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Natural Realizations of Seesaw in Mini-Warped Minimal SO(10)

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Fermions in Standard Model

- Standard Model includes 3 families for fermions. For every family, we have $\mathrm{SU}(2)_\mathrm{L}$ doublets $\mathrm{Q}^\mathrm{i} = (\mathrm{u}, \mathrm{d}), \mathrm{L} = (\nu, \mathrm{e})$, and $\mathrm{SU}(2)_\mathrm{L}$ singlets u_R^i, d_R^i, e_R. The total number of one family of fermions is 15. Note there is no right-handed neutrino. C and P are broken maximally.
- Neutrinos are massless and there is no mixing in the leptonic sector if we only consider the renormalizable operators.

Experimental results for Neutrino Mass and Mixing

[Maltoni, Schwetz, Tortola and Valle 2006]

- Mass (3σ)

$$7.1 \times 10^{-5}\mathrm{eV}^2 \leq \Delta\mathrm{m}^2_{21} \leq 8.9 \times 10^{-5}\mathrm{eV}^2$$
$$2.0 \times 10^{-3}\mathrm{eV}^2 \leq \Delta\mathrm{m}^2_{31} \leq 3.2 \times 10^{-3}\mathrm{eV}^2$$

- Mixing angles (3σ)

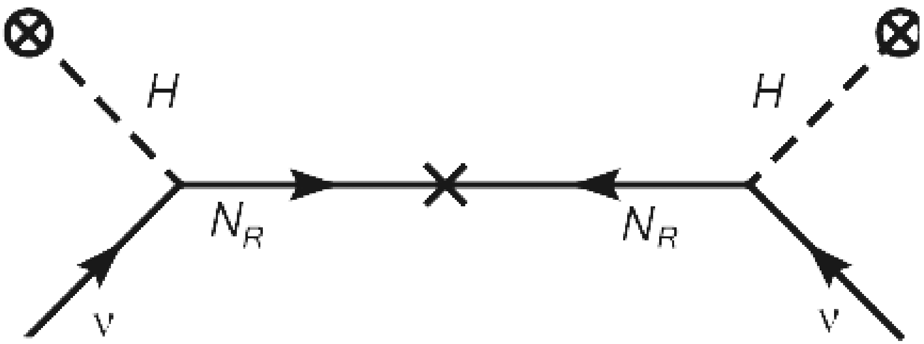
$$0.24 \leq \sin\theta^2_{12} \leq 0.40$$
$$0.34 \leq \sin\theta^2_{23} \leq 0.68$$
$$\sin\theta^2_{13} \leq 0.04$$

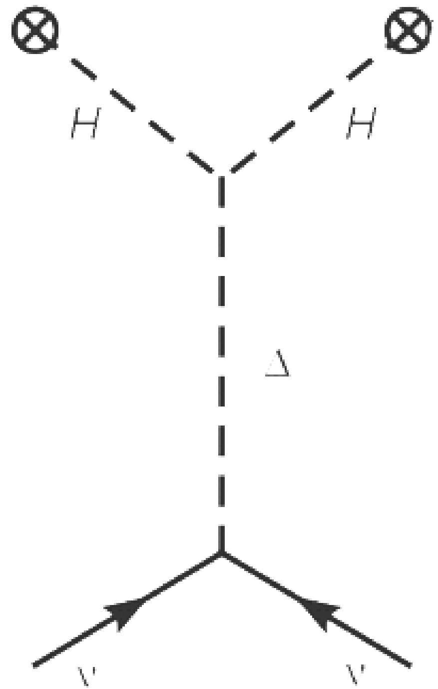
How to understand the tiny neutrinos masses?

- Experiments tell us neutrinos are massive and there is mixing in the leptonic sector. But their masses are tiny. $m_e/m_\nu \sim 10^6$, $m_t/m_\nu \sim 10^{11}$.
- In SM, neutrinos are massless. But we can write down $\frac{1}{2}\frac{(LH)^2}{\Lambda}$, if the light neutrino mass scale is ~ 0.1 eV, $\Lambda \sim 10^{14}$ GeV. Λ is the scale of new physics. From effective field point of view, neutrino mass indicates new physics.
- Seesaw mechanism is used to explain the smallness of masses of light neutrinos by introducing heavy Majorana right-handed neutrinos or $SU(2)$ triplet.

