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Yukawa textures in SO(10) with vector-like matter

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Yukawa sector of SO(10) with vector matter multiplet

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Outline

- Yukawa sector of minimal SUSY SO(10) GUT
- Non-decoupling effects of vector matter multiplet(s) (S. Barr)
- Sample 3+1 semi-realistic toy model :
    - No flavour symmetry (neutrino sector of course elusive)
    - SU(5) breaking propagate at renormalizable level
    - SUSY compatible (all at tree level, $SU(2)_R$ close to M_G)
    - irreducible (testing GUT)
    - easily refutable (parameter counting)

Yukawa sector of minimal SUSY SO(10)

At renormalizable level : $16 \otimes 16 = 10 + 120 + 126$

$$\boxed{16_F Y_{10} 16_F 10_H}$$

- single 10_H : $Y_u = Y_l = Y_d$
- multiple 10_H : still $Y_l = Y_d$

$$\boxed{16_F Y_{10} 16_F 10_H + 16_F Y_{120} 16_F 120_H}$$

- in general $Y_l \neq Y_d$, but not satisfactory

Grimus, Kuhbock, Lavoura 2006

$$\boxed{16_F Y_{10} 16_F 10_H + 16_F Y_{126} 16_F \overline{126}_H}$$

Minimal "potentially realistic" renormalizable setting of a SUSY SO(10) :

- effective Higgs mixing : $10 \otimes \overline{126} = 210 \oplus 1050$ Aulakh, Bajc, Melfo, Senjanovic, Vissani 2003
- D-flattness : $\overline{126} + 126$

All together : $10_H \oplus \overline{126}_H \oplus 126_H \oplus 210_H$

Yukawa sector of minimal SUSY SO(10)

Problems:

- proximity of the Landau pole
- potentially large GUT thresholds
- Neutrino challenge :

Aulakh, 2005

Bertolini, M.M., Schwetz 2006

Higher $SU(2)_R$ breaking scale ⇨ neutrinos too light

(M_M^ν comes from $16_F Y_{126} 16_F \overline{126}_H$ at renormalizable level)

Lower $SU(2)_R$ breaking scale ⇨ thresholds spoil gauge unification

The culprit seems to be the $\overline{126}_H$ required by the Yukawa sector consistency

Is there any other option to disentangle charged leptons and down quarks at renormalizable level without resorting to large Higgs representations ?

Effects of a vector-like matter multiplet

$$16_F = \overset{L_L}{(1,2,-1)} \oplus \overset{D^c_L}{(\overline{3},1,+2/3)} \oplus \overset{Q_L}{(3,2,+1/3)} \oplus \overset{U^c_L}{(\overline{3},1,-4/3)} \oplus \overset{N^c_L}{(1,1,0)} \oplus \overset{E^c_L}{(1,1,+2)}$$

$$10_F = \overset{\Lambda_L}{(1,2,-1)} \oplus \overset{\Delta^c_L}{(\overline{3},1,+2/3)} \oplus \overset{\Lambda^c_L}{(1,2,+1)} \oplus \overset{\Delta_L}{(3,1,-2/3)}$$

Vector multiplets percieve the GUT breaking from smaller representations like $45_H, 54_H$!

How to mix 16_F with 10_F into the effective light states ?

- spinorial masses protected by symmetries while vectors unprotected !

$$Y_{10} 16_F 16_F 10_H \qquad\qquad M_{10} 10_F 10_F$$

- however, without $\overline{126}_H \oplus 126_H$ the $SU(2)_R$ must be broken by $\overline{16}_H \oplus 16_H$

$$F 16_F 10_F 16_H \qquad\qquad V_R = \langle 1,1,0 \rangle_{16,\overline{16}}$$

- gauge coupling unification suggests V_R not far from M_G
- with a bit of conspiracy $M_{10} \sim M_G$ and the mixing is huge ⇒ no decoupling !

