The Abdus Salam International Centre for Theoretical Physics

United Nations Educational, Scientific and Cultural Organization

IAEA International Atomic Energy Agency

SMR/1849-6

Conference and School on Predictability of Natural Disasters for our Planet in Danger. A System View; Theory, Models, Data Analysis

25 June - 6 July, 2007

Natural Modes of Variability: Linear and Nonlinear Perspectives II. Scale Interactions

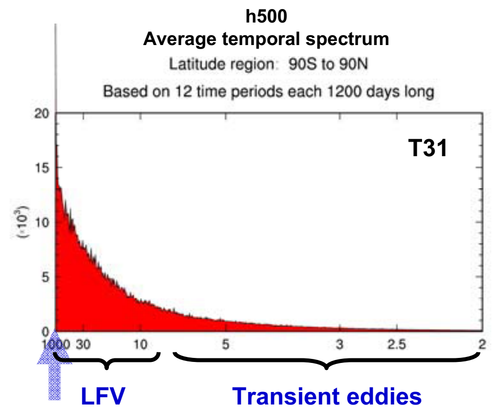
Grant Branstator

National Center for Atmospheric Research
Boulder, CO, U.S.A.

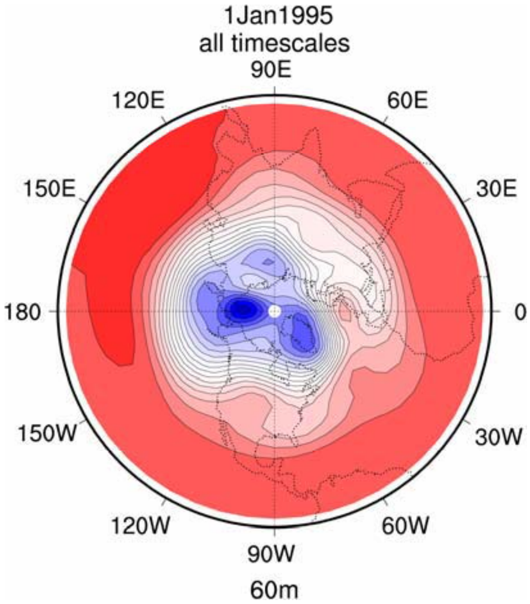
Natural Modes of Variability:
Linear and Nonlinear Perspectives

Grant Branstator
National Center for Atmospheric Research

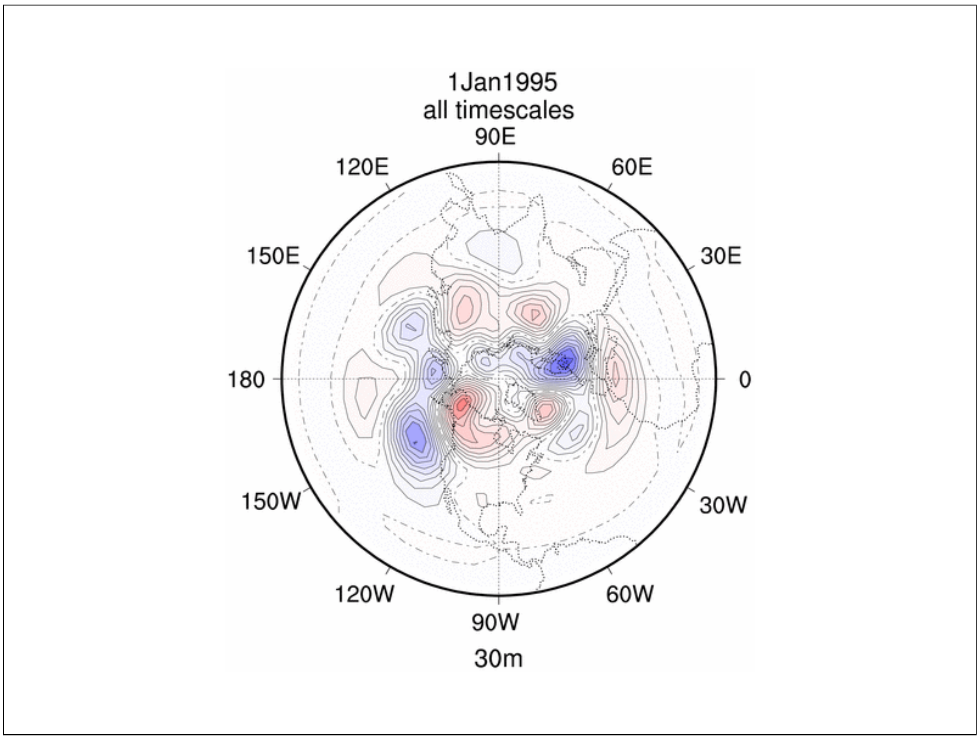
II. Scale Interactions



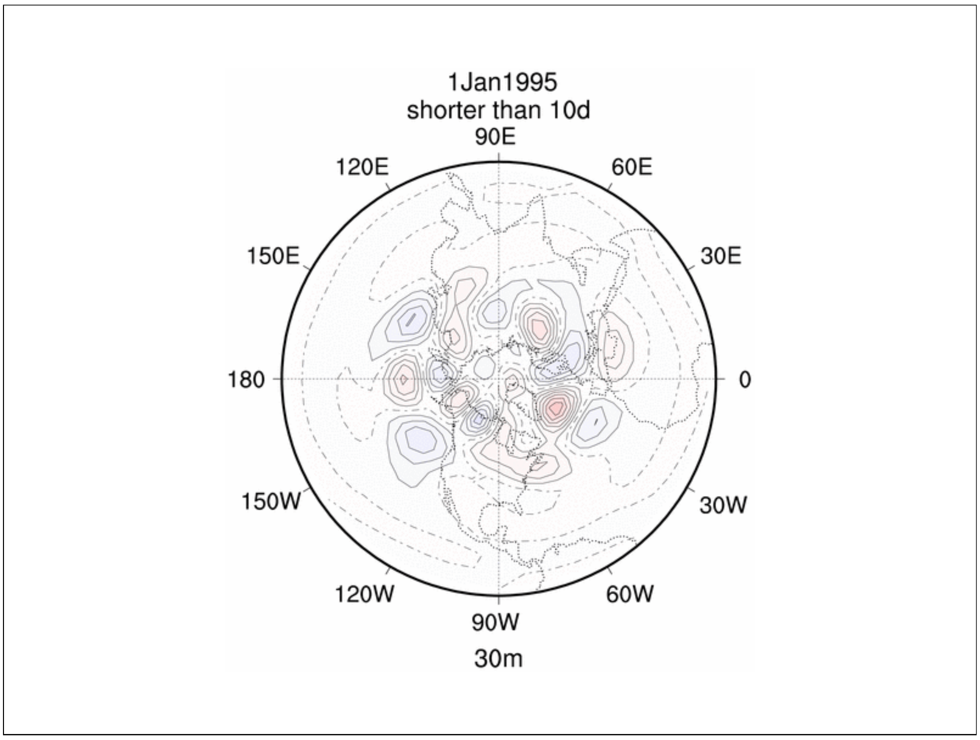
Modes of variability are recurring patterns that are prominent in atmospheric variability that takes place on timescales longer than a week or two. We call this low-frequency variability. An important property of these modes and this timescale is that it interacts very strongly with other timescales, namely the time mean state and the transient (or synoptic) eddies. In this lecture we show evidence of this interaction and demonstrate how it can be accounted for and understood using linear approximations even though it results from nonlinear terms in the governing equations.



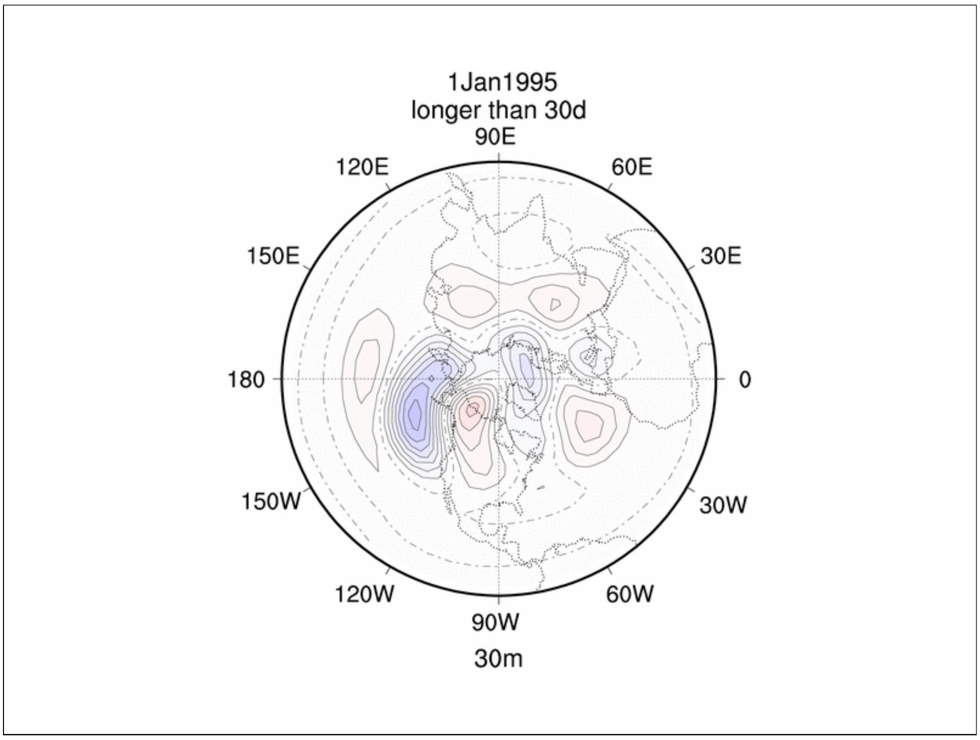
To get a feeling for the timescales we are talking about, we look at an example from nature. From this movie of 500kPa geopotential heights, we see that the circulation is dominated by a time mean component. Our eye tends to primarily notice the eastward propagating perturbations that are superimposed on this time mean state.



