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# Magnetic monopoles: guts of GUTs

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# Monopoles in field theory

$$G \to H = \frac{[SU(3) \times U(1)]}{Z_3}$$

If $\pi_2(G) = \pi_1(G) = 1$ condition for monopoles to exist is

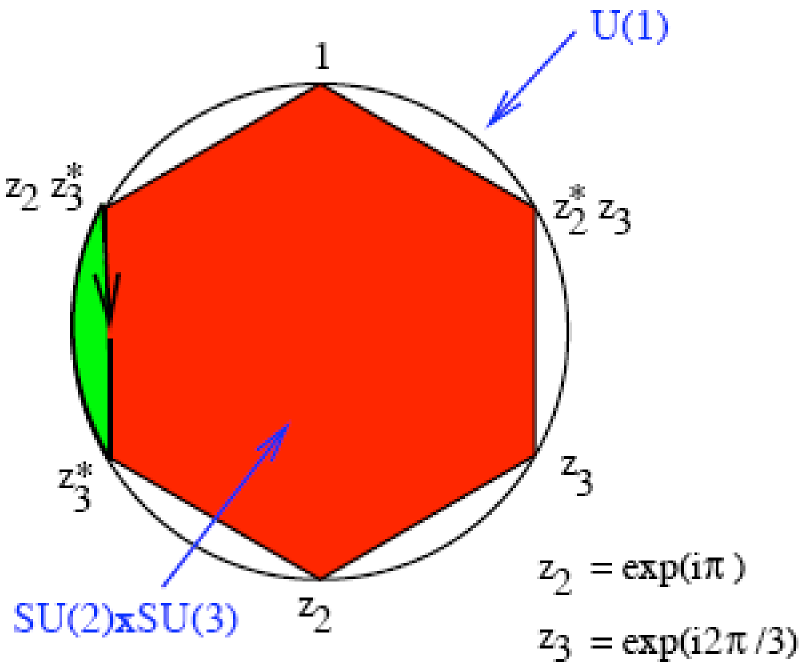
$$\pi_2(G/H) = \pi_1(H) = Z$$

Therefore monopoles are a generic (model independent) prediction of Grand Unified Theories.

If $\pi_2(G) \neq 1$ or $\pi_1(G) \neq 1$ then need to examine topology more explicitly.

# E.g. SU(5) unification

$$SU(5) \to \frac{[SU(3) \times SU(2) \times U(1)]}{Z_3 \times Z_2}$$




$$\pi_1 \left( \frac{[SU(3) \times SU(2) \times U(1)]}{Z_3 \times Z_2} \right) = Z$$

Fundamental monopole has SU(3), SU(2) and U(1) charge.

# Spectrum of stable monopoles

## Topology doesn't imply stability: 2 = 1+1, etc.

$$V(r) = \frac{1}{4\alpha r}[n_1 n_1' \mathrm{Tr}(Y^2)(1 - e^{-\mu_0 r}) + n_3 n_3' \mathrm{Tr}(\lambda_i \lambda_j)(1 - e^{-\mu_3 r}) + n_8 n_8' \mathrm{Tr}(T_a T_b)(1 - e^{-\mu_8 r})]$$

Gardner & Harvey, 1984

| $n$ | $n_3$ | $n_2$ | $n_1$ | $d_m$ |
|---|---|---|---|---|
| +1 | 1/3 | 1/2 | +1/6 | 6 |
| -2 | 1/3 | 0 | -1/3 | 3 |
| -3 | 0 | 1/2 | -1/2 | 2 |
| +4 | 1/3 | 0 | +2/3 | 3 |
| -6 | 0 | 0 | -1 | 1 |

| | $SU(3)_c$ | $SU(2)_L$ | $U(1)_Y$ | $d_f$ |
|---|---|---|---|---|
| $(u,d)_L$ | 1/3 | 1/2 | +1/6 | 6 |
| $d_R$ | 1/3 | 0 | -1/3 | 3 |
| $(\nu,e)_L$ | 0 | 1/2 | -1/2 | 2 |
| $u_R$ | 1/3 | 0 | +2/3 | 3 |
| $e_R$ | 0 | 0 | -1 | 1 |

"dual standard model"

TV, 1996; Liu & TV, 1997

# Monopoles as particles

Goddard-Nyuts-Olive (GNO) conjecture --

Magnetic monopoles form representations of a dual symmetry group.

(Only fundamental representations in examples I have seen.)

