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The Structure Formation of Coulomb Clusters

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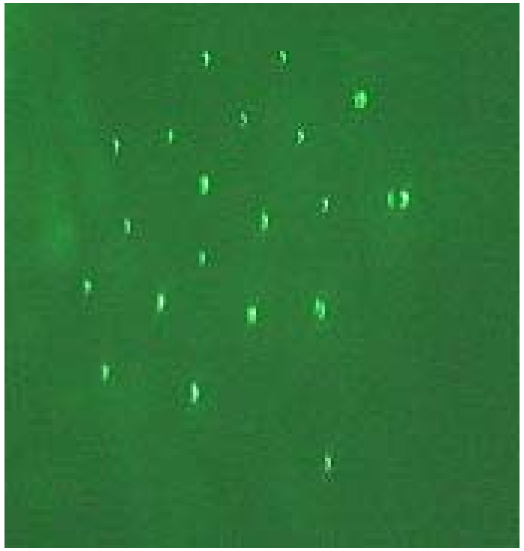
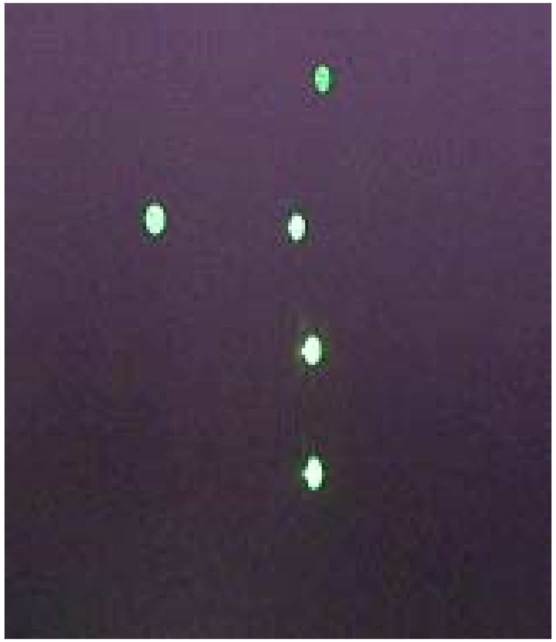

Summer College on Plasma Physics
Abdus Salam International Centre for Theoretical Physics (ASICTP)
Trieste, Italy August 20 – August 24, 2007

The Structure Formation of Coulomb Clusters

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Configurations of Coulomb Clusters in Complex Plasma

1. Introduction
   -historical survey: crystal structure
2. Dusts confined in a plasma
3. Hamiltonian of our system
4. Spherically symmetric potential
   CME (configuration of minimum energy)
   Shell structure
5. Non-spherical potential
   2D flat structure to spindle-like structure
6. Lattice oscillation
7. Conclusions

Historical background on crystal structures

D. I. Mendeleev 1869

Periodic table of elements

J. J. Thomson 1883,1904, 1921

Atomic system - as a stable arrangement of a mixture of a positive charge and a number of electrons.

E. Madelung 1918

Ionic crystals based on the electrostatic energy

E. Wigner 1934

Lattice structure of electrons in a metal as a result of the Coulomb forces acting among electrons.

Periodic Table of elements

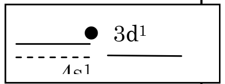
Mendeleev 1869

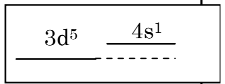
-63 elements

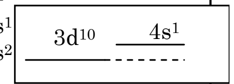
	I	II	III	IV	V	VI	VII	VIII
	H							
Representative element	Li	Be	B	C	N	O	F	
1-1	Na	Mg	Al	Si	P	S	Cl	
2	K	Ca	①	Ti	V	Cr	Mn	Fe, Co, Ni, Cu
2-3	(Cu)	Zn	②	③	As	Se	Br	Ru, Rh, Pb, Ag
4	Rb	Sr	(Yt)	Zr	Nb	Mo	④	
3-5	(Ag)	Cd	In	Sn	Sb	Te	I	Os, Ir, Pt, Au
6	Cs	Ba	-	Ce	-	-	-	
4-7	-	-	⑤	-	-	-	-	
8	-	-	-	-	Ta	W	-	
5-9	(Au)	Hg	Tl	Pb	Bi		-	
10	-	-		Tn		Ur	-	

Periodic Table of Elements

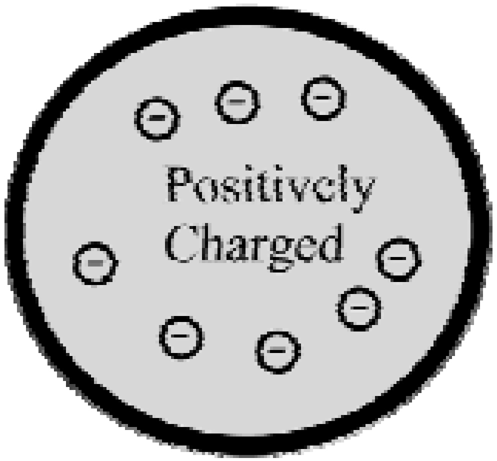
Z	element	No. of electrons K	L	M	N	Electron configuration
1	H	1				$1s^1$
2	He	2				$1s^2$
3	Li	[He]	1			$[He]2s^1$
4	Be		2			$2s^2$
5	B		2 1			$2s^2 2p^1$
6	C		2 2			$2s^2 2p^2$
7	N		2 3			$2s^2 2p^3$
8	O		2 4			$2s^2 2p^4$
9	F		2 5			$2s^2 2p^5$
10	Ne		2 6			$2s^2 2p^6$
11	Na	[Ne]	1			$[Ne]3s^1$
12	Mg		2			$3s^2$
13	Al		2 1			$3s^2 3p^1$
14	Si		2 2			$3s^2 3p^2$
15	P		2 3			$3s^2 3p^3$
16	S		2 4			$3s^2 3p^4$
17	Cl		2 5			$3s^2 3p^5$
18	Ar		2 6			$3s^2 3p^6$
19	K			[Ar]	1	$[Ar]4s^1$
20	Ca				2	$4s^2$
21	Sc			1	2	$3d^1 4s^2$
22	Ti			2	2	$3d^2 4s^2$
23	V			3	2	$3d^3 4s^2$
24	Cr			5	1	$3d^5 4s^1$
25	Mn			5	2	$3d^5 4s^2$
26	Fe			6	2	$3d^6 4s^2$
27	Co			7	2	$3d^7 4s^2$
28	Ni			8	2	$3d^8 4s^2$
29	Cu			2	1	$3d^{10} 4s^1$
30	Zn			3	2	$3d^{10} 4s^2$
31	Ga			10	2 1	$3d^{10} 4s^2 4p^1$
32	Ge			10	2 2	$3d^{10} 4s^2 4p^2$
33	As			10	2 3	$3d^{10} 4s^2 4p^3$
34	Se			10	2 4	$3d^{10} 4s^2 4p^4$
35	Br			10	2 5	$3d^{10} 4s^2 4p^5$
36	Kr			10	2 6	$3d^{10} 4s^2 4p^6$







J.J. Thomson 1898



- First atomic model

  Plum-pudding model

  (raison muffin model)

  -atom is considered of a positive matrix with negatively charged electrons floating inside

$$F \sim \frac{q}{r^2}\left(1 - \frac{r_0}{r}\right)$$

Ishihara, O., 1998, *Polygon structures of plasma crystals,* Phys. Plasmas 5, 357-364.

Ishihara, O., 1998, *Plasma crystals – structure and stability*, Physica Scripta T75, 79-83.

Study of structure of charged particles

- **In a neutral plasma, available free energy always drives a plasma unstable and recombination makes a plasma hard to be confined by static electric and magnetic fields.**

1970s: Nonneutral plasmas: a group of particles with a single sign of charge can be confined by static electric and magnetic fields - a pure electron plasma.

The electron-plasma oscillations observed in a plasma involve only the motion of electrons and no ions are involved in the oscillations. The electron oscillations also appear in a group of electrons without ions if they are trapped electromagnetically. Positive ions in a plasma play a role to push displaced electrons back to equilibrium positions, while electric or magnetic potential keeps electrons in a confined space.

Rahman, A. and Schiffer, J.P., 1986, *Structure of a one-component plasma in an external field: A molecular-dynamics study of particle arrangement in a heavy-ion storage ring*, Phys. Rev. Lett. 57, 1133-1136.

Dubin, H.E. and O'Neil, T.M., 1999, *Trapped nonneutral plasmas, liquids, and crystals (the thermal equilibrium states)*, Rev. Mod. Physics 71, 87-172.

Study of structure of charged particles (continued)

1978 Laser cooling technique was successfully used to observe the cooling of Mg^+ ions at 40 K in a Penning trap in which a confining magnetic field serves as a neutralizing background to charged particles, and the cooling of Ba^+ ions in a Paul trap in which an RF electric field confines ions.

1979: Electrons on the surface of liquid helium were observed to form Wigner crystals

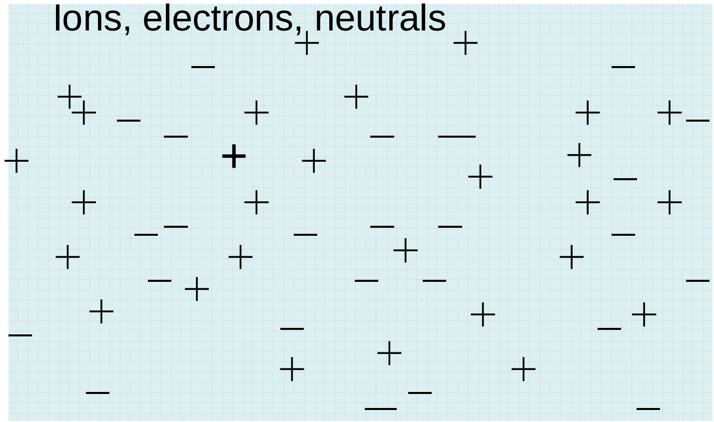
Rafac, R., Schiffer, J.P., Hangst, J. S., Dubin, D. H.E. and Wales, D.J., 1991, *Stable configurations of confined cold ionic systems*, Proc. Natl. Acad. Sci. USA **88** , 483-486.

Complex Plasma (Dusty Plasma)

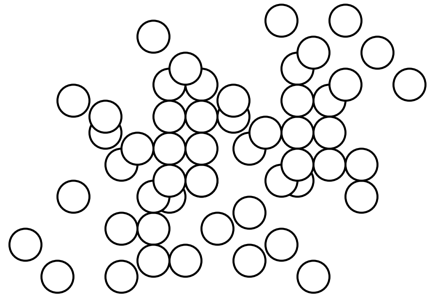
- Dust particles are charged in a plasma.
- The plasma effectively confine charged dust particles.
- Dust particles can be seen by naked eyes.
- Complex plasma provides a novel tool to study formation of structures of charged particles.

2. Dusts confined in a plasma

Complex plasma (Dusty plasma)



Plasma



Dusts, Fine Particles

Normalized charge

$$z = \frac{|Z_d| e^2}{4\pi\varepsilon_0 a k_B T_e}$$

Z=2~4

Havnes ordering parameter

$$P = \frac{|Z_d| n_d}{n_e}$$

P=10^{-4}~10^{-2}

Coupling parameter

$$\Gamma = \frac{Z_d^2 e^2}{4\pi\varepsilon_0 d k_B T_d} \exp(-\kappa)$$

Γ<< 1 , Γ>> 1

Dusts in a Plasma

$$\rho(\mathbf{x}) = Z_d e n_d(\mathbf{x}) + \sum_{s=i,e} z_s e n_s(\mathbf{x})$$

$$n_d(\mathbf{x}) = \frac{1}{4\pi a^2} \sum_{j=1}^{N} \delta\left(\left|\mathbf{x} - \mathbf{x}_j\right| - a\right)$$

N dust particle

$$H_{\text{int}} = \sum_{j=1}^{N} Z_d e \phi\left(\mathbf{x}_j\right), \qquad \nabla^2 \phi(\mathbf{x}) = -\frac{\rho(\mathbf{x})}{\varepsilon_0}$$

.

Dust charge $Q = Z_d e = 4\pi\varepsilon_0 a \phi_0$

Dust radius $= a$

Dust surface potential $= \phi_0$

Dust-Plasma Interaction

$$H_{d-b} = \sum_{s} Z_d z_s e^2 \sum_{i=1}^{N} \int d^3 r \frac{n_s(\mathbf{r}) e^{-k_D |\mathbf{r}-\mathbf{x}_i|}}{4\pi\varepsilon_0 |\mathbf{r}-\mathbf{x}_i|}$$

$$= \sum_{s} \sum_{i=1}^{N} \frac{Z_d z_s e^2 \overline{n}_s}{4\pi\varepsilon_0} \int d^3 r \frac{e^{-k_D |\mathbf{r}-\mathbf{x}_i|}}{|\mathbf{r}-\mathbf{x}_i|}$$

$$+ \sum_{s} \sum_{i=1}^{N} \frac{Z_d z_s^2 e^3 \overline{n}_s}{4\pi\varepsilon_0 k_B T_s} \int d^3 r \frac{\left(\overline{\phi} - \phi(\mathbf{r})\right) e^{-k_D |\mathbf{r}-\mathbf{x}_i|}}{|\mathbf{r}-\mathbf{x}_i|}$$

$$\overline{n}_s = \frac{1}{V} \int_V d^3 x \, n_s(\mathbf{x})$$

$$H_{d-b} = \sum_{i=1}^{N} \Phi_{eff}(\mathbf{x}_i)$$

$$\Phi_{eff}(\mathbf{x}_i) = \sum_{s} Z_d z_s e^2 \int d^3 r \frac{n_s(\mathbf{r}) e^{-k_D |\mathbf{r}-\mathbf{x}_i|}}{4\pi\varepsilon_0 |\mathbf{r}-\mathbf{x}_i|} \approx \sum_{s} \frac{Z_d z_s e^2 \overline{n}_s}{4\pi\varepsilon_0} \int d^3 r \frac{e^{-k_D |\mathbf{r}-\mathbf{x}_i|}}{|\mathbf{r}-\mathbf{x}_i|}$$

Effective Confining Potential

$$\Phi_{eff}(\mathbf{x}_i) = \sum_s \frac{Z_d z_s e^2 \overline{n}_s}{4\pi\varepsilon_0} \int d^3r \frac{e^{-k_D|\mathbf{r}-\mathbf{x}_i|}}{|\mathbf{r}-\mathbf{x}_i|}$$

$$\frac{1}{|\mathbf{r}-\mathbf{x}_i|} = \sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{\ell} \frac{4\pi}{2\ell+1} \frac{r_<^\ell}{r_>^\ell} Y_{lm}^*(\theta',\phi') Y_{lm}(\theta,\phi), \qquad V = \frac{4\pi}{3} R^3$$

For $|\mathbf{x}_i| < R,$

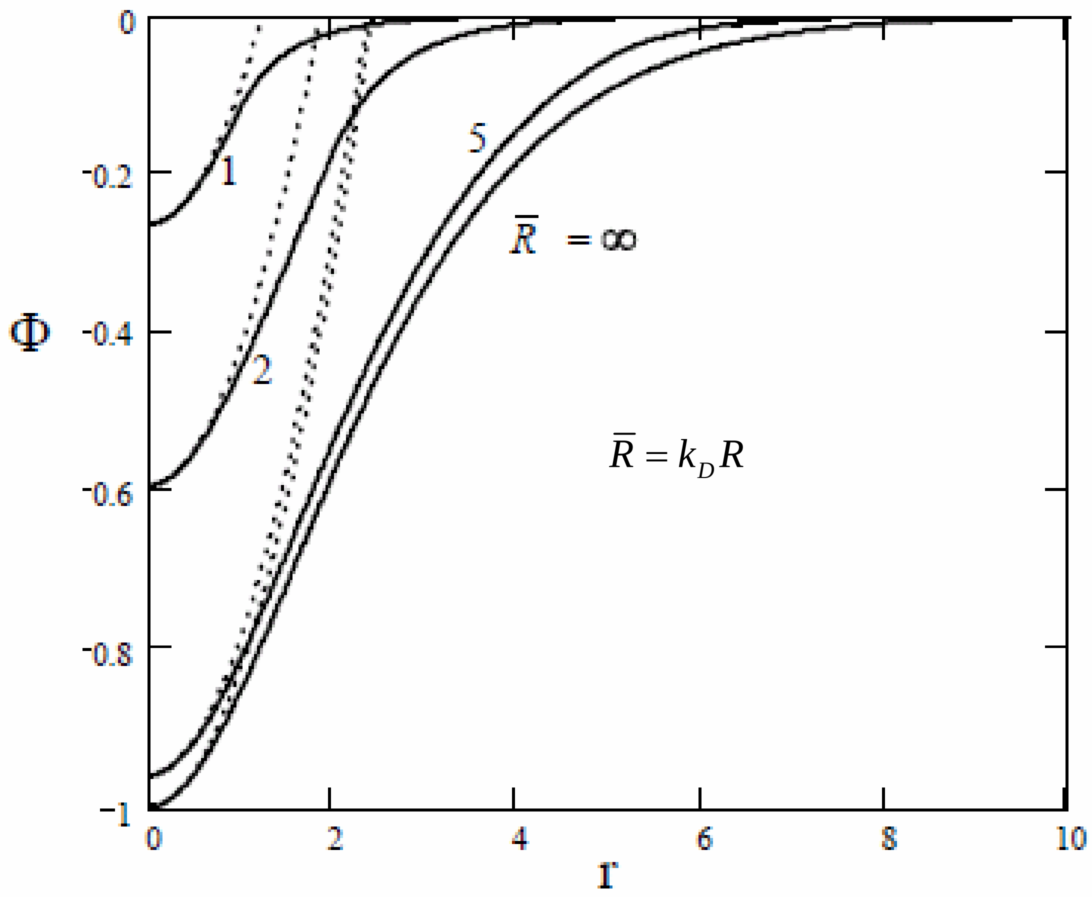
$$\Phi_{eff}(\mathbf{x}_i) = -\frac{Z_d^2 e^2 \overline{n}_d}{\varepsilon_0 k_D^2} \left[e^{-k_D|\mathbf{x}_i|} \left(1 + k_D|\mathbf{x}_i| + \frac{1}{3}\left(k_D|\mathbf{x}_i|\right)^2 \right) - e^{-k_D R}\left(1 + k_D R\right) \right]$$

For $|\mathbf{x}_i| > R,$

$$\Phi_{eff}(\mathbf{x}_i) = -\frac{Z_d^2 e^2 \overline{n}_d}{\varepsilon_0} \frac{R^3}{3|\mathbf{x}_i|} e^{-k_D|\mathbf{x}_i|}$$

2. Dusts confined in a plasma

Parabolic potential near the plasma center.

