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Generation Symmetry and E6 unification

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Generation Symmetry and E_6

with Zurab Tavartkiladze

Aim : To reach a consistent scheme which connects

the many observables in particle physics,

in particular, the masses and mixings of

ALL light and heavy fermions

GUTs like $SO(10)$ and E_6 give a general understanding of structure and quantum numbers of the standard model and suggest new ideas.

Chiral Symmetry:
Lagrangians for a single generation are invariant under chiral transformations;
For more generations one needs Generation Symmetry

Two possibilities:

i) To give Higgs fields generation indices;

ii) Identify the coupling matrices as VEVs of new scalar fields.

We use the <u>second</u> alternative
3 Generations $\rightarrow$ coupling matrices are 3×3 matrices
$\rightarrow$ 9 scalar fields
Expressed in terms of hermitian 3×3 matrix $\Phi(x)$

(1) $$\Phi_{\alpha\beta}(x) = \chi_{\alpha\beta}(x) + \mathrm{i}\,\xi_{\alpha\beta}(x) \qquad \alpha, \beta = 1, 2, 3$$

- Generation Symmetry: $SO(3)_g \times \mathcal{P}_g$,
  $\mathcal{P}_g$: generation parity

all fermion fields: 3 vectors in generation space with
$\mathcal{P}_g = -1$
$\Longrightarrow$ under $SO(3)_g$ χ fields transform as $\mathbf{1} + \mathbf{5}$, ξ fields as **3**
with $\mathcal{P}_g = +1$

Immediate consequence:

only <u>two</u> generation matrices

$\langle \chi_{\alpha\beta} \rangle$ can be taken diagonal, 3 entries
responsable for the <u>hierarchy</u> of generations

$i\,\langle \xi_{\alpha\beta} \rangle$ hermitian <u>antisymmetric</u> matrix, 3 entries
responsable for mixing and CP violation for
all fermions: quarks, charged leptons and
light and heavy neutrinos

Our starting symmetry at the GUT scale:

