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New pathways to SO(10) unification

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New Pathways to $SO(10)$ Unification

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Contents

Grand unifcation-status

- Grand unification ($\boldsymbol{SU(5)}$, $\boldsymbol{SO(10)}$, $\boldsymbol{E_6}$..) has desirable features: quantization of charge, gauge coupling unification, ..

  $\boldsymbol{SO(10)}$ **:** One gen of fermions in one IR rep. Unlike E6 not too many exotics.

  $\boldsymbol{SO(10)}$ **:** Can generate small neutrino masses via see-saw

- Too many Higgs representations to break SO(10). Many 'minimal' models.

Conventional schemes

- Conventional scenarios: several Higgs reps needed
  - One needs at least $\mathbf{45}$, $\mathbf{54}$ or $\mathbf{210}$ and $\mathbf{16}+\overline{\mathbf{16}}$ or $\mathbf{126}+\overline{\mathbf{126}}$ to break $SO(10)$ down to $SU(3)_C \times SU(2)_L \times U(1)_Y$, and further a 10 plet to break it to $SU(3)_C \times U(1)_{em}$.
  - The arbitrariness of the Higgs sector allows for many possibilities for building models.
- An ideal scenario
  - Unification of 3 generation of matter.
  - Unification of the Higgs sector.

A new path to SO(10) unfication
Babu, Gogoladze, PN, Syed

- Quite remarkably it is possible to break $SO(10)$ with a single irreducible rep. This is done by use of a vector-spinor 144 which under $SU(5) \times U(1)$:

$$\mathbf{144} = \bar{\mathbf{5}}_{\mathbf{3}} + \mathbf{5}_{\mathbf{7}} + \mathbf{10}_{-\mathbf{1}} + \mathbf{15}_{-\mathbf{1}} + \mathbf{24}_{-\mathbf{5}} + \mathbf{40}_{-\mathbf{1}} + \overline{\mathbf{45}}_{-\mathbf{3}}$$

  The 24-plet has a $U(1)$ charge which means that once the 24-plet gets a VEV, there is a change in the rank.

- The self couplings of the 144 are quartic or higher

$$(\mathbf{144} \times \overline{\mathbf{144}}).(\mathbf{144} \times \overline{\mathbf{144}}).$$

  Similarly the couplings of the 144 with quarks and leptons in the 16-plets are at least quartic

$$(\mathbf{16} \times \mathbf{16}).(\mathbf{144} \times \mathbf{144})$$

Spontaneous breaking of SO(10)

The superpotential contains only couplings involving the $\mathbf{144}$ and $\overline{\mathbf{144}}$ of Higgs

$$W = M(144.\overline{144}) + \sum_{i=1,45,210} \frac{\lambda_i}{M'}(144.\overline{144})_i(144.\overline{144})_i$$

Symmetry breaking generates VEVs for the 24 plets

$$< 24_{144} >= q\ diag(2,2,2,-3,-3) = \frac{q}{p} < 24_{\overline{144}} >$$

$$\frac{MM'}{qp} = 116\lambda_{45_1} + 7\lambda_{45_2} + 4\lambda_{210}$$

With the above VEV's break the symmetry to SM

$$SO(10) \longrightarrow SU(3)_C \times SU(2)_L \times U(1)_Y$$

Electroweak symmetry breaking

- The 144-plet and $\overline{\mathbf{144}}$ contains SM Higgs doublets.

$$\mathbf{144}: Q_i(\bar{5}) + Q^i(5) + Q^k_{ij}(\overline{45})$$

$$\overline{\mathbf{144}}: P_i(\bar{5}) + P^i(5) + P^{ij}_k(45)$$

The 45 plet in $SU(3) \times SU(2) \times U(1)_Y$ decomposition

$$\mathbf{45} = (1,2)(3) + ..$$

$$\overline{\mathbf{45}} = (1,2)(-3) + ..$$

- Thus we get 3 pairs of Higgs doublets. One can arrange one pair of Higgs doublets to be light.
- This allows the breaking in one step

$$SO(10) \rightarrow SU(3)_C \times U(1)_{em}.$$

Calculational tools: Oscillator method + Basic Theorem

Mohapatra, Sakita; Wilczek, Zee ; **PN, R. Syed**

Oscillator method: $SO(10)$ representations may be decomposed in term of $SU(5)$ using harmonic oscillators

$$\{b_i, b_j^\dagger\} = \delta_{ij}, \quad \{b_i, b_j\} = 0$$

Define SO(10)operators $\Gamma_\mu (\mu = 1, 2, .., 10)$

$$\Gamma_{2i} = (b_i + b_i^\dagger), \quad \Gamma_{2i-1} = -i(b_i - b_i^\dagger)$$

$$\{\Gamma_\mu, \Gamma_\nu\} = 2\delta_{\mu\nu}$$

$\overline{\mathbf{16}}$ may be expanded as follows:

$$\overline{\mathbf{16}} : |\Psi_{(-)}> = (\mathrm{P} + b_i^\dagger b_j^\dagger \mathrm{P}^{ij} + \epsilon^{ijklm} b_j^\dagger b_k^\dagger b_l^\dagger b_m^\dagger \mathrm{P_i})|0>$$

Similarly $\mathbf{16}$.