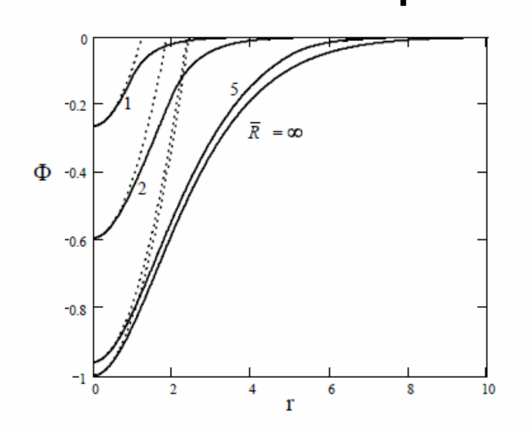


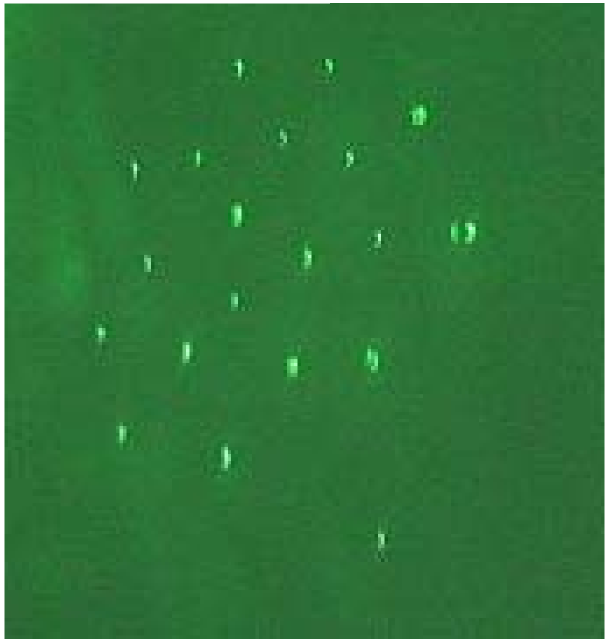
$$\Phi = \Phi_{eff} / \left[\bar{n}_d \left(Z_d e \right)^2 / \varepsilon_0 k_D^2 \right],$$

Hamaguchi et al, 1994, J. Chem. Phys;Totsuji et al, 2005, Phys. Rev. E.
Ishihara, 2007, J. Phys. D: Appl. Phys.

2. Dusts confined in a plasma

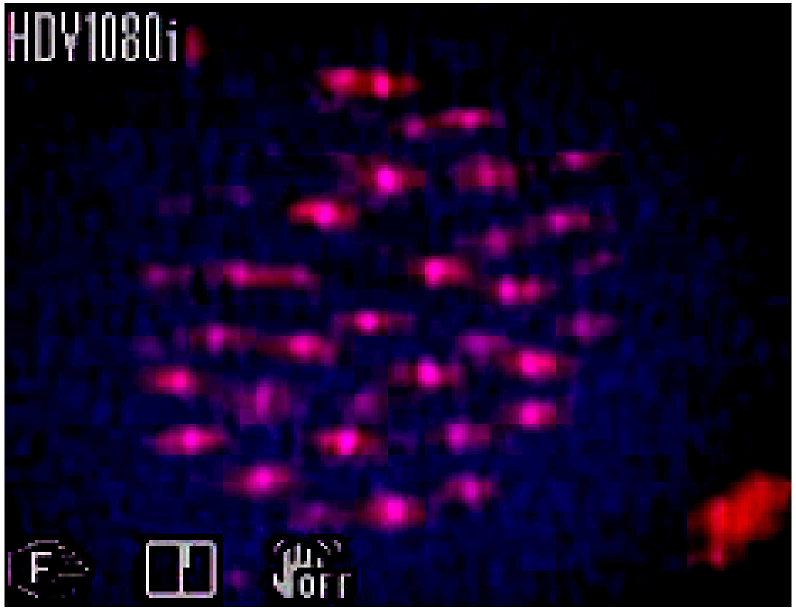
Dust particles can be trapped by the background plasma





Dust particles, illuminated by green laser, floating in a helium plasma

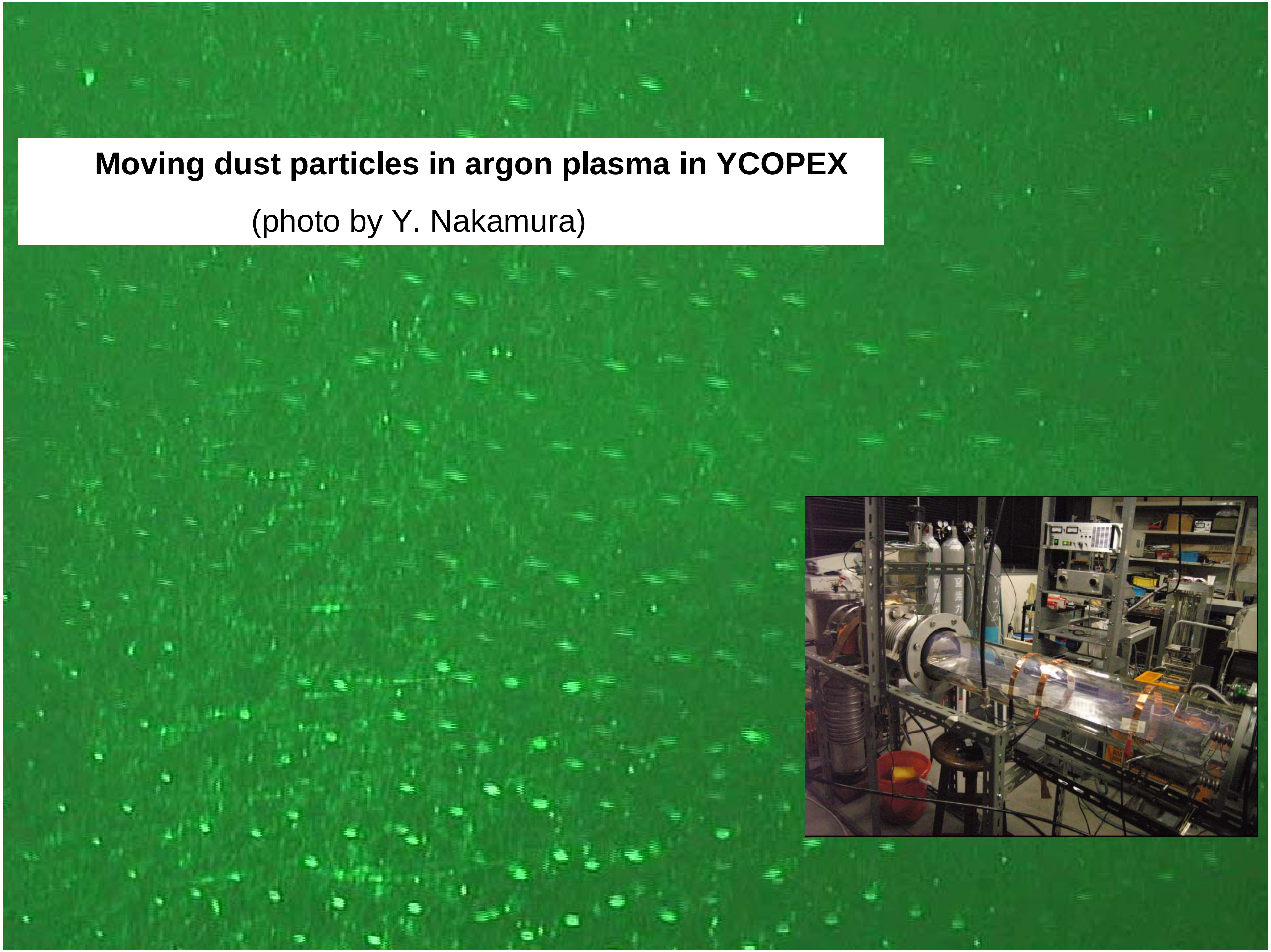
(YD1, photo by C. Kojima)



Dust particles, illuminated by red laser, floating in a helium plasma
(YD1, photo by M. Kugue)

Moving dust particles in argon plasma in YCOPEX

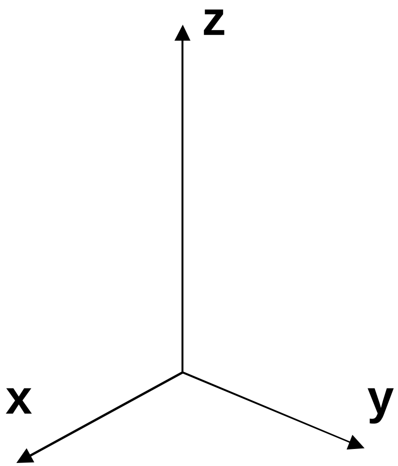
(photo by Y. Nakamura)



Hamiltonian for our numerical study

$$F = -k\Delta r$$

$$\Phi \sim ax^2 + by^2 + cz^2$$



$k = m\omega^2$ in x-y plane

$k = \alpha m\omega^2 \;(\alpha \geq 0)$ in z direction.

$$\Phi(\mathbf{r}_i) = \frac{1}{2} m\omega^2 \left[r_i^2 - (1-\alpha) z_i^2 \right]$$

$$H = \sum_{i}^{N} \Phi(\mathbf{x}_i) + \frac{Q^2}{4\pi\varepsilon_0} \sum_{\substack{i,j \\ (j>i)}}^{N} \frac{1}{|\mathbf{x}_i - \mathbf{x}_j|}$$

$$Q = Z_d e, \quad \Phi = \frac{1}{2} m\omega_0^2 (x_i^2 + y_i^2 + \frac{1}{\kappa} z_i^2)$$

3. Hamiltonian of our system

$$H = \sum_{i}^{N} \frac{1}{2} m\omega_0^2 (x_i^2 + y_i^2 + \frac{1}{\kappa} z_i^2) + \frac{Q^2}{4\pi\varepsilon_0} \sum_{\substack{i,j \\ (j>i)}}^{N} \frac{1}{|\mathbf{x}_i - \mathbf{x}_j|}$$

$$\ell_0 \equiv (2Q^2 / 4\pi\varepsilon_0 m\omega_0^2)^{1/3}$$

$$E_0 \equiv m\omega_0^2 \ell_0^2 / 2$$

$$H = \sum_{i=1}^{N} \left(x_i^2 + y_i^2 + \frac{z_i^2}{\kappa} \right) + \frac{1}{2} \sum_{i=1}^{N} \sum_{i=1}^{N} \frac{1}{|\mathbf{x}_i - \mathbf{x}_j|}$$

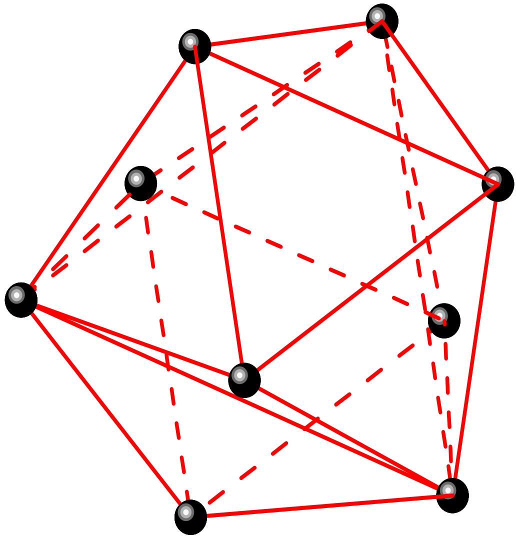
$$H = H(\kappa, N)$$

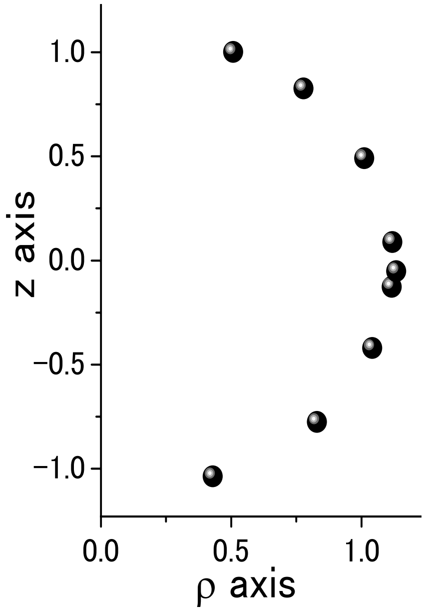
K=elongation parameter

N=number of dust particles

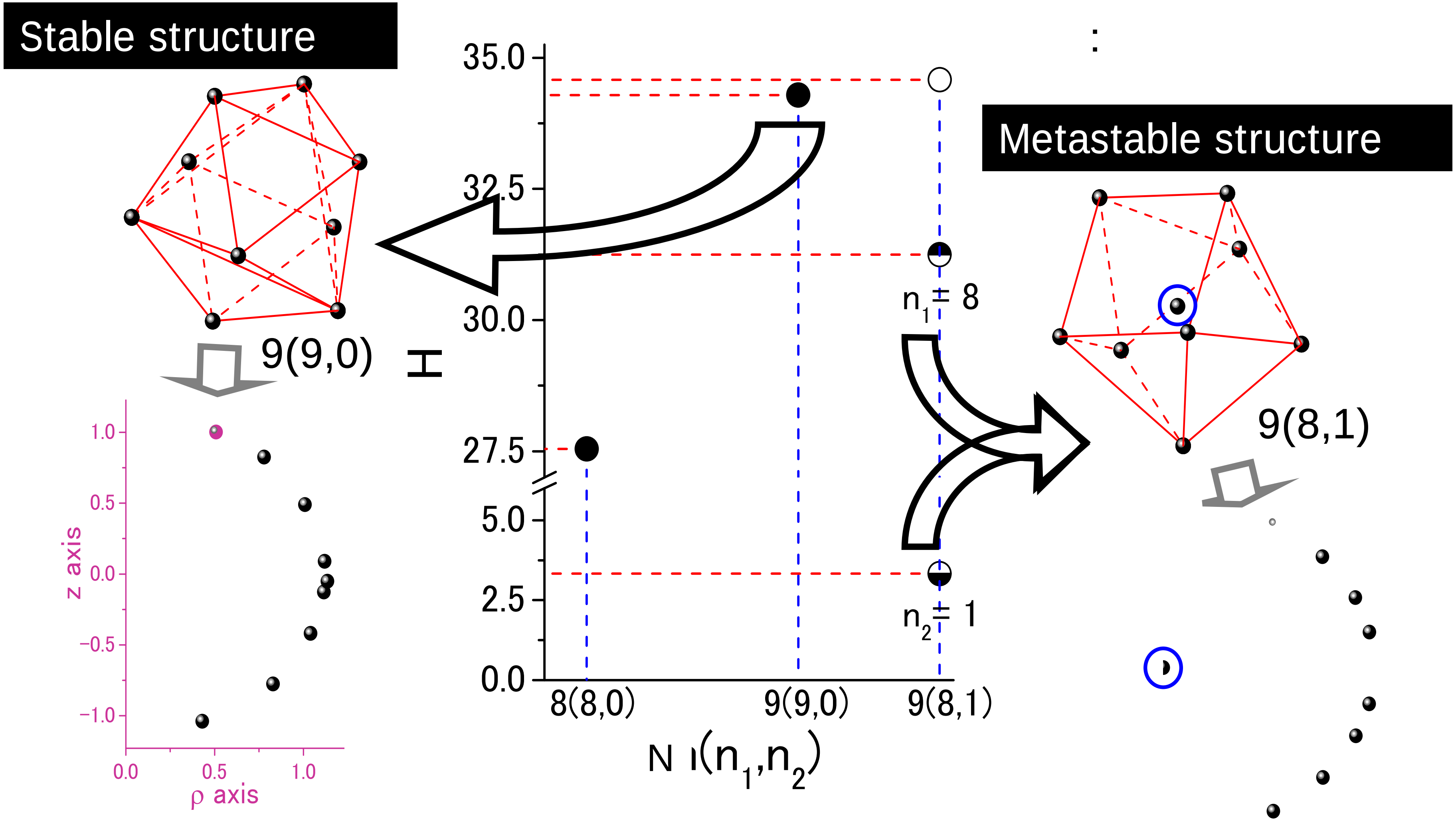
Numerical solution

$$\frac{d^2\mathbf{r}_i}{dt^2} = -\nabla_i H - \nu \frac{d\mathbf{r}_i}{dt}, \quad (i = 1 \sim N)$$