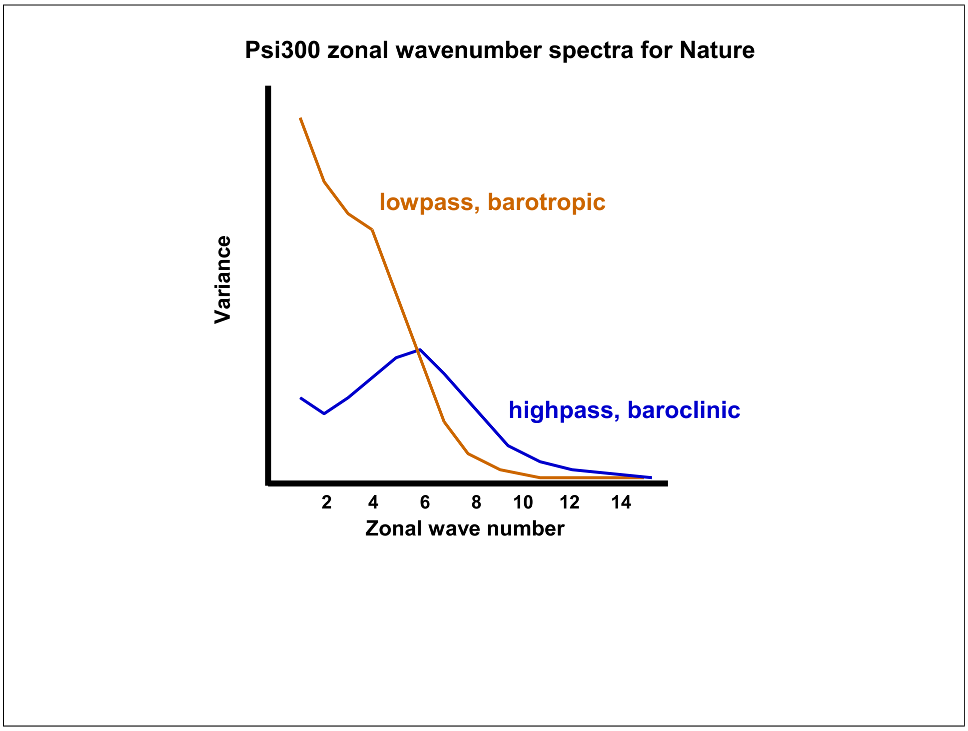
By removing the time mean, the propagating features are easier to see in this movie. Circulation anomalies consist of a variety of shapes that evolve in seemingly unorganized ways.



In this movie, where the fields are filtered to remove all but the synoptic timescale features, it is the eastward propagating features that remain. Furthermore the space scales are rather small.

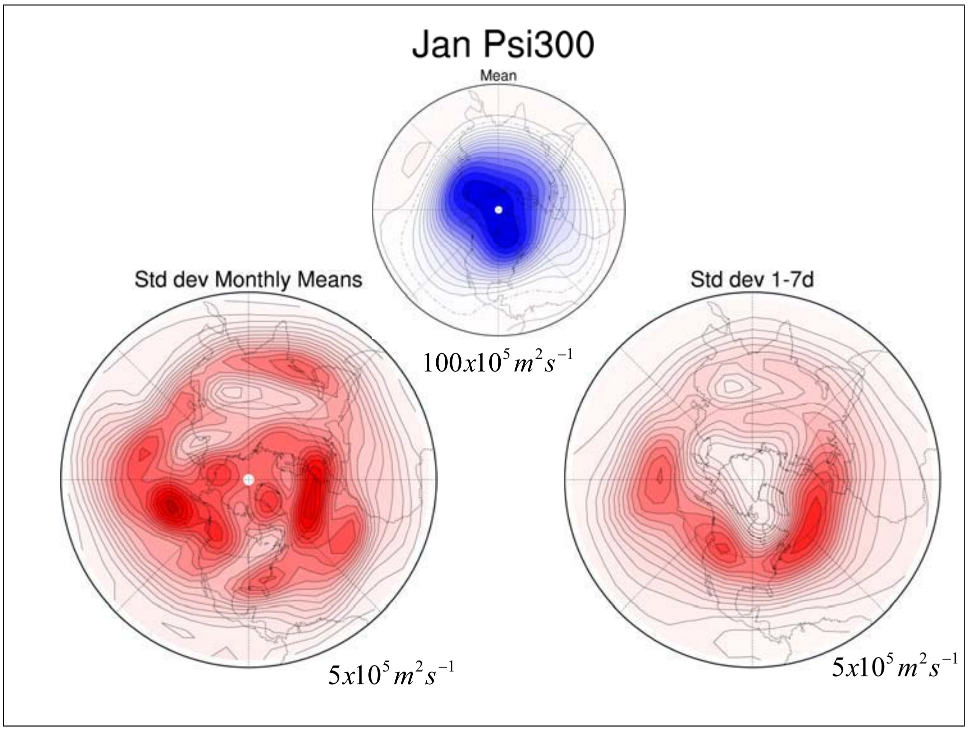


In this movie, which concentrates on the slowly evolving component of the circulation, more organization is evident than for the synoptic scales. But it is difficult to pick out a preferred direction of phase propagation. Some features propagate westward, others eastward. With this temporal filter the space scales are larger than for the synoptic timescale filter.



If we look at the variance in the two temporal bands as a function of zonal wavenumber, we see the distinctions in space scale noticed in the movies. LFV is concentrated at much smaller wavenumbers. (Here low pass is running 5d means and highpass is departures from 5d means.)

And it turns out that the two bands have a second distinction, namely the LFs are much more equivalent barotropic than are the bandpass eddies – that is LFV has much weaker vertical tilts to its phase lines.



Both frequency bands tend to have their strongest variability in midlatitudes but at somewhat different longitudes.

Two-way Timescale Interactions

$$\frac{\partial \varsigma}{\partial t} = -\vec{v}_\psi \cdot \nabla(\varsigma + f) - ...$$

$$() = \frac{1}{T}\int_0^T () dt + ()' = (\hat{\bullet}) + (\bullet)'$$

$$\frac{\partial \hat{\varsigma}}{\partial t} + \frac{\partial \varsigma'}{\partial t} = -\hat{\vec{v}}_\psi \cdot \nabla(\hat{\varsigma} + f) - \hat{\vec{v}}_\psi \cdot \nabla \varsigma' - \vec{v}'_\psi \cdot \nabla(\hat{\varsigma} + f) - \vec{v}'_\psi \cdot \nabla \varsigma' ...$$

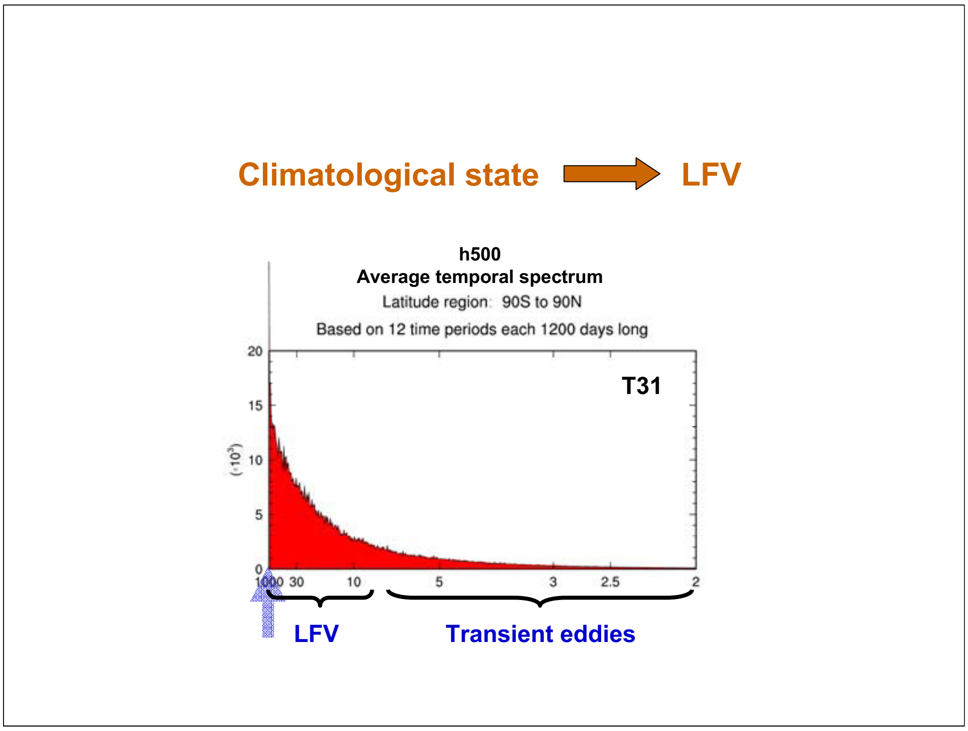
Time averaging gives:

$$\frac{\partial \hat{\varsigma}}{\partial t} = -\hat{\vec{v}}_\psi \cdot \nabla(\hat{\varsigma} + f) - \widehat{\vec{v}'_\psi \cdot \nabla \varsigma'} ...$$

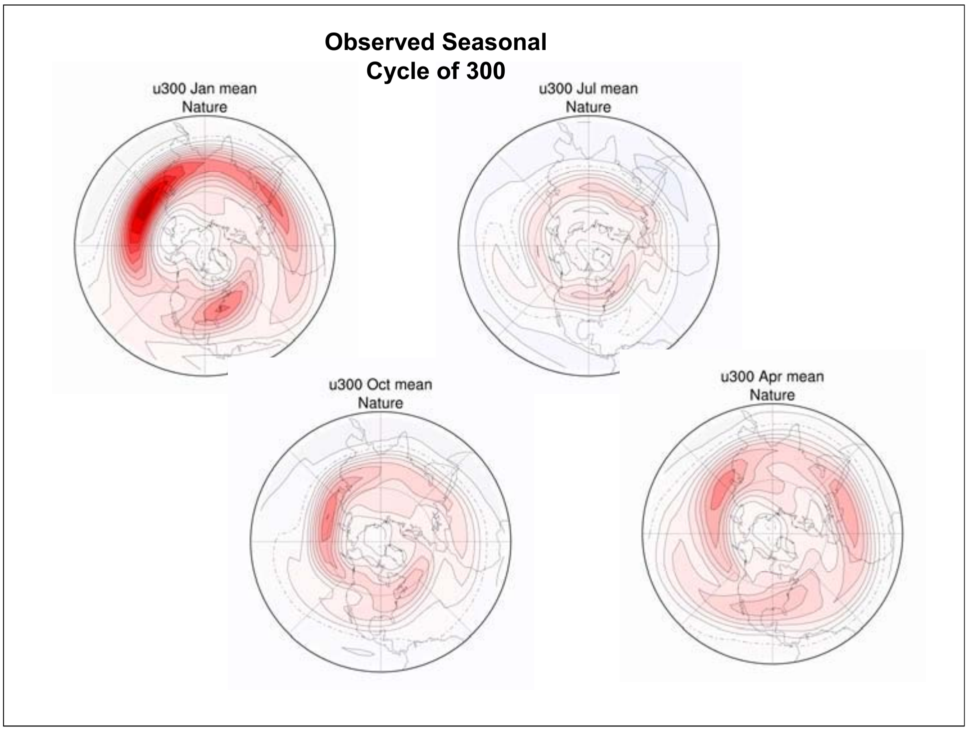
Subtracting time average equation from full equation gives:

$$\frac{\partial \varsigma'}{\partial t} = -\hat{\vec{v}}_\psi \cdot \nabla \varsigma' - \vec{v}'_\psi \cdot \nabla(\hat{\varsigma} + f) - (\vec{v}'_\psi \cdot \nabla \varsigma' - \widehat{\vec{v}'_\psi \cdot \nabla \varsigma'}) ...$$

