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Coherent Whistler Waves: PIC simulations of Oscilliton Formation

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Coherent Whistler Waves: PIC simulations of Oscilliton Formation

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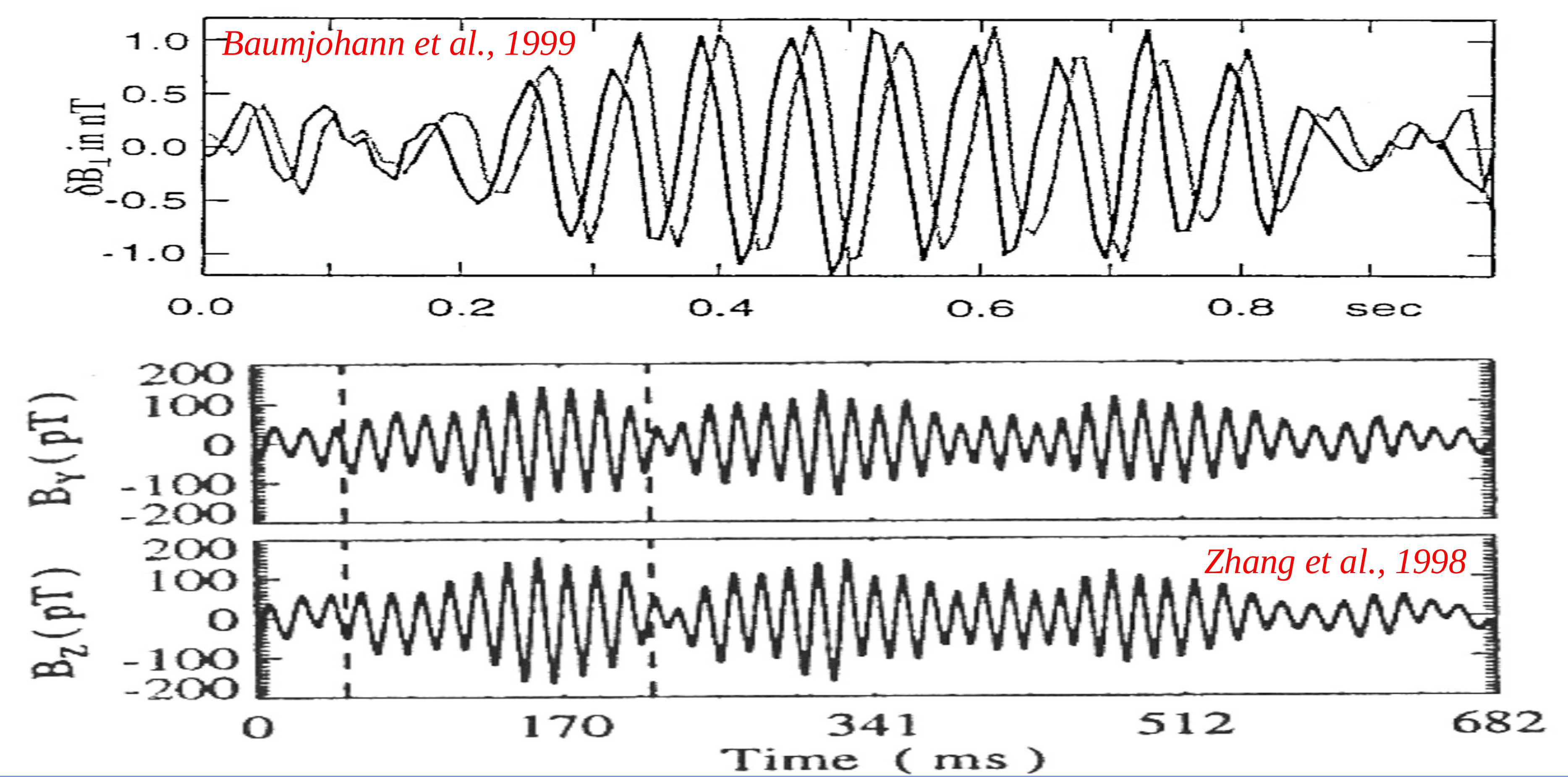
Coherent Whistler Waves: PIC simulations of Oscilliton Formation

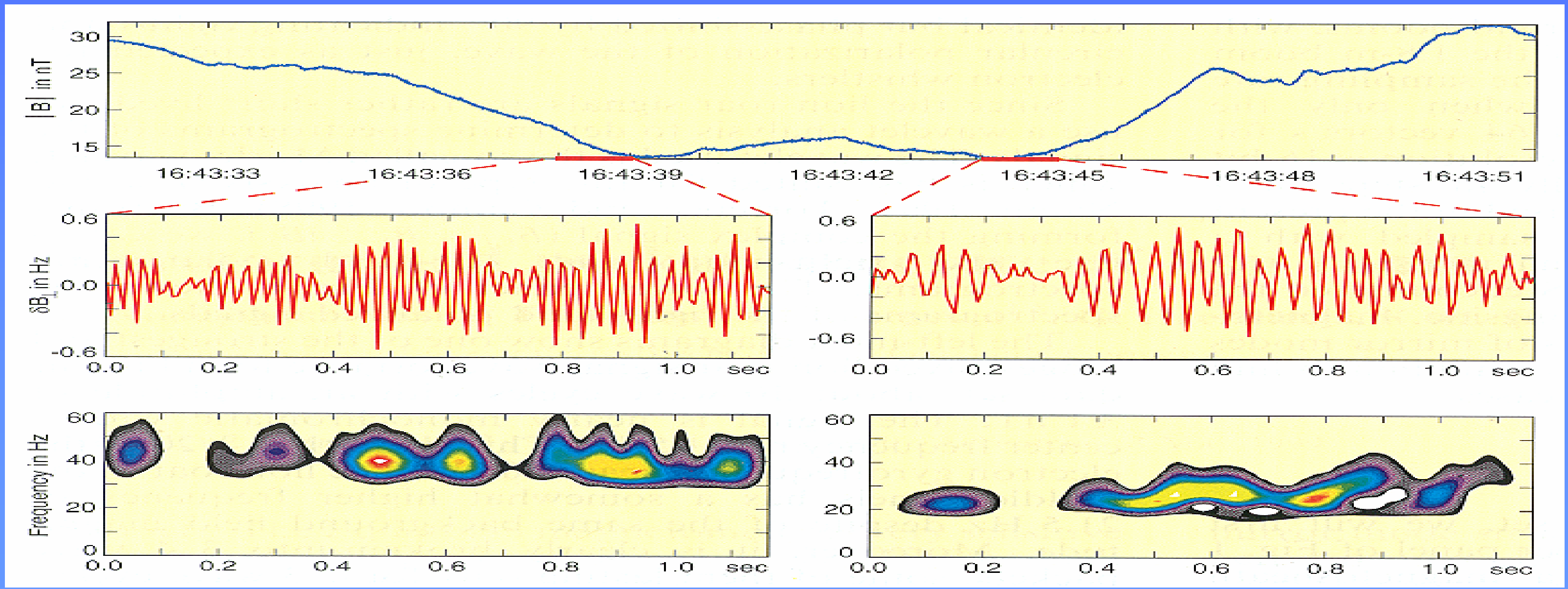
- Introduction:
- Observation of coherent whistlers
- The concept of nonlinear stationary waves:
  Solitons und Oscillitons
- Kinetic simulationen to Whistler Oscillitons
- Summary

