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Nonlinear mirror waves in non-Maxwellian space plasmas: collapse or solitons?

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Nonlinear mirror waves in non-Maxwellian space plasmas: collapse or solitons?

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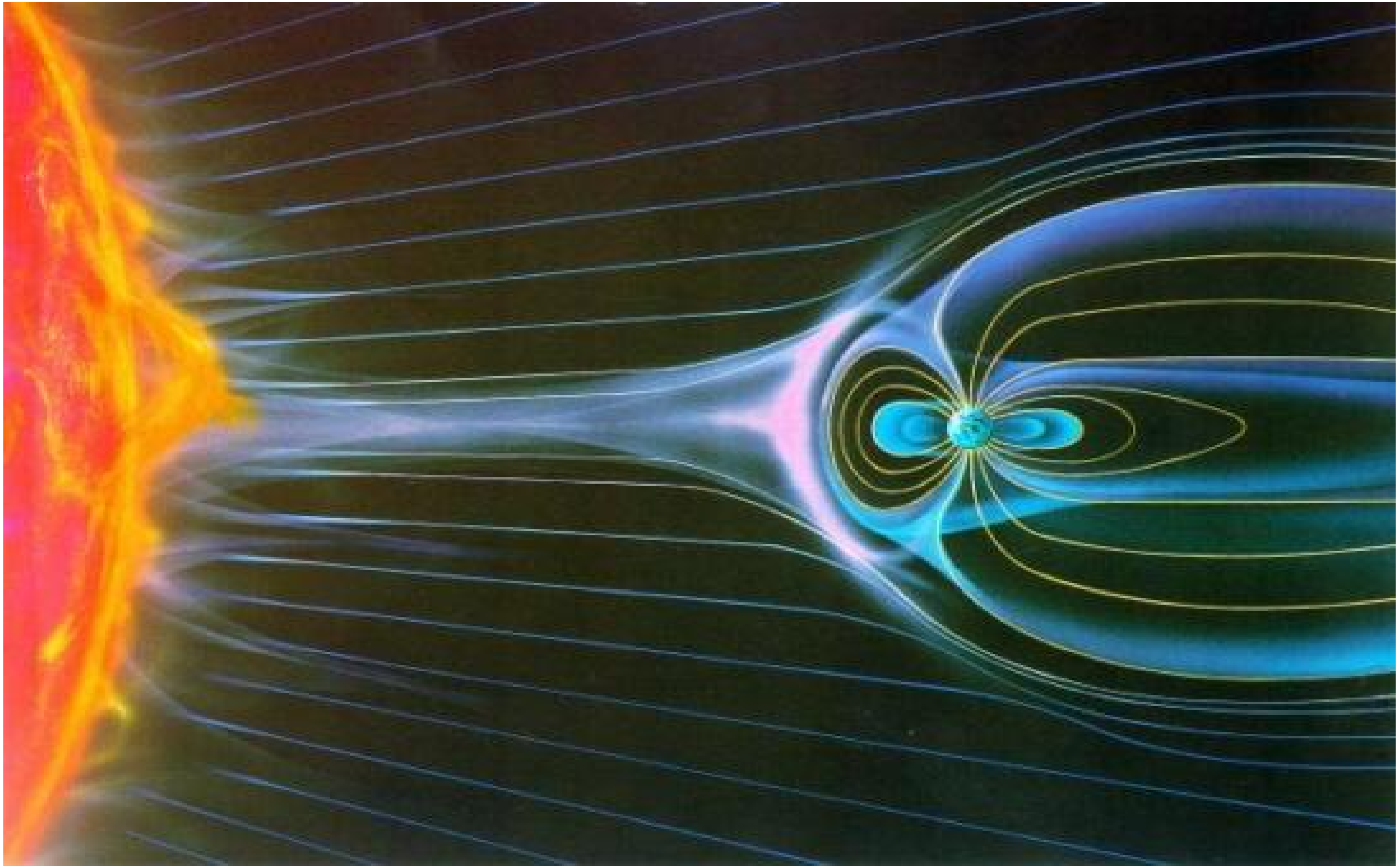
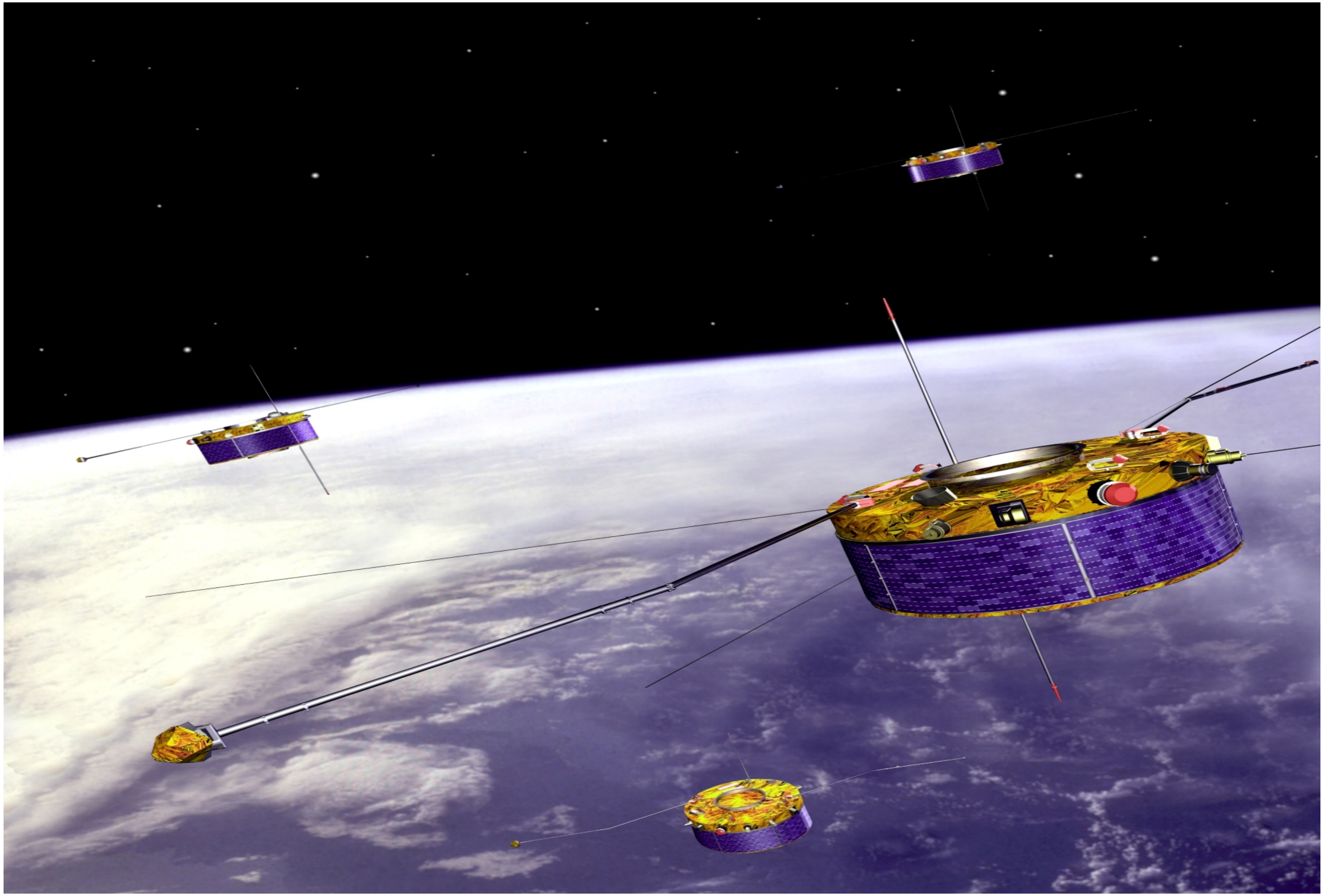
Mirror waves are found in:

- Ring-current plasma
- Earth’s magnetosheath
- Planetary magnetosheaths
- Cometary comas
- Wake of Io
- Solar wind
- Laboratory plasmas (teta-pinches)

