



**The Abdus Salam  
International Centre for Theoretical Physics**



**1856-61**

**2007 Summer College on Plasma Physics**

*30 July - 24 August, 2007*

**Parametric Interaction of Alfvén  
Waves with Convective Cells in  
the IAR**

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# **Parametric Interaction of Alfvén Waves with Convective Cells in the IAR**

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# Short history

- The idea of IAR was suggested by Polyakov [1976]
- First experimental evidence: Polyakov and Rapoport [1981], Belyaev et al. [1987]
- Recent experimental studies: Belyaev et al. [1999], Demekhov et al. [2000], Bosinger et al. [2002]
- Theoretical model of the IAR was first developed by Trakhtengertz and Feldstein [1987,1991] and Lysak [1991]
- Recent progress in the theory: Pokhotelov et al. [2000, 2001, 2003, 2004], Lysak and Song [2002, 2003], Pilipenko et al. [2002], Streltsov et al. [2002], Surkov et al. [2004]

# Ground and satellite observations

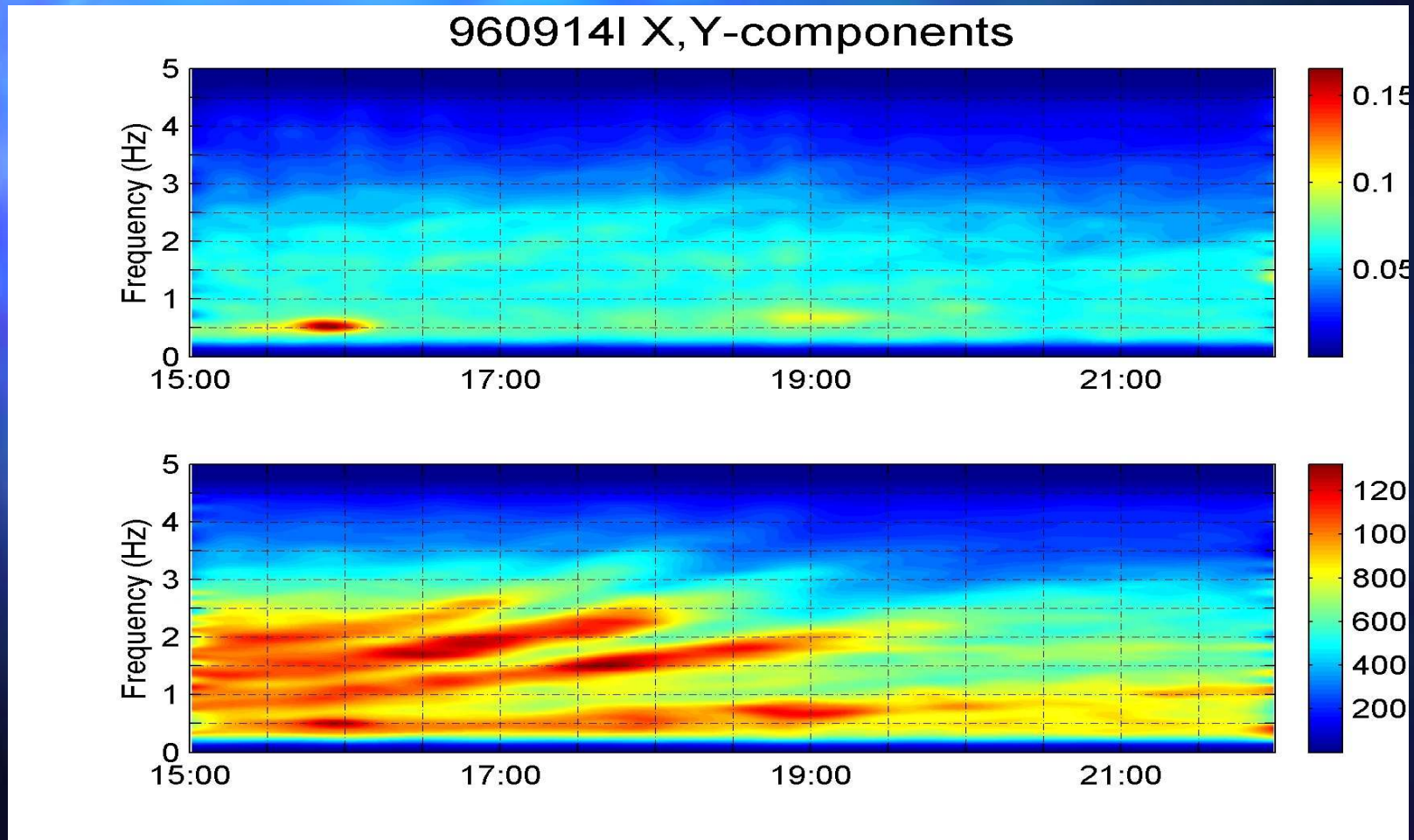
- Nizniy-Novgorod (Middle Russia)
- Borok (Middle Russia)
- Mondy (Siberia, Russia)
- Karimshino (Kamchatka, Russia)
- Sodankyla (Finland)
- Crete (Greece)
- Table Mountain obs., USA
- FREJA satellite
- FAST satellite
- CLUSTER satellites

# Sources of free energy for the IAR excitation

- High-latitudes - Magnetospheric convection (feedback instability)
- Middle-latitudes - Thunderstorm activity
- Neutral winds, Subauroral Polarization Streams (SAPS)

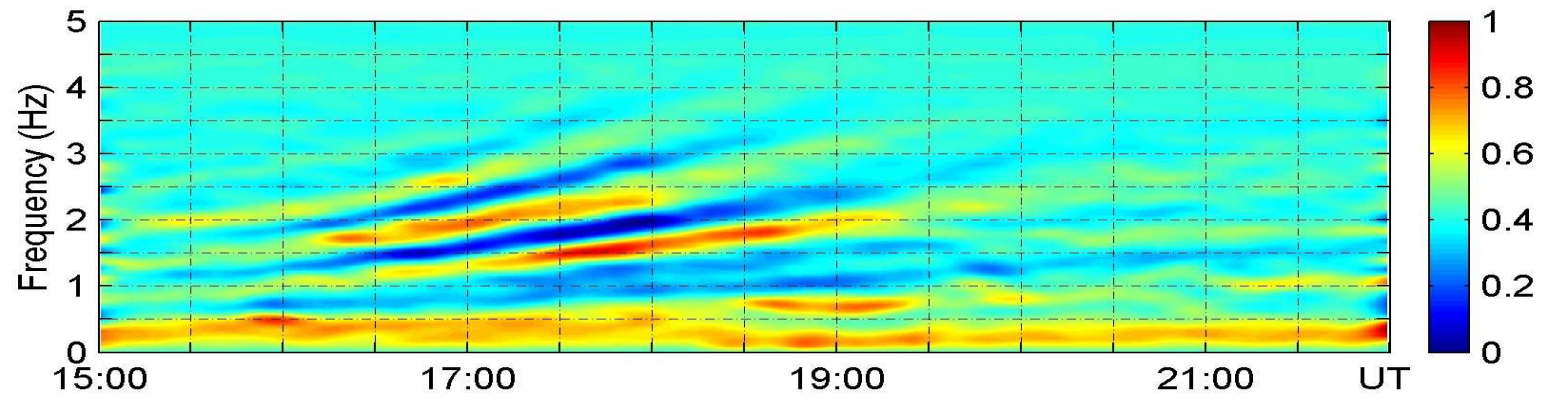
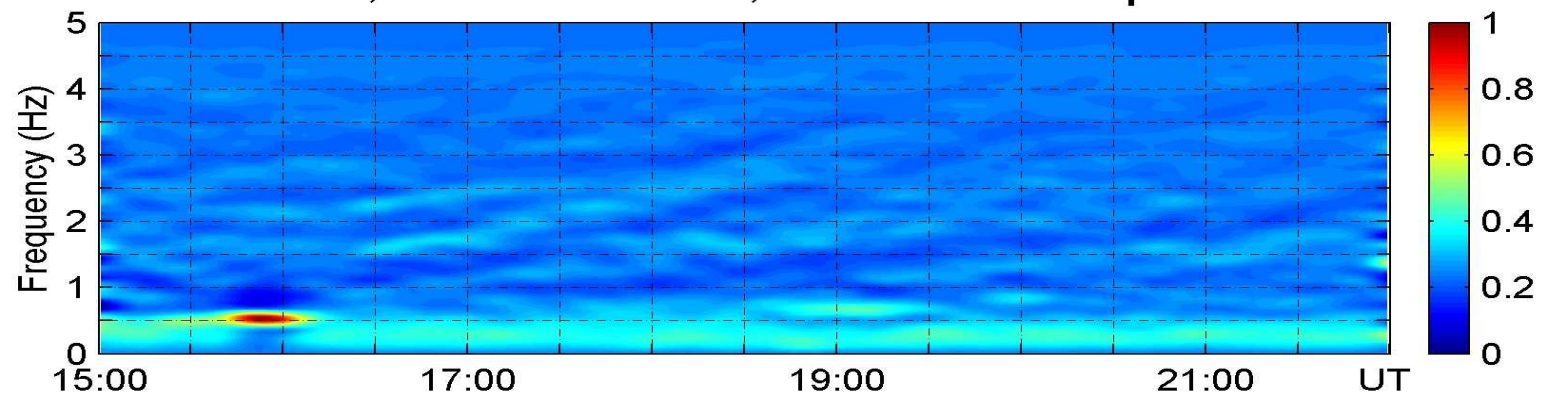