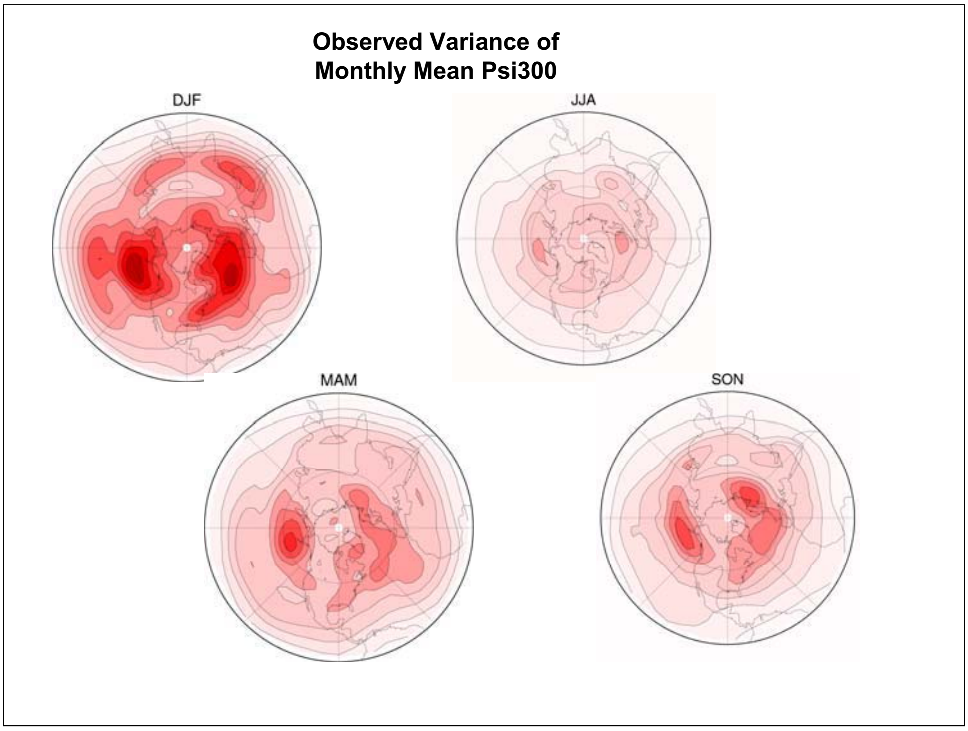
That there is potential for two-way interactions between these various timescales can be seen by considering the vorticity equation. Similar interactions occur because of nonlinearities in the other governing equations as well.



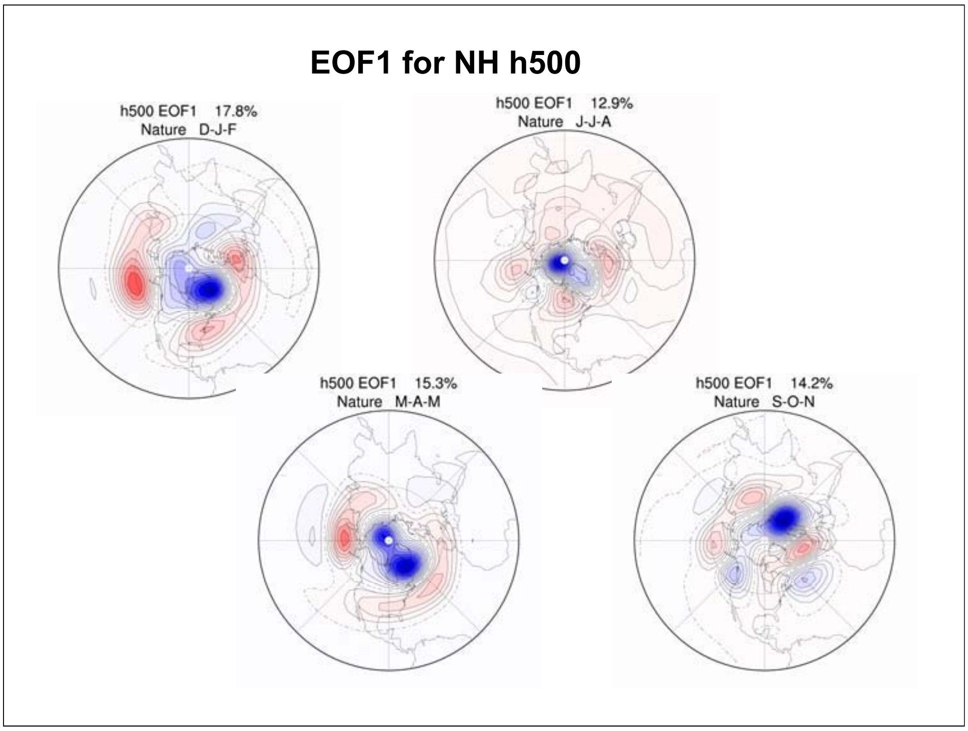
Now we consider interactions between each of the three timescales by quantifying and characterizing those interactions. First consider the effect of the climatological state on LFV.



Note there is also a strong seasonal cycle in the zonal mean with the latitude and strength of the westerlies changing with the seasons.



Accompanying the seasonal cycle of the mean state is a seasonal cycle of the interannual variability. Given the potential for two-way interactions, and the influence that we saw that the mean state can have on the linear modes of the system in lecture I, it is possible that the seasonality of the mean state causes the seasonality of the variance of low-frequency variability.



The structure of the most prominent patterns also changes with the seasons. Again, this may be a result of the seasonality of the mean state.

$$\frac{\partial \varsigma}{\partial t} = -\vec{v}_{\psi} \cdot \nabla(\varsigma + f) - ...$$

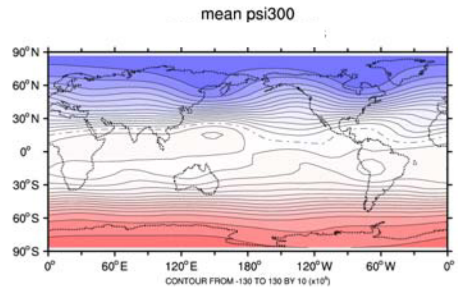
$$() = \frac{1}{T}\int_0^T () dt + ()'$$

$$\frac{\partial \varsigma'}{\partial t} = -\bar{\vec{v}}_{\psi} \cdot \nabla(\bar{\varsigma} + f) - \bar{\vec{v}}_{\psi} \cdot \nabla\varsigma' - \vec{v}'_{\psi} \cdot \nabla(\bar{\varsigma} + f) - \vec{v}'_{\psi} \cdot \nabla\varsigma' ...$$

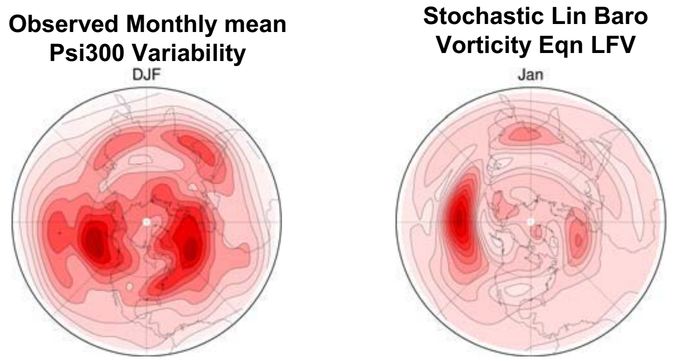
$$\bar{\vec{v}}_{\psi} \cdot \nabla(\bar{\varsigma} + f) = -\overline{\vec{v}'_{\psi} \cdot \nabla\varsigma'} + ...$$

$$\frac{\partial \varsigma'}{\partial t} = -\bar{\vec{v}}_{\psi} \cdot \nabla\varsigma' - \vec{v}'_{\psi} \cdot \nabla(\bar{\varsigma} + f) - (\vec{v}'_{\psi} \cdot \nabla\varsigma' - \overline{\vec{v}'_{\psi} \cdot \nabla\varsigma'}) ...$$

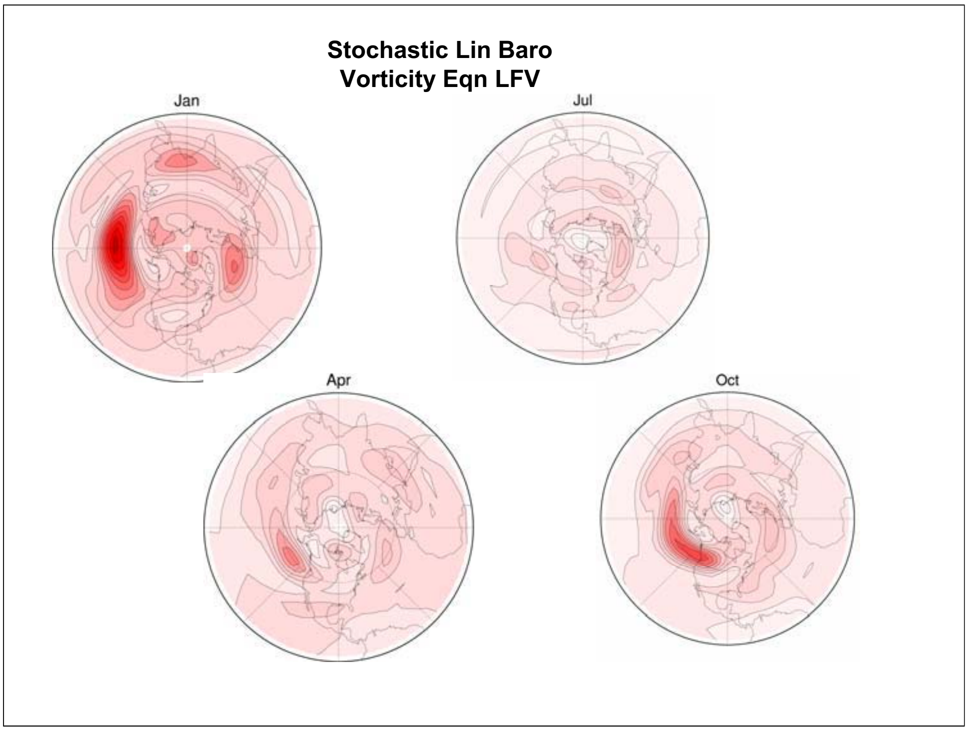
$$= -\bar{\vec{v}}_{\psi} \cdot \nabla\varsigma' - \vec{v}'_{\psi} \cdot \nabla(\bar{\varsigma} + f) + damping + noise ...$$



To test whether the seasonality of the mean state is an example of the mean state influencing low-frequency variability, we do a calculation in which the only influence on low-frequency variability is the effect of the mean state. Assuming that conservation of angular momentum is the controlling constraint on low-frequency midlatitude variability, we linearize the barotropic vorticity equation about the January mean streamfunction at 300 hPa and replace nonlinearities with linear damping and stochastic forcing that is white in space and time.