Effects of a vector-like matter multiplet

The vector part brings in the sensitivity to the effects of SU(5) breaking via e.g. $45_H, 54_H$

$$SO(10) \xrightarrow{45_H} SU(3)_c \otimes SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L}$$
$$SO(10) \xrightarrow{54_H} SU(4)_{PS} \otimes SU(2)_L \otimes SU(2)_R$$

Full Yukawa sector : $W_Y = Y_{10} 16_F 16_F 10_H + F 16_F 10_F 16_H + \lambda 10_F 10_F 54_H + M_{10} 10_F 10_F$

$M_u = Y_{10} v_u^{10}$

M_d	D_L^c	Δ_L^c
D_L	$Y_{10} v_d^{10}$	0
Δ_L	FV^{16}	$M_{10} - \lambda V^{54}$

M_l	E_L^c	Λ_L^c
E_L	$Y_{10} v_d^{10}$	FV^{16}
Λ_L	0	$M_{10} + \frac{3}{2}\lambda V^{54}$

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- this is not enough, because one can rotate the off-diagonalities away with no effect on spectrum

The zeros are lifted upon EWSB !

- an electroweak VEV is induced onto $(1,2,-1)_{16_H}$ due to the mixing term $10_H 16_H 16_H$ along the same lines like $10_H \overline{126}_H 210_H$ in the minimal SUSY SO(10)

Mixing $16_F \oplus 10_F$

Down sector matrices in full glory :

$$\boxed{\begin{array}{c|cc} M_d & D_L^c & \Delta_L^c \\ \hline D_L & Y_{10} v_d^{10} & -F v_d^{16} \\ \Delta_L & F V^{16} & M_{10} - \lambda V^{54} \end{array}} \qquad \boxed{\begin{array}{c|cc} M_l & E_L^c & \Lambda_L^c \\ \hline E_L & Y_{10} v_d^{10} & F V^{16} \\ \Lambda_L & -F v_d^{16} & M_{10} + \frac{3}{2}\lambda V^{54} \end{array}}$$

Integrating out the heavy degrees of freedom :

$$\mathcal{L}_Y \ni \Delta_L \underbrace{[F V^{16} D_L^c + (M_{10} - \lambda V^{54})\Delta_L^c]}_{M_\Delta \tilde{\Delta}_L^c} \equiv M_\Delta \Delta_L \tilde{\Delta}_L^c \qquad \gamma \equiv F\frac{V^{16}}{M_\Delta} \qquad \delta \equiv \frac{M_{10} - \lambda V^{54}}{M_\Delta}$$

$\tilde{\Delta}_L^c = \gamma D_L^c + \delta \Delta_L^c$ is a part of the unitary transformation from defining to the physical basis, i.e.

$$\begin{pmatrix} d_L^c \\ \tilde{\Delta}_L^c \end{pmatrix} = \underbrace{\begin{pmatrix} \alpha & \beta \\ \gamma & \delta \end{pmatrix}}_{U} \begin{pmatrix} D_L^c \\ \Delta_L^c \end{pmatrix} \quad \Rightarrow \quad \begin{pmatrix} D_L^c \\ \Delta_L^c \end{pmatrix} = \begin{pmatrix} \alpha^* & \gamma^* \\ \beta^* & \delta^* \end{pmatrix} \begin{pmatrix} d_L^c \\ \tilde{\Delta}_L^c \end{pmatrix}$$

$$\mathcal{L}_Y \ni \underbrace{D_L}_{d_L}(Y_{10} D_L^c v_d^{10} - F \Delta_L^c v_d^{16}) = d_L \left(Y_{10} v_d^{10} \alpha^* - F v_d^{16} \beta^*\right) d_L^c + \ldots$$

$$\Rightarrow \quad \boxed{M_d = Y_{10} v_d^{10} \alpha^* - F v_d^{16} \beta^*}$$