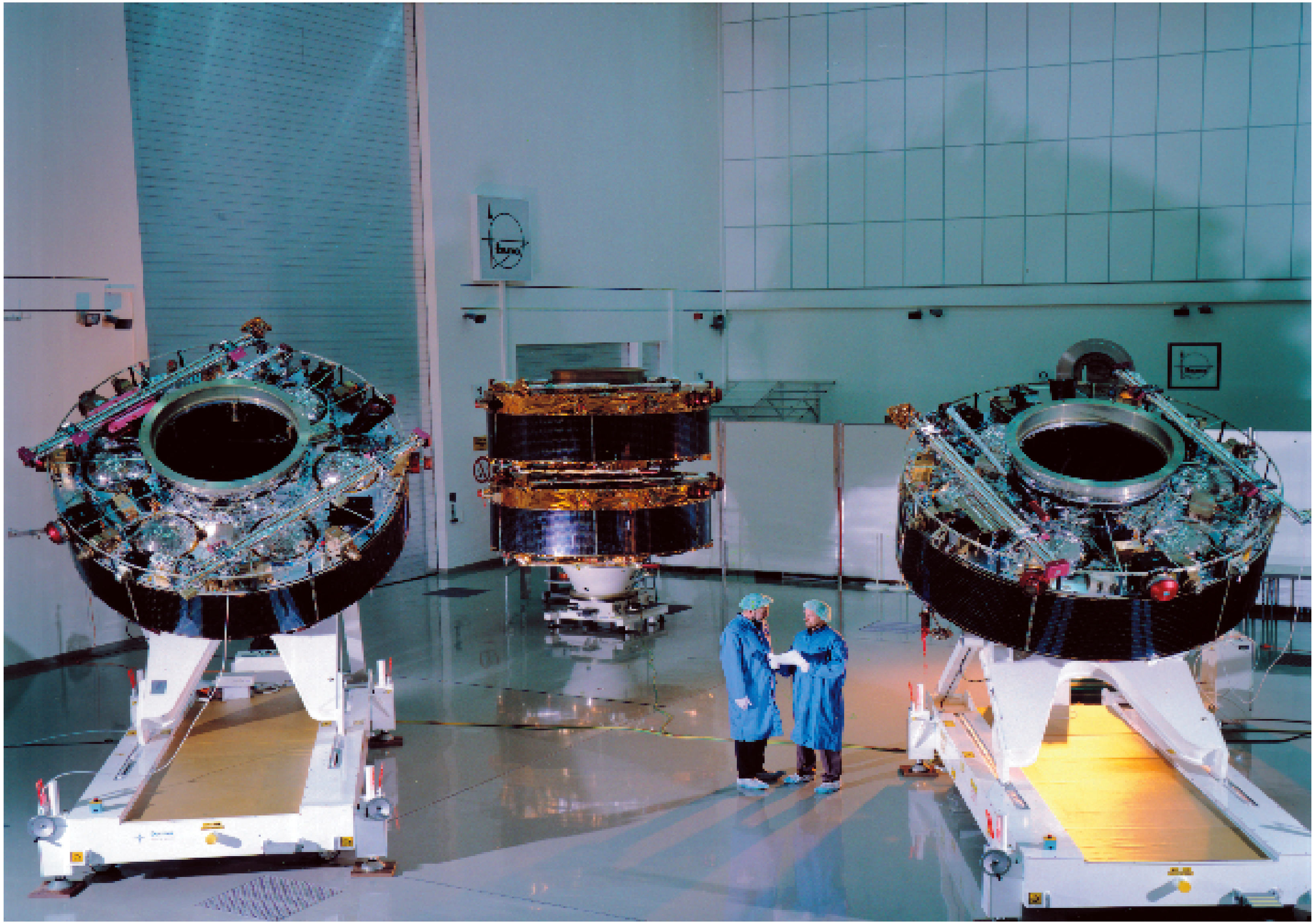
Solar-Terrestrial Interactions



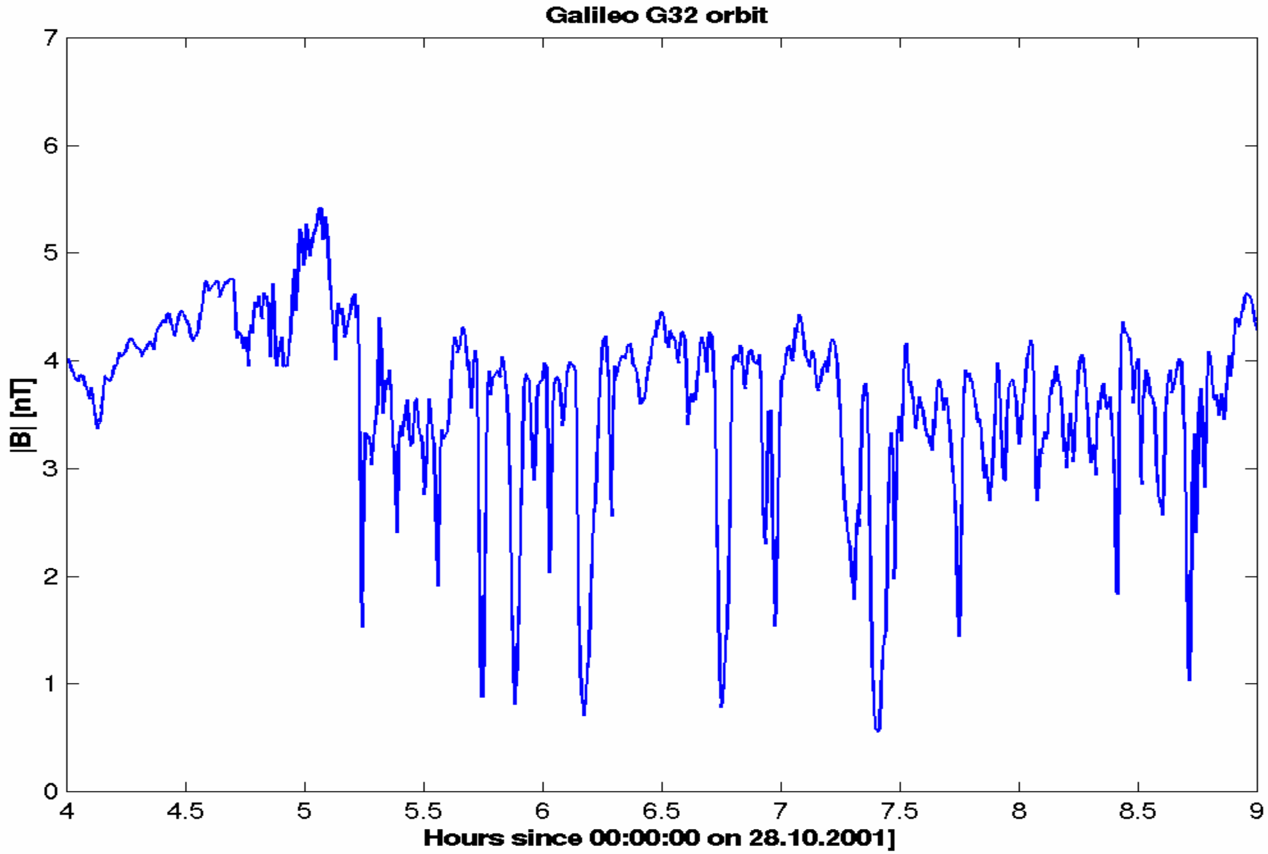
CLUSTER MISSION



Satellites



Galileo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|B| [nT]
Hours since 00:00:00 on 28.10.2001]
0
1
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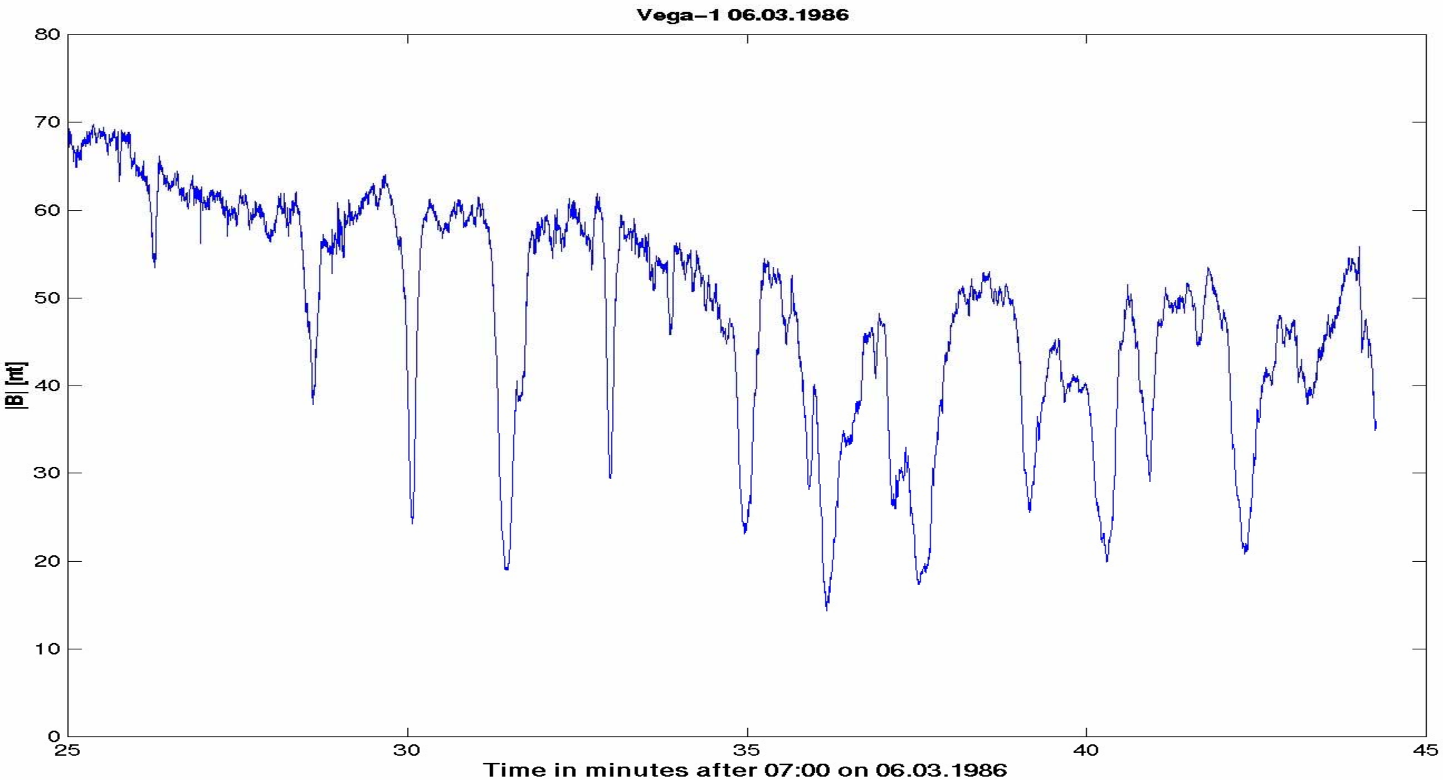
9

Vega


Vega-1 06.03.1986
|B| [nt]
Time in minutes after 07:00 on 06.03.1986
80
70
60
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35
40
45

Short history

- The mirror instability (MI) has been theoretically identified at the end of 50th [Rudakov & Sagdeev,1958; Vedenov & Sagdeev, 1958; Chandrasekhar et al., 1958].
- Physics of linear MI was discussed by Southwood and Kivelson [1993]
- Incorporation of the parallel electric field in MI theory [Pantellini & Schwartz, 1995; Pokhotelov et al., 2000]
- Non-Maxwellian effects (lose-cone, kappa-distributions etc.) - Pokhotelov and Pilipenko [1976], Leubner and Schupfer [2000, 2001], Gedalin et al. [2001], Pokhotelov et al. [2002]
- Multi-component content (Gedalin et al., 2001)
- Incorporation of the plasma inhmogeneity (Hasegawa, 1969 and numerous variations).
- Global modes (Johnson and Cheng, 1997)
- FLR effects (Pokhotelov et al.,2004)
- The first attempt to describe the nonlinear evolution of MI in terms of RPA has been proposed, long ago, by Shapiro & Shevchenko [1964].
- Nonlinear MI saturation (Kivelson and Southwood,1996; Pantellini, 1998)
- Wave breaking (collapse) effects (Kuznetsov et al.,2007)

Mirror instability dispersion relation

General dispersion relation

