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Workshop on Grand Unification and Proton Decay

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Unravelling the Missing Link of Grand Unification: Proton Decay

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UNRAVELLING THE MISSING LINK OF GRAND UNIFICATION: PROTON DECAY

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REFERENCES:

JCP: Review Talk: "Grand Unification As a Bridge ..." Aug, 2006, hep-ph/0606089

I. Introduction

A) Improved Studies of

① Proton Decay

② ν oscillations

2 indispensable tools to probe nature at the shortest distance $\sim 10^{-30}$ cm.

In a broader sense, these two processes get intimately connected with each other within Grand Unification.

In fact both are predictions of a well-motivated class of Grand Unification symmetries.

I will therefore begin by first listing evidence which has built up over the years in favor of Grand Unification, in particular for a certain class of Grand Unification Symmetries.

SM Now Brilliantly Successful

Since 1972, both the SU(2) × U(1) EW Theory & QCD have been confirmed by numerous experiments

→ A Triumph of Gauge Principle & SSB

→ Waiting To see Higgs at LHC/Fermilab

But ∃ Clear Evidence For Physics Beyond SM

1) ν Masses $\left(\sqrt{\Delta m^2(\nu)_{23}} \sim 1/20\,\text{eV}\right)$

2) Higgs Mass Fine Tuning

3) Need For Inflation → Horizon, → Flatness $\Omega = 1$

4) Dark Matter (Cold) : $\Omega_{DM} \approx 0.23$

5) Baryogenesis ↔ Leptogenesis

} All Five Go Well With SUSY Grand Unification!

⇓

There exists PHYSICS BEYOND SM

6) Dark Energy : Cosm. Const.

$$\Omega_\Lambda \approx 0.72$$

} A MYSTERY FOR ALL THEORIES

II. Grand Unification: Alternative Routes & Evidence

A) Main Idea (1972-74)

① Unify (q & l)

&

② Unify (Weak, EM & Strong) Forces

$SU(2)_L \times U(1) \times SU(3)^c$

Purely on aesthetic grounds

B) Alt. Routes To Grand Unification

$$G(213) = SU(2)_L \times U(1)_Y \times SU(3)^c$$

$$\begin{pmatrix} u_r & u_y & u_b \\ d_r & d_y & d_b \end{pmatrix}_L^{1/3}, \quad (u_{r,y,b})_R^{4/3}, \quad (d_{r,y,b})_R^{-2/3}, \quad \begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L^{-1}, \quad (e^-_R)^{-2}$$

5 disconn. multiplets // Arb. Y_W, $SU(2)$, $SU(3)^c$ Q. NOS.

$$\downarrow$$

$$G(224) = SU(2)_L \times SU(2)_R \times SU(4)^c_{L+R} \times (L \leftrightarrow R)$$

$$F^e_{L,R} = \begin{bmatrix} u_r & u_y & u_b & \nu_e \\ d_r & d_y & d_b & e^- \end{bmatrix}_{L,R} \qquad \begin{matrix} F^e_L = (2,1,4) \\ F^e_R = (1,2,4) \end{matrix}$$

$$Q_{em} = I_{3L} + I_{3R} + \frac{B-L}{2}$$

Advantages of G(224)

- All 16 in one L-R Conj. multiplet // L-R symm
- Explain Y_W & All Q. Nos. // Quantize Q_{em} //
- $Q_{e^-} = -Q_p$ // ν_R // B-L ← Need For Seesaw & Leptogenesis

$$\downarrow$$

SO(10) : 16 (one coupling)

All Advantages of G(224) retained by SO(10), not SU(5)

SU(5): $\bar{5} + 10$
NO ν_R, NO B-L } → Problem with ν Masses & Leptogenesis.

II C. EVIDENCE FOR GRAND UNIFICATION

① Quantum nos. of members in a Family

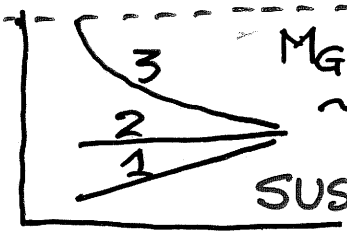
② Quantization of Q_{em}

③ $Q_{e-} = -Q_p$

④ Gauge Coupling Unif.

$$\Updownarrow$$

$\sin^2\theta_W$



$M_{GUT} \sim 2\times10^{16}$ GeV

$\Updownarrow$

SUSY ~ 1TeV

⑤ $\boxed{\nu \text{ osc. } \sqrt{\Delta m^2(\nu)_{23}} \approx 1/20\,\text{eV}} \leftrightarrow SU(4)^C$

$\Downarrow$

ν_R

$\Downarrow$

Seesaw

⑥ Baryogenesis ↔ Leptogenesis → ν_R, B-L

⑦ $m_b^{(0)} = m_\tau$

⑧ $\boxed{m(\nu^\tau)_{DIRAC} = m_{top}^{(0)}}$

→ SU(4) color

Agreement of all eight non-trivial

$$\Downarrow$$

strong support for SUSY Grand Unif.
& an Eff. Symmetry like SO(10) or minimally
a string-derived G(224).

II. Evidence Favoring Grand Unification

1) Family-Multiplet Structure // All 5 sub-multiplets → ONE MULTIPLET } ⇒ G(224)/SO(10) $\cancel{SU}(5)$

2) Quantization of Q_{em} → SU(5)/G(224)/SO(10

3) Coupling Unification ($M_X \sim 2 \times 10^{16}$ GeV) → SU(5)/SO(10)/String G(224)
← SUSY ——→

4) $m(\nu_\tau) \sim 1/20$ eV (SuperK) → String G(224)/SO(10) $\cancel{SU}(5)$

5) $\theta(\nu_\mu - \nu_\tau)$ Large ↔ V_{bc} small → G(224)/SO(10)

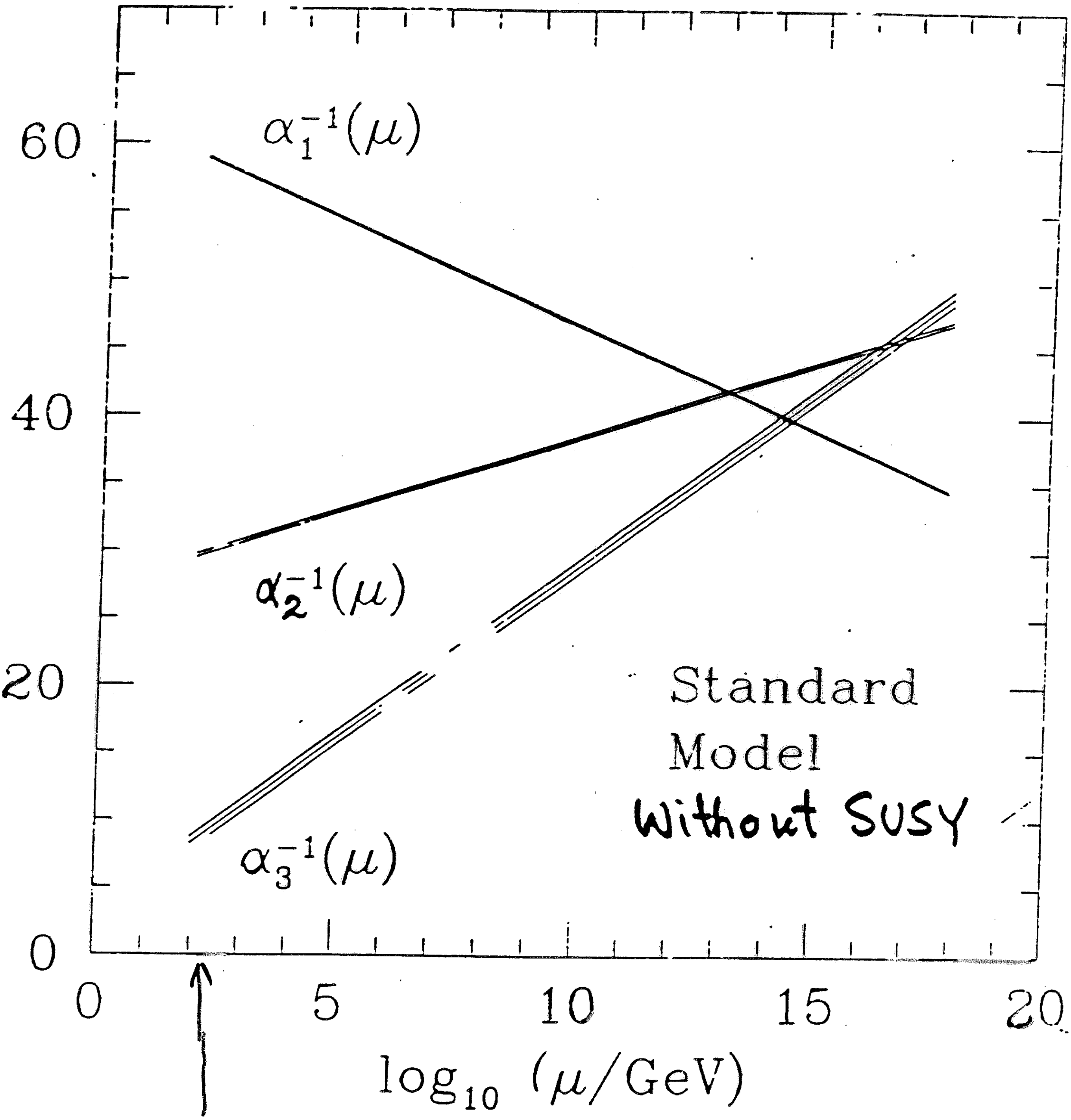
6) $m_b^0 \approx m_\tau^0$ → G(224)/SO(10)/SU(5)

7) Lepto/Baryogenesis → Spont $\cancel{B-L}$ → G(224)/SO(10) $\cancel{SU}(5)$

THE SUCCESS OF THESE SEVEN FEATURES SEEM NON-TRIVIAL. TOGETHER THEY MAKE A STRONG CASE FOR SUSY GRAND UNIFICATION & SIMULTANEOUSLY FOR A STRING-DERIVED G(224)/SO(10) ROUTE TO SUCH UNIFICATION.