- $$E_6 \times SO(3)_g \times \mathcal{P}_g$$

fermion fields in **27** representation of E_6 $(27, 3)$

$$\psi_r^\alpha(x) \qquad \alpha = 1, 2, 3 \;, \qquad r = 1, \cdots, 27$$

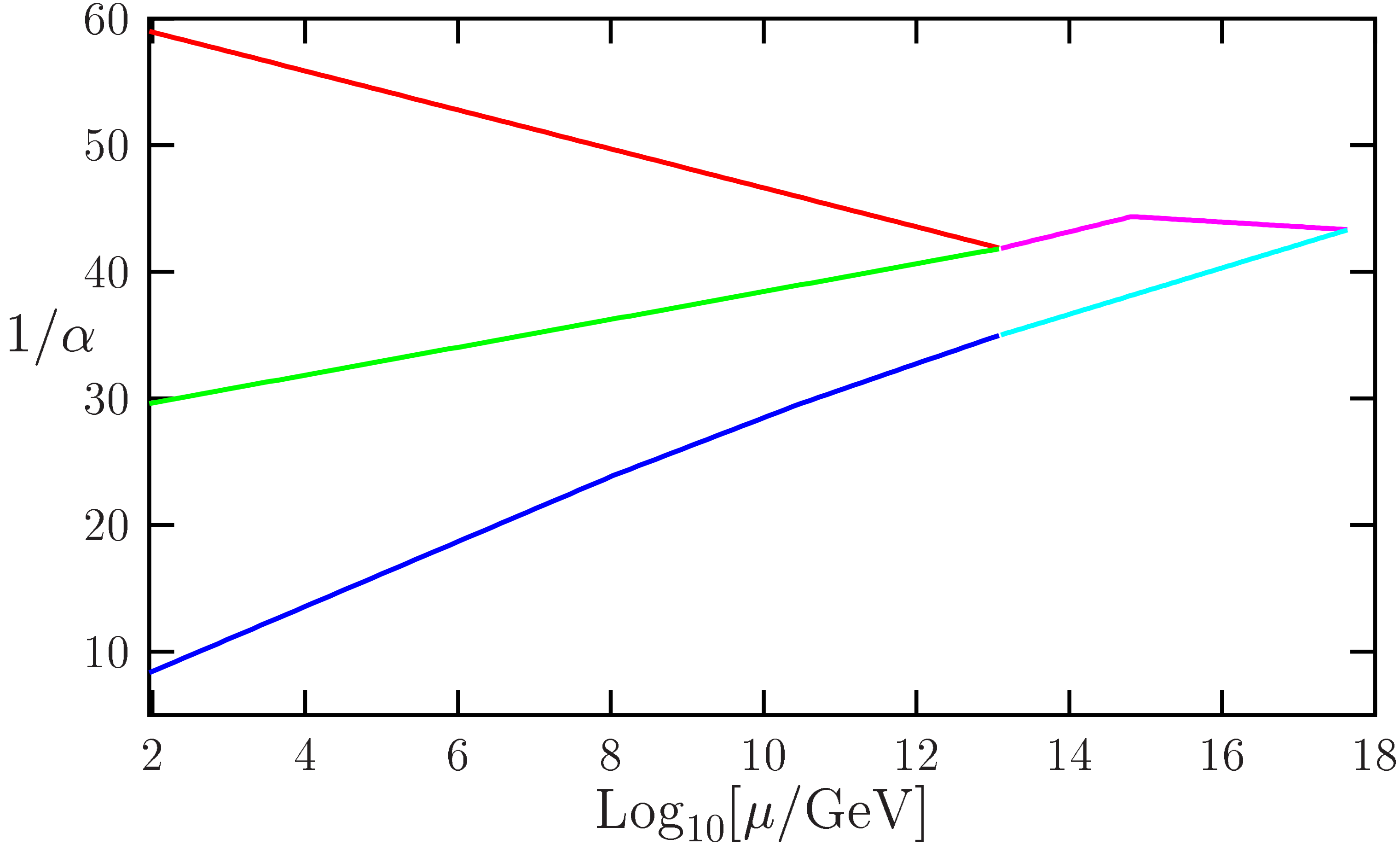
- Intermediate symmetry of E_6:

$$G_{333} \equiv SU(3)_L \times SU(3)_R \times SU(3)_C$$

in non SUSY approach $M_I \simeq 10^{13} - 10^{14}$ GeV fixed
where gauge couplings g_1 and g_2 unify! (B.S, Z.T. '03)

- $M_I =$ scale for heavy neutrinos!

‘Concorde’



details $\Longrightarrow$ talk by Zurab

single generation for fermions $\psi(27)$

$$(Q_L)^a_i = \begin{pmatrix} u^a \\ d^a \\ D^a \end{pmatrix}, \quad L^i_k = \begin{pmatrix} L^1_1 & E^- & e^- \\ E^+ & L^2_2 & \nu \\ e^+ & \hat{\nu} & L^3_3 \end{pmatrix}, \quad (Q_R)^k_a = \left(\hat{u}_a,\ \hat{d}_a,\ \hat{D}_a\right)$$

$$Q_L(x) = (3, 1, \bar{3}) \ , \qquad L(x) = (\bar{3}, 3, 1) \ , \qquad Q_R(x) = (1, \bar{3}, 3)$$

mixing: $d \leftrightarrow D$ $\mathcal{U}_L$-spin $\hat{d} \leftrightarrow \hat{D}$ $\mathcal{U}_R$-spin

$$\langle H(27) \rangle = \mathrm{Diag}\left(e^1_1 \ , \ e^2_2 \ , \ e^3_3\right) \ \ \text{choice of basis} : e^2_3 = e^3_2 = 0$$

Phenomenology

Yukawa interaction with $H(27)$ $\langle H(27)\rangle \sim (\bar{3}, 3, 1)$

$$\mathcal{L}_Y^{\rm eff} = G_{\alpha\beta}\left(\psi^{\alpha T} H(27)\psi^{\beta}\right) + \text{h.c.} \qquad \langle H(27)\rangle_1^1 = m_t$$