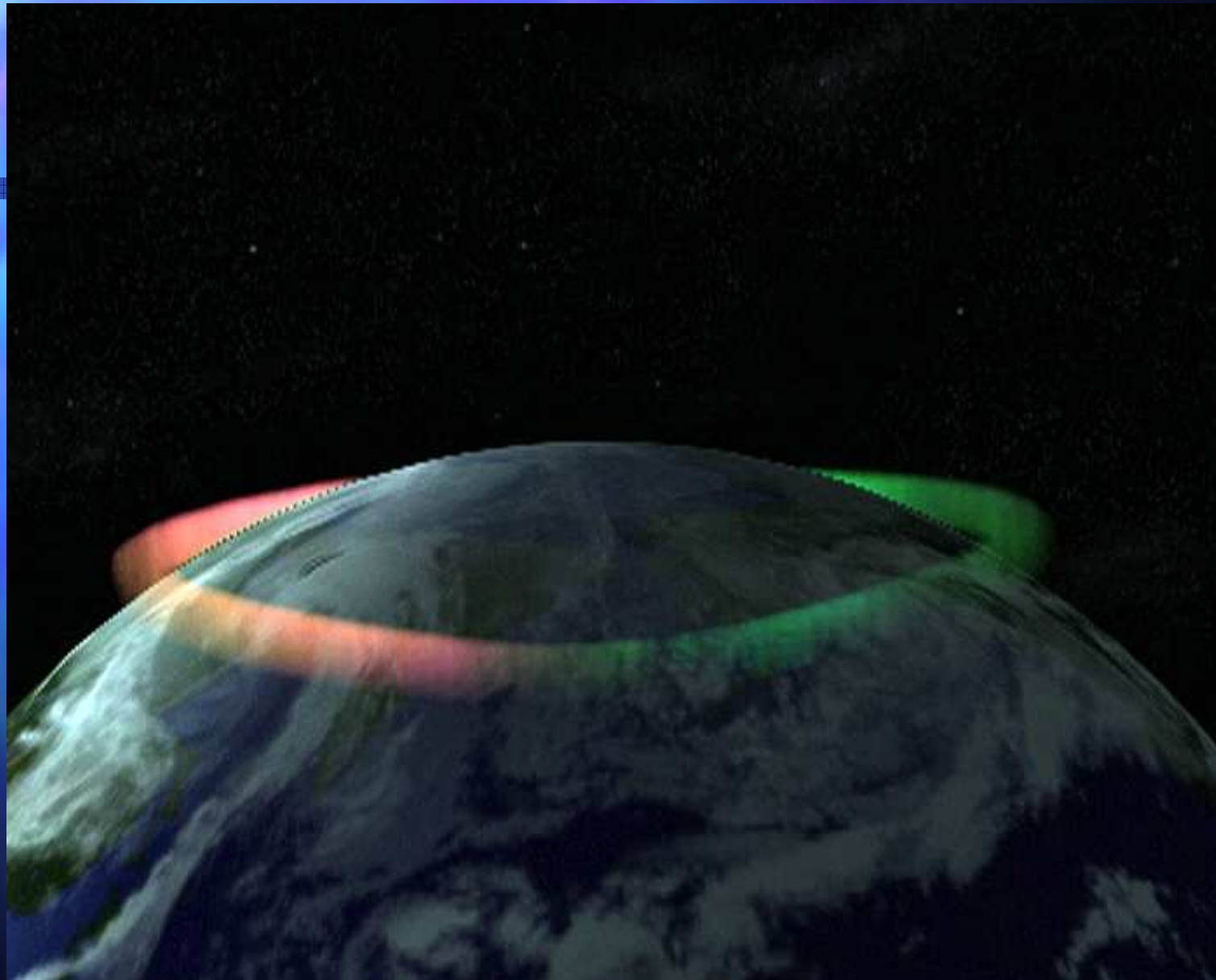
# Ground observations of SRS



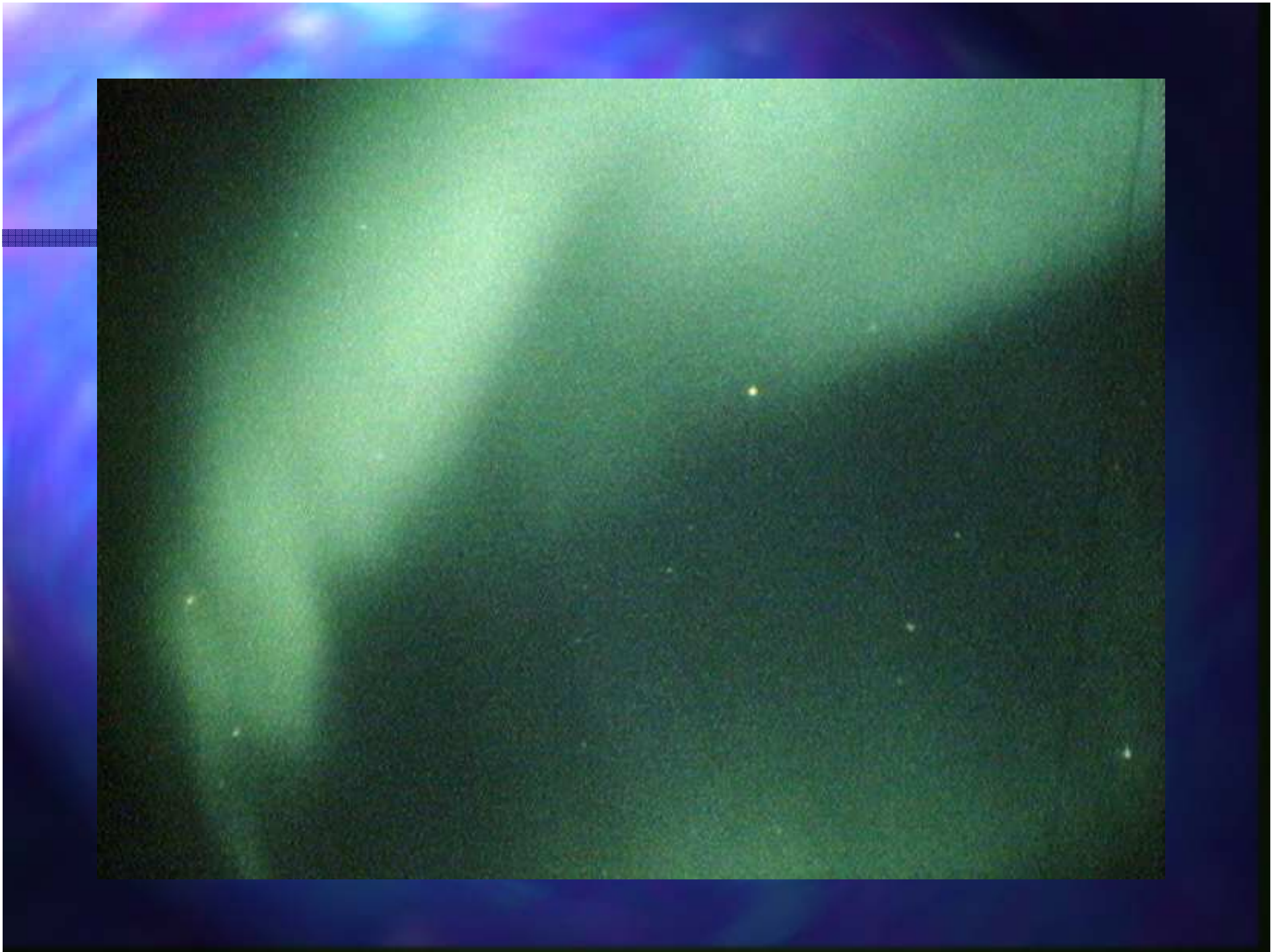
# Ground observations of SRS

BOROK, date:960914I, X