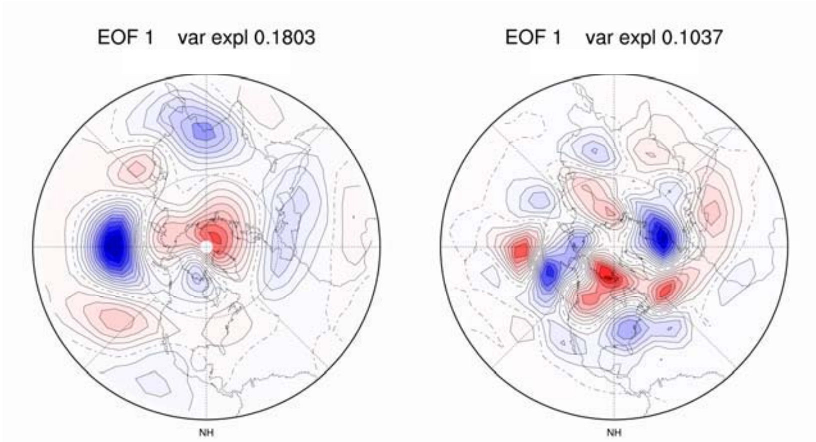


The variability of monthly means from this linear stochastic integration has a distribution much like that observed in nature. Hence it appears that the influence of the January mean state on low-frequency variability does have a large influence on its organization.



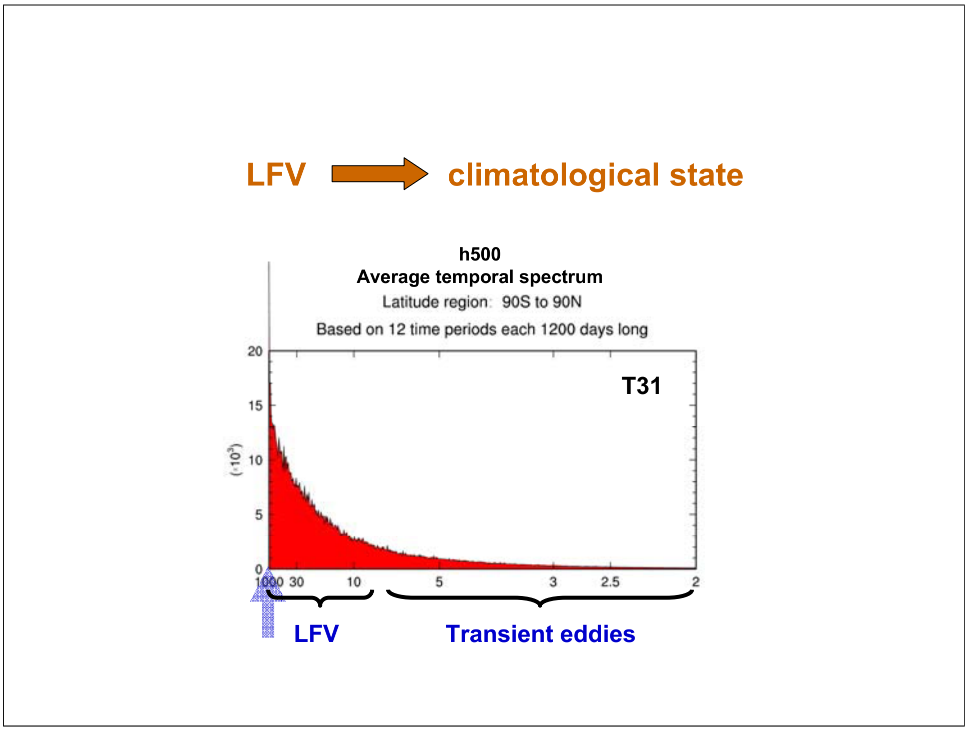
As a further test, we carry out a similar calculation for each of the other three seasons and find in a qualitative sense the resulting seasonal cycle of low-frequency variance matches the observed seasonal cycle. Thus we have further evidence of the mean state have strong control over LFV.

Stochastically Driven Linear
Barotropic Vorticity Equation

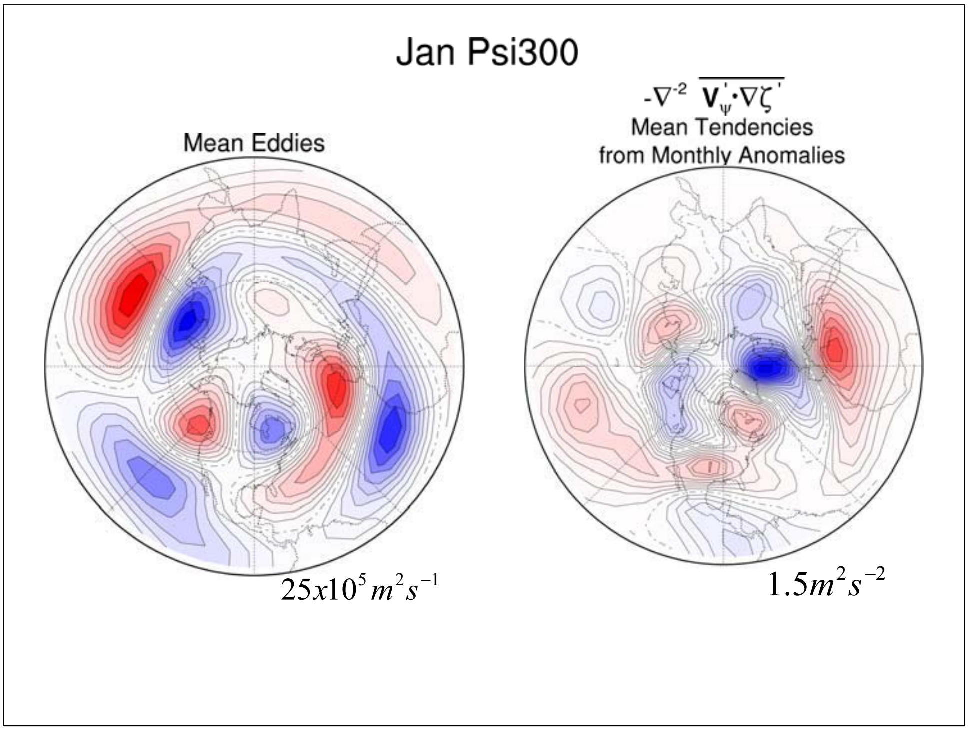


January Basic State ***July Basic State***

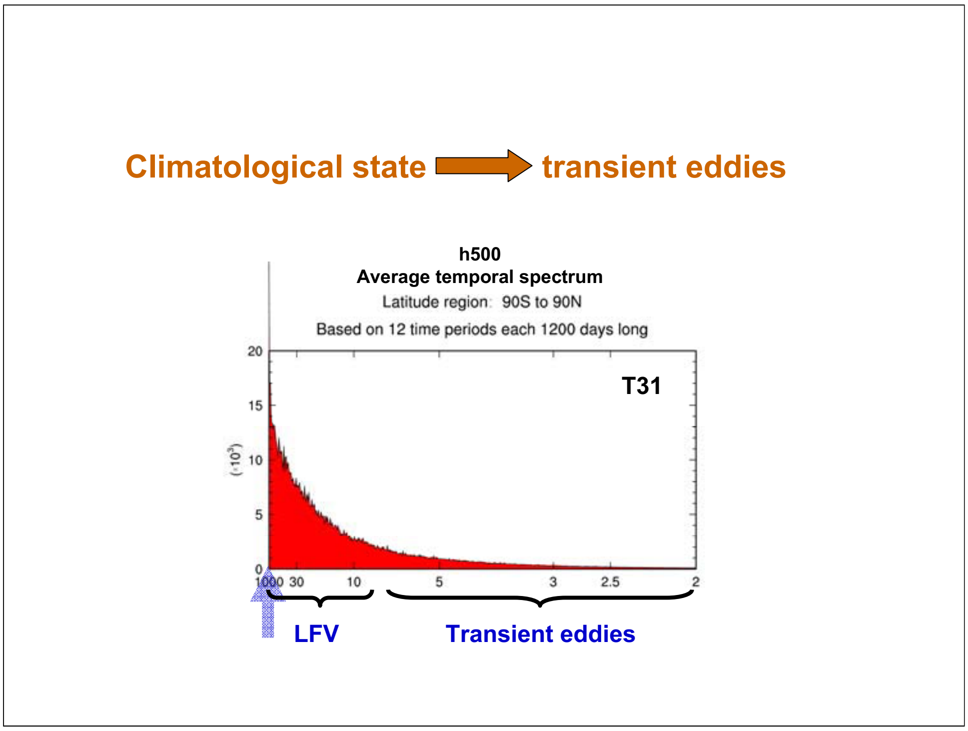
Finally, when we decompose the monthly mean variability in the stochastic integrations into EOFs, we find that some of the important features in the seasonality of the leading structures seen in nature is reproduced. Here, for example, we find that for the January basic state, the leading pattern has much larger scale and is not as confined near the pole as it is when a July basic state is used. Hence another manifestation of the control of the mean state on LFV perturbations is that it appears to have an influence on the structure of the leading modes of the system and their seasonality.



Recall when we decomposed the vorticity equation into various timescales we found there could be two-way interactions between timescales. Next we see if in addition to the climatological state influencing LFV, does LFV affect the climatological state.

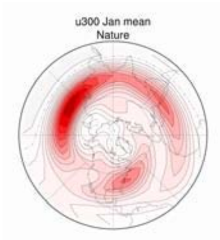


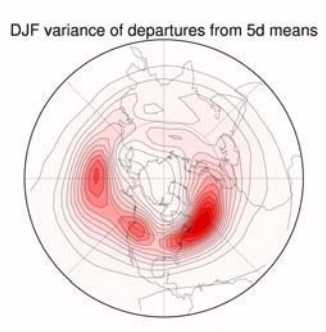
That LFV affects the mean state can be seen most distinctly in northern winter. There the LF anomalies are shaped in such a way as to produce fluxes that tend to damp the climatological mean waves. That can be seen in this figure where there tends to be anticorrelation between the mean waves and streamfunction tendencies resulting from fluxes by LFV. Note the tendencies associated with LFV would tend to spin down the mean waves in about 15 days.



Next we consider the influence of the climatological state on the transient eddies.