Magnetospheric lion roars: Equator-S and Geotail observations: $\omega \leq \Omega_e$



Waveform and packet structure of lion roars

Magnetic field measurements aboard Equator-S in the equatorial dayside
(Baumjohann et al., 1999)



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Magnetospheric lion roars: Equator-S observations

Baumjohann et al., 1999

Main characteristics:

- Frequency range ~**0.1 Ω_e** ,
- Nearly parallel propagation
- Right-hand circular polarization
- whistlers

Lion roars in the magnetosheath: Geotail observations

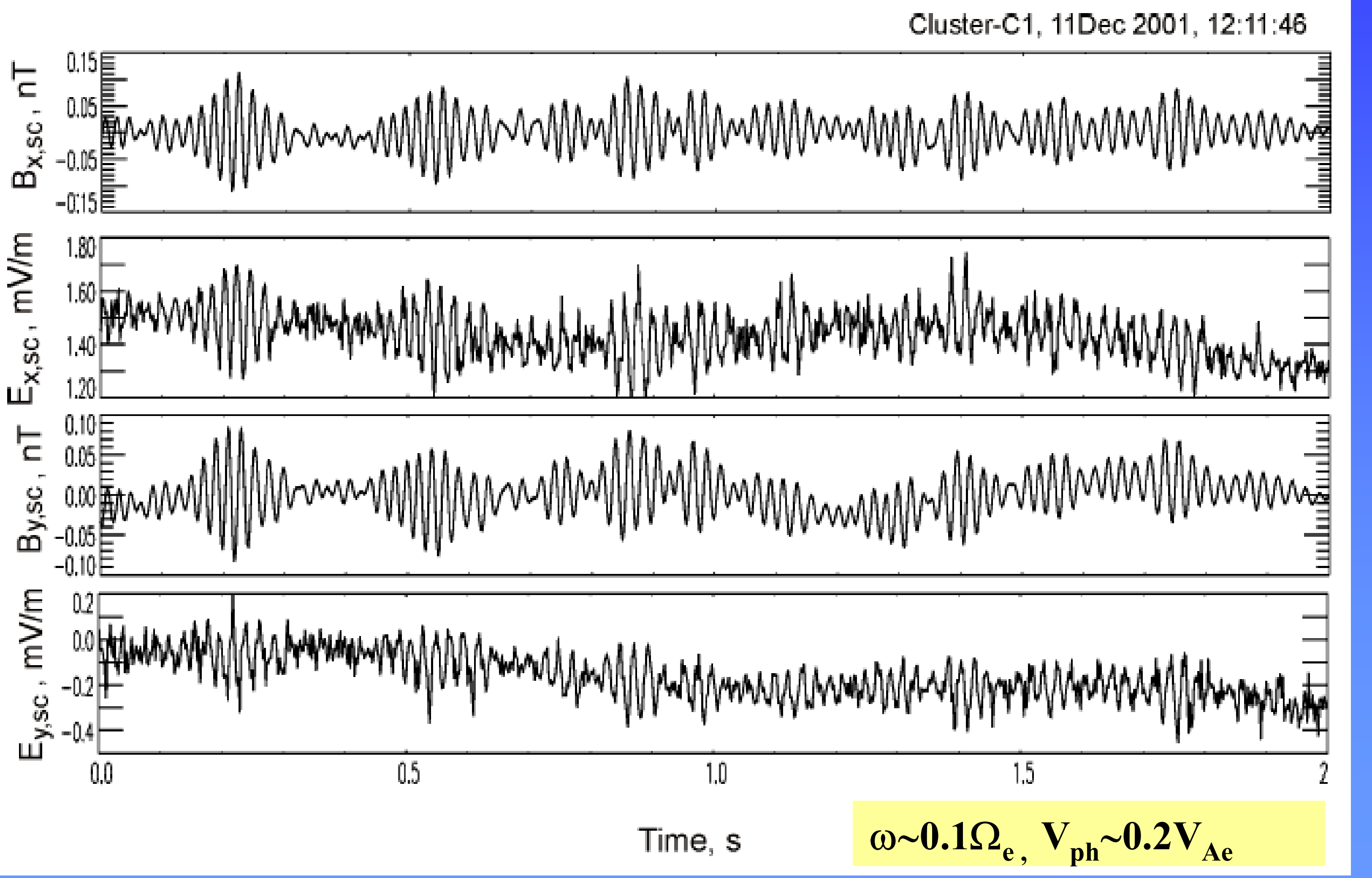
Zhang et al., 1998

Main characteristics:

- Frequency range **0.02 Ω_e - 0.75 Ω_e** ,
- Propagation slightly oblique ($\Theta \sim 10^0$)

