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Planet in Danger. A System View; Theory, Models, Data Analysis**

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**Predictability of ENSO
Part III: Pacific MM as an optimal
stochastic forcing—Hypothesis test**

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Part III: Pacific MM as an optimal stochastic forcing—Hypothesis test

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Linear Stochastic Climate Dynamics

Hasselmann, K., 1976: Stochastic climate models. *Tellus*, **28**, 473-385.

Penland, M. C., and P. Sardeshmukh, 1995: The optimal growth of tropical sea surface temperature anomalies. *J. Climate*, **8**, 2000-2024.

Farrell, B. F., and P. J. Ioannou, 1996: Generalized stability. Part I: Autonomous operators. *J. Atmos. Sci.*, **53**, 2025-2040.

Chang, P., R. Saravanan, T. DelSole, F. Wang, and L. Ji, 2003: Predictability of Linear Coupled Systems. Part I: Theory, *J. Climate*, **17**, 1474-1486.

Coupled Systems

Equations of Motion :

1. Start from a set of deterministic equations based on 1st principle.

$$\frac{\partial s_a}{\partial t} = f(s_a, sst), \quad (1)$$

$$\frac{\partial s_o}{\partial t} = g(s_o, \tau), \quad (2)$$

where

- s_a : state vector of atmospheric variables,
- sst : sea-surface temperature,
- s_o : state vector of oceanic variables,
- τ : atmospheric fluxes at sea surface.

2. Linearize equations of motion about a mean climatic state. Define anomalies, $s'_o = s_o - \bar{s}_o$ and $s'_a = s_a - \bar{s}_a$:

$$\frac{\partial s'_a}{\partial t} = \frac{\partial f}{\partial s_a} \Big|_{\bar{s}_a} s'_a + \frac{\partial f}{\partial sst} \Big|_{\bar{sst}} sst' + N_a, \quad (3)$$

$$\frac{\partial s'_o}{\partial t} = \frac{\partial g}{\partial s_o} \Big|_{\bar{s}_o} s'_o + \frac{\partial g}{\partial \tau} \Big|_{\bar{\tau}} \tau' + N_o. \quad (4)$$

1

Linear Stochastic Coupled Model

Assumption : Atmosphere has a fast adjustment time scale, so that its variability can be divided into a quasi-equilibrium response to SST and a quasi-random part due to nonlinear processes.

Set $\frac{\partial s'_a}{\partial t} = 0, \Rightarrow \tau' = Csst' + N_a$; **C** a coupling matrix.

Linear Coupled Model :

$$\frac{\partial s'_o}{\partial t} = \mathbf{A}s'_o + N. \quad (1)$$

where

- **A**: System matrix or dynamic operator,
- **N**: Uncoupled atmos. and oceanic processes.

Linear Stochastic Model :

1. The coupled dynamics **A** is stable
2. The uncoupled processes **N** is a white noise

Predictability depends on A and N !

1

Analysis of Linear Stochastic Climate Model

$$\frac{d\vec{\theta}}{dt} = \mathbf{A}\vec{\theta} + \mathbf{F}\vec{\eta},$$

where $\vec{\theta}$ is a state vector which can be, for example, a grid representation of sea-surface temperature (SST) or other geophysical field, \mathbf{A} is the dynamic operator matrix of the system, characterizing the deterministic dynamics of the system, \mathbf{F} is a forcing matrix, consisting of spatial distributions of all the components of the random noise forcing, which can be, for example, EOFs or other representations of the noise forcing, $\vec{\eta}$ is a temporally Gaussian white-noise forcing vector with ensemble covariance $\langle \eta_i(t_1)\eta_j^*(t_2) \rangle = \delta_{ij}\delta(t_1 - t_2)$.

Solution:

$$\vec{\theta} = \underbrace{e^{\mathbf{A}(t-t_0)}}_{\text{propagator}} \underbrace{\vec{\theta}(t_0)}_{\text{initial condition}} + \int_{t_0}^t e^{\mathbf{A}(t-s)} \mathbf{F} \vec{\eta}(s) ds,$$

Signal $\vec{\theta}_s$
Noise $\vec{\theta}_e$

predictable component
unpredictable component

Variance and Predictability Measure

- Total variance or climatological variance:

$$\sigma_{\infty}^2 = \text{trace}(\mathbf{C}(\infty)) = \langle \vec{\theta}_0^* \cdot \vec{\theta}_0 \rangle = \langle \vec{\theta}_s^* \cdot \vec{\theta}_s \rangle + \langle \vec{\theta}_e^* \cdot \vec{\theta}_e \rangle$$

- Normalized error variance:

$$\epsilon^2(\tau) = \frac{\sigma_e^2(\tau)}{\sigma_{\infty}^2} = \frac{\langle \vec{\theta}_e^* \cdot \vec{\theta}_e \rangle}{\langle \vec{\theta}_0^* \cdot \vec{\theta}_0 \rangle}.$$

Optimal Noise Forcing

- Is there a particular **spatial structure** \vec{f}_1 the noise forcing under which the system variance or predictability can be maximized?

$$\frac{d\vec{\theta}}{dt} = \mathbf{A}\vec{\theta} + \vec{f}_1\eta_1.$$

Stochastic dynamic operator

$$\sigma_{\infty}^2 = \vec{f}_1^* \mathbf{B}(\infty) \vec{f}_1, \quad \sigma_e^2(\tau) = \vec{f}_1^* \mathbf{B}(\tau) \vec{f}_1, \quad \mathbf{B}(\tau) = \int_0^{\tau} e^{\mathbf{A}^*s} e^{\mathbf{A}s} ds.$$

$$\epsilon^2(\tau) = \frac{\sigma_e^2(\tau)}{\sigma_{\infty}^2} = \frac{\vec{f}_1^* \mathbf{B}(\tau) \vec{f}_1}{\vec{f}_1^* \mathbf{B}(\infty) \vec{f}_1}.$$

- \vec{f}_{ϵ^2} that minimizes the normalized error variance is given by the follow eigenvalue problem:

$$\mathbf{B}(\infty) \vec{f}_{\epsilon^2} = \nu \mathbf{B}(\tau) \vec{f}_{\epsilon^2}$$

- \vec{f}_{ϵ^2} is called stochastic optimal. At small lead time, it optimizes the total variance.

Hypothesis

The Pacific Meridional Mode (MM) is a *stochastic optimal* of ENSO. Therefore, ENSO is especially responsive to MM-like forcing. When “noise” projects strongly onto the MM, ENSO is more energetic and more predictable.

First Test Hypothesis

Noise Filter Experiment:

- 1) $\tau^x = (\tau^x)_{S \leftarrow SST} + (\tau^x)_N$
 $\tau^y = (\tau^y)_{S \leftarrow SST} + (\tau^y)_N$
 $q = (q)_{S \leftarrow SST} + (q)_N$
- 2) Find signal / noise optimal to construct a noise filter
- 3) Apply noise filter at each coupled step, only allowing $(\tau^x)_{S \leftarrow SST}$, $(\tau^y)_{S \leftarrow SST}$, $(q)_{S \leftarrow SST}$ to interact with ocean
- 3) Since noise is suppressed, MM variance is reduced and projection onto the stochastic optimal is reduced. ENSO variance should decrease.