Stormtracks and the Mean State





As we saw earlier, the synoptic eddies are highly organized into stormtracks, especially in winter. It is thought that this organization is a manifestation of the climatological mean state's controlling influence on high frequency variability. In particular, the mean zonal wind tends to guide the propagation of the storms and the main energy sources for the storms are the meridional temperature gradients associated with the mean vertical wind shear under the upper tropospheric jet.

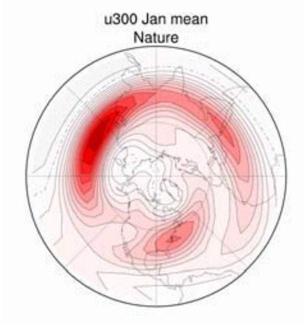
Linear Stormtrack Model

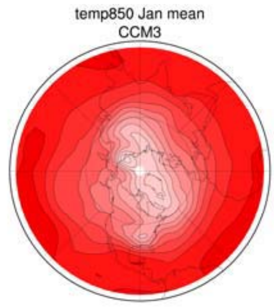
$$\frac{\partial \varsigma'}{\partial \mathrm{t}} = -\overline{\vec{v}} \cdot \nabla \varsigma' - \vec{v}' \cdot \nabla \overline{\varsigma} + \ldots + damping$$

$$\frac{\partial T'}{\partial t} = -\overline{\vec{v}} \cdot \nabla T' - \vec{v}' \cdot \nabla \overline{T} + \ldots + damping$$

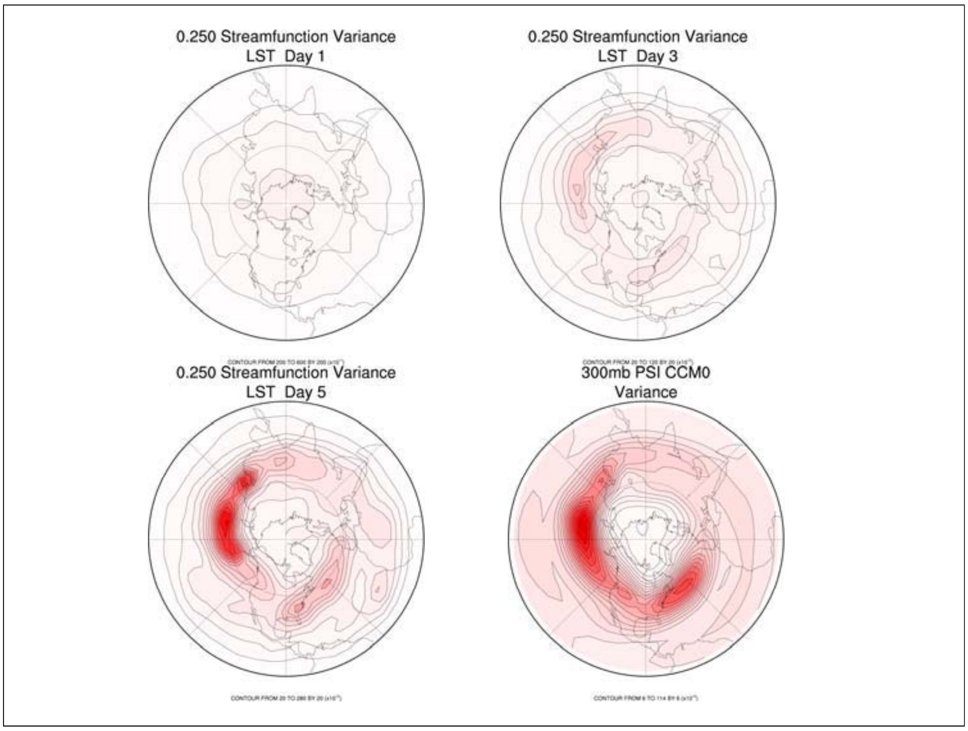
$$\frac{\partial D'}{\partial t} = \ldots$$

$$\frac{\partial p_s}{\partial t} = \ldots$$

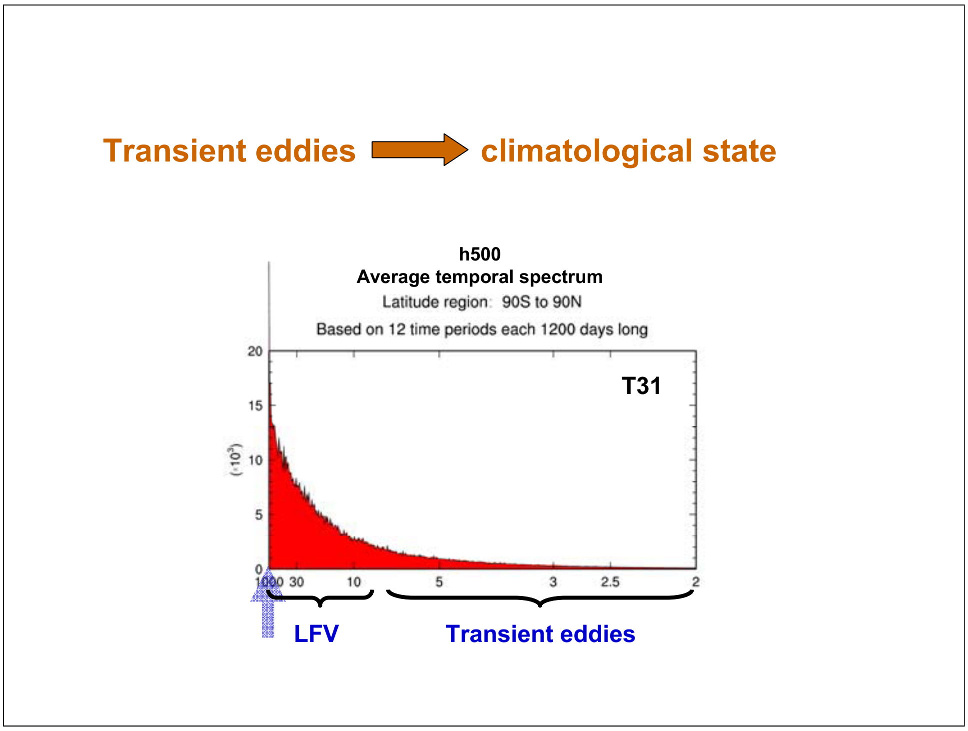




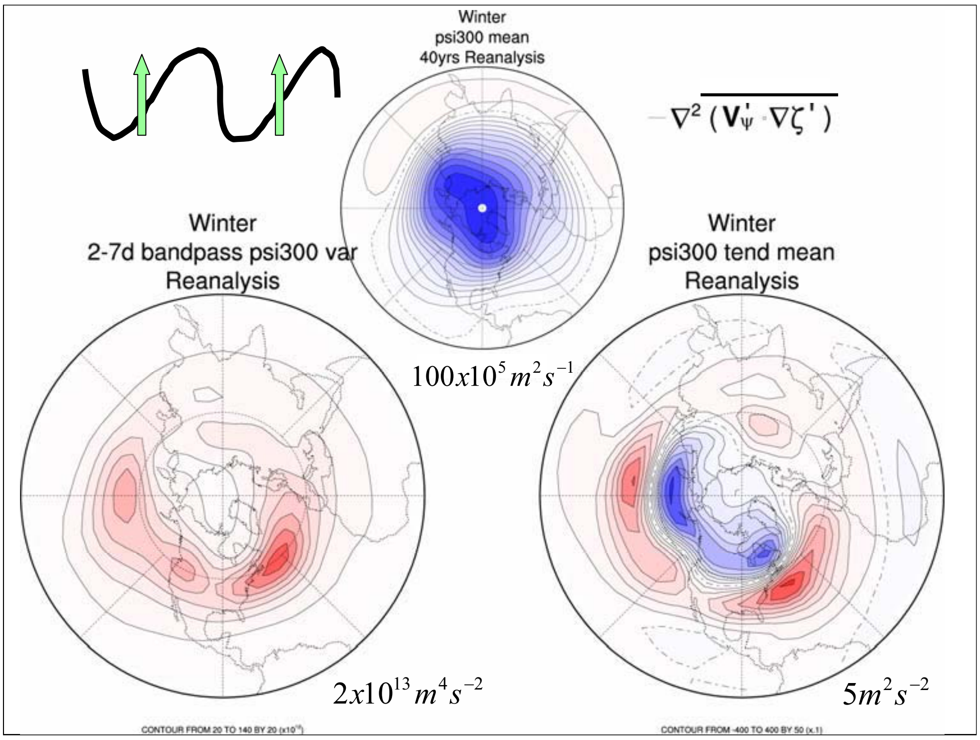
To confirm that the stormtracks are an example of the influence of the climatological state on transient eddies, we use an approach that is related to our test of the climatological state on LFV. But since synoptic eddies rely on thermal gradients for their energy, in this case we linearize the full, three-dimensional primitive equations about climatological conditions..



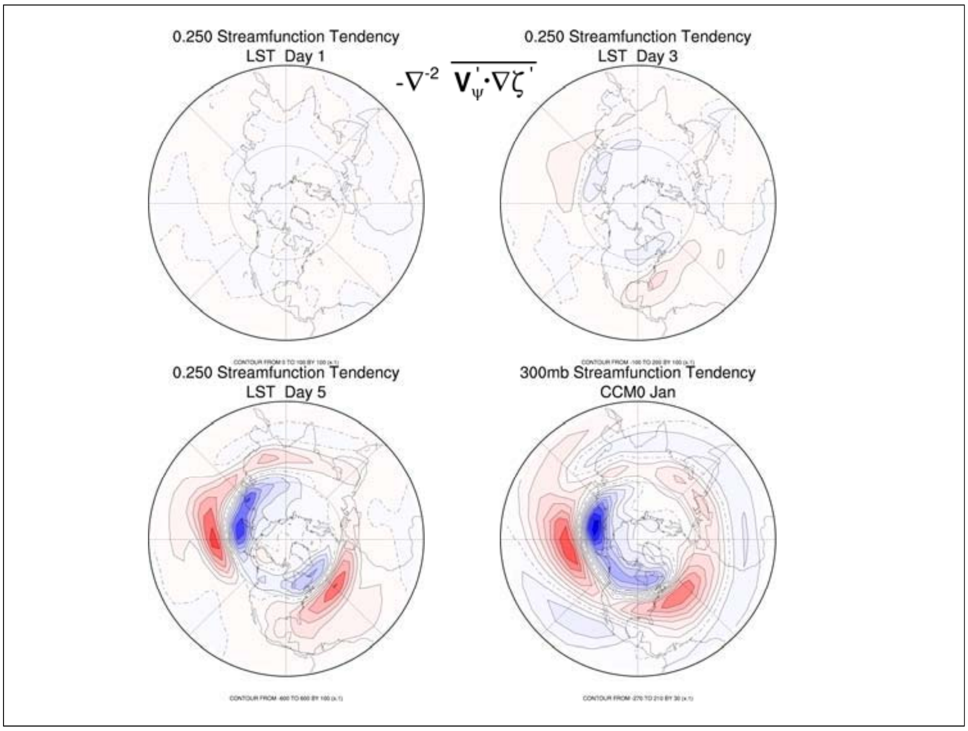
To determine the effect of the mean state on short-lived rapidly growing structures like midlatitude storms, we introduce random perturbations into the linearized equations as initial conditions and integrate them forward in time. If we do this many times and collect variance statistics, we find that within just a few days, the influence of the background state is very apparent and the observed stormtracks form. Thus the hypothesis that the stormtracks are a manifestation of the influence of the climatological state is supported.