Seesaw mechanism

[Minkovski 1977, Yanagida 1979, Gell-Mann,Rammond and Slansky 1980, Glashow 1980, Mohapatra and Senjanovic 1980,Mohapatra and Senjanovic 1981,Lazarides,Shafi and Wetterich 1981]





Look for more fundamental theory

- Seesaw needs new degree of freedom, SM singlet right-handed neutrinos or $\mathrm{SU}(2)_{\mathrm{L}}$ Higgs triplet or both.
- What determines the Seesaw scale? What is the new physics associated with Seesaw scale?
- How to understand the leptonic mixing pattern? Flavor symmetry, flavor anarchy?

$\mathrm{SO}(10)$ GUT model as a good candidate for more fundamental theory

- Low scale measurements support supersymmetric grand unification.
- The **16** dimensional spinor representation of $\mathrm{SO}(10)$ includes right-handed neutrino that is an essential part of the Seesaw mechanism. Right-handed neutrino is a singlet under $\mathrm{SU}(5)$.
- The Seesaw scale which is close to the GUT scale receives natural explanation as the GUT symmetry breaking scale. $\mathrm{U}(1)_{\mathrm{B-L}}$ is gauged as subgroup of $\mathrm{SO}(10)$. $\mathrm{SU}(5)$ does not have this feature. Note either B or L itself is not anomaly free. Even B+L is anomalous. Majorana mass term breaks B-L.
- $\mathrm{SO}(10) \supset \mathrm{SU}(5) \times \mathrm{U}(1), \mathrm{SU}(2)_{\mathrm{L}} \times \mathrm{SU}(2)_{\mathrm{R}} \times \mathrm{SU}(4)$.

Minimal SO(10)

[Aulakh and Mohapatra 1983, Babu and Mohapatra 1993]

- Minimal supersymmetric $\mathrm{SO}(10)$ model includes matter field $\mathbf{16}$ for each generation, Higgs fields $\mathbf{126}, \overline{\mathbf{126}}, \mathbf{10}$ and $\mathbf{210}$.
- Renormalizable operators $\mathbf{16}\cdot\mathbf{16}\cdot\mathbf{10}$ and $\mathbf{16}\cdot\mathbf{16}\cdot\overline{\mathbf{126}}$ generate fermions masses.
- $\mathbf{126}$ is needed for D-term cancellation to keep the SUSY down to the TeV scale.
- $\mathbf{210}$ is needed to break $\mathrm{SO}(10)$ to SM group.

Decompositions of SO(10) multiplets under $SU(5) \times U(1)_X$

$$
\begin{aligned}
\mathbf{16} &= \mathbf{1}_{-5} \oplus \overline{\mathbf{5}}_{+3} \oplus \mathbf{10}_{-1} \\
\mathbf{210} &= \mathbf{1}_0 \oplus \mathbf{5}_{-8} \oplus \overline{\mathbf{5}}_8 \oplus \mathbf{10}_4 \oplus \overline{\mathbf{10}}_{-4} \oplus \mathbf{24}_0 \oplus \mathbf{75}_0 \oplus \mathbf{40}_{-4} \oplus \overline{\mathbf{40}}_4 \\
\mathbf{126} &= \mathbf{1}_{-10} \oplus \overline{\mathbf{5}}_{-2} \oplus \mathbf{10}_{-6} \oplus \overline{\mathbf{15}}_{+6} \oplus \mathbf{45}_{+2} \oplus \overline{\mathbf{50}}_{-2} \\
\mathbf{10} &= \mathbf{5}_2 \oplus \overline{\mathbf{5}}_{-2}
\end{aligned}
$$

$\mathbf{16} \cdot \mathbf{16} \cdot \overline{\mathbf{126}} \supset \mathbf{1}_{-5}\mathbf{1}_{-5}\mathbf{1}_{+10}$ gives right-handed neutrino mass once $\mathbf{1}_{+10}$ gets VEV which breaks B-L.

Why minimal SO(10)?

[Bajc,Senjanovic and Vissani 2003; Goh,Mohapatra and Ng 2004]

- The $\mathrm{U}(1)_{\mathrm{B-L}}$ breaking VEV carries $B - L = 2$, so R-parity is conserved.
- The model is very predictive. There are only 13 parameters as input(6 quark masses,3 CKM angles, 3 charged lepton masses and overall scale). Neutrino sector are completely determined up to an overall scale. Note there are total 18 low scale observables without phase.
- Large atmospherical mixing angle and small reactor angle are naturally explained by $b - \tau$ unification if type II Seesaw dominates the contribution to the light neutrino mass. Very natural explanation. No flavor symmetry needed.

