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**Non trivial Quark-Lepton Complementarity
from Grand Unified Theory:
Phenomenology and Theoretical consequences**

Marco PICARIELLO
*INFN
Lecce, Italy*

Non trivial Quark-Lepton Complementarity from Grand Unified Theory: Phenomenology and Theoretical consequences

Marco Picariello

Chauhan, Pulido, Torrente-Lujan

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Università degli Studi di Lecce - Dipartimento di Fisica

Istituto Nazionale di Fisica Nucleare - Sezione di Lecce

Instituto Superior Técnico - Centro de Física Teórica das Partículas

Non trivial Quark-lepton complementarity

- Phenomenology and Non trivial quark-lepton complementarity
- Prediction for θ_{13}^{PMNS}
- Prediction for neutrino CP violating parameters
- The role of the complementarity in grand unified theory with flavor symmetry (at page 17)
- Predictions in SUSY: $\mu \rightarrow e\gamma$

Non trivial Quark-lepton complementarity

The disparity that nature indicates between quark and lepton mixing angles has been viewed in terms of a 'Quark-Lepton **complementarity**'

$$\underline{\theta_{12}^{PMNS} + \theta_{12}^{CKM} \simeq 45^\circ}; \quad \underline{\theta_{23}^{PMNS} + \theta_{23}^{CKM} \simeq 45^\circ}.$$

A detailed analysis shows that the correlation matrix

$$\underline{V_M = U_{CKM} \Omega U_{PMNS}}$$

is phenomenologically compatible with a tribimaximal pattern, and only marginally with a bimaximal pattern.

From actual experimental evidences a non trivial Quark-Lepton **complementarity** arises.

The clear non trivial structure fo V_M and gauge coupling unification allow us to obtain in a straightforward way constraints on the high energy spectrum too.

Quark-lepton complementarity with lepton and quark mixing data predict $\theta_{13}^{PMNS} = (9_{-2}^{+1})^\circ$

The **complementarity** between the quark and lepton mixing matrices is shown to provide a robust **prediction** for the neutrino mixing angle θ_{13}^{PMNS} .

We obtain this **prediction** by first showing that the matrix V_M , product of the CKM and PMNS mixing matrices, may have a zero (1,3) entry which is favored by experimental data.

Hence models with bimaximal or tribimaximal forms of the correlation matrix V_M are quite possible. Any theoretical model with a vanishing (1,3) entry of V_M that is in agreement with quark data, solar, and atmospheric mixing angle leads to $\theta_{13}^{PMNS} = (9_{-2}^{+1})^\circ$. This value is consistent with the present 90% CL experimental upper limit.

Quark-lepton complementarity with lepton and quark mixing data predict $\theta_{13}^{PMNS} = (9_{-2}^{+1})^\circ$ (cont'd)

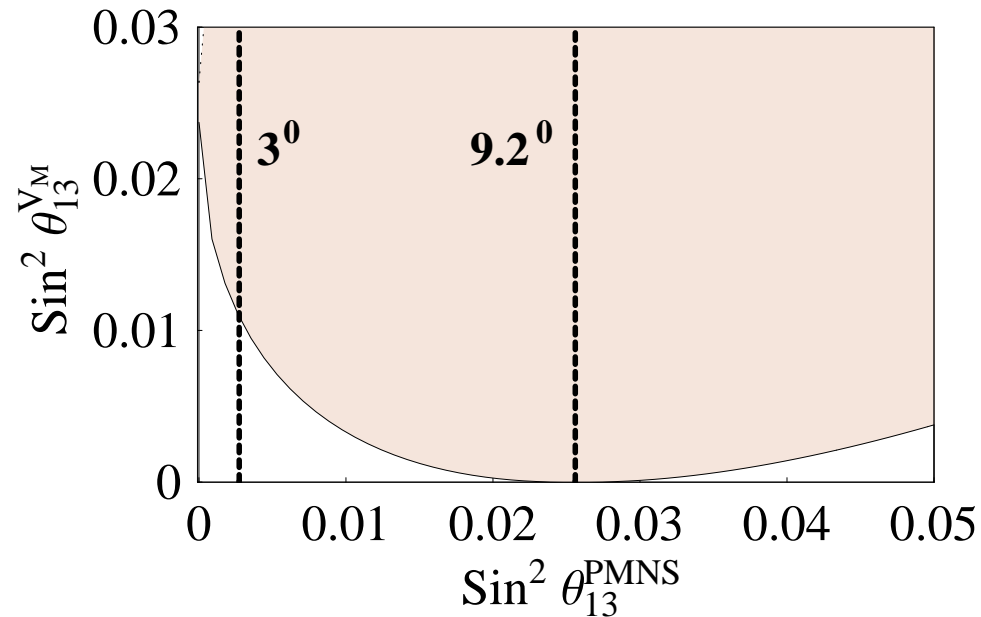


Figure 1: The minimum value allowed for $\sin^2 \theta_{13}^{\text{VM}}$ as a function of $\sin^2 \theta_{13}^{\text{PMNS}}$. All the other CKM and PMNS mixing parameters are fixed at their best fit points. We also report the values of $\theta_{13}^{\text{PMNS}} = 3.0^\circ$ and 9.2° .

Quark-lepton complementarity with lepton and quark mixing data predict $\theta_{13}^{PMNS} = (9_{-2}^{+1})^\circ$ (cont'd)

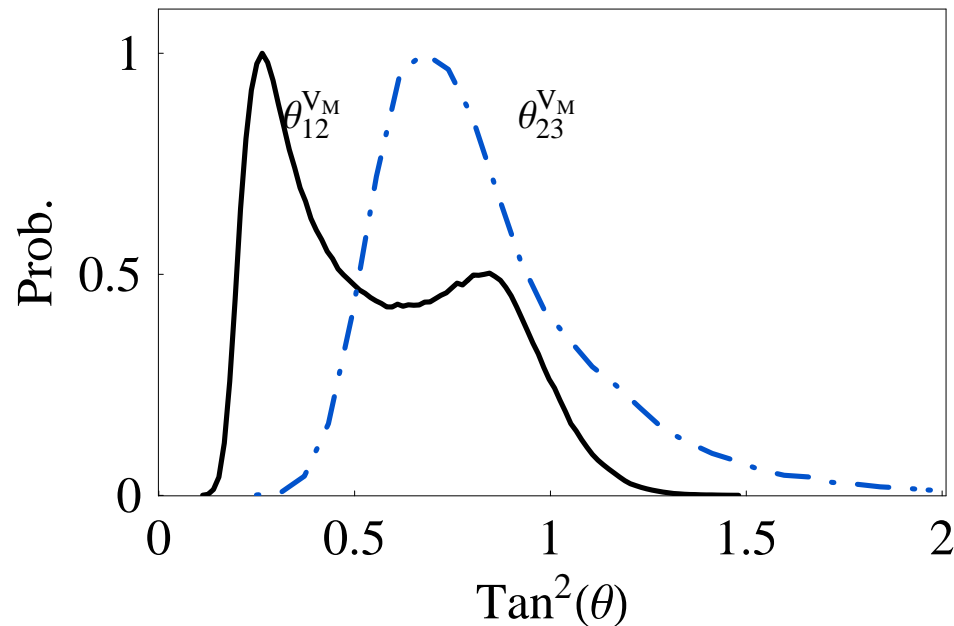


Figure 2: The distributions, normalized to one at the maximum, of $\tan^2 \theta_{12}^{V_M}$ (continuous), and $\tan^2 \theta_{23}^{V_M}$ (dot-dashed) obtained from the definition of the correlation mixing matrix V_M by using a Monte Carlo simulation of all the experimental data.

