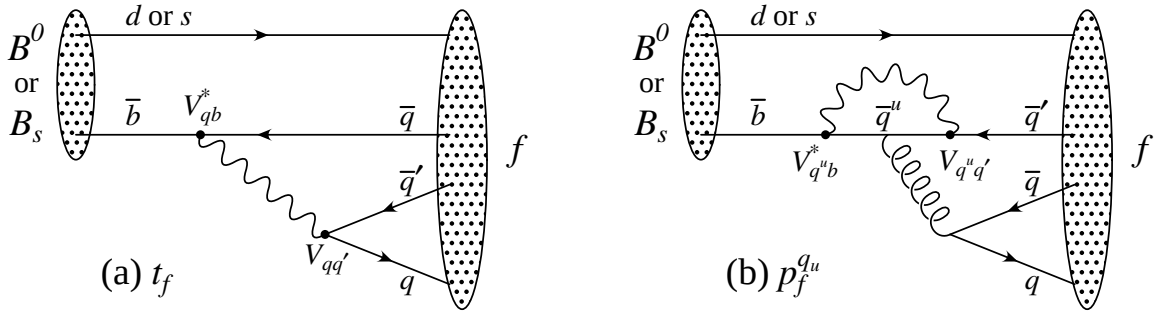
$$\frac{\overline{A}_{\psi K_S}}{A_{\psi K_S}} = -\frac{(V_{cb}V^*_{cs})\,T_{\psi K} + (V_{ub}V^*_{us})\,P^u_{\psi K}}{(V^*_{cb}V_{cs})\,T_{\psi K} + (V^*_{ub}V_{us})\,P^u_{\psi K}} \times \frac{V^*_{cd}V_{cs}}{V_{cd}V^*_{cs}}. \tag{127}$$

For $q = s$ or d, there are only penguin contributions to A_f, that is, $t_f = 0$ in Eq. (123). (The tree $\bar{b} \to \bar{u}u\bar{q}'$ transition followed by $\bar{u}u \to \bar{q}q$ rescattering is included below in the P^u terms.) Again, CKM unitarity allows us to write A_f in terms of two CKM combinations. For example, for $f = \phi K_S$, which proceeds via $\bar{b} \to \bar{s}s\bar{s}$ transition, we can write

$$\frac{\overline{A}_{\phi K_S}}{A_{\phi K_S}} = -\frac{(V_{cb}V^*_{cs})\,P^c_{\phi K} + (V_{ub}V^*_{us})\,P^u_{\phi K}}{(V^*_{cb}V_{cs})\,P^c_{\phi K} + (V^*_{ub}V_{us})\,P^u_{\phi K}} \times \frac{V^*_{cd}V_{cs}}{V_{cd}V^*_{cs}}, \tag{128}$$

where $P^c_{\phi K} = p^c_{\phi K} - p^t_{\phi K}$ and $P^u_{\phi K} = p^u_{\phi K} - p^t_{\phi K}$.

FIG. 3: Feynman diagrams for (a) tree and (b) penguin amplitudes contributing to $B^0 \to f$ or $B_s \to f$ via a $\bar{b} \to \bar{q}q\bar{q}'$ quark-level process.



Since the amplitude A_f involves two different weak phases, the corresponding decays can exhibit both CP violation in the interference of decays with and without mixing, $S_f \neq 0$, and CP violation in decays, $C_f \neq 0$. [At the present level of experimental precision, the contribution to C_f from CP violation in mixing is negligible, see Eq. (119).] If the contribution from a second weak phase is suppressed, then the interpretation of S_f in terms of Lagrangian CP-violating parameters is clean, while C_f is small. If such a second contribution is not suppressed, S_f depends on hadronic parameters and, if the relevant strong phase is large, C_f is large.

TABLE I: Summary of $\bar{b} \to \bar{q}q\bar{q}'$ modes with $q' = s$ or d. The second and third columns give examples of final hadronic states. The fourth column gives the CKM dependence of the amplitude A_f, using the notation of Eqs. (124,126,128), with the dominant term first and the sub-dominant second. The suppression factor of the second term compared to the first is given in the last column. "Loop" refers to a penguin versus tree suppression factor (it is mode-dependent and roughly $\mathcal{O}(0.2-0.3)$) and $\lambda = 0.22$ is the expansion parameter of Eq. (53).

$\bar{b} \to q\bar{q}\bar{q}'$	$B^0 \to f$	$B_s \to f$	CKM dependence of A_f	Suppression
$\bar{b} \to \bar{c}c\bar{s}$	ψK_S	$\psi\phi$	$(V_{cb}^* V_{cs})T + (V_{ub}^* V_{us})P^u$	loop $\times\, \lambda^2$
$\bar{b} \to \bar{s}s\bar{s}$	ϕK_S	$\phi\phi$	$(V_{cb}^* V_{cs})P^c + (V_{ub}^* V_{us})P^u$	λ^2
$\bar{b} \to \bar{u}u\bar{s}$	$\pi^0 K_S$	K^+K^-	$(V_{cb}^* V_{cs})P^c + (V_{ub}^* V_{us})T$	λ^2/loop
$\bar{b} \to \bar{c}c\bar{d}$	D^+D^-	ψK_S	$(V_{cb}^* V_{cd})T + (V_{tb}^* V_{td})P^t$	loop
$\bar{b} \to \bar{s}s\bar{d}$	$\phi\pi$	ϕK_S	$(V_{tb}^* V_{td})P^t + (V_{cb}^* V_{cd})P^c$	$\lesssim 1$
$\bar{b} \to \bar{u}u\bar{d}$	$\pi^+\pi^-$	$\pi^0 K_S$	$(V_{ub}^* V_{ud})T + (V_{tb}^* V_{td})P^t$	loop

A summary of $\bar{b} \to \bar{q}q\bar{q}'$ modes with $q' = s$ or d is given in Table I. The $\bar{b} \to \bar{d}d\bar{q}$ transitions lead to final states that are similar to the $\bar{b} \to \bar{u}u\bar{q}$ transitions and have similar phase dependence. Final states that consist of two vector-mesons ($\psi\phi$ and $\phi\phi$) are not CP eigenstates, and angular analysis is needed to separate the CP-even from the CP-odd contributions.

The cleanliness of the theoretical interpretation of S_f can be assessed from the information in the last column of Table I. In case of small uncertainties, the expression for S_f in terms of CKM phases can be deduced from the fourth column of Table I in combination with Eq. (122) (and, for $b \to q\bar{q}s$ decays, the example in Eq. (127)). In the next three sections, we consider three interesting classes.

For B_s decays, one has to replace Eq. (122) with

$$e^{-i\phi_{B_s}} = (V_{tb}^* V_{ts})/(V_{tb} V_{ts}^*). \tag{129}$$

Note that one expects $\Delta\Gamma_s/\Gamma_s = \mathcal{O}(0.1)$, and therefore y_{B_s} should not be put to zero in the expressions for the time dependent decay rates, but $|q/p| = 1$ is expected to hold to an even better approximation than for B mesons. The CP asymmetry in $B_s \to D_s^+ D_s^-$

(or in $B_s \to \psi\phi$ with angular analysis to disentangle the CP-even and CP-odd components of the final state) will determine $\sin 2\beta_s$, where β_s is defined in Eq. (63). Since the SM prediction is that this asymmetry is small [see Eq. (67)], $\sin 2\beta_s \sim 0.036$, an observation of a $S_{B_s \to D_s^+ D_s^-} \gg 0.04$ will provide evidence for new physics.

VIII. $b \to c\bar{c}s$ TRANSITIONS

For $B \to J/\psi K_S$ and other $\bar{b} \to \bar{c}c\bar{s}$ processes, we can neglect the P^u contribution to $A_{\psi K}$, in the SM, to an approximation that is better than one percent:

$$\lambda_{\psi K_S} = -e^{-2i\beta} \;\Rightarrow\; S_{\psi K_S} = \sin 2\beta, \quad C_{\psi K_S} = 0 \; . \tag{130}$$

(Below the percent level, several effects modify this equation [75, 76].) The experimental measurements give the following ranges [77]:

$$S_{\psi K_S} = 0.69 \pm 0.03, \quad C_{\psi K_S} = 0.02 \pm 0.05 \; . \tag{131}$$

The consistency of the experimental results (131) with the SM predictions means that the KM mechanism of CP violation has successfully passed its first precision test. For the first time, we can make the following statement based on experimental evidence:
Very likely, the Kobayashi-Maskawa mechanism is the dominant source of CP violation in flavor changing processes.

There are three qualifications implicit in this statement, and we now explain them in little more detail [78].

- *'Very likely'*: It could be that the success is accidental. It could happen, for example, that $\sin 2\beta$ is significantly different from the SM value and that, at the same time, there is a significant CP violating contribution to the $B^0 - \overline{B}^0$ mixing amplitude, and the sum of $M_{12}^{\rm SM} + M_{12}^{\rm NP}$ accidentally carries the same phase as the one predicted by the SM alone. It could also happen that the size of NP contributions to $b \to d$ transitions is small, or that its phase is similar to the SM one, but that in $b \to s$ transitions the deviation is significant.

- *'Dominant'*: While $S_{\psi K}$ is measured with an accuracy of order 0.04, the accuracy of the SM prediction for $\sin 2\beta$ is only at the level of 0.2. Thus, it is quite possible that there is a new physics contribution at the level of $|M_{12}^{\rm NP}/M_{12}^{\rm SM}| \lesssim \mathcal{O}(0.2)$.

- *'Flavor changing'*: It may well happen that the KM phase, which is closely related to flavor violation through the CKM matrix, dominates meson decays while new, flavor diagonal phases (such as the two unavoidable phases in the universal version of the MSSM) dominate observables such as electric dipole moments by many orders of magnitude.