Fermions mass sum rule and neutrino mass in minimal SO(10) model

The general neutrino mass formulae $M_\nu = fv_L - M_D^T (fv_R)^{-1} M_D$

$$M_u = h\kappa_u + fv_u, M_d = h\kappa_d + fv_d$$
$$M_\ell = h\kappa_d - 3fv_d, M_D = h\kappa_u - 3fv_u$$

The type II contribution:$M_\nu \sim (M_d - M_l)$.

$M_d \sim m_b \begin{bmatrix} \lambda^4 & \lambda^5 & \lambda^3 \\ \lambda^5 & \lambda^2 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{bmatrix}$,and $M_l \sim M_d$ for $m_b \simeq m_\tau$,

$$M_\nu \simeq m_0 \begin{bmatrix} \lambda^4 & \lambda^5 & \lambda^3 \\ \lambda^5 & \lambda^2 & \lambda^2 \\ \lambda^3 & \lambda^2 & \lambda^2 \end{bmatrix}$$

Problems

- Gauge couplings blow up beyond GUT scale due to large representations.
- To explain discrepancy between Seesaw scale and GUT scale, one needs to tune(adjust) the parameters.
- In the minimal $\mathrm{SO}(10)$ model, type II Seesaw can not dominate contribution to the light neutrino mass!!!

The type II contribution can be written as

$$M_\nu \sim f v_{wk}^2 / M_T$$

where M_T is SU(2) triplet mass.

- If $f \sim 1$,we need $M_T \sim 10^{14}\mathrm{GeV}$.
- $\overline{\mathbf{126}} = \mathbf{1}_{+10} \oplus \mathbf{5}_{+2} \oplus \mathbf{10}_{+6} \oplus \mathbf{15}_{-6} \oplus \mathbf{45}_{-2} \oplus \mathbf{50}_{+2}$. This triplet belongs to **15**, whose mass has the same order as masses of **45** and **50**.
- This model is so constraint that there is no way to achieve type-II dominance even by tuning the parameters if we want to keep the theory perturbative up to GUT scale.

A brief review of original Randall-Sundrum model

[Randall and Sundrum 1999]

- Hierarchy problem. $m_{phy}^2 = m_{bare}^2 - \frac{|y|^2}{8\pi^2}\Lambda_{UV}^2$.If Λ_{UV} is $M_P = (8\pi G_{Newton})^{-/2} = 2.4 \times 10^{18}\mathrm{GeV}$, SM is extremely fine-tuned theory.
- In the RS model, because of special geometry, the mass scale on the IR brane is warped down by factor $e^{-k\pi r_c}$. To generate hiearchy between electroweak scale and Planck scale, one just needs $kr_c \simeq 12$.

RS metric

$$ds^2 = e^{-2kr_c|y|}\eta_{\mu\nu}dx^\mu dx^\nu - r_c^2 dy^2$$

$$\begin{aligned} & \int d^4x\sqrt{-G}(g_{in}^{\mu\nu}\partial_\mu H\partial_\nu H - m^2H^2) \\ = & \int d^4x e^{-4kr_c\pi}(e^{2kr_c\pi}\eta^{\mu\nu}\partial_\mu H\partial_\nu H - m^2H^2) \\ \rightarrow & \int d^4x(\eta^{\mu\nu}\partial_\mu H\partial_\nu H - m^2e^{-2kr_c\pi}H^2) \end{aligned}$$

Wavefuction overlapping

$$\Phi(y,x) = f_0(y)\phi_0(x) + \sum_{n\geq 1} f_n(y)\phi_n(x)$$

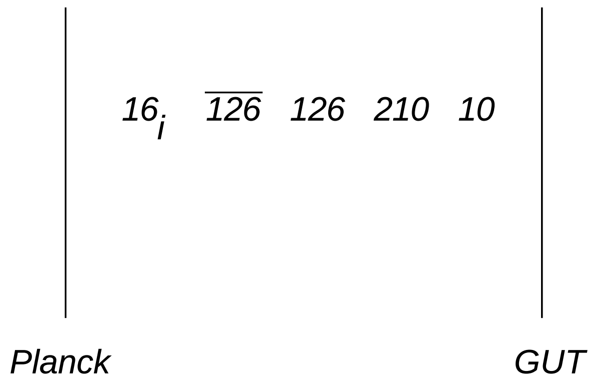
$$\int d^4x dy Y_{5D} \Phi_1(y,x)\Phi_2(y,x) = \int d^4x Y_{eff}\phi_1(x)\phi_2(x),$$

where the 4D effective coupling is $Y_{eff} \equiv \int dy Y_{5D} f_{1,0}(y) f_{2,0}(y)$, which is so-called wave-function overlapping.