The Basic Theorem

PN, Raza Syed: PLB 506,68(2001); NPB, 618, 138(2001); NPB 676, 64(2004)

The basic theorem allows one to decompose the $SO(2N)$ vertices in representations of $SU(N)$.

$$\Gamma_\mu \phi_\mu = \Gamma_{2i}\phi_{2i} + \Gamma_{2i-1}\phi_{2i-1} = b_i^\dagger \phi_{c_i} + b_i \phi_{\bar{c}_i}$$

$$\phi_{ci} = \phi_{2i} + i\phi_{2i-1}, \ \phi_{\overline{ci}} = \phi_{2i} - i\phi_{2i-1}$$

$$\phi_{c_i c_j \bar{c}_k \ldots} = \phi_{2i c_j \bar{c}_k \ldots} + i\phi_{2i-1 c_j \bar{c}_k \ldots} \quad 2^N \text{terms}$$

$\phi_{c_i c_j \bar{c}_k \ldots}$ transforms like a reducible rep of $SU(N)$. Thus if we can express SO(2N) couplings in terms of $\phi_{c_i c_j \bar{c}_k \ldots}$ etc, then we need only to decompose $\phi_{c_i c_j \bar{c}_k \ldots}$ is terms of irreducible reps of $SU(N)$.

The Basic Theorem continued

Vertex expansion at

$$\Gamma_\mu \Gamma_\nu \Gamma_\lambda .. \Gamma_\sigma \phi_{\mu\nu\lambda..\sigma} = b_i^\dagger b_j^\dagger b_k^\dagger .. b_n^\dagger \phi_{c_i c_j c_k ... c_n}$$

$$+ (b_i b_j^\dagger b_k^\dagger .. b_n^\dagger \phi_{\bar{c}_i c_j c_k ... c_n} + \ perms)$$

$$+ (b_i b_j b_k^\dagger .. b_n^\dagger \phi_{\bar{c}_i \bar{c}_j c_k .. c_n} + \ perms)$$

$$+ .... + (b_i b_j b_k .. b_{n-1}^\dagger b_n \phi_{c_i c_j c_k .. c_{n-1} c_n} + \ perms) +$$

$$+ b_i b_j b_k .... b_n \phi_{\bar{c}_i \bar{c}_j \bar{c}_k ... \bar{c}_n}$$

where one must take into account all possible permutations of b and $b^\dagger$ in writing the expansion.

$16 - 16 - 120$ coupling example

$$\frac{1}{3!} f_{ab} \tilde{\Psi}_a B C^{-1} \Gamma_\mu \Gamma_\nu \Gamma_\lambda \Psi_b \phi_{\mu\nu\lambda}$$

The vertex can be expanded using the basic theorem

$$\Gamma_\mu \Gamma_\nu \Gamma_\lambda \phi_{\mu\nu\lambda} = b_i b_j b_k \phi_{\bar{c}_i \bar{c}_j \bar{c}_k} + (b_i b_j b_k^\dagger \phi_{\bar{c}_i \bar{c}_j c_k} + perms)$$
$$+ (b_i b_j^\dagger b_k^\dagger \phi_{\bar{c}_i c_j c_k} + perms) + b_i^\dagger b_j^\dagger b_k^\dagger \phi_{c_i c_j c_k}$$

Decompose 120 into SU(5) reps:

$$120 = 5 + \bar{5} + 10 + \bar{10} + 45 + \bar{45}$$

A direct computation gives

$$W^{(120)} = i \frac{2}{\sqrt{3}} f^{(-)}_{\acute{a}\acute{b}} [2(1_{\acute{a}} 5_{i\acute{b}} 5^i_H) + 10^{ij}_{\acute{a}} 1_{\acute{b}} 10_{Hij} + 5_{i\acute{a}} 5_{j\acute{b}} 10^{ij}_H$$
$$-10^{ij}_{\acute{a}} \bar{5}_{i\acute{b}} \bar{5}_{Hj} + \bar{5}_{i\acute{a}} 10^{jk}_{\acute{b}} \bar{45}^i_{Hjk} - \frac{1}{4} \epsilon_{ijklm} 10^{ij}_{\acute{a}} 10^{mn}_{\acute{b}} 45^{kl}_{Hn}]$$

