The Abdus Salam International Centre for Theoretical Physics

United Nations Educational, Scientific and Cultural Organization

IAEA International Atomic Energy Agency

1856-60

2007 Summer College on Plasma Physics

30 July - 24 August, 2007

Plasma effects in cold atom physics

J.T. Mendonca
Instituto Superior Tecnico, Lisbon, Portugal

Plasma effects in cold atom physics

J. T. Mendonça

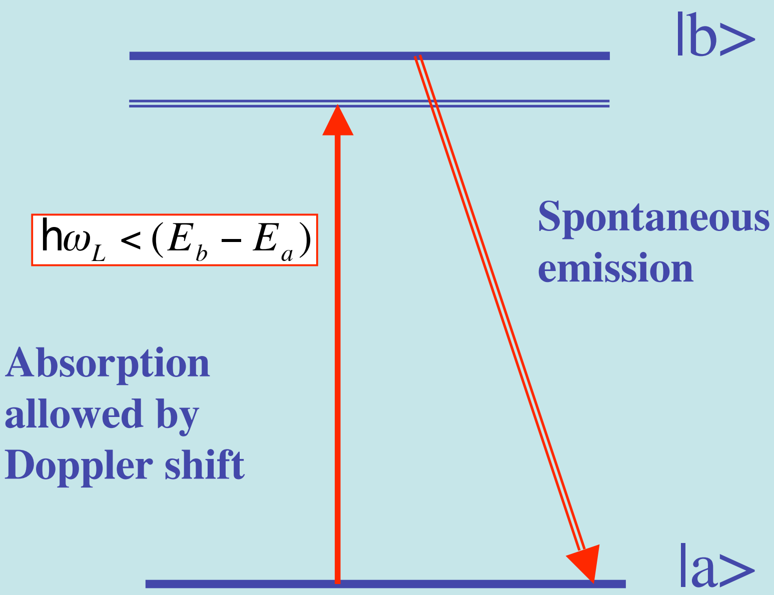
GoLP and CFIF, Instituto Superior Tecnico, Lisboa

Outline:

1. Laser cooling forces;
2. From wave equation to kinetic equation;
3. Collective oscillations of a cold atom gas;
4. Mie resonance (observations?);
5. Plasma-acoustic mode;
6. Coulomb-like explosions;
7. Tonks-Dattner resonances;
8. Rydberg plasmas;
9. Waves in could quantum plasmas;
10. Conclusions.

1. Principle of laser cooling

Energy picture



The atom looses kinetic energy at each absorption-emission cycle

Momentum picture

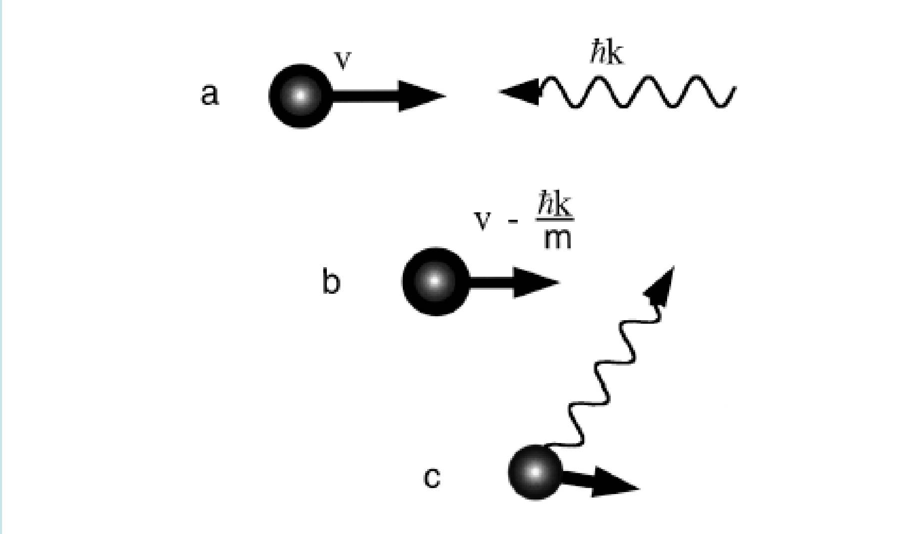


FIG. 1. (a) An atom with velocity v encounters a photon with momentum $\hbar k = h/\lambda$; (b) after absorbing the photon, the atom is slowed by $\hbar k/m$; (c) after re-radiation in a random direction, on average the atom is slower than in (a).