Electric and magnetic field variations in whistler wave packets

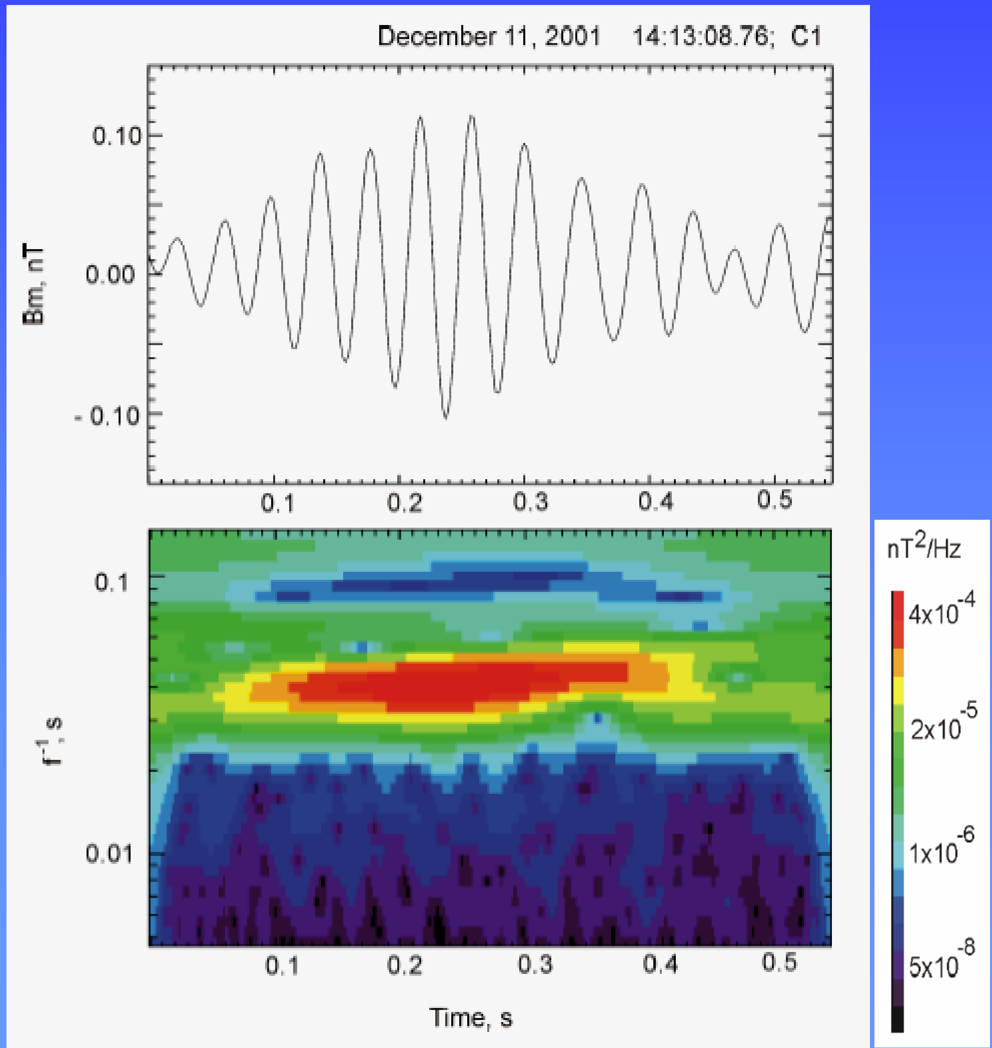
Dubinin et al., 2007



$\omega \sim 0.1\Omega_e$, $V_{ph} \sim 0.2 V_{Ae}$

Single whistler wave packet from Cluster measurements

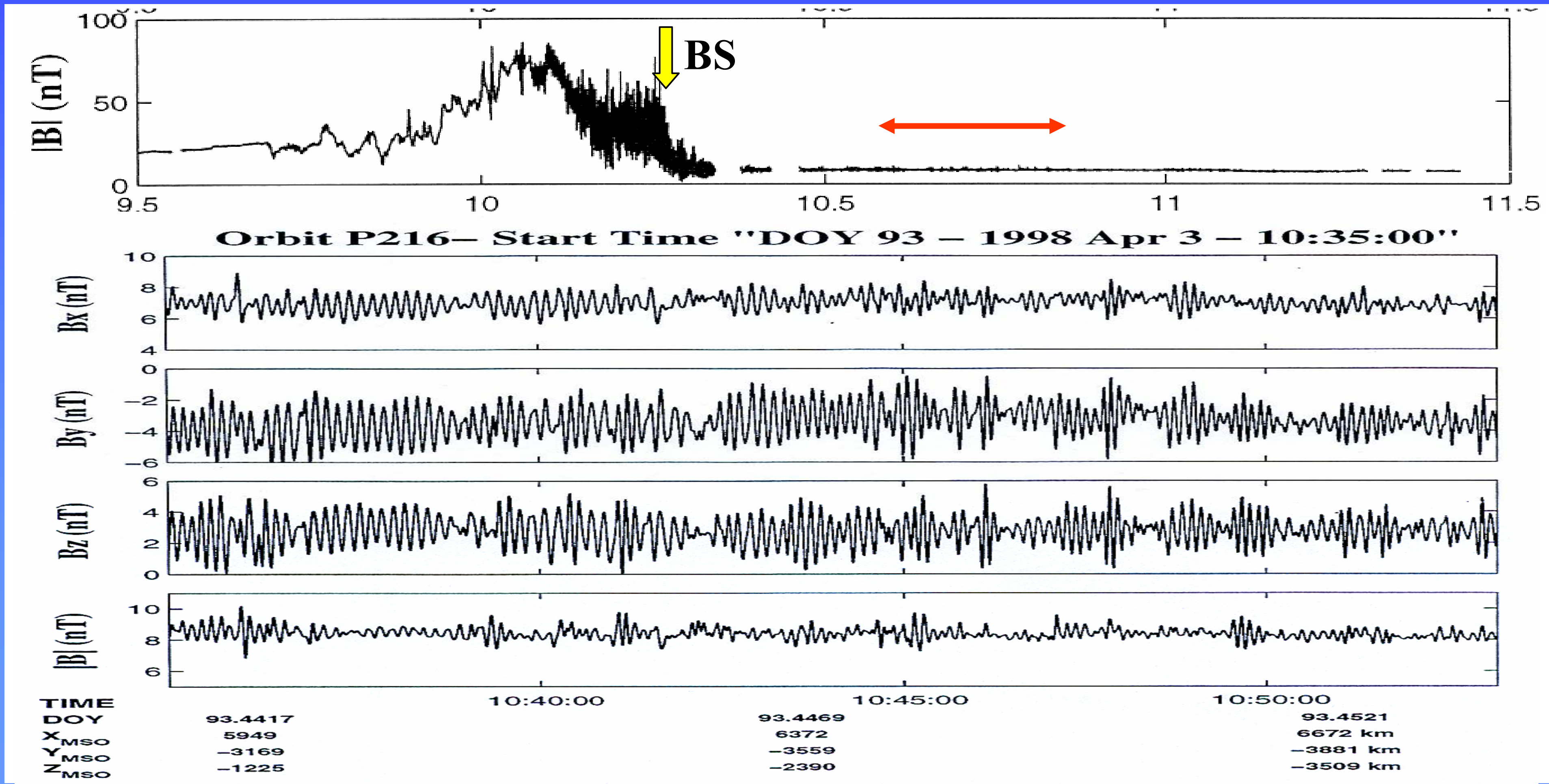
Dubinin et al., 2007



Magnetic field variation

Wavelet spectrogram of the observed emission: f~25 Hz