and Y components



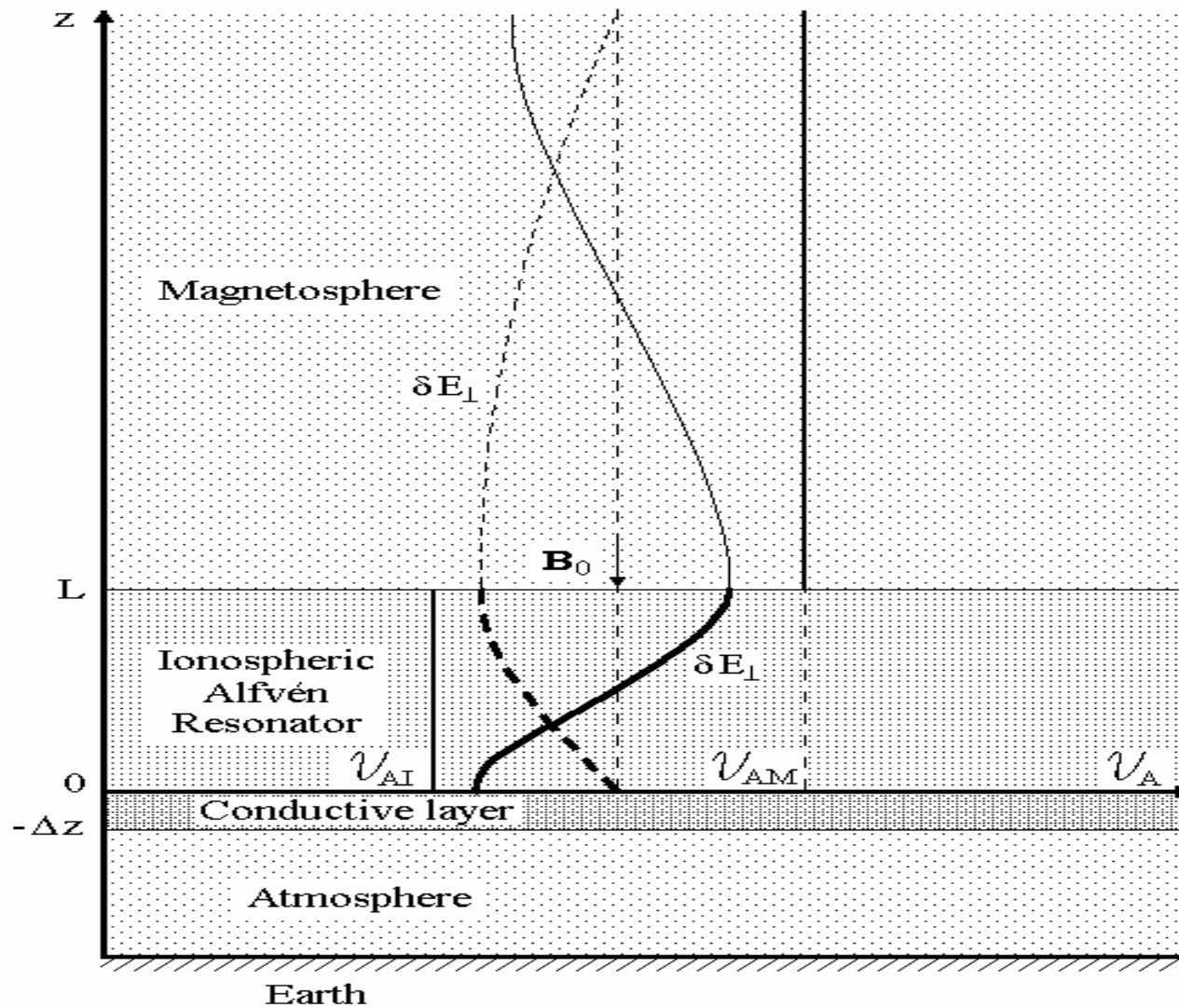






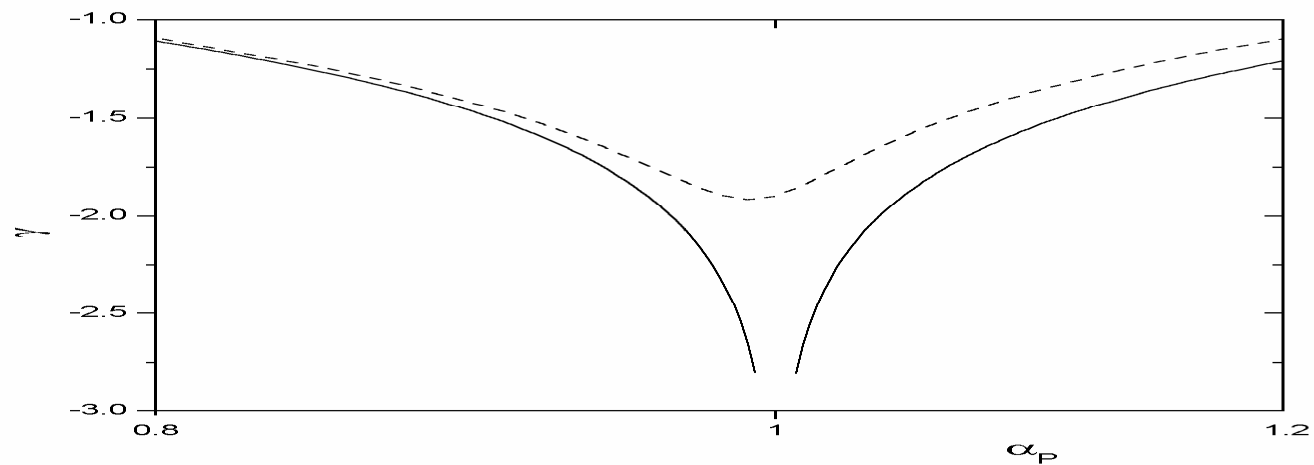
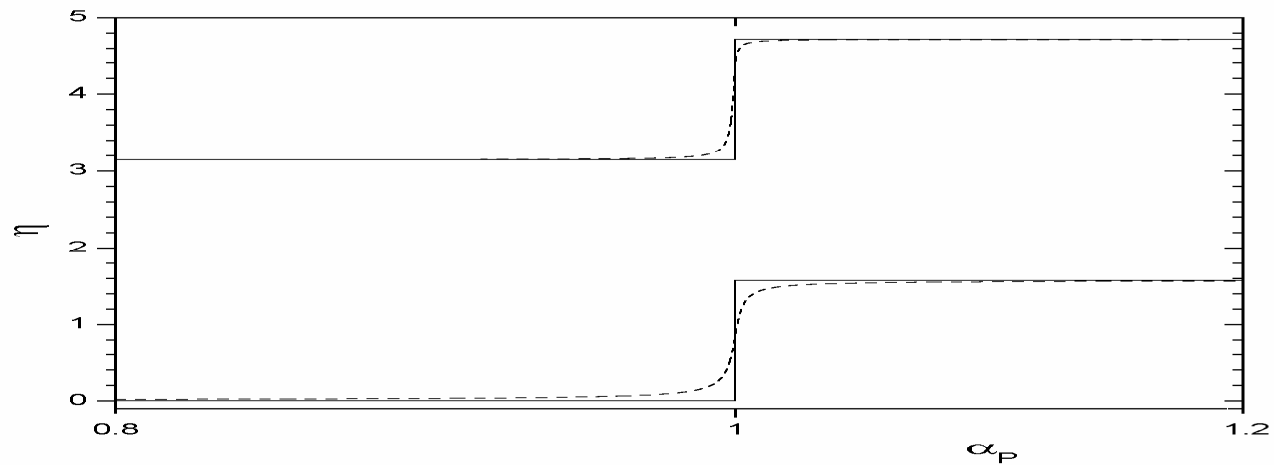