Motivation

[Mohapatra, Okada and HBY 2007]

- Mini-Warping provides the solution to the mini hierarchy between M_P and M_{GUT}. $M_{GUT} = e^{-k\pi r_c} M_P$ is the cutoff of 4D theory. No GUT running beyond M_{GUT}.
- Overlapping of of bulk fields configuration provides a way to understand mini-tuning needed in 4D minimal SO(10).
- Particularly it may be helpful for the realization of type II dominance or mixed case.


16$_i$
$\overline{126}$
126
210
10
Planck
GUT

Warped extra dimension

As in the RS model, we use the warped metric,

$$ds^2 = e^{-2kr_c|y|}\eta_{\mu\nu}dx^\mu dx^\nu - r_c^2 dy^2 \ ,$$

with $-\pi \leq y \leq \pi$ and $\eta_{\mu\nu} = (+,-,-,-)$. In the above expression, k is the AdS curvature, and r_c and y are the radius and the angle of S^1, respectively.

SUSY in 5D

We still use supersymmetry to solve the hierarchy problem. We need to consider the supersymmetry in the 5D,

$$\mathcal{L} = \int dy \left\{ \int d^4\theta \; r_c \; e^{-2kr_c|y|} \left(H_i^\dagger e^{-Q_i V} H_i + H_i^c e^{Q_i V} H_i^{c\dagger} \right) + \int d^2\theta e^{-3kr_c|y|} H_i^c \left[\partial_y - (1 + C_i)\, kr_c \epsilon(y) - Q_i \frac{\chi}{\sqrt{2}} \right] H_i + h.c. \right\}$$

$$V = -\theta\sigma^\mu\bar{\theta}A_\mu - i\bar{\theta}^2\theta\lambda_1 + i\theta^2\bar{\theta}\bar{\lambda}_1 + \tfrac{1}{2}\theta^2\bar{\theta}^2 D.$$
$$\chi = \tfrac{1}{\sqrt{2}}(\Sigma + iA_5) + \sqrt{2}\theta\lambda_2 + \theta^2 F.$$

- Assign H^c and χ to be Z2 odd. They are projected out on both branes. N=2 SUSY breaks to N=1 SUSY.
- Adjoint field χ develop a VEV, and break gauge symmetry. $\langle \Sigma \rangle = 2\alpha k r_c \epsilon(y)$. This can be realized by brane field.

Zero mode

The zero mode wave function of H_i satisfies the following equation of motion:

$$\left[\partial_y - (1 + C_i + Q_i\alpha)\, kr_c\epsilon(y)\right] H_i = 0$$

which yields $H_i = \frac{1}{\sqrt{N_i}} e^{(1+C_i+Q_i\alpha)kr_c|y|}\, h_i(x^\mu)$, where $h_i(x^\mu)$ is the chiral multiplet in four dimensions. Here, N_i is a normalization constant, $\frac{1}{N_i} = \frac{2(C_i+Q_i\alpha)k}{e^{2(C_i+Q_i\alpha)kr_c\pi}-1}$. $(C_i + Q_i\alpha)$ determines the location of fifth dimensional wave function configuration.

Setup of mini-warped minimal $\mathrm{SO}(10)$

- Put matter fields and Higgs fields in the bulk. Write the superpotential on both UV brane and IR brane. All of the coupling constants are $\mathcal{O}(1)$.
- Bulk adjoint field gets VEV, and breaks $\mathrm{SO}(10)$ to $\mathrm{SU}(5) \times \mathrm{U}(1)_{\mathrm{X}}$. $\mathrm{SU}(5)$ submultiplets within one $\mathrm{SO}(10)$ multiplet get different effective 5D mass parameters due to different $\mathrm{U}(1)_{\mathrm{X}}$ charge. This is crucial to generate mass difference of the submultiplets within one multiplet.
- Choose 5D mass parameters for $\mathrm{SO}(10)$ multiplets C_i and bulk adjoint field VEV parameter α.
- Exam the GUT breaking.