Up quarks can not mix with heavy states

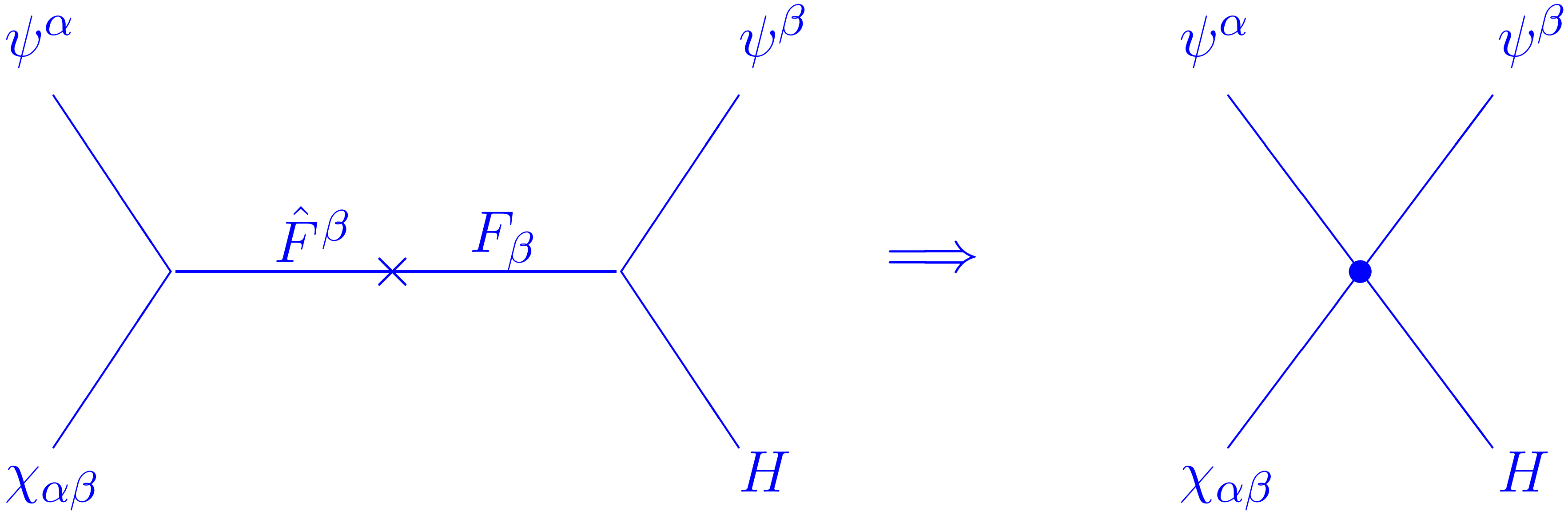
$$G_{\alpha\beta} = \begin{pmatrix} m_u & 0 & 0 \\ 0 & m_c & 0 \\ 0 & 0 & m_t \end{pmatrix} \frac{1}{m_t} \simeq \begin{pmatrix} \sigma^4 & 0 & 0 \\ 0 & \sigma^2 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \qquad \begin{array}{c} \sigma = 0.058 \\ \text{from experiment} \end{array}$$

interpretation: $$G_{\alpha\beta} = \frac{\langle \chi_{\alpha\beta}\rangle}{M}$$