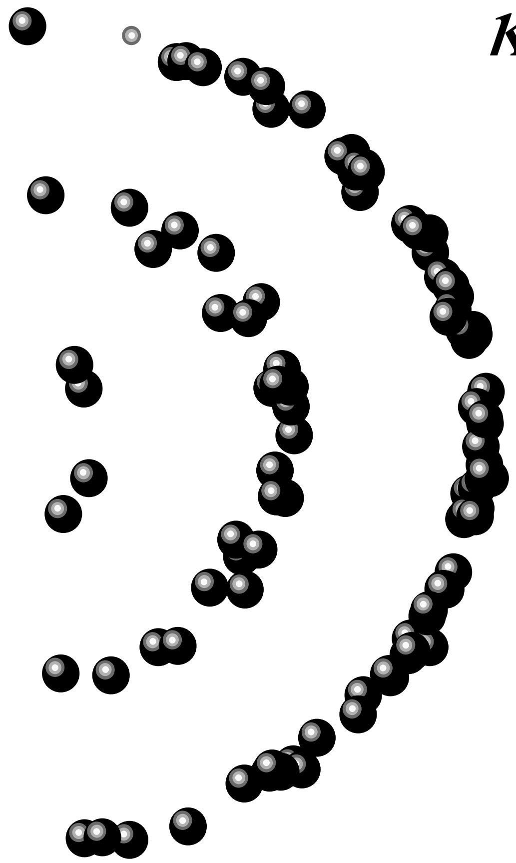
Stable structure and metastable structure (κ=1)



● : H for a shell, ○ : H for a shell with a dust particle at the center

◐ : H for the outer cluster, ◐ H for the inner cluster

4. Spherically symmetric potential



$\kappa = 1$

Shell structure

(onion-like structure)

T. Yamanouchi, M. Shindo, O. Ishihara and T. Kamimura,
Thin Solid Films **506-507**, 642-646 (2006).

Configuration, total energy and energy state

H=total energy S : Stable M : Metastable

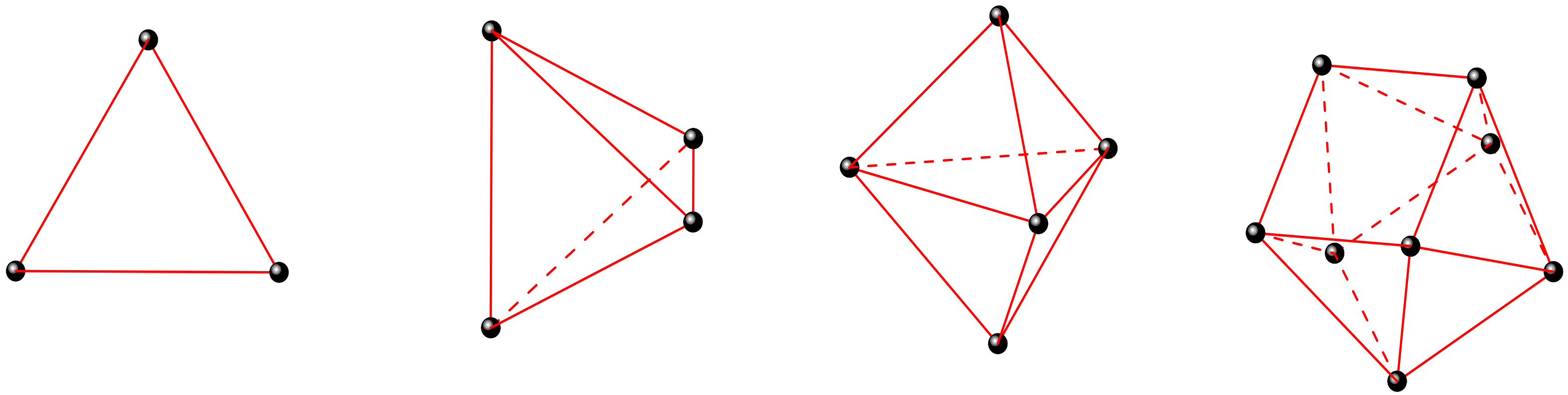
N	(n_1,n_2)	H	S/M
1	(1,0)	0.0	S
2	(2,0)	1.5	S
3	(3,0)	3.93111	S
4	(4,0)	7.1433	S
5	(5,0)	11.22594	S
6	(6,0)	15.92423	S
7	(7,0)	21.4493	S
8	(8,0)	27.54728	S
9	(9,0)	34.28804	S
	(8,1)	34.58274	M
10	(10,0)	41.6499	S
	(9,1)	41.86979	M
11	(11,0)	49.64603	S
	(10,1)	49.75467	M
12	(12,0)	58.0676	S
	(11,1)	58.25173	M
13	(12,1)	67.16838	S
	(13,0)	67.23417	M
14	(13,1)	76.80282	S
	(14,0)	76.85183	M
15	(14,1)	86.88141	S
	(15,0)	87.01687	M
16	(15,1)	97.49474	S
	(16,0)	97.68994	M
17	(16,1)	108.6064	S
	(17,0)	108.8744	M
18	(17,1)	120.2189	S
	(18,0)	120.5539	M
19	(18,1)	132.3188	S
	(19,0)	132.7687	M
20	(19,1)	144.9436	S
	(18,2)	145.0289	M
	(20,0)	145.3899	M
21	(20,1)	157.9699	S
	(19,2)	158.068	M
	(21,0)	158.5229	M
22	(21,1)	171.5003	S
	(20,2)	171.5235	M
	(22,0)	172.1092	M
23	(21,2)	185.4622	S
	(22,1)	185.4771	M
	(20,3)	185.5668	M
	(23,0)	186.2033	M

Compiled by T. Yamanouchi

4. Spherically symmetric potential: CME (configuration of minimum energy)

CME（Configuration of Minimum Energy） State

$$\Phi = x_i^2 + y_i^2 + \frac{z_i^2}{\kappa} \quad (\kappa = 1)$$



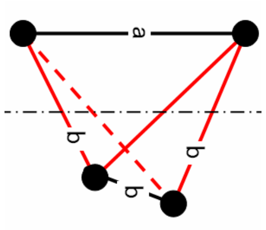
n=3 n=4 n=5 n=8

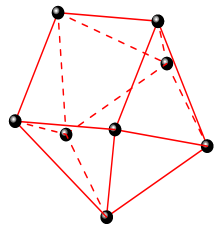
Triangle --------------------- Polyhedron -----------------------

4. Spherically symmetric potential: CME (configuration of minimum energy)

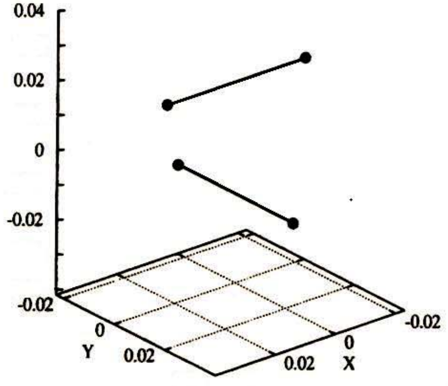
Fundamental CME (unit) configurations

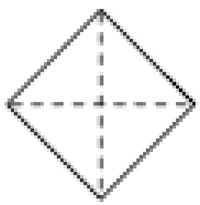
in a spherical harmonic potential (κ = 1)



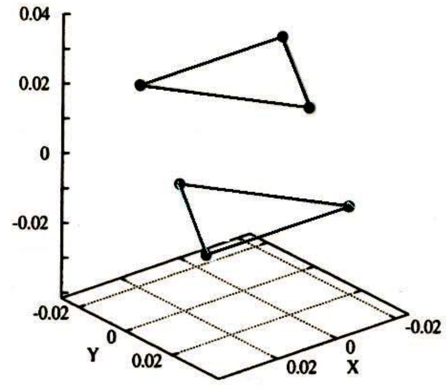


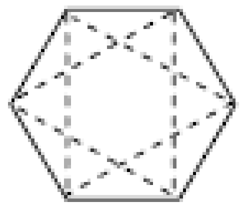
N=4



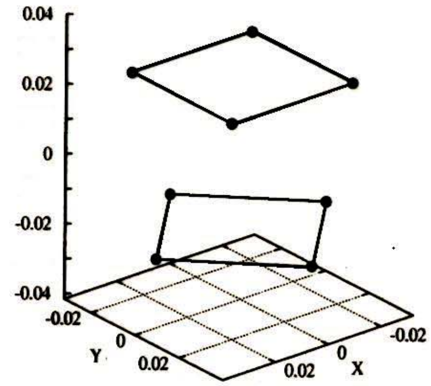


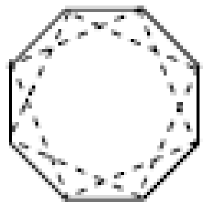
N= 6





N=8





4. Spherically symmetric potential: CME (configuration of minimum energy)

Tetrahedron (N=4),Octahedron(N=6),---

N=4: A tetrahedron. two sets of 2 particles at right angles to each other - projection -a square.

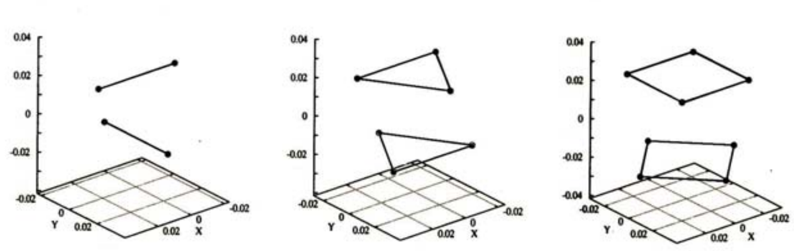
N=6: An octahedron. two sets of 3 particles, forming an equilateral (regular) triangle each, twisted by the angle 60 degrees

- a regular hexagon

N=8: two sets of 4 particles, twisted by an angle 45 degrees - projection - a regular octagon.

The essential features of the configurations for N=4, 6 and 8 are the fact that there are two sets of separated configurations in parallel plane and they make angles each other by the amount

$$\theta = \frac{2\pi}{N}$$



$$\theta = \frac{2\pi}{4} = \frac{\pi}{2}, \qquad \frac{2\pi}{6} = \frac{\pi}{3}, \qquad \frac{2\pi}{8} = \frac{\pi}{4}$$

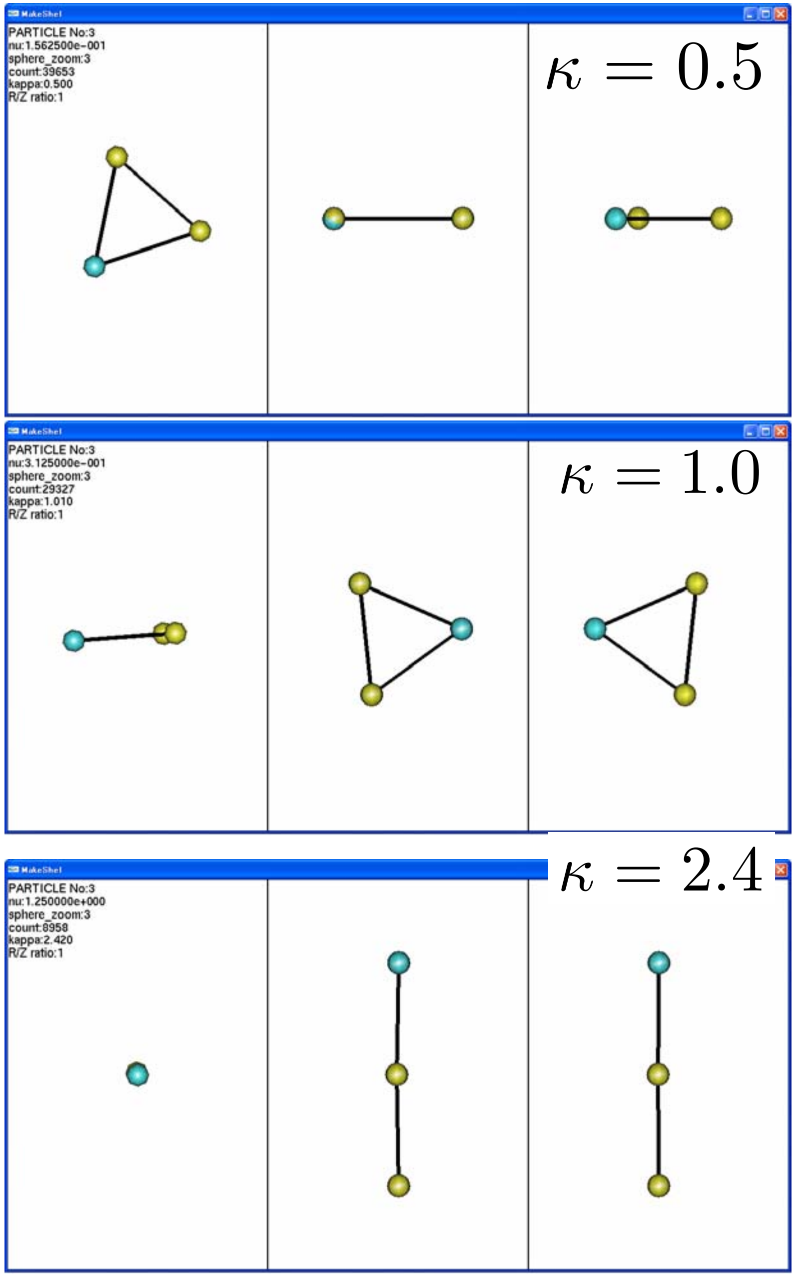
Non-spherical potential

$$\Phi = x_i^2 + y_i^2 + \frac{z_i^2}{\kappa}$$

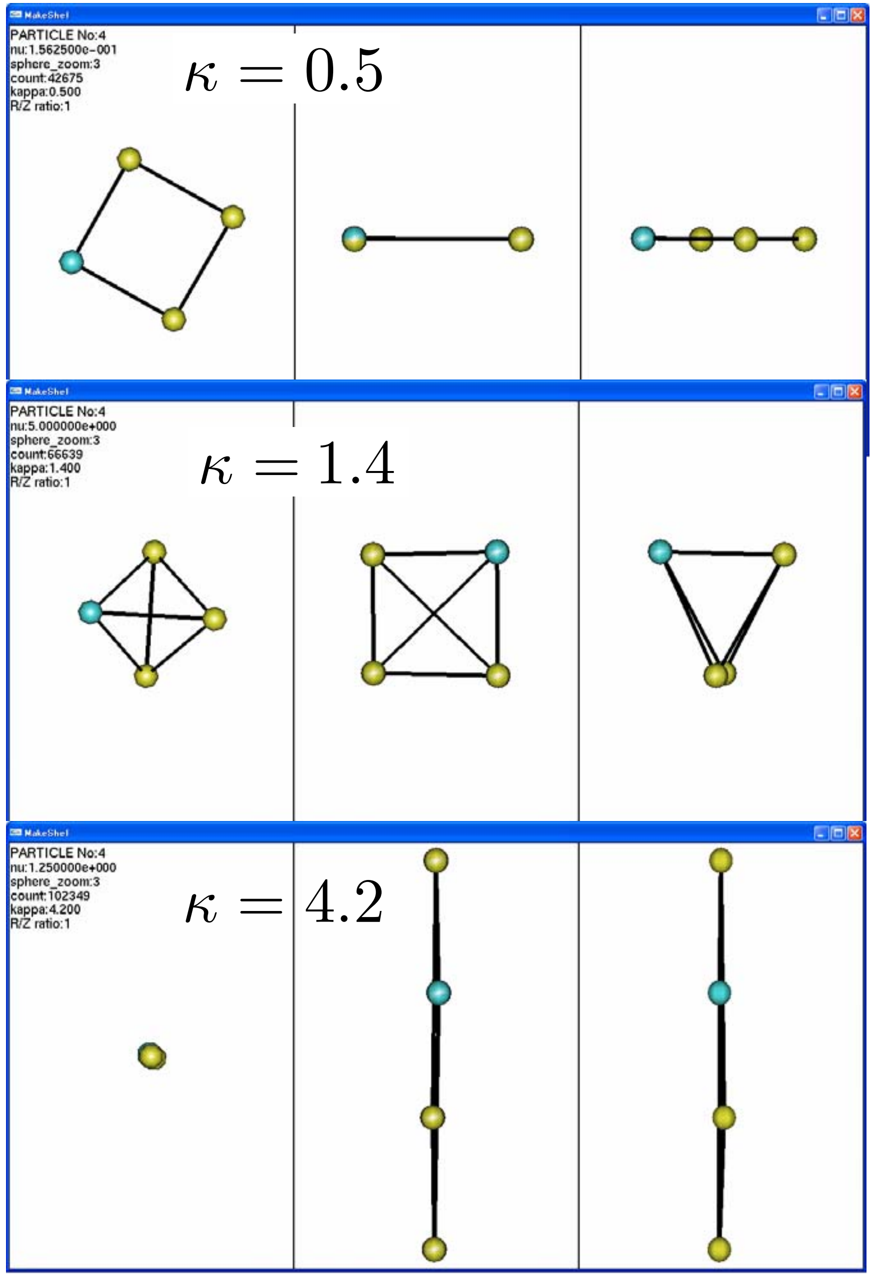
$$\kappa << 1, \quad \kappa >> 1$$

5. Non-spherical potential : 2D flat structure to spindle-like structure

N=3



N=4



5. Non-spherical potential : 2D flat structure to spindle-like structure

Non-spherical potential (κ<<1, κ>>1,)

