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Workshop on Grand Unification and Proton Decay

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Family Unification with SO(10)

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Family Unification with SO(10)

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Workshop on Grand Unification and Proton Decay
ICTP, Trieste
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Matter Unification

in 16 of SO(10)

u_1	:	$\vert\uparrow\downarrow\uparrow\uparrow\downarrow>$
u_2	:	$\vert\uparrow\downarrow\uparrow\downarrow\uparrow>$
u_3	:	$\vert\uparrow\downarrow\downarrow\uparrow\uparrow>$
d_1	:	$\vert\downarrow\uparrow\uparrow\uparrow\downarrow>$
d_2	:	$\vert\downarrow\uparrow\uparrow\downarrow\uparrow>$
d_3	:	$\vert\downarrow\uparrow\downarrow\uparrow\uparrow>$
u_1^c	:	$\vert\downarrow\downarrow\uparrow\downarrow\downarrow>$
u_2^c	:	$\vert\downarrow\downarrow\downarrow\uparrow\downarrow>$
u_3^c	:	$\vert\downarrow\downarrow\downarrow\downarrow\uparrow>$
d_1^c	:	$\vert\uparrow\uparrow\uparrow\downarrow\downarrow>$
d_2^c	:	$\vert\uparrow\uparrow\downarrow\uparrow\downarrow>$
d_3^c	:	$\vert\uparrow\uparrow\downarrow\downarrow\uparrow>$
ν	:	$\vert\uparrow\downarrow\downarrow\downarrow\downarrow>$
e	:	$\vert\downarrow\uparrow\downarrow\downarrow\downarrow>$
e^c	:	$\vert\downarrow\downarrow\uparrow\uparrow\uparrow>$
ν^c	:	$\vert\uparrow\uparrow\uparrow\uparrow\uparrow>$

u_1	:	$\mid\uparrow\downarrow\uparrow\uparrow\downarrow>$
u_2	:	$\mid\uparrow\downarrow\uparrow\downarrow\uparrow>$
u_3	:	$\mid\uparrow\downarrow\downarrow\uparrow\uparrow>$
d_1	:	$\mid\downarrow\uparrow\uparrow\uparrow\downarrow>$
d_2	:	$\mid\downarrow\uparrow\uparrow\downarrow\uparrow>$
d_3	:	$\mid\downarrow\uparrow\downarrow\uparrow\uparrow>$
u_1^c	:	$\mid\downarrow\downarrow\uparrow\downarrow\downarrow>$
u_2^c	:	$\mid\downarrow\downarrow\downarrow\uparrow\downarrow>$
u_3^c	:	$\mid\downarrow\downarrow\downarrow\downarrow\uparrow>$
d_1^c	:	$\mid\uparrow\uparrow\uparrow\downarrow\downarrow>$
d_2^c	:	$\mid\uparrow\uparrow\downarrow\uparrow\downarrow>$
d_3^c	:	$\mid\uparrow\uparrow\downarrow\downarrow\uparrow>$
ν_e	:	$\mid\uparrow\downarrow\downarrow\downarrow\downarrow>$
e	:	$\mid\downarrow\uparrow\downarrow\downarrow\downarrow>$
e^c	:	$\mid\downarrow\downarrow\uparrow\uparrow\uparrow>$
ν_e^c	:	$\mid\uparrow\uparrow\uparrow\uparrow\uparrow>$