Renormalizable interaction: introduce massive spinor fields

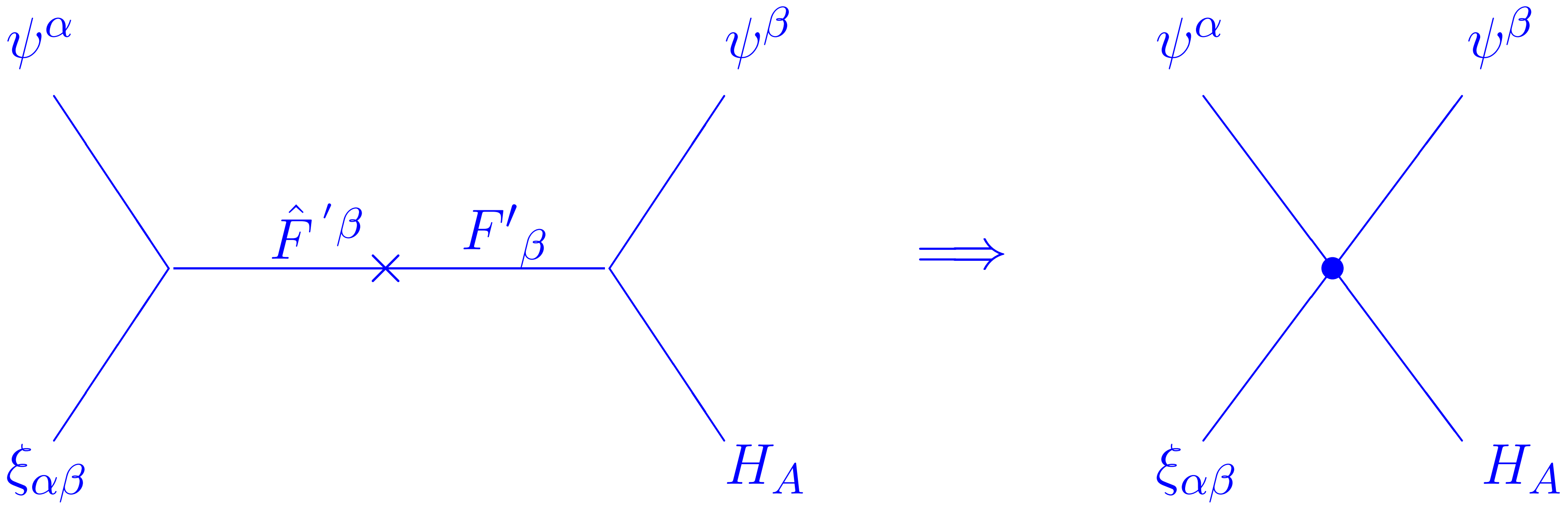
$F^{\alpha}(27,3), \hat{F}^{\alpha}(\overline{27},3); \qquad F'^{\alpha}(27,3), \hat{F}'^{\alpha}(\overline{27},3)$

$$\text{Vertices :} \qquad \chi_{\alpha\beta}\,\psi^{\alpha}\hat{F}^{\beta} + \psi^{\alpha} H F_{\alpha} + M F_{\alpha}\hat{F}^{\alpha}$$



$$\frac{\langle \chi_{\alpha\beta} \rangle}{M}\left(\psi^{\alpha} H(27)\psi^{\beta}\right)$$

Vertices : $\quad \mathrm{i}\xi_{\alpha\beta}\,\psi^{\alpha}\hat{F}'^{\beta} + \psi^{\alpha}H_A(351)F'_{\alpha} + M'F'_{\alpha}\hat{F}'^{\alpha}$



$$\mathrm{i}\frac{\langle\xi_{\alpha\beta}\rangle}{M'}\left(\psi^{\alpha}H_A(351)\psi^{\beta}\right)$$

$\frac{\langle\chi_{\alpha\beta}\rangle}{M}\left(\psi^{\alpha} H(27)\psi^{\beta}\right)$ provides masses for all fermions except $L^3_2 = \hat{\nu}$ and L^3_3:

$$m_u = Ge^1_1\,, \quad m_d = m_l = Ge^2_2\,, \quad m_D = m_L = Ge^3_3 \text{ (Dirac masses)}$$

$$m(L^3_2) = m(L^3_3) = 0$$

for fermion mixings we need $\frac{\mathrm{i}\langle\xi_{\alpha\beta}\rangle}{M'} = A$

$$A_{\alpha\beta} = \mathrm{i}\begin{pmatrix} 0 & \sigma & -\sigma \\ -\sigma & 0 & \frac{1}{2} \\ \sigma & -\frac{1}{2} & 0 \end{pmatrix} \quad \text{from CKM fit} \quad \text{(B.S. and Z.T.)}$$