$$K - \frac{k_{\parallel}^{2}}{k_{\perp}^{2}\beta_{\perp}}\left(1 + \frac{2\beta_{\perp}\left\langle mv_{\parallel}^{2} J_{1}^{'2}\mu \frac{\partial F}{\partial \mu}\right\rangle}{p_{\perp}}\right) + \frac{i\pi^{2}\omega D}{mk_{\parallel} p_{\perp}} = 0$$

Low - frequency (mirror approximat ion)
$\omega << k_{\parallel} v_{T_{\parallel}}$

where the abbreviati ons are

$$K = A^{FLR} - \frac{1}{\beta_{\perp}}, \quad D = -\int_{0}^{\infty}\left(\frac{2J_{1}}{\xi}\right)^{2}\frac{\partial F_{res}}{\partial W} W^{2} dW$$

with anisotropy factor given by

$$A^{FLR} = \frac{\left\langle \mu^{2} B^{2}\left(\frac{2J_{1}}{\xi}\right)^{2}\frac{\partial F}{B\,\partial\mu}\right\rangle}{2p_{\perp}}$$

Mirror istability in non-Maxwellian plasmas

Optimal dimensions

$$\left(\frac{k_{\|}}{k_{\perp}}\right)^2_{max} = \frac{K}{3}\frac{\beta_{\perp}}{1+2\beta_{\perp}p_{\perp}^{-1}\left\langle m v_{\|}^2 J_1'^2 \mu \partial F/\partial\mu\right\rangle}$$

Maximum growth rate

$$\gamma_{max} = \frac{2}{3^{3/2}\pi^2}\frac{1}{D}\frac{k_{\perp}p_{\perp}m\beta_{\perp}^{1/2}K^{3/2}}{\left[1+2\beta_{\perp}p_{\perp}^{-1}\left\langle m v_{\|}^2 J_1'^2 \mu \partial F/\partial\mu\right\rangle\right]^{1/2}}$$

General kinetic mirror instability condition with FLR effect

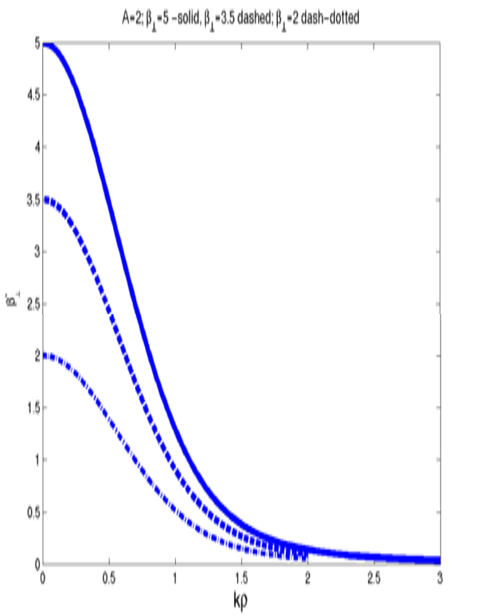
Arbitrary ion distribution

$$K = \frac{\left\langle \mu^2 B^2 \left(\frac{2J_1}{\xi} \right)^2 \frac{\partial F}{B \partial \mu} \right\rangle}{2 p_\perp} - \frac{1}{\beta_\perp} > 0$$

Example : Bi - Maxwellian ion distribution

$$\frac{T_\perp}{T_{II}} - 1 > \frac{1}{\beta_\perp^*}$$

where $\beta_\perp^* = \beta_\perp [I_0(z) - I_1(z)] e^{-z}, z = (k_\perp \rho_i)^2 / 2$