Taken from W.D. Phillips, RMP (1998)

Laser cooling forces

1) Induced light pressure force
[Ashkin, PRL (1970)]

$$F = F_0 - \beta v + O(v^2)$$

$$F_0 = \hbar q \Gamma I \frac{\Gamma^2}{\Gamma^2 + \Delta^2},$$

$$\beta = \frac{4MI}{[1+(\Delta/\Gamma)^2]^2} \left(\frac{\varepsilon}{\Gamma}\right) \Delta$$

Recoil energy $\varepsilon = \frac{\hbar q^2}{2M}$

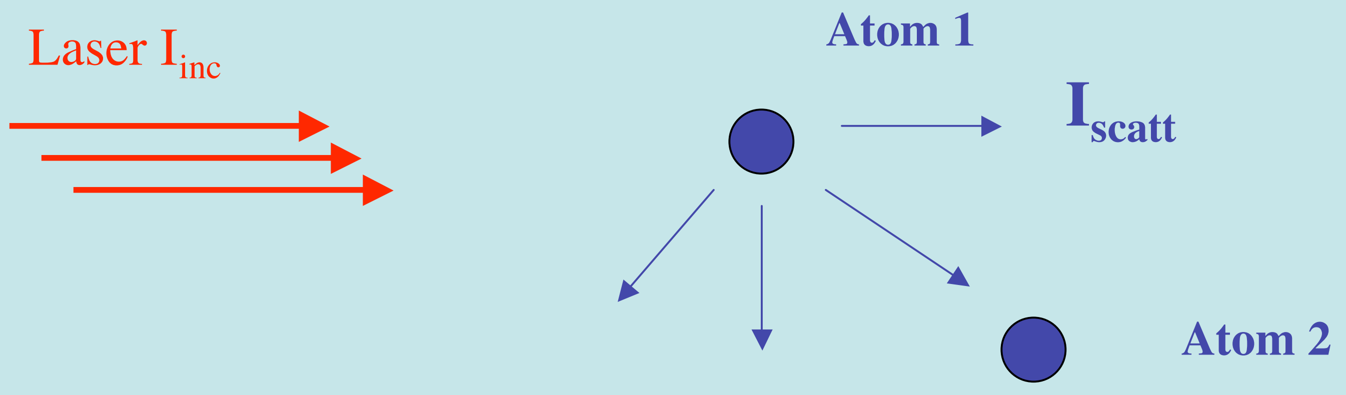
2) Shadow effect or absorption force
[Dalibard, Opt.Commun. (1988)

$$\vec{\nabla} \cdot [\vec{F}_A(\vec{r})] = -\sigma_L^2 \frac{I}{c} n(\vec{r})$$

3) Repulsive effect or radiation trapping force [Sesko et al., JOSA B (1990)]

$$\vec{\nabla} \cdot [\vec{F}_R(\vec{r})] = \sigma_R \sigma_L \frac{I}{c} n(\vec{r})$$