Field theoretic description of vector spinor

Babu, Gogoladze, PN, Syed: hep-ph/hep-ph/0506312

To generate a 144 ($\overline{\mathbf{144}}$) we start with a unconstrained vector spinor $\mathbf{160}(\overline{\mathbf{160}})$ and then obtain a constrained vector-spinor.

$$\mathbf{160}: |\Psi_{(+)\mu}>= (\mathbf{P}_\mu + b_i^\dagger b_j^\dagger \mathbf{P}_\mu^{ij} + \epsilon^{ijklm} b_j^\dagger b_k^\dagger b_l^\dagger b_m^\dagger \mathbf{P}_{i\mu})|0>$$

$\mathbf{144}, \overline{\mathbf{144}}$ ($|\Upsilon_{(\pm)\mu}>$) are constrained vector spinors

$$\Gamma_\mu |\Upsilon_{(\pm)\mu}>= |\Psi_\mp>= 0$$

The constraint require that $\mathbf{16}_\mp$ fields must be removed from the $\mathbf{160}_\pm$ reducing it to $\mathbf{144}_\pm$.

Vector-spinor continued

To get the $\overline{144}$ spinor impose $\Gamma_\mu|\Upsilon_{(-)\mu}>=0$ which gives

$$\overline{144}=\overline{144}^n(79)+\overline{144}_n(65)$$

In oscillator decomposition 144=79 + 65 components are

$$|\overline{144}^n>=|0>\mathrm{P}^n+\frac{1}{2}b_i^\dagger b_j^\dagger|0>\left[\epsilon^{ijklm}\mathrm{P}^n_{klm}-\frac{1}{6}\epsilon^{ijnlm}\mathrm{P}_{lm}\right]$$

$$+\frac{1}{24}\epsilon^{ijklm}b_j^\dagger b_k^\dagger b_l^\dagger b_m^\dagger|0>\mathrm{P}_i^n;\quad 5+\overline{10}+\overline{40}+24=79$$

$$|\overline{144}_n>=|0>\mathrm{P}_n+\frac{1}{2}b_i^\dagger b_j^\dagger|0>\left[\mathrm{P}_n^{ij}+\frac{1}{4}\left(\delta_n^i\mathrm{P}^j-\delta_n^j\mathrm{P}^i\right)\right]$$

$$+\frac{1}{24}\epsilon^{ijklm}b_j^\dagger b_k^\dagger b_l^\dagger b_m^\dagger|0>\left[\frac{1}{2}\mathrm{P}_{in}+\frac{1}{2}\mathrm{P}^{(S)}_{in}\right];\ \bar{5}+\overline{15}+45=65$$

Couplings with fermions

Matter-Higgs couplings are at least quartic

$$(\mathbf{16}\times\mathbf{16})_{\mathbf{10}}(\mathbf{144}\times\mathbf{144})_{\mathbf{10}},\quad (\mathbf{16}\times\mathbf{16})_{\mathbf{10}}(\overline{\mathbf{144}}\times\overline{\mathbf{144}})_{\mathbf{10}}$$

$$(\mathbf{16}\times\mathbf{16})_{\mathbf{120}}(\mathbf{144}\times\mathbf{144})_{\mathbf{120}},\quad (\mathbf{16}\times\mathbf{16})_{\mathbf{120}}(\overline{\mathbf{144}}\times\overline{\mathbf{144}})_{\mathbf{120}}$$

$$(\mathbf{16}\times\mathbf{16})_{\overline{\mathbf{126}}}(\mathbf{144}\times\mathbf{144})_{\mathbf{126}},\quad (\mathbf{16}\times\mathbf{16})_{\overline{\mathbf{126}}}(\overline{\mathbf{144}}\times\overline{\mathbf{144}})_{\mathbf{126}},\cdots$$

Quark-lepton-neutrino masses arise as follows

$$\mathbf{up-quarks:}\quad \mathbf{10}_M\mathbf{10}_M\frac{\mathbf{24}_H}{M}\mathbf{5}_H$$

$$\mathbf{down-quark-lepton:}\quad \mathbf{10}_M\bar{\mathbf{5}}_M\frac{\mathbf{24}_H}{M}\bar{\mathbf{5}}_H$$

$$\mathbf{RR-\nu\ mass:}\quad \mathbf{1}_M\mathbf{1}_M\frac{\mathbf{24}_H}{M}\mathbf{24}_H$$

$$\mathbf{LR-\nu\ mass:}\quad \bar{\mathbf{5}}_M\mathbf{1}_M\frac{\mathbf{24}_H}{M}(\mathbf{5}_H,\mathbf{45}_H)$$

$$\mathbf{LL-\nu\ mass:}\quad \bar{\mathbf{5}}_M\bar{\mathbf{5}}_M\frac{\mathbf{5}_H}{M}\mathbf{5}_H$$