# Schematic view



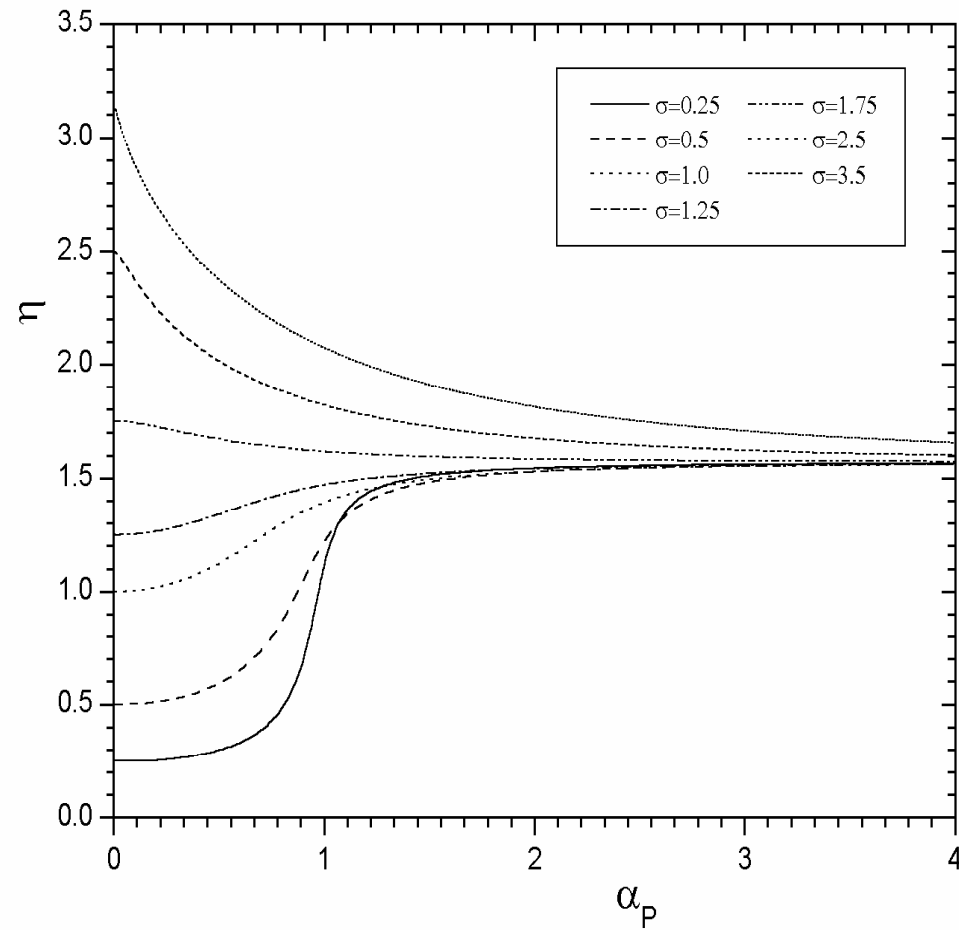


# IAR Eigenmodes and damping rates

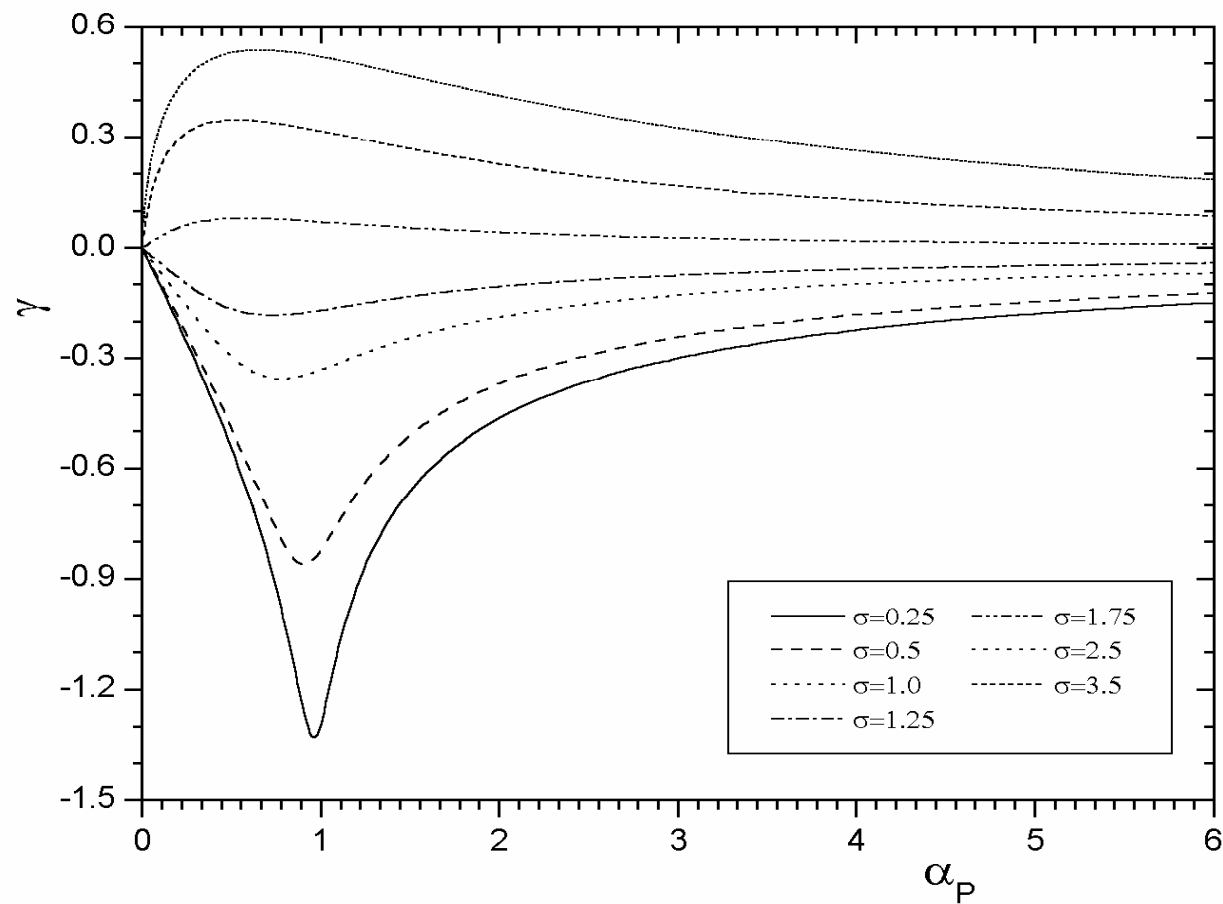




# Eigen-frequencies



# Variation of the damping/growth rate in the presence of magnetospheric convection



# More complex model

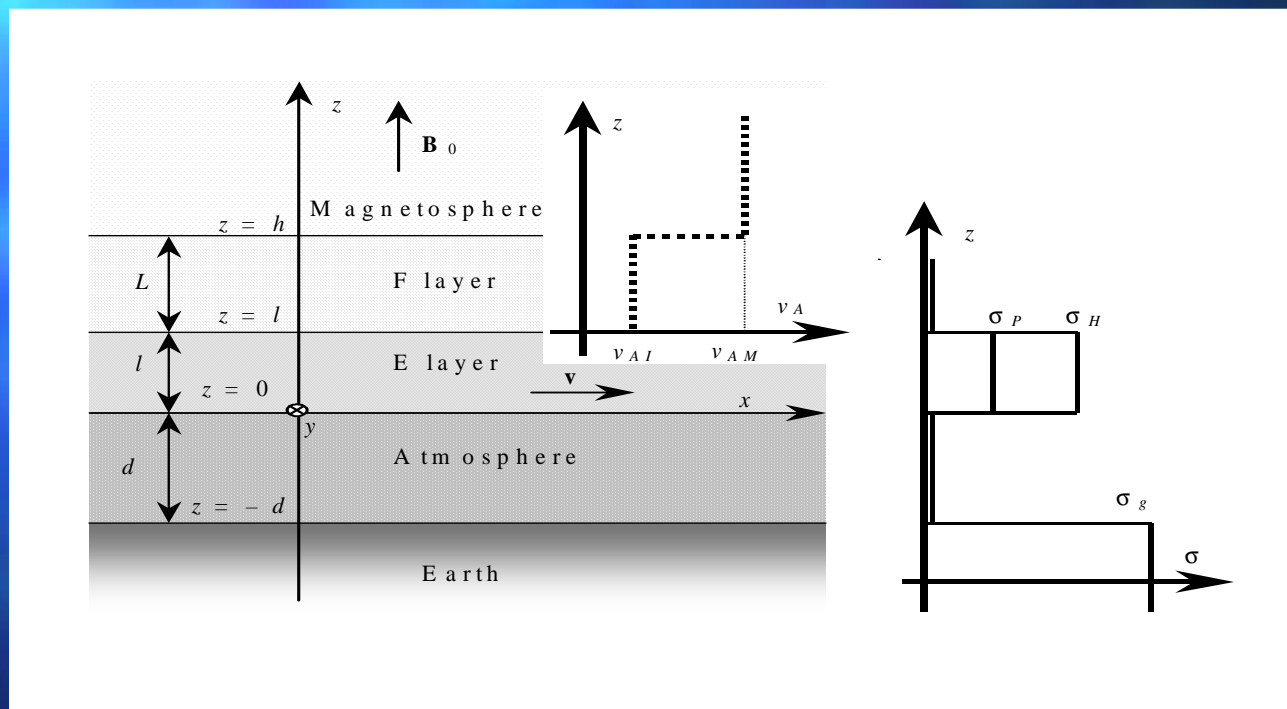


Fig. 1

# Excitation of the IAR by neutral winds.

## Potentials

$$\mathbf{E}_{\perp} = -i\omega \left( \frac{\mathbf{k}\Phi}{\omega} - i\mathbf{k} \times \hat{\mathbf{z}} \Psi \right)$$

$$\delta\mathbf{B} = \partial_z \left( \frac{\mathbf{k} \times \hat{\mathbf{z}}}{\omega} \Phi + i\mathbf{k}\Psi \right) + k^2 \Psi \hat{\mathbf{z}}$$