$2 \leftrightarrow 3$ generation exchange: $A \to -A$

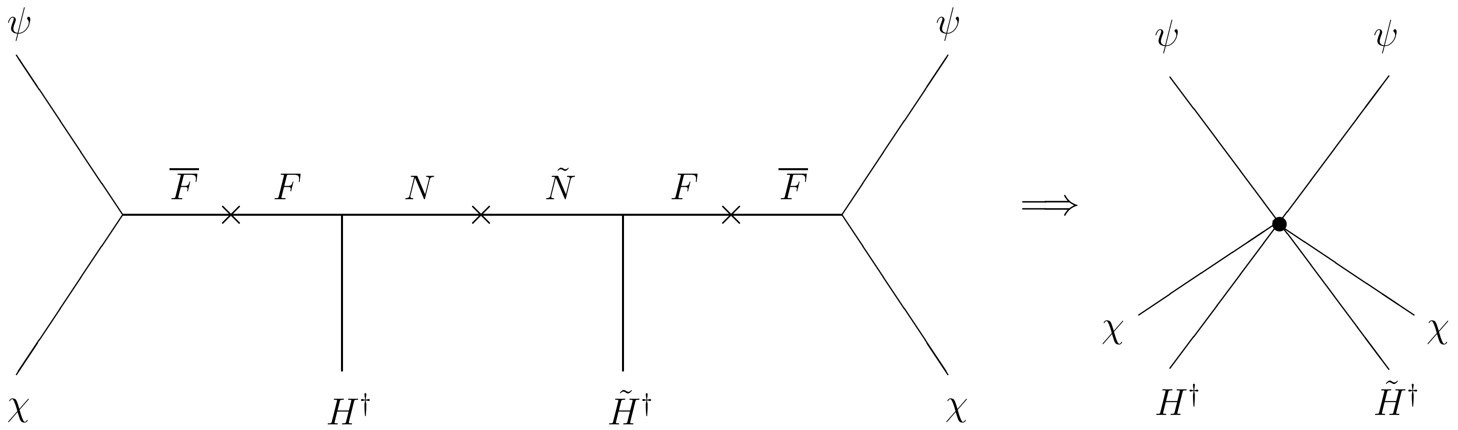
Masses for $\underline{L_2^3 = \hat{\nu}}$ and $\underline{L_3^3}$: 'right handed' neutrinos

$L_3^3 L_3^3 \sim (\bar{6}, 6, 1)$ is not contained in $H(27)$

• Coupling with two $H(27)$ needed

via a massive spinor E_6 singlet fields $N^\alpha(1,3)$, $\tilde{N}^\alpha(1,3)$

$$\text{Vertices}: \quad F^\alpha H^\dagger N^\alpha \; + \; F^\alpha \tilde{H}^\dagger \tilde{N}^\alpha \; + \; M_N N^\alpha \tilde{N}^\alpha$$



$$\Longrightarrow \frac{\langle \chi_{\alpha\beta} \rangle}{M} \frac{1}{M_N} \left(\psi^\alpha H^\dagger \tilde{H}^\dagger \psi^\gamma \right) \frac{\langle \chi_{\gamma\beta} \rangle}{M}$$

This gives masses for leptons, dominantly for L_2^3 and L_3^3 from $\langle H\rangle_3^3$, $\langle\tilde{H}\rangle_2^3$ and $\langle\tilde{H}\rangle_3^3 \implies$ two mass terms:

$$F^{23}G^2\, L_2^3 L_3^3 \quad - \text{Dirac mass} \qquad F^{33}G^2\, L_3^3 L_3^3 \quad - \text{Majorana mass}$$

- Superstrong hierarchy $\quad \sigma^8 : \sigma^4 : 1 \simeq 10^{-10} : 10^{-5} : 1 \;\; !$

- $$\mathcal{L}_Y^{\text{eff}} = G_{\alpha\beta}\left(\psi^\alpha H \psi^\beta\right) \;+\; A_{\alpha\beta}\left(\psi^\alpha H_A \psi^\beta\right) \;+$$
$$\frac{1}{M_N}(G^2)_{\alpha\beta}\left(\psi^\alpha H^\dagger \tilde{H}^\dagger \psi^\beta\right)$$