∴ CONSIDER PROTON DECAY WITHIN THIS FRAMEWORK.

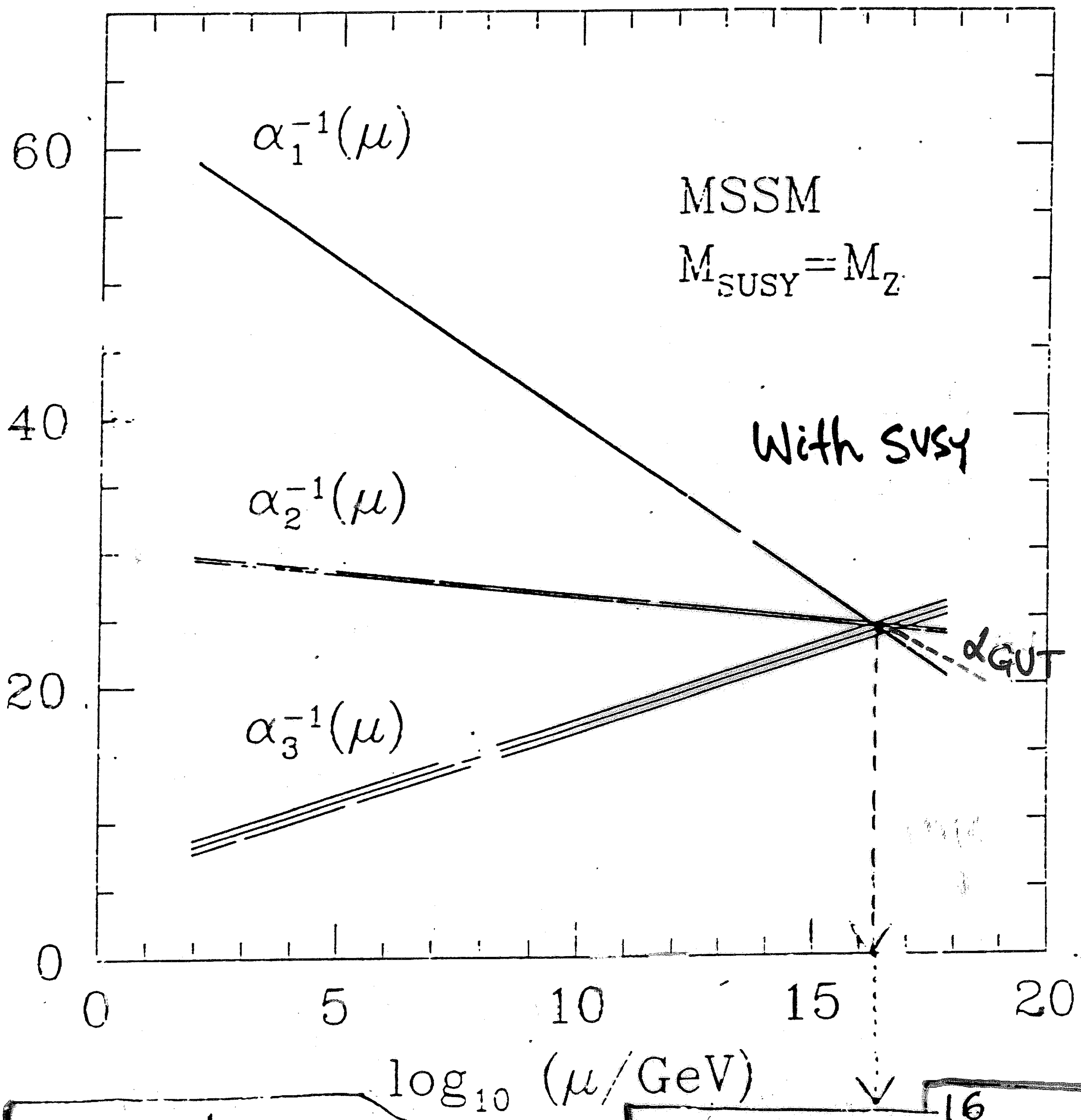
Coupling Unification Without SUSY



α_i's Measured at LEP
at ~ 100 GeV

8.1 $\frac{1}{27.2}$

Gauge Coupling Unification With SUSY



Supports SUSY Unification

$M_{GUT} \approx 2 \times 10^{16}$ Gev

$\sin^2\Theta_W(m_Z)_{th} = 0.2315 \pm .003$

$\sin^2\Theta_W(m_Z)_{EXPT} = 0.23124 \pm .00017$

$\alpha_3(m_Z)^0_{theory} \simeq 0.125-0.13$

$\alpha_3(m_Z)_{obs} = 0.118 \pm .003$

$\updownarrow$

GUT THRESHOLD CORRECTION

TWO GENERAL PREDICTIONS OF GUT SO(10)/
SUSY G(224)

① ν OSC

② PROTON DECAY

↕

Correlated With Fermion Masses & Mixings

∴ Discuss First Fermion Masses & Mixings within G(224)/SO(10)

III SO(10) Breaking : Alternative Higgs

① Low Dim. Higgs

$\{45_H, 16_H, \overline{16}_H, 10_H\}$

Babu, Barr // Albright, Babu, Barr // Albright, Barr (Lop sided) // Dermisek, Mafi, Raby // Babu, Pati, Wilczek (Hierarchical)

② Large Dim. Higgs

$126, \overline{126}, (210 // 120 // 54), 10$

Aulukh, Mohapatra // Babu, Mohapatra // Aulukh, Bajc, Melfo, Rasin, Senjanovic // Clark, Kuo, Nakagawa // Chang, Mohapatra, Parida // Chen, Mahantthapa // Aulukh // Bando et al // Fukuyama et al //

A Good Comparative analysis:
Melfo, Senjanovic (2005).

Low Dim. Higgs	Large Dim. Higgs
$45_H, 16_H, \overline{16}_H, 10_H$	$126, \overline{126}, (210 // 120 // 54), 10_H$
$16_i 16_j 10_H$; $\boxed{16_i 16_j 10_H \cdot \frac{45_H}{M} ; 16_i 16_j 16_H \frac{16_H}{M} ; 16_i 16_j \overline{16}_H \overline{16}_H / M}$ $\Uparrow$ • Non-Ren. Terms // Should be Kept if allowed by Symm	$16_i 16_j (10_H, 126, 120)$ $\Uparrow$ • All Ren. Terms
• $\lvert \Delta(B-L)_{16_H} \rvert = 1$ $\Downarrow$ R-Parity Broken, but discrete Symm naturally Present	• $\lvert \Delta(B-L)_{126_H} \rvert = 2$ $\Downarrow$ R-Parity Preserved
• GUT Threshold Corr & RG effects incl. D-T splitting Well under Control // Coupling Unif. demonstrated Sign & Magn.	• Threshold Corr. Large // RG studies & Coupling Unif ↔ M_{GUT} ?
$m_b^{(0)} = m_\tau$, Yet $m_\mu \neq m_s$ & $V_{cb} \ll \theta^{\nu}_{23}$	$m_b^{(0)} = m_\tau$ Lost
Fermion Mass Correl Proton Decay Concretely Studied (BPW)	?

c) Insight From SuperK Result: $\sqrt{\Delta m^2_{23}} \approx 1/20 \text{ eV}$

SeeSaw

ignore mixing for a moment

$$\boxed{m(\nu_L^\tau) \approx \frac{m(\nu^\tau_{Dirac})^2}{M(\nu_R^\tau)}}$$

(a) $\boxed{m(\nu^\tau_{Dirac}) \approx m_t(M_X) \approx 120 \text{ GeV}} \Leftarrow$ SU(4) - Color

$SU(5)$, $[SU(3)]^3$

$\boxed{m_b^\circ \approx m_\tau^\circ}$

(b) Get $M(\nu_R^\tau)$ from SUSY Unif. Scale: $M_X \approx 2 \times 10^{16}$ GeV

$$f_{33} \frac{16_3\, 16_3 \langle \overline{16}_H \rangle \langle \overline{16}_H \rangle}{M} \Rightarrow M(\nu_R^\tau) \sim \frac{(2 \times 10^{16} \text{ GeV})^2}{10^{18} \text{ GeV}}$$

(≈ 1) $M \rightarrow \sim 10^{18}$ GeV

$$\approx \boxed{4 \times 10^{14} \text{ GeV } (1/2 - 2)}$$

$$\Downarrow$$

$$\boxed{m(\nu_L^\tau) \sim \frac{(120 \text{ GeV})^2}{4 \times 10^{14} \text{ GeV}} \approx \left(\frac{1}{30} \text{ eV}\right)\left(\frac{1}{2} \text{ to } 2\right)}$$

Also get $m(\nu_L^\mu) \sim \frac{m(\nu_L^\tau)}{10} \Rightarrow \sqrt{\Delta m^2(\nu_2 \nu_3)}_{Th} \approx \left(\frac{1}{30} \text{eV}\right)\left(\frac{1}{2} - 2\right)$