Coherent large-amplitude waves near the proton cyclotron frequency - MGS results



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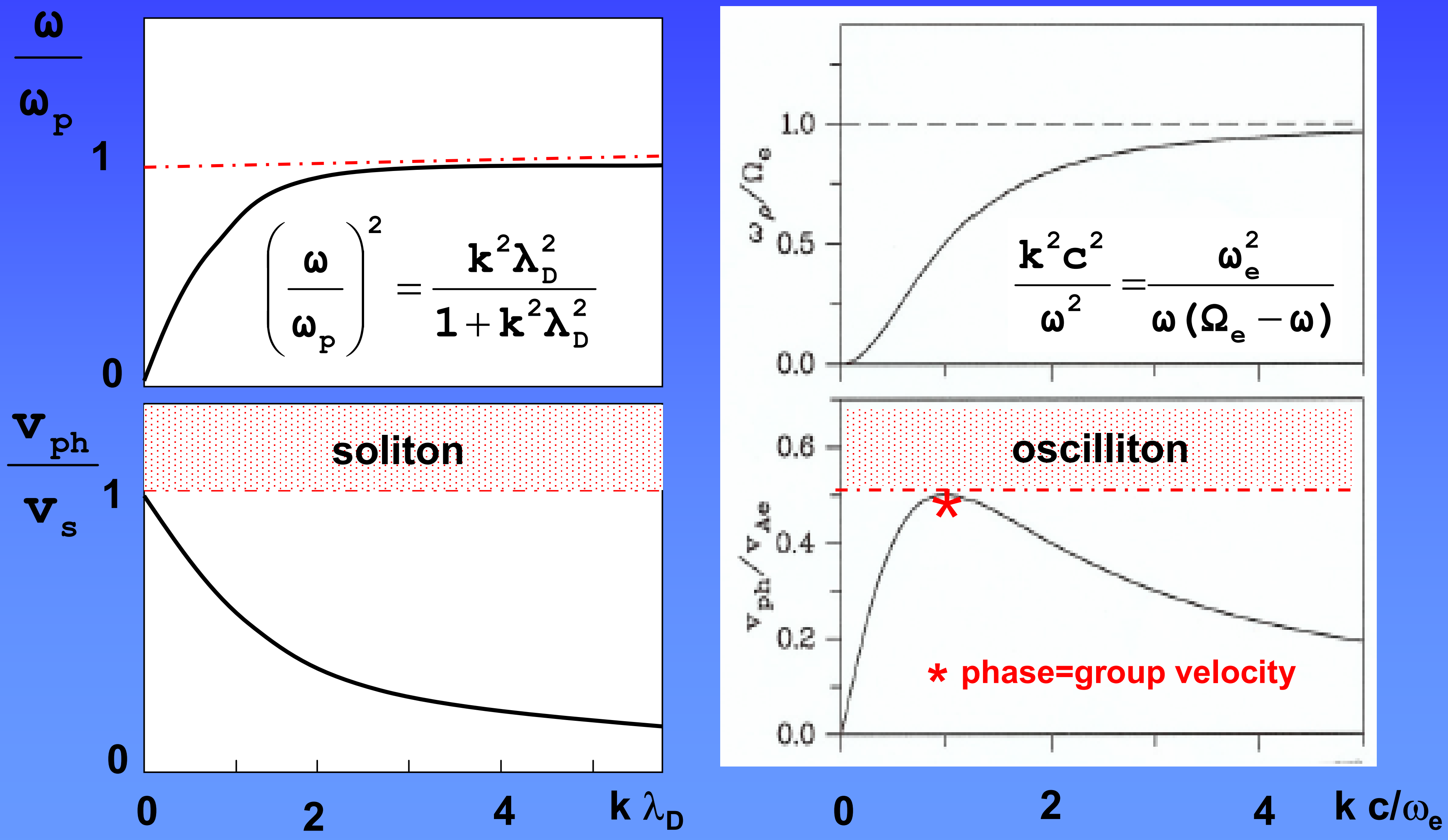
Oscilliton concept:

Coherent plasma structures are signatures of stationary nonlinear waves (oscilitons) which appear in plasmas with particular dispersion: As a necessary condition there must be a (ω, k) point where phase and group velocity coincide.