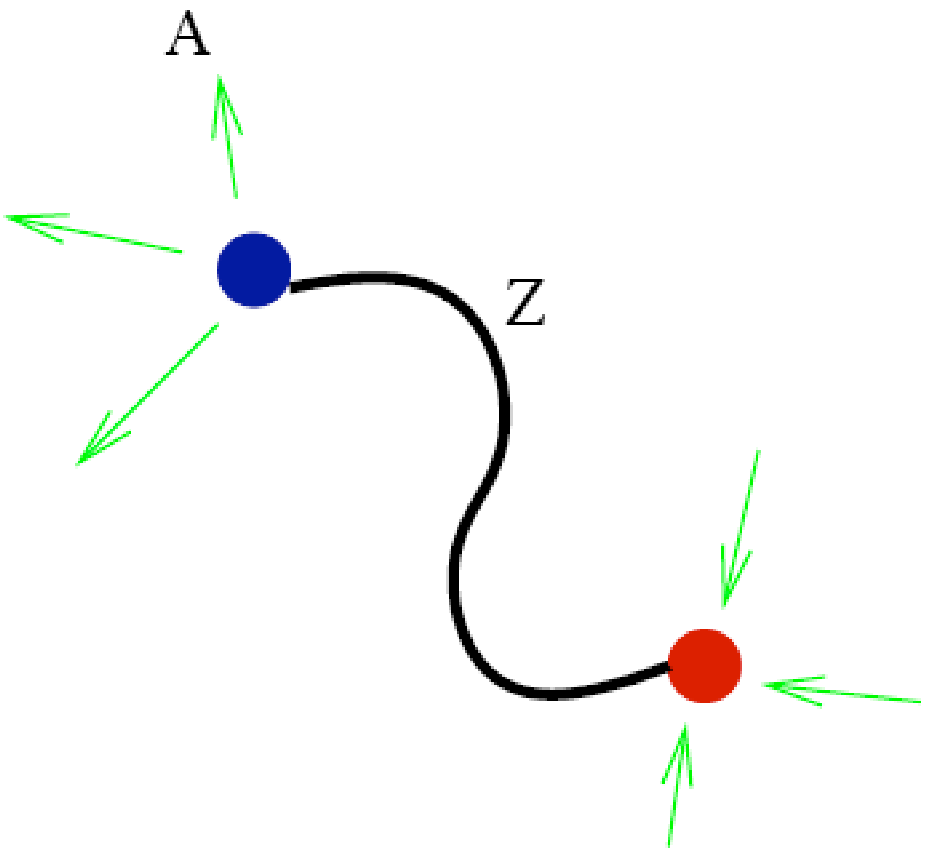
Are non-Abelian magnetic monopole solutions stable to classical/quantum effects? Maor, Mathur & TV (ongoing)

*Classical analysis:* only lowest charge monopole solutions are stable. Brandt & Neri, 1984

# Electroweak monopoles

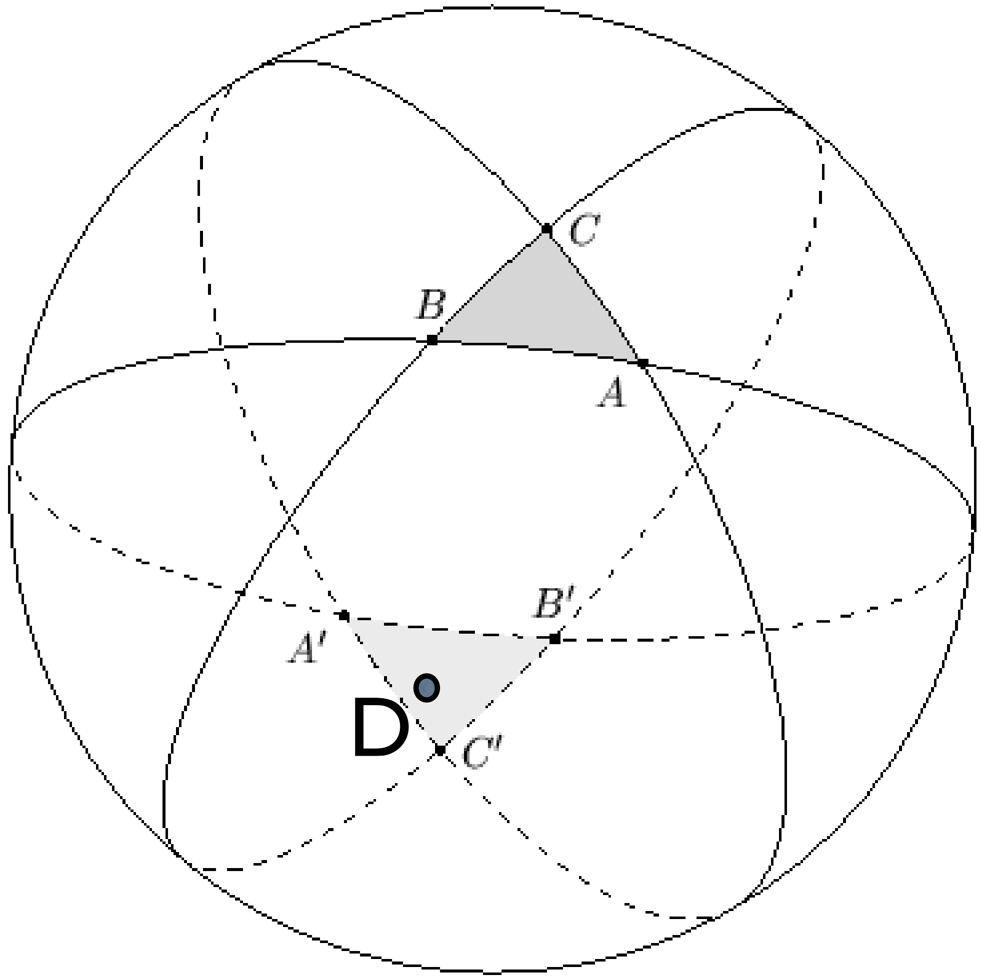
Nambu, 1977

$$\Phi = \begin{pmatrix} \cos(\theta/2) \\ \sin(\theta/2)e^{i\phi} \end{pmatrix}$$




about 7 TeV

# Monopoles in phase transitions

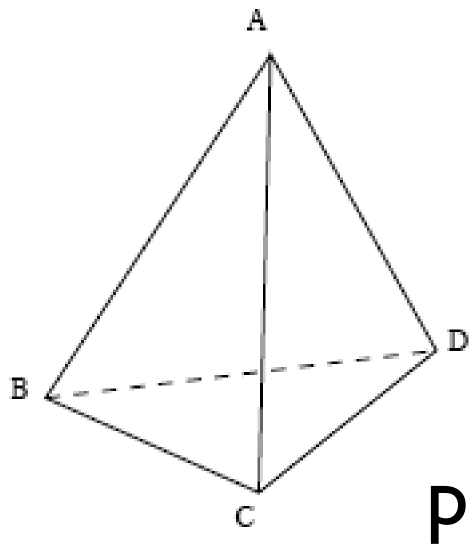



Vacuum manifold

Uncorrelated Higgs vacuum expectation values

"Kibble mechanism"

$\xi \sim \frac{1}{eT_c}$ in 2nd order phase transition




physical space

# GUT vacuum manifolds

$$SU(3) \to \frac{[SU(2) \times U(1)]}{Z_2}$$ Vacuum manifold is CP(2).