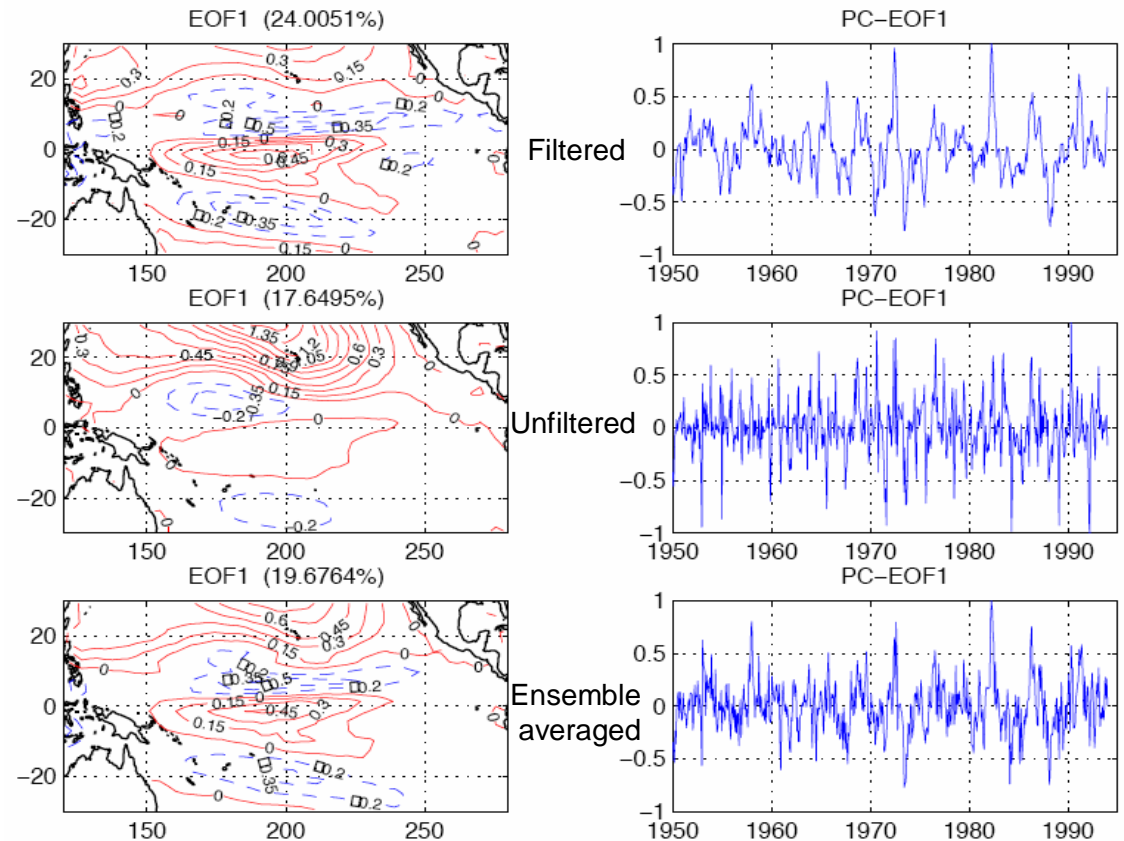
Noise Filter

Procedure:

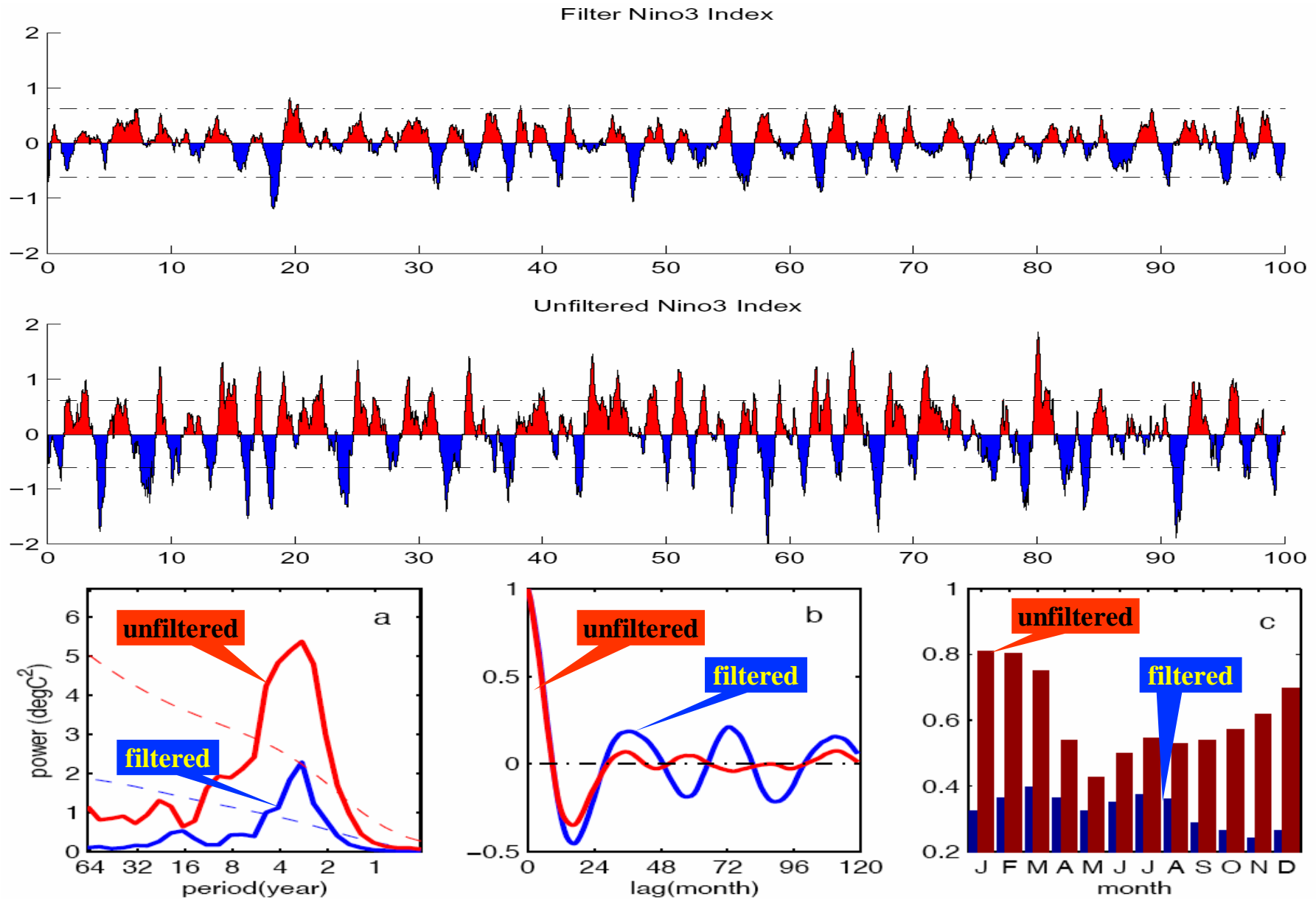
- Performing an ensemble of AMIP runs forced with SST from CCM3-RGO.
- Formulating ensemble mean covariance matrix C_M and noise covariance matrix C_N .
- Estimating “true” SST forced response using a s/n optimization procedure:

$$\mathbf{X}_S = (C_M - \frac{1}{m}C_N)C_M^{-1}\mathbf{X}_M = (\mathbf{I} - \frac{1}{m}C_N C_M^{-1})\mathbf{X}_M$$