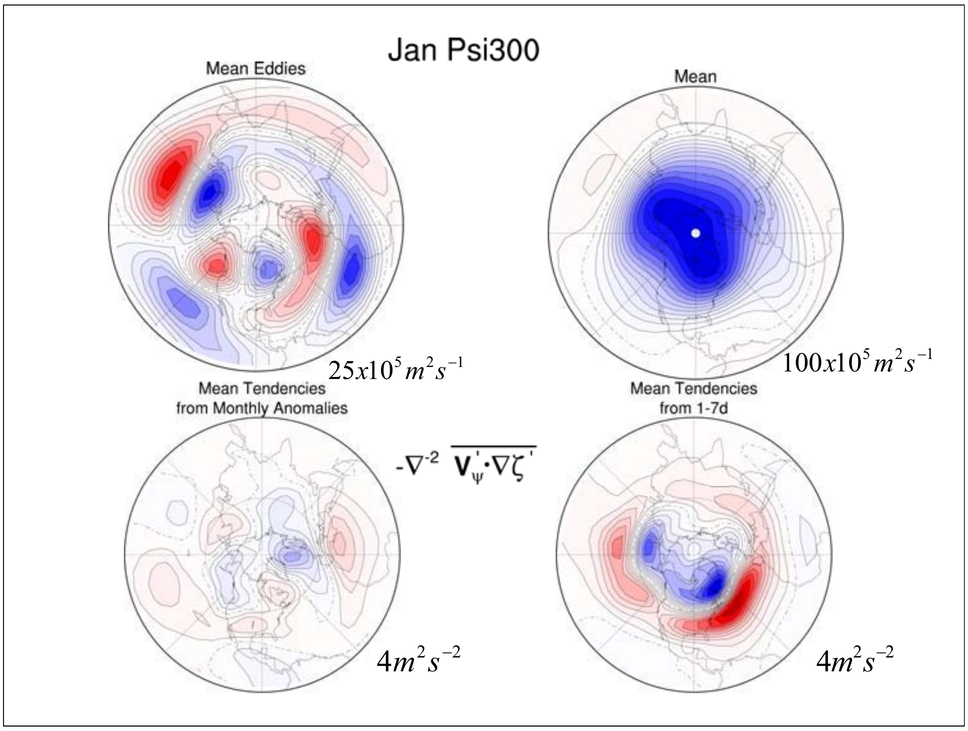
Again, because of the potential for two-way interactions, we also consider the effects of the transient eddies on the climatological state.



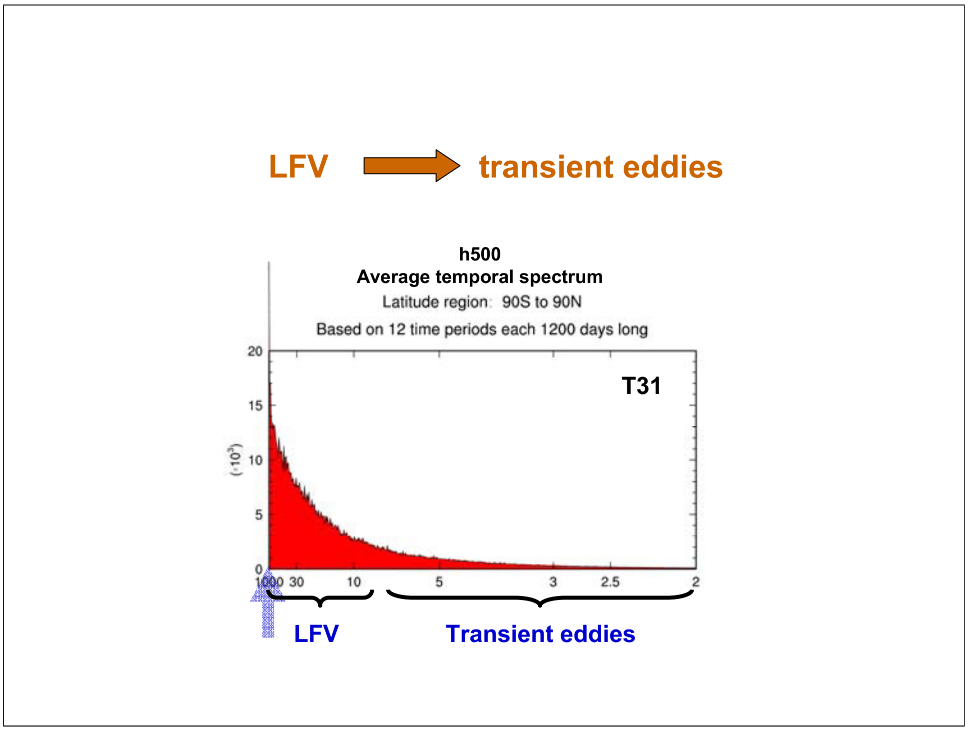
Not only are the high frequency transients concentrated in certain areas, their structure is also highly organized. Through horizontal tilts they systematically transport momentum. This is evident for climatological conditions with tilts south of the midlatitude jets preferentially fluxing zonal momentum northward and tilts north of the jet fluxing momentum southward. This has the effect of maintaining and elongating the jet eastward.



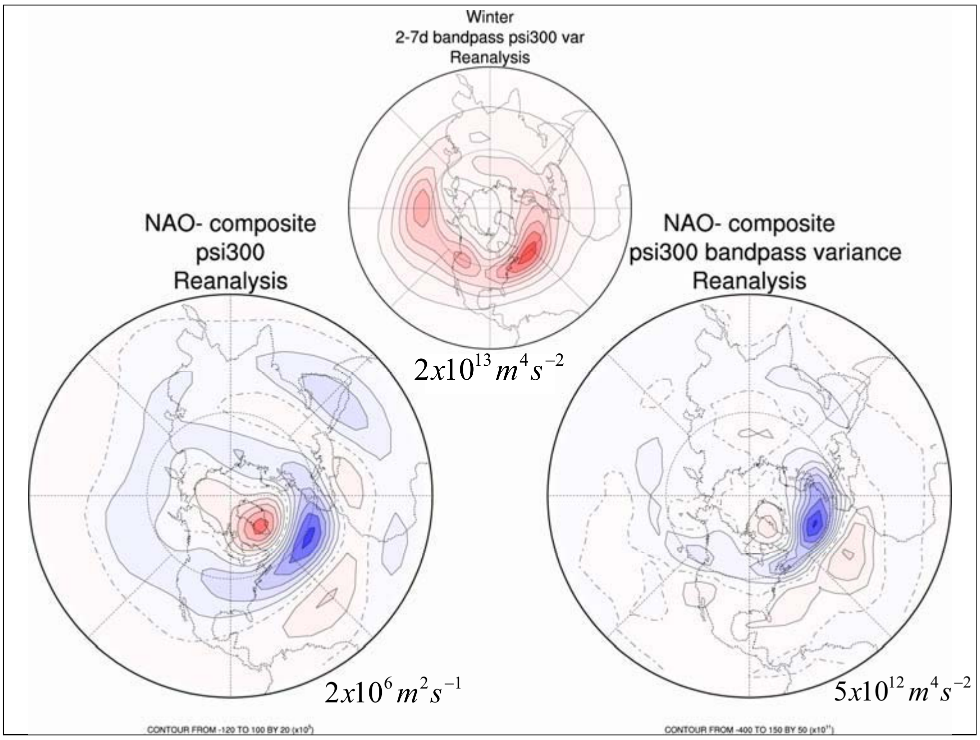
Interestingly this maintenance of the mean eastern parts of the jets by the eddy momentum fluxes is an another manifestation of the control of the mean state on the synoptic eddies. As seen here, the storms that form in the linear stormtrack model, have the observed tilts that produce the observed jet-maintaining momentum fluxes.



We have seen an important distinction between LF and bandpass eddies. The former tend to reduce the amplitude of the climatological mean circulation while the latter tends to enhance it.



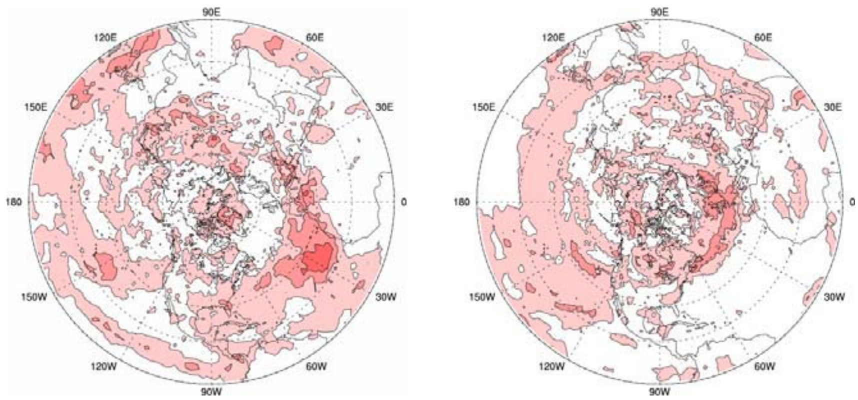
Finally we consider interactions between LFV and the transient eddies. If one thinks of a LF state as being quasi-stationary relative to bandpass timescales, then it would not be surprising if it turns out that the relation between LFV and transient eddies is similar to the relationship between the climatological mean and transient eddies.



The situation for the climatological mean state does carry over to persistent departures from this state. When there is a persistent anomaly, then there is in effect a new climatology, and the synoptic eddies play the same role for this new quasi-equilibrium state as they do for the climatological state. As seen in this case for a commonly occurring LF anomalous state, the NAO, the presence of the large-scale persistent anomaly coincides with a redistribution of the synoptic eddies.