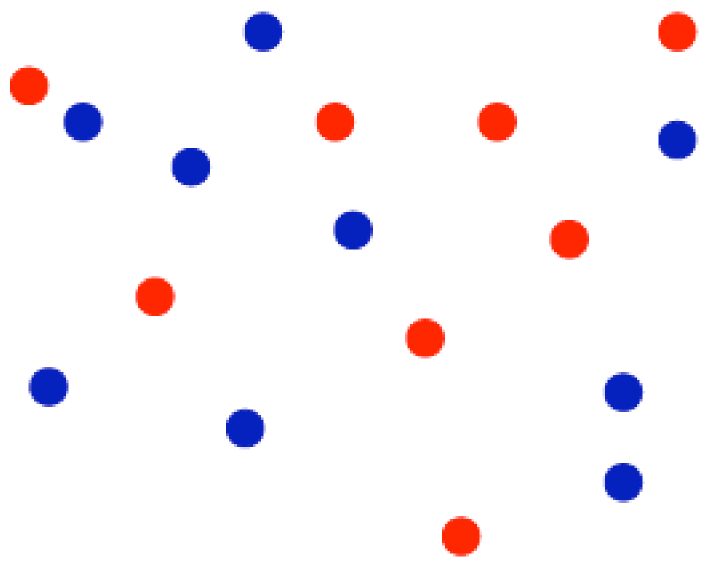
$$SU(5) \to \frac{[SU(3) \times SU(2) \times U(1)]}{Z_3 \times Z_2}$$ 12 dimensional

Similar to CP(N) but properties are not known in detail.

SO(10) GUTs?

# Monopoles in Cosmology

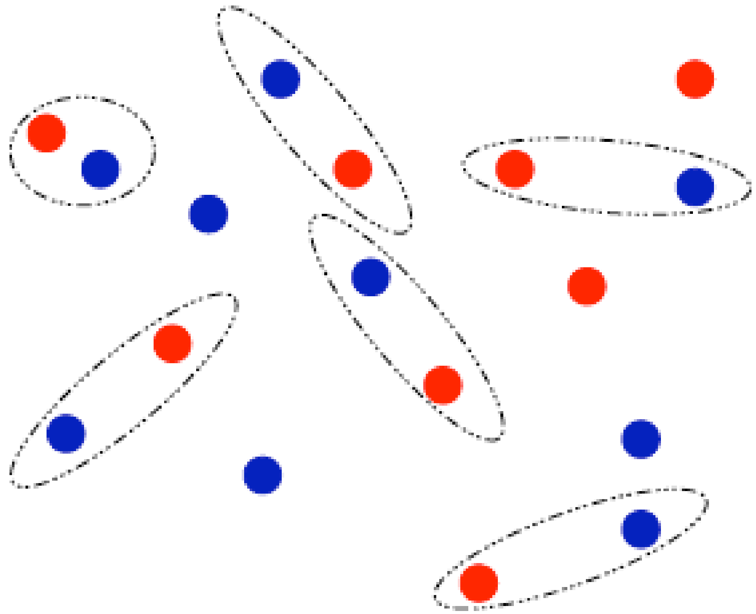
Kibble mechanism implies on order 0.1 monopole per correlation volume.



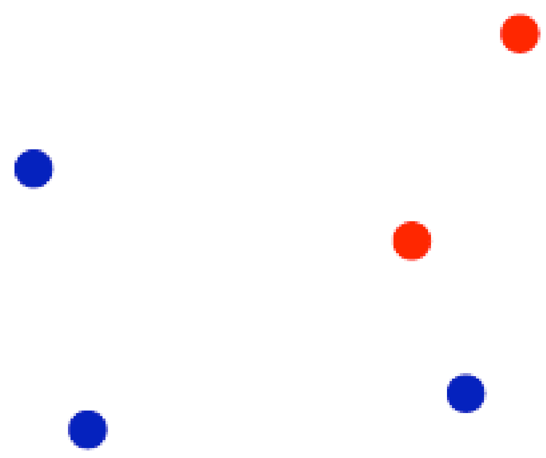
$$\xi \sim \frac{1}{eT_c}$$

$$n_M \sim \frac{0.1}{\xi^3}$$

A large number of monopoles and antimonopoles will annihilate in this dense gas of monopoles.



But some monopoles will get isolated and freeze-out.



*Even if there is 1 monopole/horizon at the GUT epoch...*

Their density then redshifts as dust and quickly dominates over the ambient radiation leading to the cosmological monopole over-abundance problem.