Generation structure is now fixed

Spontaneous symmetry breaking of $SO(3)_g \times \mathcal{P}_g$

Can it lead to $\langle \chi \rangle \sim \begin{pmatrix} \sigma^4 & 0 & 0 \\ 0 & \sigma^2 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ and simultaneously to

$$\mathrm{i}\langle \xi \rangle \sim \mathrm{i} \cdot \begin{pmatrix} 0 & \sigma & -\sigma \\ -\sigma & 0 & \frac{1}{2} \\ \sigma & -\frac{1}{2} & 0 \end{pmatrix} \quad ??$$

$$\mathcal{L}_\Phi = \frac{1}{2} \mathrm{Tr} \left| \partial_\mu \Phi - \mathrm{i} e [B_\mu, \Phi] \right|^2 - V_{SO(3)}(\Phi)$$

$$B_\mu = B_\mu^i t^i \qquad t^i : \; SO(3)_g \text{ generators } i = 1, 2, 3$$

$SO(3)_g$ invariance allows to choose χ diagonal

$$\Phi = \begin{pmatrix} \chi_1 & 0 & 0 \\ 0 & \chi_2 & 0 \\ 0 & 0 & \chi_3 \end{pmatrix} + \frac{\mathrm{i}}{\sqrt{2}} \begin{pmatrix} 0 & \xi_3 & -\xi_2 \\ -\xi_3 & 0 & \xi_1 \\ \xi_2 & -\xi_1 & 0 \end{pmatrix}$$

$V_{SO(3)}$ can be taken such that one gets minima for

$$\mathrm{Tr}(\chi)\ , \quad \mathrm{Tr}\left(\chi - \frac{1}{3}\mathrm{Tr}\chi\right)^2 \ \text{and} \ \mathrm{Tr}(\xi^2)$$

remaining symmetry $\Longrightarrow$ subgroup of $SO(3)_g \times \mathcal{P}_g$

- Subgroup must keep χ diagonal
- Discrete symmetry

S_4? detailed study: Lindner, Hagedorn, Mohapatra'06

S_4 permutation symmetry of 4 distinct objects

$\psi \sim 3_2 =$ analogue of vector

but $[3_2 \times 3_2] \sim 3_2$ is <u>not</u> the analogue of pseudo vector

only: even permutations give $\sim 3_2 = 3_1 = 3$

- $\Longrightarrow$ <u>A_4</u> symmetry <u>works</u> $\psi \sim \mathbf{3}$ $F, F' \sim \mathbf{3}$
  χ_1, χ_2, χ_3 singlets, $\xi \sim \mathbf{3}$
- $V = V_{SO(3)} + V_{A4}$
- V can be chosen (tuned) such that $\langle \Phi \rangle = \langle \chi \rangle + \mathrm{i} \langle \xi \rangle$

  has the structure we found phenomenological **!**

A_4- invariant potential

$$R = \frac{\chi_1+\chi_2+\chi_3}{\sqrt{3}}, \quad f_1 = \frac{\chi_2-\chi_3}{\sqrt{2}}, \quad f_2 = \frac{\chi_2+\chi_3-2\chi_1}{\sqrt{6}}$$

$$\begin{aligned} V_{A4} &= \frac{9}{16}(f_1^2+f_2^2)^2 + (\xi_1^2+\xi_2^2+\xi_3^2)^2 + \frac{f_2(f_2^2-3f_1^2)R}{2\sqrt{2}} + R^4 \\ &- \frac{4}{\sqrt{3}}\xi_1\xi_2\xi_3 R s^4 - \frac{(f_1^2+f_2^2)(2+s^2-8s^4)}{(4+8s^2)} + \cdots \end{aligned}$$

For $s = 0.058$, V_{A4} has an absolute minimum at

$\chi_1 = s^4$, $\chi_2 = s^2$, $\chi_3 = 1$,

$\xi_1 = \frac{1}{2}$, $\xi_2 = s$, $\xi_3 = s$

Mass Matrices

A) Quark mass matrix
Down quarks

$\langle H_A \rangle \sim (\bar{3}, 3, 1)$

$$M_{d,D} = \begin{array}{c} \\ d \\ D \end{array} \begin{array}{c} \begin{array}{cc} \hat{d} & \hat{D} \end{array} \\ \begin{pmatrix} m_b^0 G + h_2^2 A\ , & h_3^2 A \\ h_2^3 A\ , & e_3^3 G \end{pmatrix} \end{array},$$

$$h_2^2 = \langle H_A \rangle_2^2$$
$$h_3^2 = \langle H_A \rangle_3^2 = h_0 / x_h$$
$$h_2^3 = \langle H_A \rangle_2^3 = x_h \sigma^3 e_3^3$$