Thus SuperK result brings to light the existence of ν_R // reinforces the ideas of a) SeeSaw // (b) SU(4) Color // & (c) SUSY Unif

IV Assume (q, ℓ, ν) Masses & Mixings in a predictive G(224) or SO(10)-Framework

Babu, Pati, Wilczek (98, 2000)

Minimal Higgs For SO(10) Breaking

$\boxed{45_H ,\ 16_H ,\ \overline{16}_H ,\ 10_H}$ $\cancel{126}_H$, $\cancel{120}_H$, ..

$$\begin{tikzcd}
SO(10) \arrow[d, "\langle 45_H \rangle \propto B-L"', "\sim \langle 16_H \rangle \sim M_{GUT}"] \\
SU(2)_L \times U(1)_Y \times SU(3)^c \arrow[d, "\langle \text{``}10_H\text{''} \rangle"', "\simeq v_{EW}"] \\
U(1)_{em} \times SU(3)^c
\end{tikzcd}$$

Only allowed Cubic Coupling

$16_i\, 16_j\, 10_H \rightarrow$ B-L Indep // symmetric $i \leftrightarrow j$ //

No CKM

These arise from ⇐ Higher Dim Operators

$16_i\, 16_j\, 16_H\, 16_H / M$, $16_i\, 16_j\, 10_H\, 45_H / M$

FLAVOR SYMMETRY: $SO(10) \times U(1)_F$

Assume Hierarchical Yukawa Couplings

"33" ≫ "23" ~ "32" ≫ "22" ≫ "12" etc.

↑

Due to Flavor symmetry $U(1)_F$

$$\begin{array}{lcccccccc} & 16_3 & 16_2 & 16_1 & 10_H & 16_H & \overline{16}_H & 45_H & S \\ U(1)_F \Rightarrow & a & a+1 & a+2 & -2a & -a-\frac{1}{2} & -a & 0 & -1 \end{array}$$

Thus, $\underset{a}{16_3}\ \underset{a}{16_3}\ \underset{-2a}{10_H} \rightarrow$ Allowed ~ 1

$$16_3\, 16_2\, 10_H \left\langle S/M \right\rangle \rightarrow \left(\frac{M_{GUT}}{M_{st}} \sim 1/10 \right)$$

$\langle S \rangle \sim M_{GUT}$, $M \sim M_{string}$

A Concrete Example: Minimal Higgs: $\{45_H, 16_H, \overline{16}_H, 10_H\}$ + "S"

st/ consider Only μ & τ Families (Flavor sym: $\mu \neq \tau$)

$$\begin{aligned}
&\mathcal{L}_{mass} = h_{33}\, 16_3\, 16_3\, \langle 10_H \rangle \longrightarrow \text{3rd Family}: m_b^0 = m_\tau^0 \\
\textcircled{1} \dashleftarrow\ & \\
\textcircled{\sigma} \dashleftarrow\ & + \tilde{h}_{23}\, 16_2\, 16_3\, \langle 10_H \rangle \langle S/M \rangle \longrightarrow \text{2nd Family} \\
& \qquad\qquad\qquad\qquad\qquad\qquad \underbrace{\text{ANTISYM}}\text{METRIC}, \propto B-L \\
\textcircled{\epsilon} \dashleftarrow\ & + a_{23}\, 16_2\, 16_3 \frac{\langle 10_H \rangle \langle 45_H \rangle}{M} \rightarrow m_\mu^0 \neq m_s^0 \\
\textcircled{\hat{\eta}} \dashleftarrow\ & + g_{23}\, 16_2\, 16_3\, \underline{\langle 16_H \rangle}\, \langle 16_H \rangle^D_{EW} / M
\end{aligned}$$

↳ CKM ≠ 1

$$10_H \rightarrow (2 \leftrightarrow 2) \ll (2 \leftrightarrow 3) \ll (3,3) \text{ Flavor}_{\text{sym}}$$

$$U = \begin{array}{cc} c & t \\ \left(0 \right. & \left. \epsilon+\sigma \right) \\ \left(-\epsilon+\sigma \right. & \left. 1 \right) \end{array} m_U^0 \qquad D = \begin{array}{cc} s & b \\ \left(0 \right. & \left. \epsilon+\eta \right) \\ \left(-\epsilon+\eta \right. & \left. 1 \right) \end{array} m_D^0$$

$$N_{Dirac} = \begin{array}{cc} \nu_\mu & \nu_\tau \\ \left(0 \right. & \left. -3\epsilon+\sigma \right) \\ \left(+3\epsilon+\sigma \right. & \left. 1 \right) \end{array} m_U^0 \qquad L = \begin{array}{cc} \mu & \tau \\ \left(0 \right. & \left. -3\epsilon+\eta \right) \\ \left(3\epsilon+\eta \right. & \left. 1 \right) \end{array} m_D^0$$

Note $q-\ell$ correlation (SU(4)-color)

up-down " $(SU(2)_L \times SU(2)_R)$

$$\boxed{\eta \equiv \hat{\eta} + \sigma}$$

Dirac Masses (3 Families)

$$U = \begin{pmatrix} 0 & \epsilon' & 0 \\ -\epsilon' & 0 & \epsilon+\sigma \\ 0 & -\epsilon+\sigma & 1 \end{pmatrix} m_U^0 \; ; \; D = \begin{pmatrix} 0 & \epsilon'+\eta' & 0 \\ -\epsilon'+\eta' & 0 & \epsilon+\eta \\ 0 & \epsilon+\eta & 1 \end{pmatrix} m_D^0$$

$$N = \begin{pmatrix} 0 & -3\epsilon' & 0 \\ 3\epsilon' & 0 & -3\epsilon+\sigma \\ 0 & 3\epsilon+\sigma & 1 \end{pmatrix} m_U^0 \; ; \; L = \begin{pmatrix} 0 & \epsilon'+\eta' & 0 \\ -\epsilon'+\eta' & 0 & -3\epsilon+\eta \\ 0 & +3\epsilon+\eta & 1 \end{pmatrix} \times m_D^0$$

---

2 New param (ϵ', η'), but 5 new observables just in (q, $\bar{\ell}$) System ⇒ 3 New predictions for (q, ℓ) // With $\epsilon' = 0$ → $m_u \to 0$

ν Majorana Masses

Saw before, $M_R \approx 10^{15}$ GeV

Expect $y \sim 1/10$

$$M_R^{\nu} = \begin{pmatrix} x & 0 & z \\ 0 & 0 & y \\ z & y & 1 \end{pmatrix} M_R$$

Note same hier. pattern as in Dirac Secto

$$f_{ij}\, 16_i\, 16_j \langle \overline{16}_H \rangle \langle \overline{16}_H \rangle / M_{st}$$

$$\downarrow$$

$$f_{ij}\, \nu_{iR}^T\, \bar{C}'\, \nu_{jR}\, \langle \overline{16}_H \rangle^2 / M_{st}$$

$$(M_R^{\nu})_{ij} = f_{ij} \langle \overline{16}_H \rangle^2 / M_{st}$$

Including $m_U^0 \to$ 7 param $(\eta, \epsilon, \sigma, \eta', \epsilon', m_U^0, m_D^0)$

describing $9 \times 4 = 36$ entries → Will it work ?

Input: Assume all param real for a moment

$$\boxed{\begin{array}{l} m_t^{phys} = 174\ \text{GeV}\ ;\ m_c(m_c) = 1.37\ \text{GeV}, \\ m_s(1\,\text{GeV}) = 116\ \text{MeV},\ m_\mu, m_\tau, m_u(M_X) = 1.5\ \text{MeV}, m_e \end{array}}$$

$$\Downarrow$$

$$\boxed{\begin{array}{l} \sigma \simeq 0.110,\ \eta \simeq 0.151,\ \epsilon \simeq -0.095, \\ \epsilon' = \sqrt{m_u/m_c}\,(m_c/m_t) \simeq 2 \times 10^{-4}\ ;\ \eta' \simeq \sqrt{m_e/m_\mu}\left(\frac{m_c}{m_t}\right) \simeq 4 \times 10^{-3} \\ m_U^0 = m_t(M_X) \simeq 110\ \text{GeV}\ ;\ m_D^0 \simeq 1.5\ \text{GeV} \end{array}}$$

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Majorana Mass of ν_R's : $f_{ij}\, 16_i\, 16_j\, \overline{16}_H\, \overline{16}_H / M$

$\Rightarrow f_{ij}\, (\nu_R^{iT} C^{-1} \nu_R^j)\, \langle 16_H \rangle^2 / M$

$$M_R^\nu = \begin{pmatrix} x & 0 & z \\ 0 & 0 & y \\ z & y & 1 \end{pmatrix} \boxed{M_R}$$

determined by $m_{\nu_2}/m_{\nu_3} \approx 1/6$

Calculated $\simeq 5 \times 10^{14}$ GeV.