Effective charged sector Yukawa sum-rules :

$$\gamma_d \equiv \frac{FV^{16}}{M_\Delta} \qquad \delta_d \equiv \frac{M_{10} - \lambda V^{54}}{M_\Delta}$$

$$\gamma_l \equiv \frac{FV^{16}}{M_\Lambda} \qquad \delta_l \equiv \frac{M_{10} + \frac{3}{2}\lambda V^{54}}{M_\Lambda}$$

$$\boxed{\begin{array}{rcl} M_u & = & Y_{10} v_d^{10} \\ M_d & = & Y_{10} v_d^{10} \alpha_d^* - F v_d^{16} \beta_d^* \\ M_l & = & Y_{10} v_d^{10} \alpha_l^* - F v_d^{16} \beta_l^* \end{array}}$$

$$M_\Lambda \equiv \sqrt{|FV^{16}|^2 + |M_{10} + \tfrac{3}{2}\lambda V^{54}|^2}$$

$$M_\Delta \equiv \sqrt{|FV^{16}|^2 + |M_{10} - \lambda V^{54}|^2}$$

$$U \equiv \begin{pmatrix} \alpha & \beta \\ \gamma & \delta \end{pmatrix}$$ unitary

Understanding the game :

$V^{16} \ll V^{54} \sim M_{10} :$ $\quad \gamma_{d,l} \to 0 \Rightarrow \beta_{d,l} \to 0 \Rightarrow \alpha_d \sim \alpha_l \to 1 \quad M_u, M_d \sim M_l \propto Y_{10}$

- intermediate Pati-Salam stage & 10_F decouples, the subsequent P-S breakdown not transmitted O.K.

$V^{54} \ll V^{16} \sim M_{10} :$ $\quad \gamma_{d,l} \to 1 \Rightarrow \beta_{d,l} \to 1 \Rightarrow \alpha_d \sim \alpha_l \to 0 \quad M_u \propto Y_{10},$ while $M_d \sim M_l \propto F$

- intermediate SU(5) stage, Clebsch of 10_F screened, the subsequent SU(5) breakdown invisible O.K.

……

Toy model : generalization to 3+1 case, i.e. $16_F^1, 16_F^2, 16_F^3, 10_F$:

$$\vec{C}_{d,l} \equiv \frac{\vec{F}V^{16}}{M_{\Delta,\Lambda}}$$

$$D_{d,l} \equiv \frac{M_{10} + c_{d,l}V^{54}}{M_{\Delta,\Lambda}}$$

$$\boxed{\begin{array}{rcl} M_u & = & Y_{10} v_u^{10} \\ M_d & = & Y_{10} A_d^\dagger v_d^{10} - \vec{F} \otimes \vec{B}_d^* v_d^{16} \\ M_l & = & A_l^* Y_{10} v_d^{10} - \vec{B}_l^* \otimes \vec{F} v_d^{16} \end{array}}$$

$$U_{d,l} \equiv \begin{pmatrix} A_{d,l} & \vec{B}_{d,l} \\ \vec{C}_{d,l}^T & D_{d,l} \end{pmatrix}$$ unitary

- CKM mixing small up to 12
- b-tau unification

Neutrino sector

At renormalizable level : $W_Y = Y_{10} 16_F 16_F 10_H + F 16_F 10_F 16_H + \lambda 10_F 10_F 54_H + M_{10} 10_F 10_F$

$$\begin{array}{c|cccc} M_\nu & N_L & N_L^c & \Lambda_L^{c0} & \Lambda_L^0 \\ \hline N_L & 0 & Y_{10} v_u^{10} & F V^{16} & 0 \\ N_L^c & Y_{10}^T v_u^{10} & 0 & F v_d^{16} & 0 \\ \Lambda_L^{c0} & F^T V^{16} & F^T v_d^{16} & \lambda\, w & M_{10} + \frac{3}{2}\lambda V^{54} \\ \Lambda_L^0 & 0 & 0 & M_{10} + \frac{3}{2}\lambda V^{54} & \lambda\, w \end{array}$$

triplets in 54

- the large 13 entry can be rotated away by means of 14 - rotation.
After that, the full 34 block decouples while the 12 block reads :

$$\begin{array}{c|cc} M_\nu & \tilde{N}_L & N_L^c \\ \hline \tilde{N}_L & \sin^2\alpha\, \lambda w & Y_{10} v_u^{10} \\ N_L^c & Y_{10}^T v_u^{10} & 0 \end{array}$$

$\alpha \sim V^{16}/M_\Lambda$

☹ Pseudodirac neutrinos around the EW scale.