Ion distribution functions

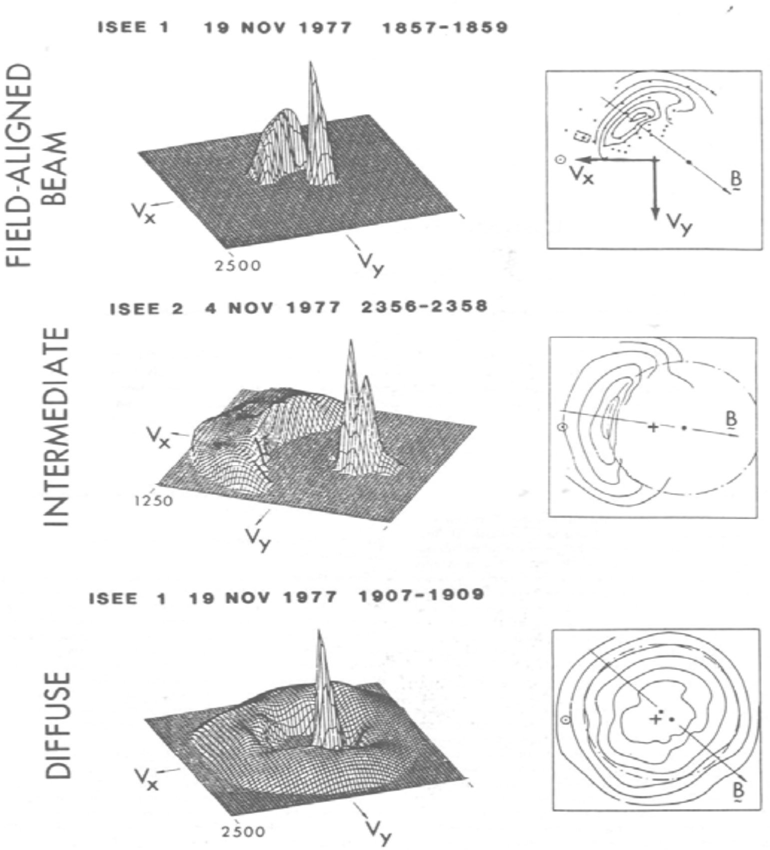


Fig. 1. Examples of three types of suprathermal upstream ion distributions (adapted from *Paschmann et al.* [1981]). On the left are relief plots of count rate in two-dimensional velocity space (integrated over ±55° of elevation about the ecliptic plane), and on the right are contours of constant phase space density in the same two-dimensional velocity space for the same events.

Halo instability (isotropic plasma)

Growth rate

$$\gamma = -\frac{k_{\parallel} k^2 v_A^2}{4\pi^2 \omega_{ci}^2} \frac{\langle F \rangle}{\int_0^\infty \left(J_1^2 + \xi J_1 J_1^{'} \right) F_{res} v dv}$$

Instability condition

$$I = \int_0^\infty \left(J_1^2 + \xi J_1 J_1^{'} \right) F_{res} v dv < 0$$

Bump on the tail ion distribution

$$F_{res} = \frac{n}{\pi^{3/2} v_{T_i}^3} \exp(-v^2 / v_{T_i}^2) + \frac{n_h}{4\pi v_0^2} \delta(v - v_0)$$

Summary

- The general low-frequency dispersion relation in high-beta non-Maxwellian plasmas accounting for the FLR effect admitts for two distinct instabilities. One of them corresponds to the classical mirror instability that arises when plasma pressure anisotropy exceeds a certain critical value. Another one, refers to the halo instability that can arise even in isotropic plasma when the ion distribution function possess a bump on the tail.
- The conditions for the halo instability can be found in the region upstream to the Earth's bow shock where the ion distributions possess the core plasma of the solar wind origin and an admixture of nearly isotropic suprathermal ion population. The halo instability arises when the halo velocity by an order in value exceeds the ion thermal velocity
- The 4-point simultaneous measurement capabilities of the CLUSTER fleet possess the potential to resolve some of the problems related to the interpretation of the mirror waves in space plasmas.

NL perpendicular plasma pressure balance condition

FLR effect

$$\delta p_{\perp} + \frac{B_0 \delta B_z}{\mu_0} - \frac{3}{2} \rho_i^2 \nabla_{\perp}^2 \frac{B_0 \delta B_z}{\mu_0} + \frac{\delta B_z^2}{2\mu_0}$$

$$= -\left(1 + \frac{\beta_{\perp} - \beta_{II}}{2}\right) \nabla_{\perp}^{-2} \frac{\partial^2}{\partial z^2} \frac{B_0 \delta B_z}{\mu_0}$$

Magnetic field line bending

Distribution function

Mirror effect

Resonant particles

Linear response

$$\delta f^{(1)} = -\mu \frac{\partial F}{\partial \mu}\frac{\delta B_z}{B_0} - \frac{\omega\mu B_0}{\omega - k_{\|} v_{\|}}\frac{\partial F}{\partial W}\frac{\delta B_z}{B_0}$$

NL response

$$\frac{\partial\, \delta f^{(2)}}{\partial z} = -\frac{\delta B_x}{B_0}\mu\frac{\partial\, \delta f^{(1)}}{\partial x} + \frac{\partial b}{\partial z}\mu\frac{\partial\, \delta f^{(1)}}{\partial \mu}$$
$$+\left(b\frac{\partial b}{\partial z} - \frac{\delta B_x}{B_0}\frac{\partial b}{\partial x}\right)\mu\frac{\partial F}{\partial \mu}$$

Particular case : $\delta f \propto f(x + \alpha z)$

$$\delta f^{(2)} = b^2\left(\mu\frac{\partial F}{\partial \mu} + \frac{1}{2}\mu^2\frac{\partial F}{\partial \mu^2}\right)$$

Plasma pressure variations

$$\delta p_\perp = \delta p_\perp^{(1)} + \delta p_\perp^{(2)}$$

where

Resonant paricles

Mirror effect

$$\delta p_\perp^{(1)} = -2Ap_\perp b - \frac{2\pi^2 \int_0^\infty W^2 \frac{\partial F_{res}}{\partial W}}{m\,|k_\parallel|}\frac{\partial b}{\partial t}$$

$$A = -\frac{\left\langle \mu^2 B_0^2 \partial F / \partial W \right\rangle}{2p_\perp} - 1; \quad \mathrm{b} = \frac{\delta \mathrm{B}_z}{B_0}$$

$$\delta p_\perp^{(2)} = A + \frac{\left\langle \mu^3 \partial^2 F / \partial \mu^2 \right\rangle}{4\langle \mu F \rangle}$$