Quark-lepton complementarity with lepton and quark mixing data predict $\theta_{13}^{PMNS} = (9_{-2}^{+1})^\circ$ (cont'd)

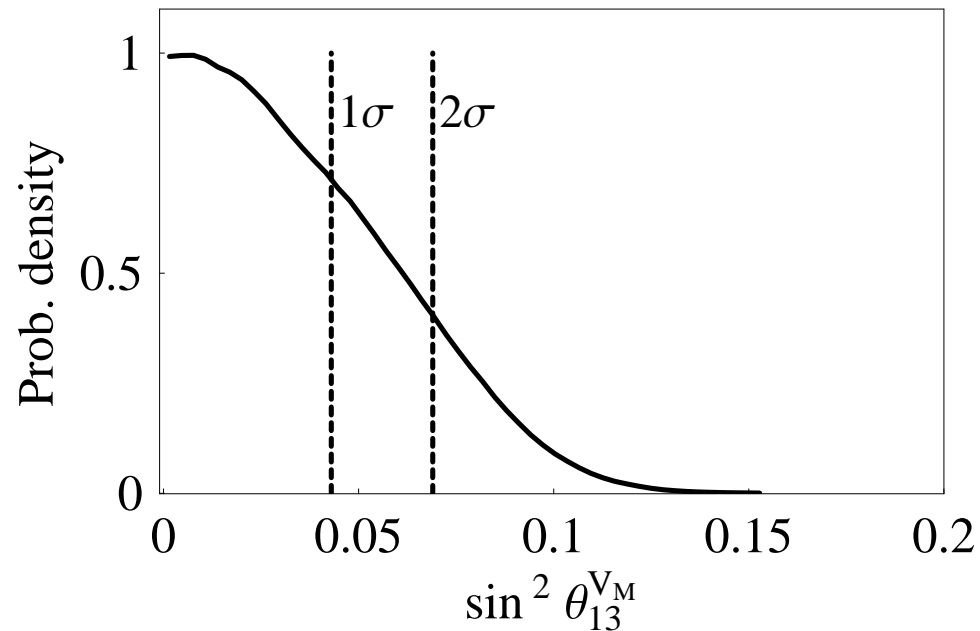


Figure 3: The distribution, normalized to one at the maximum, of $\sin^2 \theta_{13}^{V_M}$ obtained from the definition of the correlation mixing matrix V_M by using a Monte Carlo simulation of all the experimental data. We also plot the 1σ and the 2σ lines.

Quark-lepton complementarity with lepton and quark mixing data predict $\theta_{13}^{PMNS} = (9_{-2}^{+1})^\circ$ (cont'd)

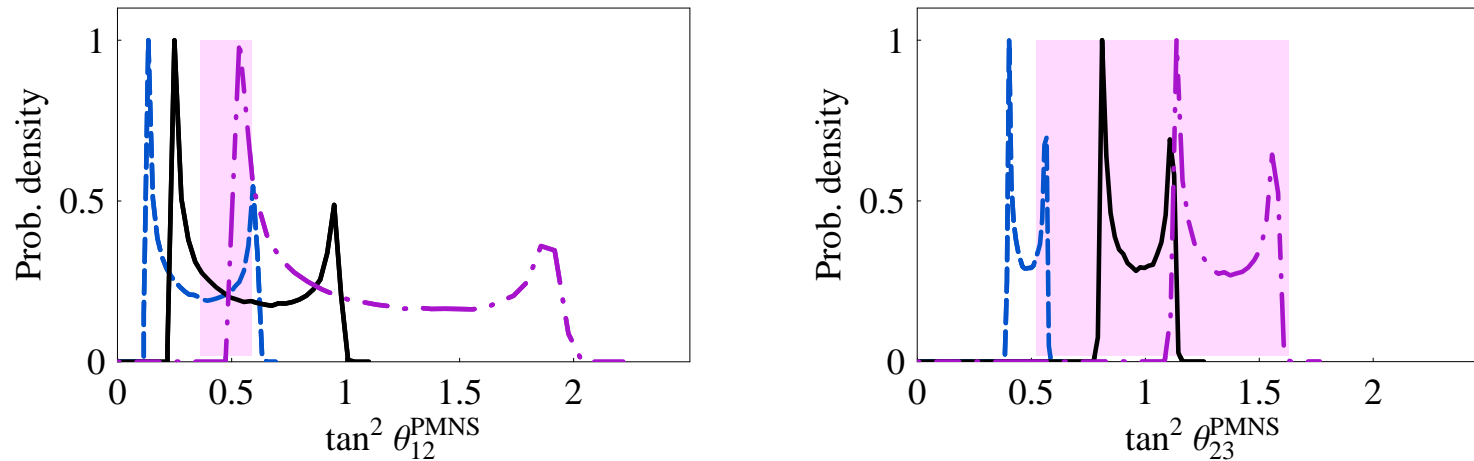


Figure 4: The distribution of $\tan^2 \theta_{12}^{PMNS}$ (left), and $\tan^2 \theta_{23}^{PMNS}$ (right) for the CKM experimental data and for values of the correlation matrix V_M respectively given by (left) $\tan^2 \theta_{12}^{V_M} = 0.3$ (dashed), 0.5 (continuous), 1.0 (dot-dashed), $\tan^2 \theta_{23}^{V_M} = 1.0$, and $\sin^2 \theta_{13}^{V_M} = 0$; (right) $\tan^2 \theta_{23}^{V_M} = 0.5$ (dashed), 1.0 (continuous), 1.4 (dot-dashed), $\tan^2 \theta_{12}^{V_M} = 0.5$, $\sin^2 \theta_{13}^{V_M} = 0$; The shaded areas represent the experimentally allowed regions for each case.