Lifting zeros beyond renormalizable level :

$$\begin{array}{c|cccc} M_\nu & N_L & N_L^c & \Lambda_L^{c0} & \Lambda_L^0 \\ \hline N_L & 0 & Y_{10} v_u^{10} & F V^{16} & 0 \\ N_L^c & Y_{10}^T v_u^{10} & M_M^\nu & F v_d^{16} & \omega \\ \Lambda_L^{c0} & F^T V^{16} & F^T v_d^{16} & \lambda\, w & M_{10} + \frac{3}{2}\lambda V^{54} \\ \Lambda_L^0 & 0 & \omega & M_{10} + \frac{3}{2}\lambda V^{54} & \lambda\, w \end{array}$$

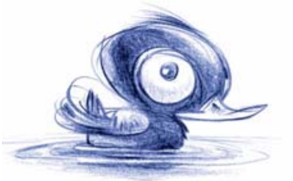
$\omega : \frac{1}{\Lambda} 16_F 10_F 16_H 10_H$

$M_M^\nu : \frac{1}{\Lambda} 16_F 16_F 16_H 16_H$

$\vdots$

Seesaw: $$\boxed{m_\nu \doteq \lambda \vec{B}_l^* \otimes \vec{B}_l^* w - (A_l^* Y_{10} v_u^{10} + \vec{\omega} \otimes \vec{B}_l^* v_u^{10}) {M_M^\nu}^{-1} (A_l^* Y_{10} v_u^{10} + \vec{\omega} \otimes \vec{B}_l^* v_u^{10})^T}$$

$M_M^\nu \sim (V^{16})^2/\Lambda$



Charged sector analysis

(Renormalizable)

3+1 toy model : $W_Y = Y_{10}^{ij} 16_F^i 16_F^j 10_H + F^i 16_F^i 10_F 16_H + \lambda 10_F 10_F 54_H + M_{10} 10_F 10_F$

$$U_{d,l} \equiv \begin{pmatrix} A_{d,l} & \vec{B}_{d,l} \\ \vec{C}_{d,l}^T & D_{d,l} \end{pmatrix}$$

$$\boxed{\begin{aligned} M_u &= Y_{10} v_u^{10} \\ M_d &= Y_{10} A_d^\dagger v_d^{10} - \vec{F} \otimes \vec{B}_d^* v_d^{16} \\ M_l &= A_l^* Y_{10} v_d^{10} - \vec{B}_l^* \otimes \vec{F} v_d^{16} \end{aligned}}$$

$$\vec{C}_{d,l} \equiv \frac{\vec{F} V^{16}}{M_{\Delta,\Lambda}}$$

$$D_{d,l} \equiv \frac{M_{10} + c_{d,l} V^{54}}{M_{\Delta,\Lambda}}$$

- $U_{d,l}$ are 4 x 4 unitary matrices (6 angles, 10 phases)

- given $\vec{C}_{d,l}, D_{d,l}$ unitarity admits only a partial reconstruction of $A_{d,l}, \vec{B}_{d,l}$

- at the effective level, there is an extra irrelevant 3 x 3 unitary RH rotation in the quark sector

- only 6 - 3 = 3 angles, 10 - 6 = 4 phases in U_d (i.e. also in $\vec{C}_d, D_d$ and $A_d, \vec{B}_d$) physically relevant

- fixing $\vec{C}_d, D_d$ from the quark sector fit, one easily obtains $\vec{C}_l, D_l$ for the lepton sector

- it is convenient to work with diagonal M_u and with transposed $\boxed{M_l^T = Y_{10} A_l^\dagger v_d^{10} - \vec{F} \otimes \vec{B}_l^* v_d^{16}}$