Basic principle of the repulsive force

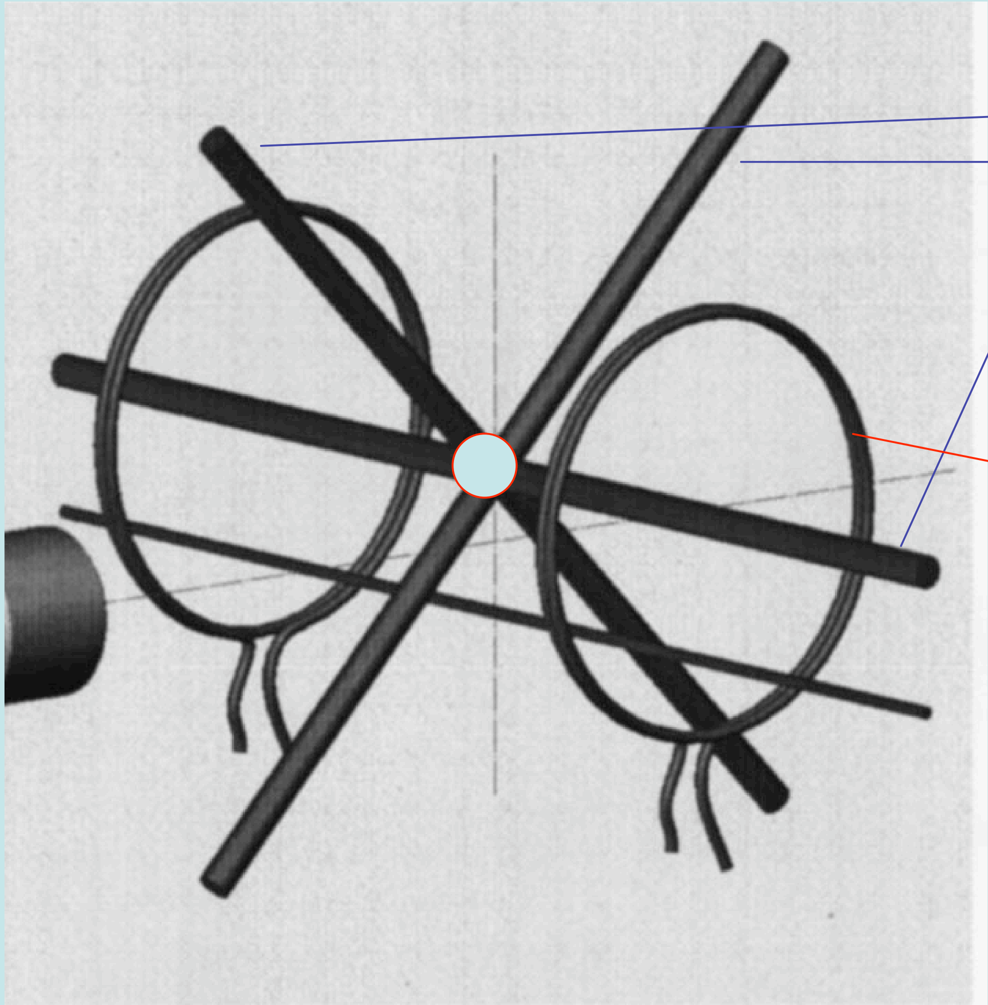


Atomic repulsion results from radiation pressure of the scattered radiation ($I_{scatt} \sim 1 / r^2$)

$$\nabla \cdot \left[\vec{F}(\vec{r}) \right] = Q n(\vec{r}), \qquad Q = (\sigma_R - \sigma_L)\sigma_L I / c$$

Competing effect: repulsive force dominates over shadow effect

Magneto-optical traps (MOTs)



3 pairs of laser beams, for cooling

Helmotz coils, for magnetic confinement

Rubidium, the most popular cold gas

$5S_{1/2} \rightarrow 5P_{3/2}$ ^{85}Rb transition

2. From the Wave Equation ...

Schrödinger's equation

$$i\hbar\frac{\partial}{\partial t}|\vec{r},\vec{R}> = H|\vec{r},\vec{R}>$$

$$H \equiv H(\vec{r},\vec{R},t) = \frac{1}{2m}(\vec{p}+e\vec{A})^2 + \frac{1}{2M}(\vec{P}-Ze\vec{A})^2 + V(r)$$

Potentials

$$V(r) = -\frac{Z_{eff}e^2}{4\pi\varepsilon_0 r},$$

$$\vec{A}(\vec{R},t) = \vec{A}_0\exp(i\vec{k}\cdot\vec{R}-i\omega t) + \vec{A}_S(\vec{R},t) + \vec{A}_C(\vec{R})$$

Laser cooling beam

Scattered radiation

Confining static field

2... to Wave Kinetic equation

Wigner matrix

$$W_{nk}(\vec{R},\vec{q},t) = \int \Phi_n^*(\vec{R} + \vec{s}/2,t)\Phi_k(\vec{R} - \vec{s}/2,t)\exp(-i\vec{q}\cdot\vec{s})d\vec{s}$$

Wigner-Moyal equation

$$\left(\frac{\partial}{\partial t} + \frac{\hbar\vec{q}}{M}\cdot\frac{\partial}{\partial\vec{R}}\right)W_{nn} = \sum_k h_{nk}(\omega)[W_{nk}^{(-)} - W_{nk}^{(+)}]\exp(i\vec{k}\cdot\vec{R} - i\Delta\omega t)$$

$$\Delta\omega = \omega - \omega_{nk}, \qquad h_{nk}(\omega) = \frac{\omega}{\hbar}A_0 p_{nk}, \qquad W_{nk}^{(\pm)} = W_{nk}(\vec{R},\vec{q} \pm \vec{k}/2,t)$$

Quasi-classical approximation

$$W_{nk}^{(\pm)} \approx W_{nk} \pm \frac{\vec{k}}{2}\cdot\frac{\partial}{\partial\vec{q}}W_{nk} + \frac{\vec{k}\vec{k}}{2^3}\cdot\frac{\partial^2}{\partial\vec{q}^2}W_{nk}$$

Force term

Diffusion term

3. Collective forces in cold atom gas

Wave kinetic equation in the quasi-classical limit

$$\left[\frac{\partial}{\partial t} + \vec{v}\cdot\frac{\partial}{\partial \vec{r}} + \frac{1}{M}\left(\vec{F}_{conf} + \vec{F}\right)\cdot\frac{\partial}{\partial \vec{v}}\right]W = 0$$

Collective (shadow - repulsive) force

$$\nabla\cdot\vec{F} = Qn(\vec{r},t) \equiv Q\int W(\vec{v})d\vec{v}$$

Coulomb-like atom-atom interaction

$$Q = (\sigma_R - \sigma_L)\sigma_L I/c$$

INSTITUTO SUPERIOR TÉCNICO

Equilibrium

$$\vec{F}_{conf} + \vec{F}_0 = 0, \qquad \nabla \cdot \vec{F}_0 = Q n_0(\vec{r})$$

Perturbation

$$\delta\vec{F} = \vec{F}_{conf} + \vec{F} \propto \exp(i\vec{k}\cdot\vec{r} - i\omega t),$$
$$W(\vec{r},\vec{v},t) = W_0(\vec{v}) + \tilde{W}(\vec{v})\exp(i\vec{k}\cdot\vec{r} - i\omega t)$$

Linearized evolution equations

$$\tilde{W} = -\frac{i}{M}\frac{\delta\vec{F}\cdot\partial W_0/\partial\vec{v}}{(\omega - \vec{k}\cdot\vec{v})}$$
$$i\vec{k}\cdot\delta\vec{F} = Q\int \tilde{W}(\vec{v})\,d\vec{v}$$