$+$

c_1	:	$\mid\uparrow\downarrow\uparrow\uparrow\downarrow>$
c_2	:	$\mid\uparrow\downarrow\uparrow\downarrow\uparrow>$
c_3	:	$\mid\uparrow\downarrow\downarrow\uparrow\uparrow>$
s_1	:	$\mid\downarrow\uparrow\uparrow\uparrow\downarrow>$
s_2	:	$\mid\downarrow\uparrow\uparrow\downarrow\uparrow>$
s_3	:	$\mid\downarrow\uparrow\downarrow\uparrow\uparrow>$
c_1^c	:	$\mid\downarrow\downarrow\uparrow\downarrow\downarrow>$
c_2^c	:	$\mid\downarrow\downarrow\downarrow\uparrow\downarrow>$
c_3^c	:	$\mid\downarrow\downarrow\downarrow\downarrow\uparrow>$
s_1^c	:	$\mid\uparrow\uparrow\uparrow\downarrow\downarrow>$
s_2^c	:	$\mid\uparrow\uparrow\downarrow\uparrow\downarrow>$
s_3^c	:	$\mid\uparrow\uparrow\downarrow\downarrow\uparrow>$
ν_μ	:	$\mid\uparrow\downarrow\downarrow\downarrow\downarrow>$
μ	:	$\mid\downarrow\uparrow\downarrow\downarrow\downarrow>$
μ^c	:	$\mid\downarrow\downarrow\uparrow\uparrow\uparrow>$
ν_μ^c	:	$\mid\uparrow\uparrow\uparrow\uparrow\uparrow>$

$+$

t_1	:	$\mid\uparrow\downarrow\uparrow\uparrow\downarrow>$
t_2	:	$\mid\uparrow\downarrow\uparrow\downarrow\uparrow>$
t_3	:	$\mid\uparrow\downarrow\downarrow\uparrow\uparrow>$
b_1	:	$\mid\downarrow\uparrow\uparrow\uparrow\downarrow>$
b_2	:	$\mid\downarrow\uparrow\uparrow\downarrow\uparrow>$
b_3	:	$\mid\downarrow\uparrow\downarrow\uparrow\uparrow>$
t_1^c	:	$\mid\downarrow\downarrow\uparrow\downarrow\downarrow>$
t_2^c	:	$\mid\downarrow\downarrow\downarrow\uparrow\downarrow>$
t_3^c	:	$\mid\downarrow\downarrow\downarrow\downarrow\uparrow>$
b_1^c	:	$\mid\uparrow\uparrow\uparrow\downarrow\downarrow>$
b_2^c	:	$\mid\uparrow\uparrow\downarrow\uparrow\downarrow>$
b_3^c	:	$\mid\uparrow\uparrow\downarrow\downarrow\uparrow>$
ν_τ	:	$\mid\uparrow\downarrow\downarrow\downarrow\downarrow>$
τ	:	$\mid\downarrow\uparrow\downarrow\downarrow\downarrow>$
τ^c	:	$\mid\downarrow\downarrow\uparrow\uparrow\uparrow>$
ν_τ^c	:	$\mid\uparrow\uparrow\uparrow\uparrow\uparrow>$

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b_2^c	:	$\|\uparrow\uparrow\downarrow\uparrow\downarrow>$
b_3^c	:	$\|\uparrow\uparrow\downarrow\downarrow\uparrow>$
ν_τ	:	$\|\uparrow\downarrow\downarrow\downarrow\downarrow>$
τ	:	$\|\downarrow\uparrow\downarrow\downarrow\downarrow>$
τ^c	:	$\|\downarrow\downarrow\uparrow\uparrow\uparrow>$
ν_τ^c	:	$\|\uparrow\uparrow\uparrow\uparrow\uparrow>$

Family Unification

$$G = SO(10) \times SO(10) \times SO(10) \times \mathcal{F}$$

- $\mathcal{F}$: Family Parity
- G: Maximally symmetric unification group with 3 families – with no exotics
- For one family, $SO(10)$ is the maximally symmetric unification group

  Anomaly free

  Chiral
- Can realistic models with three family unification be constructed?

