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Spin Plasma Dynamics

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Spin Plasma Dynamics

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Overview

- Why look at quantum plasma effects?
- Schrödinger’s description.
- Non-relativistic single electron dynamics.
- Paramagnetic electrons.
- From micro to macro physics.
- MHD regime.
- Conclusions - what the future might bring.

Why quantum plasmas?

- Manifold applications:
  - Solid state systems.
  - Astrophysical environments.
  - Ultracold plasmas (Rydberg atoms).
  - Nanostructured materials.
  - Laser-plasmas...
- Interesting fundamental aspects of matter dynamics.
- Collective & nonlinear effects + quantum mechanics

⬇

New physics!

Schrödinger's description

Electron properties described by complex scalar wavefunction ψ ($|\psi|^2 =$ probability)

$$i\hbar\frac{\partial\psi}{\partial t} = H\psi$$

where we have the Hamiltonian operator

$$H = -\frac{\hbar^2}{2m_e}\nabla^2 - e\phi$$

and ϕ is the *external* electrostatic potential and e being the magnitude of the electron charge.

Schrödinger's description

- Nice approach: allows for easy generalizations, new interactions can be incorporated in Hamiltonian.
- Microscopic equations of motion

$$\frac{dF}{dt} = \frac{\partial F}{\partial t} + \frac{1}{i\hbar}[F, H]$$

for some operator F, $[F, H]$ Poisson brackets.

Example:

$$\boldsymbol{v} \equiv \frac{d\boldsymbol{x}}{dt} = [\boldsymbol{x}, H] = \frac{\boldsymbol{p}}{m_e}$$

for previous scalar electron description.

Incorporating the spin

Electron properties described by complex *spinor* wavefunction (spin degrees of freedom)

$$\psi = \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix}$$

and the Pauli Hamiltonian operator

$$H = \frac{1}{2m_e}\left(\frac{1}{i\hbar}\boldsymbol{\nabla} + \frac{e}{c}\boldsymbol{A}\right)^2 + \mu_B \boldsymbol{B}\cdot\boldsymbol{\sigma} - e\phi$$

Here $\boldsymbol{A}$ is the vector potential, $\boldsymbol{\sigma}$ is the spin operator, $\mu_B = e\hbar/2m_e$ is the Bohr magneton.

Operator equations of motion

$$\boldsymbol{v} = \frac{1}{m_e}\left(\boldsymbol{p} + \frac{e}{c}\boldsymbol{A}\right)$$

$$m_e \frac{d\boldsymbol{v}}{dt} = -e(\boldsymbol{E} + \boldsymbol{v} \times \boldsymbol{B}) - \frac{2\mu_B}{\hbar}\boldsymbol{\nabla}(\boldsymbol{B} \cdot \boldsymbol{S})$$

$$\frac{d\boldsymbol{S}}{dt} = \frac{2\mu_B}{\hbar}\boldsymbol{B} \times \boldsymbol{S}$$

where $\boldsymbol{S} = (\hbar/2)\boldsymbol{\sigma}$.

Para- vs. diamagnetism

- Energy of a magnetic dipole $\boldsymbol{m}$ in magnetic field $\boldsymbol{B}$

$$E = -\boldsymbol{m} \cdot \boldsymbol{B}$$

- Parallel magnetic dipole minimizes the energy $\Longrightarrow$ *paramagnetic* behaviour, typical of electrons.
- Electron magnetic dipole moment in terms of spin angular momentum

$$\boldsymbol{m} = -\frac{2\mu_B}{\hbar}\boldsymbol{S}$$

opposite direction of magnetic moment due to electron charge.

- Electron *orbital* angular momentum acts as *diamagnet*.
- Any applied magnetic field alters the electron orbit, thus altering the atomic current, in such a way as to counteract the applied magnetic field.
- Well known in classical plasmas.
- In materials having both para- and diamagnetic properties, paramagnetism dominates.

From micro to macro

- Allow the electron microstates to interact via electromagnetic fields.
- *N*-body problem seldom tractable in plasmas.
- Develop a macroscopic model from the underlying microscopic theory.
- Neglecting spin: Let $\psi_\alpha = \sqrt{n_\alpha}\,\exp(iS_\alpha/\hbar)$, where α enumerates the *N* wave functions. It is then straightforward to add up the resulting conservation laws for n_α and $\boldsymbol{v}_\alpha = \boldsymbol{\nabla} S_\alpha/m_e$.
- Fluid equations with Bohm potential:

$$m_e \frac{d\boldsymbol{v}_e}{dt} = -e(\boldsymbol{E} + \boldsymbol{v}_e \times \boldsymbol{B}) - \frac{\boldsymbol{\nabla} p_e}{n_e} + \frac{\hbar^2}{2m_e}\boldsymbol{\nabla}\left(\frac{\nabla^2\sqrt{n_e}}{\sqrt{n_e}}\right)$$

From micro to macro

- For spin systems: Decompose spinor wave function

$$\psi_\alpha = \sqrt{n_\alpha}\,\exp(iS_\alpha/\hbar)\varphi_\alpha$$

  Electron spin: unit spinor φ_α.