6 New observables.

Summary on Fermion Masses & Mixings (Babu, Pati Wilczek)

Predictions	Observations
$m_b(m_b) \approx (4.7-4.9)\,\text{GeV}$	$\approx 4.2\,\text{GeV}$
$m(\nu_\tau) \sim (\frac{1}{24}\text{eV})(\frac{1}{2}-2)$	$\approx (1/15 - 1/25)\,\text{eV}$ ⊛
$V_{cb} \approx 0.043$	≈ 0.04
$\updownarrow$	$\updownarrow$
$\sin^2 2\Theta^{osc}_{\nu_\mu\nu_\tau} \approx \underset{\text{SMA}}{\boxed{0.92}} \leftrightarrow \underset{\text{LMA}}{\boxed{0.99}}$	$\approx 0.92 \leftrightarrow 1$
$V_{us} \approx 0.21$	≈ 0.22
$\lvert V_{ub}\rvert \simeq 0.0032$	$\approx 0.003-0.004$
$m_d(1\,\text{GeV}) \simeq 8\,\text{MeV}$	$\approx 8-10\,\text{MeV}$

$$m(\nu_\mu) \approx (2-10)\times10^{-3}\,\text{eV} \leftrightarrow \left\{ \begin{array}{l} \text{SMA} \sim 3\times10^{-3}\,\text{eV} \\ \boxed{\text{LMA} \simeq 7\times10^{-3}\,\text{eV}} \end{array} \right)$$

$\updownarrow$ Consistent with the framework

$$m(\nu_e) \sim (1 \text{ to few})\times10^{-3}\,\text{eV}$$

$$M(``\nu_R^\tau", ``\nu_R^\mu", \boxed{\nu_R^e}) \approx (10^{15}, 2\times10^{12}, (\tfrac{1}{3}-3)\times10^{10})\,\text{GeV}$$

Just right for Lepto/Baryogenesis

NOTE: ν_L Masses Necessarily Hierarchical.

16.1
(cont'd)

$$M_{3R} \approx M_R \approx 10^{15}\,\mathrm{GeV}\,(\tfrac{1}{2}-1)$$

$$M_{2R} \approx |y^2|\,M_{3R} \approx 2.5\times 10^{12}\,\mathrm{GeV}\,(\tfrac{1}{2}-1)$$

$$M_{1R} \approx |x - Z^2|\,M_{3R} \approx (\tfrac{1}{2}-2)\times 10^{-5}\,M_{3R}$$

$$\approx 10^{10}\,\mathrm{GeV}\,(\tfrac{1}{4}-2)$$

Predictions

Writing only for 2×2 (for Simplicity)

$$U = \begin{array}{c} \begin{array}{cc} c & t \end{array} \\ \begin{pmatrix} 0 & \epsilon+\sigma \\ -\epsilon+\sigma & 1 \end{pmatrix} \end{array} m_U^0 \qquad D = \begin{array}{c} \begin{array}{cc} s & b \end{array} \\ \begin{pmatrix} 0 & \epsilon+\eta \\ -\epsilon+\eta & 1 \end{pmatrix} \end{array} m_D^0$$

$$N = \begin{pmatrix} 0 & -3\epsilon+\sigma \\ 3\epsilon+\sigma & 1 \end{pmatrix} m_U^0 \qquad L = \begin{array}{c} \begin{array}{cc} \mu & \tau \end{array} \\ \begin{pmatrix} 0 & -3\epsilon+\eta \\ 3\epsilon+\eta & 1 \end{pmatrix} \end{array} m_D^0$$

$$\Downarrow$$

$$m_b^0 \simeq m_\tau^0(1-8\epsilon^2) \Rightarrow \boxed{m_b(m_b) \simeq 4.7 \text{ GeV}}$$

$$V_{cb} = \left| \underbrace{\sqrt{\frac{m_s}{m_b}}}_{(0.156)} \underbrace{\left(\frac{\eta+\epsilon}{\eta-\epsilon}\right)^{1/2}}_{(1/2.2)} - \sqrt{\frac{m_c}{m_t}} \left(\frac{\sigma+\epsilon}{\sigma-\epsilon}\right)^{1/2} \right| = |\sigma-\eta|$$

$$\boxed{= 0.042}$$

√ suppressed

ENHANCED ≃ 1.8

$$\Theta^{osc}_{\nu_\mu\nu_\tau} = |\theta^l_{\mu\tau} - \theta^\nu_{\mu\tau}| \approx \left| \sqrt{\frac{m_\mu}{m_\tau}} \left(\frac{\eta-3\epsilon}{\eta+3\epsilon}\right)^{1/2} + \sqrt{\frac{m_{\nu_2}}{m_{\nu_3}}} \right|$$

$$= 0.437 + \sqrt{m_{\nu_2}/m_{\nu_3}} \longrightarrow \boxed{\approx 0.3}$$

Expt

$$\Rightarrow \boxed{\sin^2 2\Theta^{osc}_{\nu_\mu\nu_\tau} = 0.92 \longleftrightarrow 0.99}$$

$$\downarrow$$

$.92 \leftrightarrow 1$

$$m_{\nu_2}/m_{\nu_3} = 1/15 \longleftrightarrow 1/7$$

SMA or LMA ?

$f_{ij}\; 16_i\, 16_j\, \overline{16}_H\, \overline{16}_H / M$

Just with Standard See-Saw ν_L-masses, SMA rather generic

$$m(\nu_L^e) \sim 2\times10^{-5} - 2\times10^{-6}\,\text{eV} \;//\; m(\nu_L^\mu) \sim 3\times10^{-3}\,\text{eV}$$

$$\Theta^{osc}_{\nu_e\nu_\mu} = \Theta^{\ell}_{e\mu} - \Theta^{\nu}_{e\mu} \approx 0.05$$

Situation alters once allow for direct Maj. masses of ν_L's — Most likely to arise through Higher Dim. op. involving GUT & EW VEV's - Through tiny $\sim 10^{-3}$ eV entries → IMPORTANT For $(\nu_e - \nu_\mu)$

$$W \supset g_{12}\, \underbrace{16_1\, 16_2}_{g_{12}(\nu_L^e \nu_L^\mu)}\, \underbrace{16_H\, 16_H}_{(\langle \tilde{\nu}_{RH}\rangle / M_{GUT})^2}\, \underbrace{10_H\, 10_H}_{(v_u^2)} / M^3_{GUT}$$

$$(\sim 175\ \text{GeV})^2$$

$$\boxed{\sim g_{12}(\nu_L^e \nu_L^\mu)\,(1.5-6)\times10^{-3}\,\text{eV}} \quad (\langle 16_H\rangle \approx (1\text{-}2)\, M_{GUT})$$

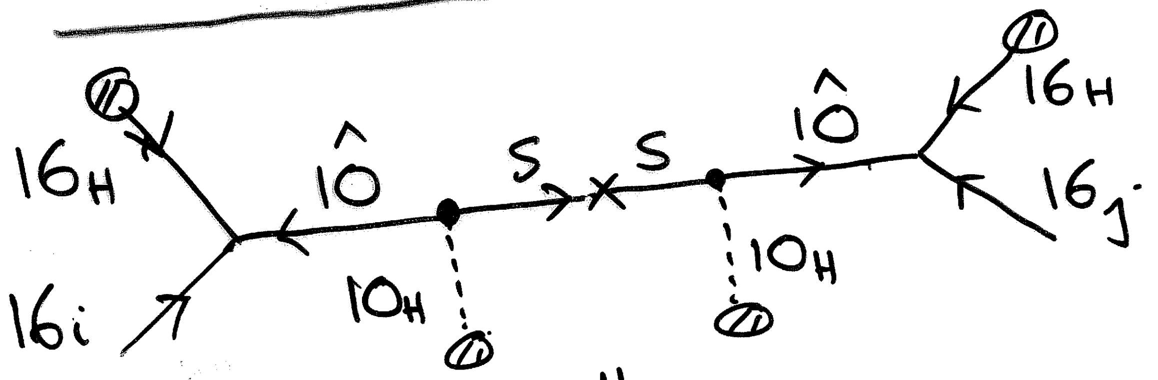
$$\begin{array}{cc} \nu_L^e & \nu_L^\mu \\ \end{array}$$

$$\begin{bmatrix} \approx 0 & (3-4) \\ (3-4) & 7 \end{bmatrix} \times 10^{-3}\,\text{eV} \Longrightarrow \begin{bmatrix} \Theta^{\nu}_{\nu_e\nu_\mu} \approx 1/2 \\ \sin^2 2\Theta^{osc}_{\nu_e\nu_\mu} \sim 0.7 \end{bmatrix}$$ ~~Quite~~ Plausible

Thus LMA not strictly a prediction, but perfectly plausible within the framework.

16.4.~~24.3~~

GUT-SCALE EFFECTIVE OPERATORS



$$\Downarrow$$

$$g_{ij}\ 16_i\, 16_j\, 16_H\, 16_H\ 10_H\, 10_H / (M^2_{\hat{10}} M_S)$$

Θ_{13}

$$m(\nu_L^e \nu_L^\tau)_{\text{Non-seesaw}} \sim (2-6) \times 10^{-3}\,\text{eV}$$

$$\Theta_{13} \sim \frac{(2-6) \times 10^{-3}\,\text{eV}}{5 \times 10^{-2}\,\text{eV}}$$

$$\sim .03 - .1$$

ν-Less 2β decay : $\Delta L = \pm 2$

$$m_{ee} = \left| \sum_i m_i U_{ei}^2 \right|$$

$m_1 \sim$ few $\times 10^{-3}$ eV, $m_2 \approx (6-8) \times 10^{-3}$ eV

$m_3 \approx 5 \times 10^{-2}$ eV

$\Theta_{12} \approx 1/2$, $\Theta_{13} \sim 0.03 - 0.1$

$$\Downarrow$$

$$m_{ee} \sim (1 \text{ to } 6) \times 10^{-3}\,\text{eV}$$

Summary on Fermion Masses & Mixings in the G(224)/SO(10) Framework

Given the bizarre pattern of masses & mixings of quarks, charged leptons and neutrinos, it seems remarkable that the simple pattern of fermion mass matrices*, motivated in large part by the group th of G(224)/SO(10) and the assumption of minimality of the Higgs system makes 7 predictions in agreement with observation.