Third generation fermion masses

Babu, Gogoladze, PN, Syed: hep-ph/0607244

- 3rd gen cubic couplings from additional 10+45 of matter

$$W_{F_3} = 10^2 + 45^2 + 16.10.144 + 16.45.\overline{144}$$

- $b - \tau$ and $b - t - \tau$ unification (a bit more tricky!)
  - Conventional 10-plet breaking of EW symmetry

$$h_\tau = h_b = h_t$$

  - The modified relation

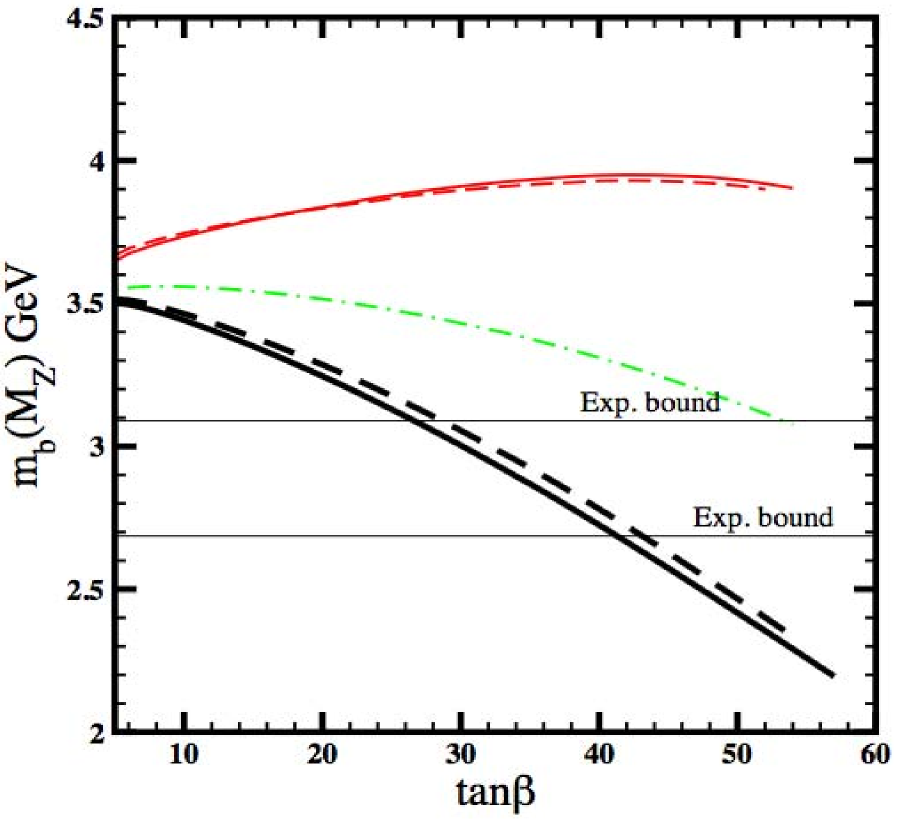
$$|h_\tau| \approx h_b \approx \frac{z}{6}|h_t|, \; z = O(1)$$

$b - t - \tau$ unification can be achieved with a $\tan\beta$ as low as 10 instead of $\tan\beta = 50$.