Dispersion relation for cold atom gas (infinite geometry)

$$1 + \frac{Q}{Mk^2}\int \frac{\vec{k}\cdot\partial W_0/\partial\vec{v}}{(\omega - \vec{k}\cdot\vec{v})}\,d\vec{v} = 0$$

Dispersion relation similar to that of electrostatic waves in a plasma

$$1 + \chi(\omega, \vec{k}) = 0$$

Mono-kinetic distribution

$$W_0(\vec{v}) = N_0 \delta(\vec{v} - \vec{v}_0)$$

$$1 - \frac{Q N_0}{M(\omega - \vec{k} \cdot \vec{v}_0)^2} = 0$$

For $v_0 = 0$, cold atom oscillations similar to plasma oscillations (compare with ω_{pe})

$$\omega = \omega_P \equiv \sqrt{\frac{Q N_0}{M}}$$

Effective atomic charge

$$q_{eff} = \sqrt{\varepsilon_0 Q}$$

Typical experimental value, $q_{eff} = 10^{-6}$ e

4. Mie resonance

Centre of mass position

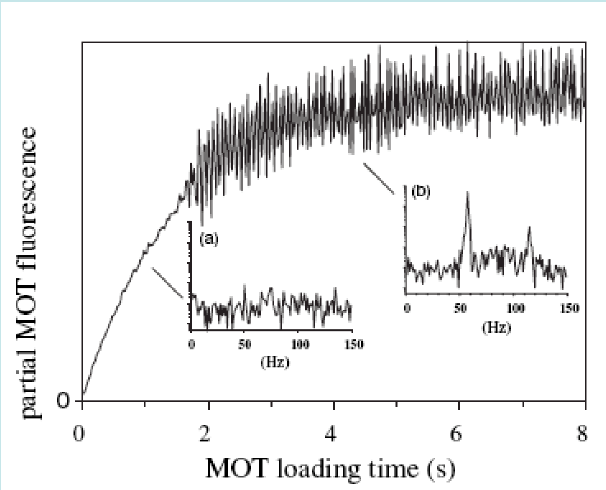
$$\vec{R}(t) = \frac{1}{N}\int\int d\vec{r}\, d\vec{v}\, W(\vec{r},\vec{v},t)\,\vec{r}$$

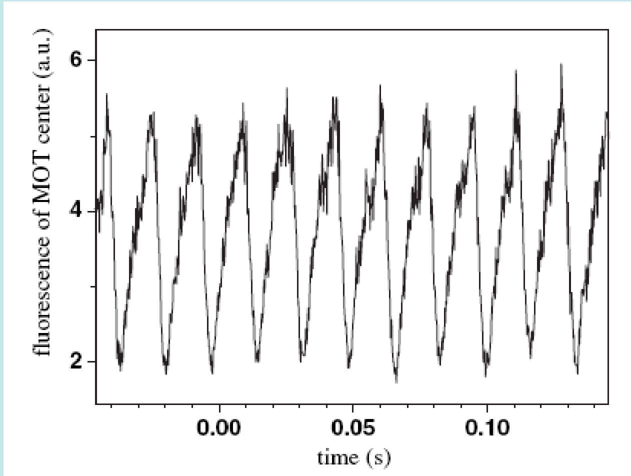
Global equation of motion

$$\frac{d^2\vec{R}}{dt^2} - \omega_M^2 \vec{R} = \vec{f}(t)$$

Mie frequency (similar to dust Mie frequency in a plasma)

$$\omega_M = \sqrt{\frac{QN_0}{3M}}$$





Experiments by G. Labeyrie et al., PRL (2006)

5. Plasma-acoustic mode

Fluid equations for the cold gas

$$\frac{\partial \vec{v}}{\partial t} + \vec{v} \cdot \nabla \vec{v} = -\frac{\nabla P}{Mn} + \frac{\vec{F}}{M}$$