→ Study Proton Decay // Leptogenesis // $\not{C}\not{P}$ within this framework.

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* Need to understand the origin of flavor Symmetries.
→ Hierarchical entries.

V CP & Flavor Violations

Question: Can observed CP & Flavor Viols. emerge consistently within the SUSY G(224)/SO(10)-Framework, while preserving its successes wrt fermion masses & ν-oscills?

→ A Non-trivial Challenge (Many SO(10)-models do not satisfy both Sets of constraints).

⑤ RESULTS

(Babu, Pati, Rastogi, ph/0410200)

Find For natural phases (~10-40%) in the Dirac mass-parameters $(\sigma, \eta, \epsilon', \cdots)$, of the same G(224)/SO(10)-framework, Can get observed CP & Flavor Viols while preserving the successes in fermion masses & ν-oscillations!

(a) For phases in a natural range, get

$$\boxed{\hat{\eta}_W \approx 0.30-0.37;\ \hat{\rho}_W \approx 0.15-0.18}$$

BPR-Values

$$\updownarrow$$

Close To SM CKM }

$$\bar{\eta}_W \approx 0.34 \quad \bar{\rho}_W \approx 0.18$$

SM CKM Values.

New With Phases Babu, Pati, Rastogi	No Phases Babu, Pati, Wilczek
$\eta = 0.12 - 0.05\,i$	$\eta = 0.15$
$\sigma = 0.1 - 0.012\,i$	$\sigma = 0.11$
$\epsilon = -0.0954$	$\epsilon = -0.095$
$\epsilon' = 1.8 \times 10^{-4}\, e^{i[0.4\pi]}$	$\epsilon' = 2 \times 10^{-4}$
$\zeta_{22}^{d} = 1.1 \times 10^{-2}\, e^{-i[0.8\pi]}$	$\zeta_{22}^{d} \lesssim (1/3) \times 10^{-2}$
$\eta' = 2.5 \times 10^{-3}$	$\eta' = 4.4 \times 10^{-3}$

$\Downarrow$

Preserves all the Predictions of BPW for fermion masses/mixings & ν-OSC

AND USING SUSY YIELDS RIGHT CP & Flavor Viols:

Δm_K, ϵ_K, Δm_{B_d}, Δm_{B_s}, $S(B_d \to J/\psi\, K_S)$, $S(B_d \to \phi K_s)$, $b \to s\gamma$,

→ MANY PREDICTIONS: $\mu \to e\gamma$, $\tau \to \mu\gamma$, $(EDM)_{e,n}$

Ample Further Tests

① $B(\mu \to e\gamma) \approx 10^{-9} - 10^{-13}$; Expt $< 10^{-11}$

② $B(\tau \to \mu\gamma) \approx 10^{-7} - 10^{-9}$; $< 7 \times 10^{-8}$ (BABAR)

③ $(edm)_n \approx (1.6 - 1) \times 10^{-26}$ ecm ; $< 6.3 \times 10^{-26}$ ecm

$\tan\beta = 5, 10$

④ $(edm)_e \approx \dfrac{1.1 \times 10^{-28} \text{ ecm}}{\tan\beta}$

The Two Notable Missing Pieces

⑤ SUSY Particles $\lesssim$ 1TeV $\longrightarrow$ LHC

⑥ PROTON DECAY

$$\left\{ \begin{array}{l} \Gamma^{-1}(p \to e^{+}\pi^{0}) \approx 10^{35\pm1} \text{ yrs} \\ \Gamma^{-1}(p \to \bar{\nu} K^{+}) \lesssim 10^{34} \text{ yrs.} \end{array} \right\}$$

NEED MEGATON SIZE DETECTOR

VI ν Masses ⟷ Matter-Antimatter Asymmetry

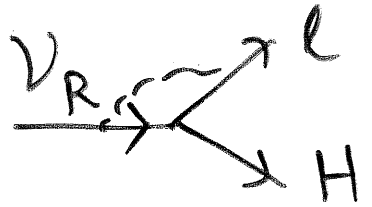
Starting with a baryon-symmetric Universe at $t=0$ (Big Bang) or just at the beginning of reheating following inflation, what eventually caused

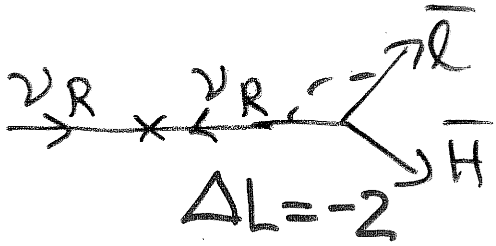
$$Y_B \equiv \frac{n_B - n_{\bar{B}}}{s} = 9 \times 10^{-11}$$ (WMAP)

Sakharov (
Need (1967)
① C & CP Non-Conservation
② $\Delta B \neq 0$ Processes
③ Thermodynamic Inequilibrium

Most Promising Explanation

$M(\nu_R)\,\nu_R^T\,\nu_R + h.c.$ ($\Delta L = \pm 2$)





$\Delta(B-L) \neq 0$

Fukugita, Yanagida

$\Longrightarrow$ $\boxed{n_\ell \neq n_{\bar{\ell}}}$ With $T_{RH} \approx 10^6 - 10^8$ GeV

$Y_L \neq 0 \xrightarrow{SU(2)\times U(1)} Y_B \neq 0$ (Kuzmin, Rubakov, Saposhnikov

G(224)/SO(10) Theory

JCP, Phys. Rev 2003, hep-ph/0209160

Note All Vertices & Dirac Phases determined.

$$\nu_R \to \ell + H \;;\; \bar{\ell}\,\bar{H} \;\Big\}\; \Delta(B-L) \neq 0$$

$$\Delta B = \boxed{\frac{\Delta(B+L)}{2}} + \frac{\Delta(B-L)}{2}$$

ERASED BY EW SPHALERONS

$$\frac{(Y_B)_{\text{Thermal}}}{\sin 2\phi_{21}} \simeq (10-30)\times 10^{-11} \quad \text{THEORY}$$

$$\text{Non-Thermal} \simeq (10-100)\times 10^{-11} \quad \text{THEORY}$$

$$\boxed{\text{WMAP} \approx 9 \times 10^{-11}}$$

OBSERVED

Agrees with obs. For natural values of $\phi_{21} \approx 1 - 1/10$!

A Simple & Unified Picture of fermion masses, ν oscillations, CP Violation & Baryo-Leptogenesis IN Full Accord With Observations !

EXISTENCE OF ν_R, B-L & SU(4)-Color CRUCIAL ! AS ALSO EW $SU(2)_L \times U(1)_Y$!

Summary

2 Alt. Scenarios $\begin{cases}\nearrow \text{Thermal} \\ \searrow \text{Non-Thermal}\end{cases}$

within G(224)/SO(10) Framework

↓

Unified description of not only fermion masses & ν oscill (Consistent with maximal atmospheric & LMA) but also of baryogenesis via leptogenesis.

- The Exist of RH neutrinos
- B-L Local Symmetry
- q-ℓ unif SU(4)-Color
- See Saw Mechanism
- & SUSY unif Scale

Crucial roles in this unified description.

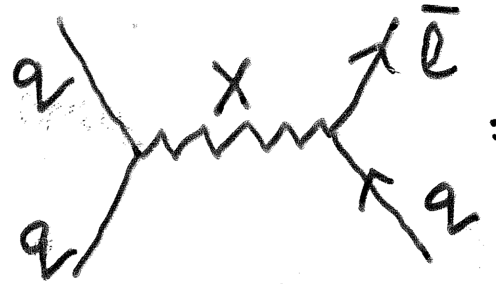
⟹ Relevance of G(224)/SO(10) in 4D below String Scale.

→ $\boxed{\text{CP Phases} \rightarrow \text{Sign of } Y_B \text{ a Challenge}}$

VII PROTON DECAY : THREE MECHANISMS

① d = 6 Gauge Med. SUSY SU(5)/SO(10)

$M_X \approx M_Y$
$\approx 10^{16}$ GeV
$(1 \pm 25\%)$



$\Rightarrow$ Amp $(p \to e^+ \pi^0) \propto g^2/M_X^2$

Chiral Lag Param (D+F) $\simeq 1.25$ // $A_R \simeq 3.4$

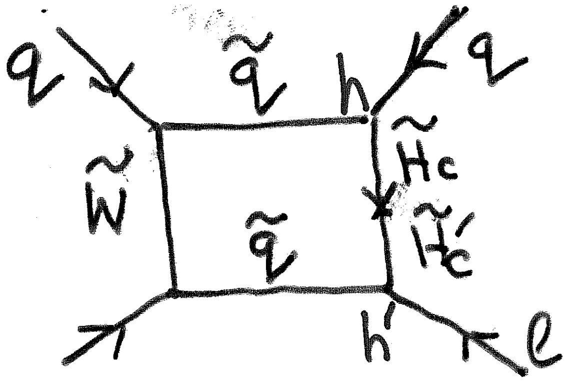
$\boxed{H \simeq 0.01 \text{ GeV}^3 \; (1/\sqrt{2} - \sqrt{2})}$

$\Gamma^{-1}(p \to e^+ \pi^0) \simeq 10^{35 \pm 1}$ yrs (Theory)

Can be as short as $\sim 10^{34}$ yrs.