Charged sector analysis

(Renormalizable)

$$A = \tilde{U}^3 V$$

$$\begin{aligned} M_u &= Y_{10} v_u^{10} \\ M_d \tilde{U}_d^3 &= Y_{10} V_d^\dagger v_d^{10} + (\vec{F}^T \otimes \vec{C}_d D_d^*) V_d^{-1} v_d^{16} \\ M_l^T \tilde{U}_l^3 &= Y_{10} V_l^\dagger v_d^{10} + (\vec{F}^T \otimes \vec{C}_l D_l^*) V_l^{-1} v_d^{16} \end{aligned}$$

$$\vec{C}_{d,l} \equiv \frac{\vec{F} V^{16}}{M_{\Delta,\Lambda}}$$

$$D_{d,l} \equiv \frac{M_{10} + c_{d,l} V^{54}}{M_{\Delta,\Lambda}}$$

- $V_{d,l}$ are "nonunitary" parts of $A_{d,l}$, $\tilde{U}_d^3$ (irrelevant) and $\tilde{U}_l^3$ (relevant) the unitary pieces

Parametrization : $\alpha_1, \alpha_2, \alpha_3$ are the 3 relevant angles and $\psi_1, \psi_2, \psi_3; \psi_4$ the 4 phases

$$\begin{aligned} \vec{C}^T &= e^{i\psi_4}(\sin\alpha_1 e^{-i\psi_1}, \cos\alpha_1 \sin\alpha_2 e^{-i\psi_2}, \cos\alpha_1 \cos\alpha_2 \sin\alpha_3 e^{-i\psi_3}) \\ D &= e^{i\psi_4} \cos\alpha_1 \cos\alpha_2 \cos\alpha_3 \end{aligned}$$

$$\propto \vec{F}^T$$

$$V = e^{i\psi_4} \begin{pmatrix} \cos\alpha_1 & -e^{i(\psi_1-\psi_2)} \sin\alpha_1 \sin\alpha_2 & -e^{i(\psi_1-\psi_3)} \cos\alpha_2 \sin\alpha_1 \sin\alpha_3 \\ 0 & \cos\alpha_2 & -e^{i(\psi_2-\psi_3)} \sin\alpha_2 \sin\alpha_3 \\ 0 & 0 & \cos\alpha_3 \end{pmatrix}$$

- small angles correspond to Pati-Salam limit $\vec{F} \to 0$, $V \to \mathbb{1}$
- the only difference between α_i^d, ψ_i^d and α_i^l, ψ_i^l stems from the single Clebsch in $M_{\Delta,\Lambda}$!

Charged sector analysis

(Renormalizable)

$$\vec{C}_{d,l} \equiv \frac{\vec{F}V^{16}}{M_{\Delta,\Lambda}}$$

Parameter counting : $\vec{C}^T = e^{i\psi_4}\underbrace{(\sin\alpha_1 e^{-i\psi_1}, \cos\alpha_1 \sin\alpha_2 e^{-i\psi_2}, \cos\alpha_1\cos\alpha_2\sin\alpha_3 e^{-i\psi_3})}_{\vec{f}}$

$$\vec{C}_l = \underbrace{\frac{M_\Delta}{M_\Lambda}}_{s\in\mathbb{R}}\vec{C}_d \qquad \Rightarrow \psi_4^d - \psi_i^d = \psi_4^l - \psi_i^l \Rightarrow \psi_i^l = \psi_i^d - \underbrace{(\psi_4^l - \psi_4^d)}_{\phi}$$

for phases

$$\begin{aligned} \sin\alpha_1^l &= s\sin\alpha_1^d, \\ \cos\alpha_1^l\sin\alpha_2^l &= s\cos\alpha_1^d\sin\alpha_2^d, \\ \cos\alpha_1^l\cos\alpha_2^l\sin\alpha_3^l &= s\cos\alpha_1^d\cos\alpha_2^d\sin\alpha_3^d \end{aligned}$$