# GUT monopole density

One monopole per horizon at GUT phase transition:

$$\left.\frac{\rho_M}{\rho_r}\right|_{t_{\rm GUT}} \sim \left(\frac{T_{\rm GUT}}{T_{\rm Planck}}\right)^3 \sim 10^{-9}$$

Then at present epoch

$$\left.\frac{\rho_M}{\rho_\gamma}\right|_{t_0} \sim 10^{-9}\frac{T_{\rm GUTs}}{T_0} \sim 10^{19}$$

# Bounds

Cosmological bounds on GUT monopole flux:

$$F < 3 \times 10^{-12} h_0^2 \beta \ \mathrm{cm}^{-2}\mathrm{s}^{-1}\mathrm{sr}^{-1}$$

Parker bound on GUT monopoles from galactic field:

$$F < 10^{-15} \ \mathrm{cm}^{-2}\mathrm{s}^{-1}\mathrm{sr}^{-1}$$

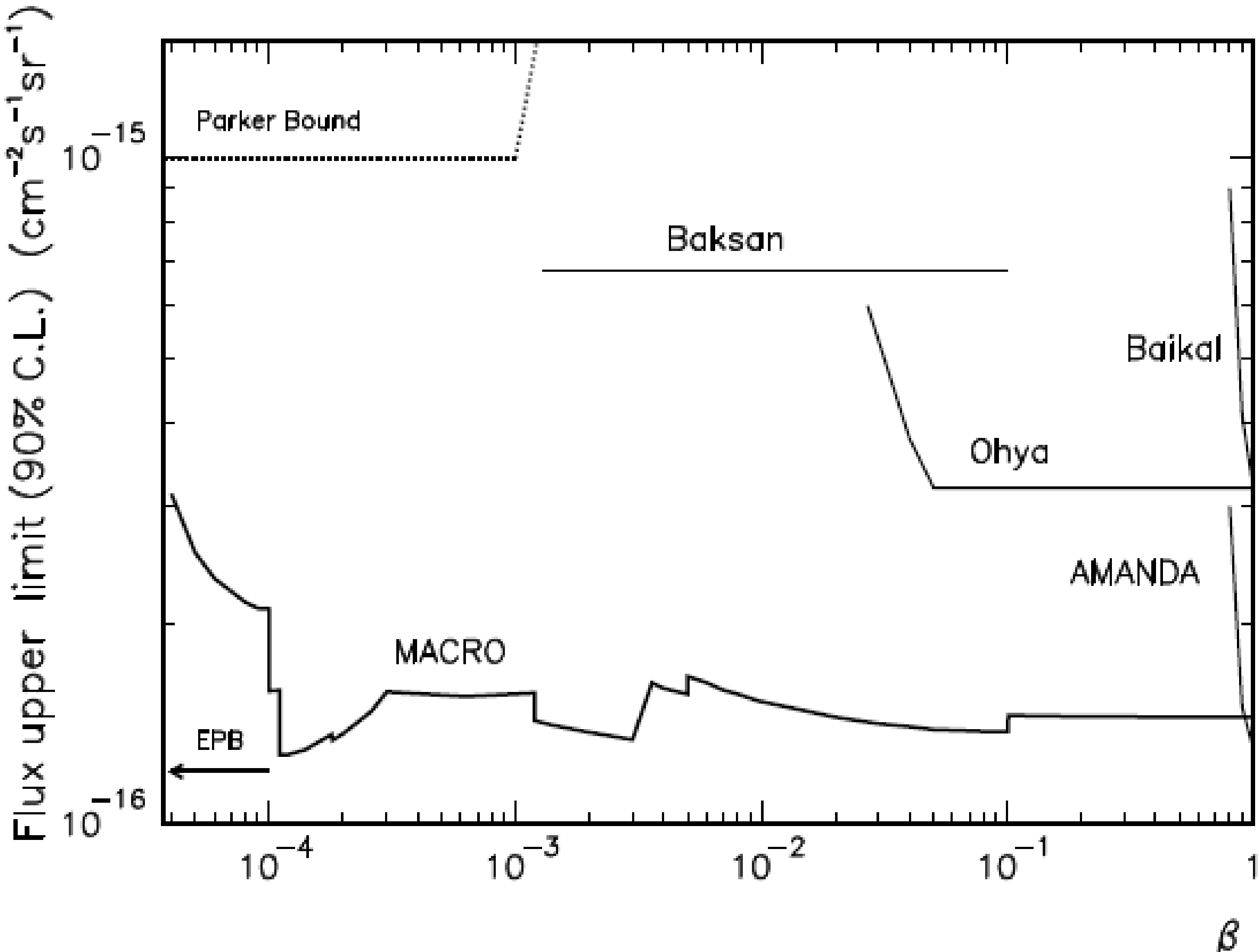
Parker bound on GUT monopoles from seed fields:

$$F < 10^{-16} \ \mathrm{cm}^{-2}\mathrm{s}^{-1}\mathrm{sr}^{-1}$$

Several other bounds -- stellar evolution, proton decay,...

# Direct bounds

Giacomelli & Patrizii, 2002



# Cosmology vs. GUTs

Monopoles of mass $m_M > 10^{11}$ GeV are a problem and need a solution.

Cosmological solution -- GUT or post-GUT inflation.

Guth, 1981

*Other possibilities:*

Monopoles connected by strings -- Langacker & Pi, 1980
then annihilation more efficient as every m has an mbar.