- Electron fluid equations (continuity eq. unaffected)

$$m_e\left(\frac{\partial}{\partial t} + \boldsymbol{v}_e\cdot\boldsymbol{\nabla}\right)\boldsymbol{v}_e = -e\left(\boldsymbol{E} + \boldsymbol{v}_e\times\boldsymbol{B}\right) - \frac{\boldsymbol{\nabla}p_e}{n_e} - \frac{2\mu_B}{\hbar}S_a\boldsymbol{\nabla}B^a - \frac{\hbar^2}{2m_e}\left(\frac{\nabla^2\sqrt{n_e}}{\sqrt{n_e}}\right) + \text{nonlinear spin terms}$$

$$\left(\frac{\partial}{\partial t} + \boldsymbol{v}\cdot\boldsymbol{\nabla}\right)\boldsymbol{S} = \frac{2\mu_B}{\hbar}\boldsymbol{B}\times\boldsymbol{S} + \text{thermal and nonlinear spin terms}$$

Maxwell's equation

Due to the intrinsic magnetization, given by

$$\boldsymbol{M} = -\frac{2\mu_B n_e}{\hbar}\boldsymbol{S}\,,$$

Ampère's law is modified according to

$$\boldsymbol{\nabla} \times \boldsymbol{B} = \mu_0(\boldsymbol{j} + \boldsymbol{\nabla} \times \boldsymbol{M}) + \frac{1}{c^2}\frac{\partial \boldsymbol{E}}{\partial t}.$$

Gives dynamic spin contribution to Maxwell's equations.

MHD regime

Single fluid dynamics (for lowest order coherent spin)

$$\rho \frac{d\boldsymbol{v}}{dt} = -\boldsymbol{\nabla}\left(\frac{B^2}{2\mu_0} - \boldsymbol{M}\cdot\boldsymbol{B}\right) + \boldsymbol{B}\cdot\boldsymbol{\nabla}\left(\frac{1}{\mu_0}\boldsymbol{B} - \boldsymbol{M}\right) - \boldsymbol{\nabla}p + \frac{\hbar^2\rho}{2m_e m_i}\left(\frac{\nabla^2\sqrt{\rho}}{\sqrt{\rho}}\right)$$

$$\frac{\partial \boldsymbol{B}}{\partial t} = \boldsymbol{\nabla}\times\left\{\boldsymbol{v}\times\boldsymbol{B} - \frac{[\boldsymbol{\nabla}\times(\boldsymbol{B}-\mu_0\boldsymbol{M})]\times\boldsymbol{B}}{e\mu_0 n_e} - M_a\boldsymbol{\nabla}B^a\right\}$$

Model magnetization using Brillouin function for spin-1/2 particles

$$\boldsymbol{M} = \mu_B n_e \tanh x\, \hat{\boldsymbol{B}}$$

where the Zeeman energy $x = \mu_B B / k_B T_e$ gives the degree of alignment through the Brillouin function. For high temp. magnetization $\longrightarrow$ 0.

Example:Dispersion relation

Linearizing the equations around a weak static magnetic field, we get the dispersion relation

$$\left(\omega^2 - k_z^2 \widetilde{C}_A^2\right)\left[\left(\omega^2 - k^2 \widetilde{C}_A^2 - k_x^2 \widetilde{c}_s^2\right)(\omega^2 - k_z^2 c_s^2) + k_x^2 k_z^2 \widetilde{c}_s^4\right] = 0$$

where we have the modified Alfvén and acoustic speeds

$$\widetilde{C}_A = C_A \left(1 - \frac{\hbar^2 \omega_{pe}^2}{mc^2 k_B T}\right)^{1/2}$$

$$\widetilde{c}_s = c_s \left[1 - \left(\frac{\hbar \omega_{ce}}{k_B T}\right)^2\right]^{1/2}$$

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Spin modified shear Alfvén wave

Spin modified fast and slow magnetosonic waves

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Example: Magnetic action

Ferrofluids   Nanostructured paramagnetic fluids. Formalism similar to the above applicable.

Normal field instability - saturated by gravity and surface tension (Cowley & Rosensweig 1967).

Movies from http://mrsec.wisc.edu/Edetc/cineplex/ff/text.html

Conclusions

- New important effects appear from collective quantum domain.
- Wide ranging possibilities for applications.
  - Nanomaterials.
  - Astroplasmas.
  - Ultracold plasmas...
- Interesting future possibilities:
  - Theoretical development: Dense, relativistic plasmas using computationally viable models.
  - Solitons and plasmonics (see C. S. Liu's talk, Aug 19).