for angles

$$\boxed{\begin{aligned} M_d\tilde{U}_d^3 &= \left\{M_u\left[1-P(\vec{f})\right]r + qe^{i\psi_q}(\vec{f}^T\otimes\vec{f})\sqrt{1-|\vec{f}|^2}\right\}\left[N(\vec{f})\right]^{-1} \\ M_l^T\tilde{U}_l^3 &= \left\{M_u\left[1-P(s\vec{f})\right]r + s\,qe^{i(\psi_q-\phi)}(\vec{f}^T\otimes\vec{f})\sqrt{1-s^2|\vec{f}|^2}\right\}\left[N(s\vec{f})\right]^{-1} \end{aligned}}$$

$$P(\vec{f}) = \begin{pmatrix} |f_1|^2 & 0 & 0 \\ f_1 f_2{}^* & |f_1|^2+|f_2|^2 & 0 \\ f_1 f_3{}^* & f_2 f_3{}^* & |f_1|^2+|f_2|^2+|f_3|^2 \end{pmatrix}$$

$$N(\vec{f}) = \mathrm{diag}\left(n_1(\vec{f}), n_2(\vec{f}), n_3(\vec{f})\right)$$

$$\begin{aligned} n_1(\vec{f}) &= \sqrt{1}.\sqrt{1-|f_1|^2} \\ n_2(\vec{f}) &= \sqrt{1-|f_1|^2}\sqrt{1-|f_1|^2-|f_2|^2} \\ n_3(\vec{f}) &= \sqrt{1-|f_1|^2-|f_2|^2}\sqrt{1-|f_1|^2-|f_2|^2-|f_3|^2} \end{aligned}$$

10 Independent parameters : $q, r, s;\ \underbrace{\alpha_1^d, \alpha_2^d, \alpha_3^d;\ \psi_1^d, \psi_2^d, \psi_3^d}_{\in\vec{f}}, \psi_q, \phi$, but ψ_q can be absorbed

Given M_u , there is 10 physical parameters to extract - is the solution trivial ? NOT AT ALL !

Charged sector analysis

2 x 2 case (2nd and 3rd generation only)

Parameter counting : $q, r, s; \alpha_1^d, \alpha_2^d, \alpha_3^d, \psi_1^d, \psi_2^d, \psi_3^d, \phi$

Charged sector analysis

2 x 2 case (2nd and 3rd generation only)

Parameter counting : $q, r, s; \xcancel{\alpha_1^d}, \alpha_2^d, \alpha_3^d, \xcancel{\psi_1^d}, \psi_2^d, \psi_3^d, \phi$ - 5 real numbers + 3 phases

Charged sector analysis

2 x 2 case (2nd and 3rd generation only)

Parameter counting : $q, r, s; \cancel{\alpha_1^d}, \alpha_2^d, \alpha_3^d, \cancel{\psi_1^d}, \psi_2^d, \psi_3^d, \phi$

- 5 real numbers + 3 phases

Parameters to fit : $m_s, m_b, m_\mu, m_\tau, \sin\phi_{23}$

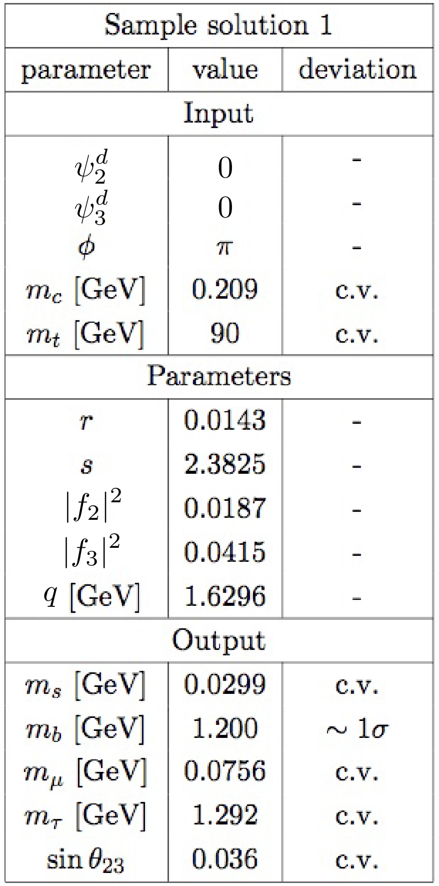
- 5 real numbers (no phases)