Results



Suppression of Atmosphere Internal Variability



Second Test Hypothesis

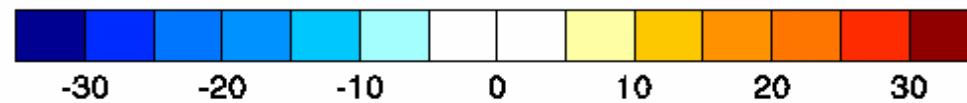
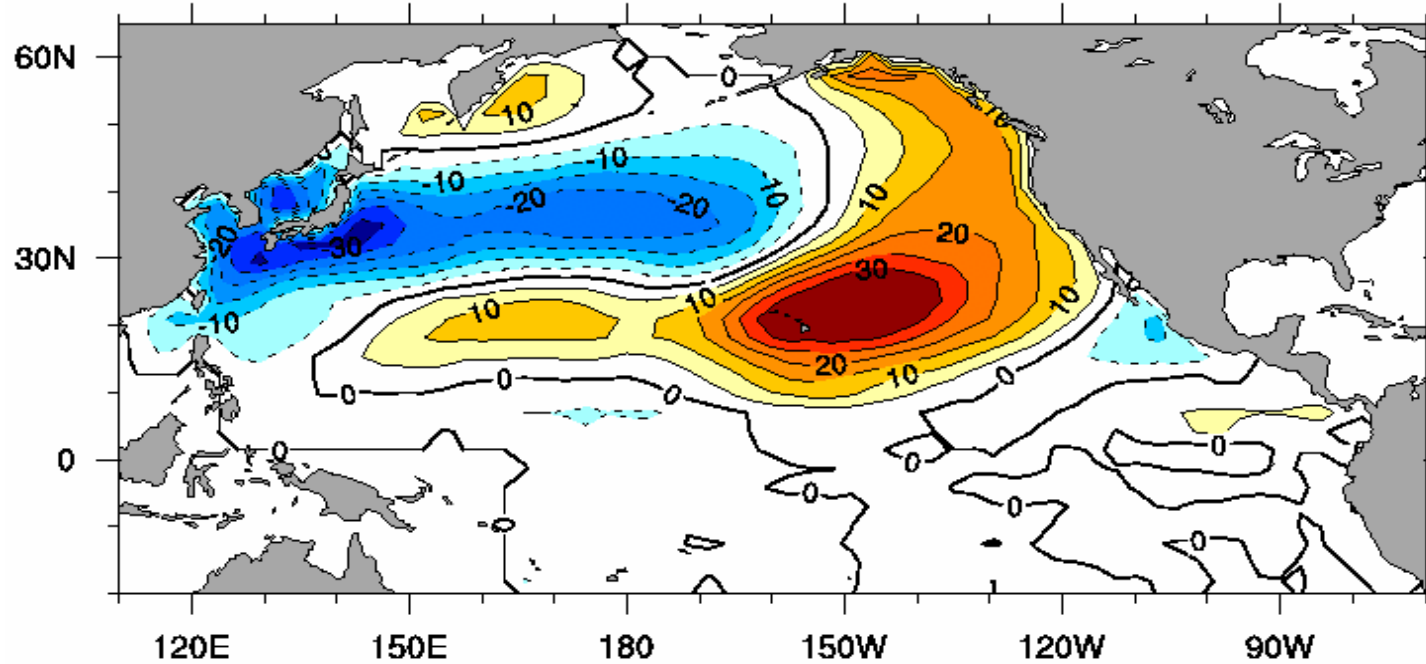
Heat Flux Forcing Experiment (Alexander et al):

- 1) Determine the heat flux pattern that forces the MM.
- 2) Force the coupled model with the heat flux anomaly from Nov to March and then let the model evolve freely.
- 3) An ensemble of 50 runs with different initial conditions.
- 4) Since the heat flux forcing is chosen to enhance MM, MM variance is increased and projection onto the stochastic optimal is also increased. ENSO variance should increase.

CCM3 Heatflux Anom (Vimont*2) positive down

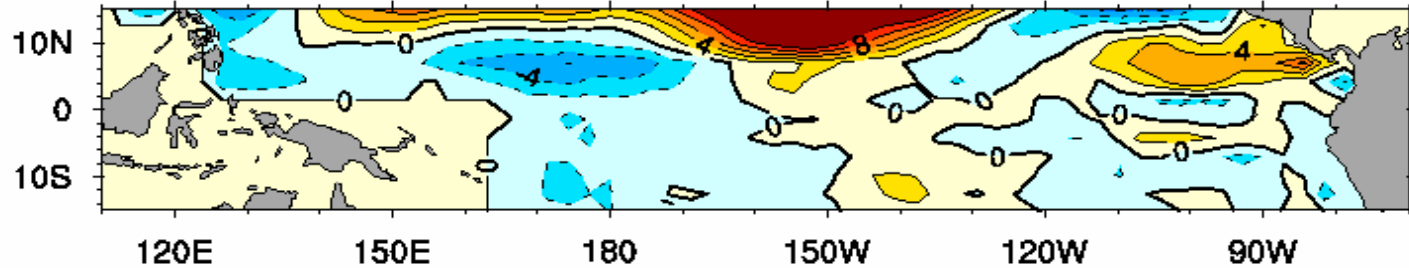
Anomalous forcing for NDJFM(0)

Wm⁻²

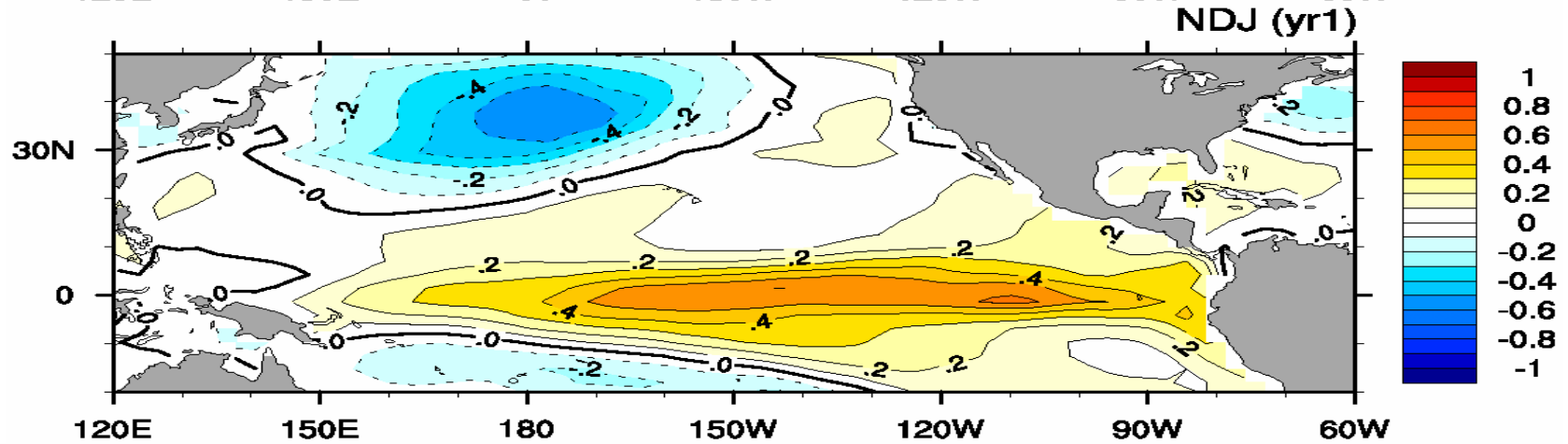
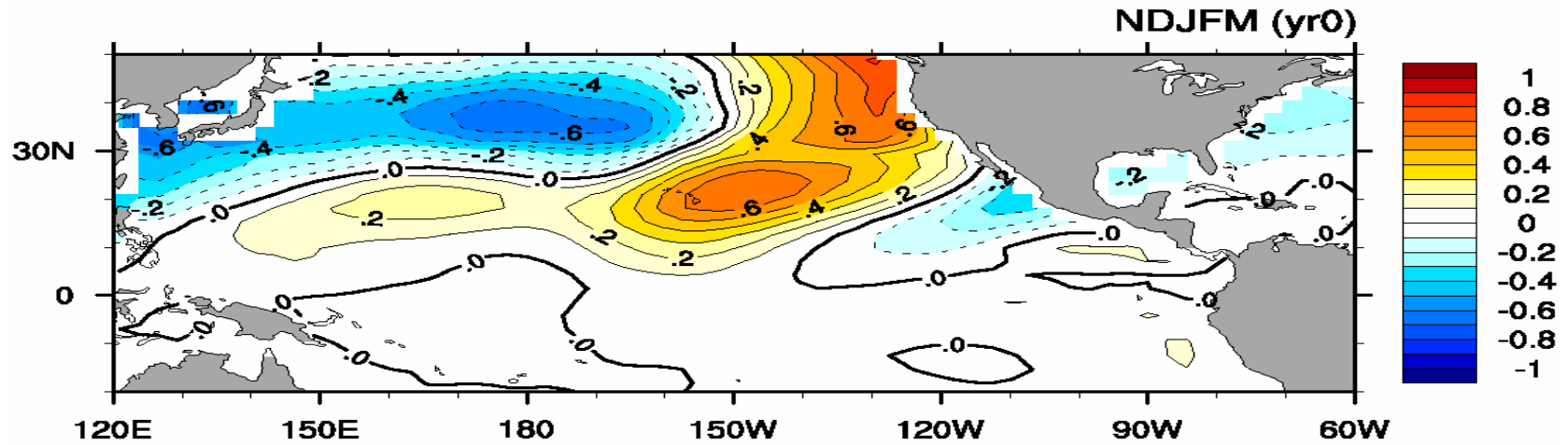