## Basic equations

$$\partial_z^2 \Phi + i\mu_0 \omega \sigma_P \Phi + \mu_0 \omega^2 \sigma_H \Psi$$

$$= \frac{B\mu_0 \omega}{k^2} [\sigma_P (\mathbf{k} \times \mathbf{v}) + \sigma_H (\mathbf{k} \cdot \mathbf{v})]$$

$$\partial_z^2 \Psi - (k^2 - i\mu_0 \omega \sigma_P) \Psi + \mu_0 \sigma_H \Phi$$

$$= -i \frac{B\mu_0}{k^2} [\sigma_H (\mathbf{k} \times \mathbf{v}) - \sigma_P (\mathbf{k} \cdot \mathbf{v})]$$

Here  $\sigma_H$  and  $\sigma_P$  are the Hall and Pedersen ionospheric conductivities



# Magnetic field perturbation on the ground

$$\delta \mathbf{B} = \left( i \mathbf{k} \kappa + k^2 \hat{\mathbf{z}} \right) \Psi(-d)$$

where  $\kappa^2 = k^2 - i \mu_0 \omega \sigma_g$

**Driving force**

$$\delta \mathbf{B} / B = \frac{Q \left( \hat{\mathbf{z}} + i \mathbf{k} \kappa / k^2 \right)}{\beta_3 [(is + x_0 \alpha_P)(\beta_1 + \alpha_P) + x_0 \alpha_H^2]}$$

**FM**

**AM**

**Mode Coupling**

where

$$Q = Lv_{AI}^{-1} [(\mathbf{k} \cdot \mathbf{v})(\alpha_H^2 + \alpha_P^2 + \beta_1 \alpha_P) - (\mathbf{k} \times \mathbf{v})_z \beta_1 \alpha_P]$$

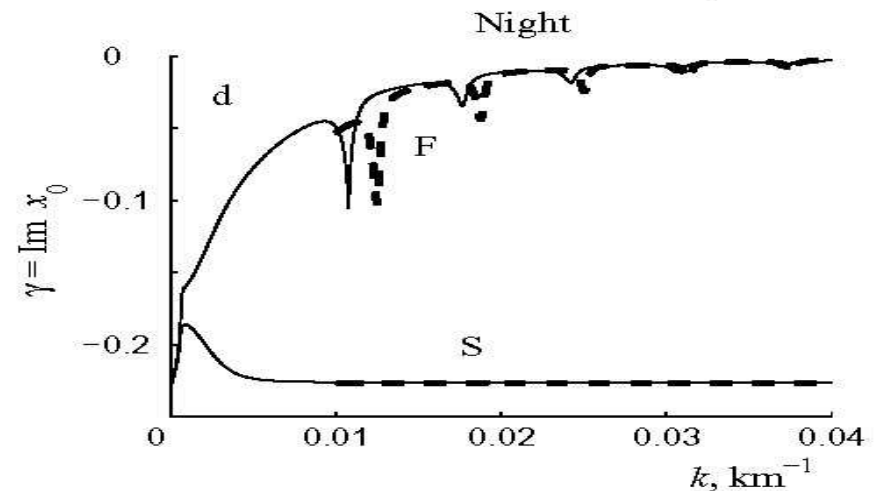
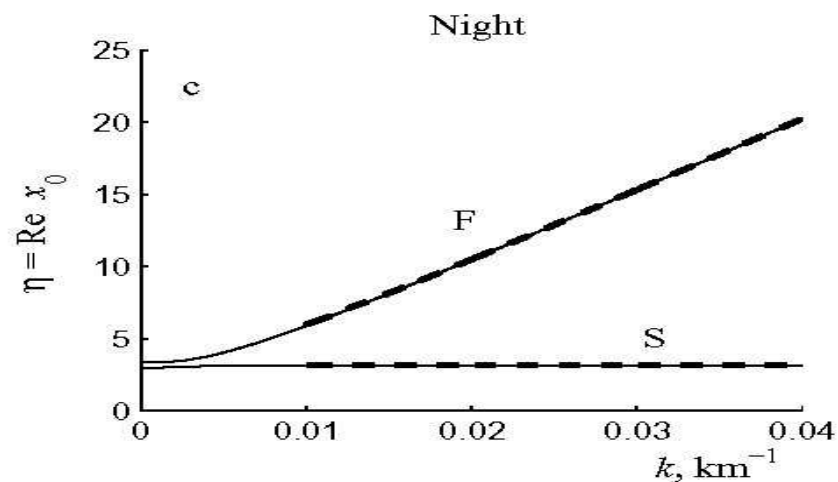
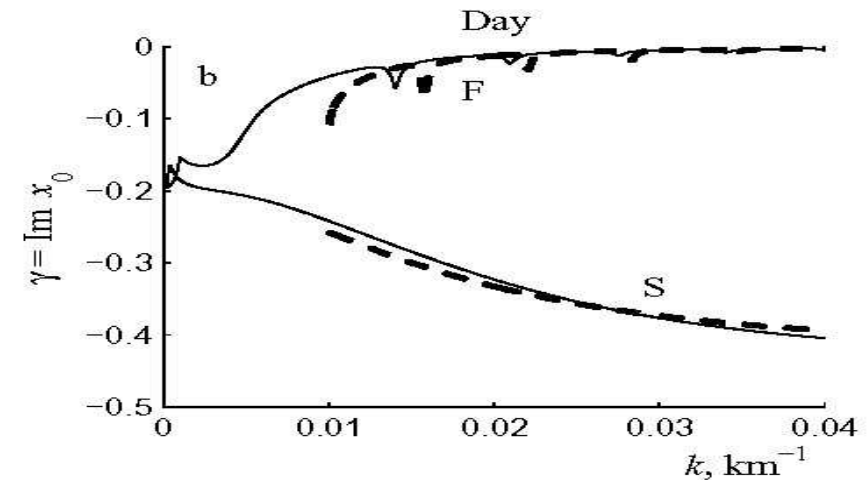
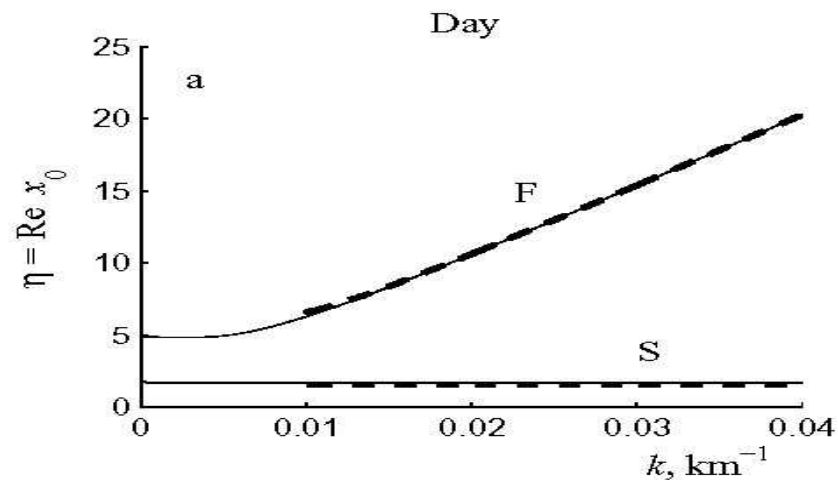
$$s = kL \frac{\kappa + k \tanh(kd)}{k + \kappa \tanh(kd)} - \lambda_I \beta_2, \quad \beta_3 = \cosh(kd) + i \frac{\kappa}{k} \sinh(kd)$$

$$\beta_1 = \frac{1 + \varepsilon - (1 - \varepsilon) \exp(2ix_0)}{1 + \varepsilon + (1 - \varepsilon) \exp(2ix_0)}, \quad \beta_2 = \frac{\lambda_I + \lambda_M - (\lambda_I - \lambda_M) \exp(2\lambda_I)}{\lambda_I + \lambda_M + (\lambda_I - \lambda_M) \exp(2\lambda_I)}$$