$$\frac{\partial n}{\partial t} + \nabla \cdot (n\vec{v}) = 0$$

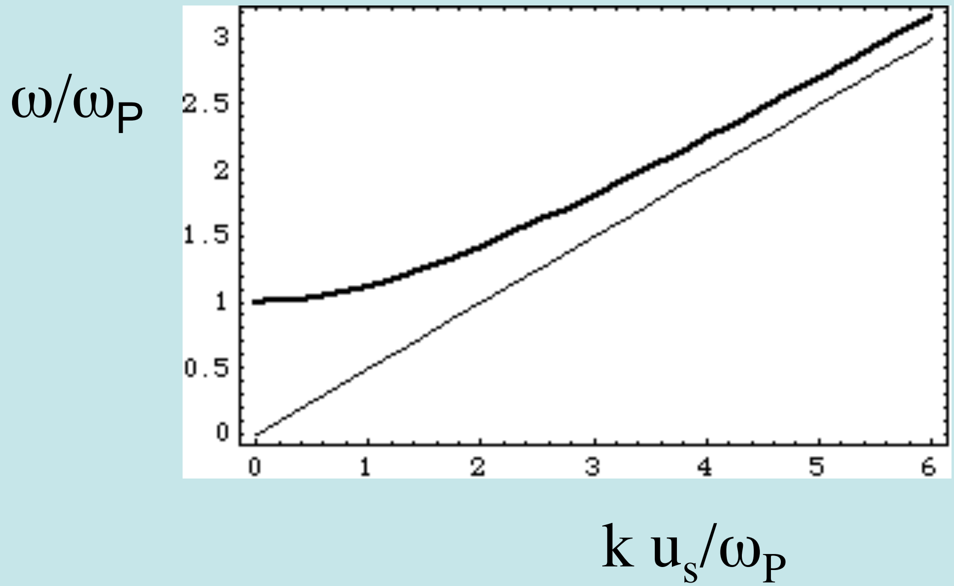
$$\nabla \cdot \vec{F} = Qn$$

$$P \propto n^{\gamma}$$

$$\frac{\partial^2 \tilde{n}}{\partial t^2} + \omega_P^2 \tilde{n} - u_s^2 \nabla^2 \tilde{n} = 0$$

Dispersion relation

$$\omega^2 = \omega_P^2 + k^2 u_s^2$$



Sound speed

$$u_s^2 = \frac{5}{3}\frac{P_0}{Mn_0}$$

6. Coulomb-like explosions

Fluid equations

$$\frac{\partial \vec{v}}{\partial t} + \vec{v}\cdot\nabla\vec{v} = -\frac{\nabla P}{Mn} + \frac{\vec{F}}{M} - \alpha\vec{v}$$

$$\frac{\partial n}{\partial t} + \nabla\cdot(n\vec{v}) = 0, \qquad \nabla\cdot\vec{F} = Qn$$

High viscosity limit

$$\vec{v} \approx \frac{\vec{F}}{\alpha}$$

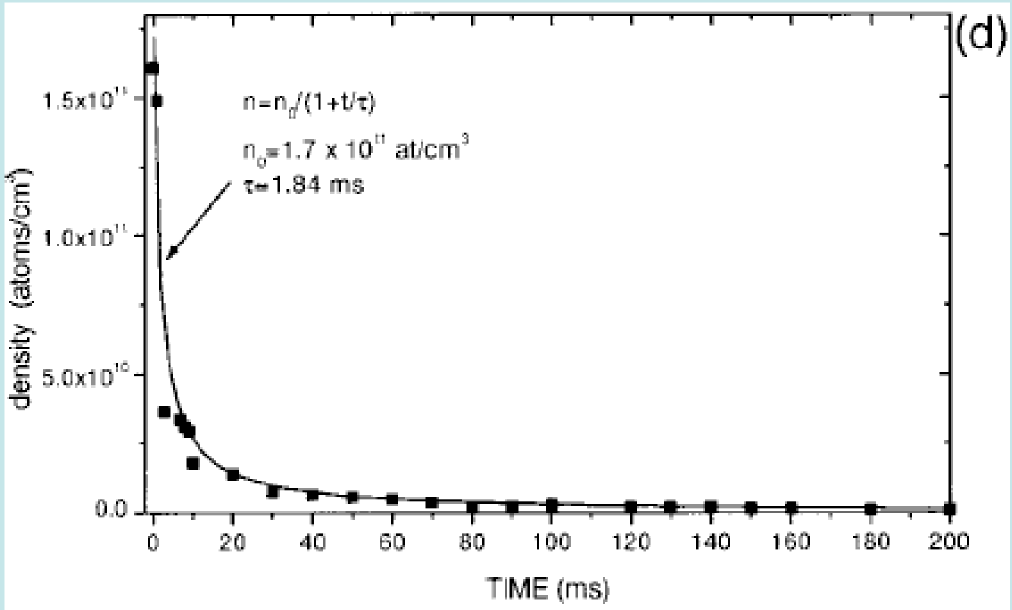
$$\frac{\partial n}{\partial t} = -\frac{1}{\alpha}\nabla\cdot\left[\vec{F}(\vec{r})\,n(\vec{r},t)\right]$$