Example

We take $\alpha = -1/4$ and C_i as follows

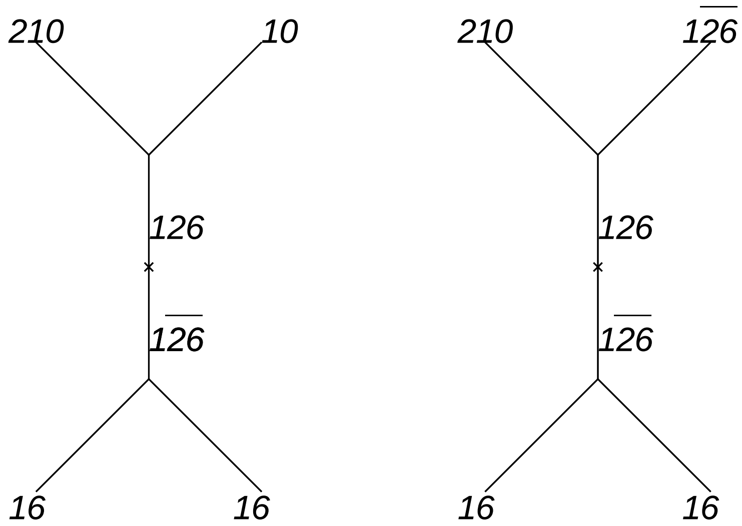
H_i components	C_i
$\mathbf{16}$	$1/2$
$\mathbf{10}$	$1/2$
$\overline{\mathbf{126}}$	1
$\mathbf{126}$	0
$\mathbf{210}$	-2

Masses of submultiplets of $\overline{\mathbf{126}}$

$$\overline{\mathbf{126}} = \mathbf{1}_{+10} \oplus \mathbf{5}_{+2} \oplus \mathbf{10}_{+6} \oplus \mathbf{15}_{-6} \oplus \mathbf{45}_{-2} \oplus \mathbf{50}_{+2}$$

$$\mathbf{15} \sim \omega^{3/2} M_{GUT} \sim 10^{13}\mathrm{GeV}$$
$$\mathbf{45} \sim \omega^{1/2} M_{GUT} \sim 10^{15}\mathrm{GeV}$$
$$\mathbf{50} \sim \ M_{GUT} \ \sim 10^{16}\ \mathrm{GeV}$$

Note $\omega \equiv e^{-k\pi r_c} = M_{GUT}/M_P$ factors are due to fifth dimensional wave function overlapping. This mass splitting leaves MSSM gauge coupling unification unchanged, and $\alpha_{GUT} \sim 0.2$.


210
10
126
$\overline{126}$
16
16
210
$\overline{126}$
126
$\overline{126}$
16
16
(a) (b)

Type II Seesaw contribution is estimated as

$$M_\nu^{II} \simeq \frac{2(f_1)_{33}\omega^{1/2} v_{10} v_{210} \alpha_2}{M_{GUT}\omega^{3/2}},$$

where $\omega = M_{GUT}/M_P \sim 10^{-2}$. If we take $(f_1)_{33} \sim 1$, $\alpha_2 \sim 0.5$ and assume $v_{10} \simeq v_{210} \sim 100$ GeV, we get the reasonable value for the atmospheric neutrino oscillation data, $M_\nu^{II} \simeq 0.05$ eV.
The type I Seesaw contribution

$$M_\nu^I = M_D^T M_R^{-1} M_D \simeq \frac{m_t^2 \omega^{1/2}}{2(f_1)_{33} M_{GUT}\omega^{3/2}},$$

$m_t \sim 100$ GeV at the GUT scale, the type I Seesaw gives the contribution to the "heaviest" light neutrino mass as $m_3 \simeq 0.025$ eV.

Conclusions

- Minimal SO(10) model is a good candidate for GUT, which provides way to understand neutrino mass, mixing and also predictive.
- There are some problems in the minimal SO(10) models. Gauge coupling blows up beyond GUT scale. Type II Seesaw can not be realized in minimal model if we want to keep perturbation of the theory.
- Embedding 4D minimal SO(10) into mini-warped 5D space provides a way to solve them. In this theory, the fundamental scale is M_P, GUT scale is cutoff of 4D theory. The 5D wave function overlapping solves the fine-tuning of parameters needed in 4D.