The measurement of $S_{\psi K}$ provides a significant constraint on the unitarity triangle. In the $\rho - \eta$ plane, it reads:

$$\sin 2\beta = \frac{2\eta(1-\rho)}{\eta^2 + (1-\rho)^2} = 0.69 \pm 0.03. \tag{132}$$

One can get an impression of the impact of this constraint by looking at Fig. 2, where the blue region represents $\sin 2\beta = 0.69 \pm 0.03$. An impression of the KM test can be achieved by observing that the blue region has an excellent overlap with the region allowed by all other measurements. A comparison between the constraints in the $\rho - \eta$ plane from CP conserving and CP violating processes is provided in Fig. 4. The impressive consistency between the two allowed regions is the basis for our statement that the KM mechanism has passed its first precision tests. The fact that the allowed region from the CP violating processes is more strongly constrained is related to the fact that CP is a good symmetry of the strong interactions and that, therefore, various CP violating observables – in particular $S_{\psi K}$ – can be cleanly interpreted.

IX. PENGUIN DOMINATED $b \to s$ TRANSITIONS

A. General considerations

The present experimental situation concerning CP asymmetries in decays to final CP eigenstates dominated by $b \to s$ penguins is summarized in Table II.

For $B \to \phi K_S$ and other $\bar{b} \to \bar{s}s\bar{s}$ processes, we can neglect the P^u contribution to A_f, in the Standard Model, to an approximation that is good to order of a few percent:

$$\lambda_{\phi K_S} \approx -e^{-2i\beta} \;\Rightarrow\; S_{\phi K_S} \approx \sin 2\beta, \quad C_{\phi K_S} \approx 0\ . \tag{133}$$

In the presence of new physics, both A_f and M_{12} can get contributions that are comparable in size to those of the Standard Model and carry new weak phases [36]. Such a situation gives several interesting consequences for $\bar{b} \to \bar{s}s\bar{s}$ decays:

FIG. 4: Constraints in the $\rho - \eta$ plane from (a) CP conserving or (b) CP violating loop processes.

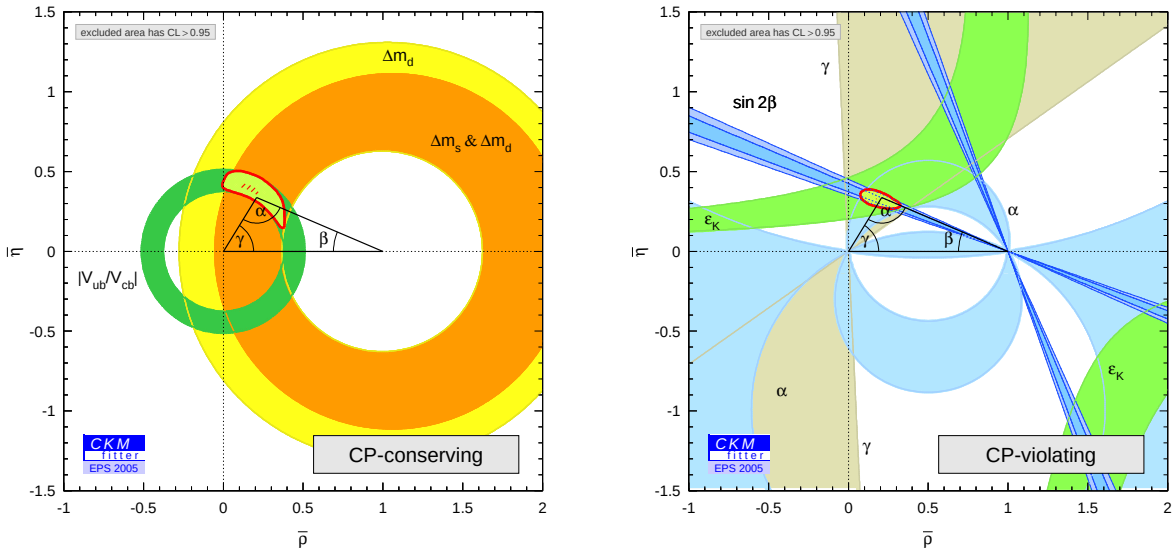


TABLE II: CP asymmetries in $b \to s$ penguin dominated modes.

$f_{\rm CP}$	$-\eta_{f_{\rm CP}} S_{f_{\rm CP}}$	$C_{f_{\rm CP}}$
ϕK_S	$+0.47 \pm 0.19$	-0.09 ± 0.15
$\eta' K_S$	$+0.50 \pm 0.09(0.13)$	$-0.07 \pm 0.07(0.10)$
$f_0 K_S$	$+0.75 \pm 0.24$	$+0.06 \pm 0.21(0.23)$
$\pi^0 K_S$	$+0.31 \pm 0.26$	-0.02 ± 0.13
ωK_S	$+0.63 \pm 0.30$	-0.44 ± 0.24
$K_S K_S K_S$	$+0.61 \pm 0.23$	$-0.31 \pm 0.17(0.20)$

1. A new CP violating phase in the $b \to s$ decay amplitude will lead to a deviation of $-\eta_f S_f$ from $S_{\psi K}$.

2. The S_f's will be different, in general, among the various f's. Only if the new physics contribution to A_f dominates over the SM we should expect a universal S_f.

3. A new CP violating phase in the $b \to s$ decay amplitude in combination with a strong phase will lead to $C_f \neq 0$.

B. Calculating the deviations from $S_f = S_{\psi K}$

It is important to understand how large a deviation from the approximate equalities in Eq. (133) is expected within the SM. The SM contribution to the decay amplitudes, related to $\bar{b} \to \bar{q}q\bar{s}$ transitions, can always be written as a sum of two terms, $A_f^{\rm SM} = A_f^c + A_f^u$, with $A_f^c \propto V_{cb}^* V_{cs}$ and $A_f^u \propto V_{ub}^* V_{us}$. Defining the ratio $a_f^u \equiv e^{-i\gamma}(A_f^u/A_f^c)$, we have

$$A_f^{\rm SM} = A_f^c(1 + a_f^u e^{i\gamma}). \tag{134}$$

The size of the deviations from Eq. (133) is set by a_f^u. For $|a_f^u| \ll 1$, we obtain

$$\begin{aligned} -\eta_f S_f &\simeq \sin 2\beta + 2\cos 2\beta\ \mathcal{R}e(a_f^u)\sin\gamma, \\ C_f &\simeq -2\mathcal{I}m(a_f^u)\sin\gamma. \end{aligned} \tag{135}$$

For charmless modes, the effects of the a_f^u terms (often called 'the SM pollution') are at least of order $|(V_{ub}^* V_{us})/(V_{cb}^* V_{cs})| \sim$ a few percent.

To calculate them explicitly, we use the operator product expansion (OPE). We follow the notations of ref. [79]. We consider the following effective Hamiltonian for $\Delta B = \pm 1$ decays:

$$\mathcal{H}_{\rm eff} = \frac{G_F}{\sqrt{2}} \sum_{p=u,c} V_{ps}^* V_{pb} \left(C_1 O_1^p + C_2 O_2^p + \sum_{i=3}^{10} C_i O_i + C_{7\gamma} O_{7\gamma} + C_{8g} O_{8g} \right) + \text{h.c.}, \tag{136}$$

with

$$\begin{aligned} O_1^p &= (\bar{p}b)_{V-A}(\bar{s}p)_{V-A}, & O_2^p &= (\bar{p}_\beta b_\alpha)_{V-A}(\bar{s}_\alpha p_\beta)_{V-A}, \\ O_3 &= (\bar{s}b)_{V-A}\sum_q (\bar{q}q)_{V-A}, & O_4 &= (\bar{s}_\alpha b_\beta)_{V-A}\sum_q (\bar{q}_\beta q_\alpha)_{V-A}, \\ O_5 &= (\bar{s}b)_{V-A}\sum_q (\bar{q}q)_{V+A}, & O_6 &= (\bar{s}_\alpha b_\beta)_{V-A}\sum_q (\bar{q}_\beta q_\alpha)_{V+A}, \\ O_7 &= \frac{3}{2}(\bar{s}b)_{V-A}\sum_q e_q(\bar{q}q)_{V+A}, & O_8 &= \frac{3}{2}(\bar{s}_\alpha b_\beta)_{V-A}\sum_q e_q(\bar{q}_\beta q_\alpha)_{V+A}, \\ O_9 &= \frac{3}{2}(\bar{s}b)_{V-A}\sum_q e_q(\bar{q}q)_{V-A}, & O_{10} &= \frac{3}{2}(\bar{s}_\alpha b_\beta)_{V-A}\sum_q e_q(\bar{q}_\beta q_\alpha)_{V-A}, \\ O_{7\gamma} &= -\frac{e m_b}{8\pi^2}\bar{s}\sigma^{\mu\nu}(1+\gamma_5)F_{\mu\nu}b, & O_{8g} &= -\frac{g_s m_b}{8\pi^2}\bar{s}\sigma^{\mu\nu}(1+\gamma_5)G_{\mu\nu}b, \end{aligned} \tag{137}$$

where $(\bar{q}_1 q_2)_{V\pm A} = \bar{q}_1\gamma_\mu(1 \pm \gamma_5)q_2$, the sum is over active quarks, with e_q denoting their electric charge in fractions of $|e|$ and α, β are color indices. The decay amplitudes can be calculated from this effective Hamiltonian:

$$A_f = \langle f|\mathcal{H}_{\rm eff}|B^0\rangle, \quad \overline{A}_f = \langle f|\mathcal{H}_{\rm eff}|\overline{B}^0\rangle. \tag{138}$$

The electroweak model determines the Wilson coefficients while QCD (or, more practically, a calculational method such as QCD factorization) determines the matrix elements $\langle f|O_i|B^0(\overline{B}^0)\rangle$.