Spherically expanding gas cloud

$$\frac{1}{n(t)} = \frac{1}{n_0} + \frac{Q}{\alpha}t$$

$$V/V_0 = 1 + t/\tau,$$

$$n/n_0 = (1 + t/\tau)^{-1}$$



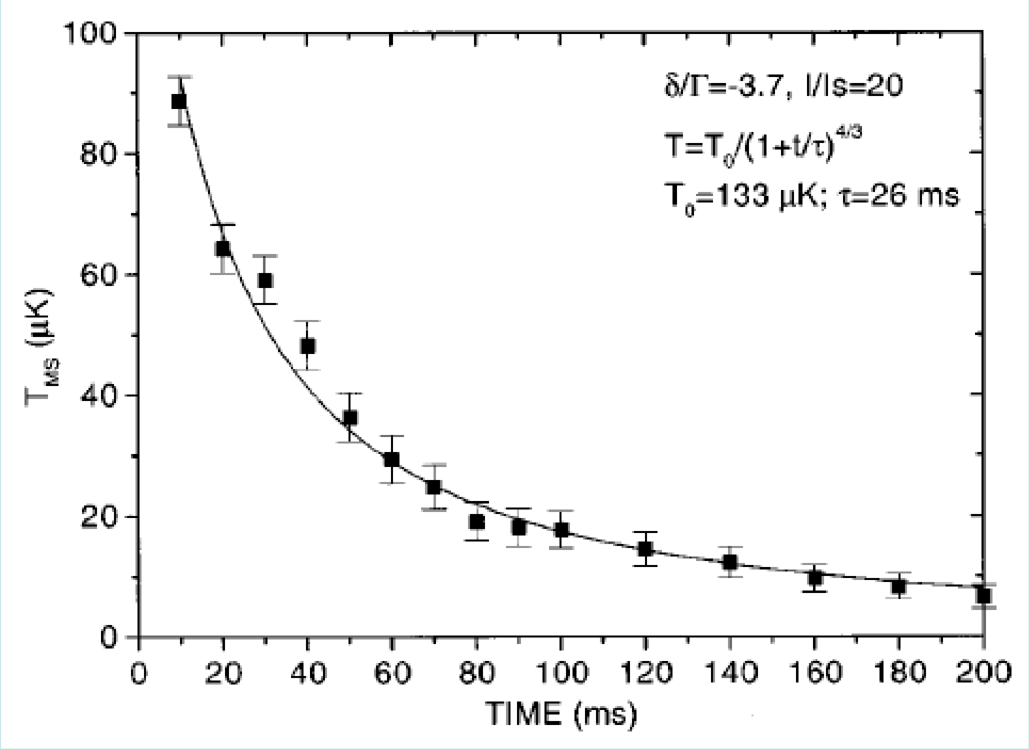
L.Pruvost et al, PRA(2000)

Atom cooling results from cloud expansion

Similar but slower than Coulomb explosion in plasmas

$$T_{MS} = \frac{T_0}{(1 + t/\tau)^{4/3}}$$

$$k_B T_{MS} = m \langle v^2 \rangle$$



Question: can collective effects lead to new cooling processes, and to more effective BE condensation?

7. Tonks-Dattner resonances

Internal oscillations in a Nonuniform cold gas

$$\nabla^2 \tilde{n} + k^2(\vec{r})\tilde{n} = \frac{\delta \vec{F}}{M u_S^2} \nabla n_0 + \frac{\nabla n_0}{M} \nabla \tilde{n},$$
$$k^2(\vec{r}) = [\omega^2 - \omega_P^2(\vec{r})]/u_S^2$$

a) Uniform slab

$$\frac{d^2\tilde{n}}{dx^2} + \frac{1}{u_S^2}[\omega^2 - \omega_P^2(x)]\tilde{n} \approx 0$$

$$\omega_m^2 = \omega_P^2 \left[1 + \left(m + \frac{1}{2} \right)^2 \pi^2 \frac{\lambda_D^2}{L} \right]$$

m = 0, 1, 2, ...

b) Cylindrical geometry (plasma)

Parker, Nickel and Gould, PoF (1964)

c) Spherical geometry (neutral cold atom gas)

Terças, Mendonca (2007)

8. Rydberg Plasmas

a. Creation of ultracold plasmas by photoionization of laser cooled Xe atoms [T.C. Killian et al., PRL (1999)]

b. Spontaneous evolution of a Rydberg could Xe gas, into a plasma

Creation of very could plasmas (an apparent contradiction)

$T_i \sim 30\ \mu K$, $T_e < 100\ mK$
(instead of 1-10 eV)

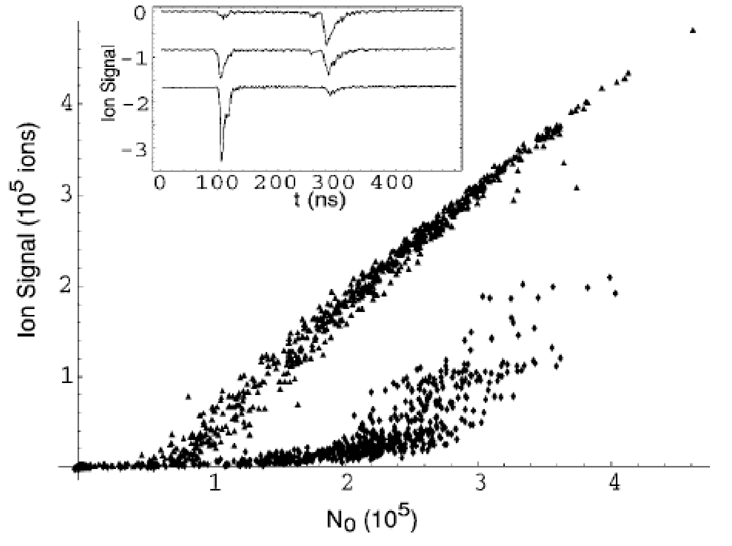


FIG. 1. Ion signals observed for different initial populations of the Rb 36*d* state. The two curves show the ion signals 2 μs (◆) and 12 μs (▲) delays after the dye laser excitation. The inset shows the time resolved signals obtained for $N_0 = 1.9 \times 10^5$ atoms at delays of 2 μs (upper trace), 5 μs (middle trace), and 12 μs (lower trace). In the upper trace there is no early ion signal and a large late atom signal while the reverse is true in the lower trace, indicating the formation of the plasma by 12 μs after laser excitation.