Earlier Work

Gell-Mann, Ramond, Slansky, 1979; Wilczek, Zee, 1982

- $SO(4n+2)$ groups have complex spinors

  dimension of spinor $d = 2^{(2n)}$

$$n = 2 : SO(10) \Rightarrow d = 16$$
$$n = 3 : SO(14) \Rightarrow d = 64$$
$$n = 4 : SO(18) \Rightarrow d = 256$$

$SO(14) \to SO(10) \times SO(4)$:

$$64 \to (16, 2_+) + (\overline{16}, 2_-)$$

$\Rightarrow$ 2 families and 2 antifamilies

SO(18) Unification

$$SO(18) \to SO(10) \times SO(8):$$

$$256 \to (16, 8_+) + (\overline{16}, 8_-)$$

$$\Rightarrow \text{8 families and 8 antifamilies}$$

♦Can 8 antifamilies and 5 families be removed from low energy spectrum?

$$SO(8) \to SO(5):$$
$$8_+ \to 5 + 1 + 1 + 1$$
$$8_- \to 4 + 4$$

♦If $SO(5)$ is somehow confining, 8 antifamilies and 5 families will acquire large mass
Leaves 3 unconfined 16–plets

- Confinement mechanism for $SO(5)$ has not been realized

- In higher dimensions, upon compactification unwanted matter can be projected out (K.B, Barr, Kyae, 2002)

- Here wish to stay in 4 d

- Focus on the 3 family maximal symmetry group

$$\boxed{G = SO(10) \times SO(10) \times SO(10) \times \mathcal{F}}$$

$$G = SO(10) \times SO(10) \times SO(10) \times \mathcal{F}$$

Fermion content

$$\{(16,1,1) + (1,16,1) + (1,1,16)\}$$

With Family Parity $\mathcal{F}$ all 48 components of fermions are indistinguishible

- How does G break down to SM?
- Can realistic fermion masses and mixings arise?
- What predictions?

Symmetry Breaking

♦Assume supersymmetry

♦Majorana neutrino mass generation and R parity conservation motivates use of a bispinor Higgs:

$$\Delta : \{(16, 16, 1) + (1, 16, 16) + (16, 1, 16)\}$$

$$\overline{\Delta} : \{(\overline{16}, \overline{16}, 1) + (1, \overline{16}, \overline{16}) + (\overline{16}, 1, \overline{16})\}$$

$\langle \Delta \rangle + \left\langle \overline{\Delta} \right\rangle$ break

$$\boxed{SO(10)^3 \times \mathcal{F} \to SU(5)^3 \times \mathcal{F}}$$

$$SO(10)^3 \times \mathcal{F} \rightarrow SU(4)_c \times SU(2)_L \times SU(2)_R$$

Bifundamental Higgs:

$$\Omega_i = \{(10, 10, 1) + (1, 10, 10) + (10, 1, 10)\}$$

Combined effect:

$$SO(10)^3 \times \mathcal{F} \rightarrow SU(3)_c \times SU(2) \times U(1)_Y$$

Two Ω fields needed for $\mathcal{F}$ breaking
and natural doublet-triplet splitting

Fundamental Higgs:

$$H = \{(10, 1, 1) + (1, 10, 1) +)1, 1, 10)\}$$

for electroweak symmetry breaking

Superpotential for Symmetry Breaking

$$W = W(\Delta) + W(\Omega) + W(\Delta, \Omega)$$

$$\begin{aligned} W(\Delta) &= \lambda_1 [S_1(\Delta_{12}\overline{\Delta}_{12} - M^2) + S_2(\Delta_{23}\overline{\Delta}_{23} - M^2) \\ &+ S_3(\Delta_{31}\overline{\Delta}_{31} - M^2)] \end{aligned}$$

$$\begin{aligned} W(\Omega) &= \mu(\Omega_{12}^2 + \Omega_{23}^2 + \Omega_{31}^2) + \mu'((\Omega'_{12})^2 + (\Omega'_{23})^2 + (\Omega'_{31})^2) \\ &+ \lambda_2 \Omega_{12}\Omega_{23}\Omega_{31} + \lambda_3 \Omega_{12}\Omega'_{23}\Omega'_{31} + \lambda_4 \Omega'_{12}\Omega_{23}\Omega'_{31} + \lambda_5 \Omega'_{12}\Omega'_{23}\Omega_{31} \end{aligned}$$

$$W(\Delta, \Omega) = \lambda_6 \Delta\Delta\Omega + \lambda_7 \overline{\Delta\Delta}\Omega$$

The mixed terms do not affect minimzation, but give masses to all would-be Goldston bososn