Take, for example, the $B^0 \to K^0\pi^0$ decay amplitude. It can be written as follows (for simplicity, we omit the contributions from O_{7-10}):

$$A^c_{K^0\pi^0} \approx iV^*_{cb}V_{cs}\frac{G_F}{2}f_K F^{B\to\pi}(m_K^2)(m_B^2-m_\pi^2)\,(a_4+r_\chi a_6)\,, \tag{139}$$
$$A^u_{K^0\pi^0} \approx iV^*_{ub}V_{us}\frac{G_F}{2}\Big[f_K F^{B\to\pi}(m_K^2)(m_B^2-m_\pi^2)\,(a_4+r_\chi a_6) - f_\pi F^{B\to K}(m_\pi^2)(m_B^2-m_K^2)a_2\Big]\,,$$

where $r_\chi = 2m_K^2/[m_b(m_s+m_d)]$. The a_i parameters are related to the Wilson coefficients as follows:

$$a_i \equiv C_i + \frac{1}{N_c}C_{i\pm1} \text{ for } i = \text{odd, even}. \tag{140}$$

Within the SM, at leading order,

$$C_1(m_W) = 1, \quad C_{i\neq1}(m_W) = 0. \tag{141}$$

(Strictly speaking, $C_{7\gamma}(m_W)$ and $C_{8g}(m_W)$ are also different from zero. Their contributions to the decay processes of interest occur, however, at next-to-leading order which we neglect here for simplicity.) To run the Wilson coefficients from the weak scale m_W to the low scale of order m_b, we use

$$\vec{C}(\mu) = [\alpha_s(m_W)/\alpha_s(\mu)]^{\gamma/2\beta_0}, \tag{142}$$

where $\beta_0 = (33-2f)/3$, with $f = 5$ for $m_b \le \mu \le m_W$, and γ is the 12-dimensional leading-log anomalous dimension matrix which can be found, for example, in ref. [80]. The bottom line is the following set of values for the relevant a_i parameters at the scale $\mu = m_b$:

$$a_1 = 1.028, \quad a_2 = 0.105, \quad a_4 = -0.0233, \quad a_6 = -0.0314. \tag{143}$$

We use the following values for the relevant hadronic parameters:

$$f_\pi = 131\ MeV, \quad f_K = 160\ MeV, \quad F^{B\to\pi}(0) = 0.28, \quad F^{B\to K}(0) = 0.34, \quad r_\chi(m_b) = 1.170. \tag{144}$$

Thus we can estimate $a^u_{\pi K}$:

$$a^u_{\pi K} \approx \lambda^2 R_u\left(1 - \frac{f_\pi}{f_K}\frac{F^{B\to K}}{F^{B\to\pi}}\frac{a_2}{a_4+r_\chi a_6}\right) \approx 2.75\lambda^2 R_u \approx 0.052. \tag{145}$$

TABLE III: The a_f^u parameters, calculated in QCD factorization at leading log and to zeroth order in Λ/m_b (except for chirally enhanced corrections), and the SM values of S_f for $\mu = m_b$ and in parentheses the respective values for $\mu = 2m_b$ (first) and $\mu = m_b/2$ (second) if different from the central one. In the last column, the results of ref. [83], using QCD factorization at NLO, are given. Taken from [80].

f	a_f^u [80]	$-\eta_{\rm CP} S_f$ [80]	$-\eta_{\rm CP} S_f$ [83]
ψK_S	0	0.69	0.69
ϕK_S	0.019	0.71	0.71 ± 0.01
$\pi^0 K_S$	0.052 $[0.094, 0.021]$	0.75 $[0.79, 0.72]$	$0.76^{+0.05}_{-0.04}$
ηK_S	0.08 $[0.16, 0.02]$	0.78 $[0.84, 0.72]$	$0.79^{+0.11}_{-0.07}$
$\eta' K_S$	0.007 $[-0.006, 0.019]$	0.70 $[0.68, 0.71]$	0.70 ± 0.01
ωK_S	0.22 $[0.37, 0.04]$	0.88 $[0.94, 0.74]$	0.82 ± 0.08
$\rho^0 K_S$	-0.16 $[-0.32, 0.005]$	0.45 $[0.15, 0.70]$	$0.61^{+0.08}_{-0.12}$

We learn that the SM and factorization predict that $-S_{\pi^0 K_S} - S_{\psi K_S} \approx +0.05$.

In Table III we give the values of the a_f^u parameter (obtained in ref. [80] by using factorization [79, 81, 82]) for all relevant modes.

An examination of Table III shows that the SM pollution is small (that is, at the naively expected level of $|(V_{ub}V_{us}^*)/(V_{cb}V_{cs}^*)| \sim$ a few percent) for $f = \phi K_S$, $\eta' K_S$ and $\pi^0 K_S$. It is larger for $f = \eta K_S$, ωK_S and $\rho^0 K_S$. In these modes, a_f^u is enhanced because, within the QCD factorization approach, there is an accidental cancellation between the leading contributions to A_f^c. The reason for the suppression of the leading A_f^c piece in $f = \rho K$, ωK versus $f = \pi^0 K$ is that the dominant QCD-penguin coefficients a_4 and a_6 appear in $A^c_{(\rho,\omega)K}$ as $(a_4 - r_\chi a_6)$ and in $A^c_{\pi^0 K}$ as $(a_4 + r_\chi a_6)$. Since $r_\chi \simeq 1$ and, within the Standard Model, $a_4 \sim a_6$, there is a cancellation in $A^c_{(\rho,\omega)K}$ while there isn't one in $A^c_{\pi^0 K}$. The suppression for $A^c_{\eta K}$ with respect to $A^c_{\eta' K}$ has a different reason: it is due to the octet-singlet mixing, which causes destructive (constructive) interference in the $\eta(\eta')K$ penguin amplitude [84].

TABLE IV: CP asymmetries in $b \to c\bar{c}d$ (above line) or $b \to u\bar{u}d$ (below line) modes.

$f_{\rm CP}$	$-\eta_{f_{\rm CP}} S_{f_{\rm CP}}$	$C_{f_{\rm CP}}$
$\psi\pi^0$	$+0.69 \pm 0.25$	-0.11 ± 0.20
D^+D^-	$+0.29 \pm 0.63$	$+0.11 \pm 0.35$
$D^{*+}D^{*-}$	$+0.75 \pm 0.23$	-0.04 ± 0.14
$\pi^+\pi^-$	$+0.50 \pm 0.12(0.18)$	$-0.37 \pm 0.10(0.23)$
$\pi^0\pi^0$		-0.28 ± 0.39
$\rho^+\rho^-$	$+0.22 \pm 0.22$	-0.02 ± 0.17

X. $b \to u\bar{u}d$ TRANSITIONS

The present experimental situation concerning CP asymmetries in decays to final CP eigenstates via $b \to d$ transitions is summarized in Table IV.

For $B \to \pi\pi$ and other $\bar{b} \to \bar{u}u\bar{d}$ processes, the penguin-to-tree ratio can be estimated using SU(3) relations and experimental data on related $B \to K\pi$ decays. The result is that the suppression is of order $0.2 - 0.3$ and so cannot be neglected. The expressions for $S_{\pi\pi}$ and $C_{\pi\pi}$ to leading order in $R_{PT} \equiv (|V_{tb}V_{td}|P^t_{\pi\pi})/(|V_{ub}V_{ud}|T_{\pi\pi})$ are:

$$\begin{aligned} \lambda_{\pi\pi} &= e^{2i\alpha}\left[(1 - R_{PT}e^{-i\alpha})/(1 - R_{PT}e^{+i\alpha})\right] \;\Rightarrow \\ S_{\pi\pi} &\approx \sin 2\alpha + 2\,\mathcal{R}e(R_{PT})\cos 2\alpha \sin\alpha, \quad C_{\pi\pi} \approx 2\,\mathcal{I}m(R_{PT})\sin\alpha. \end{aligned} \tag{146}$$

R_{PT} is mode-dependent and, in particular, could be different for $\pi^+\pi^-$ and $\pi^0\pi^0$. If strong phases can be neglected then R_{PT} is real, resulting in $C_{\pi\pi} = 0$. As concerns $S_{\pi\pi}$, it is clear from (146) that the relative size and strong phase of the penguin contribution must be known to extract α. (Only one of the two is required if both $C_{\pi\pi}$ and $S_{\pi\pi}$ are measured.) This is the problem of penguin pollution.

The cleanest solution involves isospin relations among the $B \to \pi\pi$ amplitudes. Let us derive this relation step by step. The $SU(2)$-isospin representations of the $\pi\pi$ states are as follows:

$$\begin{aligned} \langle\pi^+\pi^-| &= \sqrt{\frac{1}{2}}\langle(1,+1)(1,-1) + (1,-1)(1,+1)| = \sqrt{\frac{1}{3}}\,\langle 2,0| + \sqrt{\frac{2}{3}}\,\langle 0,0|, \\ \langle\pi^0\pi^0| &= \langle(1,0)(1,0)| = \sqrt{\frac{2}{3}}\,\langle 2,0| - \sqrt{\frac{1}{3}}\,\langle 0,0|, \end{aligned}$$

$$\langle\pi^+\pi^0| \;=\; \sqrt{\frac{1}{2}}\langle(1,+1)(1,0)+(1,0)(1,+1)| = \langle 2,+1|. \tag{147}$$

The Hamiltonian, with its four quark operators, has two features that are important for our purposes:

1. There are $\Delta I = 1/2$ and $\Delta I = 3/2$ pieces, but no $\Delta I = 5/2$ one. The absence of the latter gives isospin relations among the $B \to \pi\pi$ amplitdues.