$b-\tau$ unification

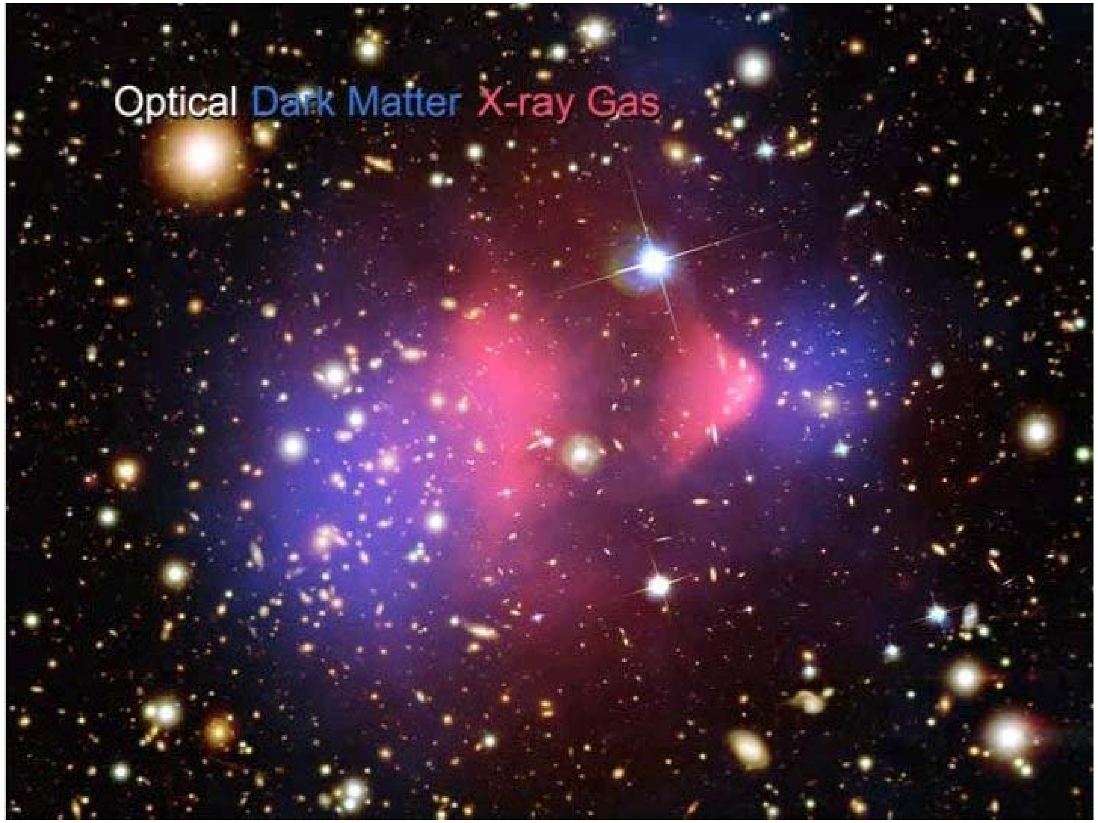
Gomez, Ibrahim, PN, Skadhauge

(i) $g_\mu - 2$ favors $\mu > 0$
(ii) $b-\tau$ unification favors $\mu < 0$.



"Direct Empirical Evidence For the Existence of Dark Matter" astro-ph/0608407

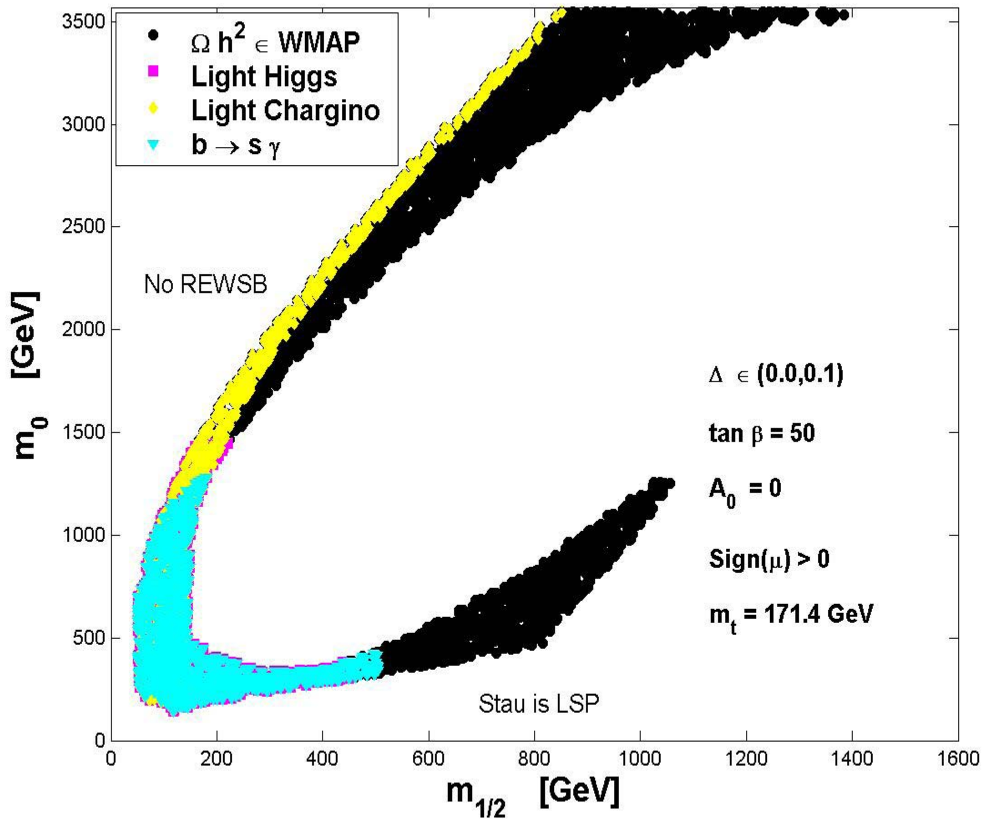
- Collison of galaxy clusters (1E 0657-56 = bullet cluster)
- optical image: Magellan and Hubble ST shows galaxies in orange and white.
- Chandra X-ray Observ. Hot gas, normal matter (pink)