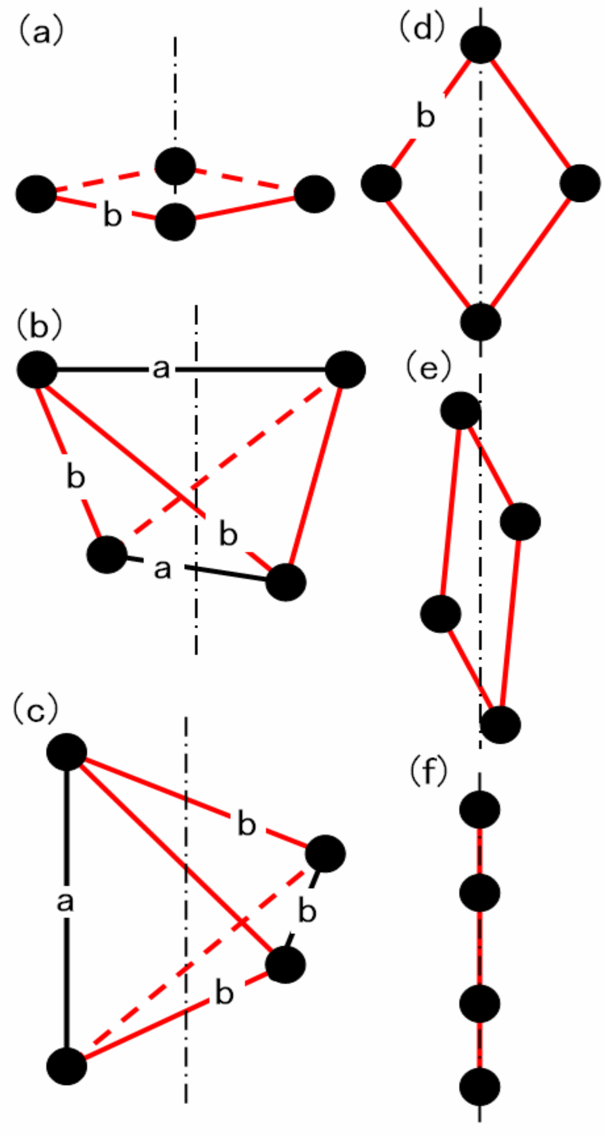
$$\Phi = x_i^2 + y_i^2 + \frac{z_i^2}{\kappa}$$

N=4

$\kappa < 0.68$

$0.68 < \kappa < 1.52$

$1.52 < \kappa < 1.67$



$1.67 < \kappa < 2.86$

$2.86 < \kappa < 4.17$

$\kappa > 4.17$

5. Non-spherical potential : 2D flat structure to spindle-like structure

Total potential energy does not change substantially, while the structure changes drastically.

To identify quantitatively the structural change, we separate potentials into radial component and vertical component:

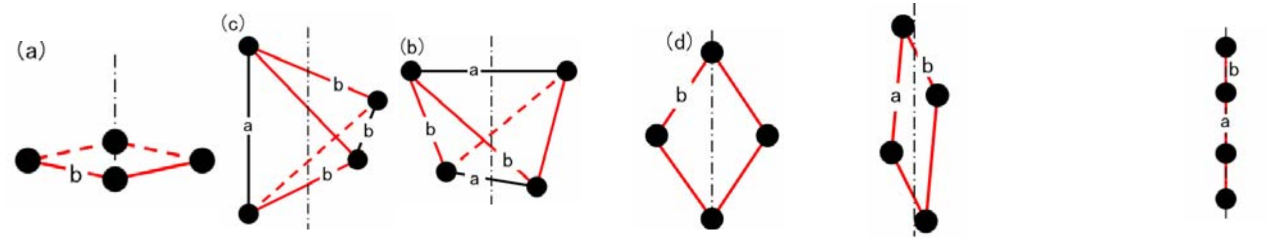
$$\Phi = \Phi_R + \Phi_Z$$

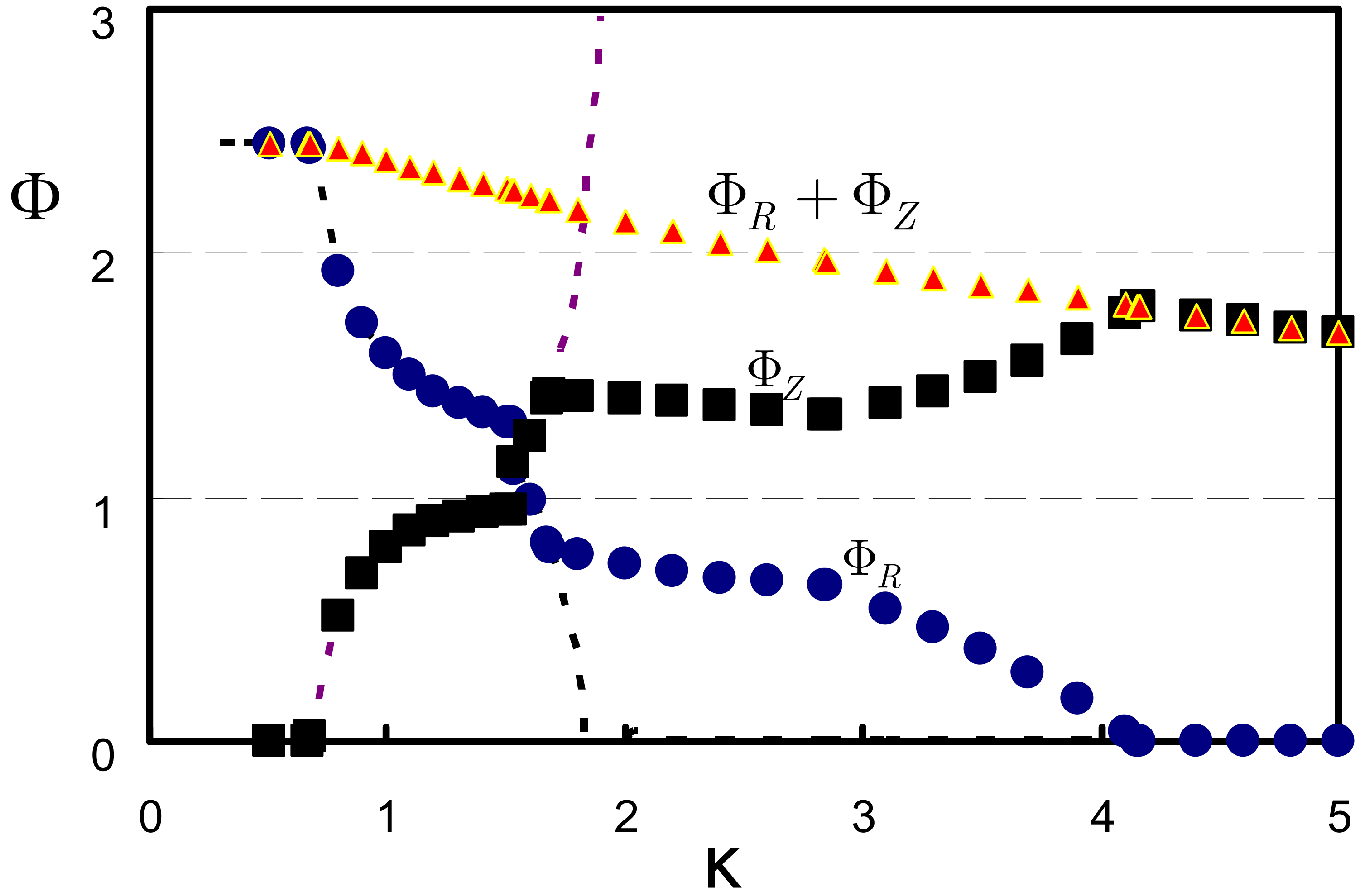
$$\Phi_R = \sum_i^N \left(r_i^2 - z_i^2 \right) = \sum_i^N \left(x_i^2 + y_i^2 \right)$$

$$\Phi_Z = \sum_i^N \frac{z_i^2}{\alpha^{-1}}$$

5. Non-spherical potential : 2D flat structure to spindle-like structure

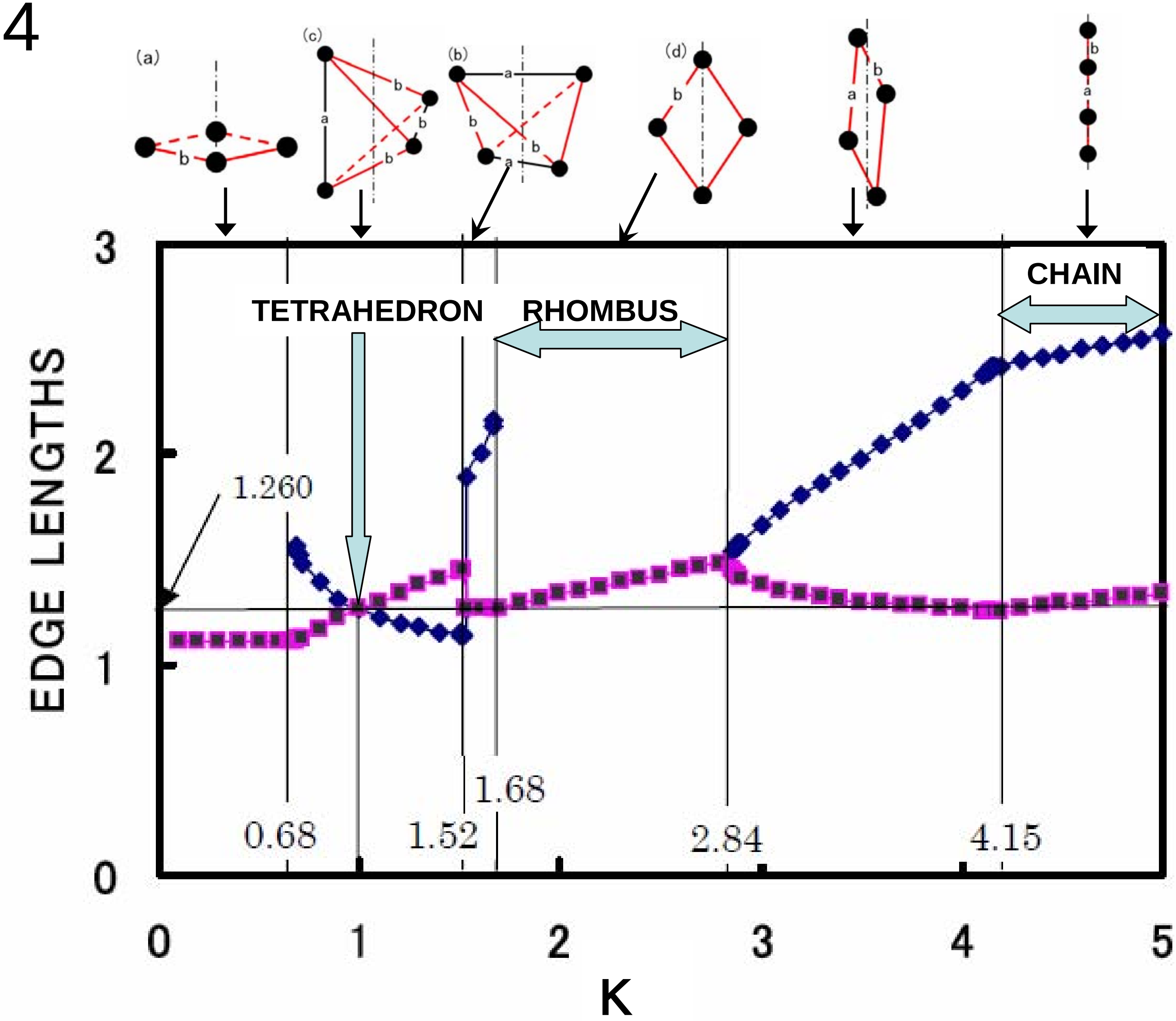
N=4





5. Non-spherical potential : 2D flat structure to spindle-like structure

N=4



5. Non-spherical potential : 2D flat structure to spindle-like structure

Force balance determines the length of sides.

N=4

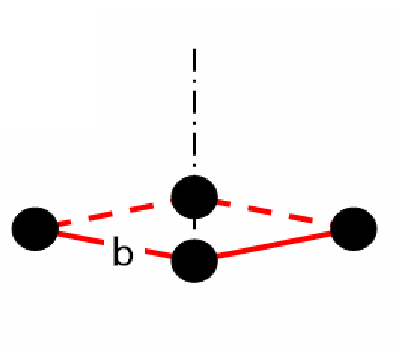
$$f_1 + 2 f_2 \cos\left(\pi / 4\right) = f_{atr}$$

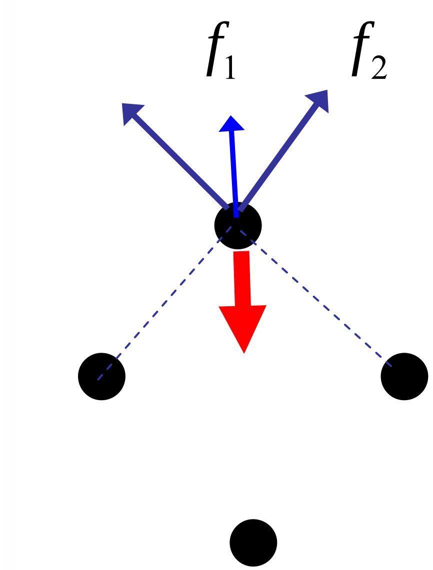
$$f_1 = 1/2b^2$$

$$f_2 = 1/b^2$$

$$f_{atr} = 2(b/\sqrt{2})$$

$$b = \left[\left(4+\sqrt{2}\right)/4\right]^{1/3} = 1.106$$





5. Non-spherical potential : 2D flat structure to spindle-like structure

$$H = \sum_{i}^{N}\left[x_i^2 + y_i^2 + \frac{1}{\kappa} z_i^2\right] + \frac{1}{2}\sum_{i}^{N}\sum_{j}^{N}\frac{1}{\left|\mathbf{r}_i - \mathbf{r}_j\right|}$$

$$\nabla H = 0$$

(1) $For\ \kappa < \left(4+\sqrt{2}\right)/8 = 0.6768,$

$$\Phi_R = (1+2\sqrt{2})^{2/3} = 2.447$$

$$\Phi_Z = 0$$

(2) For $0.68 < \kappa < 1.52$

$$\Phi_R = \left(\frac{2\kappa}{2\kappa - 1}\right)^{2/3}$$

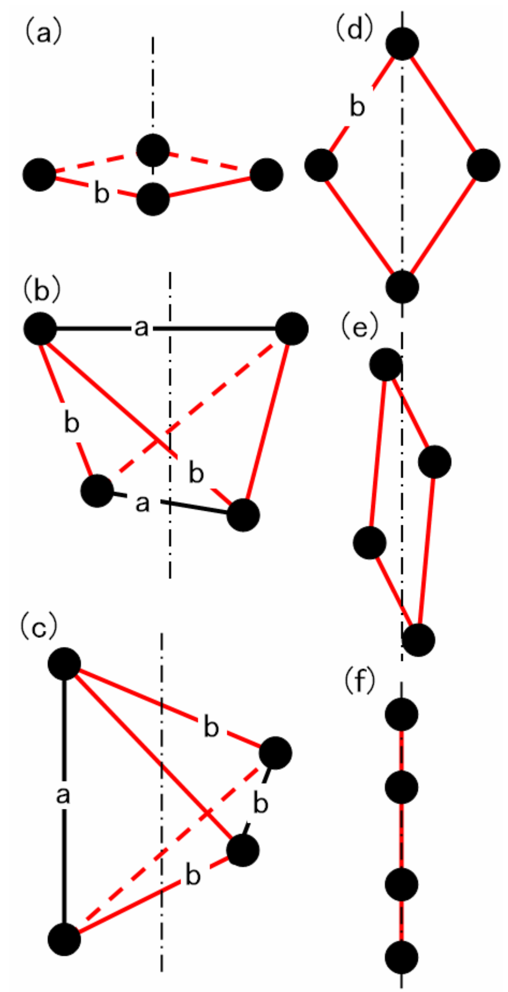
$$\Phi_Z = \left(\frac{\kappa}{4}\right)^{-1/3}\left(1 - \frac{1}{2}(2\kappa-1)^{-2/3}\right)$$