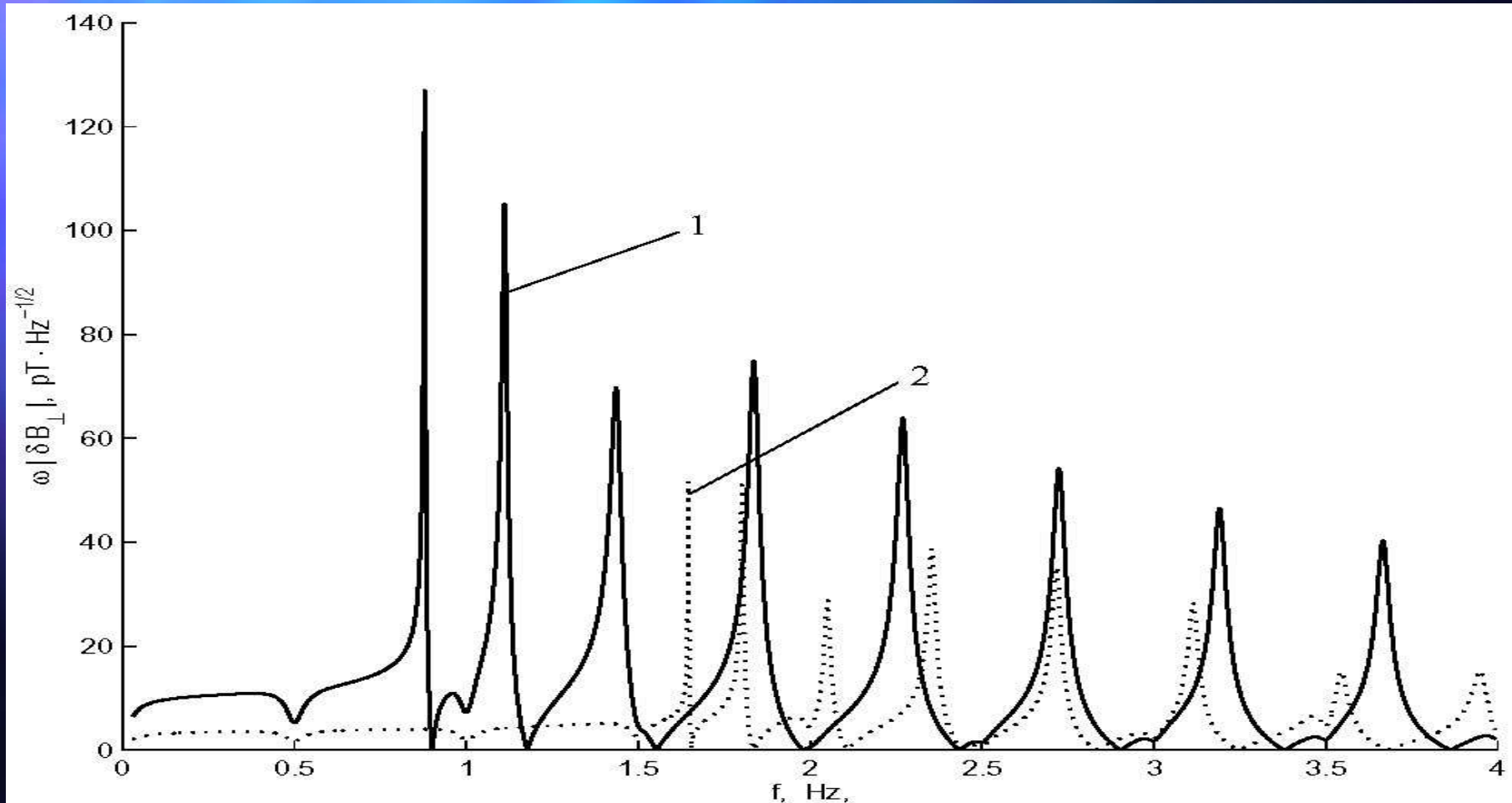
and

$$x_0 = \omega L / v_{AI}, \quad \varepsilon = v_{AI} / v_{AM}, \quad \lambda_I = (k^2 L^2 - x_0^2)^{1/2}, \quad \lambda_M = (k^2 L^2 - x_0^2 \varepsilon^2)^{1/2}$$

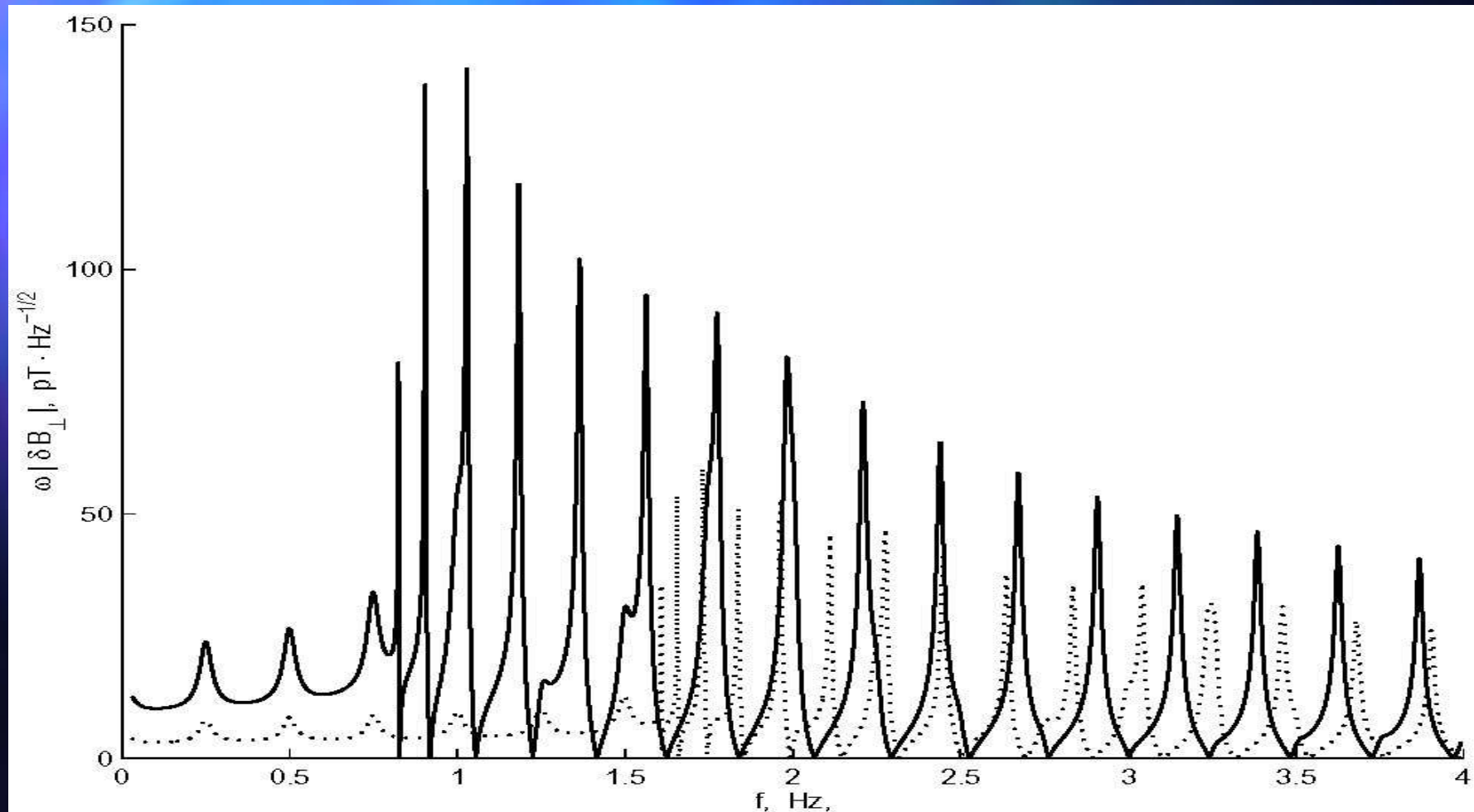
# Eigenfrequencies (a,c) and damping rates (b,d) for fundamental mode