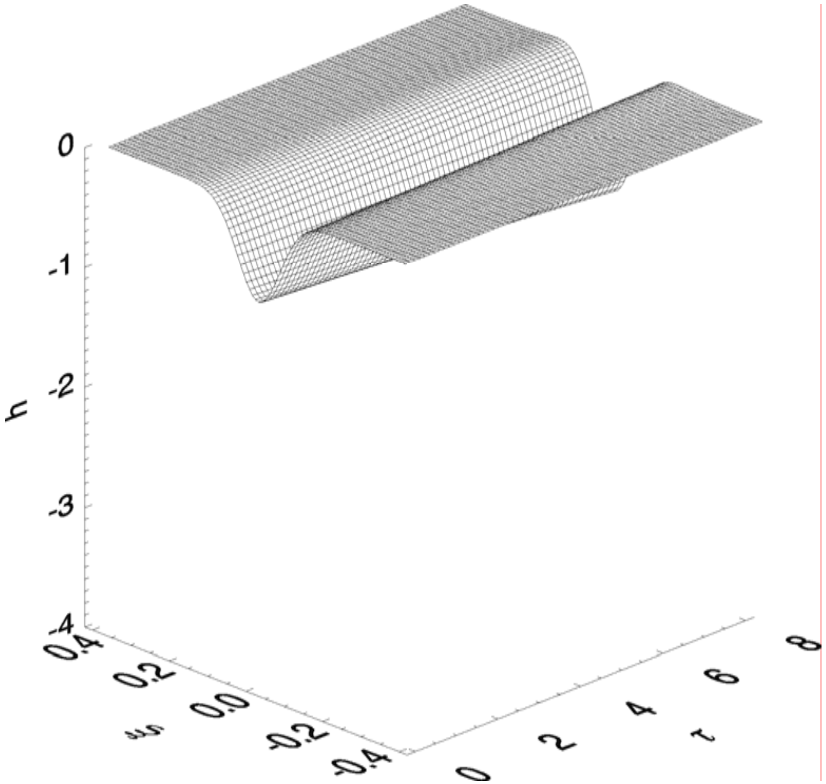
Dimensionless NL mirror equation

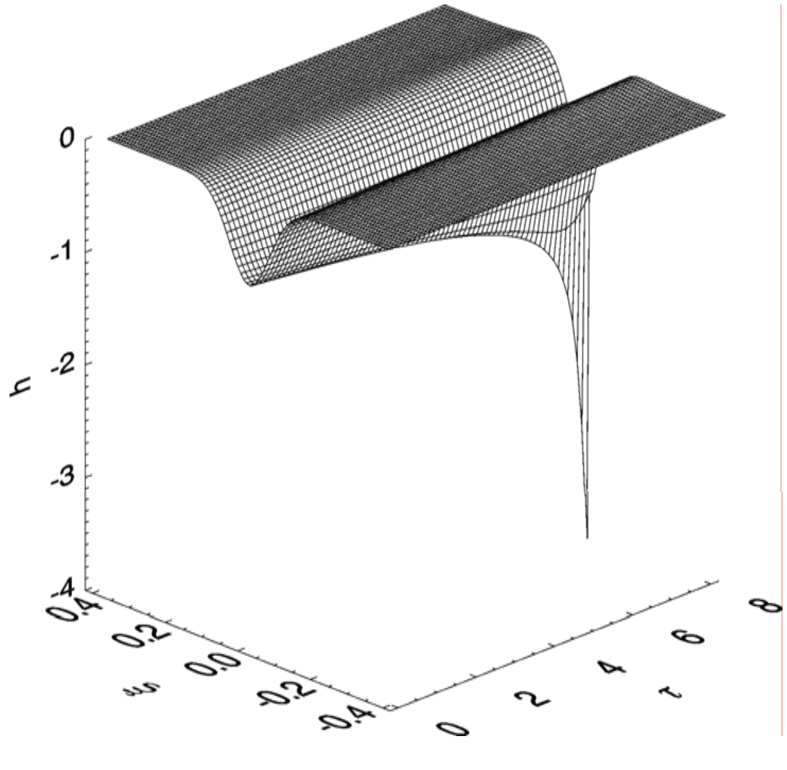
$$\frac{\partial h}{\partial \tau} = \hat{k}_{\xi}\left[\left(1+\frac{\partial^2}{\partial \xi^2}\right)h - h^2\right]$$

Here $\hat{k}_{\xi} = \hat{H}\,\partial / \partial \xi$, where $\hat{H}$ is Hilbert transform

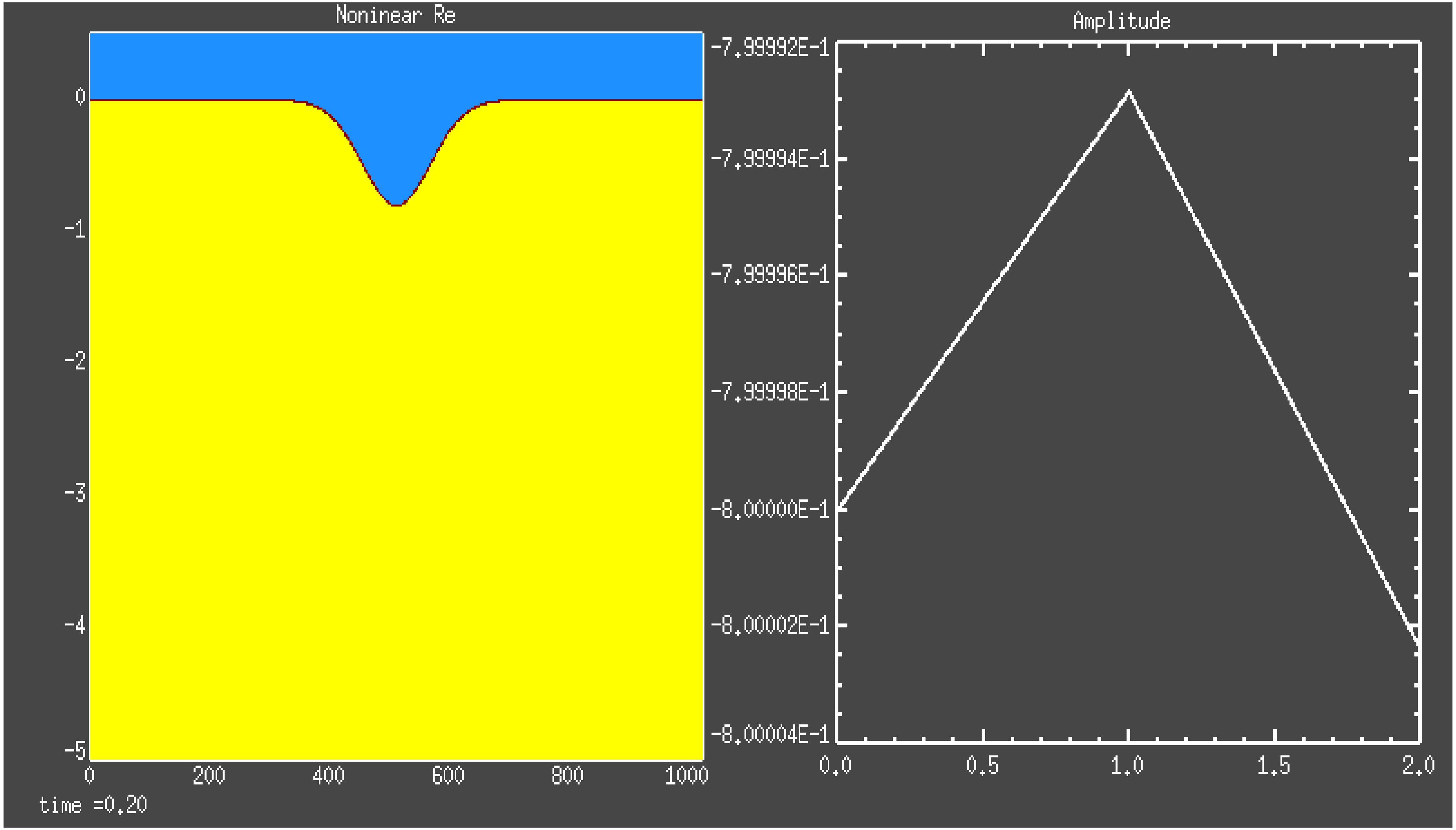
$$\hat{H}\; f(\varsigma) = \frac{1}{\pi} P \int_{-\infty}^{\infty} \frac{f(\varsigma')}{\varsigma - \varsigma'} d\varsigma'$$

3D plots





Magnetic collapse

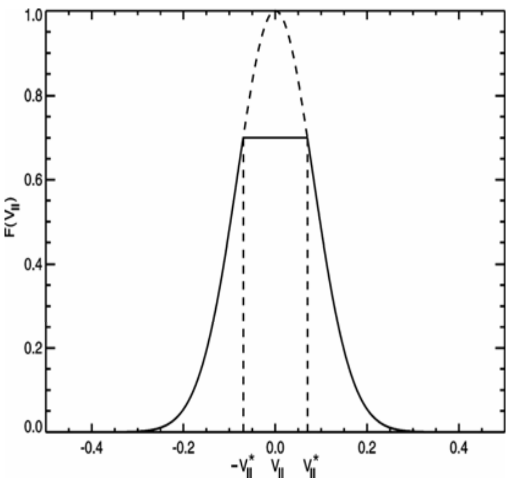


NL effects associated with trapped particles

$$\delta p_{\perp}^{res} = -\pi m \frac{\partial b}{\partial t} \int_0^{\infty} v_{\perp}^5 dv_{\perp} \lim_{\nu \to 0} \int_{v_{\parallel}^*}^{\infty} d v_{\parallel} \frac{\nu}{\nu^2 + k_{\parallel}^2 v_{\parallel}^2} \frac{\partial F}{v_{\parallel} \partial v_{\parallel}}$$

where $v_{\parallel}^* = v_{\perp} |b|^{1/2}$ and F is

$$\mathrm{F} = \frac{n}{\pi^{3/2} v_{T_{\perp}}^2 v_{T_{\parallel}}} e^{-\frac{v_{\perp}^2}{v_{T_{\perp}}^2} - \frac{v_{\parallel}^2}{v_{T_{\perp}}^2}}$$