(3) For $1.52 < \kappa < 1.68$

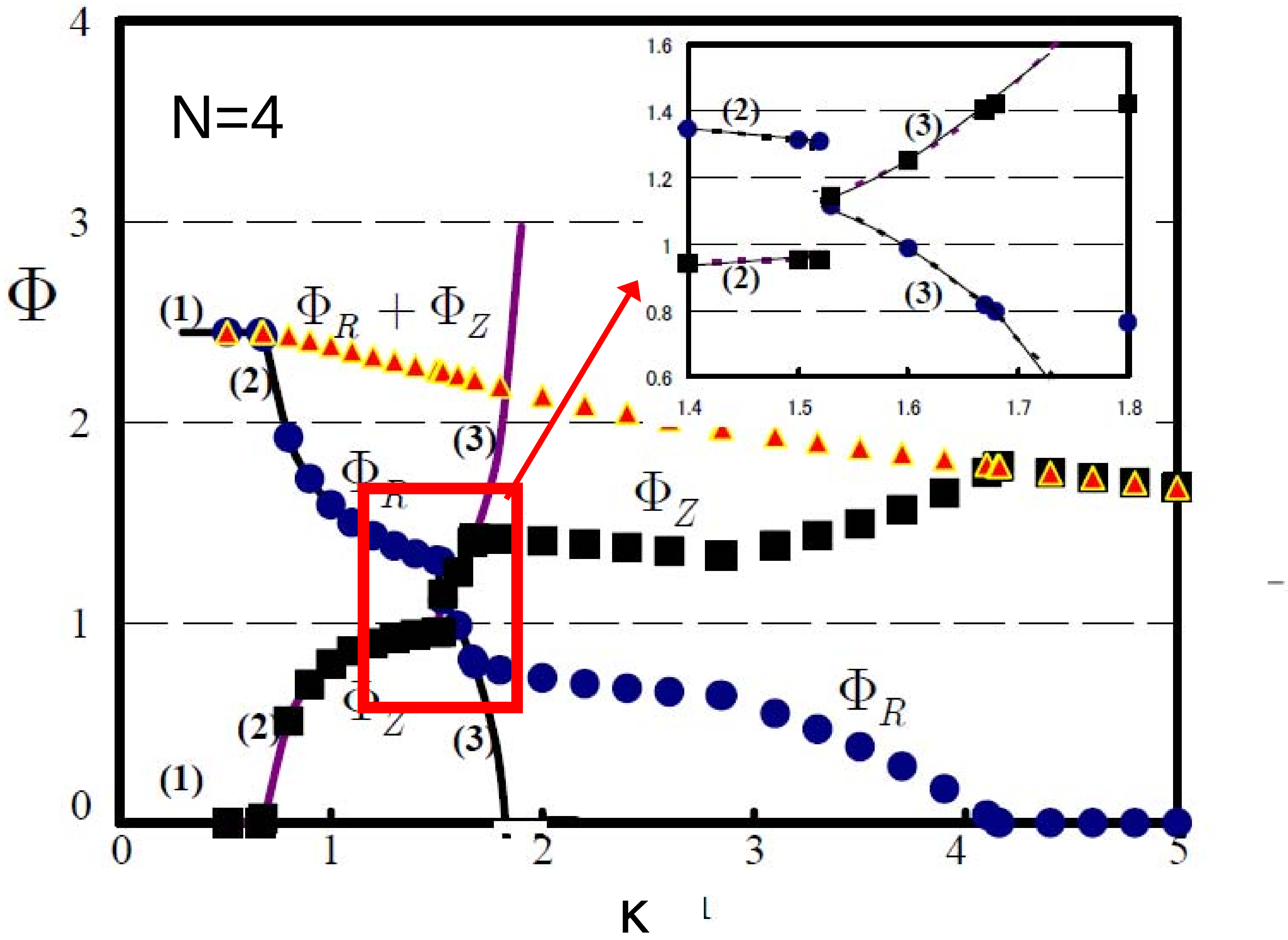
$$\Phi_R = 2^{-4/3}\left[5 - \kappa^{2/3}(2-\kappa)^{-2/3}\right]$$

$$\Phi_Z = (2\kappa)^{-1/3}(2-\kappa)^{-2/3}$$

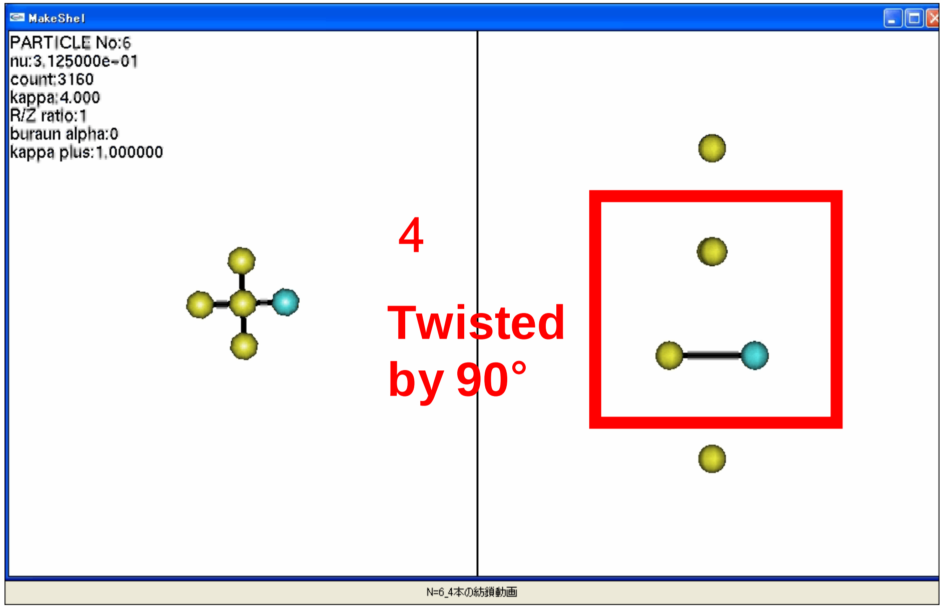
N=4



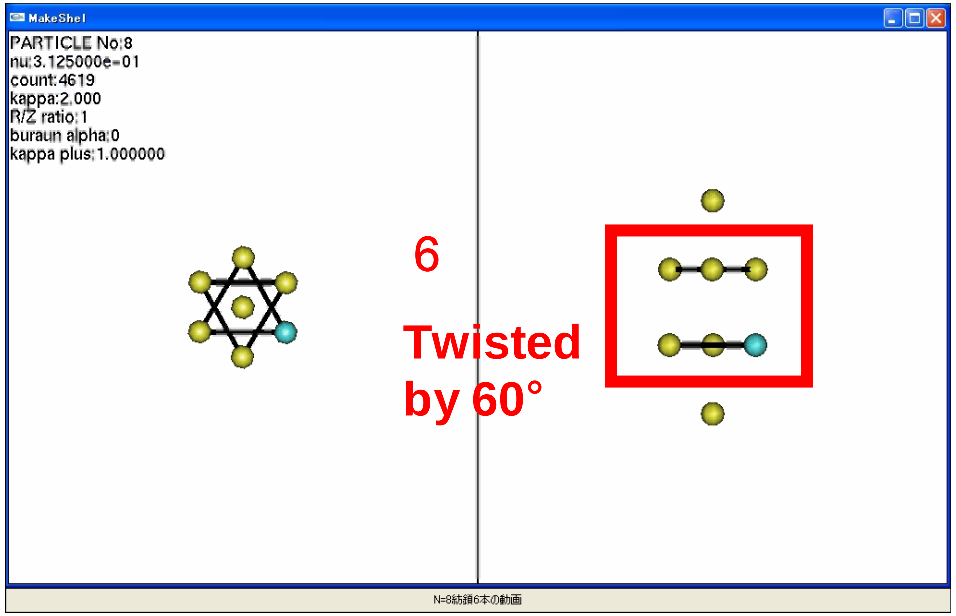
5. Non-spherical potential : 2D flat structure to spindle-like structure