No cosmological GUT phase transition Dvali, Melfo & Senjanovic, 1995

Sweeping scenarios -- Dvali, Liu & TV

# Walls and monopoles in SU(5)

$$V(H,\phi) = V(H) + V(\phi) + \lambda_4(\mathrm{Tr}H^2)\phi^\dagger\phi + \lambda_5(\phi^\dagger H^2 \phi)$$

$$V(H) = -m_1^2\mathrm{Tr}(H^2) + \lambda_1(\mathrm{Tr}(H^2))^2 + \lambda_2\mathrm{Tr}(H^4) + \gamma\mathrm{Tr}(H^3)$$

$$V(\phi) = -m_2^2\phi^\dagger\phi + \lambda_3(\phi^\dagger\phi)^2$$

If cubic coupling is small, symmetry breaking is

$$SU(5) \times Z_2 \to \frac{[SU(3) \times SU(2) \times U(1)]}{Z_3 \times Z_2}$$

giving both monopoles and walls at the GUT phase transition.

# Naive domain wall

$Z_2$ wall $\qquad \phi = \eta \tanh\left(\frac{x}{w}\right)$

$$H = \eta \tanh\left(\frac{x}{w}\right) \mathrm{diag}(2, 2, 2, -3, -3)$$

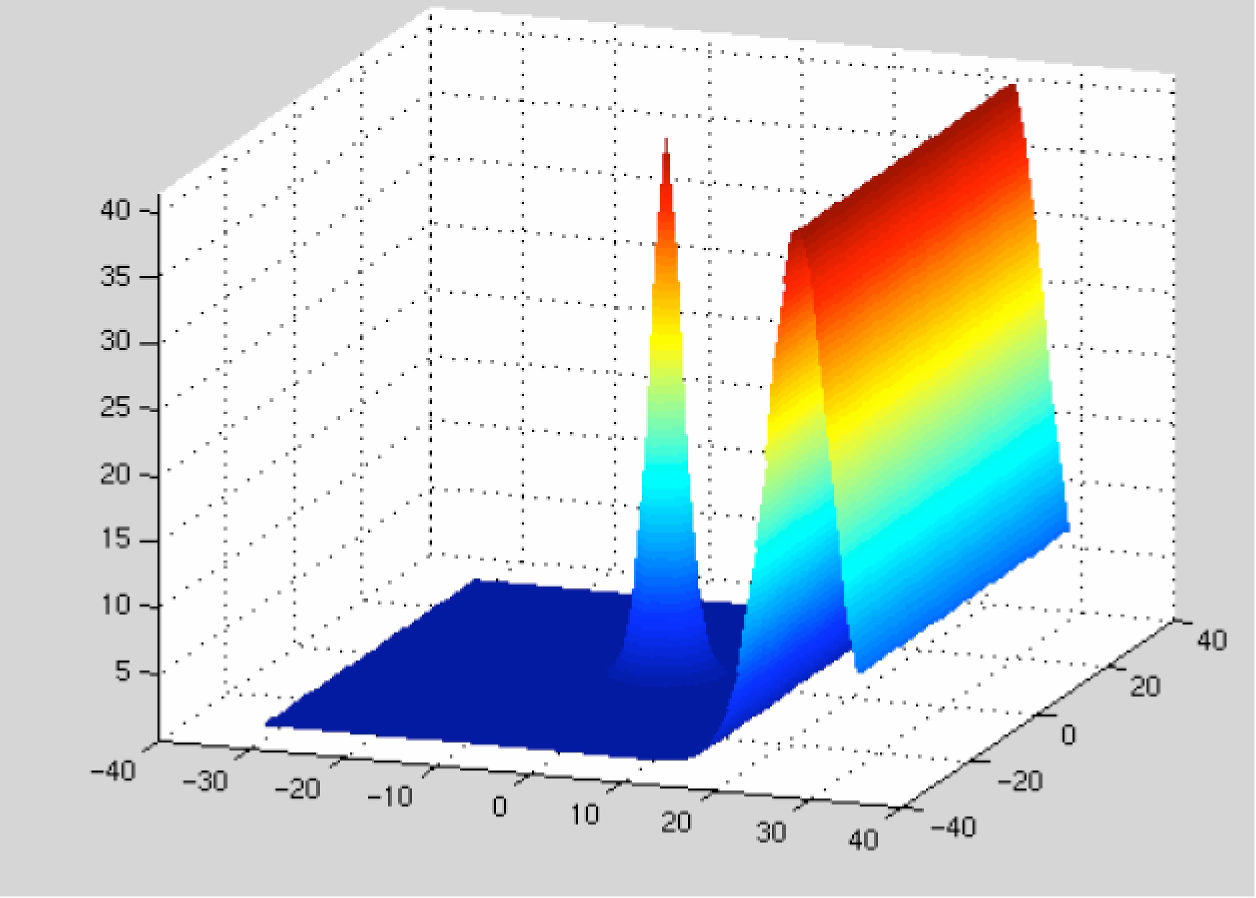
"q=0" wall

Full SU(5) restored within q=0 wall.