Dispersion of ion-acoustic waves and whistlers: a comparison



Dispersion of whistler propagating parallel to the magnetic field: $\omega > \Omega_p$

cold electrons:
$\omega_e > \Omega_e$

$$D(\omega, k) = \frac{k^2 c^2}{\omega^2} - \frac{\omega_e^2}{\omega(\Omega_e - \omega)} = 0$$

$\longrightarrow$

$$D(x, y) = x^2 + \frac{y}{y-1} = 0$$

$x = kc/\omega_e$

$y = \omega/\Omega_e$

Transition to stationary whistlers:

$\omega = k U$ → $y = x M$

$$D(x,y) = x^2 + \frac{y}{y-1} = 0$$

↓

$$D(x,U) = x^2 - \frac{x}{M} + 1 = 0$$

↓

$$x_{1,2} = \frac{1}{2M}\left(1 \pm \sqrt{1-4M^2}\right) = 0$$

$x = kc/\omega_e$

$y = \omega/\Omega_e$

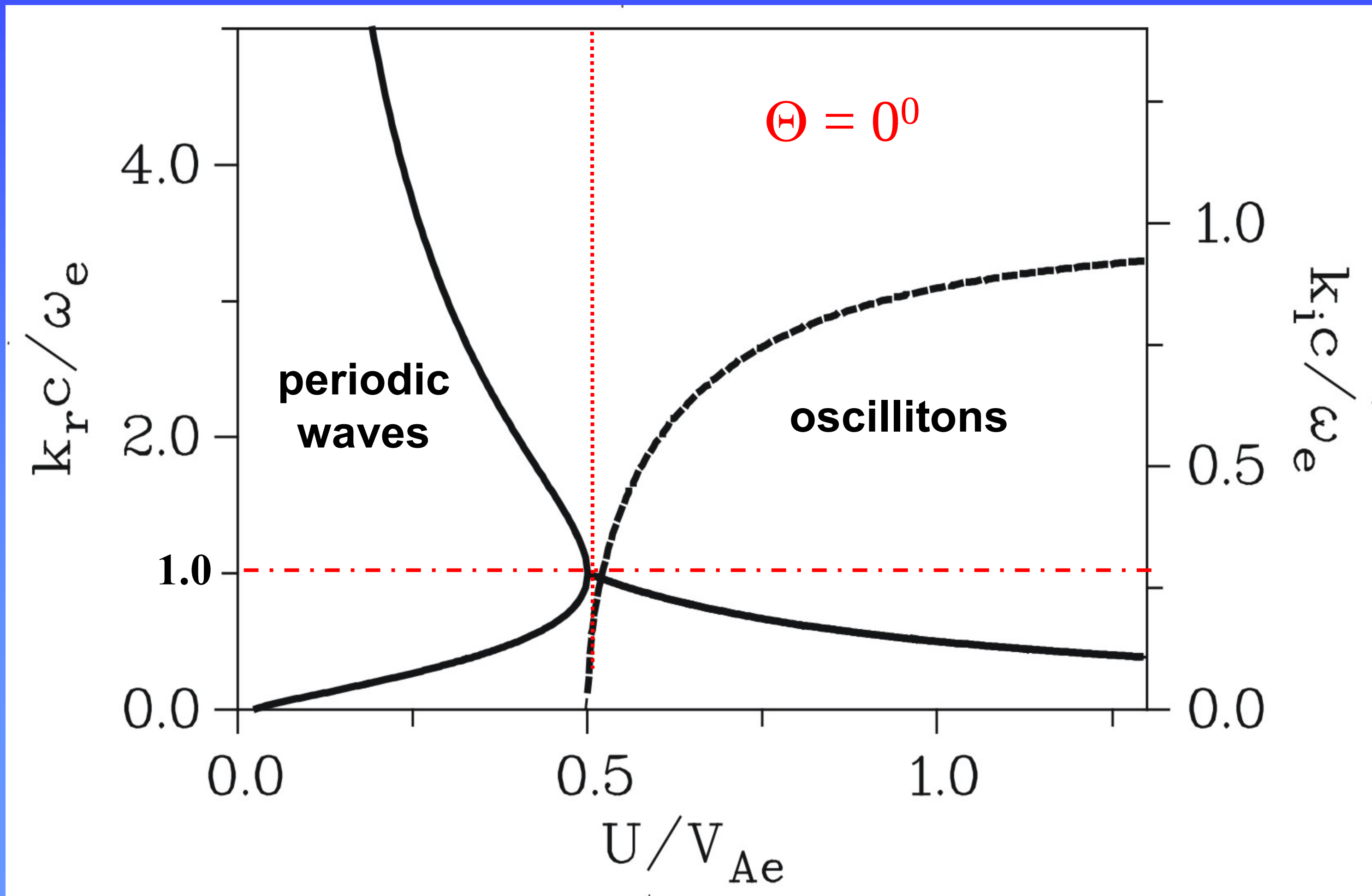
$M = U/V_{Ae}$

x=1 if M=0.5

x: complex if M > 0.5

Dispersion of stationary whistler waves

$D(\omega = k \cdot U, k) = 0 \rightarrow k = k(U)$



Stationary <u>nonlinear</u> equations:

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Basic equations are:

(1) Equations of motion for electrons and protons

$$\frac{\partial}{\partial \mathbf{t}} \longrightarrow -\mathbf{U}\frac{\partial}{\partial \mathbf{x}}$$

(2) Ampere‘s law and

(3) Faraday‘s law

Governing equations of whistler oscillitons

Equations of motion for electrons and protons: i=e,p

$$\frac{\partial u_{iy}}{\partial x} = -\frac{1}{m_i}(E_y - u_{ix}B_z + u_{iz}B_x)/(M - v_{ix})$$

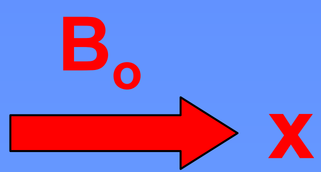
$$\frac{\partial u_{iz}}{\partial x} = -\frac{1}{m_i}(E_z + u_{ix}B_y - u_{iy}B_x)/(M - v_{ix})$$

Ampere's law:

$$\frac{\partial B_y}{\partial x} = -n_e v_{ez} + n_p v_{pz} \qquad \frac{\partial B_z}{\partial x} = +n_e v_{ey} + n_p v_{py}$$