Anomalous forcing for NDJFM(0)

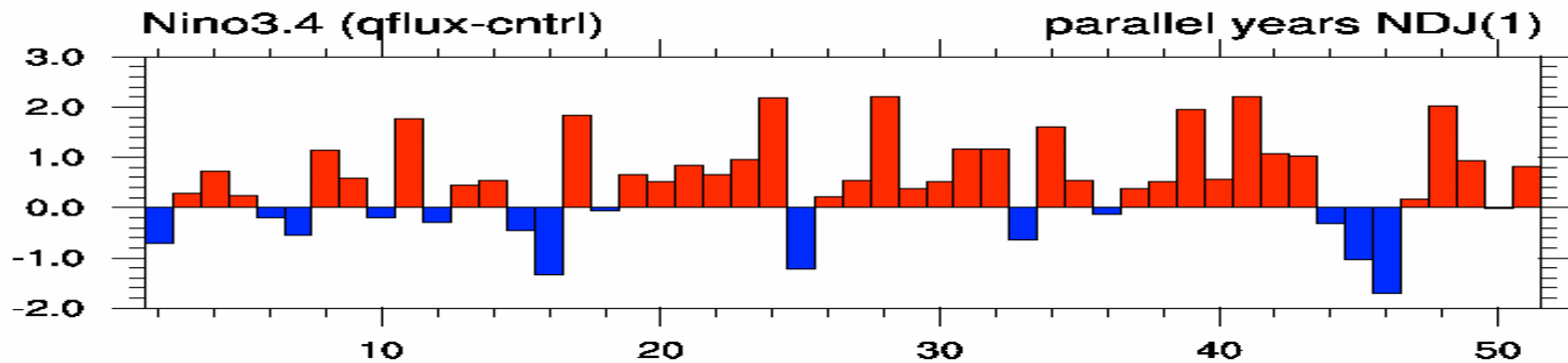
Wm⁻²



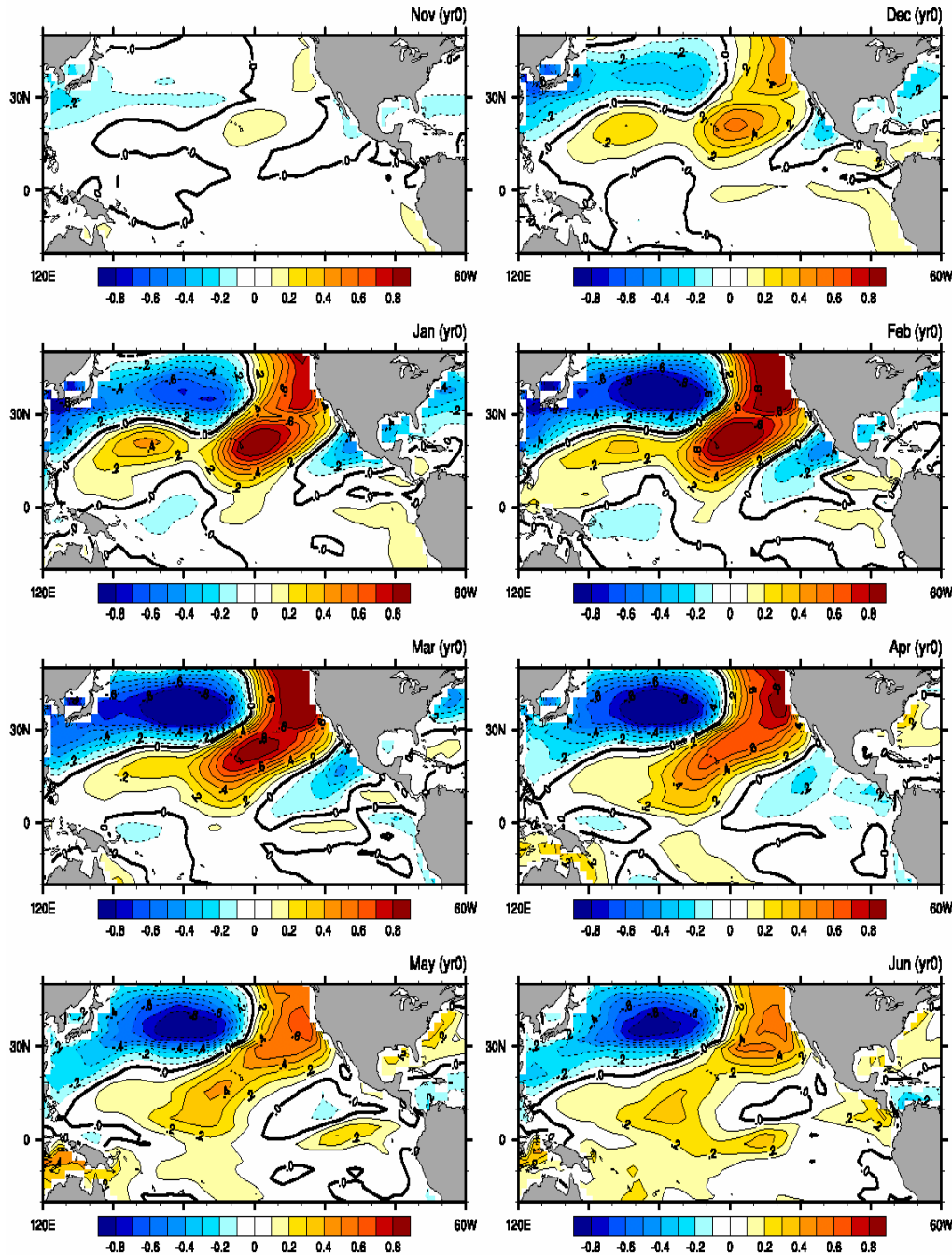
SST Qflux(50)-CNTRL(parallel years)