# A daytime power spectrum

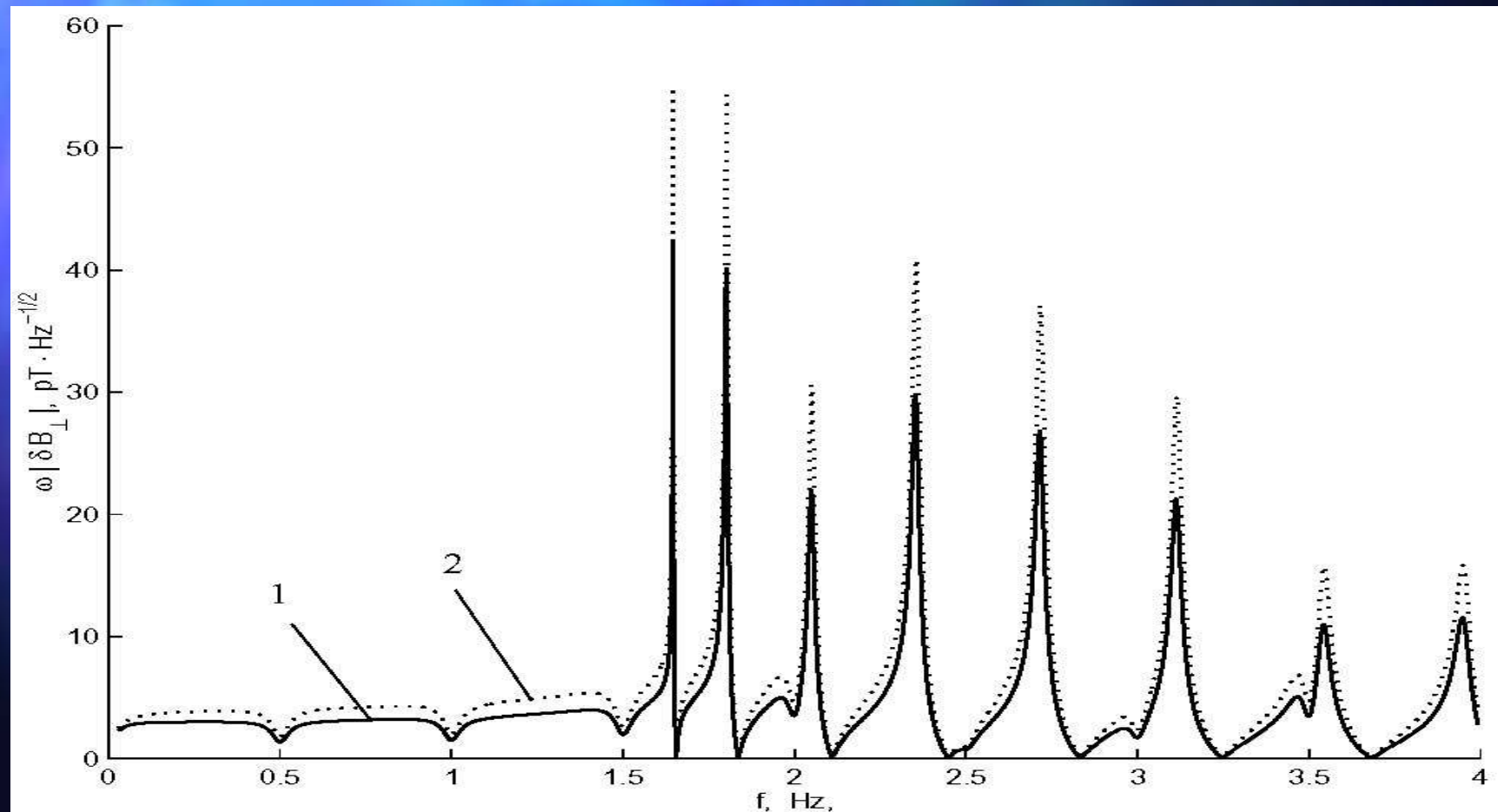


The nighttime spectrum. The low frequency spikes at 0.25, 0.5 and 0.75 Hz are due the shear Alfvén mode





# The dependence of the spectrum on the ground conductivity



## Recent observations

- Observations of the IAWs provided by LAPD experiments (UCLA). A special emphasis was given to structures of the order of the electron skin depth. It was demonstrated that the phenomena observed in laboratory experiments show striking similarities to those detected in space plasmas
- FAST and Freja observations show that the energies of the electrons which are accelerated by IAWs are higher than those found in numerical simulations including the linear inertial effects

# Basic NL equations

## Potentials

$$\mathbf{E}_z = \mathbf{E} \cdot \hat{\mathbf{z}} = -\partial_z \varphi - \partial_t A$$

$$\mathbf{E}_\perp = -\nabla_\perp \varphi,$$

$$\mathbf{B}_\perp = \nabla A \times \hat{\mathbf{z}}$$

$$E_\parallel = E_z + \frac{\mathbf{B}_\perp \cdot \mathbf{E}_\perp}{B_0} = D_t A - \partial_t \varphi$$

$$D_t = \partial_t + B_0^{-1} \{ \varphi, \dots \}$$

$$\{A, B\} = (\partial_x A) \partial_y B - (\partial_y A) \partial_x B$$

## Nonlinear equations

$$D_t \nabla_\perp^2 \varphi + v_A^2 D_z \nabla_\perp^2 A = 0,$$

$$D_z \equiv \partial_z + B_0^{-1} \mathbf{B}_\perp \cdot \nabla$$

$$D_t (1 - \lambda_e^2 \nabla_\perp^2) A + \partial_z \varphi = 0.$$

# Multiscale expansion

$$\varphi = \varphi + \psi$$

$$A = a + A_h$$

HF

LF

Reynolds Stresses

$$\partial_t \nabla_{\perp}^2 \varphi = -B_0^{-1} (\langle \{\psi, \nabla_{\perp}^2 \psi\} \rangle - v_A^2 \langle \{A, \nabla_{\perp}^2 A\} \rangle)$$

$$\partial_t (1 - \lambda_e^2 \nabla_{\perp}^2) a = -B_0^{-1} \langle \{\psi, (1 - \lambda_e^2 \nabla_{\perp}^2) A\} \rangle$$



# Quasi-monochromatic approach

$$(\phi, a) = (\phi_{\mathbf{q}}, A_{\mathbf{q}}) \exp[i(\mathbf{q} \cdot \mathbf{r} - \Omega t)] + c.c.$$

$$\psi = \psi_0 + \psi_+ + \psi_-$$

$$A = A_0 + A_+ + A_-$$

$$(\psi_0, A_0) = (\psi_{\mathbf{k}}, A_{\mathbf{k}}) \exp[i(\mathbf{k} \cdot \mathbf{r} - \omega_{\mathbf{k}} t)] + c.c.$$

$$(\psi_{\pm}, A_{\pm}) = (\psi_{\mathbf{k}_{\pm}}, A_{\mathbf{k}_{\pm}}) \exp[i(\mathbf{k}_{\pm} \cdot \mathbf{r} - \omega_{\mathbf{k}_{\pm}} t)] + c.c.$$

$$\omega_{\mathbf{k}_{\pm}} = \omega_{\mathbf{k}} \pm \Omega$$

$$\mathbf{k}_{\pm} = \mathbf{k} \pm \mathbf{q}$$

# Convective cells generation

Dispersion relation

$$\Omega_{\pm} = q \cdot v_g \pm i [b(q \times k)_z^2 |\psi_0|^2 B_0^{-2} - \delta\omega^2]^{1/2}$$

$$b = k^2 \lambda_e^2 (1 + k^2 \lambda_e^2)^{-1}$$

$$\delta\omega \approx b \omega_k q^2 / 2k^2$$

Optimal dimensions

$$\left(\frac{q}{k}\right)_{\max}^2 = \frac{2(1 + \lambda_e^2)^3}{k_z^2 \lambda_e^2} \frac{|B_k|^2}{B_0^2}$$

Maximum growth rate

$$\frac{\gamma_{\max}}{\omega_k} = (1 + \lambda_e^2)^3 \frac{k^2}{k_z^2} \frac{|B_k|^2}{B_0^2}$$

# Competing mechanisms

$$\text{IAW} = \text{IAW} + \text{IAW}$$

$$\gamma_{AA} = 0.15 v_A \lambda_e k_{\perp}^2 \left| \frac{\mathbf{B}_k}{B_0} \right|^{-1}$$

$$\frac{\gamma_{AA}}{\gamma_{cell}} = 0.15 \frac{k_z \lambda_e}{(1 + k_{\perp}^2 \lambda_e^2)^{3/2}} \left| \frac{\mathbf{B}_k}{B_0} \right|^{-1}$$

$$\frac{k_z}{k(1 + k^2 \lambda_e^2)} > \left| \frac{\mathbf{B}_k}{B_0} \right| > 0.15 \frac{k_z \lambda_e}{(1 + k^2 \lambda_e^2)^{3/2}}$$