When $v_{\parallel}^* << v_{T_{\parallel}}$, i.e. $|b| << 1$

$$\delta p_{\perp}^{res} = \frac{2nm}{\pi^{1/2} |k_{\parallel}| v_{T_{\perp}}^2 v_{T_{\parallel}}^3} \frac{\partial b}{\partial t} \int_0^{\infty} v_{\perp}^5 e^{-\frac{v_{\perp}^2}{v_{T_{\parallel}}^2}} dv_{\perp} \lim_{\nu \to 0} \left(\frac{\pi}{2} - \arctan \frac{v_{\perp} |k_{\parallel}| |b|^{1/2}}{\nu} \right)$$

when amplitude is small one has the standard linear (Landau) response

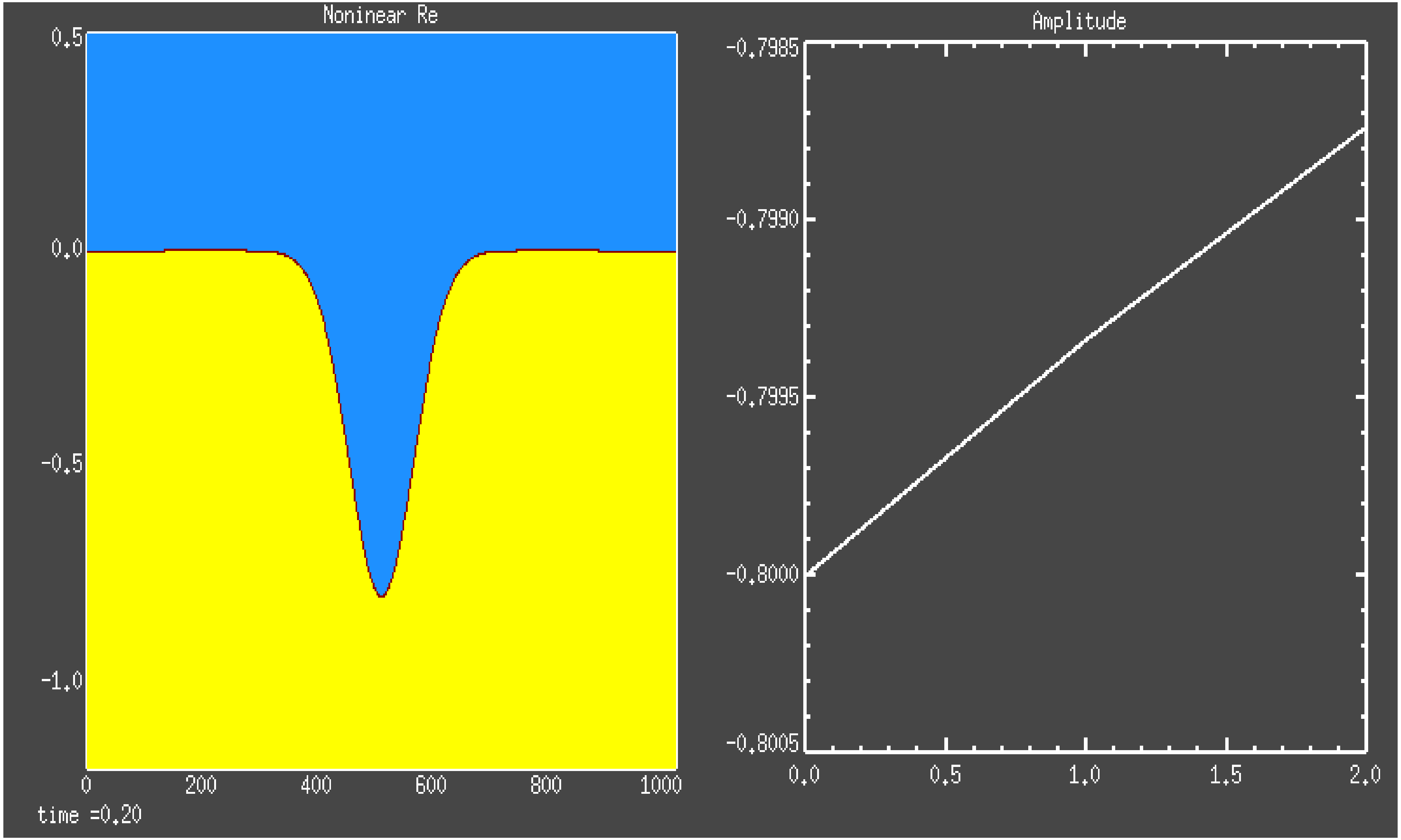
$$\delta p_{\perp}^{res} = p_{\perp} \frac{2\pi^{1/2}}{|k_{\parallel}| v_{T_{\parallel}}} \frac{T_{\perp}}{T_{\parallel}} \frac{\partial b}{\partial t}$$

when $|k_{\parallel}| v_{\parallel}^* >> \nu$ $\quad \delta p_{\perp}^{res} \to 0$

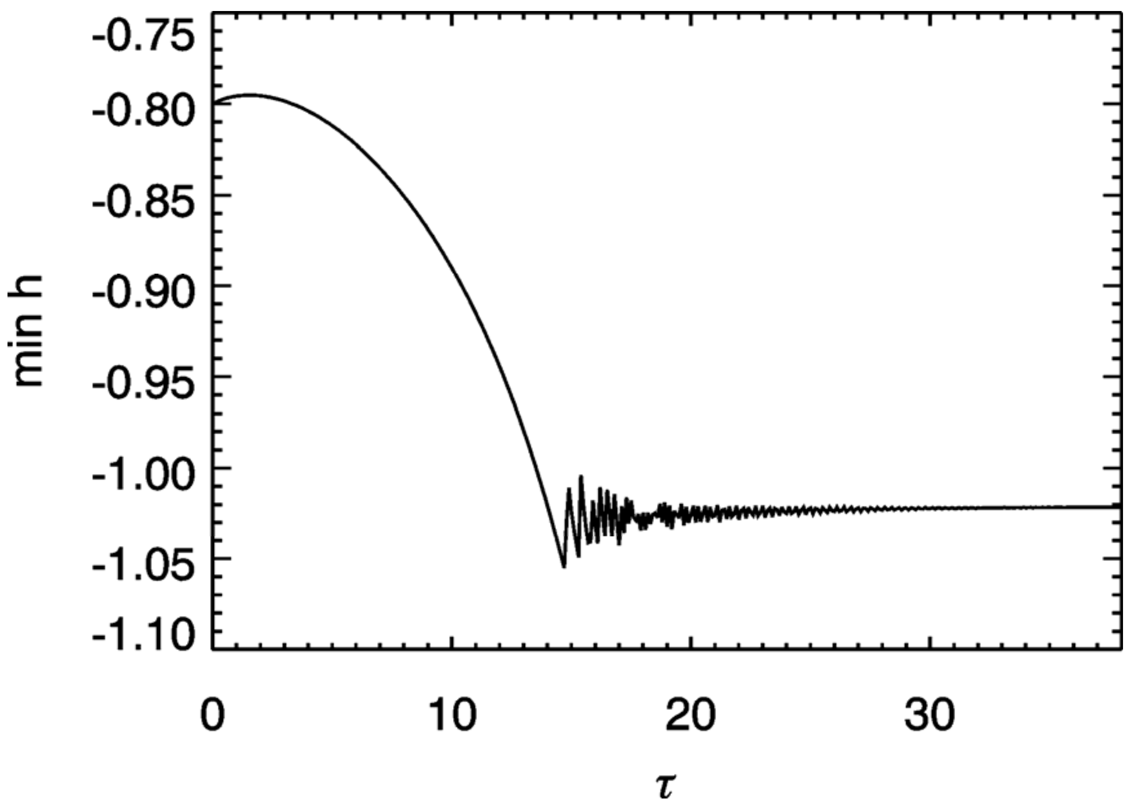
Model equation

$$\left(1 - \frac{2}{\pi} \arctan \frac{|h|^{1/2}}{\gamma}\right) \frac{\partial h}{\partial \tau} = \hat{k}_{\xi} \left[\left(1 + \frac{\partial^2}{\partial \xi^2}\right) h - h^2 \right]$$