↕

Sticking to the real case : $q, r, s;\ \alpha_1^d, \alpha_2^d$

- 5 real numbers

Sample inputs from Das, Parida 2000



Sample solution 1		
parameter	value	deviation
Input		
ψ_2^d	0	-
ψ_3^d	0	-
ϕ	π	-
m_c [GeV]	0.209	c.v.
m_t [GeV]	90	c.v.
Parameters		
r	0.0143	-
s	2.3825	-
$\lvert f_2\rvert^2$	0.0187	-
$\lvert f_3\rvert^2$	0.0415	-
q [GeV]	1.6296	-
Output		
m_s [GeV]	0.0299	c.v.
m_b [GeV]	1.200	$\sim 1\sigma$
m_μ [GeV]	0.0756	c.v.
m_τ [GeV]	1.292	c.v.
$\sin\theta_{23}$	0.036	c.v.

} real setup

} drives the hierarchy between V^{16} and the GUT scale

} need for b-tau unification

Charged sector analysis

3 x 3 case (crushing the toy)

Parameter counting : $q, r, s; \alpha_1^d, \alpha_2^d, \alpha_3^d, \psi_1^d, \psi_2^d, \psi_3^d, \phi$ - 6 real numbers + 4 phases

Parameters to fit : $m_d, m_s, m_b, m_e, m_\mu, m_\tau, \sin\phi_{12}, \sin\phi_{23}, \sin\phi_{13}, \delta_{CKM}$

- 9 real numbers + 1 phase

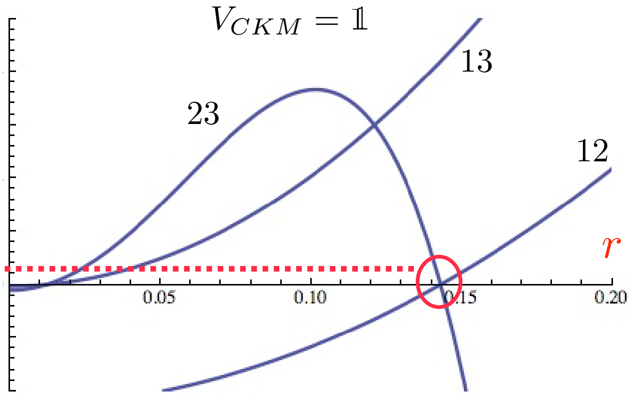
Purely quark sector fit possible (8 parameters for 7 observables) for $(r \sim 0.14)$ but charged leptons fail

Analytic understanding : quark sector yields

$$\boxed{\underset{i \neq j}{|f_i|^2|f_j|^2} = \frac{|d_{ij}|^2 - \left[d_{ii} - r^2(m_u^i)^2\right]\left[d_{jj} - r^2(m_u^j)^2\right]}{r^2 q^2 \left[(m_u^i)^2 + (m_u^j)^2 - 2m_u^i m_u^j \cos 2(\psi_i^d - \psi_j^d)\right]}}$$

$$d_{ij} \equiv [V_{CKM}.\mathrm{diag}(m_d^2, m_s^2, m_b^2).V_{CKM}^\dagger]_{ij}$$

$$d_{ij,i\neq j} \ll d_{ii} \sim (m_d^i)^2$$



$V_{CKM} \to \mathbb{1}$: $|f_i|^2|f_j|^2 \geq 0$ can never be achived $\forall(i \neq j)$!

$V_{CKM} \neq \mathbb{1}$: $|d_{ij}|^2$ lifts the curves up, r in the cross reg.

this drives $\alpha_3 \to \pi/2$ and $s < 1 \Rightarrow m_\mu < m_s$ †

Conclusions

- One can generate departures from $M_d = M_l^T$ even at renormalizable level by 10_F
- Mixing with 16_F distributes the tree-level $SU(5)$-breaking sensitivity of 10_F
- Neutrino sector elusive, calls for nonren. operators & flavour symmetry à la S. Barr
- The simplest 3+1 renormalizable model fails because of the charged Yukawa sector !

Thanks for your kind attention !