2. The penguin operatores are purely $\Delta I = 1/2$. Thus we will find that they do not contribute to the $\pi^\pm\pi^0$ modes.

We contract the Hamiltonian with with the $(B^+, B^0) = (1/2, \pm 1/2)$ states:

$$\begin{aligned}
H_{3/2,+1/2}|1/2,-1/2\rangle \;&\propto\; \sqrt{\frac{1}{2}}\;|2,0\rangle + \sqrt{\frac{1}{2}}\;|1,0\rangle,\\
H_{3/2,+1/2}|1/2,+1/2\rangle \;&\propto\; \sqrt{\frac{3}{4}}\;|2,1\rangle - \sqrt{\frac{1}{4}}\;|1,1\rangle,\\
H_{1/2,+1/2}|1/2,-1/2\rangle \;&\propto\; \sqrt{\frac{1}{2}}\;|1,0\rangle - \sqrt{\frac{1}{2}}\;|0,0\rangle,\\
H_{1/2,+1/2}|1/2,+1/2\rangle \;&\propto\; |1,0\rangle.
\end{aligned} \tag{148}$$

Combining (147) and (148), we obtain:

$$\begin{aligned}
A_{\pi^+\pi^-} \;&=\; \sqrt{1/6}\;A_{3/2} - \sqrt{1/3}\;A_{1/2},\\
A_{\pi^0\pi^0} \;&=\; \sqrt{1/3}\;A_{3/2} + \sqrt{1/6}\;A_{1/2},\\
A_{\pi^+\pi^0} \;&=\; \sqrt{3/4}\;A_{3/2}.
\end{aligned} \tag{149}$$

Analogous relation hold for the CP-conjugate amplitudes, $\overline{A}_{\pi^i\pi^j}$. These isospin decompositions lead to the Gronau-London triangle relations [85]:

$$\begin{aligned}
\frac{1}{\sqrt{2}}A_{\pi^+\pi^-} + A_{\pi^0\pi^0} \;&=\; A_{\pi^+\pi^0},\\
\frac{1}{\sqrt{2}}\overline{A}_{\pi^+\pi^-} + \overline{A}_{\pi^0\pi^0} \;&=\; \overline{A}_{\pi^-\pi^0}.
\end{aligned} \tag{150}$$

The method further exploits the fact that the penguin contribution to $P_{\pi\pi}$ is pure $\Delta I = \frac{1}{2}$ (this is not true for the electroweak penguins which, however, are expected to be small), while the tree contribution to $T_{\pi\pi}$ contains pieces which are both $\Delta I = \frac{1}{2}$ and $\Delta I = \frac{3}{2}$.

A simple geometric construction then allows one to find R_{PT} and extract α cleanly from $S_{\pi^+\pi^-}$. Explicitly, one notes that, since $A_{3/2}$ comes purely from tree contributions, we have

$$\frac{q}{p}\frac{\overline{A}_{3/2}}{A_{3/2}} = -e^{2i\alpha}. \tag{151}$$

The branching ratios of the various modes determine $|A_{\pi^i\pi^j}|$ and $|\overline{A}_{\pi^i\pi^j}|$ (with $|A_{\pi^+\pi^0}| = |\overline{A}_{\pi^-\pi^0}|$). This would determine the shape of each of the triangles (150). Defining

$$A_0 \equiv (1/\sqrt{6})\ A_{1/2}, \quad A_2 \equiv (1/\sqrt{12})\ A_{3/2}, \tag{152}$$

we can obtain $A_2 = (1/3)A_{\pi^+\pi^0}$ and $A_0 = (1/\sqrt{2})A_{\pi^+\pi^-} - A_2$. Similarly, we can obtain $\overline{A}_2$ and $\overline{A}_0$. Next, we define (and obtain)

$$\theta \equiv \arg(A_0 A_2^*), \quad \overline{\theta} \equiv \arg(\overline{A}_0 \overline{A}_2^*). \tag{153}$$

Then we have

$$\mathcal{I}m\lambda_{\pi^+\pi^-} = \mathcal{I}m\left(-e^{-2i\alpha}\frac{|\overline{A}_2| - |\overline{A}_0|e^{i\overline{\theta}}}{|A_2| - |A_0|e^{i\theta}}\right). \tag{154}$$

On the other hand, we can use the experimentally measured quantities to extract $\mathcal{I}m\lambda_{\pi^+\pi^-}$:

$$\mathcal{I}m\lambda_{\pi^+\pi^-} = \frac{S_{\pi^+\pi^-}}{1 + C_{\pi^+\pi^-}}. \tag{155}$$

The key experimental difficulty is that one must measure accurately the separate rates for $B^0, \overline{B}^0 \to \pi^0\pi^0$. It has been noted that an upper bound on the average rate allows one to put a useful upper bound on the deviation of $S_{\pi^+\pi^-}$ from $\sin 2\alpha$ [86, 87, 88]. Parametrizing the asymmetry by $S_{\pi^+\pi^-}/\sqrt{1-(C_{\pi^+\pi^-})^2} = \sin(2\alpha_{\rm eff})$, the bound reads

$$\cos(2\alpha_{\rm eff} - 2\alpha) \geq \frac{1}{\sqrt{1-(C_{\pi^+\pi^-})^2}}\left[1 - \frac{2\mathcal{B}_{00}}{\mathcal{B}_{+0}} + \frac{(\mathcal{B}_{+-} - 2\mathcal{B}_{+0} + 2\mathcal{B}_{00})^2}{4\mathcal{B}_{+-}\mathcal{B}_{+0}}\right], \tag{156}$$

where $\mathcal{B}_{ij}$ are the averages over CP-conjugate branching ratios; *e.g.*, $\mathcal{B}_{00} = \frac{1}{2}[\mathcal{B}(B^0 \to \pi^0\pi^0) + \mathcal{B}(\overline{B}^0 \to \pi^0\pi^0)]$. CP asymmetries in $B \to \rho\pi$ and, in particular, in $B \to \rho\rho$ can also be used to determine α [89, 90, 91, 92, 93]. At present, the constraints read [54]

$$\begin{aligned} |\alpha_{\rm eff}^{\pi^+\pi^-} - \alpha| &< 38^o, \quad R_{PT}^{\pi^+\pi^-} = 0.37 \pm 0.17, \\ |\alpha_{\rm eff}^{\rho^+\rho^-} - \alpha| &< 14^o, \quad R_{PT}^{\rho^+\rho^-} = 0.07^{+0.14}_{-0.07}. \end{aligned} \tag{157}$$

Using isospin analyses for all three systems ($\pi\pi$, $\rho\pi$ and $\rho\rho$), one obtains [54]

$$\alpha(\pi\pi, \pi\rho, \rho\rho) = \left[101^{+16}_{-9}\right]^o, \tag{158}$$

to be compared with the result of the CKM fit,

$$\alpha(\text{CKM fit}) = 96 \pm 16^o. \tag{159}$$

We would like to emphasize the following points:

- The consistency of (158) with (159) means that **the KM mechanism of CP violation has successfully passed a second precision test.**
- The α measurement via the $b \to u\bar{u}d$ transitions provides a significant constraint on the unitarity triangle.
- The isospin analysis determines the relative phase between the $B^0 - \overline{B}^0$ mixing amplitude and the tree decay amplitude $A_{3/2}$, independent of the electroweak model. The tree decay amplitude is unliley to bne significantly affected by new physics. Any new physics modification of the mixing amplitude is measured by $S_{\psi k}$. Thus, the combination of $S_{\psi K}$ and the isospin analysis of $S_{\pi\pi,\rho\pi,\rho\rho}$ constrains α even in the presence of new physics in $B^0 - \overline{B}^0$ mixing.

XI. $b \to c\bar{u}s, u\bar{c}s$ TRANSITIONS

An interesting set of measurements is that of $B \to DK$ which proceed via the quark transitions $\bar{b} \to \bar{c}u\bar{s}$ or $\bar{b} \to \bar{u}c\bar{s}$ (and their CP conjugates). Given the quark processes, it is clear that there is no penguin contribution here. Thus, the quark transitions are purely tree processes. The interference between the two quark transitions (if they lead to the same final states – see below) is sensitive to $\arg[(V_{ub}^* V_{us})/(V_{cb}^* V_{cs})] \approx \gamma$.

There are three variants on this method: GLW [94, 95], ADS [96] and GGSZ [97]. The simplest one to explain involves branching ratios of charged B decays, and thus $B^0 - \overline{B}^0$ mixing plays no role. Consider the decay $B^\pm \to D_1^0 K^\pm$, where $D_{1,2}^0 = \frac{1}{\sqrt{2}}(D^0 \pm \overline{D}^0)$ are the CP eigenstates. Taking into account that

$$\begin{aligned} A(B^+ \to D^0 K^+) \times A(D^0 \to D_1^0) &\propto (V_{ub}^* V_{cs}) \times (V_{cs}^* V_{us}), \\ A(B^+ \to \overline{D}^0 K^+) \times A(\overline{D}^0 \to D_1^0) &\propto (V_{cb}^* V_{us}) \times (V_{us}^* V_{cs}), \end{aligned} \tag{160}$$

we can write the relevant decay amplitudes as follows:

$$\sqrt{2} A_{D_1^0 K^+} = |A_{D^0 K^+}| e^{i(\delta+\gamma)} + |A_{\overline{D}^0 K^+}| = A_{D^0 K^+} + A_{\overline{D}^0 K^+},$$

$$\sqrt{2}A_{D_1^0K^-} \;=\; |A_{D^0K^-}|e^{i(\delta-\gamma)} + |A_{\overline{D}^0K^-}| = A_{\overline{D}^0K^-} + A_{D^0K^-}. \tag{161}$$

Measuring the rates for the six relevant decay modes ($D_1^0K^+$, D^0K^+, $\overline{D}^0K^+$ and the CP conjugate modes), one can construct an amplitude triangle for each of the two relations in Eq. (161). We can choose a phase convention where $A_{\overline{D}^0K^+} = A_{D^0K^-}$. Then, the relative angle between $A_{D^0K^+}$ and $A_{\overline{D}^0K^-}$ is 2γ.