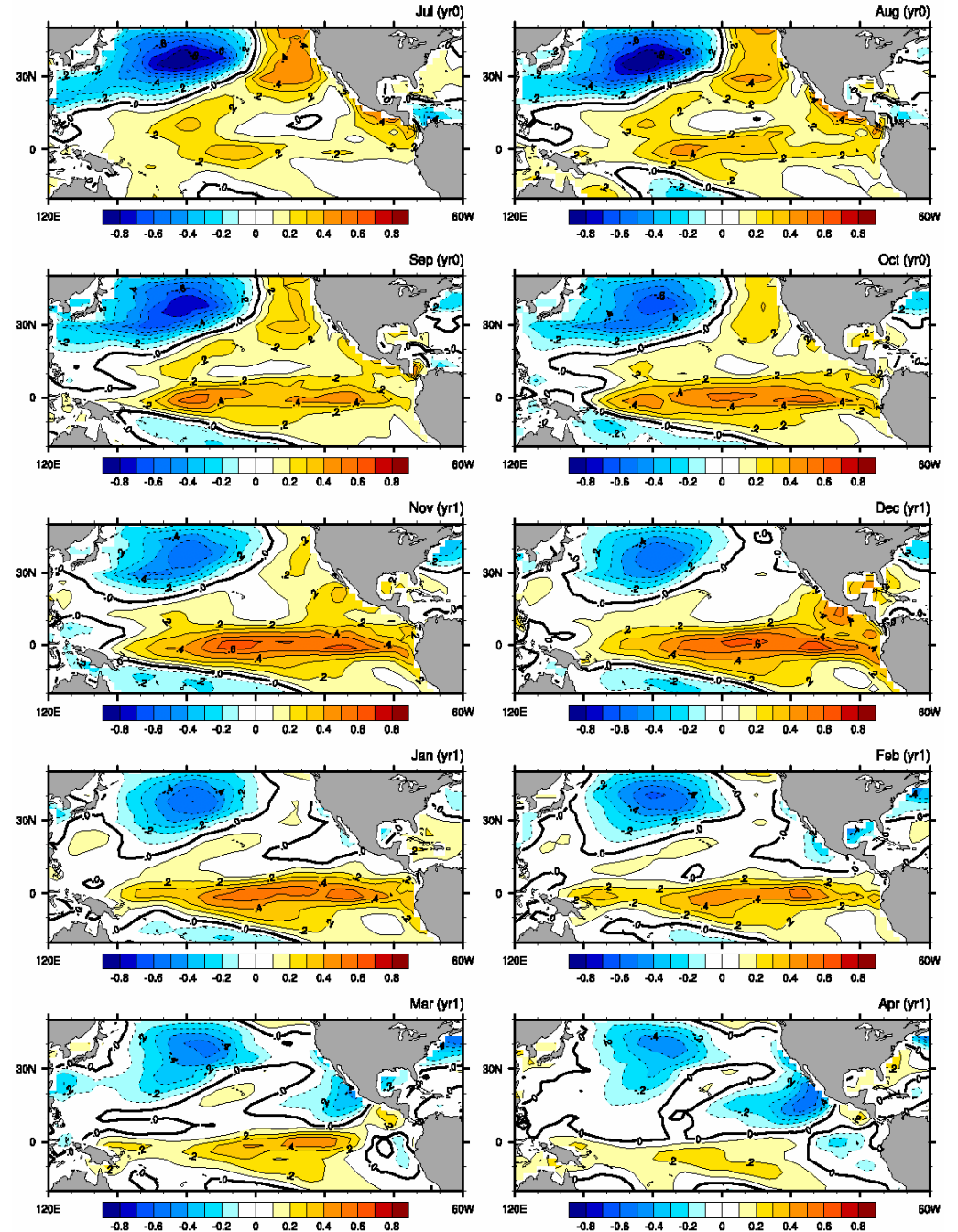
SST Qflux(50)-CNTRL(parallel years)



SST Qflux(50)-CNTRL(parallel years)



SST Qflux(50)-CNTRL(parallel years)