$5 \rightarrow 3$ parameters $\quad$ 7 observables

$$6 \times 6 \quad \Longrightarrow \quad 3 \times 3$$

- $M_d = m_b^0 G + h_2^2 A - h_0 \sigma^3 A G^{-1} A$

Mass Matrices

B) Charged lepton mass matrix $\langle H_A \rangle \sim (\bar{3}, \bar{6}, 1)$

$$M_{e,E} = \begin{matrix} & e^+ & E^+ \\ \begin{matrix} e^- \\ E^- \end{matrix} & \multicolumn{2}{c}{\begin{pmatrix} -m_\tau^0 G - g_2^2 A \,, & g_2^3 A \\ -g_3^2 A \,, & -e_3^3 G \end{pmatrix}} \end{matrix} ,$$

$$g_2^2 = \langle H_A \rangle^{2(1,3)}$$
$$g_3^2 = \langle H_A \rangle^{2(1,2)} = g_0 / x_g$$
$$g_2^3 = \langle H_A \rangle^{3(1,3)} = x_g \sigma^3 e_3^3$$

$$6 \times 6 \qquad \Longrightarrow \qquad 3 \times 3$$

- $M_e = -m_\tau^0 G - g_2^2 A - g_0 \sigma^3 A G^{-1} A$

3 parameters $\rightarrow$ 3 masses + 4 not obs. mixing angles

$$g_2^2 \approx h_2^2 \,, \quad g_3^2 \approx h_3^2 \,, \quad g_2^3 \approx h_2^3$$

Mass Matrices

C) Neutral lepton mass matrix 15×15

$$
M_L = \begin{array}{c} \\ L_3^2 \\ L_2^3 \\ L_3^3 \\ L_1^1 \\ L_2^2 \end{array}
\begin{array}{c} \begin{array}{ccccc} L_3^2 & L_2^3 & L_3^3 & L_1^1 & L_2^2 \end{array} \\
\begin{pmatrix}
0 & -e_1^1 G & 0 & -\,g_2^3 A & 0 \\
-e_1^1 G & 0 & M_1 & 0 & 0 \\
0 & M_1^T & M_2 & 0 & e_1^1 G \\
-g_2^3 A^T & 0 & 0 & 0 & e_3^3 G \\
0 & 0 & e_1^1 G & e_3^3 G & 0
\end{pmatrix} \end{array}
$$

$$
M_1 = F^{\{2,3\}} G^2 + F_A A \,, \qquad M_2 = F^{\{3,3\}} G^2 \,,
$$

New element: $F_A = \langle H_A \rangle_{\{3,3\}1}$, $\quad F^{\{3,3\}} \neq 0 \to L_3^3$ Majorana

$15 \times 15 \quad \Longrightarrow \quad 3 \times 3$ 'multiple' see-saw

(Zurab's talk)

$$m_\nu \simeq -\frac{m_t^2}{(F^{\{2,3\}})^2}\left(F^{\{3,3\}}\mathbf{1} + F^{\{2,3\}}\frac{g_2^3}{e_3^3}(A\frac{1}{G} + \frac{1}{G}A^T)\right)$$

$$+\text{corrections } (F_A, (g_2^3/e_3^3)^2)$$

$\Longrightarrow$ bimaximal neutrino mixing $+$ important corrections
diagonalization of 15 x 15 matrix

Results: Neutrinos

- **i)** Inverted hierarchy $(m_2^2 - m_1^2)/(m_2^2 - m_3^2) \simeq \frac{\sigma}{\sqrt{2}} \simeq 0.04$
- **ii)** $F^{\{2,3\}} \approx 2 \cdot 10^{13}$ GeV using $m_2^2 - m_3^2 \simeq 0.0024\ (\mathrm{eV})^2$

suggests $e_3^3 \simeq M_I \simeq F^{\{2,3\}}$ $g_1(M_I) = g_2(M_I)$ $(SU(3))^3$

Results: Neutrinos

- Fixes all masses of heavy fermions

Heavy neutrino spectrum:

≈ 2.7 TeV, 2.9 TeV $\quad (L^3_2)_1, (L^3_3)_1$, 4 states: $\approx 2.5 \cdot 10^8$ GeV,

2 states: $\approx 7 \cdot 10^{10}$ GeV, 4 states: $\approx 10^{13}$ GeV,

- $\theta_{13} \simeq 3^o \;, \qquad \theta_{12} \simeq 31^o \;, \qquad \theta_{23} \simeq 43^o$

  $-F_A \approx 10^5 \; \mathrm{GeV}$ needed to get $m_2 > m_1$

- $\langle m \rangle_{ee} \simeq m_t^2 \frac{F^{\{3,3\}}}{F^{\{2,3\}}} \approx 0.07$ eV, $\quad m_\nu \approx 0.1$ eV

Results: Charged leptons

Fit mass m_τ , $g_2^2 = 0.166$ GeV $g_0 = 1.87$ GeV

	$m_\tau = 1.75\ \mathrm{GeV}$	$m_\mu = 103\ \mathrm{MeV}$	$m_e = 0.44\ \mathrm{MeV}$
exp.	1.75	102.8	0.487

$|V_{e\mu}| = 0.085$ $|V_{\mu\tau}| = 0.083$ $|V_{e\tau}| = 0.006$

$\alpha_e = 102^o$ $\beta_e = 58^o$ $\gamma_e = 20^o$

Results: Quarks

Fit mass m_b , $h_2^2 \leq -0.214$ GeV $h_0 = 0.97$ GeV

$$m_b = 2.89 \text{ GeV} \qquad m_s = 54 \text{ MeV} \qquad m_d = 2.7 \text{ MeV}$$

exp: 2.89 ± 0.03 62 ± 12 3.3 ± 0.7

$$|V_{cd}| = 0.228 \qquad |V_{cb}| = 0.041 \qquad |V_{ub}| = 0.0042$$

$$\alpha = 100^o \qquad \beta = 25^o \qquad \gamma = 55^o$$

too large

Light D quark for $e_3^3 = F^{23} = 2.2 \cdot 10^{13}$ GeV

$m_{D_1} \simeq 250$ TeV

Summary

$$E_6 \times SO(3)_g \times P_g$$

all fermions in $(27, 3)$ representation
scalar fields in $(1, 3 \times 3)$ representation

$$\phi(x) = \chi(x) + i\,\xi(x) \qquad \text{hermitian } 3 \times 3 \text{ matrix}$$

- remaining symmetry after diagonalization of $\chi(x)$: A_4
- A_4 invariant potentials allow spontaneous symmetry breaking

$\langle\chi\rangle$ generation hierarchy, $i\ \langle\xi\rangle$ CP violating mixing matrix up quark masses $\langle H\rangle\,\langle\chi\rangle$ remain unmixed

down quarks/
charged leptons: $\langle H\rangle\,\langle\chi\rangle+\langle H_A\rangle\, i\,\langle\xi\rangle$ U-spin mixing

$\hat{\nu}=L_2^3, L_3^3$ $\langle H^\dagger\rangle\,\langle\tilde{H}^\dagger\rangle\left(\langle\chi\rangle^2+\langle H_A\rangle\, i\,\langle\xi\rangle\right)$ super hierarchy

- scale of heavy states: from $\Delta m^2_{\rm Atm.}$ of light neutrinos
identical to M_I $g_1(M_I)=g_2(M_I)$ spectrum fixed
lightest of heavy neutrinos: ≈ 2.8 TeV

- intimate connection of charged fermions with light and heavy neutrinos

- light neutrinos: inverted hierarchy

limits: $\langle\xi\rangle \rightarrow 0 \quad \Rightarrow$ degeneracy

$\langle\xi\rangle \neq 0$, $F_3^3/F_2^3 \rightarrow 0$, $F_A \rightarrow 0 \quad \Rightarrow$ bilarge mixing

- good description of all fermion masses and mixings by few parameters

Proton decay is sufficiently suppressed

Symmetry breaking of Higgs fields H and H_A not yet understood.

Hierarchy problem persists.