The method of [97] gives, at present, the most significant constraints. It allows one to determine the amplitude ratios, $r(DK) = 0.12^{+0.03}_{-0.04}$ and $r(D^*K) = 0.09^{+0.03}_{-0.04}$, and the weak phase γ [54]:

$$\gamma(DK) = (63^{+15}_{-13})^o. \tag{162}$$

This range is to be compared with the range of γ derived from the CKM fit (not including the direct γ measurements):

$$\gamma(\text{CKM fit}) = (57^{+7}_{-14})^o. \tag{163}$$

We would like to emphasize the following points:

- The consistency of (162) with (163) means that **the KM mechanism of CP violation has successfully passed a third precision test.**
- The γ measurement via the $b \to c\bar{u}s, u\bar{c}s$ transitions provides yet another constraint on the unitarity triangle. The constraint will become more significant when the experimental precision improves.
- The determination of γ here relies on tree decay amplitudes. Thus, the analysis of $B \to DK$ decays constrains γ even in the presence of new physics in loop processes.

XII. CP VIOLATION AS A PROBE OF NEW PHYSICS

We have argued that the Standard Model picture of CP violation is unique and highly predictive. We have also stated that reasonable extensions of the Standard Model have a very different picture of CP violation. Experimental results are now starting to decide between the various possibilities. Our discussion of CP violation in the presence of new physics is aimed to demonstrate that, indeed, models of new physics can significantly modify the Standard Model predictions and that present and near future measurements have therefore a strong impact on the theoretical understanding of CP violation.

To understand how the Standard Model predictions could be modified by New Physics, we focus on CP violation in the interference between decays with and without mixing. As explained above, this type of CP violation may give, due to its theoretical cleanliness, unambiguous evidence for New Physics most easily. We now demonstrate what type of questions can be (or have already been) answered when these observables are measured.

I. Consider $S_{\psi K_S}$, the CP asymmetry in $B \to \psi K_S$. This measurement cleanly determines the relative phase between the $B^0 - \overline{B}^0$ mixing amplitude and the $b \to c\bar{c}s$ decay amplitude ($\sin 2\beta$ in the SM). The $b \to c\bar{c}s$ decay has Standard Model tree contributions and therefore is very unlikely to be significantly affected by new physics. On the other hand, the mixing amplitude can be easily modified by new physics. We parametrize such a modification as follows:

$$r_d^2 \, e^{2i\theta_d} = \frac{M_{12}}{M_{12}^{\rm SM}}. \tag{164}$$

Then the following observables provide constraints on r_d^2 and $2\theta_d$:

$$\begin{aligned} S_{\psi K_S} &= \sin(2\beta + 2\theta_d), \\ \Delta m_B &= r_d^2 (\Delta m_B)^{\rm SM}, \\ \mathcal{A}_{\rm SL} &= -\mathcal{R}e\left(\frac{\Gamma_{12}}{M_{12}}\right)^{\rm SM} \frac{\sin 2\theta_d}{r_d^2} + \mathcal{I}m\left(\frac{\Gamma_{12}}{M_{12}}\right)^{\rm SM} \frac{\cos 2\theta_d}{r_d^2}. \end{aligned} \tag{165}$$

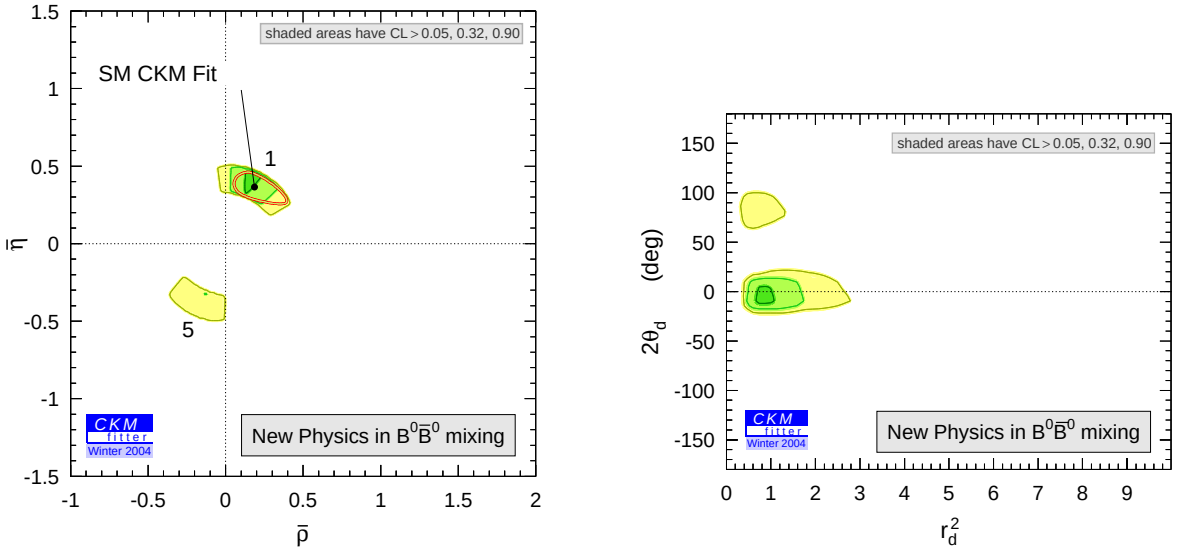
Examining whether $S_{\psi K_S}$, Δm_B and $\mathcal{A}_{\rm SL}$ fit the SM prediction, that is, whether $\theta_d \neq 0$ and/or $r_d^2 \neq 1$, we can answer the following question (see *e.g.* [98]):

(i) *Is there new physics in* $B^0 - \overline{B}^0$ *mixing*?

Thanks to the fact that quite a few observables that are related to SM tree level processes have already been measured, we are able to refer to this question in a quantitative way. The tree level processes are insensitive to new physics and can be used to constrain ρ and η even in the presence of new physics contributions to loop processes, such as Δm_B. Among these observables we have $|V_{cb}|$ and $|V_{ub}|$ from semileptonic B decays, the phase γ from $B \to DK$ decays, and the phase α from $B \to \rho\rho$ decays (in combination with $S_{\psi K}$). One can fit these observables, and the ones in Eq. (165) to the four parameters ρ, η, r_d^2 and $2\theta_d$. The resulting constraints are shown in Fig. 5.

A long list of models that require a significant modification of the $B^0 - \overline{B}^0$ mixing amplitude are excluded. We can further conclude from Fig. 5 that a new physics contribution to the $B^0 - \overline{B}^0$ mixing amplitude at a level higher than about 30% is now disfavored. Yet,

FIG. 5: Constraints in the (a) $\rho - \eta$ plane (b) $r_d^2 - 2\theta_d$ plane, assuming that NP contributions to tree level processes are negligible [54].



it is still possible that ρ and η are well outside their SM range and that NP gives $2\theta_d$ very different from zero and/or r_d^2 very different from one. In this case, the SM and the NP 'conspire' to mimic the SM values of the observables (165). This is what we meant concretely in our statement that the KM dominance of the observed CP violation is now very likely but not guaranteed.

II. Consider $S_{\phi K_S}$, the CP asymmetry in $B \to \phi K_S$. This measurement is sensitive to the relative phase between the $B - \bar{B}$ mixing amplitude and the $b \to s\bar{s}s$ decay amplitude ($\sin 2\beta$ in the SM). The $b \to s\bar{s}s$ decay has only Standard Model penguin contributions and therefore is sensitive to new physics. We parametrize the size and phase of a NP contribution as follows (for simplicity, we neglect here the a_f^u terms of Eq. (134)):

$$A_f = A_f^c \left(1 + b_f \, e^{i\phi_{bs}}\right). \tag{166}$$

Here b_f is complex only if it carries a strong phase. The effects of this new physics contribution are simple to understand in two limits:

1. The new physics contribution is dominant, $|b_f| \gg 1$. The shift in all modes where this condition is valid is universal and depends only on ϕ_{bs}:

$$-\eta_f S_f \simeq \sin(2\beta + 2\theta_d)\cos 2\phi_{bs} + \cos(2\beta + 2\theta_d)\sin 2\phi_{bs},$$

$$C_f \simeq 0. \tag{167}$$

2. The new physics contribution is small. Explicitly, $|b_f| \ll 1$. The shift is mode dependent and depends on both b_f and $\sin\phi_{bs}$:

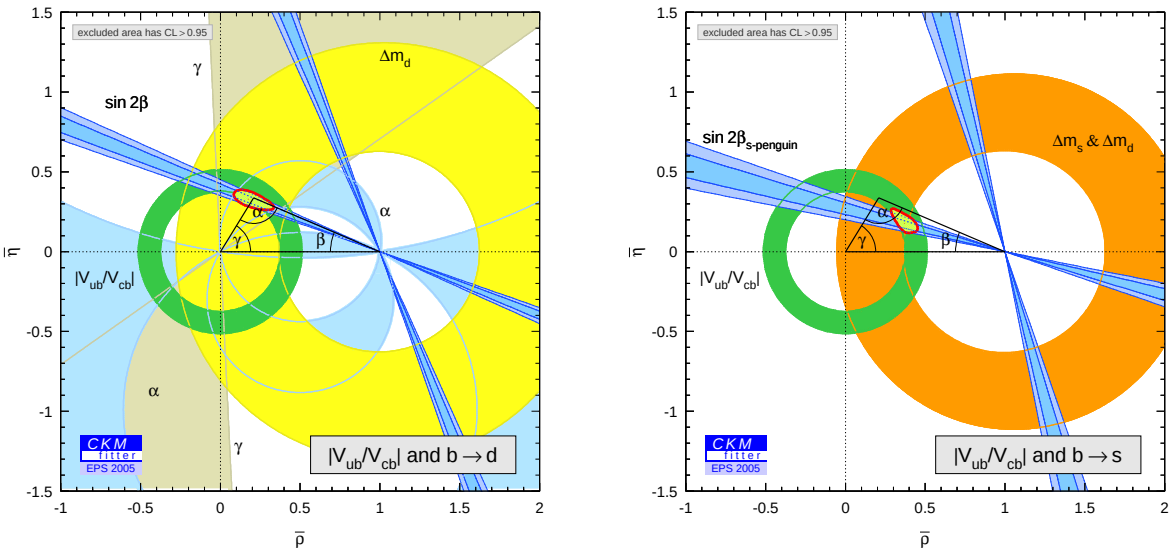
$$\begin{aligned} -\eta_f S_f &\simeq \sin(2\beta+2\theta_d) + 2\cos(2\beta+2\theta_d)\mathcal{R}e(b_f)\sin\phi_{bs}, \\ C_f &\simeq -2\mathcal{I}m(b_f^c)\sin\phi_{bs}. \end{aligned} \tag{168}$$

Note that the effect of the NP is similar to that of the SM a_f^u terms (with $b_f \leftrightarrow a_f^u$ and $\phi_{bs} \leftrightarrow \gamma$), so that the latter have to be known in order to probe the b_f terms. Once that is done, the value of $S_{\psi K}$ determines $2\beta+2\theta_d$ and one can examine whether $\phi_{bs} \neq 0$ and answer the following questions:

(ii) *Is there new physics in $b \to s$ transitions?*

So far, the experimental data – see Table II – do not provide any evidence for $\phi_{bs} \neq 0$. Yet, the experimental accuracy is still not sufficient to make qualitative statements such as we made for $b \to d$ transitions ($B^0 - \overline{B}^0$ mixing). To see this, we compare the constraints in the $\rho - \eta$ plane that arise from tree plus $b \to d$ loops (Δm_B, $S_{\psi K_S}$, $S_{\rho\rho}$, etc.) to those from tree plus $b \to s$ loops ($S_{\phi K_S}$, $S_{\eta' K_S}$, Δm_s). This is done in Fig. 6.

FIG. 6: Constraints in the $\rho - \eta$ plane from tree processes and (a) $b \to d$ or (b) $b \to s$ loop processes.



III. Together with a future measurement of $B_s - \overline{B}_s$ mixing, we may also try to answer the following question:

(iii) *Is there new physics in $\Delta B = 1$ processes? in $\Delta B = 2$? in both?*

IV. Consider $a_{\pi\nu\bar{\nu}} \equiv \Gamma_{K_L\to\pi^0\nu\bar{\nu}}/\Gamma_{K^+\to\pi^+\nu\bar{\nu}}$, see Eq. (111). This measurement will cleanly determine the relative phase between the $K^0 - \overline{K}^0$ mixing amplitude and the $s \to d\nu\bar{\nu}$ decay amplitude (of order $\sin^2\beta$ in the SM). The experimentally measured small value of ε_K requires that the phase of the $K^0 - \overline{K}^0$ mixing amplitude is not modified from the SM prediction. (More precisely, it requires that the phase of the mixing amplitude is very close to twice the phase of the $s \to d\bar{u}u$ decay amplitude [99].) On the other hand, the decay, which in the SM is a loop process with small mixing angles, can be easily modified by new physics. Examining whether the SM correlation between $a_{\pi\nu\bar{\nu}}$ and $S_{\psi K_S}$ is fulfilled, we can answer the following question:

(iv) *Is there new physics related solely to the third generation? to all generations?*

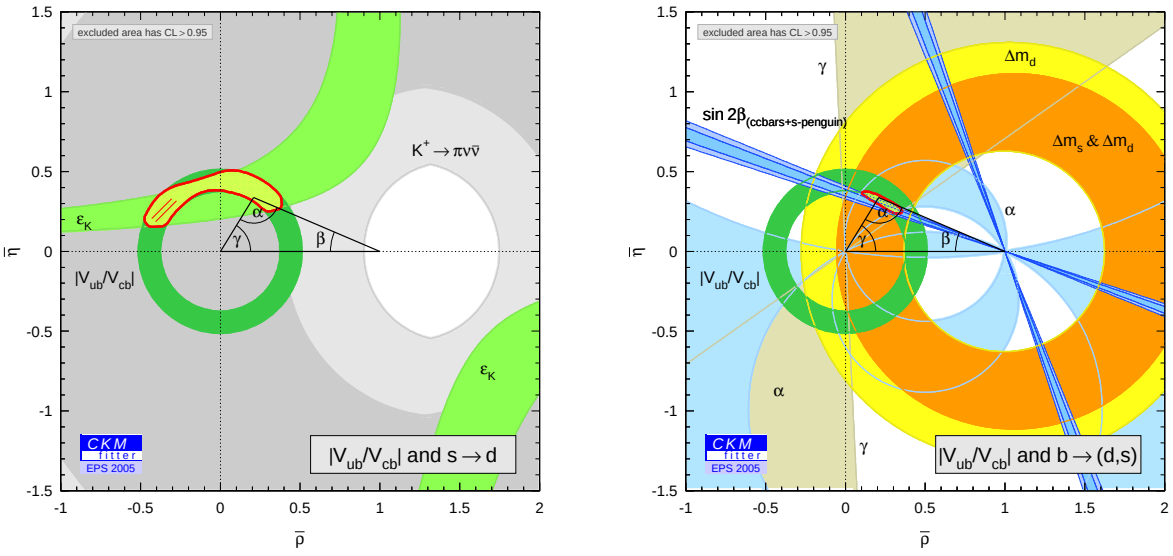
To understand the present situation, we present in Fig. 7 the constraints in the $\rho-\eta$ plane from tree plus loop processes that do not involve external third generation quarks, namely $s \to d$ transitions only (ϵ and $\mathcal{B}(K^+ \to \pi^+\nu\bar{\nu})$). This can be compared with the constraints from tree plus loop processes that do involve the third generation, namely $b \to d$ and $b \to s$ transitions. Again, one can see that there is a lot to be learnt from future measurements. (For a recent, comprehensive analysis of this question, see ref. [100].)

V. Consider ϕ_D, defined in Eq. (117), which is the relative phase between the $D^0 - \overline{D}^0$ mixing amplitude and the $c \to d\bar{s}u$ and $c \to s\bar{d}u$ decay amplitudes. Within the Standard Model, the two decay channels are tree level. It is unlikely that they are affected by new physics. On the other hand, the mixing amplitude can be easily modified by new physics. Examining whether $\phi_D \neq 0$, we can answer the following question:

(v) *Is there new physics in the down sector? in the up sector? in both?*

VI. Consider d_N, the electric dipole moment of the neutron. We did not discuss this quantity so far because, unlike CP violation in meson decays, flavor changing couplings are not necessary for d_N. In other words, the CP violation that induces d_N is *flavor diagonal*. It does in general get contributions from flavor changing physics, but it could be induced by sectors that are flavor blind. Within the SM (and ignoring $\theta_{\rm QCD}$), the contribution from $\delta_{\rm KM}$ arises at the three loop level and is at least six orders of magnitude below the experimental bound (13). If the bound is further improved (or a signal observed), we can answer the

FIG. 7: Constraints in the $\rho - \eta$ plane from tree processes and (a) $s \to d$ or (b) $b \to d$ and $b \to s$ loop processes.



following question:

(vi) *Are there new sources of CP violation that are flavor changing? flavor diagonal? both?*

It is no wonder then that with such rich information, flavor and CP violation provide an excellent probe of new physics. We next demonstrate this situation more concretely by discussing CP violation in supersymmetry.

XIII. SUPERSYMMETRIC CP VIOLATION

Supersymmetry solves the fine-tuning problem of the Standard Model and has many other virtues. But at the same time, it leads to new problems: baryon number violation, lepton number violation, large flavor changing neutral current processes and large CP violation. The first two problems can be solved by imposing R-parity on supersymmetric models. There is no such simple, symmetry-related solution to the problems of flavor and CP violation. Instead, suppression of the relevant couplings can be achieved by demanding very constrained structures of the soft supersymmetry breaking terms. There are two important questions here: First, can theories of dynamical supersymmetry breaking naturally induce such structures? Second, can measurements of flavor changing and/or CP violating

processes shed light on the structure of the soft supersymmetry breaking terms? Since the answer to both questions is in the affirmative, we conclude that flavor changing neutral current processes and, in particular, CP violating observables will provide clues to the crucial question of how supersymmetry breaks.