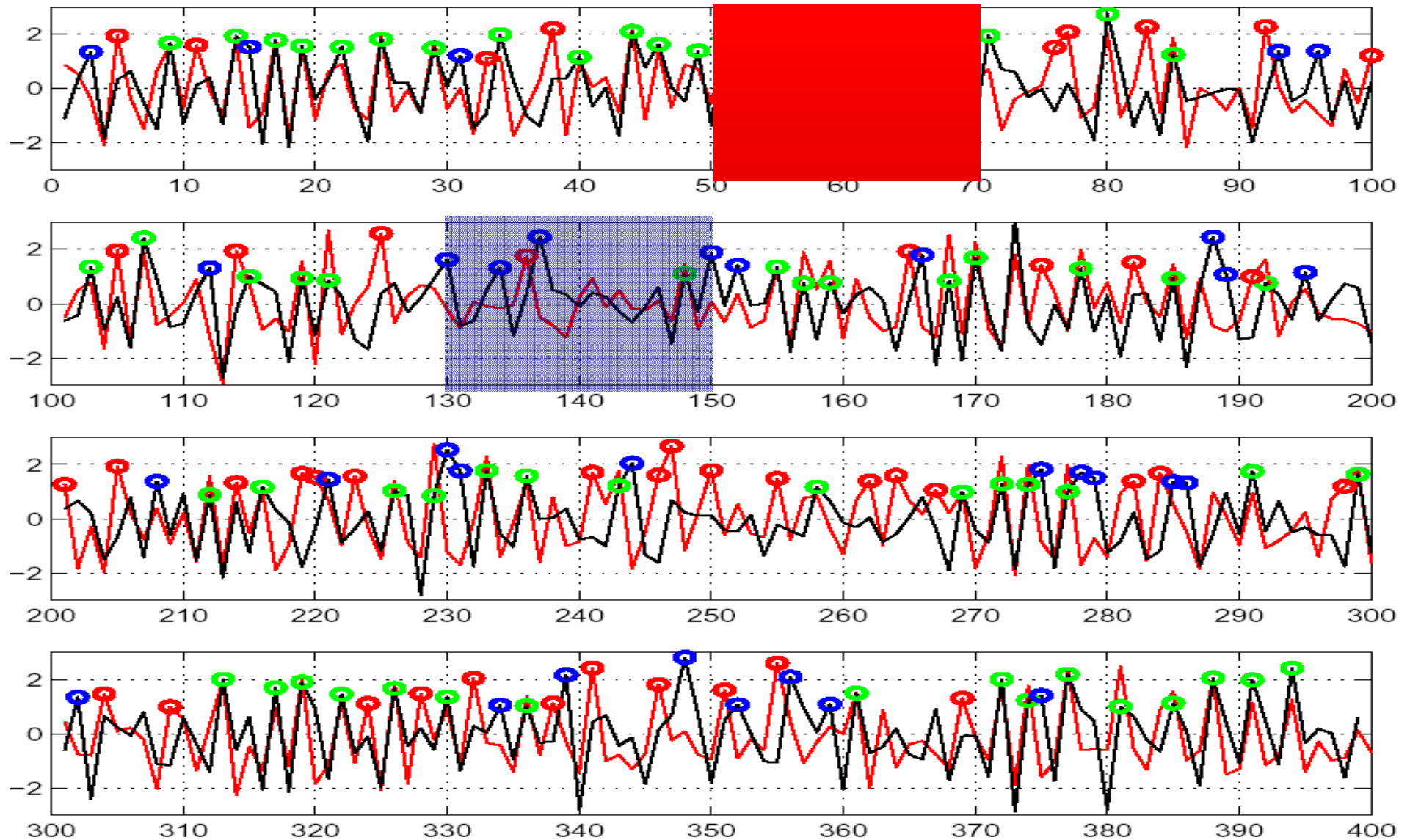


Third Test Hypothesis

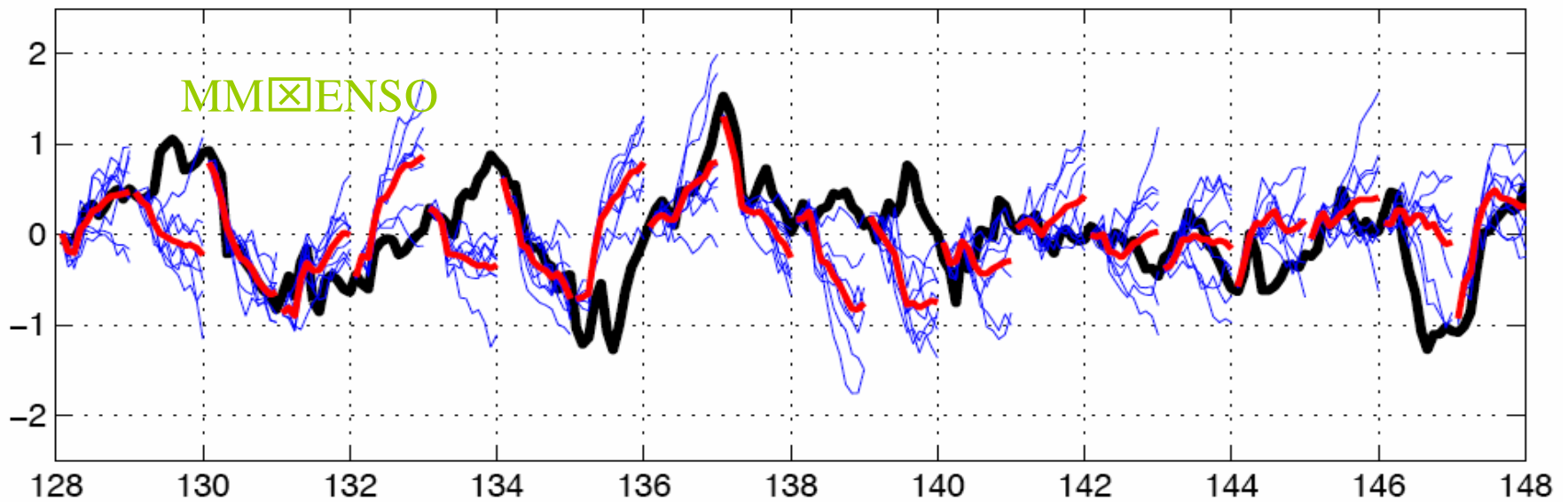
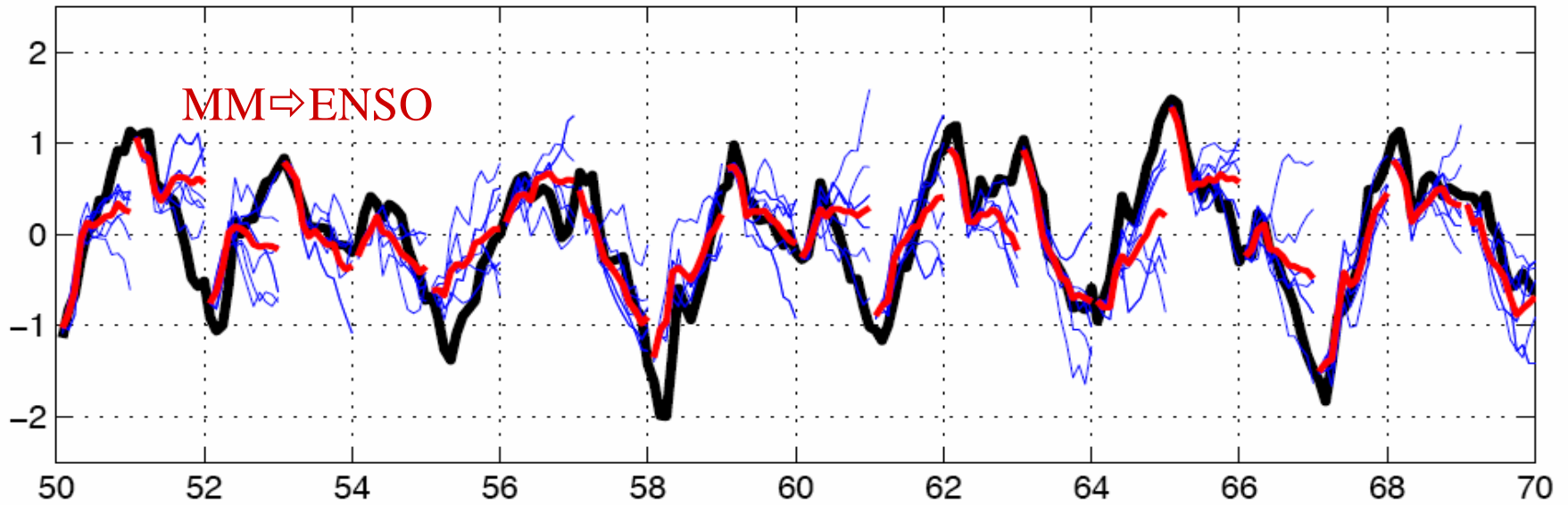
Perfect Model Prediction Experiments:

- 1) Initialize the coupled model with its own control simulation and perform an ensemble of prediction runs.
- 2) Analyze model forecast skills by validating its prediction against its own control run.
- 3) Choose two 20-year periods, one corresponds to $MM \Rightarrow ENSO$; the other corresponds to $MM \neq \Rightarrow ENSO$. An ensemble of 10 predictions starting Dec. 1.
- 3) If the MM is indeed a stochastic optimal, then the model forecast skills should be higher for the $MM \Rightarrow ENSO$ period, because the stochastic optimal should minimize normalized error variance.