Quark-lepton complementarity with lepton and quark mixing data predict $\theta_{13}^{PMNS} = (9_{-2}^{+1})^\circ$ (cont'd)

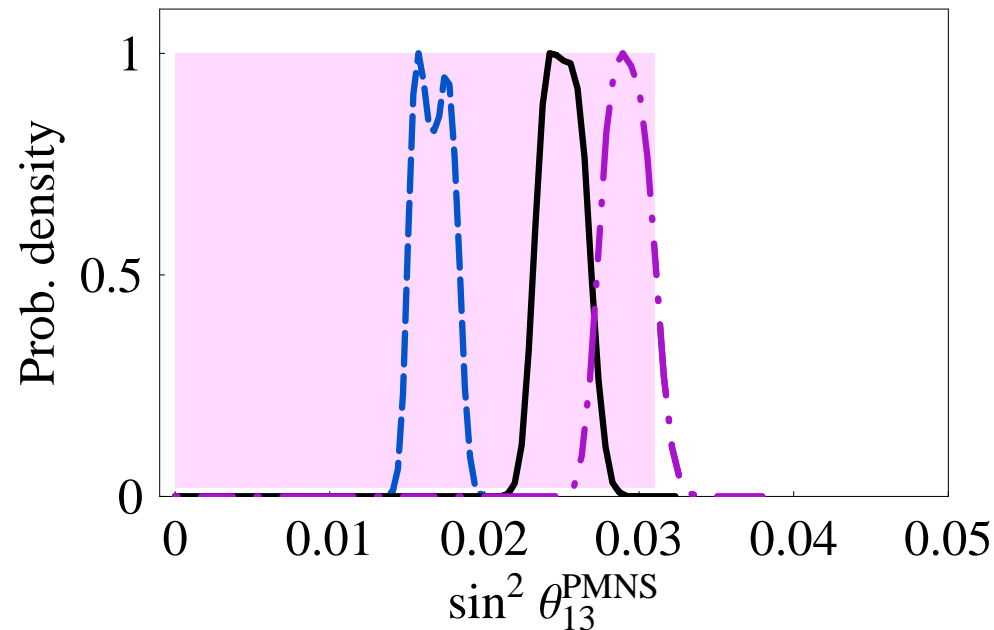


Figure 5: The distribution of $\sin^2 \theta_{13}^{PMNS}$ for values of the correlation matrix V_M given by $\tan^2 \theta_{12}^{V_M} = 0.5$, $\sin^2 \theta_{13}^{V_M} = 0$, $\tan^2 \theta_{23}^{V_M} = 0.5$ (dashed), 1.0 (continuous), 1.4 (dot-dashed). The shaded area represent the experimentally allowed region.

Quark-lepton complementarity with lepton and quark mixing data predict $\theta_{13}^{PMNS} = (9_{-2}^{+1})^\circ$ (cont'd)

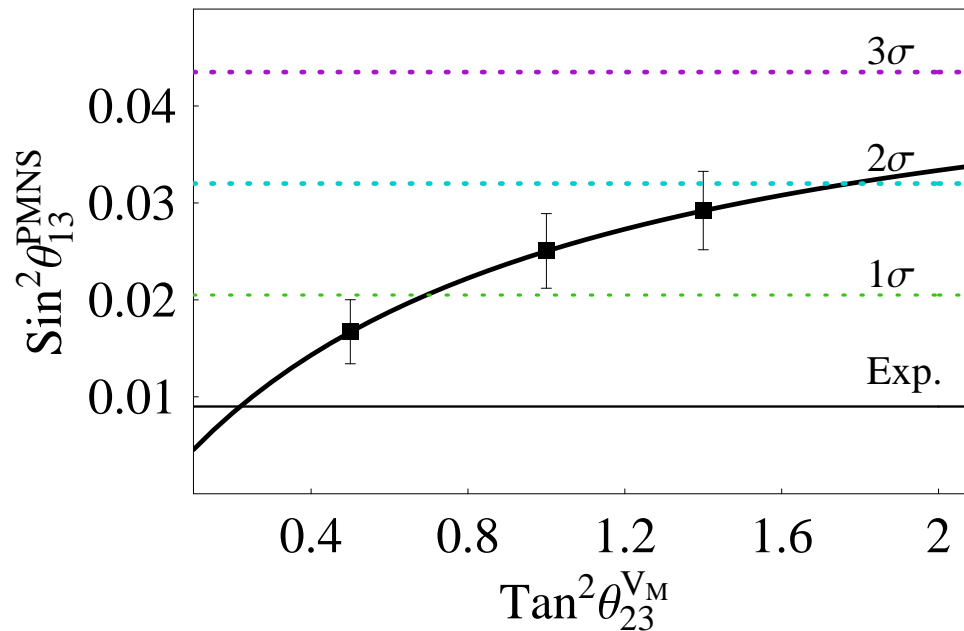


Figure 6: The allowed values for $\sin^2 \theta_{13}^{PMNS}$ as a function of $\tan^2 \theta_{23}^{VM}$ under the assumption that $\sin^2 \theta_{13}^{VM} = 0$. We report the central and 3 σ values obtained from fig.5, and the approximate analytical dependence. We also plot the experimental value.

Neutrino CP violating parameters from non trivial quark-lepton complementarity(cont'd)

There are two kind of invariants parameterizing CP violating effect

- The Jarlskog invariant J that parametrizes the effects related to the Dirac phase

$$\begin{aligned} J &= \text{Im}\{U_{\nu_e\nu_1}U_{\nu_\mu\nu_2}U_{\nu_e\nu_2}^*U_{\nu_\mu\nu_1}^*\} \\ &= \frac{1}{8} \sin 2\theta_{12} \sin 2\theta_{23} \sin 2\theta_{13} \cos \theta_{13} \sin \phi. \end{aligned}$$

- the two invariants S_1 and S_2 that parametrize the effects related to the Majorana phases

$$\begin{aligned} S_1 &= \text{Im}\{U_{\nu_e\nu_1}U_{\nu_e\nu_3}^*\} = \frac{1}{2} \cos \theta_{12} \sin 2\theta_{13} \sin(\phi + \phi_1) \\ S_2 &= \text{Im}\{U_{\nu_e\nu_2}U_{\nu_e\nu_3}^*\} = \frac{1}{2} \sin \theta_{12} \sin 2\theta_{13} \sin(\phi + \phi_2) \end{aligned}$$

Neutrino CP violating parameters from non trivial quark-lepton complementarity(cont'd)