VEV Structure

$$\langle \Delta_{12} \rangle = \langle \Delta_{23} \rangle = \langle \Delta_{31} \rangle = M$$
$$\Rightarrow SO(10)^3 \times \mathcal{F} \to SU(5)^3 \times \mathcal{F}$$

(1)

$$\langle \Omega_{ij} \rangle = (a, a, a, 0, 0)$$
$$\left\langle \Omega'_{ij} \right\rangle = (0, 0, 0, , b, b)$$

OR

(2)

$$\langle \Omega_{ij} \rangle = (a, a, a, b, b) + (a, a, a, 0, 0) + (a, a, a, 0, 0)$$
$$\left\langle \Omega'_{ij} \right\rangle = (b', b', b', 0, 0) + (b', b', b', c, c) + (b', b', b', c, c)$$

Doublet-Triplet Splitting

$$W_{DT} = \alpha[H_1 H_2 \Omega_{12} + H_2 H_3 \Omega_{23} + H_3 H_1 \Omega_{31}]$$

Doublet Mass Matrix:

$$M_D = \begin{pmatrix} 0 & \alpha b & 0 \\ \alpha b & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

A pair of doublets from H_3 remain light

Other doublets are heavy

Triplet Mass Matrix:

$$M_T = \begin{pmatrix} 0 & \alpha a & \alpha a \\ \alpha a & 0 & \alpha a \\ \alpha a & \alpha a & 0 \end{pmatrix}$$

All triplets are heavy

Fermion Masses

$$W_Y = Y[\psi_1\psi_1 H_1 + \psi_2\psi_2 H_2 + \psi_3\psi_3 H_3] + F(\psi_1\psi_2\overline{\Delta}_{12} + \psi_2\psi_3\overline{\Delta}_{23} + \psi_3\psi_1\overline{\Delta}_{31})$$

H_3 has light doublets, $H_{1,2}$ do not

$\Rightarrow$ First two family masses $= 0$
Third family mass $\neq 0$

Right handed neutrino Majorana Mass Matrix:

$$M_R = M_0 \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}$$

Eigenvalues: $M_0 \times \{-1, -1, 2\}$

$\Rightarrow$ Resonant Leptogenesis!

Light Fermion Masses

Arise from higher dimensional operators

$$W'_Y = Y_2(\psi_2\psi_2 H_3 \Omega_{23} + ..) + Y_1(\psi_1\psi_1 H_3 \Omega_{13} + ..)$$

$\Rightarrow$ First two family masses $\neq 0$

Since Ω does not break Pati-Salam symmetry, these couplings preserve $m_s = m_\mu, \; m_d = m_e$

Higgs Mixing

$$W = \Delta_{13}\Delta_{23}\overline{\Delta}_{12}H_3$$

$\Rightarrow$ Light Higgs has small component in Δ

This coupling does not upset doublet mass

Down quarks and charged lepton matrices get corrections proprtional to M_R

Georgi-Jarlskog factor not realized yet

Either $SU(5)$ or $SU(4)_c$ are unbroken

$$\boxed{W = \psi_2 \psi_3 \Delta_{23} \Omega_{23}}$$

$$\begin{array}{ccc} m_b & \simeq & m_\tau \\ m_\mu & \neq & m_s \\ Det[M_d] & \simeq & Det[M_l] \end{array}$$

Quark mixings nonzero

Other Predictions

There is no SUSY flavor problem

All scalar fields have same mass-squared
Gaugino mass is universal

Proton lifetime

Threshold corections need to be computed

Best estimate for $d = 6$ proton lifetime:
$\tau_p \sim 10^{34}$ years

Neutrino oscillation data seems consistent

$\tan\beta$ is large ~ 40

Conclusions

A new class of family unification is proposed based on $SO(10)^3 \times \mathcal{F}$

3rd family emerges to be different dynamically

Realistic fermion masses can be generated

Best estimate for $d = 6$ proton lifetime:
$\tau_p \sim 10^{34}$ years

There is no SUSY flavor problem

There is no D term problem

Many aspects need to be worked out

Thank You!