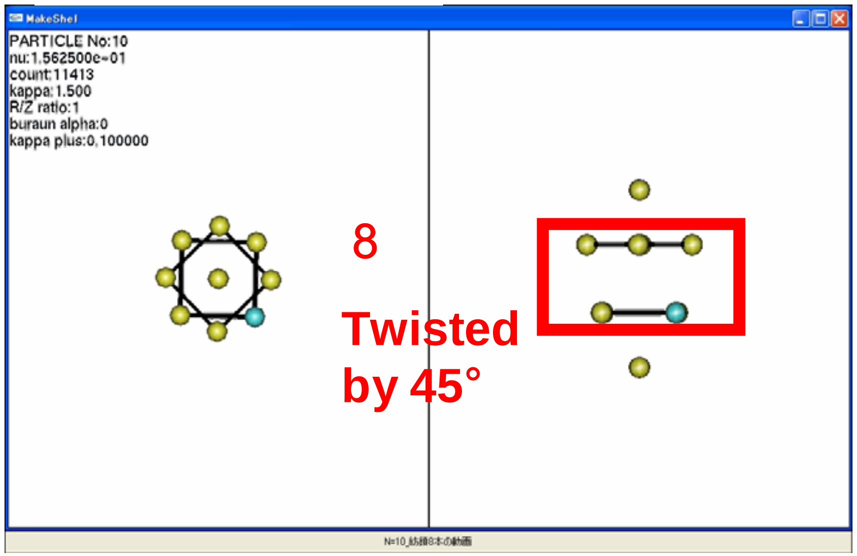
(1) N=6, κ= 4



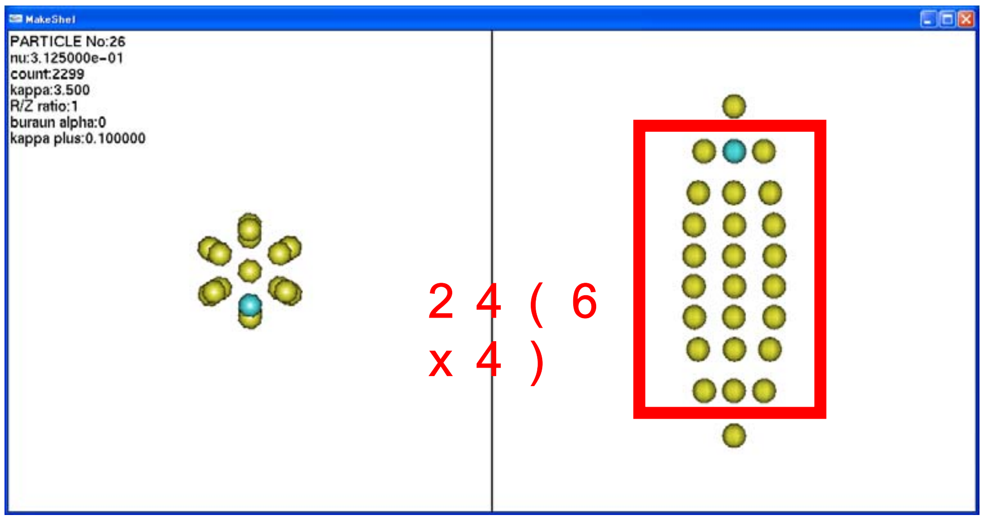
(2) N=8, κ=2.0



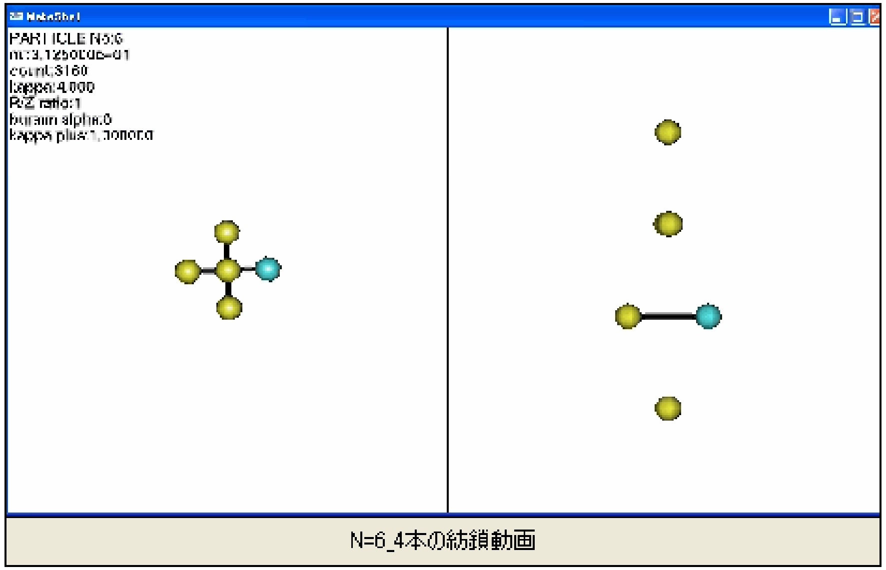
(3) N=10, κ=01.49



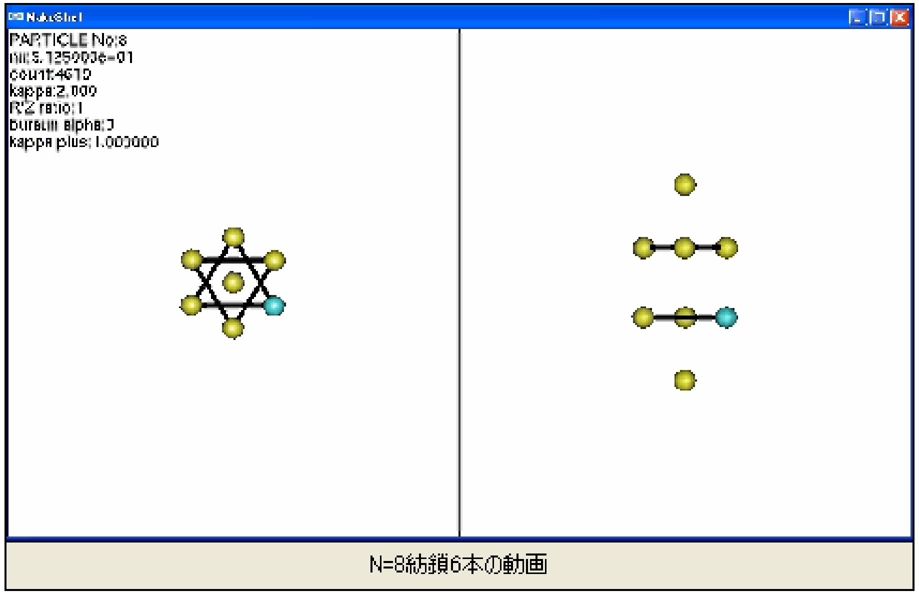
(4) N=26, κ=3.50



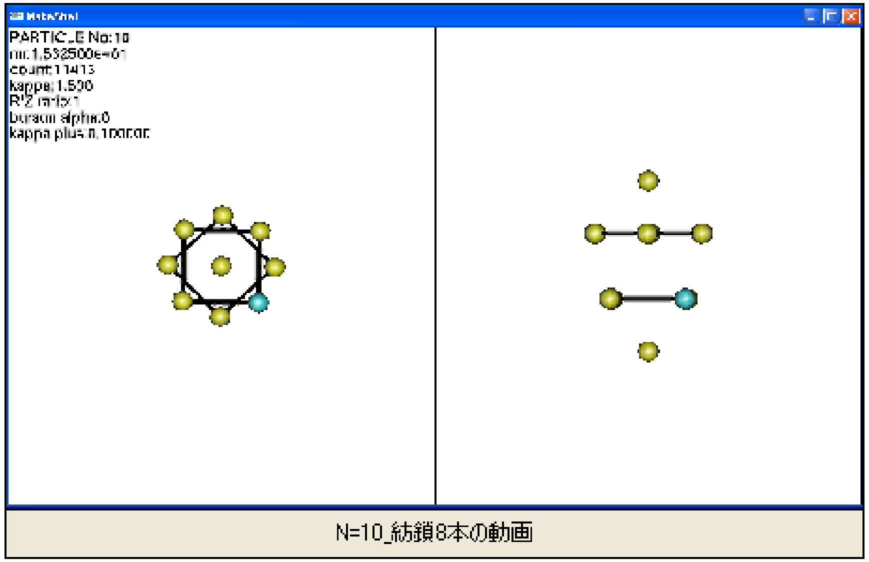
$$N = 6,\ \kappa = 4$$



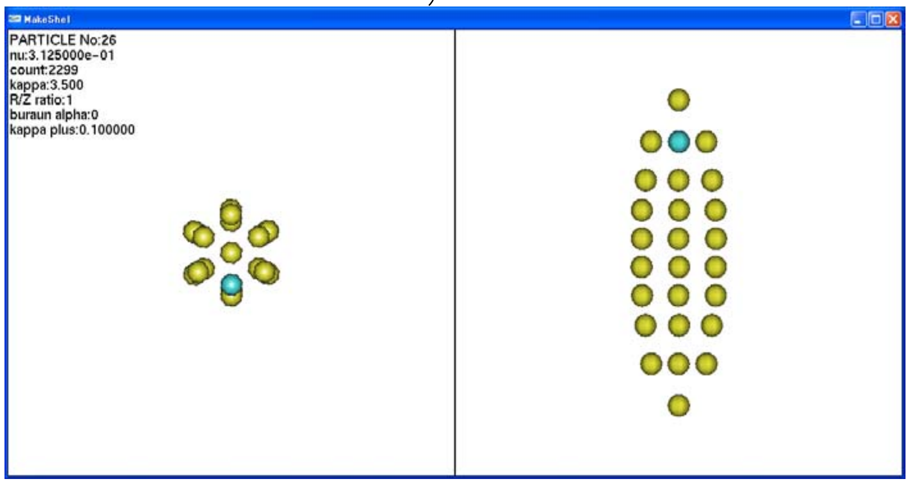
$$N = 8,\ \kappa = 2$$



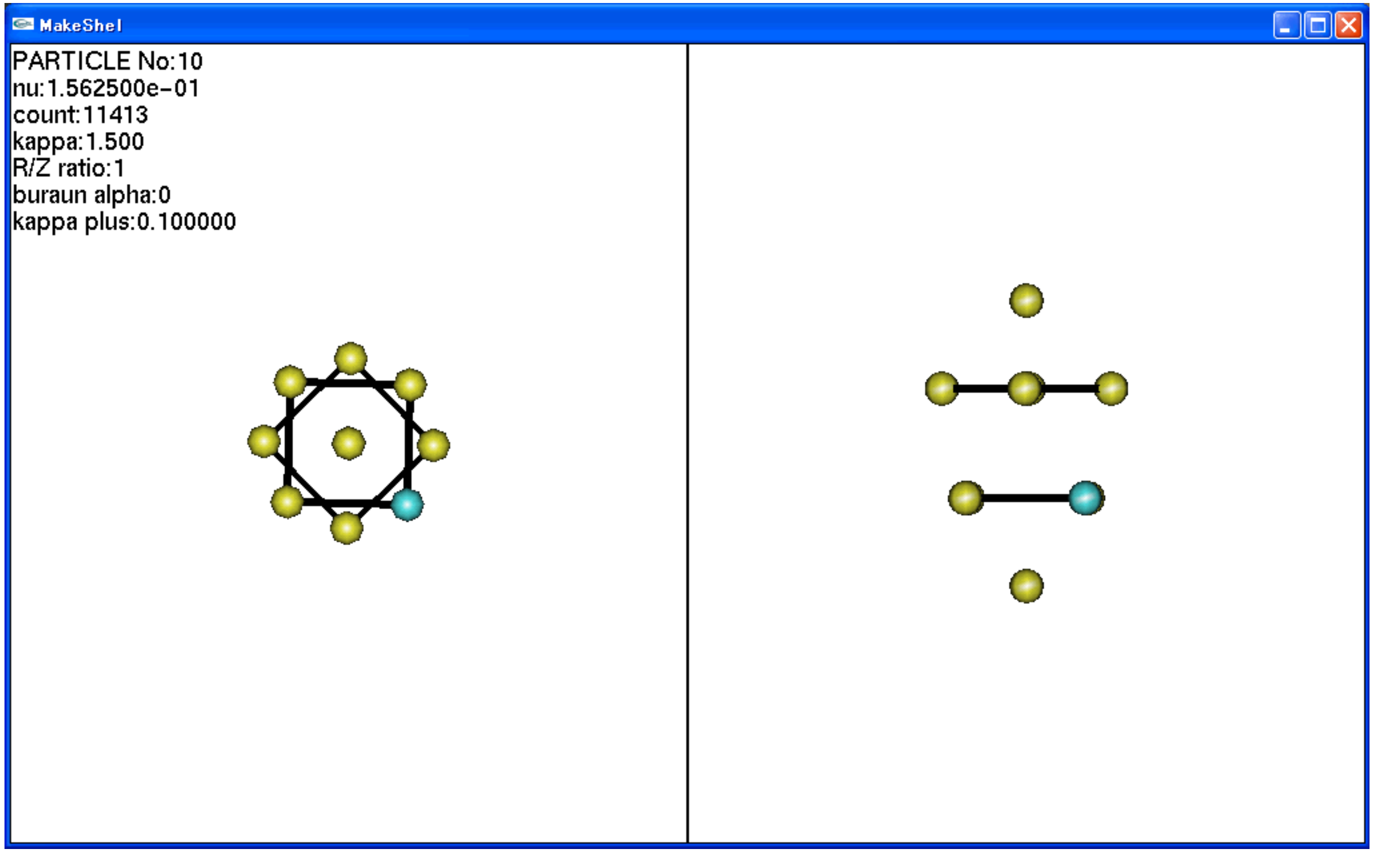
$$N = 10,\ \kappa = 1.5$$



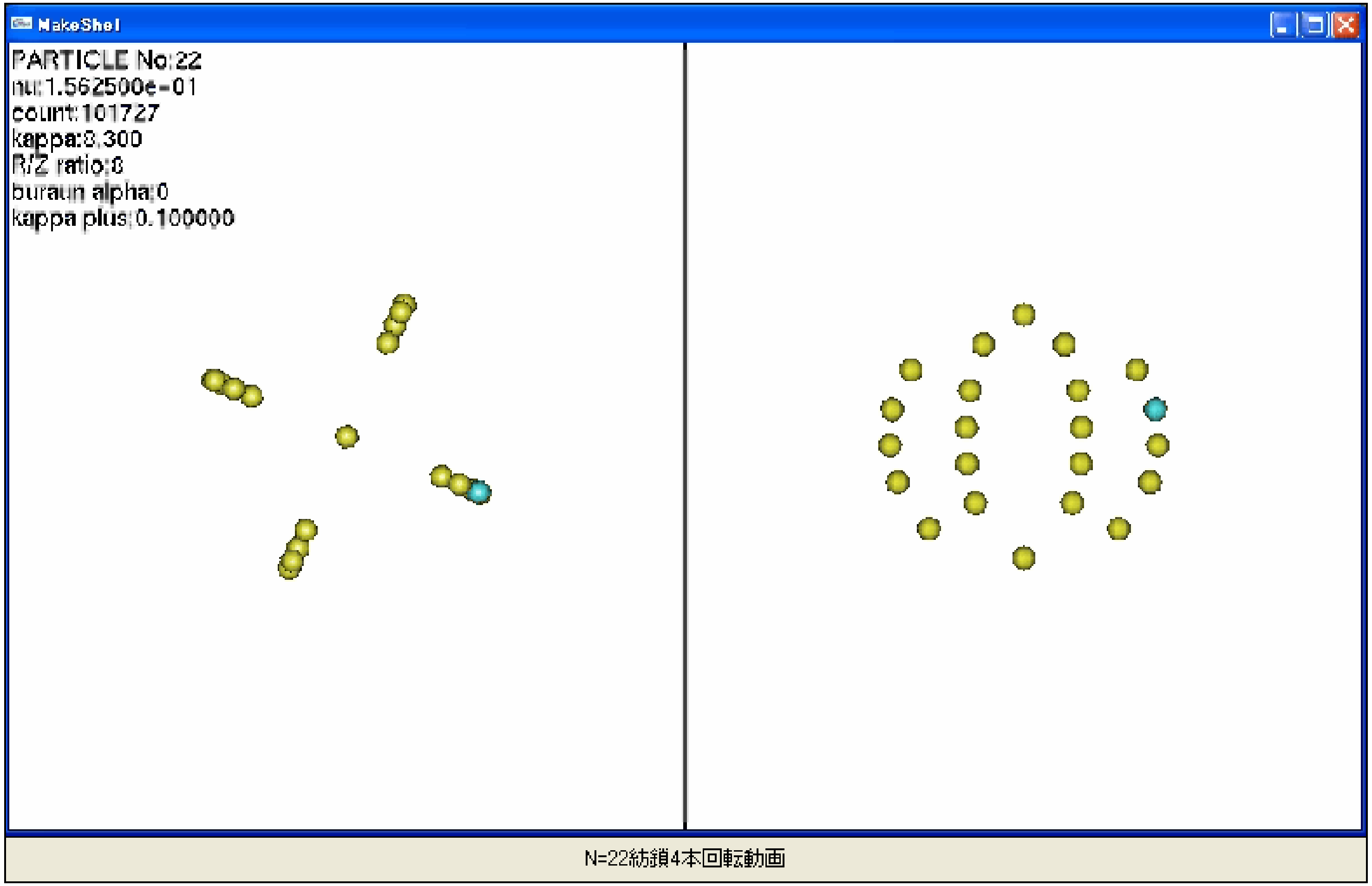
$$N = 26,\ \kappa = 3.5$$



$$N = 10, \ \kappa = 1.5$$

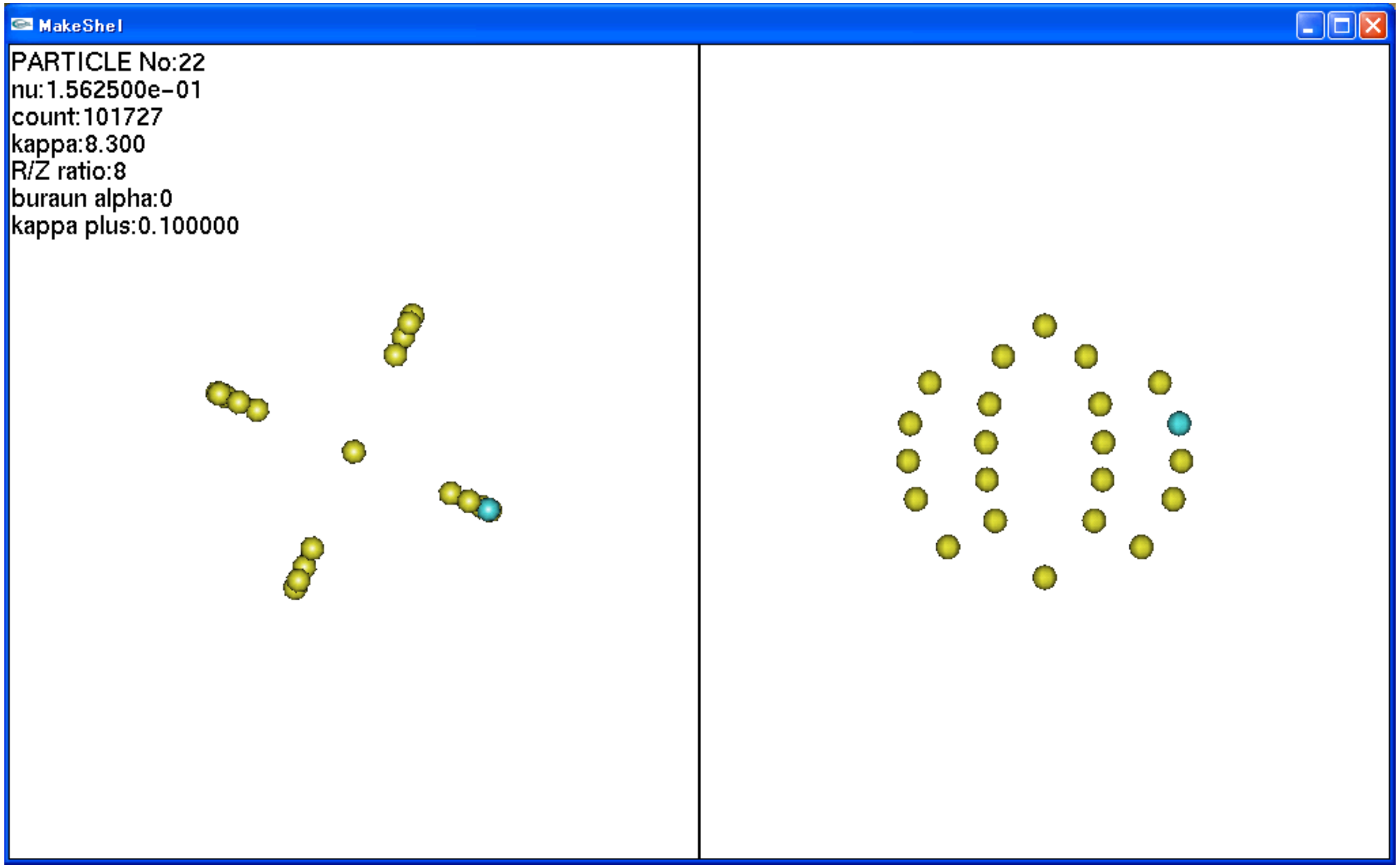

MakeShel
PARTICLE No:10
nu:1.562500e-01
count:11413
kappa:1.500
R/Z ratio:1
buraun alpha:0
kappa plus:0.100000

$$N = 22, \ \kappa = 8.3$$

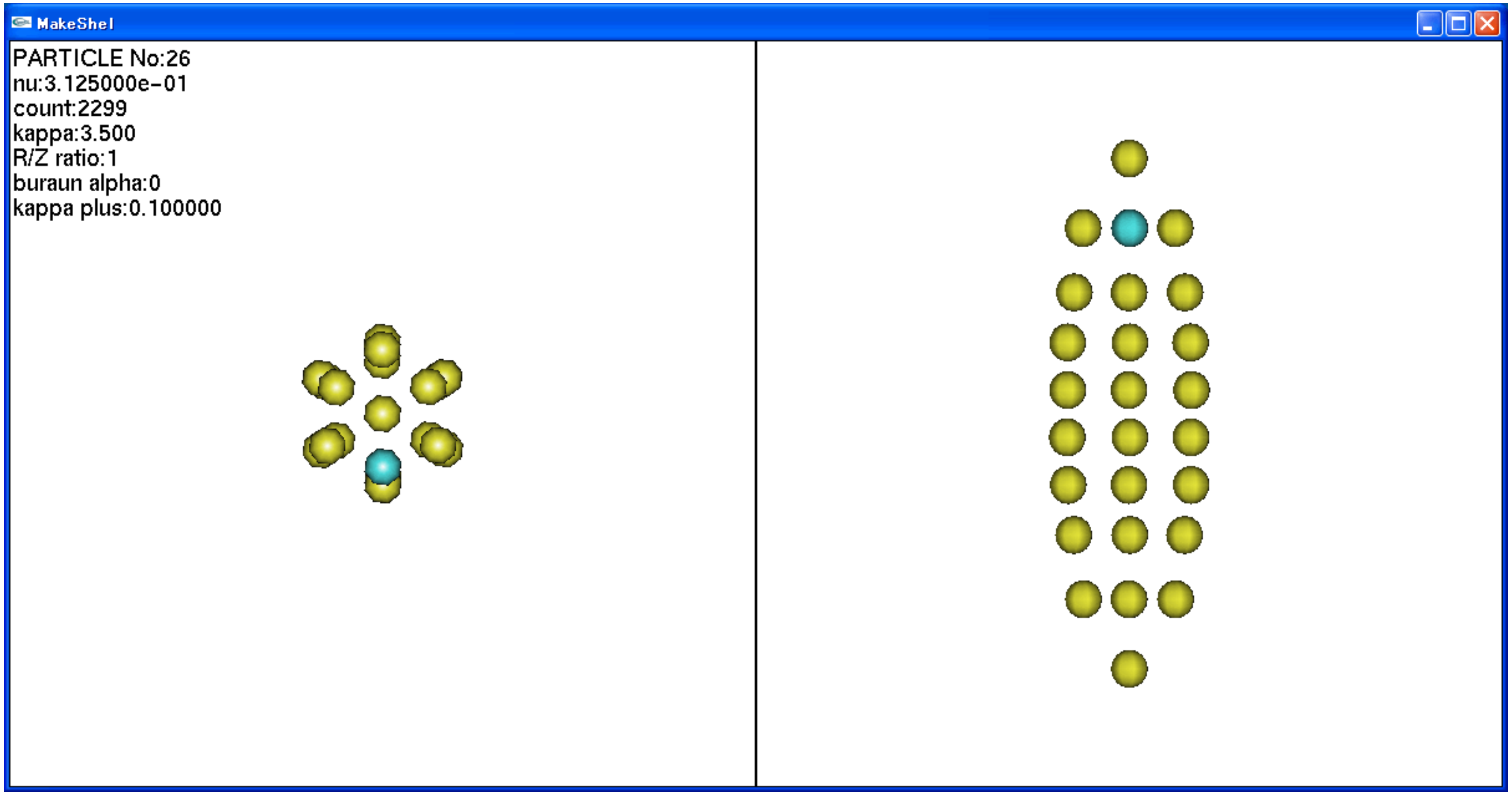


MakeShel
PARTICLE No:22
nu:1.562500e-01
count:101727
kappa:8.300
R/Z ratio:8
buraun alpha:0
kappa plus:0.100000
N=22紡錘4本回転動画