# Sweeping scenarios

q=0 wall

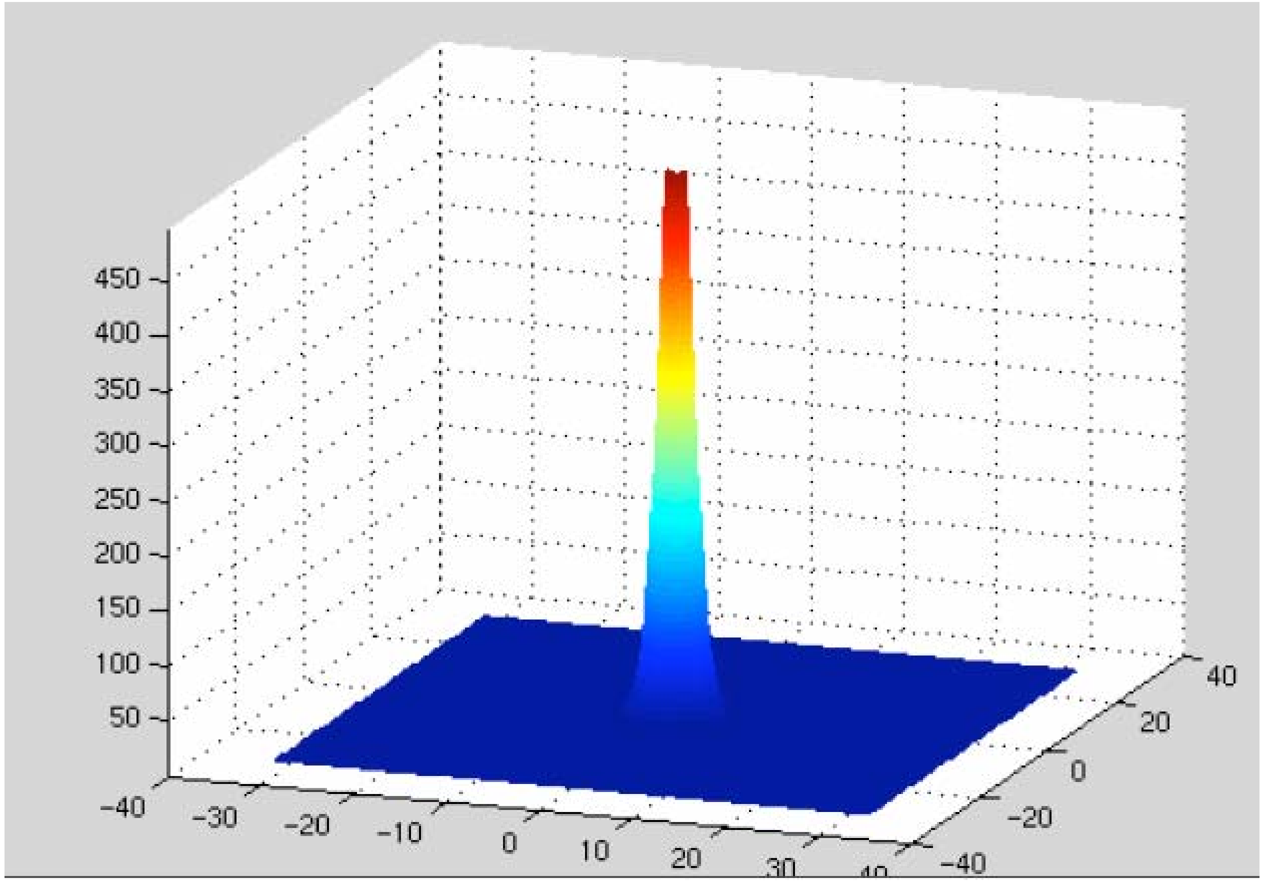
Pogosian & TV



# Sweeping scenarios

q=0 wall

Pogosian & TV



# Domain wall solution

(q=2 wall)

$$H(-\infty) = +\eta\text{diag}(2,2,2,-3,-3) \qquad H(+\infty) = -\eta\text{diag}(2,-3,-3,2,2)$$

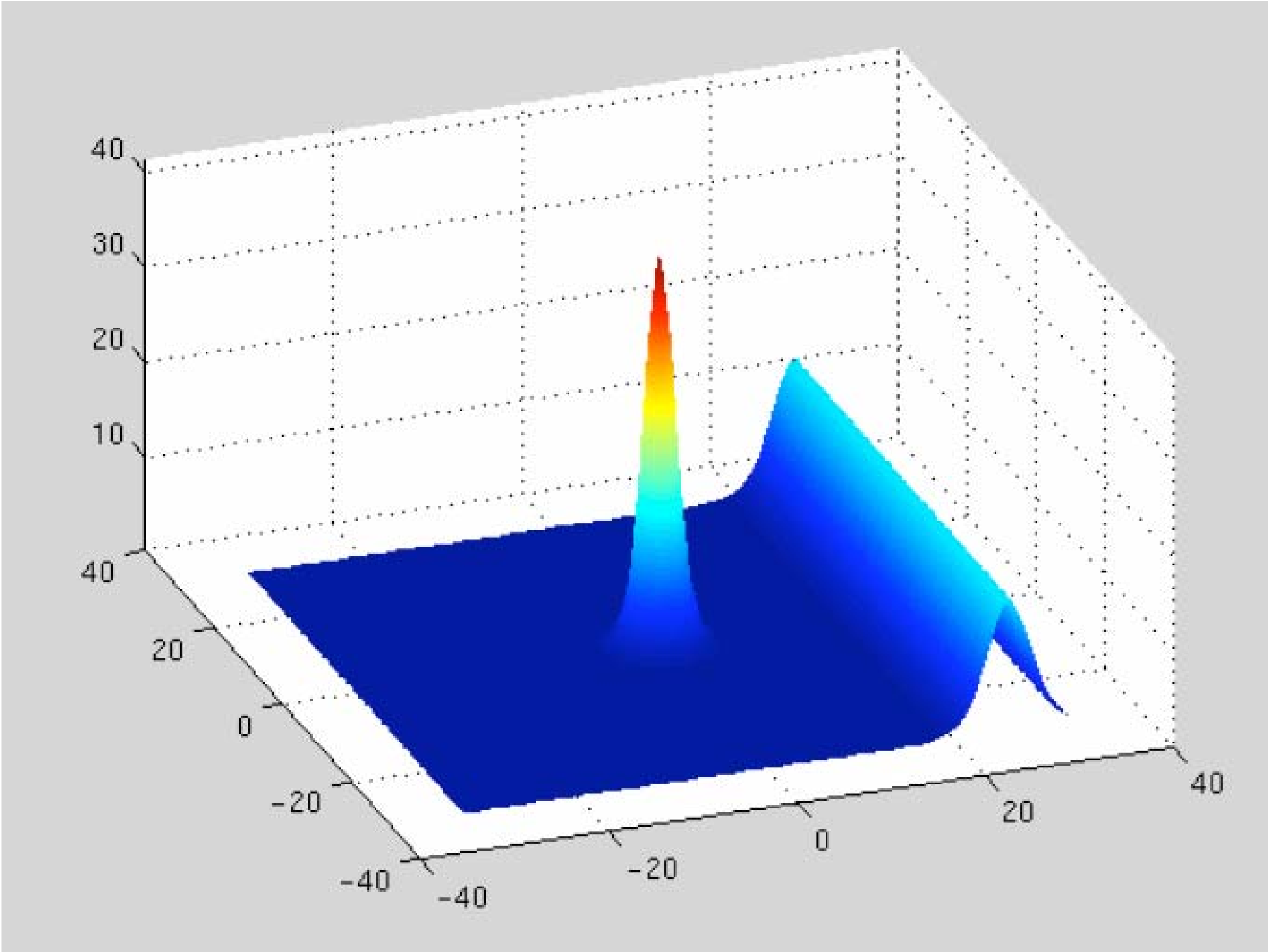
$$H(0) = +\eta\text{diag}(0,5/2,5/2,-5/2,-5/2)$$