A. CP violating parameters

A generic supersymmetric extension of the Standard Model contains a host of new flavor and CP violating parameters. (For a review of CP violation in supersymmetry see [101, 102].) It is an amusing exercise to count the number of parameters [103]. The supersymmetric part of the Lagrangian depends, in addition to the three gauge couplings of $G_{\rm SM}$, on the parameters of the superpotential W:

$$W = \sum_{i,j} \left(Y^u_{ij} H_u Q_{Li} \overline{U}_{Lj} + Y^d_{ij} H_d Q_{Li} \overline{D}_{Lj} + Y^\ell_{ij} H_d L_{Li} \overline{E}_{Lj} \right) + \mu H_u H_d. \tag{169}$$

In addition, we have to add soft supersymmetry breaking terms:

$$\begin{aligned} \mathcal{L}_{\rm soft} = & - \left(A^u_{ij} H_u \tilde{Q}_{Li} \tilde{\overline{U}}_{Lj} + A^d_{ij} H_d \tilde{Q}_{Li} \tilde{\overline{D}}_{Lj} + A^\ell_{ij} H_d \tilde{L}_{Li} \tilde{\overline{E}}_{Lj} + B H_u H_d + \text{h.c.} \right) \\ & - \sum_{\text{all scalars}} (m^2_S)_{ij} A_i \bar{A}_j - \frac{1}{2} \sum_{(a)=1}^{3} \left(\tilde{m}_{(a)} (\lambda\lambda)_{(a)} + \text{h.c.} \right). \end{aligned} \tag{170}$$

where $S = Q_L, \overline{D}_L, \overline{U}_L, L_L, \overline{E}_L$. The three Yukawa matrices Y^f depend on 27 real and 27 imaginary parameters. Similarly, the three A^f-matrices depend on 27 real and 27 imaginary parameters. The five m^2_S hermitian 3×3 mass-squared matrices for sfermions have 30 real parameters and 15 phases. The gauge and Higgs sectors depend on

$$\theta_{\rm QCD}, \tilde{m}_{(1)}, \tilde{m}_{(2)}, \tilde{m}_{(3)}, g_1, g_2, g_3, \mu, B, m^2_{h_u}, m^2_{h_d}, \tag{171}$$

that is 11 real and 5 imaginary parameters. Summing over all sectors, we get 95 real and 74 imaginary parameters. The various couplings (other than the gauge couplings) can be thought of as spurions that break a global symmetry,

$$U(3)^5 \times U(1)_{\rm PQ} \times U(1)_R \;\to\; U(1)_B \times U(1)_L. \tag{172}$$

The $U(1)_{\rm PQ} \times U(1)_R$ charge assignments are:

$$\begin{array}{cccccc} & H_u & H_d & Q\overline{U} & Q\overline{D} & L\overline{E} \\ U(1)_{\rm PQ} & 1 & 1 & -1 & -1 & -1 \\ U(1)_{\rm R} & 1 & 1 & 1 & 1 & 1 \end{array}. \tag{173}$$

Consequently, we can remove 15 real and 30 imaginary parameters, which leaves

$$124 \;=\; \begin{cases} 80 & \text{real} \\ 44 & \text{imaginary} \end{cases} \text{ physical parameters.} \tag{174}$$

In particular, there are 43 new CP violating phases! In addition to the single Kobayashi-Maskawa of the SM, we can put 3 phases in M_1, M_2, μ (we used the $U(1)_{\rm PQ}$ and $U(1)_R$ to remove the phases from μB^* and M_3, respectively) and the other 40 phases appear in the mixing matrices of the fermion-sfermion-gaugino couplings. (Of the 80 real parameters, there are 11 absolute values of the parameters in (171), 9 fermion masses, 21 sfermion masses, 3 CKM angles and 36 SCKM angles.) Supersymmetry provides a nice example to our statement that reasonable extensions of the Standard Model may have more than one source of CP violation.

The requirement of consistency with experimental data provides strong constraints on many of these parameters. For this reason, the physics of flavor and CP violation has had a profound impact on supersymmetric model building. A discussion of CP violation in this context can hardly avoid addressing the flavor problem itself. Indeed, many of the supersymmetric models that we analyze below were originally aimed at solving flavor problems.

As concerns CP violation, one can distinguish two classes of experimental constraints. First, bounds on nuclear and atomic electric dipole moments determine what is usually called the *supersymmetric CP problem*. Second, the physics of neutral mesons and, most importantly, the small experimental value of ε_K pose the *supersymmetric ε_K problem*. In the next two subsections we describe the two problems.

B. The Supersymmetric CP problem

One aspect of supersymmetric CP violation involves effects that are flavor preserving. Then, for simplicity, we describe this aspect in a supersymmetric model without additional flavor mixings, *i.e.* the minimal supersymmetric standard model (MSSM) with universal sfermion masses and with the trilinear SUSY-breaking scalar couplings proportional to the corresponding Yukawa couplings. (The generalization to the case of non-universal soft terms is straightforward.) In such a constrained framework, there are four new phases beyond the two phases of the SM ($\delta_{\rm KM}$ and $\theta_{\rm QCD}$). One arises in the bilinear μ-term of the superpotential

(169), while the other three arise in the soft supersymmetry breaking parameters of (170): $\tilde{m}$ (the gaugino mass), A (the trilinear scalar coupling) and B (the bilinear scalar coupling). Only two combinations of the four phases are physical [104, 105]:

$$\phi_A = \arg(A^*\tilde{m}), \quad \phi_B = \arg(\tilde{m}\mu B^*). \tag{175}$$

In the more general case of non-universal soft terms there is one independent phase ϕ_{A_i} for each quark and lepton flavor. Moreover, complex off-diagonal entries in the sfermion mass-squared matrices represent additional sources of CP violation.

The most significant effect of ϕ_A and ϕ_B is their contribution to electric dipole moments (EDMs). For example, the contribution from one-loop gluino diagrams to the down quark EDM is given by [106, 107]:

$$d_d = m_d \frac{e\alpha_3}{18\pi\tilde{m}^3}\left(|A|\sin\phi_A + \tan\beta|\mu|\sin\phi_B\right), \tag{176}$$

where we have taken $m_Q^2 \sim m_D^2 \sim m_{\tilde{g}}^2 \sim \tilde{m}^2$, for left- and right-handed squark and gluino masses. We define, as usual, $\tan\beta = \langle H_u\rangle/\langle H_d\rangle$. Similar one-loop diagrams give rise to chromoelectric dipole moments. The electric and chromoelectric dipole moments of the light quarks (u, d, s) are the main source of d_N (the EDM of the neutron), giving [108]

$$d_N \sim 2\left(\frac{100\,GeV}{\tilde{m}}\right)^2 \sin\phi_{A,B} \times 10^{-23}\; e\,\mathrm{cm}, \tag{177}$$

where, as above, $\tilde{m}$ represents the overall SUSY scale. In a generic supersymmetric framework, we expect $\tilde{m} = \mathcal{O}(m_Z)$ and $\sin\phi_{A,B} = \mathcal{O}(1)$. Then the constraint (13) is generically violated by about two orders of magnitude. This is *the Supersymmetric CP Problem*.

Eq. (177) shows two possible ways to solve the supersymmetric CP problem:

(i) Heavy squarks: $\tilde{m} \gtrsim 1\ TeV$;

(ii) Approximate CP: $\sin\phi_{A,B} \ll 1$.

C. The Supersymmetric ε_K problem

The supersymmetric contribution to the ε_K parameter is dominated by diagrams involving Q and $\bar{d}$ squarks in the same loop. For $\tilde{m} = m_{\tilde{g}} \simeq m_Q \simeq m_D$ (our results depend only weakly on this assumption) and focusing on the contribution from the first two squark families, one gets (see, for example, [109]):

$$\varepsilon_K = \frac{5\ \alpha_3^2}{162\sqrt{2}}\frac{f_K^2 m_K}{\tilde{m}^2\Delta m_K}\left[\left(\frac{m_K}{m_s+m_d}\right)^2 + \frac{3}{25}\right]\mathcal{I}m((\delta^d_{12})_{LL}(\delta^d_{12})_{RR}). \tag{178}$$

Here

$$(\delta^d_{12})_{LL} = \left(\frac{m^2_{\tilde{Q}_2} - m^2_{\tilde{Q}_1}}{m^2_{\tilde{Q}}}\right) K^{dL}_{12},$$
$$(\delta^d_{12})_{RR} = \left(\frac{m^2_{\tilde{D}_2} - m^2_{\tilde{D}_1}}{m^2_{\tilde{D}}}\right) K^{dR}_{12}, \qquad (179)$$

where K^{dL}_{12} (K^{dR}_{12}) are the mixing angles in the gluino couplings to left-handed (right-handed) down quarks and their scalar partners. Note that CP would be violated even if there were two families only [110]. Using the experimental value of ε_K, we get

$$\frac{(\Delta m_K \varepsilon_K)^{\rm SUSY}}{(\Delta m_K \varepsilon_K)^{\rm EXP}} \sim 10^7 \left(\frac{300\ GeV}{\tilde{m}}\right)^2 \left(\frac{m^2_{\tilde{Q}_2} - m^2_{\tilde{Q}_1}}{m^2_{\tilde{Q}}}\right) \left(\frac{m^2_{\tilde{D}_2} - m^2_{\tilde{D}_1}}{m^2_{\tilde{D}}}\right) |K^{dL}_{12} K^{dR}_{12}| \sin\phi, \qquad (180)$$

where ϕ is the CP violating phase. In a generic supersymmetric framework, we expect $\tilde{m} = \mathcal{O}(m_Z)$, $\delta m^2_{Q,D}/m^2_{Q,D} = \mathcal{O}(1)$, $K^{Q,D}_{ij} = \mathcal{O}(1)$ and $\sin\phi = \mathcal{O}(1)$. Then the constraint (180) is generically violated by about seven orders of magnitude.