Faraday's law:

$$E_y = +M\,B_z \qquad E_z = -M\,B_y$$

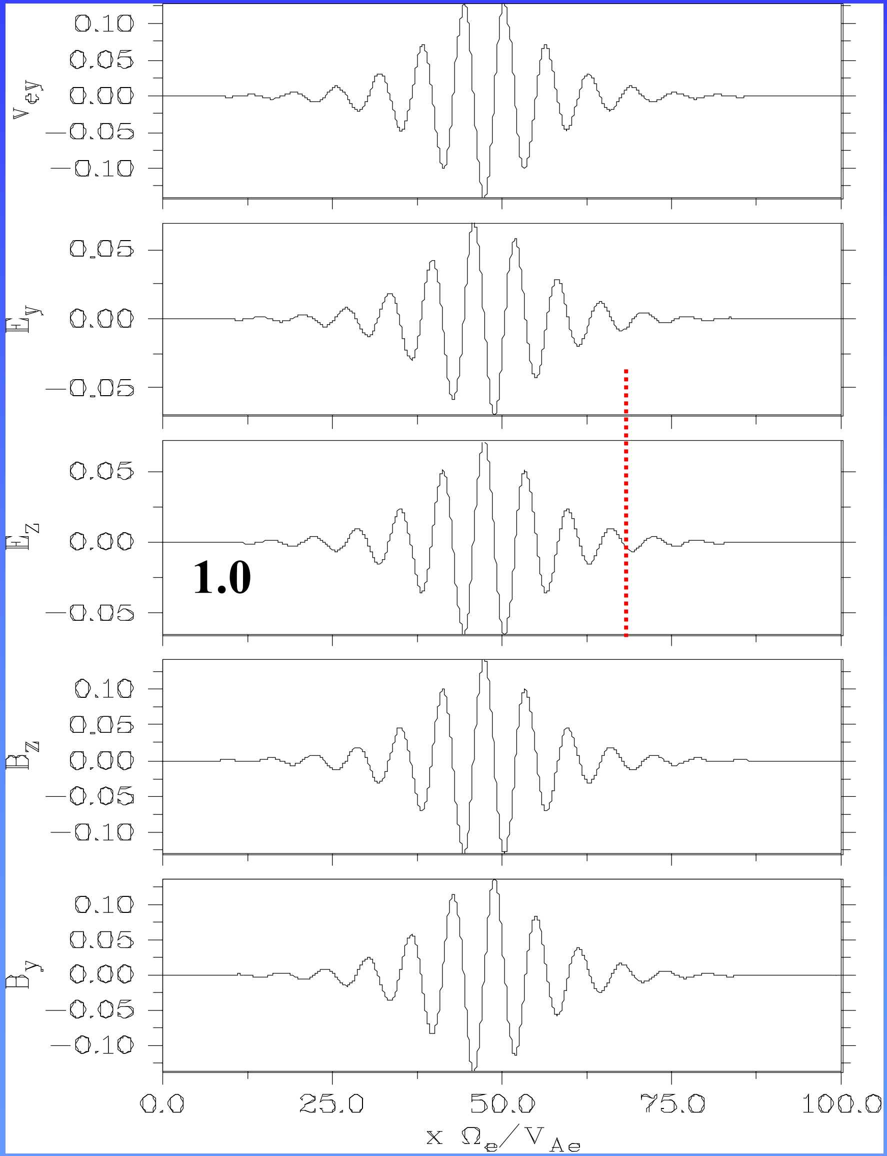


Conservation of longitudinal momentum:

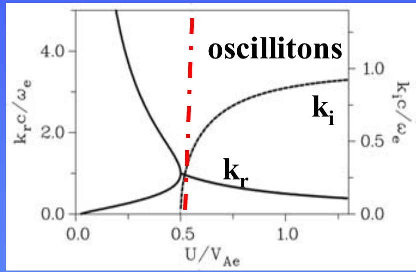
$$u_{px} \cong u_{ex} = \frac{1}{2Mm_p}(B^2 - 1)$$