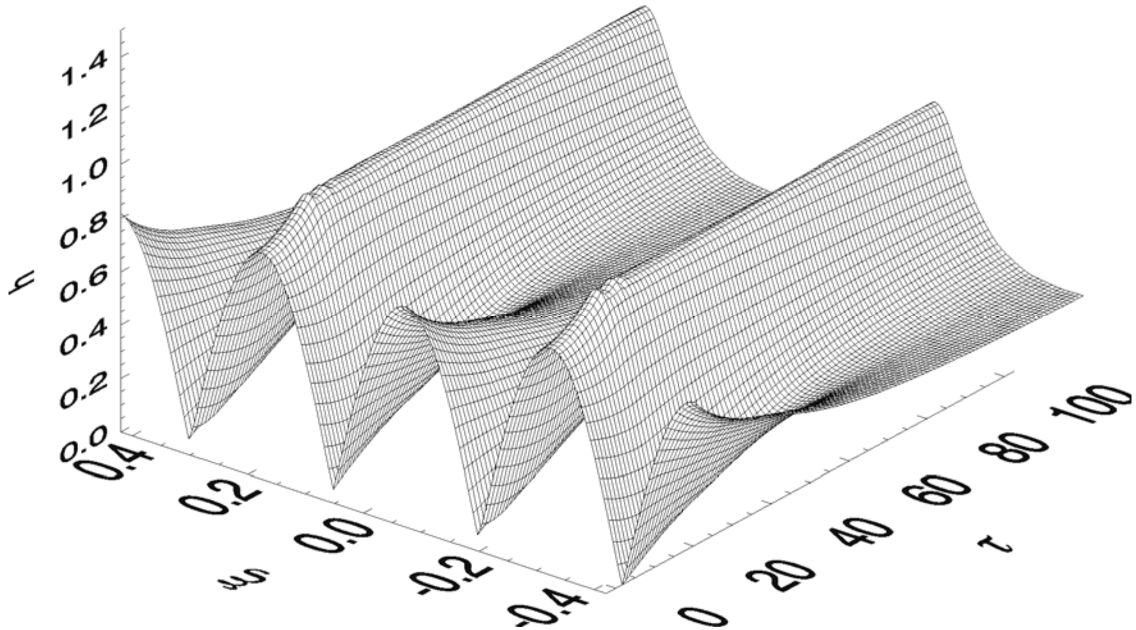
Collapse break-up



2D plot



Single-mode regime



Short history: first round

- Originally MS solitons in high beta plasmas have been discussed by Berezin [AMTF, 1965]
- MS solitons in association with collisionless shocks in high beta plasmas have been discussed by Kennel & Sagdeev [JGR, 1967]
- Basic conclusion:The MS waves have positive dispersion and thus the MS solitons represent the dark solitary structures
- Macmahon [JGR, 1968]: Kennel-Sagdeev calculations are incorrect: the MS waves possess negative dispersion
- The response: Kennel [JGR, 1968], Fredericks & Kennel [JGR, 1968] - OK
- This discussions remained unnoticed in later studies: Zhdanov and Trubnikov [JETP, 1982], Manin & Petviashvili, 1983], Berezin [1982]
- Discussion of MS solitons has been presented by Mikhailovskii and Smolyakov [1985]
- Basic conclusion: For correct description of MS solitons MHD equations should be supplemented by collisionless magnetic viscosity

Dispersion relation for MS waves

$$N_\perp^2 = \varepsilon_{22} + \frac{(\varepsilon_{12})^2}{\varepsilon_{11}}$$

where

$$\varepsilon_{22} = \frac{c^2}{v_A^2}\left(1 + \frac{\omega^2}{\omega_{ci}^2} - \frac{11}{4} k_\perp^2 \rho_i^2\right) - \frac{k_\perp^2 c^2}{\omega^2}\left(\beta_\perp - \frac{3}{2}\lambda\beta_{\perp i} k_\perp^2 \rho_i^2\right)$$

$$\varepsilon_{12} = -\varepsilon_{21} = i\frac{c^2}{v_A^2}\frac{\omega}{\omega_{ci}}\left(1 - \frac{3}{2}\frac{\omega_{ci}^2}{\omega^2} k_\perp^2 \rho_i^2\right)$$

$$\varepsilon_{11} = \frac{c^2}{v_A^2}$$

Here

$$\lambda = \frac{n\left\langle W^2 F_i \right\rangle}{2(p_{\perp i})^2}$$

Maxwellian and bi-Maxwellian Plasmas

$\lambda = 1$ and dispersion is negative

Ring type ion velocity distribution

$F \propto \delta(v_\perp - v_{\perp 0})$ and $\lambda = 1/2$. The dispersion can be positive

Generalized loss-cone DGH distribution

$$\lambda = \frac{(1 + l/2)}{(1 + l)}\frac{(\kappa - 1 - 3/2)}{(\kappa - 1 - 5/2)}$$

Partially filled loss-cone distribution of Kennel-Ashour-Abdalla

$$\lambda = \frac{\eta + (1 - \eta)(1 + \zeta + \zeta^2)}{[\eta + (1 - \eta)(1 + \zeta)]^2}$$

$$(\omega >> k_\| v_{T_\|})$$

$$\omega^2 = k_\perp^2 V_A^2 (1 - k_\perp^2 d^2)$$

where

$$d^2 = \frac{\rho_i^2}{4}\frac{1 + \beta_{\perp e} + 6\beta_{\perp i}(\lambda - 7/12)}{1 + \beta_{\perp e} + \beta_{\perp i}}$$

Here

$$V_A^2 = v_A^2 (1 + \beta_\perp)$$

Other ion velocity distributions

Generalized loss - cone DGH distribution

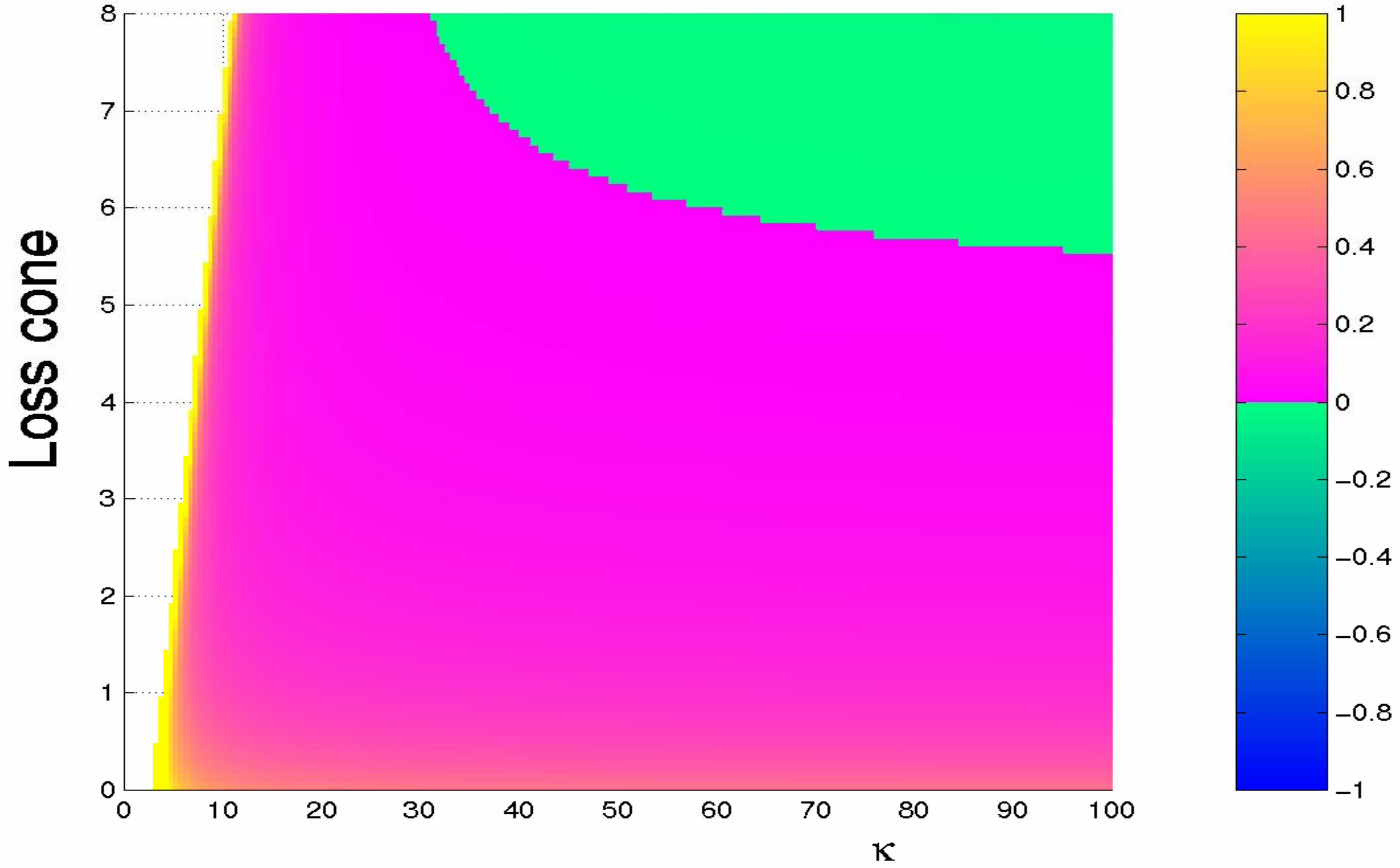
$$F_i \propto \left(\frac{v_\perp}{v_{T\perp}}\right)^{21}\left(1+\frac{v_\parallel^2}{\kappa v_{T\parallel}^2}+\frac{v_\perp^2}{\kappa v_{T\perp}^2}\right)^{-\kappa-1}$$