② SUSY SU(5) / SO(10) Standard d=5

Color Triplet Higgsino Med Sakai, Yanagida, Weinberg



$\tilde{H}_C \subset (10_H)_{SO(10)}$
$= (2,2,1) + (1,1,6)$

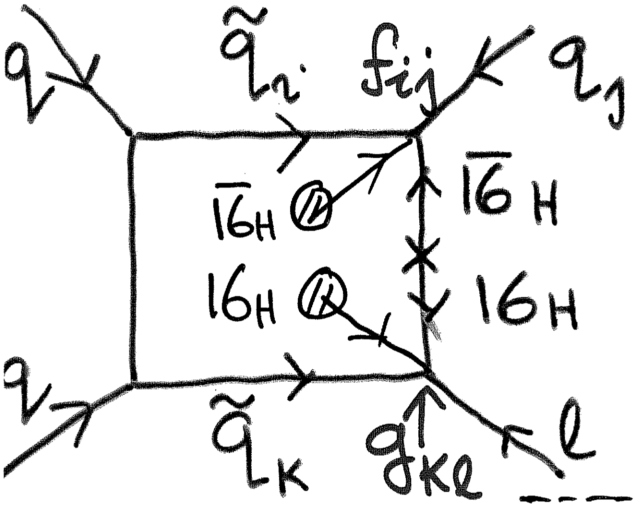
$$\text{Amp} \propto (hh'/M_{H_C})\,(m_{\tilde{W}}/m^2_{\tilde{q}})\,\alpha_2$$

$p \to \bar{\nu} K^+$ (dominant)

$\to \mu^+ K^0$ (Suppressed, BR$\sim 10^{-5}$) std. d=5

③ New d=5 ν-Mass Related operators

Babu, Pati, Wilczek



$f_{ij}\ 16_i\, 16_j\, \overline{16}_H \overline{16}_H / M$

$\Downarrow$

Maj Masses of ν_R's

$\boxed{\text{Generically } 16_i\, \overline{16}_H \text{ in } 45\ \&\ \underline{1} \text{ of } SO(10)}$

$\dashrightarrow g_{K\ell}\ 16_K\, 16_\ell\ 16_H\, 16_H / M$

↳ $V_{CKM} \neq \mathbb{1}$

Indep of tanβ

$$\boxed{\Gamma^{-1}(p \to \bar{\nu} K^+)_{\text{New } d=5} \approx 10^{33} - 10^{34}\ \text{yrs}}$$

SUSY SO(1
or
G(224)

$$\boxed{BR(p \to \mu^+ K^0)_{\text{New } d=5} \approx (10-30)\%}$$

Note these contributions from new d=5 would generically be present, even if "standard" d=5 (with color triplets ⊂ 10_H) absent, as in SUSY G(224) !

The $\mu^+ K^0$ mode a signature of this mechanism.

d = 5 proton decay rate Calculation(*)

Babu, Pati, Wilczek (99) // Dermisek, Raby (2000) // Lucas & Raby // Pati (2001 - ph/0106082) // June 2003 – ph/0305221 // KEK Talk ph/0407220) //

Incorporate Dep. of d = 5 p-decay amp. on

- Masses & Mixings of All Fermions
- Unif. Scale Threshold Effects Incl. D-T splitting ⟷ Natural Coupling Unif.
- Take

① $|\beta_H| \simeq |\alpha_H| = (0.01\,\mathrm{GeV}^3)(1/\sqrt{2} - \sqrt{2})$

② $\tan\beta \gtrsim 3$

③ $A_L \simeq 0.32$ (2 Loop)

④ $A_S \simeq 0.93$

⑤ $m_{\tilde{q}} \approx 1.2\,\mathrm{TeV}\,(1/\sqrt{2} - 2)$ (Focus Point)

⑥ $(m_{\tilde{W}}/m_{\tilde{q}}) \approx 1/6\,(1/2 - 2)$

Lattice // Domain Wall Fermions + Non-Pert Ren
Quenching error Small
$|\alpha_H| \simeq |\beta_H| \simeq 0.01\,\mathrm{GeV}^3$
Aoki et al (2006)

(*) Dimopoulos, Raby, Wilczek (82) // Ellis, Nanopoulos, Rudaz // Nath, Chamseddine, Arnowitt // Nath, Arnowitt (97) // Hissano, Murayama, Yanagida (93) // Hisano (2004) // Babu, Barr (95) //

Doublet-Triplet Splitting in SO(10) & Meff

Dimopoulos, Wilczek // Babu, Barr
Babu, Pati, Wilczek

$$W_H = \lambda\, 10_H\, 45_H\, 10'_H + M_{10}\, 10'^2_H + M_{16}\, 16_H\, \overline{16}_H$$
$$+ \lambda'\, \overline{16}_H\, \overline{16}_H\, 10_H$$

$$\langle 45_H \rangle = (a, a, a, 0, 0) \times \tau_2 \quad ; \quad a \sim M_{GUT}$$

$$\langle 45_H \rangle \propto B-L. \Rightarrow \langle (1,1,15) \rangle = (a\, a\, a\, {-3a}).$$

$$\left(\bar{5}_{10_H} \;\; \bar{5}_{10'_H} \;\; \bar{5}_{16_H} \right) \begin{bmatrix} 0 & \lambda \langle 45_H \rangle & \lambda' \langle \overline{16}_H \rangle \\ -\lambda \langle 45_H \rangle & M_{10} & 0 \\ 0 & 0 & M_{16} \end{bmatrix} \begin{pmatrix} 5_{10_H} \\ 5_{10'_H} \\ 5_{\overline{16}_H} \end{pmatrix}$$

⇒ One pair of Higgs doublets light, triplets Heavy

$$H_u = 10_u \; ; \quad H_d = \cos\gamma \; 10_d + \sin\gamma \; 16_d$$

1) $\tan\gamma = \lambda' \langle \overline{16}_H \rangle / M_{16} \Rightarrow \boxed{\tan\beta = (\cos\gamma)(m_t/m_b)}$

b) $$\boxed{A(d=5)_{std} \approx \left(h_{33}^2 / M_{eff} \right) (2 \times 10^{-5})}$$

$$\boxed{M_{eff} \equiv (\lambda a)^2 / M_{10'} \approx M_{GUT}^2 / M_{10'}}$$

$$\Rightarrow M_{eff} \neq M_{H_C} \; ; \; \text{If } M_{10'} < M_{GUT} \Rightarrow \boxed{M_{eff} > M_{GUT}}$$

M_{eff} Gets Bounded From above by Threshold Corr. & Demand of Natural Coupling Unif

$$\Delta\alpha_3(m_Z)_{DT} = \frac{\alpha_3(m_Z)^2\,(9/7)}{2\pi}\,\ln\frac{M_{eff}\cos\gamma}{M_X} > 0$$

$$\Delta\alpha_3(m_Z)_{45_H} \approx -4\%$$

$$\Delta\alpha_3(m_Z)_{\text{gauge multiplet}} \simeq -1.5 \text{ to } -2\%$$

$$\Delta\alpha_3(m_Z)|_{16_H+\overline{16}_H} \simeq \pm 2\%$$

For Coupling Unif need

$$\Delta\alpha_3(m_Z)_{Net} \approx DT + 45_H + \text{Gauge} + (16,\overline{16})_H$$

$$\approx -5 \text{ to } -8\%$$

$$\Downarrow$$

∴ For coupling Unif need

$$\Delta\alpha_3(m_Z)|_{DT} < 2\%$$

$$\Downarrow$$

$$\boxed{M_{eff} < 2\times10^{18}\ \text{GeV}}$$

$$\Downarrow$$

UPPER LIMIT ON PROTON LIFETIME

The Case of ESSM (Extended Supersymm. Standard Model)

Babu, Pati (96,97)
Babu, Pati, Stremnitzer (92,93)

Introduce 1 Pair of Vector-Like Families $(16_V + \overline{16}_V)$ at TeV-Scale & Superpartners.

→ [3 chiral 16_i + $(H_u + H_d)$ + $(16_V + \overline{16}_V)$ at ~ TeV] + SUSY Partners at 1 TeV

Consistent with (i) ν Counting // (ii) Precision EW Tests

Apriori Motivations:

(i) Removal of Unif. Mismatch

$$M_{GUT} \sim (1/2 - 2) \times 10^{17}\ \text{GeV}$$
$$\updownarrow$$
$$M_{string} \approx 3.6 \times 10^{17}\ \text{GeV (Pert.)}$$

(ii) Dilaton stabilization

(iii) $(g-2)_\mu$?