M.P. Robinson et al., PRL (2000)

Possible explanation

Existence of a small fraction of hot atoms (1% at room temperature)

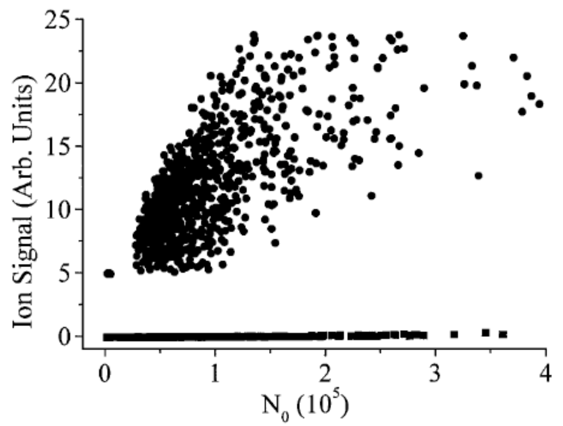


FIG. 4. Ion signals obtained with a delay of 3 μs after the excitation of the Cs 39*d* state with (●) and without (■) hot atoms. The signal hot atoms is offset by five units.

Or maybe not [T. Pohl et al. PRA (2003)]

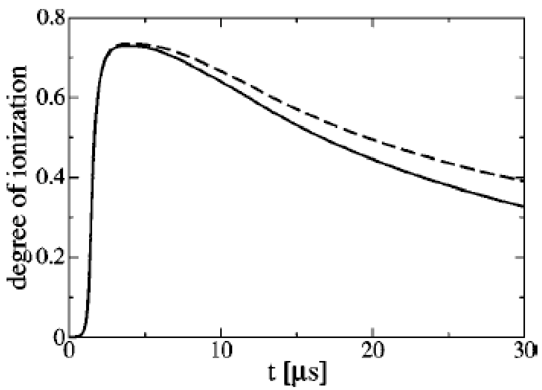


FIG. 1. Time evolution of the degree of ionization for the following initial conditions: atom density $\rho = 8\times 10^9$ cm^{-3}, atom temperature $T_a = 140$ μK, plasma width $\sigma = 60$ μm, and initial principal quantum number of the Rydberg atoms, $n_0 = 70$. Results are shown with ionic correlations (solid) and without ionic correlations (dashed).

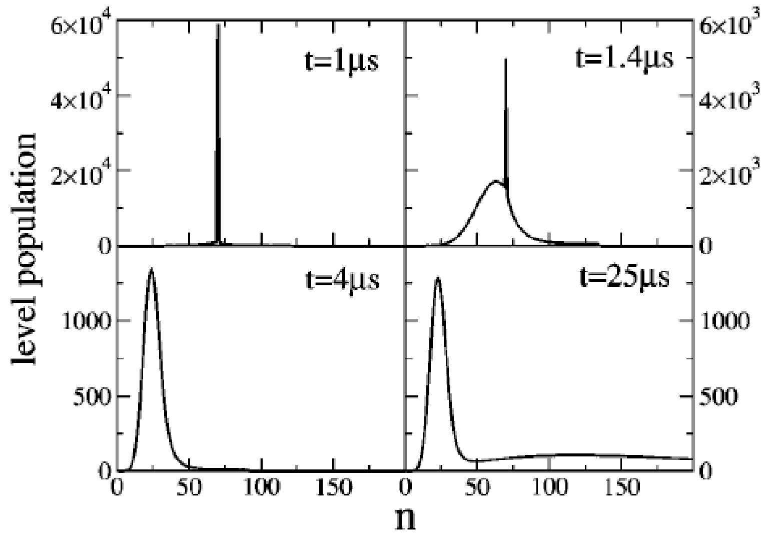


FIG. 3. Level distribution of Rydberg atoms after 1 μs, 1.4 μs, 4 μs, and 25 μs.