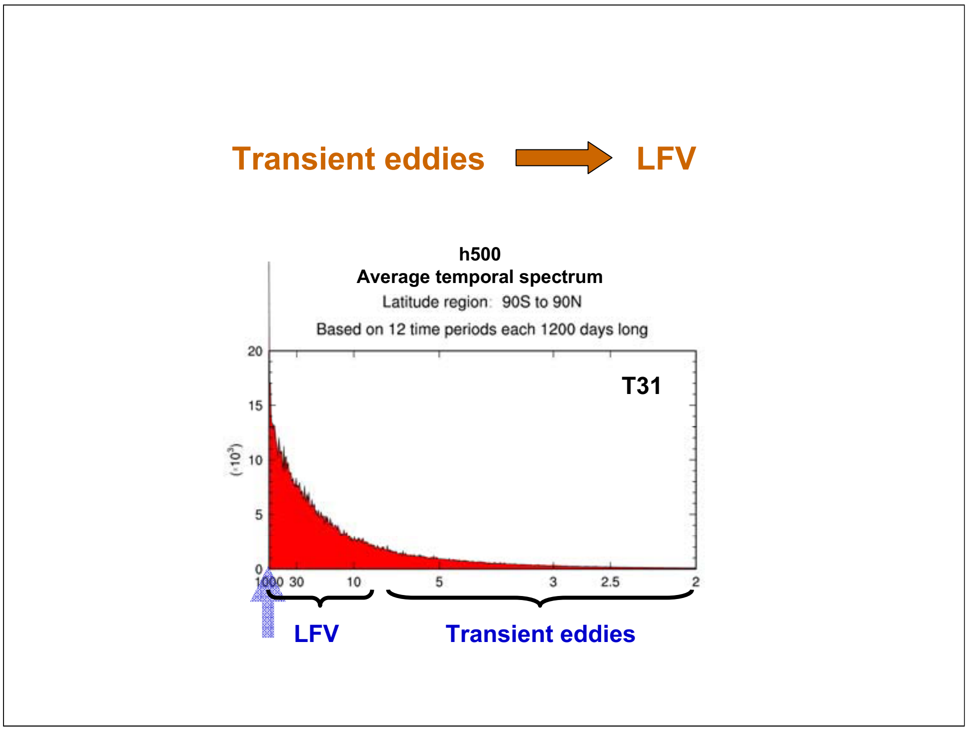
Likelihood of extreme winds in Nature
given 2σ monthly mean NAO

NAO- NAO+

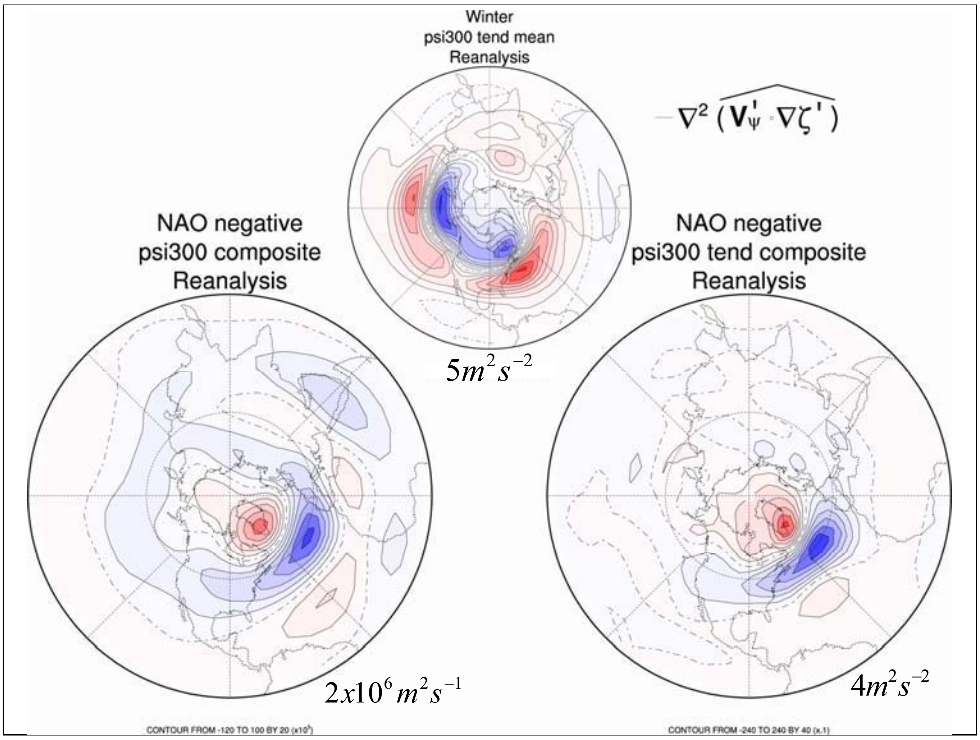


Contours at .01,.02,.03

LFV also affects the likelihood of a special class of fast timescale phenomena, namely extreme events. As seen in this plot of the likelihood of extreme 850hPa wind events during strong NAO months, negative NAO events increase the likelihood of extreme winds over the midlatitudes of the North Atlantic by a factor of three. By contrast a positive NAO makes extreme wind events more likely in the high latitudes of the North Atlantic. Here extremes are defined as daily winds exceeding their 99 percentile value.

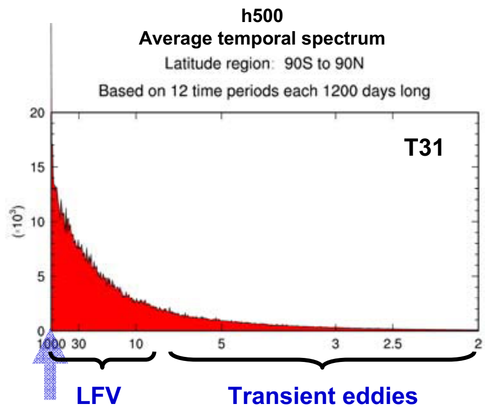


Finally, we consider how the transient eddies affect LFV.

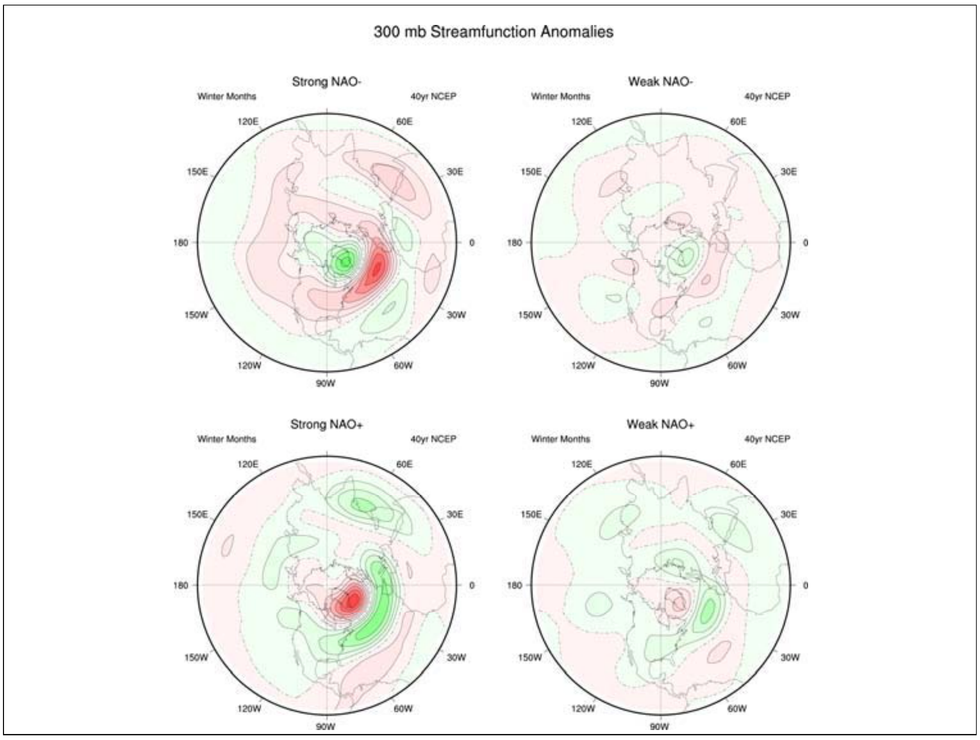


Just as with the climatological state, the synoptic eddies also reinforce LF anomalies to the mean state. In this example, the anomalous eddy fluxes that occur during NAO episodes tend to reinforce the anomalous circulation. This same processes helps to maintain many other large scale circulation anomalies. And just as with the influence of the climatological state on the synoptic eddies, the stochastically driven linear dynamics of the stormtrack model can be used to demonstrate that it is simply the presence of the low-frequency anomalies that causes the momentum fluxes from the transient eddies to take on their new configuration during low-frequency events.

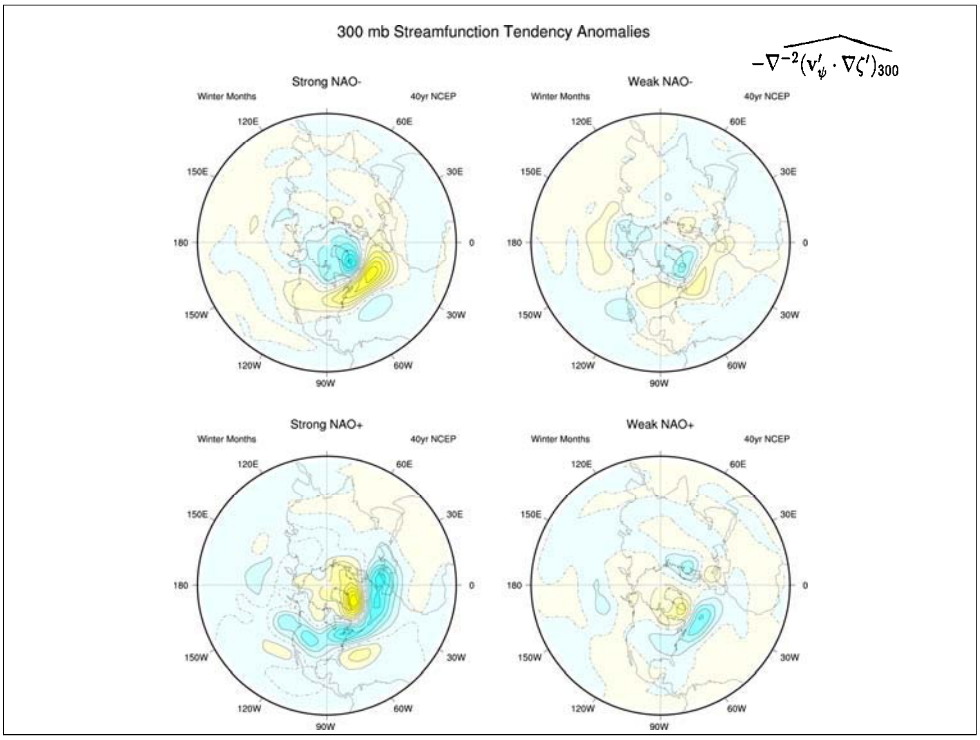
Given the two-way interaction between various timescales and the importance of fluxes, does this mean modes of variability are fundamentally nonlinear phenomena?