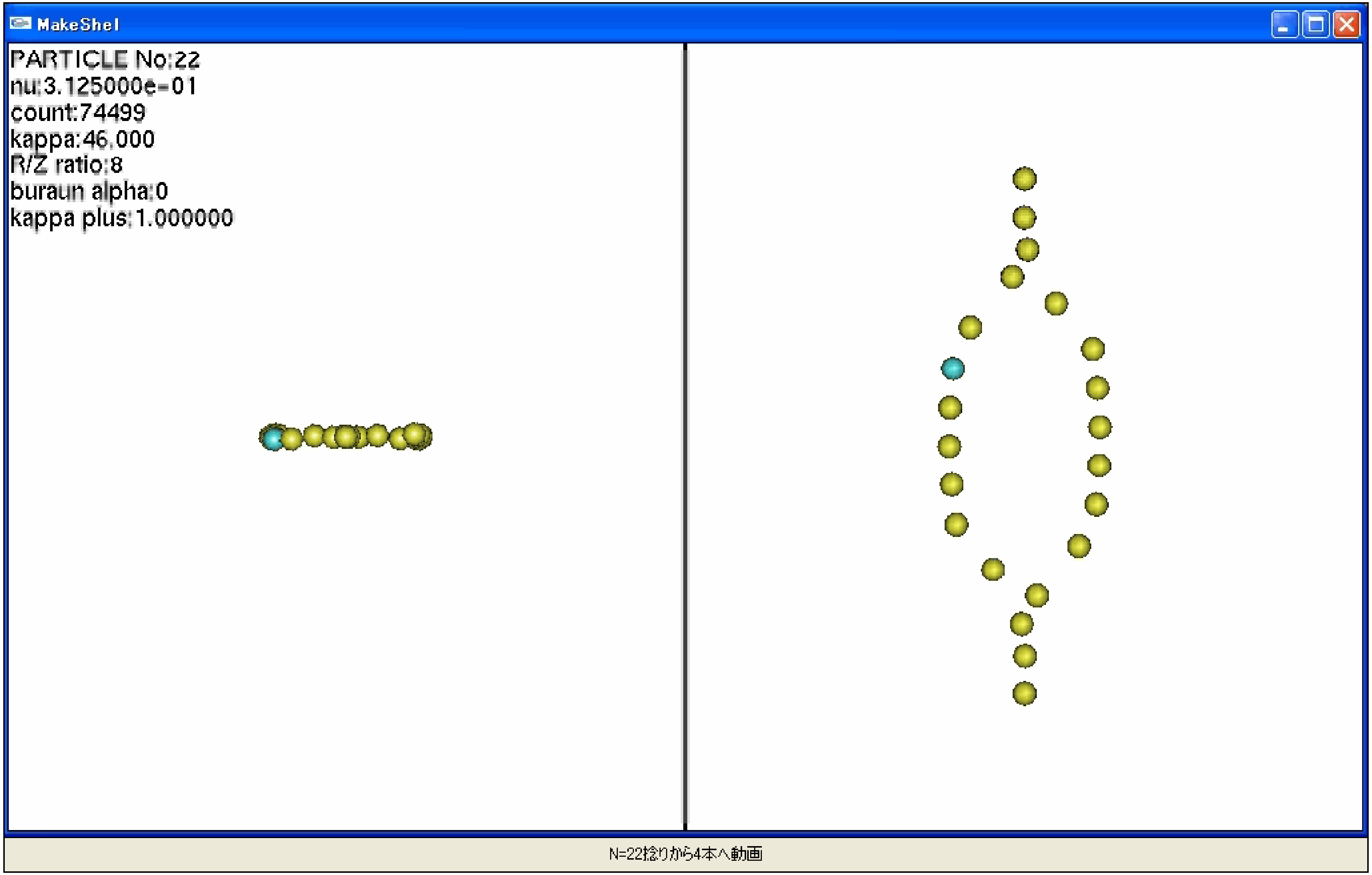
$$N = 22,\ \ \kappa = 8.3$$


MakeShel
PARTICLE No:22
nu:1.562500e-01
count:101727
kappa:8.300
R/Z ratio:8
buraun alpha:0
kappa plus:0.100000

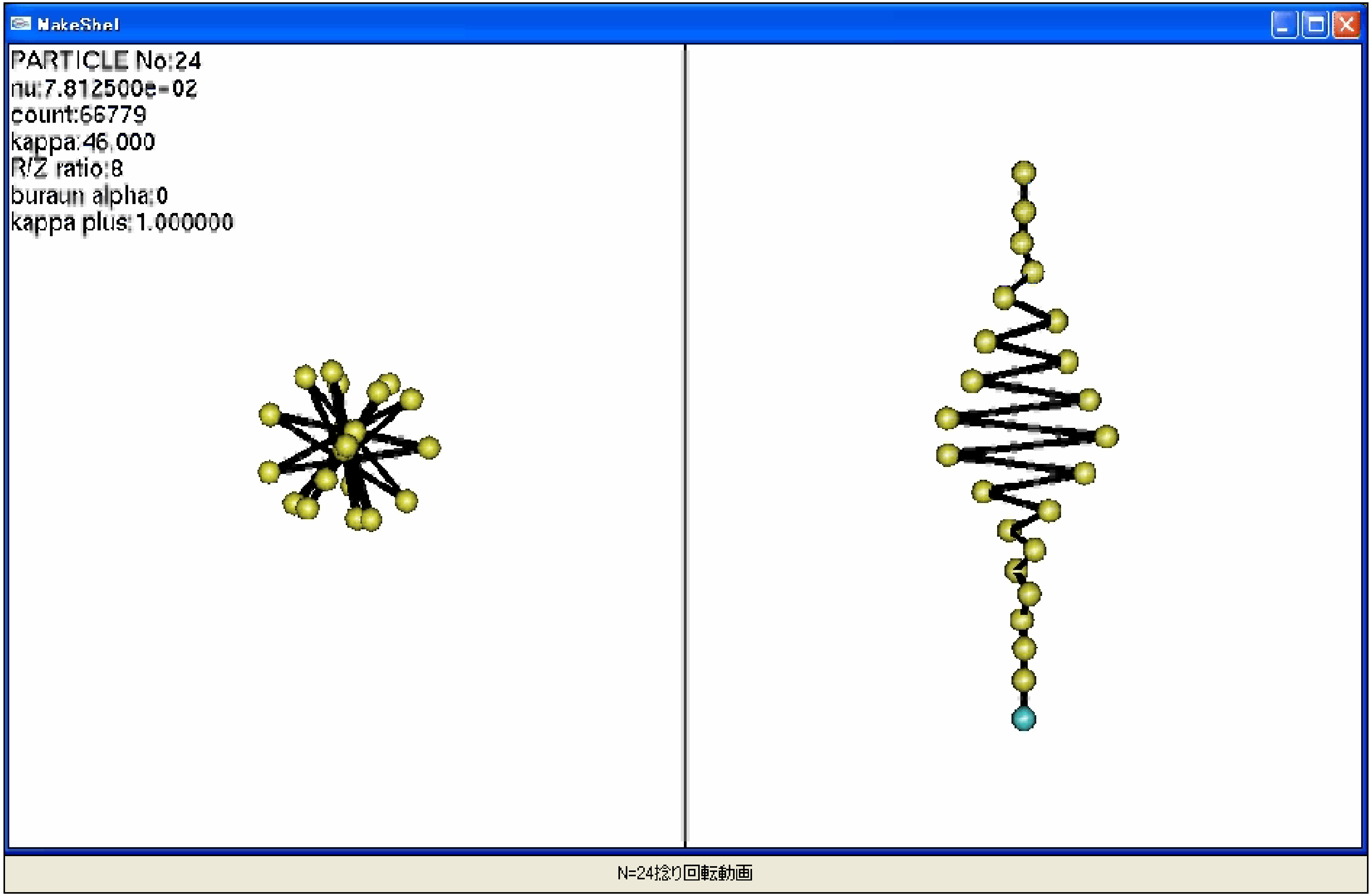
$$N = 26,\ \kappa = 3.5$$


MakeShel
PARTICLE No:26
nu:3.125000e-01
count:2299
kappa:3.500
R/Z ratio:1
buraun alpha:0
kappa plus:0.100000

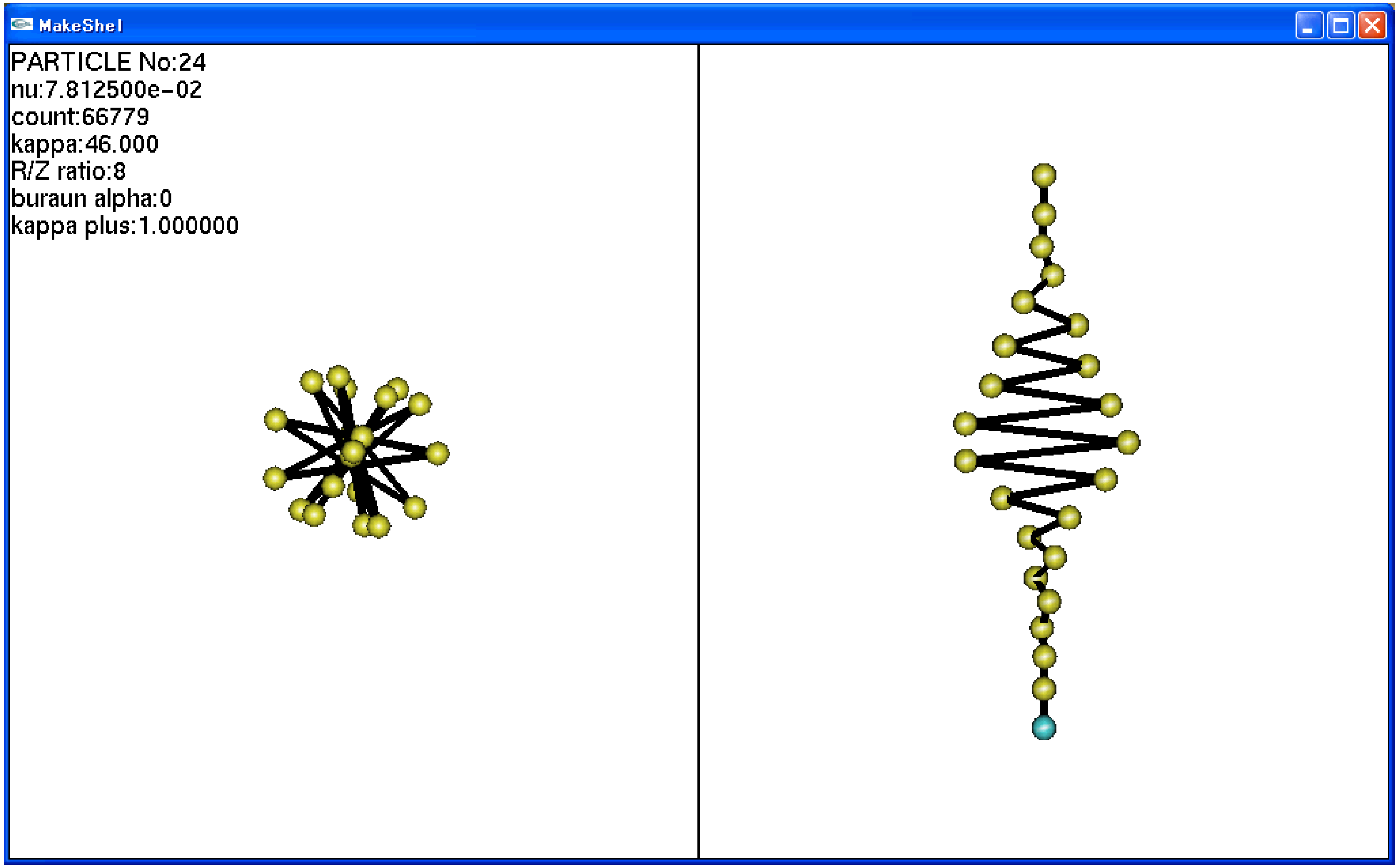
$$N = 22,\ \kappa = 46$$


MakeShel
PARTICLE No:22
nu:3.125000e-01
count:74499
kappa:46.000
R/Z ratio:8
buraun alpha:0
kappa plus:1.000000
N=22捻りから4本へ動画

$$N = 24,\ \kappa = 46$$


MakeShel
PARTICLE No:24
nu:7.812500e-02
count:66779
kappa:46.000
R/Z ratio:8
buraun alpha:0
kappa plus:1.000000
N=24捻り回転動画

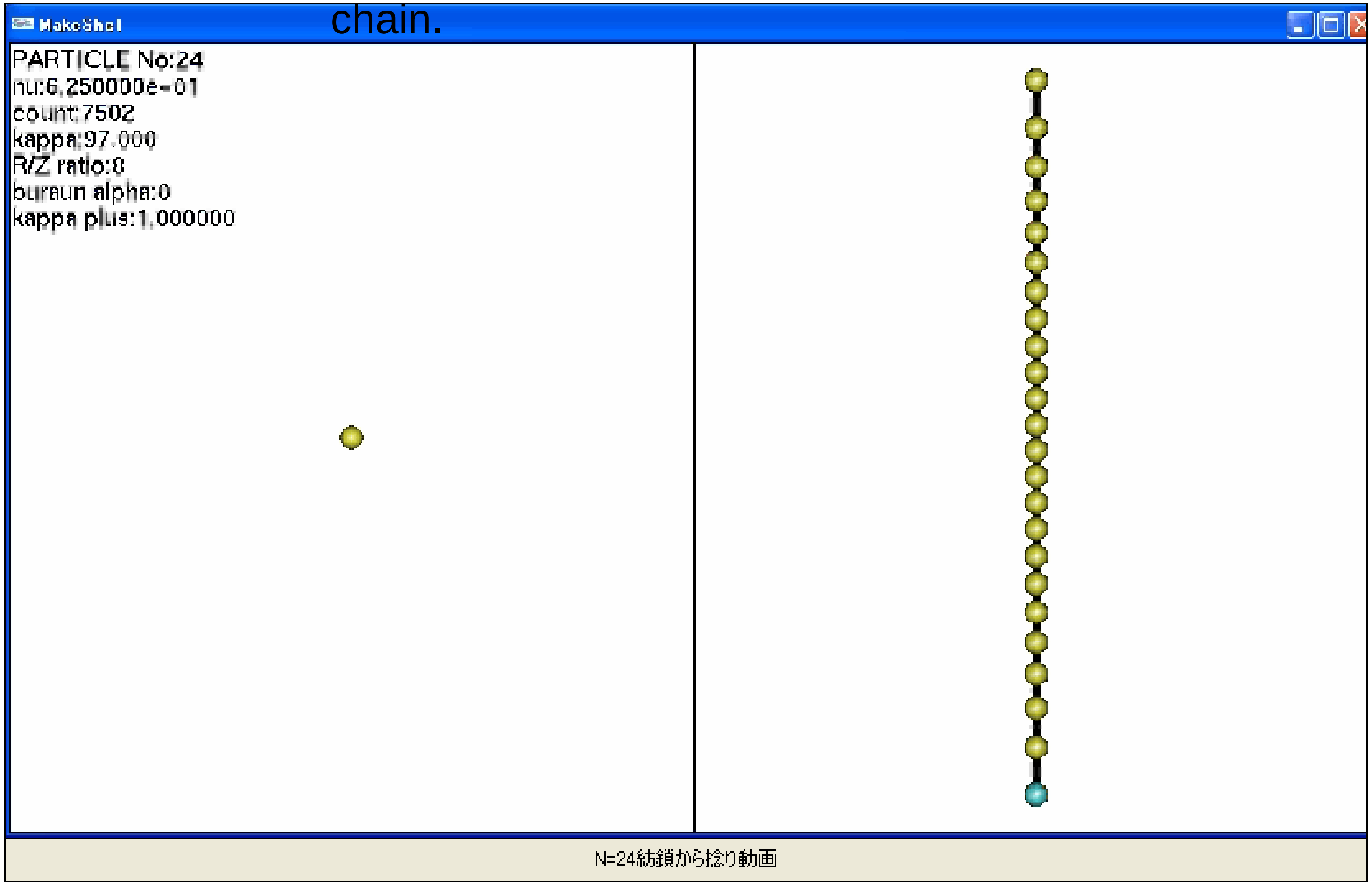
$$N = 24, \ \kappa = 46$$


MakeShel
PARTICLE No:24
nu:7.812500e-02
count:66779
kappa:46.000
R/Z ratio:8
buraun alpha:0
kappa plus:1.000000