Symmetry broken further inside the wall!

Only $SU(2)\times SU(2)\times U(1)$

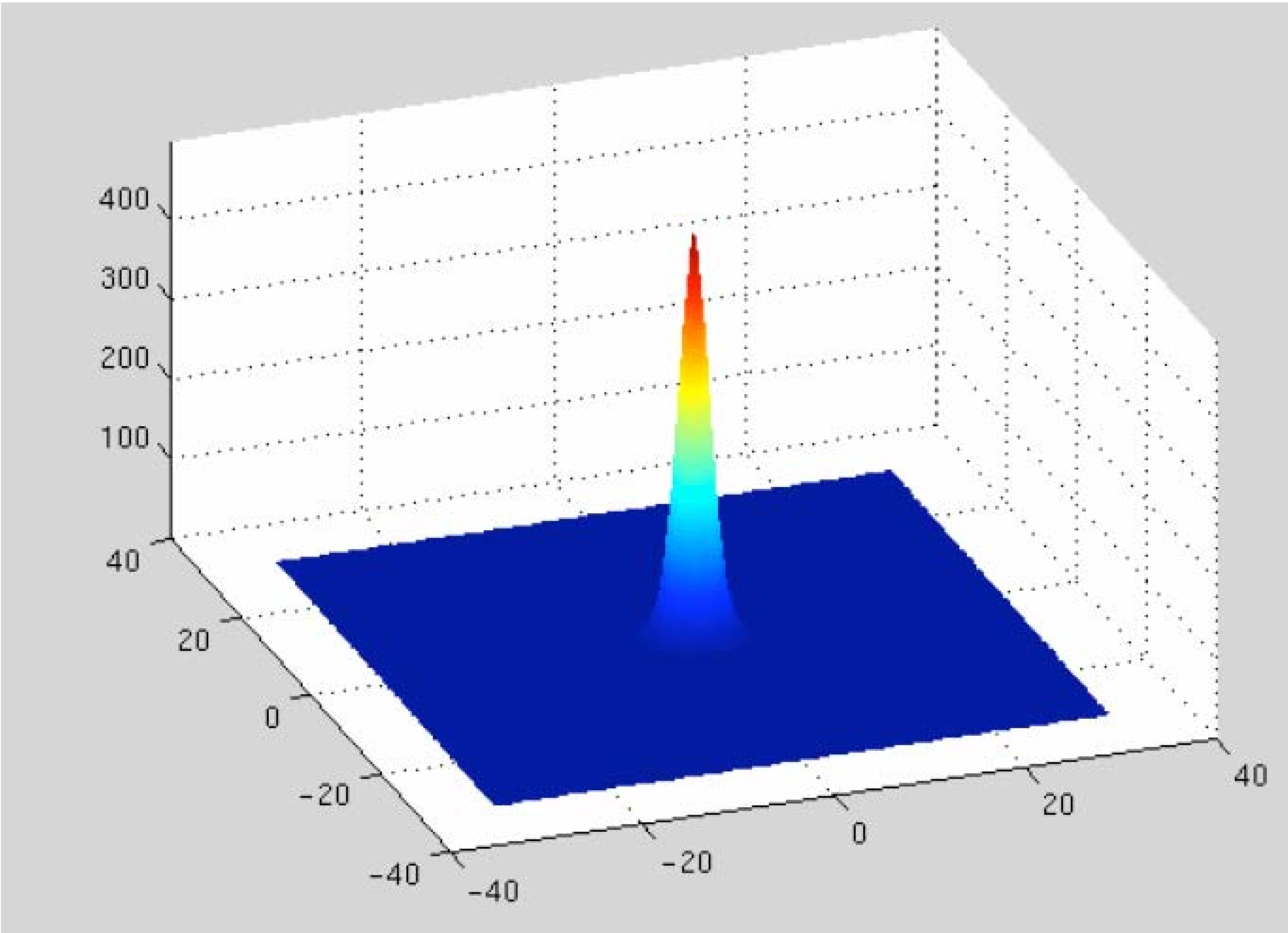
# Sweeping?

q=2 wall



# Sweeping?

## q=2 wall



# Outcome?

Depends on which monopole meets which wall.