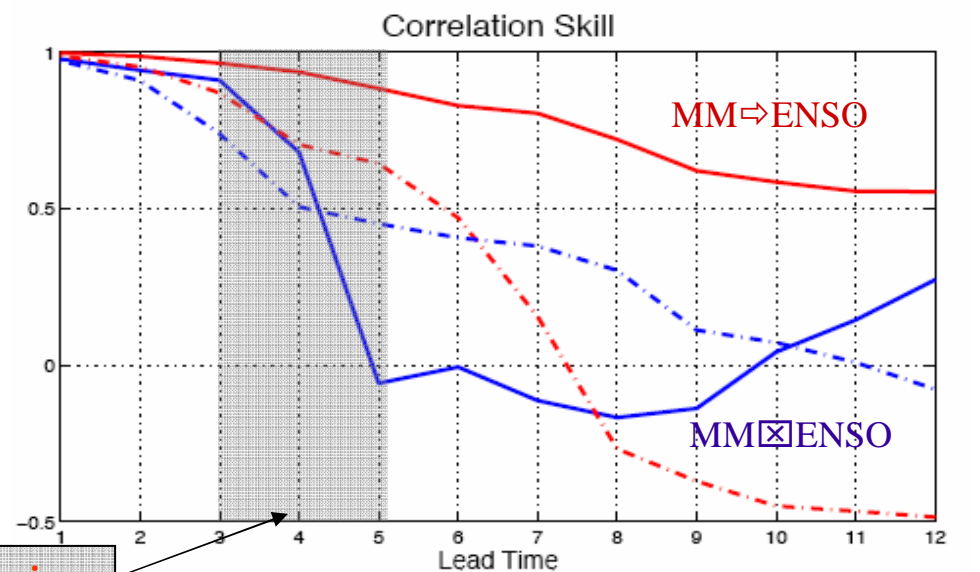
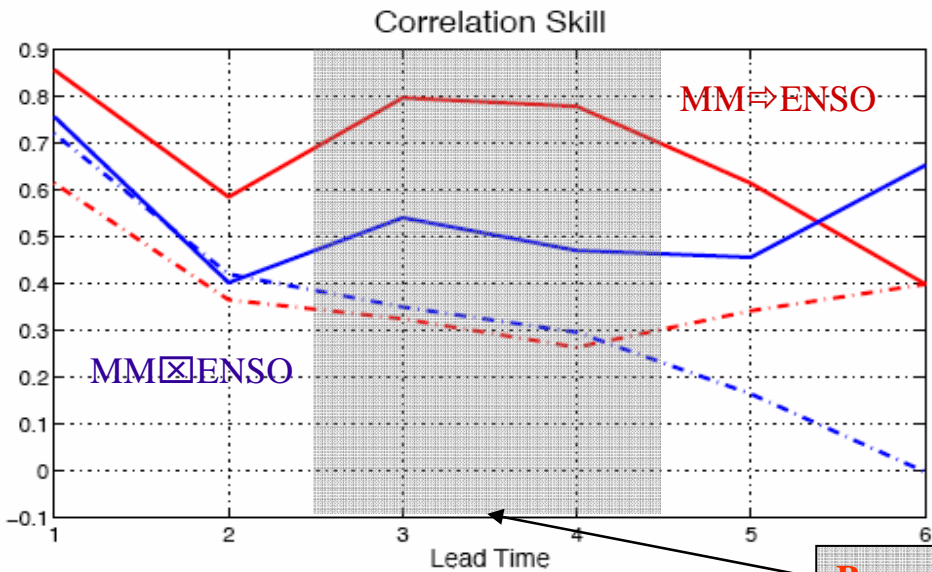
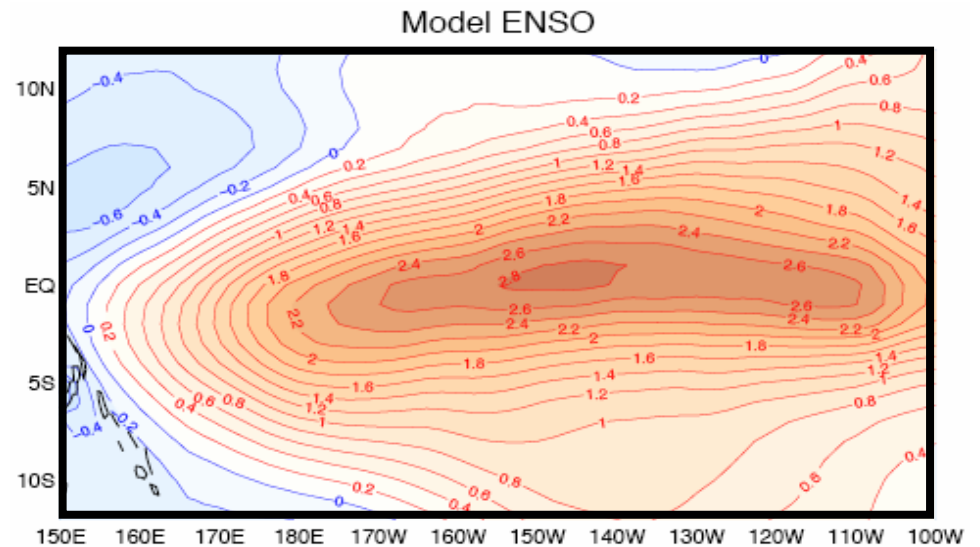
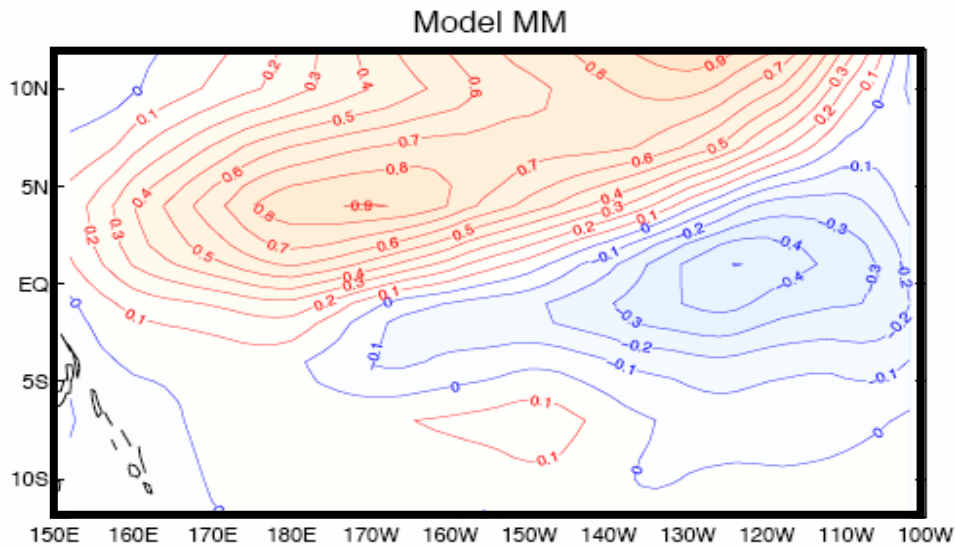
400-year FMAM MM τ^x and DJF Nino3



Ensemble of Forecasts



Skillful MM Forecast \Rightarrow Skillful ENSO Forecast



Boreal Spring

Penland and Sardeshmuk (1998) show that there exists a certain initial optimal that leads to ENSO onset. The initial optimal bears a close resemblance to the MM SST.