Plasma physics extends to new frontiers

9. Waves in Cold Quantum Plasmas

Schrödinger-Poisson system of equations, for electrons and ions (or holes or positrons)

$$i\partial_t\psi_j + A_j\nabla^2\psi_j + (s_j V - |\psi_j|^\alpha)\psi_j = 0,$$

$$\nabla^2 V = |\psi_e|^2 - |\psi_i|^2.$$

Time normalized by h/T_F, space by λ_D

$$\lambda_D = \sqrt{\epsilon_0 T_F / e^2 n_0}.$$

$s_e = +1$ and $s_i = -1$

$$A_j = m_j e^2 / 2\epsilon_0 \hbar^2 \sqrt{n_0},$$

$\alpha = 4/d$, where d is the dimension of the system

INSTITUTO SUPERIOR TÉCNICO

Wigner function for both species

$$F_j(\vec{r},\vec{k},t) = \int \psi_j(\vec{r}+\vec{s}/2,t)\,\psi_j^*(\vec{r}-\vec{s}/2,t)\,e^{i\vec{k}\cdot\vec{s}}\,d\vec{s}.$$

Wave kinetic equation

$$i\left(\partial_t + A_j\vec{k}\cdot\nabla\right)F_j = -\int V_j(\vec{q},t)[F_{j-} - F_{j+}]\,e^{i\vec{q}\cdot\vec{r}}\,\frac{d\vec{q}}{(2\pi)^3}$$

$$V_j(\vec{r},t) = s_jV - \rho_j^{\alpha/2}.$$

$$F_{j\pm} \equiv F_j(\vec{k}\pm\vec{q}/2).$$

Poisson equation

$$\nabla^2 V = \rho_e - \rho_i.$$

Probability density

$$\rho_j(\vec{r},t) = |\psi_j(\vec{r},t)|^2 = \int F_j(\vec{r},\vec{k},t)\,\frac{d\vec{k}}{(2\pi)^3}$$

INSTITUTO SUPERIOR TÉCNICO

Linear wave analysis

$$V(\vec{r},t) = \tilde{V}\exp(i\vec{q}\cdot\vec{r} - i\Omega t)$$

$$F_j(\vec{r},\vec{k},t) = F_{j0} + \tilde{F}_j\exp(i\vec{q}\cdot\vec{r} - i\Omega t)$$

$$\rho_j(\vec{r},t) = \rho_{j0} + \tilde{\rho}_j\exp(i\vec{q}\cdot\vec{r} - i\Omega t)$$

System of equations for the e-i perturbations

$$\begin{bmatrix} (q^2 - I_e) & I_e \\ I_i & (q^2 - I_i) \end{bmatrix} \begin{bmatrix} \tilde{\rho}_e \\ \tilde{\rho}_i \end{bmatrix} = 0.$$

Dispersion relation

$$(q^2 - I_e)(q^2 - I_i) - I_e I_i = 0$$

$$I_j(\Omega,\vec{q}) = \int \frac{(F_{j0-} - F_{j0+})}{(\Omega - \vec{v}_j\cdot\vec{q} - \rho_{j0}^{\alpha/2})}\frac{d\vec{k}}{(2\pi)^3}.$$

Electron oscillations

Cold quantum plasma

$$\left(\Omega - \rho_0^{\alpha/2}\right)^2 = \rho A_e + \frac{1}{4} A_e^2 q^4$$

Classical result

$$\Omega^2 = \omega_{pe}^2 (1 + 3q^2 \lambda_D^2)$$

Electron plasma frequency

$$\omega_{pe} = \sqrt{\rho_0 A_e}$$

Two types of quantum corrections

frequency shift $\rho_0^{\alpha/2}$

q^4 dispersion term

Kinetic dispersion relation

$$q^2 = \frac{1}{qA_e} \int \frac{[G_0(p - q/2) - G_0(p + q/2)]}{(p - p_0)} \frac{dp}{2\pi}$$

$$G_0(p) = \int F_0(p, \vec{k}_\perp) \frac{d\vec{k}_\perp}{(2\pi)^2}.$$

$$\epsilon_r(\Omega, q) + i\epsilon_i(\Omega, q) = 0,$$

$$\epsilon_r(\Omega, q) = q^2 - \frac{\rho_0}{A_e} \left[\frac{1}{(p_0^2 - q^2/4)} + \frac{2p_0 < p^2 >}{(p_0^2 - q^2/4)^2} \right]$$

$$\epsilon_i = -\frac{\pi}{4qA_e} [G_0(p_0 - q/2) - G_0(p_0 + q/2)]$$

Quantum Landau damping

Wave damping

$$\gamma = -\epsilon_i(\Omega_r, q)/(\partial\epsilon_r/\partial\Omega)_r$$

$$\gamma = -\frac{\pi}{8}\,\frac{\rho_0^{1/2}}{qA_e^{1/2}}\,[G_0(p_0 - q/2) - G_0(p_0 + q/2)]$$

Classical limit

$$\gamma = \frac{\pi}{2^4}\left(\frac{\rho_0}{A_e}\right)^{1/2}\left(\frac{dG_0}{dp}\right)_{p=p_0}$$

10. Conclusions

- The neutral cold atom gas can behave like a plasma;
- Collective effects are due to shadow-repulsive forces;
- Plasma-like oscillations and plasma-acoustic modes can exist;
- Mie and Tonks-Dattner resonances can be observed;
- Rydberg gas spontaneously evolves into a cold plasma;
- New areas of plasma physics can be explored.