$M = U/V_{Ae}$

Spatial profile of a whistler oscilliton

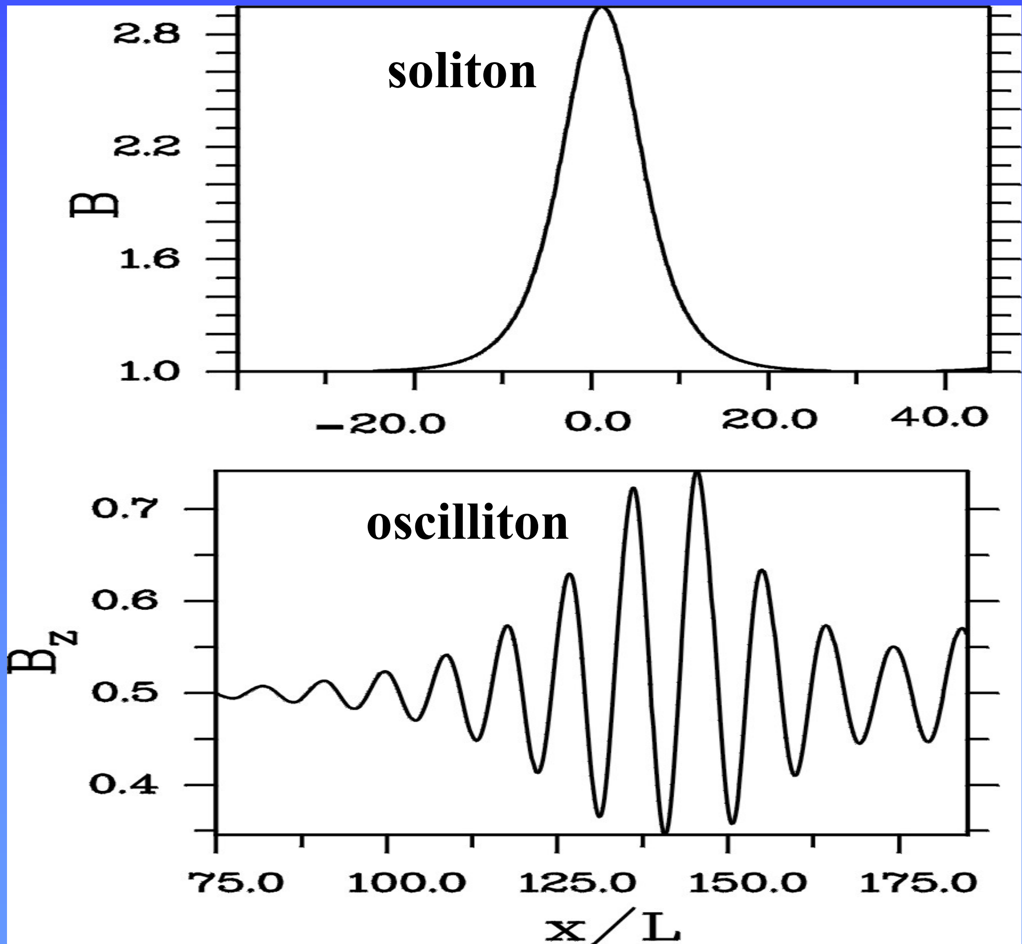


M=0.505

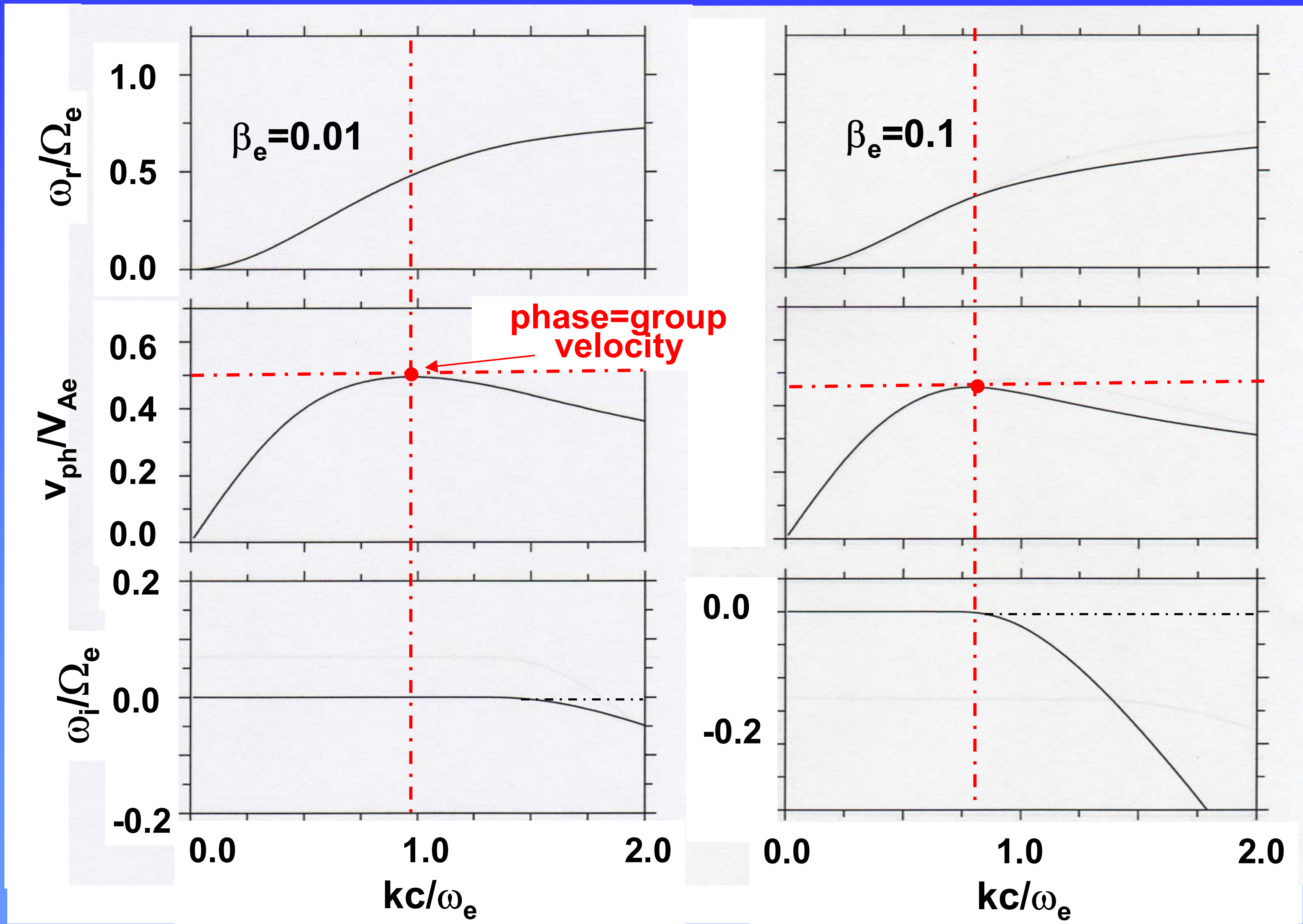


Sauer et al, 2002

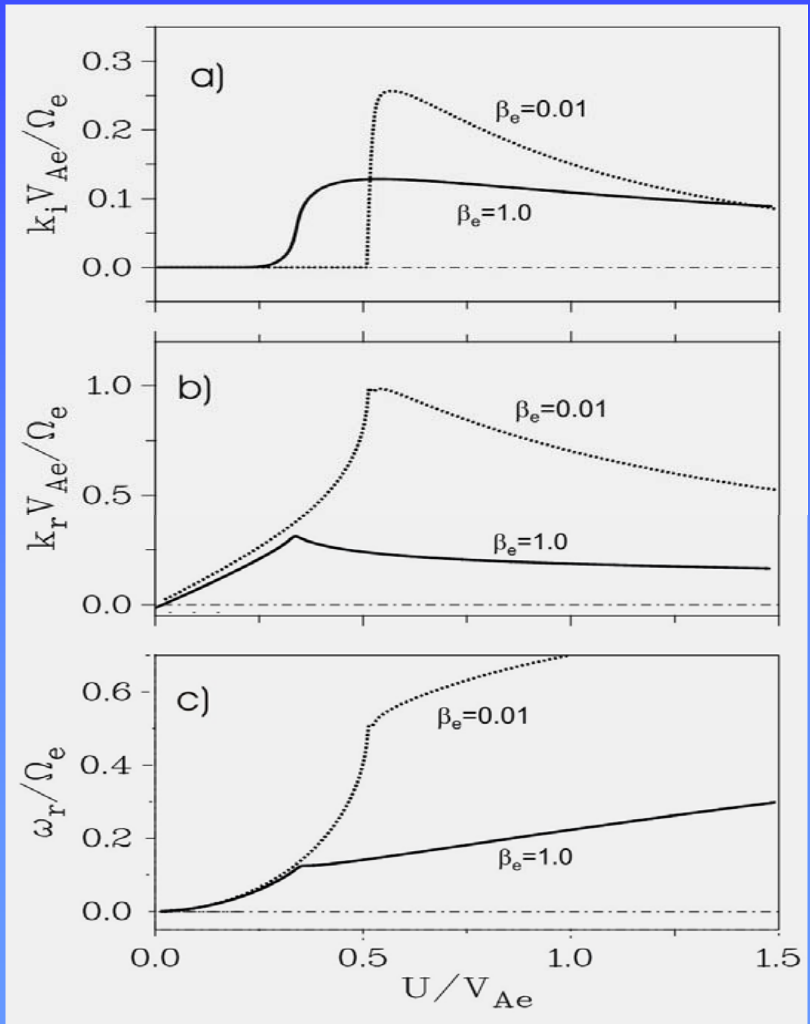
Stationary nonlinear waves: solitons and oscillitons