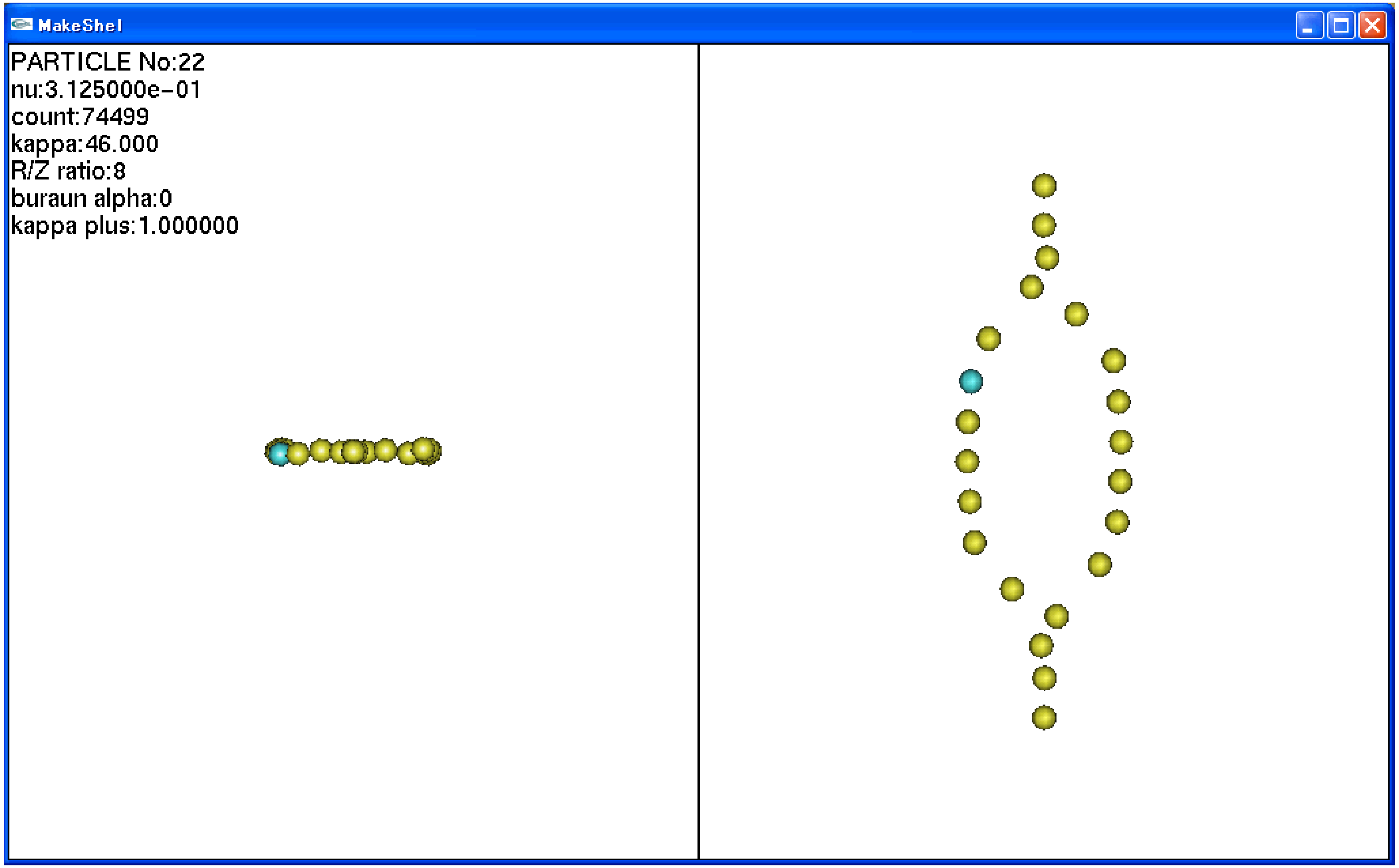
CHANGING THE ELONGATION PARAMETER

$$N = 24,\ \kappa = 97$$

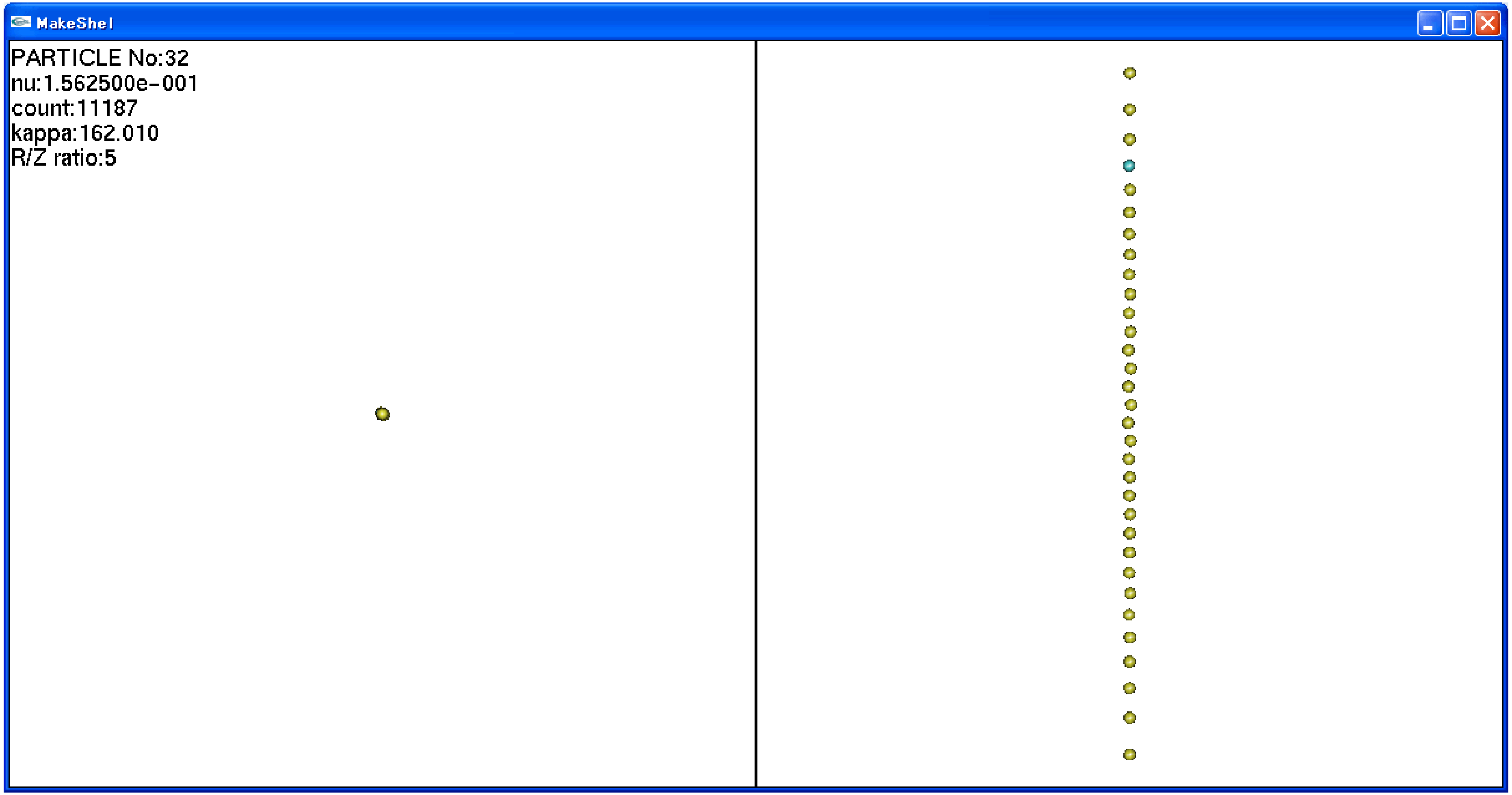
For κ = 9 7、 a straight chain.



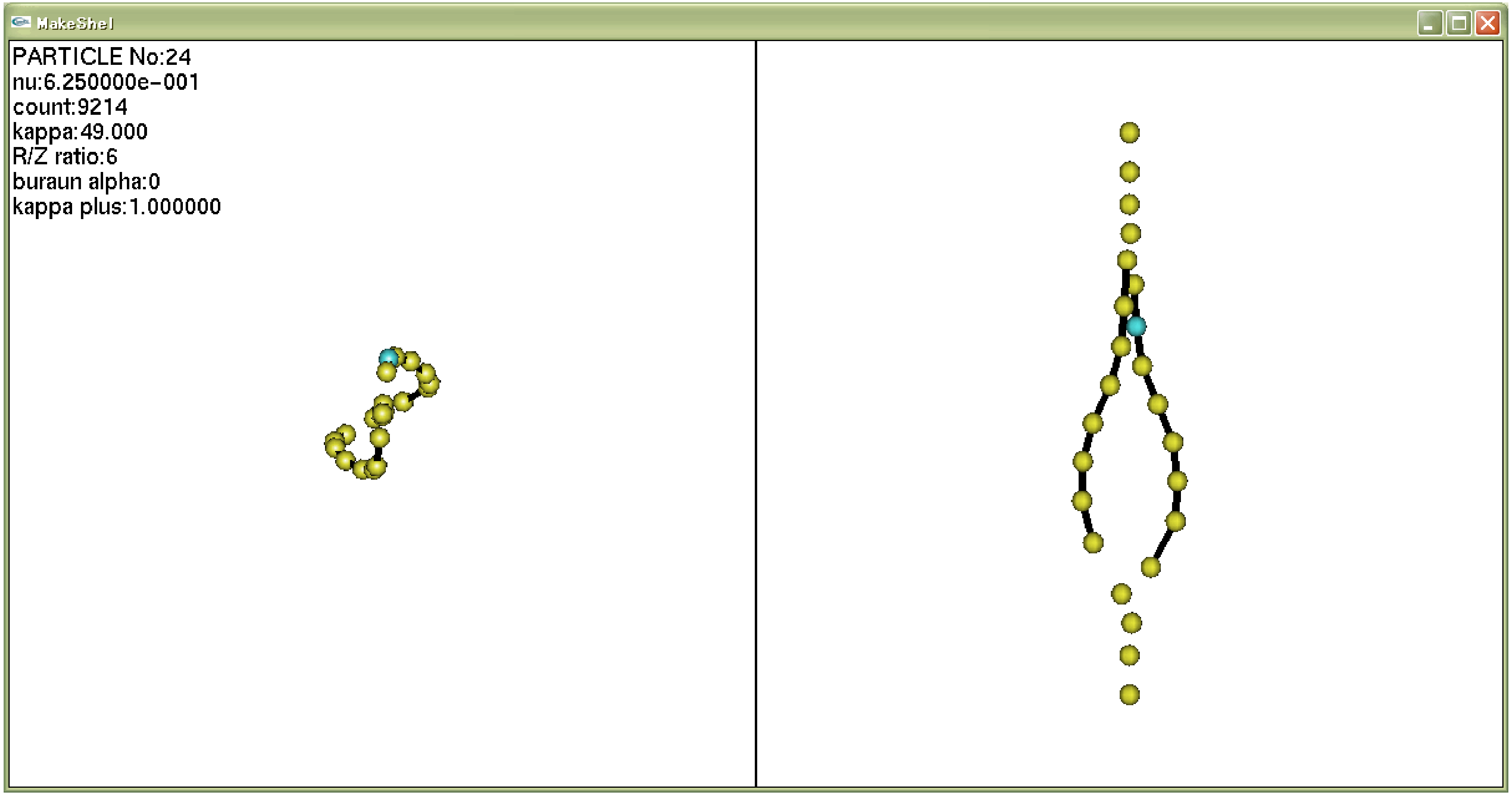
$$N = 22,\ \kappa = 46 \rightarrow 8$$



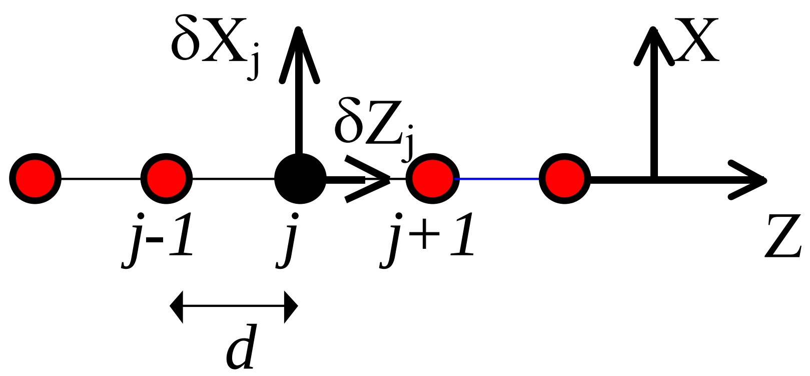
$$N = 32,\ \kappa = 162 \to 5$$



$$N = 24,\ \kappa = 49 \to 40$$



Lattice oscillation



$$\phi(r) = \frac{Q}{r} e^{-r/\lambda_D}$$

$$\mathbf{F}(\mathbf{r}) = -\kappa\delta\mathbf{r}$$

$$\mathrm{E}(\mathbf{r}) = -\nabla\phi(\mathbf{r}) = \mathrm{e}_r \frac{Q}{r^2}\left(1+\frac{r}{\lambda_D}\right)e^{-r/\lambda_D}$$

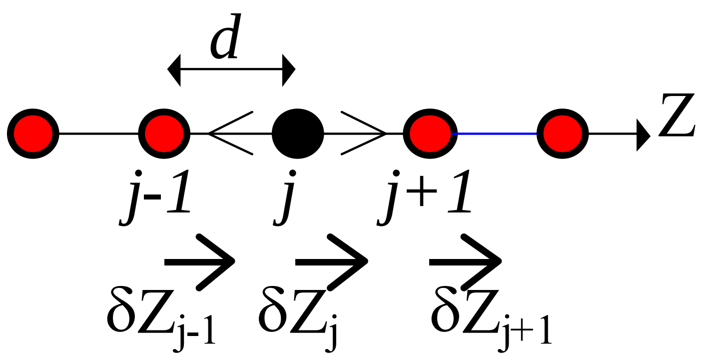
$$\mathrm{E}(\mathbf{r}+\delta\mathbf{r}) \approx \mathrm{E}(\mathbf{r}) + \delta\mathbf{r}\cdot\frac{\partial}{\partial\mathbf{r}}\mathrm{E}(\mathbf{r})$$

↓ longitudinal

↓ *transverse*

O. Ishihara and S.V.Vladimirov, Phys. Rev. E 57, 3392 (1998).

Longitudinal Oscillation



$$\delta Z_j = \delta Z_0 exp\left[-i\left(\omega t - jkd\right)\right]$$

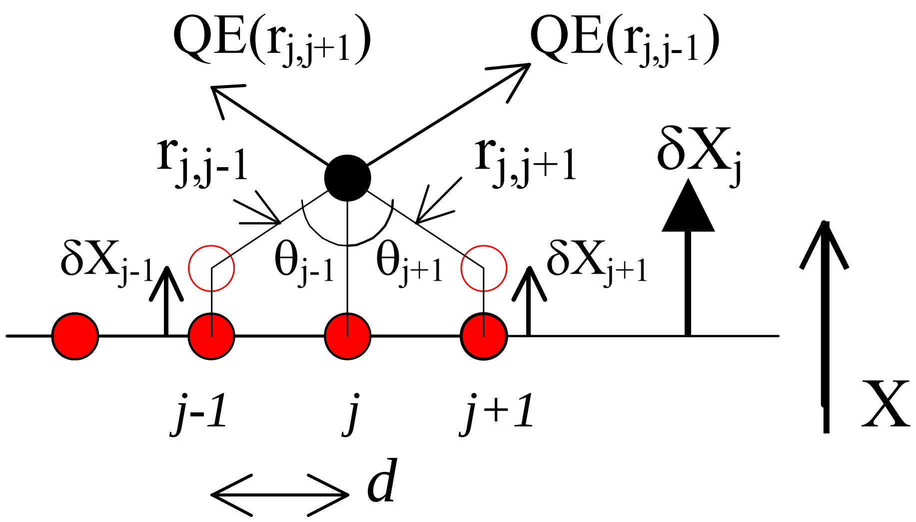
$$QE(\mathbf{r}_j) = Q\left[(\delta Z_j - \delta Z_{j-1})\left(\frac{\partial E}{\partial r}\right)_d - (\delta Z_{j+1} - \delta Z_j)\left(\frac{\partial E}{\partial r}\right)_d\right]$$

$$m_d \frac{d^2 \delta Z_j}{dt^2} = Q\left(\frac{\partial E}{\partial r}\right)_d \left(2\delta Z_j - \delta Z_{j-1} - \delta Z_{j+1}\right)$$

$$\omega^2 = \frac{2Q^2}{m_d d^3}\left(1 + \frac{d}{\lambda_D} + \frac{d^2}{2\lambda_D^2}\right)\exp\left(-\frac{d}{\lambda_D}\right)\sin^2\left(\frac{kd}{2}\right)$$

$$\omega^2 \xrightarrow[\lambda_D \to \infty]{} \frac{2Q^2}{m_d d^3}\sin^2\left(\frac{kd}{2}\right)$$

Transverse Oscillation



$$E(r_{j,j-1}) \approx E(r_{j,j+1}) \approx E(d)$$

$$\delta X_j = \delta X_0 exp\left[-i\left(\omega t - jkd\right)\right]$$

$$QE(r_j) = Q\left[E(r_{j,j-1})\cos\theta_{j-1} - E(r_{j,j+1})\cos\theta_{j+1}\right] = Q\left[E(r_{j,j-1})\frac{\delta X_j - \delta X_{j-1}}{d} - E(r_{j,j+1})\frac{\delta X_j - \delta X_{j+1}}{d}\right]$$

$$QE(r_j) = QE(d)\frac{2\delta X_j - \delta X_{j-1} - \delta X_{j+1}}{d}$$

$$\omega^2 = \frac{\kappa}{m_d} - \frac{4Q^2}{m_d d^3}\left(1 + \frac{d}{\lambda_D}\right)\exp\left(-\frac{d}{\lambda_D}\right)\sin^2\left(\frac{kd}{2}\right)$$

$$\omega^2 \xrightarrow[\lambda_D -> \infty]{} \frac{\kappa}{m_d} - \frac{4Q^2}{m_d d^3}\sin^2\left(\frac{kd}{2}\right)$$

Lattice Oscillation

$$\omega^2 = a_L(d)\, sin^2\left(\frac{kd}{2}\right), \qquad a_L(d) = \frac{2Q^2}{m_d d^3}\left(1 + \frac{d}{\lambda_D} + \frac{d^2}{2\lambda_D^2}\right) exp\left(-\frac{d}{\lambda_D}\right)$$

$$\omega^2 = \frac{\kappa}{m_d} - a_T(d) sin^2\left(\frac{kd}{2}\right), \quad a_T(d) = \frac{4Q^2}{m_d d^3}\left(1 + \frac{d}{\lambda_D}\right) exp\left(-\frac{d}{\lambda_D}\right)$$

Plasma parameters Te =1 eV、ne= 10^9 cm^{-3} (λ_D=0.23 mm)

Dust particle

a (radius)=1 μm

ρ (density) =1 g/cm^{-3}

Q (charge) =-4,000 *e*

d (interparticle distance) = 0.23mm

---> $\omega/2\pi$ = 11 Hz longitudinal frequency

Conclusions

- Complex plasma provides a good tool to study structures of charged particles.
- We studied the CME configurations of dust particles in a plasma
- Elongation parameter κ controls the con-figuration from flat 2D to spindle-like structure.
- There are fundamental structures with small numbers of particles.
- For large κ, we identify that the lattice oscillation (longitudinal /transverse) could be simulated by varying the parameter κ.

REF: O. Ishihara, J. Phys. D: Appl. Phys. **40**, R121-R147(2007).