- **Most of the mass (blue), from gravitational lensing - dominated by dark matter ; first clear separation of DM**

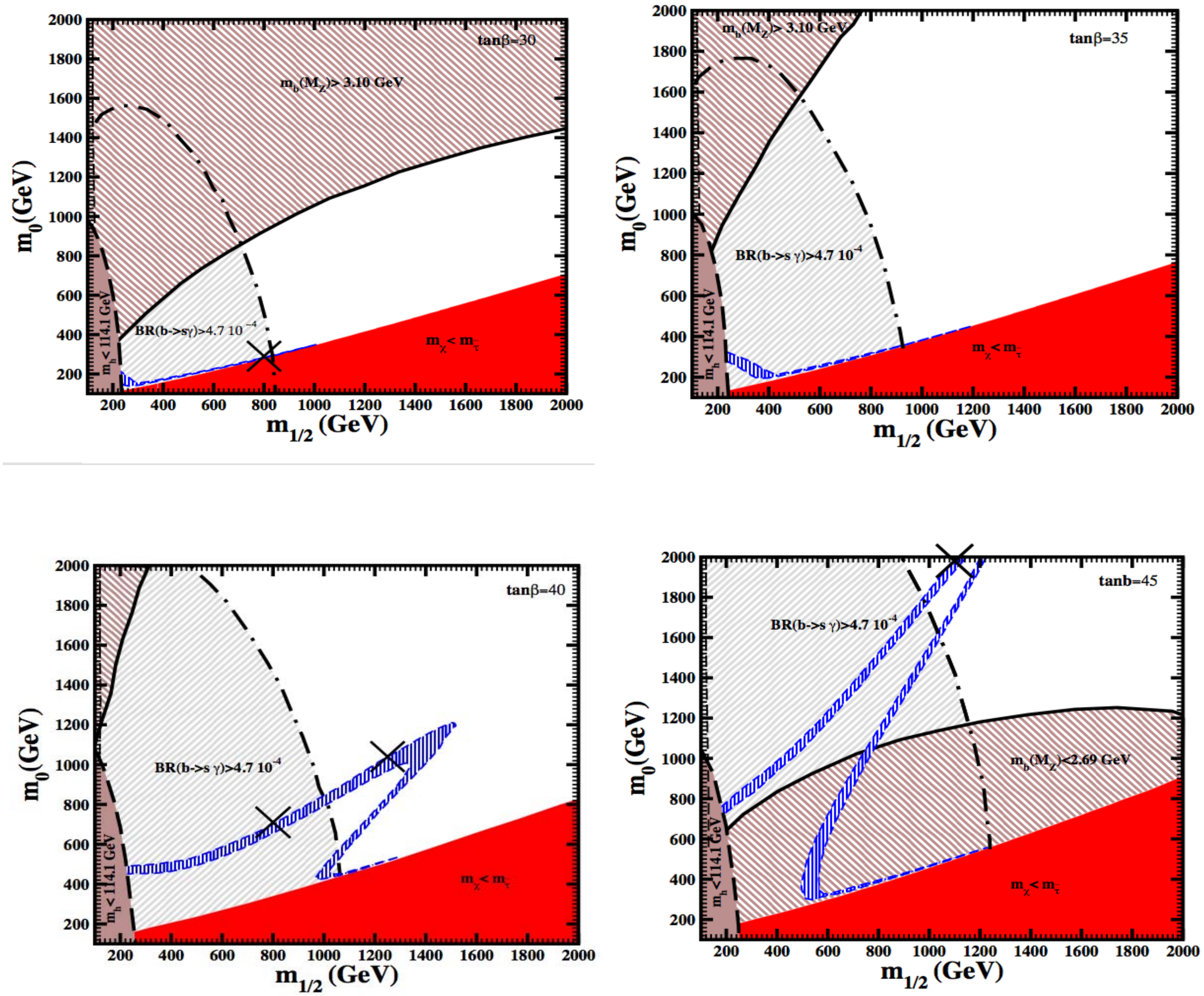
Satisfaction of relic density in $m_0 - m_{1/2}$ plane in mSUGRA