Partially filled loss cone distribution of Kennel - Ashour - Abdalla

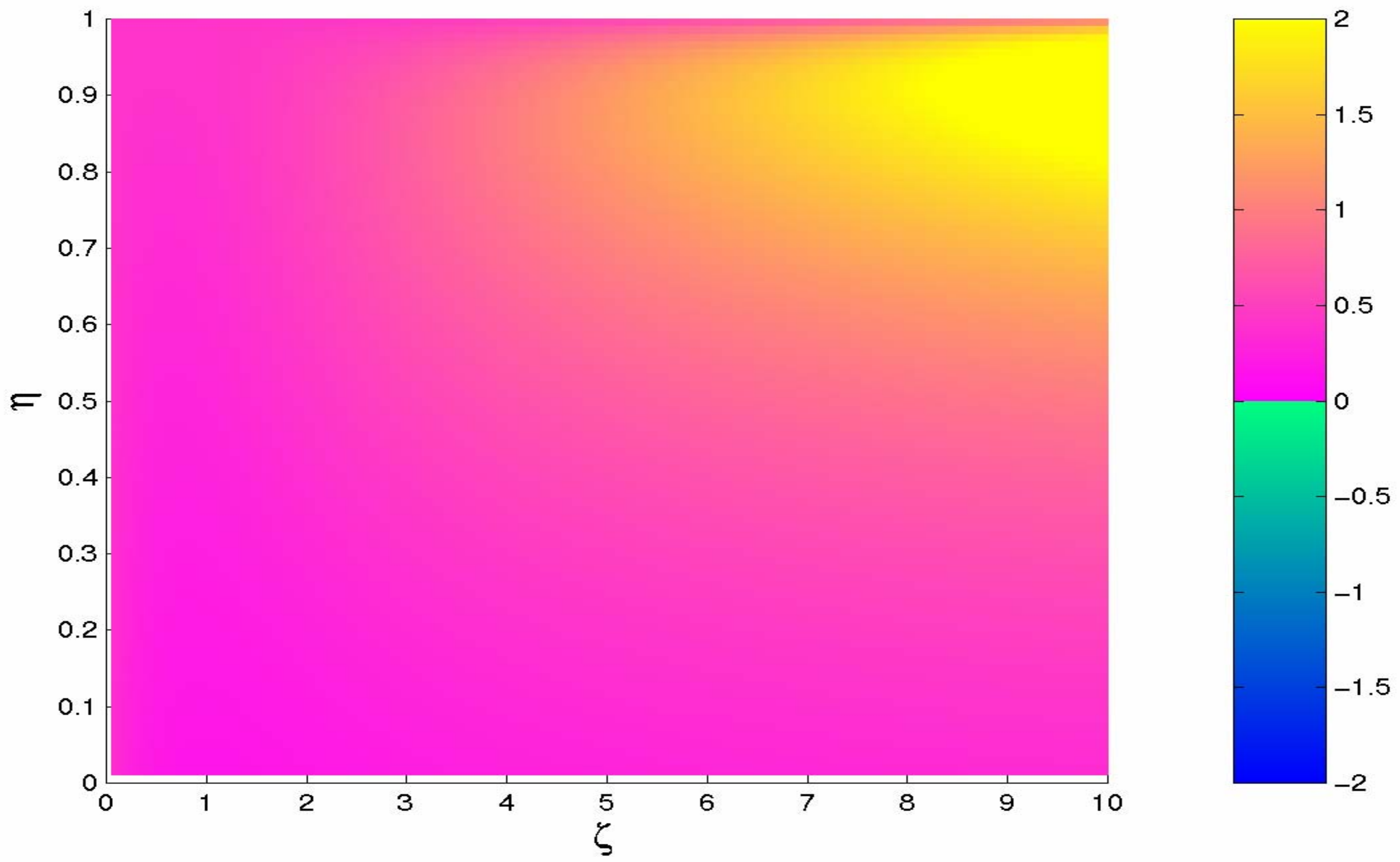
$$F_i \propto \exp\left(-v_\parallel^2/v_{T\parallel}^2\right)\left\{\eta\exp\left(-v_\perp^2/v_{T\perp}^2\right)\right.$$

$$\left.+\frac{1-\eta}{1-\zeta}\left[\exp\left(-v_\perp^2/v_{T\perp}^2\right)-\exp\left(-v_\perp^2/\zeta v_{T\perp}^2\right)\right]\right\}$$

Generalized DGH velocity distribution



Kennel-Ashour-Abdalla velocity distribution



Nonlinear effects

Ion momentum equation

$$\rho\left(\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v}\cdot\nabla)\mathbf{v}\right) = \mathbf{J}\times\mathbf{B} - \nabla\cdot\hat{\mathbf{P}}$$

Pressure tensor

$$\hat{\mathbf{P}} = p_{\perp}\hat{\mathbf{I}} + (p_{\parallel} - p_{\perp})\hat{\mathbf{b}}_i\hat{\mathbf{b}}_j$$

Ampere's law

$$V_A^2\frac{\partial b}{\partial x} + \frac{\partial v}{\partial t} + v\frac{\partial v}{\partial x} = 0$$

where $b = \delta B_z / B_0$ and $V_A = v_A(1+\beta_{\perp})^{1/2}$

For stationary solutions $(v, b) \propto f(x - ut)$

$$\frac{\partial v}{\partial t} + v\frac{\partial v}{\partial x} = -u^2\partial/\partial\xi[b - (3/2)b^2]$$

$$(1 - \frac{V_A^2}{u^2})\frac{\partial b}{\partial \xi} - 3b\frac{\partial b}{\partial \xi} = 0$$

Dispersion + Nonlinearity

$$d^2\frac{\partial^2 b}{\partial \xi^2} = (1 - \frac{V_A^2}{u^2})b - \frac{3}{2}b^2$$

$$J_y \equiv J = \frac{\rho}{B}\left(\frac{\partial v}{\partial t} + v\frac{\partial v}{\partial x}\right) + \frac{1}{B}\frac{\partial p_{\perp}}{\partial x}$$

Frozen - in condition

$$\frac{\rho}{B} = \text{const}$$

Adiabatic condition

$$\frac{p_{\perp}}{B^2} = \text{const}$$

Soliton solution

$$b = b_0/\cosh^2(\kappa\xi/2)$$

where

$$b_0 = 1 - \frac{V_A^2}{u^2} \quad \text{and} \quad \kappa = \left(\frac{1 - \frac{V_A^2}{u^2}}{d^2}\right)^{1/2}$$

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Conclusions

- The main nonlinear mechanism responsible for mirror instability saturation is associated with modification (flattening) of the shape of the background ion distribution function in the region of small parallel particle velocities.
- The nonlinear mode coupling effects are smaller and unable to take control over evolution of space profile of saturated mirror waves.
- The relevance of the theoretical results to recent satellite observations is stressed.

Conclusions

- The MS dispersion can be both negative and positive. Thus MS solitons can be with the increased or decreased magnetic field inside the structure.
- The shape of the soliton is controlled by the lambda parameter which is associated with the form of the ion velocity distribution function
- Maxwellian and bi-Maxwellian plasmas are in favour “bright” solitons. The same is true for kappa velocity distributions
- The loss cone effects can lead to the appearance of “dark” solitons

Thank you!