Given the strong evidence that there is definitely a two-interaction between low-frequency anomalies and both the time-mean state and synoptic eddies, it would appear that the modes of variability are a nonlinear phenomenon. Despite this fact, it turns out that prominent LF modes can be approximated surprisingly well in a linear fashion because all of the interactions are well-approximated by linear terms. We have already seen that the influence of the mean state on LFV is well represented linearly. On the other hand the influence of LFV on the climatological state can be approximated by simple Rayleigh damping because we saw its main influence is to damp the climatological waves. That the interaction between LFV and the high frequency transients can be approximated linearly is less obvious. But it turns out it can.



To see that the interaction between LFV and the synoptic eddies can be approximated linearly, consider four composites of the NAO. Each is distinguished from the others only in the sign and strength of the composited events.



This plot shows the anomalous streamfunction tendency from momentum fluxes by the transient eddies during the four types of NAO events. Note that the structure of the tendency is not a function of the amplitude of the NAO event, but the strength of the tendency is nearly a linear function of NAO amplitude. Thus these tendencies are essentially a linear function of the LF state. This turns out to be a common trait for midlatitude LF states.

Effect of Two-way Interaction between LFV and Synoptic Eddies

$$\frac{\partial \varsigma}{\partial t} = -\vec{v}_\psi \cdot \nabla(\varsigma + f) - \ldots$$

$$() = \frac{1}{T}\int_0^T () dt + ()'$$

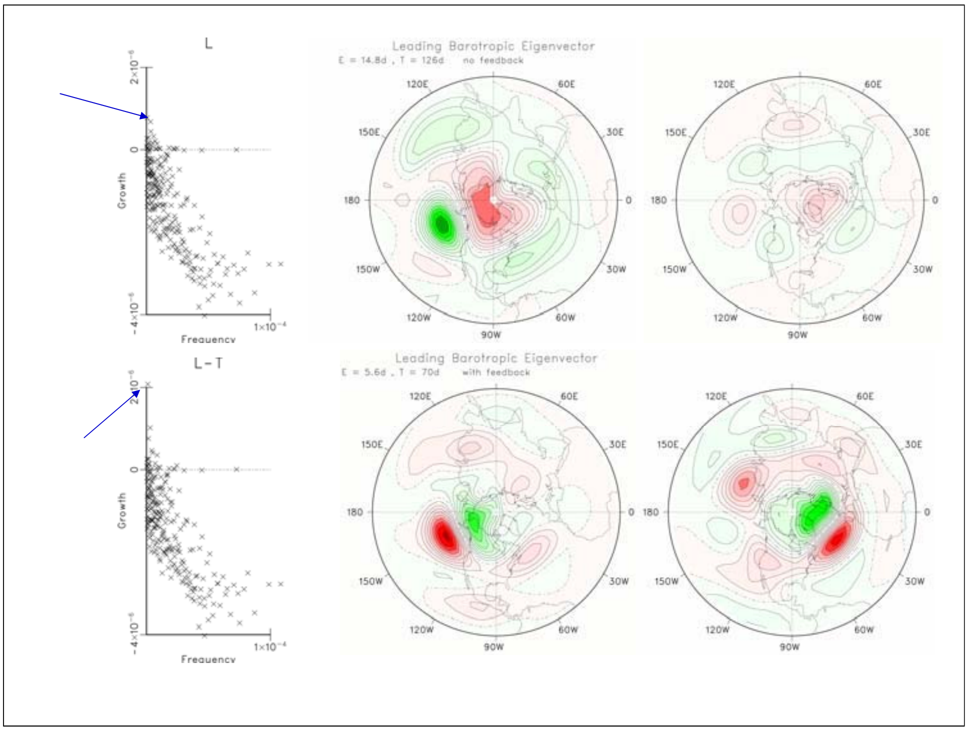
$$\frac{\partial \varsigma'}{\partial t} = -\bar{\vec{v}}_\psi \cdot \nabla(\bar{\varsigma} + f) - \bar{\vec{v}}_\psi \cdot \nabla \varsigma' - \vec{v}'_\psi \cdot \nabla \bar{\varsigma} - \vec{v}'_\psi \cdot \nabla \varsigma' \ldots$$

$$\bar{\vec{v}}_\psi \cdot \nabla(\bar{\varsigma} + f) = -\overline{\vec{v}'_\psi \cdot \nabla \varsigma'} + \ldots$$

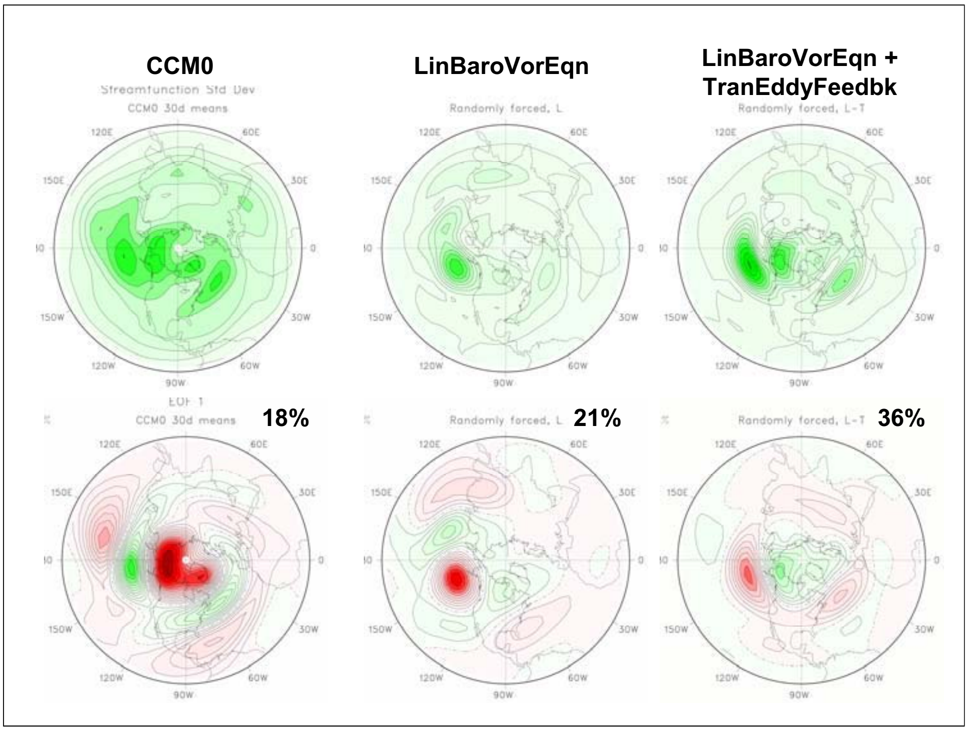
$$\frac{\partial \varsigma'}{\partial t} = -\bar{\vec{v}}_\psi \cdot \nabla \varsigma' - \vec{v}'_\psi \cdot \nabla \bar{\varsigma} - (\vec{v}'_\psi \cdot \nabla \varsigma' - \overline{\vec{v}'_\psi \cdot \nabla \varsigma'}) \ldots$$

$$= -\bar{\vec{v}}_\psi \cdot \nabla \varsigma' - \vec{v}'_\psi \cdot \nabla \bar{\varsigma} + damping + noise \rightarrow T\varsigma'$$

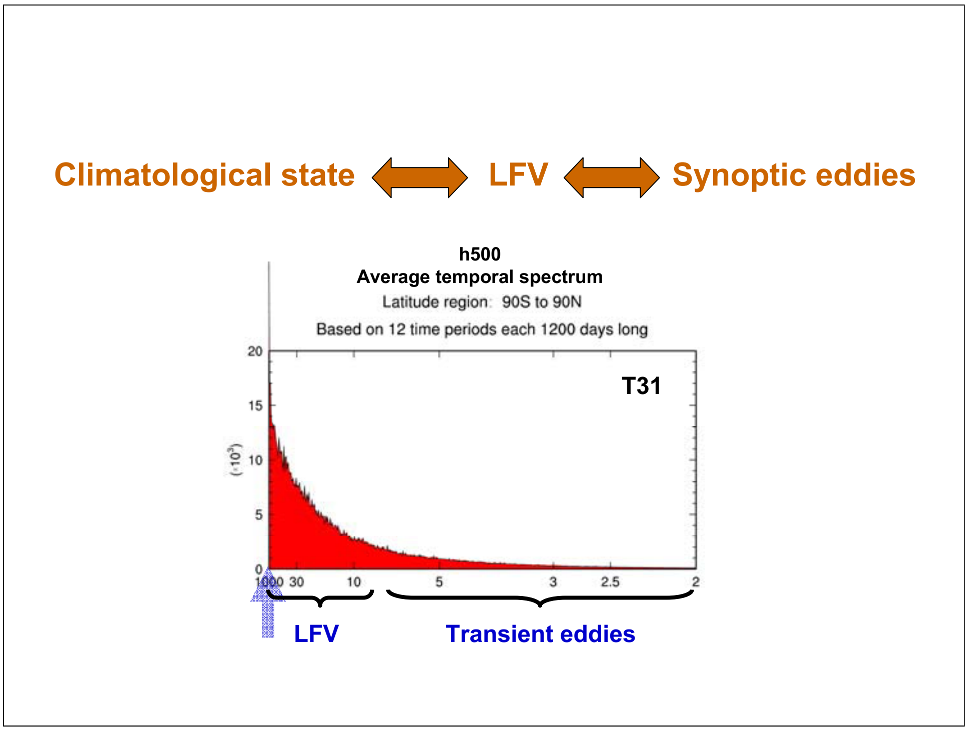
Given the linear relationship between LF state and anomalous eddy fluxes, we can introduce the two-way interaction in our stochastic planetary wave model by replacing some of the noise with a linear operator.



To see the effect of including the synoptic eddy feedback in the linear barotropic vorticity equation, we compare its attributes with and without the feedback operator. We have done this for basic state taken from a general circulation model. Inclusion of the eddy feedback increases the growth rate of the fastest growing mode by more than a factor of two and increases its strength over the Atlantic



Another way of looking at the effect of including the feedback in this model is in terms of the statistics of low-frequency structures in the model when it is stochastically forced. As seen here, inclusion of the transient eddy feedback makes the statistics of the linear model much more like those in the full GCM. The geographical distribution of variance is better because of enhanced LF variability over the North Atlantic, and the leading pattern of variability now stretches across the North Atlantic as it should.



From these results we conclude that though we have seen that there are two-way interactions between various time-scales, to a good approximation the leading modes (i.e., patterns) of variability can be approximated in a linear fashion.