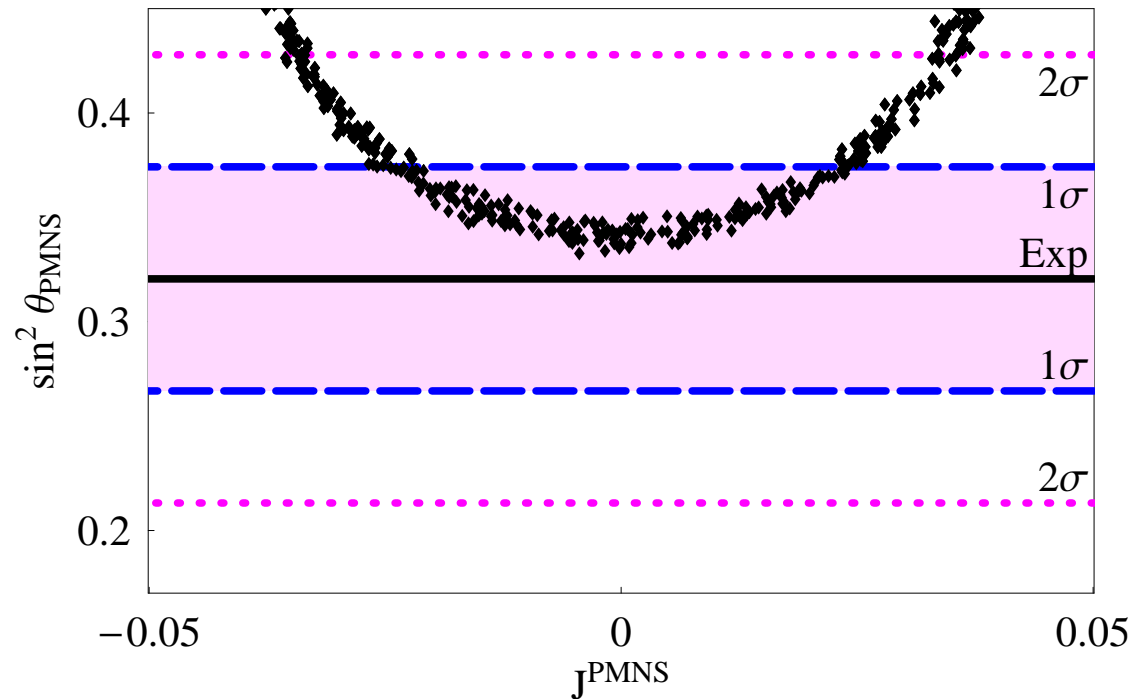


Figure 7: The correlation between the Dirac CP violating parameter \underline{J} and $\sin^2 \theta_{12}^{PMNS}$, for V_M bimaximal. We also plot the experimental central value, the 1σ , and the 2σ for $\sin^2 \theta_{12}^{PMNS}$.

Neutrino CP violating parameters from non trivial quark-lepton complementarity(cont'd)

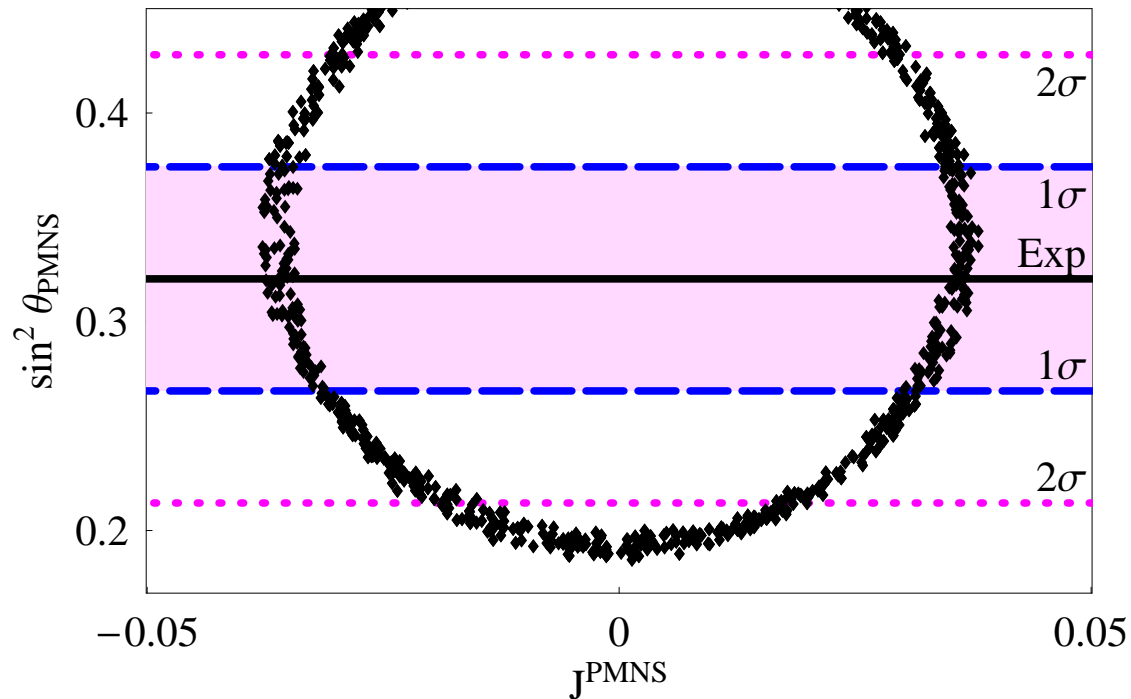


Figure 8: The correlation between the Dirac CP violating parameter \underline{J} and $\sin^2 \theta_{12}^{PMNS}$, for V_M tribimaximal. We also plot the experimental central value, the 1σ , and the 2σ for $\sin^2 \theta_{12}^{PMNS}$.

Neutrino CP violating parameters from non trivial quark-lepton complementarity(cont'd)