MSSM
0.125-0.13

(iv) $\alpha_3(m_Z)^0_{ESSM} \approx 0.112 - 0.118$
EXPT $\approx 0.118 \pm 0.003$

(V) Enhances Proton Decay Rate
$d=5$

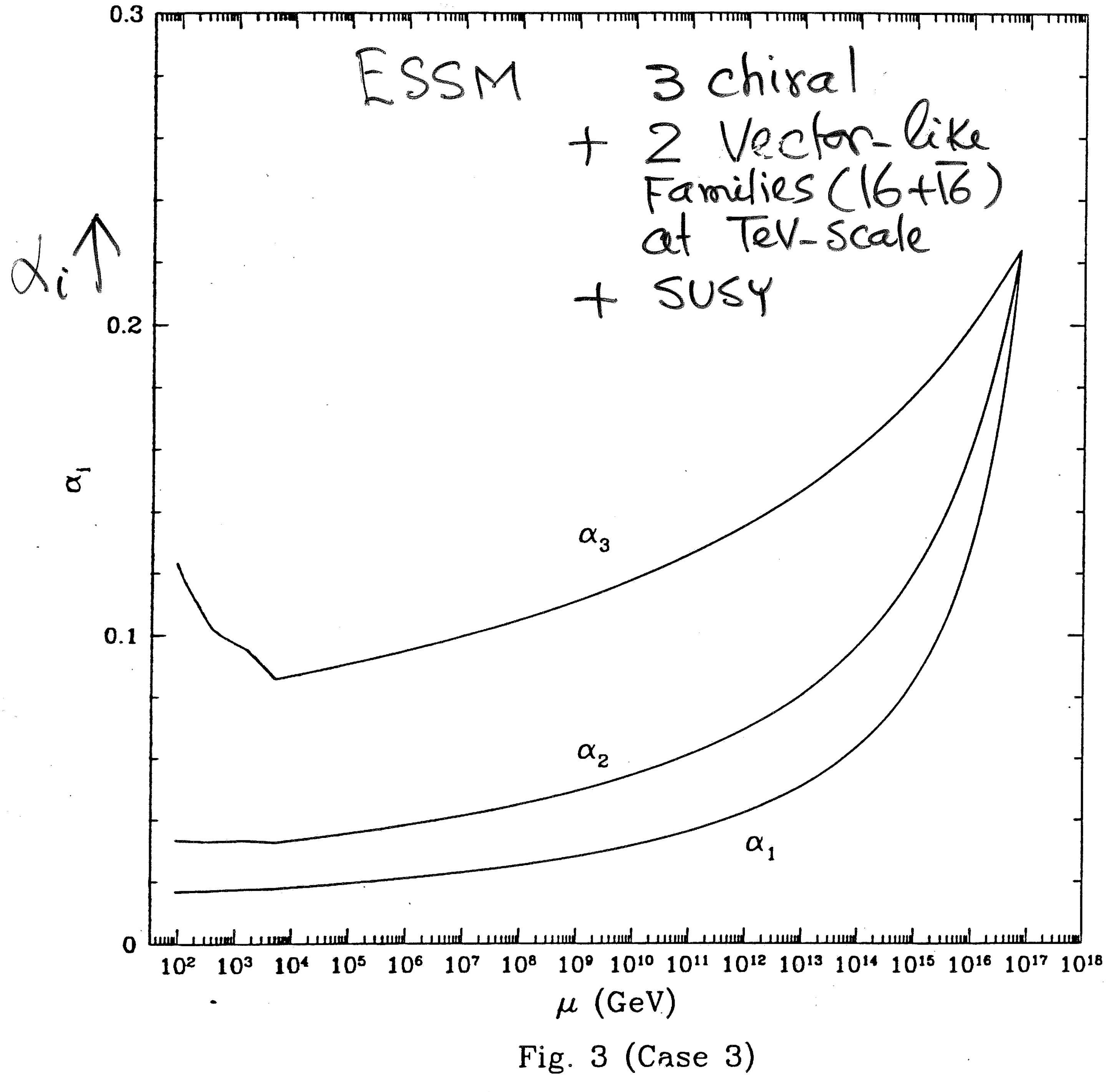


Fig. 3 (Case 3)

Babu, Pati. Phys. Lett (96)

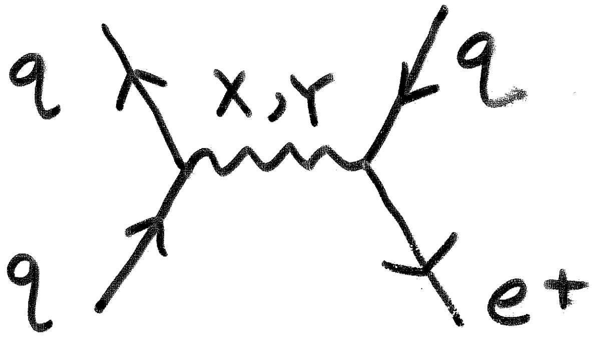
SUSY SO(10)/ESSM (Also SU(5)

α_{GUT} increases by 6-7 (Rel to MSSM)

M_{GUT} " by 2.5 - 5

$$\Gamma^{-1}(p\to e^+\pi^0)_{\substack{\text{ESSM/}\\ \text{SO(10)}\\ \text{or SU(5)}}} \approx (1\text{-}17)\,\Gamma^{-1}(p\to e^+\pi^0)_{\substack{\text{MSSM/}\\ \text{SO(10)}\\ \text{or SU(5)}}}$$

∴



SuperK Limits (50 KT Water Cerenkov)
(2005)

$$\Gamma^{-1}(p \to \bar{\nu} K^+) < 2.3 \times 10^{33} \text{ yrs}$$

$$\Gamma^{-1}(p \to e^+ \pi^0) < 6.5 \times 10^{33} \text{ yrs}.$$

Summary of Results

) $\Gamma^{-1}(\bar{\nu} K^+) \lesssim 0.33 \times 10^{33}$ yrs $\left\{\begin{matrix} \text{MSSM} \\ \downarrow \\ \text{SU(5)} \end{matrix}\right\} \rightarrow$ Excluded.

) $\Gamma^{-1}(\bar{\nu} K^+) \lesssim \frac{2.5}{1.5} \times 10^{33}$ yrs $\{\text{MSSM} \rightarrow \text{SO(10)}\}$. Tightly Constrained ↓ Disfavored.

) $\Gamma^{-1}(\bar{\nu} K^+) \approx (10^{33} - 10^{34})$ yrs. $\left\{\begin{matrix} \text{Most plausible} \\ \text{range For} \\ \text{ESSM} \rightarrow \text{SO(10)} \end{matrix}\right\}$

↑

Can realize even for nearly central values of param // & also with $m_{\tilde{q}} < 1$ TeV // $\tan\beta \approx 3 - 20$ //

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④ MSSM or ESSM → String G(224)   ONLY New op. contribute.

$$\Gamma^{-1}(\bar{\nu} K^+)^{\text{Nearly Central}}_{\text{New } d=5} \approx (1-6) \times 10^{33} \text{ yrs.}$$

$$\Gamma^{-1}(\bar{\nu} K^+) \lesssim 10^{34} \text{ Yrs.}$$

⇓

Thus Conclude,

$$\boxed{\Gamma^{-1}(\bar{\nu} K^+) \lesssim (1/2 - 2) \times 10^{34} \text{ yrs.}}$$

SUSY SO(10)/G(224)

$$\Downarrow$$

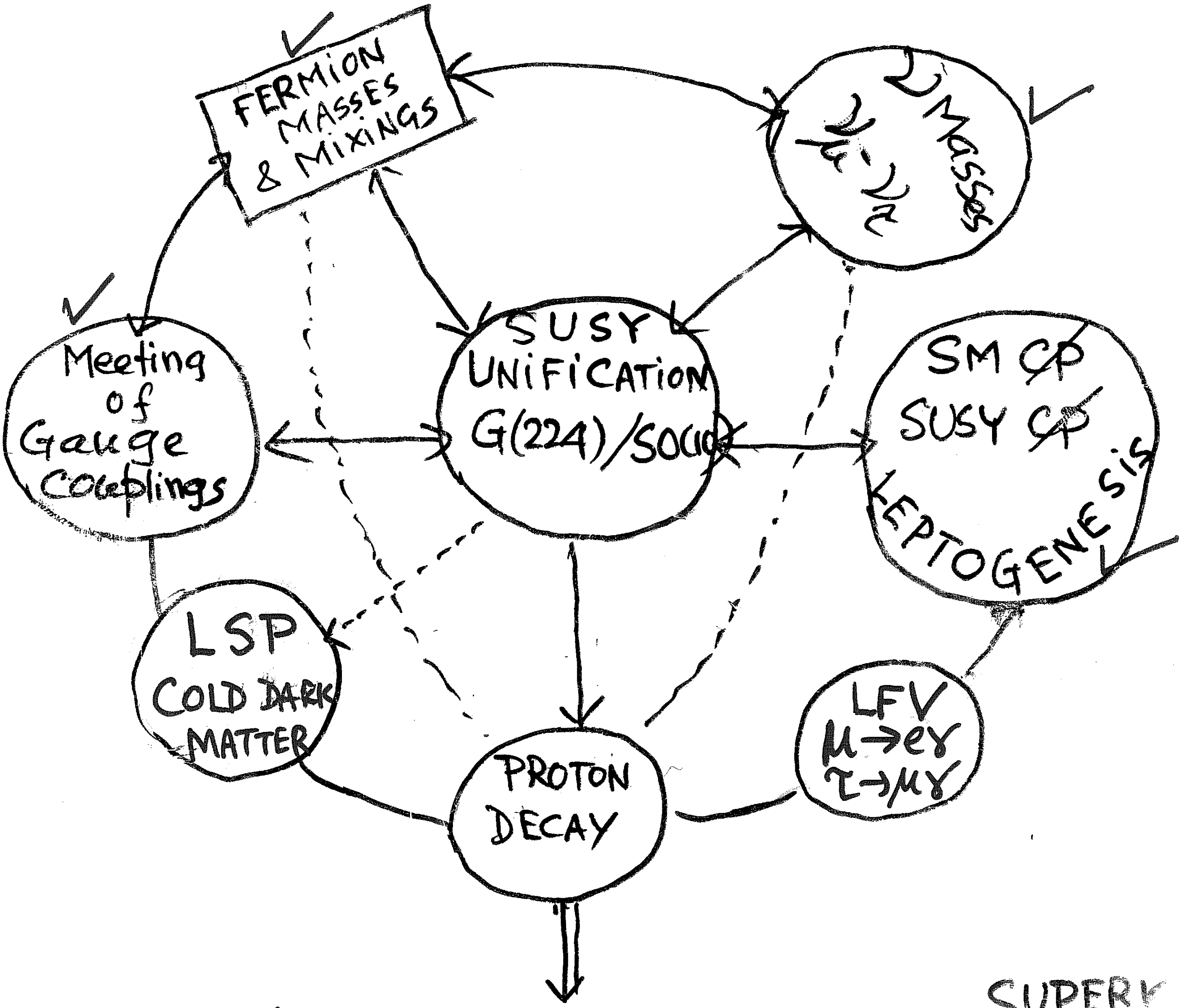
	SuperK
$\Gamma^{-1}(p \to e^{+}\pi^{0}) \simeq 10^{35\pm1\,\mathrm{yrs}}$	$> 6 \times 10^{33}$ yrs
$\Gamma^{-1}(p \to \bar{\nu} K^{+}) \simeq 10^{30-34}$ yrs.	$> 2.3 \times 10^{33}$ yrs