Feldman, Kors, PN: hep-ph/0610133



$b - \tau$ unification + $b \to s\gamma$ + relic density

Gomez, Ibrahim, PN, Skadhauge



Proton stability: review Phys. Rep. PN & PF Perez

- Nature and strength of $B\&L$ violating interactions at the GUT scale.
- Nature of soft breaking which enters in the dressing loop diagrams.
- Constraints on gauge coupling unification which constrain the heavy thresholds and the Higgsino triplet mass.
- $b - \tau$ unification, $g_\mu - 2$, and $b \to s\gamma$.
- Quark-lepton textures.
- Dark matter constraint
- Planck slop corrections.
- Gravitational warping effects
- Accuracy of effective lagrangian approximation which converts operators such as $QQQL$ and $U^C U^C D^C E^C$ into lagrangian for mesons + baryons.

Condition for suppression of proton decay

Below the unification scale in GUTs/strings one can write $SU(3)_C \times SU(2)_L \times U(1)_Y$ invariant theory, specifically the color Higgsino color triplet (anti-triplet) $H^a(H'_a)$ with charges $Q = -1/3(1/3)$ couplings with sources J_a, K^a

$$H'\mathcal{M}H + H'K + HJ$$

Suppose we are in a basis where only the Higgs H'_1, H_1 couple with matter. Then the condition for suppression of dimension five operators is

$$(\mathcal{M})^{-1}_{11} = 0$$

More general constraints for suppression of p decay

PN, R.M. Syed: 0707.1332 [hep-ph]

More generally in GUTS/strings the Higgisino triplets could also have charges -4/3(4/3)

$$\tilde{H}^a(\tilde{H}_a) : Q = -4/3(4/3)$$

which generate additional terms

$$\tilde{H}'\tilde{\mathcal{M}}\tilde{H} + \tilde{H}'\tilde{K} + \tilde{H}\tilde{J}$$

Including these the supression conditions are

$$LLLL : \ (U\mathcal{M}V^T)^{-1}_{11} + \Lambda_{\mathrm{Planck}} = 0$$

$$RRRR : \ (U\mathcal{M}V^T)^{-1}_{11} + (\tilde{U}\tilde{\mathcal{M}}\tilde{V}^T)^{-1}_{11} + \tilde{\Lambda}_{\mathrm{Planck}} = 0$$

Suppression of p decay in SU(5) GUT

Consider a model with the Higgs content $\mathbf{24, 5, \bar{5}, 45, \overline{45}}$ with the color triplets (anti-triplets) as follows

$$H^{a}(5; q = -1/3), P^{a}(45; -1/3), Q^{a}(\overline{45}; -4/3)$$

$$H'_{a}(\bar{5}; q = 1/3), Q_{a}(\overline{45}; 1/3), P_{a}(45; 4/3)$$

$$\begin{array}{cc} & \begin{array}{ccc} H^{\alpha} & P^{\alpha} & Q^{\alpha} \end{array} \\ \begin{array}{c} H'_{\alpha} \\ Q_{\alpha} \\ P_{\alpha} \end{array} & \begin{pmatrix} M_{11} & M_{12} & 0 \\ M_{21} & M_{22} & 0 \\ 0 & 0 & M_{33} \end{pmatrix} \end{array}$$

P decay suppression: Assume couplings of $\mathbf{45(\overline{45})}$ have the same flavor dependence as couplings for $\mathbf{5(\bar{5})}$: $\mathbf{45 \sim \lambda 5}$, $\mathbf{\overline{45} \sim \lambda' \bar{5}}$.

$$(i)\lambda' = 0, M_{11}^{-1} + \lambda M_{12}^{-1} = 0; \ (ii)\lambda = 0, M_{21}^{-1} + \lambda' M_{22}^{-1} = 0$$

Suppression of p decay in SO(10) GUT

- Couplings of type $\mathbf{16.16.10}$, $\mathbf{16.16.120}$, and $\mathbf{16.16.\overline{126}}$ do not give a cancellation, since 10 plet couplings symmetric, 120 plet couplings anti-symmetric and $\mathbf{\overline{126}}$ do not contribute.
- The cancellation possibility arises in $\mathbf{144(\overline{144})}$ plet couplings

$$\mathbf{144 - Higgs: 5(3) + \bar{5}(7) + \overline{45}(3) + ..}$$

$$\mathbf{\overline{144} - Higgs: \bar{5}(-3) + 5(-7) + 45(-3) + ..}$$

  - Two pairs of color triplets (anti-triplets) from $\mathbf{5(\bar{5})}$.
  - Two pairs of color triplets (anti-triplets) from $\mathbf{45(\overline{45})}$.
  - An internal cancellation can occur in the dim 5 operators originating from the two types of contributions.