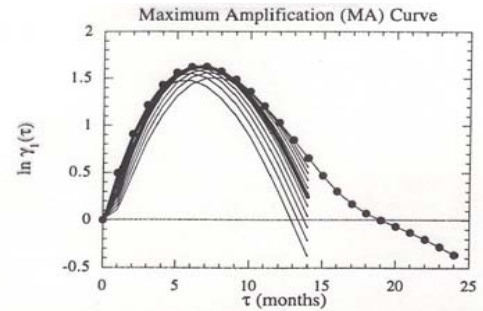
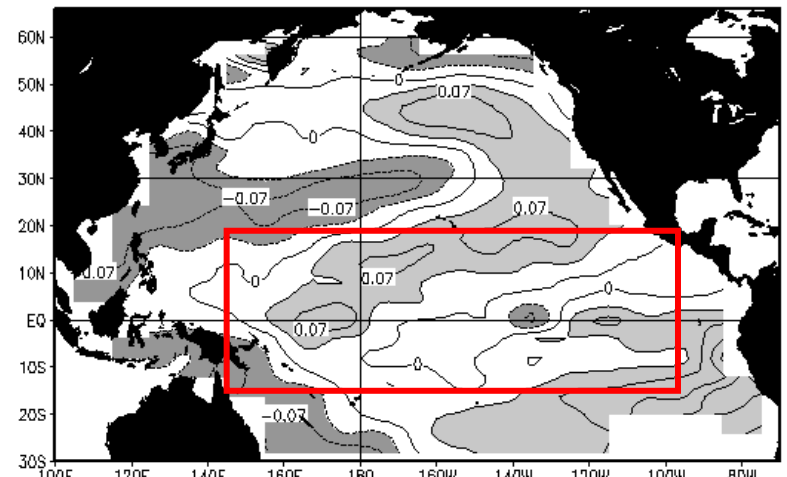
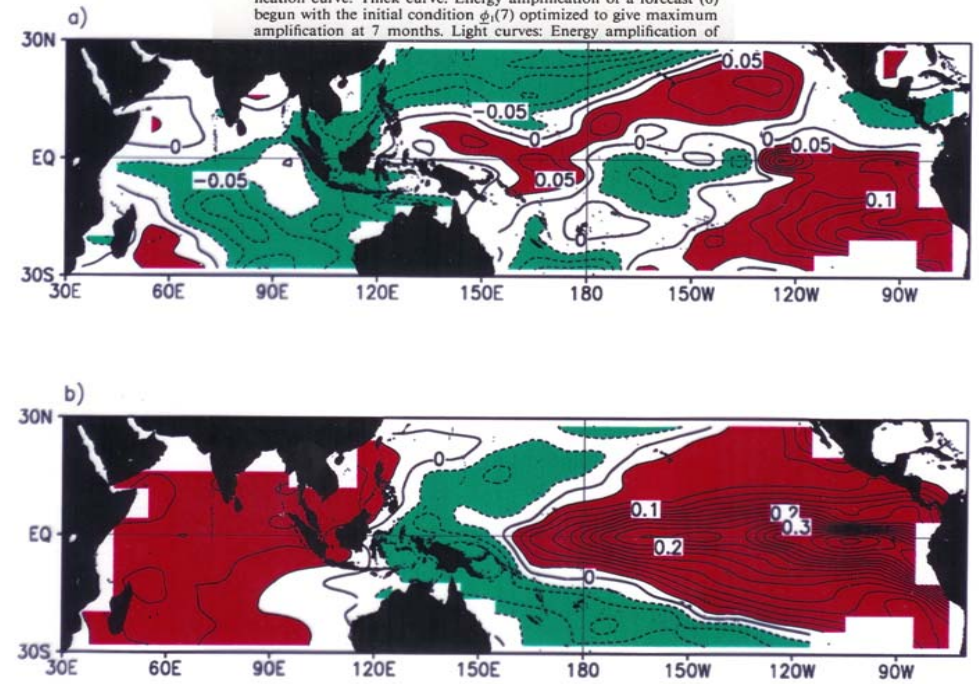
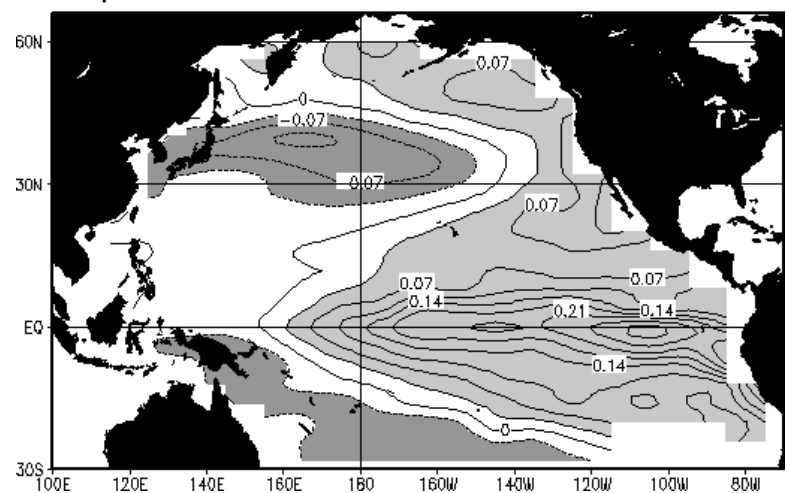


FIG. 4. (a) Thick curve with filled circles: The maximum amplification curve. Thick curve: Energy amplification of a forecast (6) begun with the initial condition $\phi_i(7)$ optimized to give maximum amplification at 7 months. Light curves: Energy amplification of

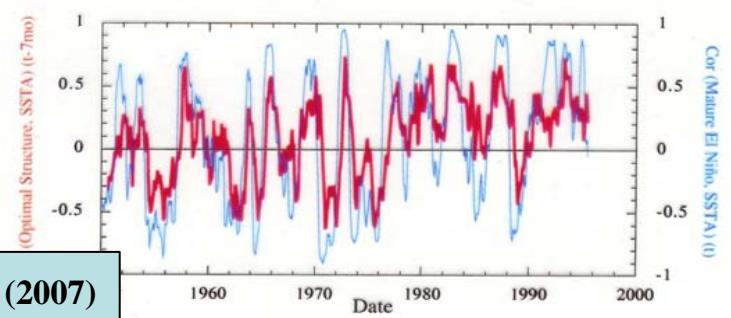
Optimal Structure in the Pacific Ocean (15 Eofs)



Optimal Structure evolves in 7 months



Alexander et al. (2007)

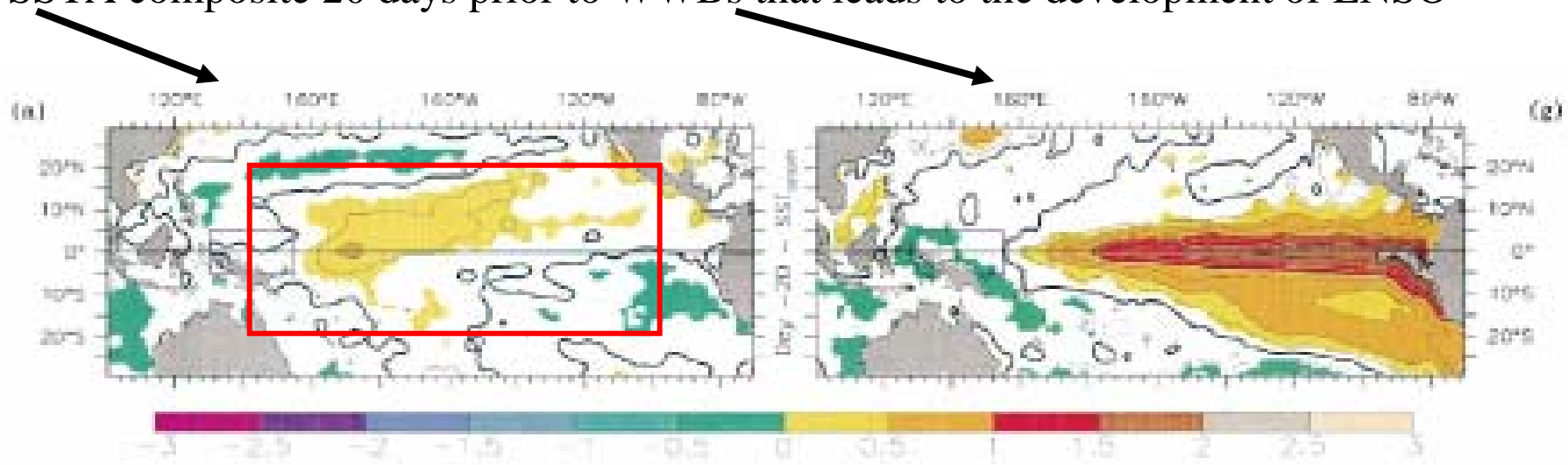


Summary

- The study confirms the existence of the meridional mode (MM) proposed by Chiang and Vimont (2004). It further shows that the MM can act as an important trigger for ENSO.
- Coupled model experiments suggest that the MM is inherent to thermodynamic coupling in ITCZ latitudes, which enhances its persistence. It is proposed that the MM is an optimal stochastic forcing of ENSO.
- This hypothesis is tested by a number of model experiments. The results thus far are consistent with the hypothesis.
- Noise-filter experiments suggest that the MM conduit effect plays an important role in the seasonal phase-locking of ENSO.
- Prediction experiments suggest that the MM-ENSO relationship may have an impact on ENSO predictability. This leads to a conjecture that ENSO prediction may be improved if the MM could be predicted, particularly during the spring barrier.

Vecchi and Harrison (2000)

SSTA composite 20 days prior to WWBs that leads to the development of ENSO



The SSTA composite shows a very similar structure to the MM SST.