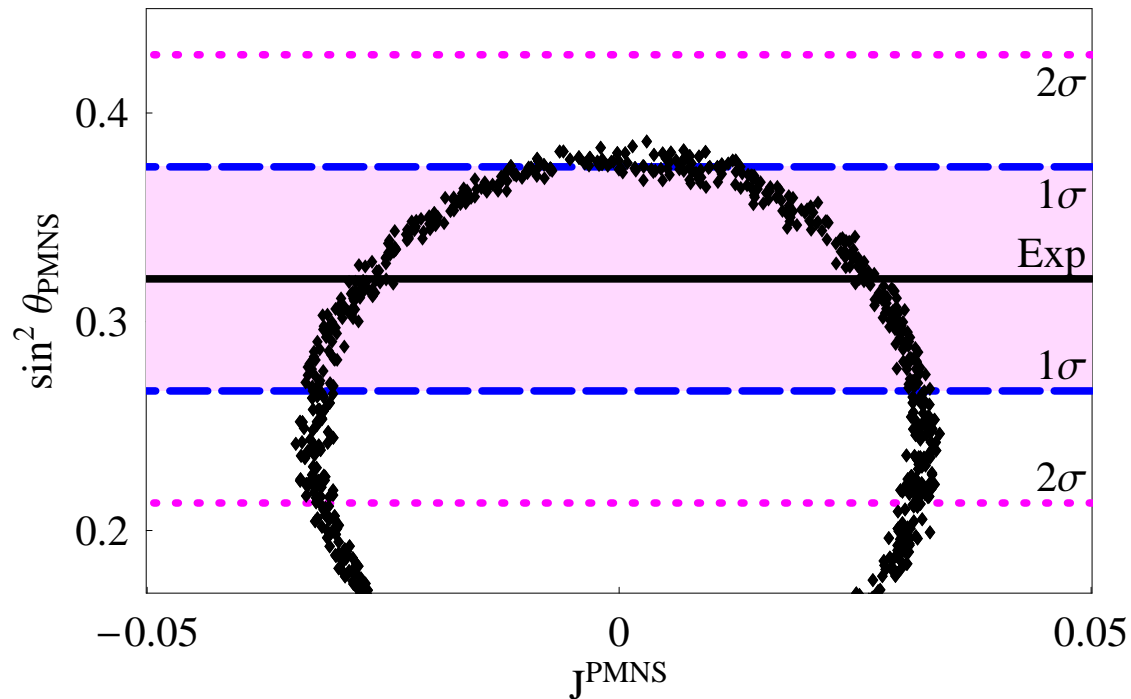


Figure 9: The correlation between the Dirac CP violating parameter \underline{J} and $\sin^2 \theta_{12}^{PMNS}$, for $\underline{V_M}$ with $\tan^2 \theta_{12}^{V_M} = 0.3$. We fixed $\tan^2 \theta_{23}^{V_M} = 1$ and $\sin^2 \theta_{13}^{V_M} = 0$.

Neutrino CP violating parameters from non trivial quark-lepton complementarity(cont'd)

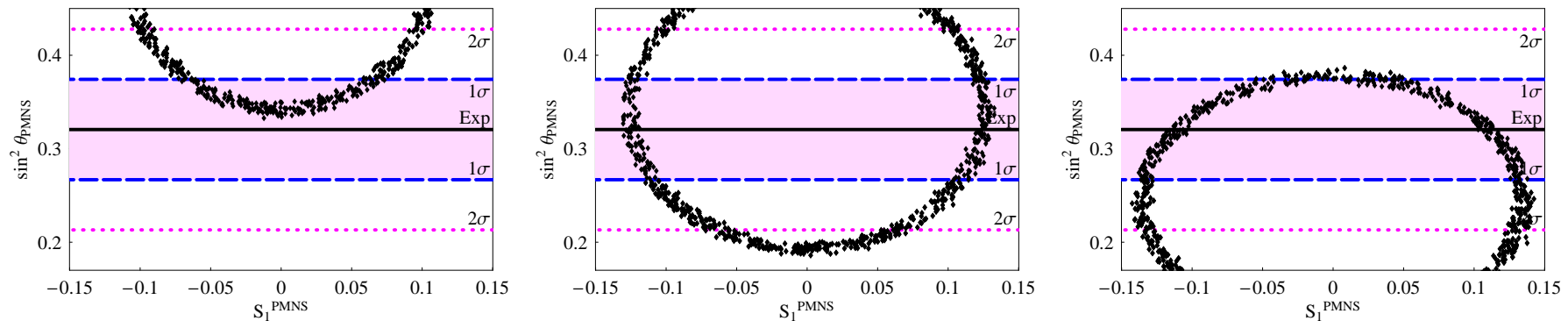


Figure 10: The correlation between the Majorana CP violating parameter $\underline{S_1}$ and $\sin^2 \theta_{12}^{PMNS}$, for V_M bimaximal, tribimaximal, and V_M with $\tan^2 \theta_{12}^{V_M} = 0.3$. We fixed $\tan^2 \theta_{23}^{V_M} = 1$ and $\sin^2 \theta_{13}^{V_M} = 0$.

Predictions in SUSY from non trivial quark-lepton complementarity, GUT and flavor symmetry(cont'd)

$$\underline{l_i \rightarrow l_j \gamma}$$

$$BR(l_i \rightarrow l_j \gamma) \propto \frac{\Gamma(l_i \rightarrow e \nu \nu)}{\Gamma(l_i)} \frac{\alpha^3}{G_f m_s^8 v_u^4} \tan^2 \beta \left(\frac{3m_0 + A_0}{8\pi^2} \right)^2 \times \left| \left(\tilde{M}_D L \tilde{M}_D^\dagger \right)_{ij} \right|^2$$

m_0 = scalar mass, A_0 = trilinear coupling,

$m_s^8 \approx 0.5 m_0^2 M_{1/2}^2 (m_0^2 + 0.6 M_{1/2}^2)^2$, $M_{1/2}$ = gaugino mass.

The experimental limit for the branching ratio of $\mu \rightarrow e \gamma$ is 1.2×10^{-11} at 90% of confidence level and it could go down to 10^{-14} as proposed by MEG collaboration.

Predictions in SUSY from non trivial quark-lepton complementarity, GUT and flavor symmetry(cont'd)

$$\mathcal{L} = \bar{\nu}_L Y_D \nu_R H + \nu_R^T C M_R \nu_R + \bar{l}_L Y_l l_R H + \bar{\nu}_L \mathcal{W} l_L$$

We introduce the following definitions

$$l'_R = V_l^\dagger l_R \quad \nu'_R = V_R^T \nu_R \quad l'_L = U_l^\dagger l_L \quad \nu'_L = U_l^\dagger \nu_L$$

Consequently we have

$$l_R = V_l l'_R, \quad \nu_R = V_R^* \nu'_R, \quad l_L = U_l l'_L \quad \text{and} \\ \nu_R^T = (\nu'_R)^T V_R^\dagger, \quad \bar{l}_L = \bar{l}'_L U_l^\dagger, \quad \bar{\nu}_L = \bar{\nu}'_L U_l^\dagger.$$

In this primed base we get

$$\mathcal{L} = \bar{\nu}'_L U_l^\dagger M_D V_R^* \nu'_R + (\nu'_R)^T C M_R^\Delta \nu'_R + \bar{l}'_L M_l^\Delta l'_R + \bar{\nu}'_L \mathcal{W} l'_L$$

Predictions in SUSY from non trivial quark-lepton complementarity, GUT and flavor symmetry(cont'd)

$$\mathcal{L} = \bar{\nu}_L Y_D \nu_R H + \nu_R^T C M_R \nu_R + \bar{l}_L Y_l l_R H + \bar{\nu}_L \tilde{W} l_L$$