p decay suppression constraints

$$
\begin{aligned}
&\left\{\zeta^{(10)(+)}_{\acute{a}\acute{b},\acute{c}\acute{d}}\right\} \quad (\mathbf{16}_{\acute{a}} \times \mathbf{16}_{\acute{b}})_{\mathbf{10}} \, (\mathbf{144}_{\acute{c}} \times \mathbf{144}_{\acute{d}})_{\mathbf{10}} \\
&\left\{\xi^{(10)(+)}_{\acute{a}\acute{b},\acute{c}\acute{d}}\right\} \quad (\mathbf{16}_{\acute{a}} \times \mathbf{16}_{\acute{b}})_{\mathbf{10}} \, (\overline{\mathbf{144}}_{\acute{c}} \times \overline{\mathbf{144}}_{\acute{d}})_{\mathbf{10}} \\
&\left\{\varrho^{(126,\overline{126})(+)}_{\acute{a}\acute{b},\acute{c}\acute{d}}\right\} \quad (\mathbf{16}_{\acute{a}} \times \mathbf{16}_{\acute{b}})_{\overline{\mathbf{126}}} \, (\mathbf{144}_{\acute{c}} \times \mathbf{144}_{\acute{d}})_{\mathbf{126}} \\
&\left\{\lambda^{(45)}_{\acute{a}\acute{b},\acute{c}\acute{d}}\right\} \quad (\mathbf{16}_{\acute{a}} \times \overline{\mathbf{144}}_{\acute{b}})_{\mathbf{45}} \, (\mathbf{16}_{\acute{c}} \times \overline{\mathbf{144}}_{\acute{d}})_{\mathbf{45}} \, , \\
&\left\{\zeta^{(120)(-)}_{\acute{a}\acute{b},\acute{c}\acute{d}}\right\} \quad (\mathbf{16}_{\acute{a}} \times \mathbf{16}_{\acute{b}})_{\mathbf{120}} \, (\mathbf{144}_{\acute{c}} \times \mathbf{144}_{\acute{d}})_{\mathbf{120}} \\
&\left\{\xi^{(120)(-)}_{\acute{a}\acute{b},\acute{c}\acute{d}}\right\} \quad (\mathbf{16}_{\acute{a}} \times \mathbf{16}_{\acute{b}})_{\mathbf{120}} \, (\overline{\mathbf{144}}_{\acute{c}} \times \overline{\mathbf{144}}_{\acute{d}})_{\mathbf{120}} \\
&\left\{\lambda^{(54)}_{\acute{a}\acute{b},\acute{c}\acute{d}}\right\} \quad (\mathbf{16}_{\acute{a}} \times \overline{\mathbf{144}}_{\acute{b}})_{\mathbf{54}} \, (\mathbf{16}_{\acute{c}} \times \overline{\mathbf{144}}_{\acute{d}})_{\mathbf{54}} \\
&\left\{\lambda^{(10)}_{\acute{a}\acute{b},\acute{c}\acute{d}}\right\} \quad (\mathbf{16}_{\acute{a}} \times \mathbf{144}_{\acute{b}})_{\mathbf{10}} \, (\mathbf{16}_{\acute{c}} \times \mathbf{144}_{\acute{d}})_{\mathbf{10}} + \text{cubic terms.}
\end{aligned}
$$

Cancellation constraints

$$
\lambda^{(45)}_{\acute{a},\acute{b}} = 4\xi^{(10)(+)}_{\acute{a}\acute{b}}, \quad \zeta^{(10)(+)}_{\acute{a}\acute{b}} = 0 = \varrho^{(126,\overline{126})(+)}_{\acute{a}\acute{b}}.
$$

Conclusions/prospects

- The attactive feature of the model with 144 plet of Higgs is that the Higgs sector is unified in one irreducible representation and one can accomplish breaking of $SO(10)$ down to $SU(3)_C \times U(1)_{em}$ in one step.
- A second attractive feature is that all the ingredients of getting quark lepton masses and see-saw masses for the neutrino are built in.
- A third attractive feature is that a cancellation mechanism can operate and one can easily enhance the proton decay lifetime.
- One issue concerns fine tuning but is acceptable in the context of current ideas of landscape.