Walls are not topological if cubic coupling is non-zero and can annihilate after some time.

Monopoles may also get attached to walls.

*Lesson 1* -- in general the GUT phase transition will be very complicated and defect interactions can be important.

*Lesson 2* -- symmetries are likely to be further *broken* within topological defects.

TV, 2003

# Confinement

Kibble, Ng & TV (ongoing)

Duality suggests non-Abelian magnetic monopole confinement.

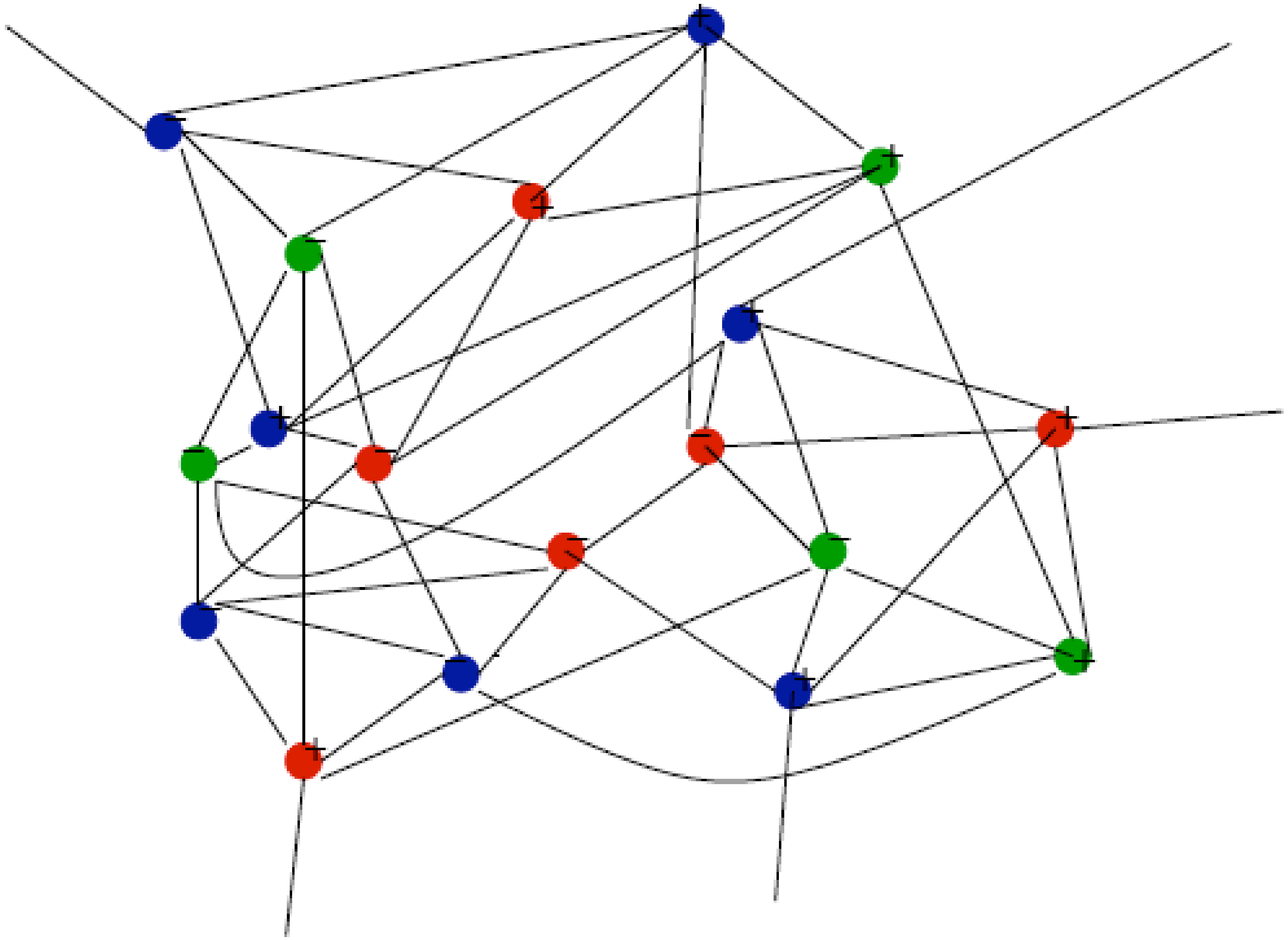
(Work under this hypothesis.)

$$SU(5) \rightarrow \frac{[SU(3) \times SU(2) \times U(1)]}{Z_3 \times Z_2}$$

Lightest monopole is non-Abelian.

Singlet monopole has topological charge 6.

# Quantum monopole network




Evolution?

Constraints?

# Intermediate mass monopoles

Can one arrange so that $\pi_1(G/H)$ is trivial until lower energy scale?

Can one have models in which only intermediate mass monopoles are present?

Or else can one prove a no-go theorem?
e.g. proton lifetime constrains mass of monopole and vice versa.

# Conclusions

- GUTs contain monopoles (consequence of G & U)
- Monopole constraints imply that the GUT model must either lead to inflation, or else contain quasi-topological walls, or Langacker-Pi strings, or GUT non-symmetry restoration, or ...
- ☆ Or perhaps GUT philosophy needs to be modified, leading only to intermediate mass monopoles.
- Experimental searches for magnetic monopoles must go on since their absence provides important constraints and if found would be truly remarkable confirmation of GUTs (at non-perturbative level).