We introduce the following definitions

$$l'_R = V_l^\dagger l_R \quad \nu'_R = V_R^T \nu_R \quad l'_L = U_l^\dagger l_L \quad \nu'_L = U_l^\dagger \nu_L$$

Consequently we have

$$l_R = V_l l'_R, \quad \nu_R = V_R^* \nu'_R, \quad l_L = U_l l'_L \quad \text{and} \\ \nu_R^T = (\nu'_R)^T V_R^\dagger, \quad \bar{l}_L = \bar{l}'_L U_l^\dagger, \quad \bar{\nu}_L = \bar{\nu}'_L U_l^\dagger.$$

In this primed base we get

$$\mathcal{L} = \bar{\nu}'_L \tilde{M}_D \nu'_R + (\nu'_R)^T C M_R^\Delta \nu'_R + \bar{l}'_L M_l^\Delta l'_R + \bar{\nu}'_L \tilde{W} l'_L$$

Predictions in SUSY from non trivial quark-lepton complementarity, GUT and flavor symmetry(cont'd)

We want now to related this \tilde{M}_D matrix to the CKM mixing matrix by using the non trivial Quark-Lepton **complementarity**.

First of all we rewrite this matrix as

$$\tilde{M}_D = U_l^\dagger M_D V_R^* = U_l^\dagger U_0 M_D^\Delta V_0^\dagger V_R^*$$

then we assume **GUT** and a **flavor symmetry**

$$Y_u \simeq Y_u^T \simeq Y_D \rightarrow U_u = U_0 \quad Y_d \simeq Y_d^T \simeq Y_l \rightarrow U_d = U_l.$$

and from $U_{CKM} = U_u^\dagger U_d = U_0^\dagger U_l$ we obtain

$$\underline{\tilde{M}_D} = \underline{U_{CKM}^\dagger M_D^\Delta V_0^\dagger V_R^*}$$

$$U_\nu \equiv U_0 V_M \rightarrow U_{PMNS} \equiv U_l^\dagger U_\nu = U_l^\dagger U_0 V_M = U_{CKM}^\dagger V_M$$

Predictions in SUSY from non trivial quark-lepton complementarity, GUT and flavor symmetry(cont'd)

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and from $\underline{U_{CKM} = U_u^\dagger U_d = U_0^\dagger U_l}$ we obtain

$$\underline{\tilde{M}_D} = \underline{U_{CKM}^\dagger M_D^\Delta V_0^\dagger V_R^\star}$$

$$U_\nu \equiv U_0 V_M \rightarrow U_{PMNS} \equiv U_l^\dagger U_\nu = U_l^\dagger U_0 V_M = U_{CKM}^\dagger V_M$$

Predictions in SUSY from non trivial quark-lepton complementarity, GUT and flavor symmetry(cont'd)

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$$\underline{\tilde{M}_D} = \underline{U_{CKM}^\dagger M_D^\Delta V_0^\dagger V_R^\star}$$

$$\underline{U_\nu \equiv U_0 V_M \rightarrow U_{PMNS} \equiv U_l^\dagger U_\nu = U_l^\dagger U_0 V_M = U_{CKM}^\dagger V_M}$$

Predictions in SUSY from non trivial quark-lepton complementarity, GUT and flavor symmetry(cont'd)

Then we notice that the matrix $V_0^\dagger V_R^*$ is related to the diagonal low energy neutrino mass matrix m_{low}^Δ and to V_M .

In **see-saw of type I** we have

$$\begin{aligned}
 m_{low}^\Delta &= U_{PMNS}^\dagger M_D \frac{1}{M_R} M_D^T U_{PMNS}^* \\
 &= U_{PMNS}^\dagger U_{CKM}^\dagger M_D^\Delta V_0^\dagger V_R^* \frac{1}{M_R^\Delta} V_R^\dagger V_0^* M_D^\Delta U_{CKM}^* U_{PMNS}^* \\
 V_M m_{low}^\Delta V_M^T &= M_D^\Delta V_0^\dagger V_R^* \frac{1}{M_R^\Delta} V_R^T V_0^* M_D^\Delta
 \end{aligned}$$

We multiply on the left and on the right both sides by $1/M_D^\Delta$ and we get

$$\underline{V_0^\dagger V_R^* \frac{1}{M_R^\Delta} V_R^T V_0^* = \frac{1}{M_D^\Delta} V_M m_{low}^\Delta V_M^T \frac{1}{M_D^\Delta}}$$

Predictions in SUSY from non trivial quark-lepton complementarity, GUT and flavor symmetry(cont'd)

One can extract $V_0^\dagger V_R^*$ by making the *square root* of the matrices

$$V_0^\dagger V_R^* \sqrt{\frac{1}{M_R^\Delta}} = \frac{1}{M_D^\Delta} V_M \sqrt{m_{low}^\Delta} R^T$$

where $R^T R = \mathbf{1}$, and one obtains that $V_0^\dagger V_R^* = \frac{1}{M_D^\Delta} V_M \sqrt{m_{low}^\Delta} R^T \sqrt{M_R^\Delta}$

Finally one concludes that

$$\begin{aligned} \tilde{M}_D &= U_l^\dagger U_0 M_D^\Delta \frac{1}{M_D^\Delta} V_M \sqrt{m_{low}^\Delta} R^T \sqrt{M_R^\Delta} \\ &= U_{PMNS} \sqrt{m_{low}^\Delta} R^T \sqrt{M_R^\Delta} \end{aligned}$$

Any information from V_M is hidden into the R matrix.

However $V_0^\dagger V_R^*$ is unequivocally fixed and then the R matrix, once we know the eigenvalues of the Dirac neutrino mass matrix,

Predictions in SUSY from non trivial quark-lepton complementarity, GUT and flavor symmetry(cont'd)

Determination of $V_0^\dagger V_R^$ and M_R^Δ :* $V_0^\dagger V_R^* \frac{1}{M_R^\Delta} V_R^T V_0^* = \frac{1}{M_D^\Delta} V_M m_{low}^\Delta V_M^T \frac{1}{M_D^\Delta}$

The experimental constraint says us that $(V_M)_{13}$ is zero:

$$V_M = \begin{pmatrix} \cos \theta_{12} & \sin \theta_{12} & 0 \\ -\sin \theta_{12} \cos \theta_{23} & \cos \theta_{12} \cos \theta_{23} & \sin \theta_{23} \\ \sin \theta_{12} \sin \theta_{23} & -\cos \theta_{12} \sin \theta_{23} & \cos \theta_{23} \end{pmatrix}$$

and the allowed ranges for $\theta_{12}^{V_M}$ and $\theta_{23}^{V_M}$ are

$$\tan^2 \theta_{12}^{V_M} \in [0.3, 1.0] \quad \text{and} \quad \tan^2 \theta_{23}^{V_M} \in [0.5, 1.4].$$

Let us denote $m_{low}^\Delta = \{m_1, m_2, m_3\}$, and $M_D^\Delta = \{M_1, M_2, M_3\}$.