# Broad-band approximation

Kinetic equation for the IAW packets

$$\frac{\partial \delta N_k}{\partial t} + \frac{\partial \omega}{\partial k} \frac{\partial \delta N_k}{\partial r} - \frac{\partial \omega}{\partial r} \frac{\partial N_k}{\partial k} = S$$

$$(\delta N_k, \phi) \propto \exp[i(qx - \Omega t)]$$

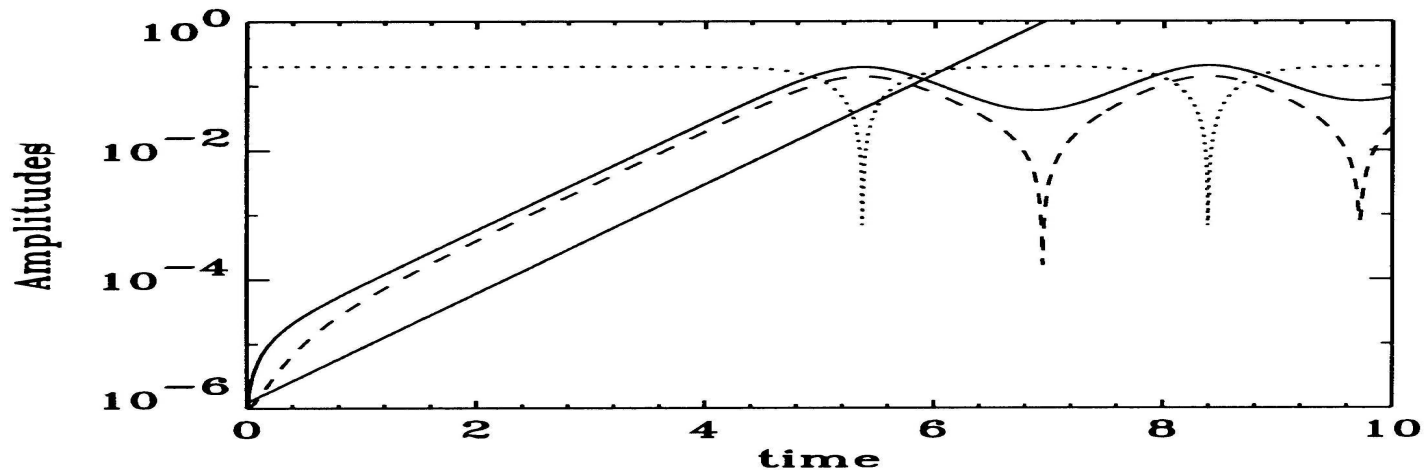
$$\delta N_k = -i \frac{c}{B_0} \frac{q^2 k_y}{\Omega - qk_y} \frac{\partial N_k}{\partial k_x} \phi$$

$$N_k \propto k^2 |\phi_k|^2 \omega_k^{-1}$$

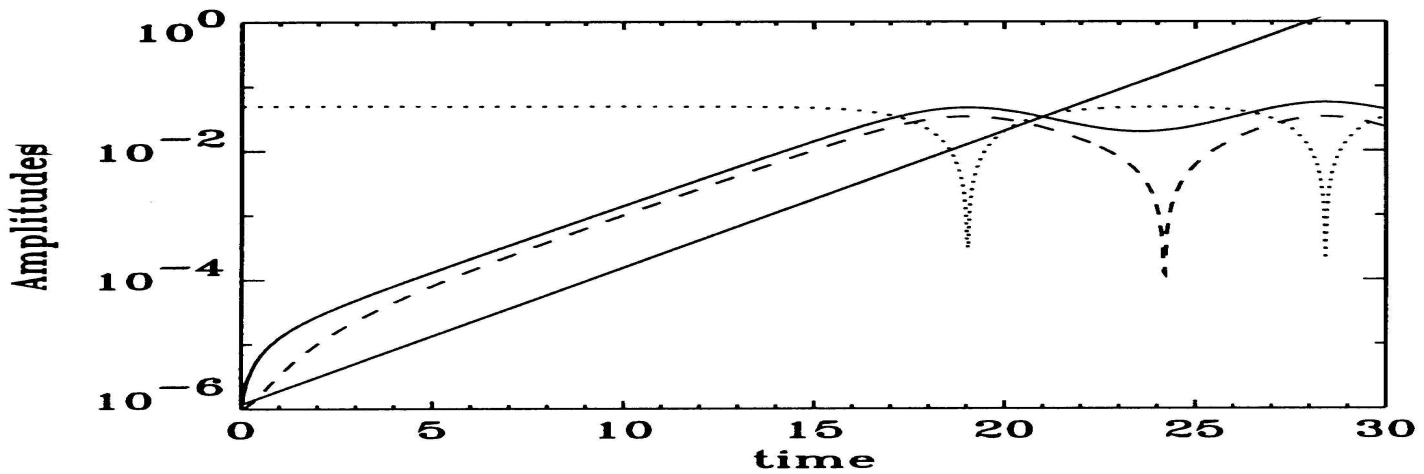
$$\omega = \omega_k + kV_0$$

$$V_0 = cB_0^{-1} z \times \nabla \phi$$

# Numerical Simulation



(a)





# Suppression of the parametric instability in the auroral cavity

$$d_t (1 - \lambda_e^2 \nabla_{\perp}^2) A + \partial_z \varphi = 0$$

$$\frac{v_A^2}{c^2} \partial_t \nabla_{\perp}^2 \varphi + d_t \nabla_{\perp}^2 \varphi + v_A^2 d_z \nabla_{\perp}^2 A = 0$$

$v_A/c \approx 1/3$

Relativity of the  
Alfven velocity

$$\gamma = \left[ 2q^2 |v_E|^2 \left( b - 2 \frac{v_A^2}{c^2} \right) - \delta \omega^2 \right]^{1/2}$$

# Kinetic Alfvén waves

$$\Omega_{\pm} = \mathbf{q} \cdot \mathbf{v}_g \pm \left[ \frac{(q \times k)_z^2}{B_0^2} |\psi_0|^2 k^2 \rho_s^2 + \delta\omega^2 \right]^{1/2}$$

$$\delta\omega = -\omega_{\mathbf{k}} b q^2 / 2k^2$$

$$b = \frac{k^2 \rho_s^2}{1 + k^2 \rho_s^2}$$

$$\omega_{\mathbf{k}}^2 = k_z^2 v_A^2 (1 + k^2 \rho_s^2)$$

# Other predator-prey models

- Hasegawa-Mima model for drift turbulence
  - Generation of zonal flows and streamers
  - Bohm-like plasma diffusion
  - Suppression of drift turbulence
- Charney model for Rossby waves
  - Generation of zonal and meridional flows

# Generation of zonal flows by Rossby waves

$$r_0^2 \nabla_{\perp}^2 \partial_t h - u_* \partial_x h + f r_0^4 \{h, \nabla_{\perp}^2 h\} = 0$$

$$u_* = g \partial_y (H_0 / f)$$

$$\Omega = \frac{f^2 q^2 r_0^4}{2u_*} \int \frac{v_g k_x}{\Omega - \mathbf{q} \cdot \mathbf{v}_g} \frac{\partial N_{\mathbf{k}}}{\partial k_y} d^2 k$$

$$\Omega_{\pm} = \mathbf{q} \cdot \mathbf{v}_g \pm \left[ \frac{v'_g k_x^2}{\omega_k} q^2 f^2 r_0^4 k^2 |h_0|^2 + \left( \frac{v'_g q^2}{2} \right)^2 \right]^{1/2}$$

# Optimal parameters of ZF

Instability condition



$$v'_g / \omega_k < 0$$

$$\mathbf{q} \perp \mathbf{k}$$

$$\frac{q^2}{k^2} = \frac{f^2}{\omega_k^2} (kr_0)^4 |h_0|^2$$

$$\gamma_{\max} = \frac{f^2}{ku_*} (kr_0)^6 |h_0|^2$$

$$f \approx 1.6 \times 10^4 \text{ s}^{-1}$$

$$r_0 \approx 2 \times 10^6 \text{ m}$$

$$h_0 \approx 5 \times 10^{-2}$$

$$u_* \approx 3 \times 10^2 \text{ m} \cdot \text{s}^{-1}$$



# Estimations

$$f = 1.6 \times 10^{-4} \text{ s}^{-1}$$

$$r_0 = 2 \times 10^6 \text{ m}$$

$$h_0 = 10^{-2}$$

$$kr_0 = 10$$

$$u_* = 3 \times 10^2 \text{ m/s}$$

$$\gamma_{\max} = 4 \times 10^{-4} \text{ s}^{-1}$$

$$\lambda_{\max} = 2\pi/q_{\max} = 1.5 \times 10^2 \text{ km}$$

$$v_{zf} = 25 \text{ m/s}$$

$$v_{zf} = \frac{(fr_0)^2}{2^{1/2} u_*} (k\rho_0)^4 |h_0|^2$$

# Conclusions

- Alfvén waves excited in the IAR practically never appear as small amplitude linear disturbances
- Parametric instability provides a substantial damping of the IAR eigenmodes and an essential mechanism of energy transfer from small-scale AWs to large and mesoscale convective motions
- The convective cells can interact with the background medium and develop 2D NL motions in the form of Kelvin-Stuart vortex streets
- Such a scenario constitutes a dynamical paradigm for intermittency in the ionospheric turbulence containing nonlinearly coupled AWs and convective motions



***Thank  
you!***