Need Improvement by Factor 10

$$\downarrow$$

A Megaton Size Detector

UNIFICATION LINKS



SUPERK

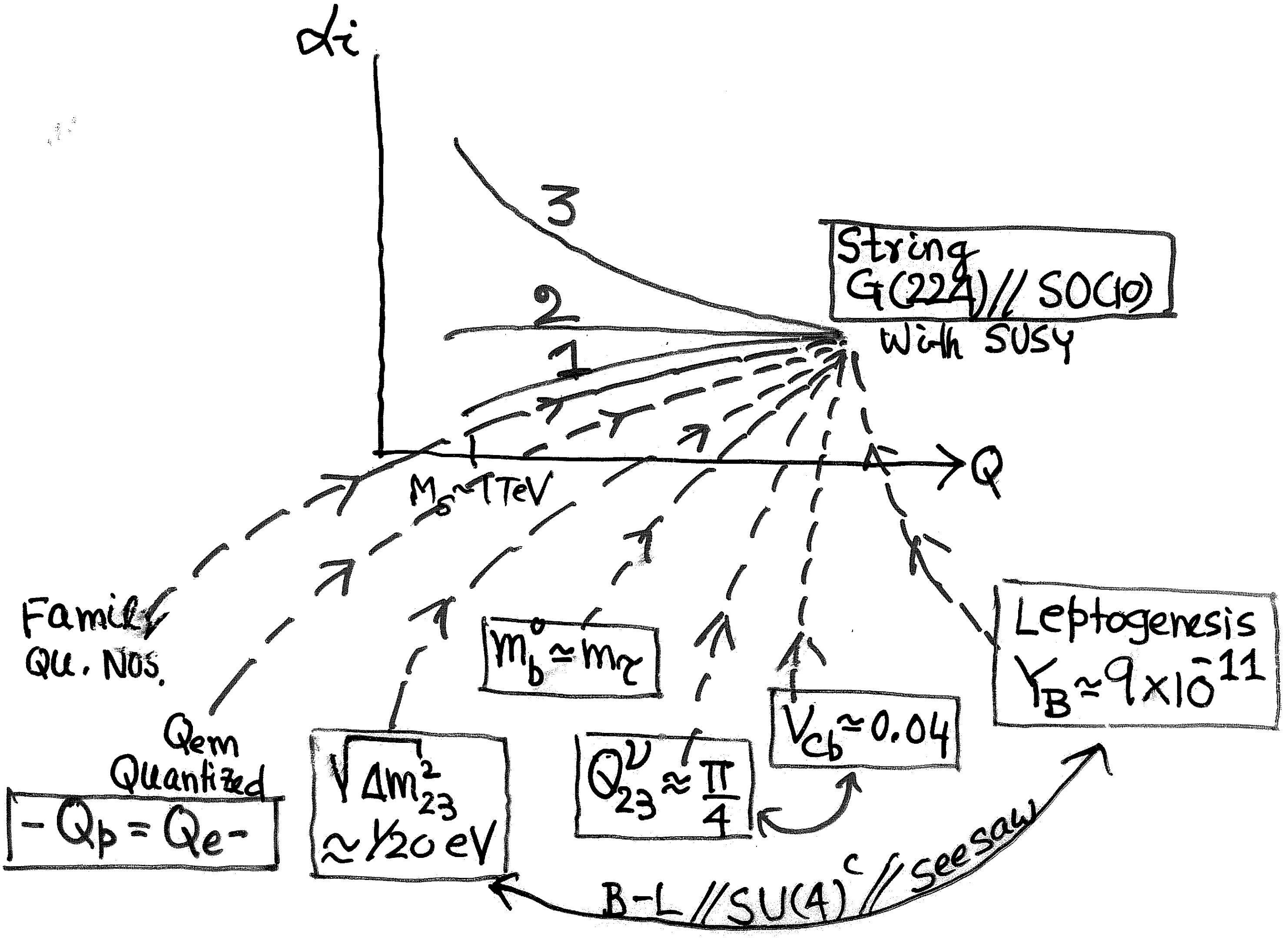
$$\Gamma^{-1}(p \to e^+\pi^0) \approx 10^{35\pm1}\ \text{yrs} \qquad > 6\times10^{33}\ \text{yrs}$$

$$\Gamma^{-1}(p \to \bar{\nu}K^+) \approx 10^{30}-10^{34}\ \text{yrs} \qquad > 2\times10^{33}\ \text{yrs}$$

PROTON DECAY AND SUPERSYMMETRY ARE THE TWO MISSING LINKS

→ NEED A MEGATON SIZE DETECTOR

VIII Rendevouz In Particle Physics



All these features hang together neatly within a Single unified framework → Hard to believe this can be a mere Coincidence.

Rendevous Incomplete → Two Missing Links

1) Supersymmetry → LHC

2) Proton Decay → Need Next Gen. Detector.

3 Important New Ideas Beyond Grand Unif

① Supersymmetry

② Cosmology // Inflation //

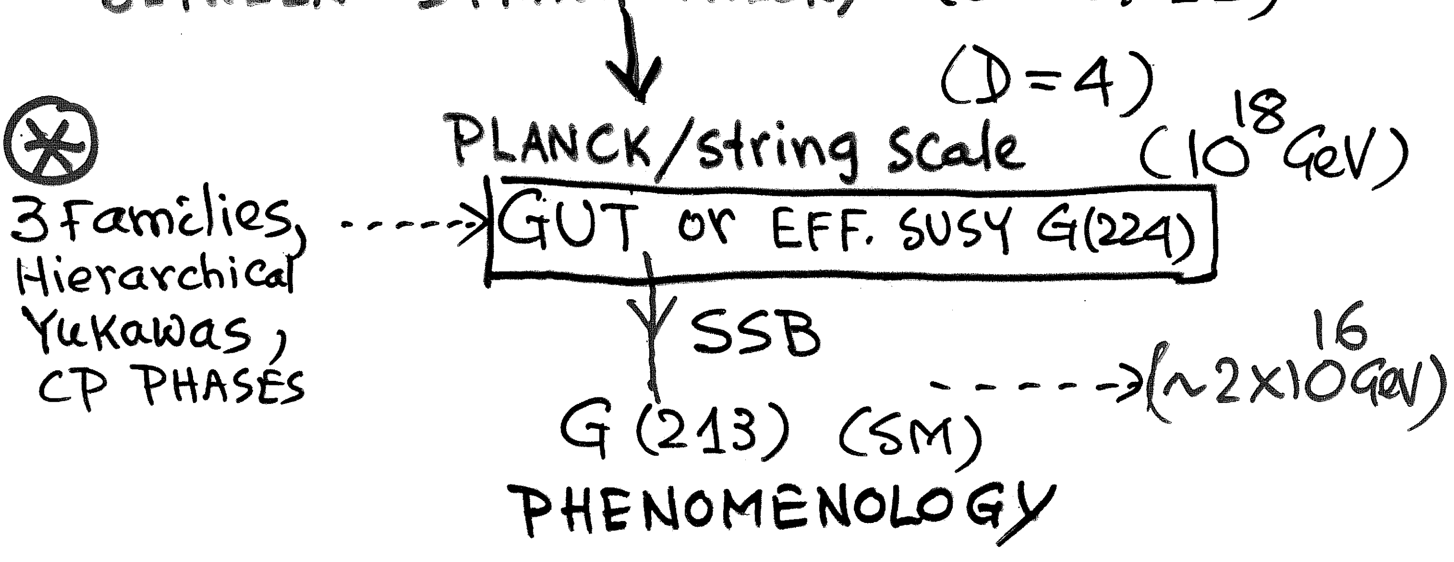
↓ GUT Symm Breaking

Monopole Soln

Leptogenesis → Baryogenesis

③ String/M Th → Extra Dim → Compactification

Grand Unif A VERY USEFUL BRIDGE BETWEEN STRING THEORY (D=10/11)



(D=4)

PLANCK/string scale (10^{18} GeV)

⊛ 3 Families, Hierarchical Yukawas, CP PHASES ----> GUT or EFF. SUSY G(224)

SSB

G(213) (SM) ----> ($\sim 2 \times 10^{16}$ GeV)

PHENOMENOLOGY

⊛ MANY INTERESTING PIECES OF WORK DERIVING EFF. SUSY G(224) with 3 Fam.

Puzzles & Challenges

TO realize a <u>TRULY UNIFIED THEORY</u> WITH GOOD QUANTUM GRAVITY - That is <u>predictive</u>, explaining at least Some of the major puzzles

Be it String/M Th or Something not yet Known

1) Why 3 Families ?
2) Hierarchical Masses/Mixings/CP ?
3) Why live in $D = 4$?
4) Dark Energy → Cosmological Constant

$$(\Lambda_{cosm}/M_{Pl})^4 \sim 10^{-120} \quad ?$$