Predictions in SUSY from non trivial quark-lepton complementarity, GUT and flavor symmetry(cont'd)

We have three cases:

1) hierarchical Dirac neutrino eigenvalues

(in this case we have very hierarchical right-handed neutrino masses)

$V_0^\dagger V_R^*$ is close to the identity

2) degenerate Dirac neutrino eigenvalues, with non degenerate low energy neutrino masses

(in this case the hierarchy of the right-handed neutrino masses is close to the one of the low energy spectrum)

3) degenerate Dirac neutrino eigenvalues and low energy neutrino spectrum

(that implies right-handed neutrino close to be degenerate)

Predictions in SUSY from non trivial quark-lepton complementarity, GUT and flavor symmetry(cont'd)

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3) degenerate Dirac neutrino eigenvalues and low energy neutrino spectrum

(that implies right-handed neutrino close to be degenerate)

$V_0^\dagger V_R^*$ is close to the identity

Predictions in SUSY from non trivial quark-lepton complementarity, GUT and flavor symmetry(cont'd)

Hierarchical Dirac neutrino eigenvalues

For any V_M , the heavy neutrino spectrum is hierarchical with ratios given by

$$M_1^R : M_2^R : M_3^R = M_1^2 : M_2^2 : M_3^2 = (\lambda^{4n})^2 : (\lambda^{2n})^2 : 1$$

Moreover the mixing matrix $V_0^\dagger V_R^*$ is close to the identity.

$$V_0^\dagger V_R^* = \begin{pmatrix} 1 - \alpha^2 \lambda^{2n} / 2 & \alpha \lambda^n & \beta \lambda^{2n} \\ -\alpha \lambda^n & 1 - (\alpha^2 + \gamma^2) \lambda^{2n} & \gamma \lambda^n \\ (-\beta + \alpha \gamma) \lambda^{2n} & -\gamma \lambda^n & 1 - \gamma^2 \lambda^{2n} / 2 \end{pmatrix} + O(\lambda^{3n})$$

In this case we have very hierarchical right-handed neutrino masses

Predictions in SUSY from non trivial quark-lepton complementarity, GUT and flavor symmetry(cont'd)

$$m_\alpha = m_1 \cos^2 \theta_{12} + m_2 \sin^2 \theta_{12} + O(\lambda^{2n}) \quad (1)$$

$$m_\beta = \frac{m_1 m_2}{m_\alpha} \cos^2 \theta_{23} + m_3 \sin^2 \theta_{23} + O(\lambda^{2n})$$

$$m_\gamma = \frac{m_1 m_2 m_3}{m_\alpha m_\beta}$$

$$\alpha = -\frac{(m_1 - m_2)}{2m_\alpha} \sin(2\theta_{12}) \cos \theta_{23} + O(\lambda^{2n})$$

$$\gamma = \frac{m_1 m_2 - m_3 m_\alpha}{2m_\alpha m_\beta} \sin(2\theta_{23}) + O(\lambda^{2n})$$

$$\beta = \frac{(m_1 - m_2)}{2m_\alpha} \sin(2\theta_{12}) \sin \theta_{23} + O(\lambda^{2n}) \quad (2)$$

the number α , β , γ are of order 1 but the corresponding angles must be computed up to order λ^{6n} to obtain the right heavy neutrino masses. The parameters m_α , m_β , m_γ are of order of the common low energy neutrino mass m . Notice that the rotation angles (1, 2) and (2, 3) in $V_0^\dagger V_R^*$ are of order λ^n while the (1, 3) angle is of order λ^{2n} .

Predictions in SUSY from non trivial quark-lepton complementarity, GUT and flavor symmetry(cont'd)

$$BR(\tau \rightarrow \mu\gamma) \propto \frac{\Gamma(\tau \rightarrow \mu\nu\nu)}{\Gamma(\tau)} \frac{\alpha^3}{G_f m_s^8 v_u^4} \tan^2 \beta \left(\frac{3m_0 + A_0}{8\pi^2} \right)^2 \hat{L}^2$$

$$\left(\frac{M_3}{m_t} \right)^4 \left| m_t^2 \log \frac{m_t^2}{m_u^2} A \lambda^2 \right|^2$$

$BR(\mu \rightarrow e\gamma)$ is suppressed by a factor λ^6 with respect to $BR(\tau \rightarrow \mu\gamma)$, and by a factor λ^2 with respect to $BR(\tau \rightarrow e\gamma)$.

Our conclusions are equivalent to the one in literature for V_M bimaximal. However our discussion is more general, in fact we shown that these results do not depend on the form of the correlation matrix V_M .

Predictions in SUSY from non trivial quark-lepton complementarity, GUT and flavor symmetry(cont'd)

Degenerate M_D and non degenerate m_{low}

$$\frac{1}{M_R^\Delta} \simeq \begin{pmatrix} m_\alpha/M_3^2 & 0 & 0 \\ 0 & m_\beta/M_3^2 & 0 \\ 0 & 0 & m_\gamma/M_3^2 \end{pmatrix}$$

$$V_0^\dagger V_R^* \simeq V_M \begin{pmatrix} 1 - \alpha^2/2 & \alpha & \beta \\ -\alpha & 1 - (\alpha^2 + \gamma^2) & \gamma \\ (-\beta + \alpha\gamma) & -\gamma & 1 - \gamma^2/2 \end{pmatrix} \equiv V_M V_\epsilon$$

In this case the hierarchy of the right-handed neutrino masses is close to the one of the low energy spectrum

Predictions in SUSY from non trivial quark-lepton complementarity, GUT and flavor symmetry(cont'd)

$$\begin{aligned}
 m_\alpha &\simeq m_1 \left(1 - \frac{\delta M_1}{M_3} \left(1 + \frac{\cos(2\theta_{12})}{2} \right) + \frac{\delta M_2}{M_3} \left(-\frac{1 - \cos(2\theta_{12})}{2} - \cos(2\theta_{23}) \sin^2 \theta_{12} \right) \right) \\
 m_\beta &\simeq m_2 \left(1 - \frac{\delta M_1}{M_3} \left(1 - \frac{\cos(2\theta_{12})}{2} \right) + \frac{\delta M_2}{M_3} \left(-\frac{1 - \cos(2\theta_{12})}{2} - \cos(2\theta_{23}) \cos^2 \theta_{12} \right) \right) \\
 m_\gamma &\simeq m_3 \left(1 - \frac{\delta M_2}{M_3} (1 + \cos(2\theta_{23})) \right) \\
 \alpha &\simeq -\frac{m_1 + m_2}{4(m_1 - m_2)} \frac{2\delta M_1 - \delta M_2 - \delta M_2 \cos(2\theta_{23})}{M_3} \sin(2\theta_{12}) \\
 \gamma &\simeq \frac{m_2 + m_3}{2(m_2 - m_3)} \frac{\delta M_2}{M_3} \sin(2\theta_{23}) \cos \theta_{12} \\
 \beta &\simeq \frac{m_1 + m_3}{2(m_1 - m_3)} \frac{\delta M_2}{M_3} \sin(2\theta_{23}) \sin \theta_{12}
 \end{aligned} \tag{3}$$

the parameters m_α , m_β , m_γ are of order of the common low energy neutrino mass m .