The Δm_K constraint on $\mathcal{R}e((\delta^d_{12})_{LL}(\delta^d_{12})_{RR})$ is about two orders of magnitude weaker. One can distinguish then three interesting regions for $\langle\delta^d_{12}\rangle = \sqrt{(\delta^d_{12})_{LL}(\delta^d_{12})_{RR}}$:

$$\langle\delta^d_{12}\rangle \begin{cases} \gg 0.003 & \text{excluded;} \\ \in [0.0002, 0.003] & \text{viable with small phases;} \\ \ll 0.0002 & \text{viable with } \mathcal{O}(1) \text{ phases.} \end{cases} \qquad (181)$$

The first bound comes from the Δm_K constraint (assuming that the relevant phase is not particularly close to $\pi/2$). The bounds here apply to squark masses of order 500 GeV and scale like $\tilde{m}$. There is also dependence on $m_{\tilde{g}}/\tilde{m}$, which is here taken to be one.

Eq. (180) also shows what are the possible ways to solve the supersymmetric ε_K problem:
(i) Heavy squarks: $\tilde{m} \gg 300\ GeV$;
(ii) Universality: $(\Delta m^2_{Q,D})_{21} \ll m^2_{Q,D}$;
(iii) Alignment: $|K^d_{12}| \ll 1$;
(iv) Approximate CP: $\sin\phi \ll 1$.

D. More on supersymmetric flavor and CP violation

The flavor and CP constraints on supersymmetric models apply to almost all flavor changing couplings. The size of supersymmetric flavor violation depends on the overall

TABLE V: Theoretical predictions for supersymmetric flavor changing couplings in viable models of alignment, and the experimental constraints.

$(\delta^q_{MN})_{ij}$	Prediction	Upper bound	$(\delta^d_{MN})_{ij}$	Prediction	Upper bound
$(\delta^d_{LL})_{12}$	$\lambda^5 - \lambda^3$	λ^3	$(\delta^d_{LR})_{12}$	$\lambda^7(m_b/\tilde{m})$	$\lambda^7(\mathcal{I}m)$
$(\delta^d_{RR})_{12}$	$\lambda^7 - \lambda^3$	$\lambda^{10}/(\delta^d_{LL})_{12}$	$(\delta^d_{RL})_{12}$	$\lambda^9(m_b/\tilde{m})$	$\lambda^7(\mathcal{I}m)$
$(\delta^d_{LL})_{13}$	λ^3	λ	$(\delta^d_{LR})_{13}$	$\lambda^3(m_b/\tilde{m})$	λ^2
$(\delta^d_{RR})_{13}$	$\lambda^7 - \lambda^3$	$\lambda^4/(\delta^d_{LL})_{13}$	$(\delta^d_{RL})_{13}$	$\lambda^7(m_b/\tilde{m})$	λ^2
$(\delta^d_{LL})_{23}$	λ^2	$\lambda^2(\mathcal{R}e) - \lambda(\mathcal{I}m)$	$(\delta^d_{LR})_{23}$	$\lambda^2(m_b/\tilde{m})$	$\lambda^4(\mathcal{R}e) - \lambda^3(\mathcal{I}m)$
$(\delta^d_{RR})_{23}$	$\lambda^4 - \lambda^2$	1	$(\delta^d_{RL})_{23}$	$\lambda^4(m_b/\tilde{m})$	λ^3
$(\delta^u_{LL})_{12}$	λ	λ			
$(\delta^u_{RR})_{12}$	$\lambda^4 - \lambda^2$	$\lambda^4/(\delta^u_{LL})_{12}$			

scale of the soft supersymmetry breaking terms, on mass degeneracies between sfermion generations, and on the mixing angles in gaugino couplings. One can choose a representative scale (say, $\tilde{m} \sim 300$ GeV) and then conveniently present the constraints in terns of the $(\delta^q_{ij})_{MN}$ parameters [see Eq. (179)]. In a given supersymmetric flavor model, one can find predictions for the $(\delta^q_{ij})_{MN}$ and test the model.

A summary of upper bounds on the supersymmetric flavor changing couplings is given in Table V. The bounds on the $\mathcal{I}m(\delta^d_{12})_{LR,RL}$ parameters are taken from [111], on δ^d_{13} from [112] and on δ^d_{23} from [113, 114]. The bounds are expressed in powers of the Wolfenstein parameter λ, which makes it easy to compare with model predictions. As an example, we give the range of these parameters that is expected in a large class of viable models of alignment [115, 116, 117].

Until some time ago, the δ^d_{23} parameters have been only weakly constrained (the improving accuracy of the measurements of $\mathcal{B}(B \to X\ell^+\ell^-)$ have strengthened the constraints considerably). Furthermore, measurements of various CP asymmetries in penguin dominated modes (particularly $S_{\phi K}$ and $S_{\eta' K}$) gave central values that were far off the expected value $\sim S_{\psi K}$ (at present the strongest discrepancy is down to the 2σ level). One may still ask whether effects of order 0.1, which is the order of the expected experimental accuracy and probably above the theoretical error on $S_{\phi K}$ and $S_{\eta' K}$, are still possible within

supersymmetric flavor models and, in particular, alignment models.

To answer this question, we use the results of ref. [113]. From their Fig. 3, we make the following estimates:

$$\begin{aligned}\frac{\Delta S_{\phi K}}{\Delta \mathcal{I}m(\delta^d_{LL})_{23}} &\sim \frac{\Delta S_{\phi K}}{\Delta \mathcal{I}m(\delta^d_{RR})_{23}} \sim 0.3,\\ \frac{\Delta S_{\phi K}}{\Delta \mathcal{I}m(\delta^d_{LR})_{23}} &\sim \frac{\Delta S_{\phi K}}{\Delta \mathcal{I}m(\delta^d_{RL})_{23}} \sim 100.\end{aligned} \tag{182}$$

Thus, for $S_{\phi K}$ to be shifted by $\mathcal{O}(0.1)$, we need at least one of the following four options:

$$\begin{aligned}\mathcal{I}m(\delta^d_{LL})_{23} &\sim \lambda, \quad \mathcal{I}m(\delta^d_{RR})_{23} \sim \lambda,\\ \mathcal{I}m(\delta^d_{LR})_{23} &\sim \lambda^4, \quad \mathcal{I}m(\delta^d_{RL})_{23} \sim \lambda^4.\end{aligned} \tag{183}$$

Examining Table V, we learn that in alignment models $\mathcal{I}m(\delta^d_{LR})_{23} \sim 7 \times 10^{-4}(350\ GeV/\tilde{m})$ is the closest to satisfying the condition in Eq. (183), though the unknown numbers of order one should be on the large side to give an observable effect.

E. Discussion

We define two scales that play an important role in supersymmetry: Λ_S, where the soft supersymmetry breaking terms are generated, and Λ_F, where flavor dynamics takes place. When $\Lambda_F \gg \Lambda_S$, it is possible that there are no genuinely new sources of flavor and CP violation. This class of models, where the Yukawa couplings (or, in the mass basis, the CKM matrix) are the only source of flavor and CP breaking, are often called 'minimal flavor violation.' The most important features of the supersymmetry breaking terms are universality of the scalar masses-squared and proportionality of the A-terms. When $\Lambda_F \lesssim \Lambda_S$, we do not expect, in general, that flavor and CP violation are limited to the Yukawa matrices. One way to suppress CP violation would be to assume that, similarly to the Standard Model, CP violating phases are large, but their effects are screened, possibly by the same physics that explains the various flavor puzzles, such as models with Abelian or non-Abelian horizontal symmetries. It is also possible that CP violating effects are suppressed because squarks are heavy. Another option, which is now excluded, was to assume that CP is an approximate symmetry of the full theory (namely, CP violating phases are all small).

We would like to emphasize the following points:

(i) For supersymmetry to be established, a direct observation of supersymmetric particles is necessary. Once it is discovered, then measurements of CP violating observables will be a very sensitive probe of its flavor structure and, consequently, of the mechanism of dynamical supersymmetry breaking.

(ii) It seems possible to distinguish between models of exact universality and models with genuine supersymmetric flavor and CP violation. The former tend to give $d_N \lesssim 10^{-31}$ e cm while the latter usually predict $d_N \gtrsim 10^{-28}$ e cm.

(iii) The proximity of $S_{\psi K_S}$ to the SM predictions is obviously consistent with models of exact universality. It disfavors models of heavy squarks such as that of ref. [118]. Models of flavor symmetries allow deviations of order 20% (or smaller) from the SM predictions. To be convincingly signalled, an improvement in the theoretical calculations that lead to the SM predictions for $S_{\psi K_S}$ will be required [119].

(iv) Alternatively, the fact that $K \to \pi\nu\bar{\nu}$ decays are not affected by most supersymmetric flavor models [120, 121, 122] is an advantage here. The Standard Model correlation between $a_{\pi\nu\bar{\nu}}$ and $S_{\psi K_S}$ is a much cleaner test than a comparison of $S_{\psi K_S}$ to the CKM constraints.

(v) The neutral D system provides a stringent test of alignment. Observation of CP violation in the $D \to K\pi$ decays will make a convincing case for new physics.

(vi) CP violation in $b \to s$ transition remains an interesting probe of supersymmetry. Deviations of order 0.1 from the SM predictions are possible if at least one of the conditions in Eq. (183) is satisfied.

XIV. LESSONS FROM THE B FACTORIES

Let us summarize the main lessons that have been learned from the measurements of CP violation in B decays:

- The KM phase is different from zero, that is, the SM violates CP.
- The KM mechanism is the dominant source of CP violation in meson decays.
- The size and the phase of new physics contributions to $b \to d$ transitions ($B^0 - \overline{B}^0$ mixing) is severely constrained ($\leq \mathcal{O}(0.2)$).
- Complete alternatives to the KM mechanism (the superweak mechanism and approximate CP) are excluded.

- Corrections to the KM mechanism are possible, particularly for $b \to s$ transitions, but there is no evidence at present for such corrections.

- There is still a lot to be learned from future measurements.

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