Predictions in SUSY from non trivial quark-lepton complementarity, GUT and flavor symmetry(cont'd)

$$BR(\mu \rightarrow e\gamma) \propto \left| M_1 M_2 \log \frac{m_\beta}{m_\alpha} \cos \alpha_{12} \cos \alpha_{23} \sin \alpha_{12} \right|^2$$

$$BR(\tau \rightarrow e\gamma) \propto \left| M_1 M_3 \log \frac{m_\beta}{m_\alpha} \cos \alpha_{12} \sin \alpha_{23} \sin \alpha_{12} \right|^2$$

$$BR(\tau \rightarrow \mu\gamma) \propto \left| M_2 M_3 \cos \alpha_{23} \sin \alpha_{23} \left(\log \frac{m_\gamma}{m_\alpha} + \sin^2 \alpha_{12} \log \frac{m_\beta}{m_\alpha} \right) \right|^2$$

The ratios among them become

$$\frac{BR(\mu \rightarrow e\gamma)}{BR(\tau \rightarrow e\gamma)} \simeq \tan^2 \alpha_{23} \in [0.5, 1.4]$$

$$\frac{BR(\mu \rightarrow e\gamma)}{BR(\tau \rightarrow \mu\gamma)} \simeq \left| \frac{\cos \alpha_{12} \sin \alpha_{12}}{\sin \alpha_{23} \left(\left(\log \frac{m_\gamma}{m_\alpha} / \log \frac{m_\beta}{m_\alpha} \right) + \sin^2 \alpha_{12} \right)} \right|^2$$

Predictions in SUSY from non trivial quark-lepton complementarity, GUT and flavor symmetry(cont'd)

Degenerate M_D and Degenerate m_{low}

$$M_1^R = \frac{m}{M^2} \left(1 - \frac{\delta M_1}{M} \left(1 + \sqrt{1 - \frac{1}{3} \frac{\sqrt{\delta m_{sol}^2}/m}{\delta M_1/M} + \frac{\delta m_{sol}^2/m^2}{(\delta M_1/M)^2}} \right) \right)$$

$$M_2^R = \frac{m}{M^2} \left(1 - 2 \frac{\delta M_2}{M} + \frac{\sqrt{\delta m_{atm}^2}}{m} \right)$$

$$M_3^R = \frac{m}{M^2} \left(1 - \frac{\delta M_1}{M} \left(1 - \sqrt{1 - \frac{1}{3} \frac{\sqrt{\delta m_{sol}^2}/m}{\delta M_1/M} + \frac{\delta m_{sol}^2/m^2}{(\delta M_1/M)^2}} \right) \right)$$

The right-handed neutrino close to be degenerate and $V_0^\dagger V_R^*$ is close to the identity

Predictions in SUSY from non trivial quark-lepton complementarity, GUT and flavor symmetry(cont'd)

We get

$$BR(\mu \rightarrow e\gamma) \propto |(M_2^2 - M_1^2)|^2 \lambda^2$$

$$BR(\tau \rightarrow e\gamma) \propto |((1 - (\rho - i\eta))M_1^2 - M_2^2 + M_3^2(\rho - i\eta))|^2 (A\lambda^3)^2$$

$$BR(\tau \rightarrow \mu\gamma) \propto |(M_3^2 - M_2^2)|^2 (A\lambda^2)^2$$

and $BR(\mu \rightarrow e\gamma) : BR(\tau \rightarrow e\gamma) : BR(\tau \rightarrow \mu\gamma) = 1 : \lambda^4 : \lambda^2$.

For example, if $M_2^2 - M_1^2 \simeq \lambda^4 M_3^2$ and M_i are of order m_t , we obtain

$$BR(\mu \rightarrow e\gamma) \propto \left(\frac{M_3}{m_t}\right)^4 |m_t^4 \lambda^5|^2, \quad BR(\tau \rightarrow e\gamma) \propto \left(\frac{M_3}{m_t}\right)^4 |m_t^4 \lambda^7|^2$$

$$BR(\tau \rightarrow \mu\gamma) \propto \left(\frac{M_3}{m_t}\right)^4 |m_t^4 \lambda^6|^2$$

Conclusions

- Phenomenology and Non trivial quark-lepton **complementarity**
- **Prediction** for θ_{13}^{PMNS}
- **Prediction** for neutrino CP violating parameters
- The role of the **complementarity** in grand unified models of flavor
- **Predictions** in SUSY: $\mu \rightarrow e\gamma$

Conclusions

$l_i \rightarrow l_j \gamma$: we have three cases:

1) hierarchical Dirac neutrino eigenvalues

(very hierarchical right-handed neutrino masses, and $V_0^\dagger V_R^\star \simeq I$)

$$BR(\mu \rightarrow e\gamma) : BR(\tau \rightarrow e\gamma) : BR(\tau \rightarrow \mu\gamma) = \lambda^6 : \lambda^4 : 1 \propto M_3^4 \lambda^4 \hat{L}.$$

2) degenerate Dirac neutrino eigenvalues, with non degenerate low energy

neutrino masses (the hierarchy of the right-handed neutrino masses is

close to the one of the low energy spectrum, and $V_0^\dagger V_R^\star \simeq V_M$)

$$BR(\mu \rightarrow e\gamma) \simeq BR(\tau \rightarrow e\gamma) \simeq BR(\tau \rightarrow \mu\gamma) \propto M_3^4$$

3) degenerate Dirac neutrino eigenvalues and low energy neutrino

spectrum (right-handed neutrino close to be degenerate, and $V_0^\dagger V_R^\star \simeq I$)

$$BR(\mu \rightarrow e\gamma) : BR(\tau \rightarrow e\gamma) : BR(\tau \rightarrow \mu\gamma) = 1 : \lambda^4 : \lambda^2 \propto M_3^4 \lambda^{10}$$