Whistler waves



Kinetic dispersion theory of stationary whistlers: $\omega \to kU$



Kinetic dispersion relation: D(ω,k)=0

$$x^2 - \frac{y}{x\sqrt{\beta_e (T_{e\perp}/T_{e||})}} W(z) - 0.5\left(\frac{T_{e\perp}}{T_{e||}} - 1\right) W'(z) = 0$$

$x = kc/\omega_e$, $y = \omega/\Omega_e$, $z = (y-1)/x$⌧β_e

Transition to stationary waves, $\omega \to kU$:

D(kU,k)=0

k=k(U)

(PIC) Kinetic simulations of whistler wave generation in an unstable plasmas with electron temperature anisotropy:

$T_{e\perp} > T_{e\parallel}$

Two parameter sets:

cold electrons:

$\beta_e = 0.01$

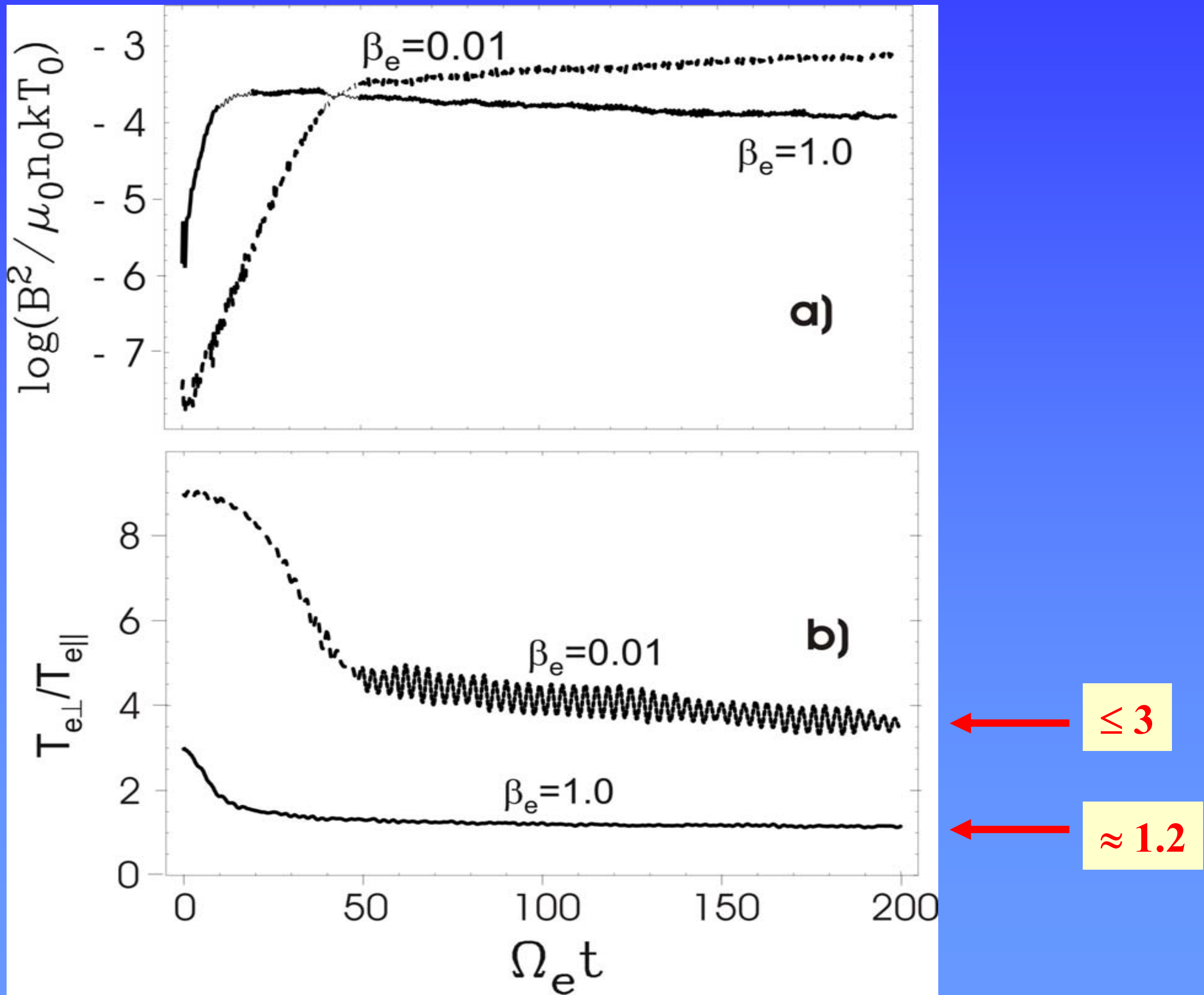
$T_{e\perp}/T_{e\parallel} = 9$ (t=0)

warm electrons:

$\beta_e = 1.0$

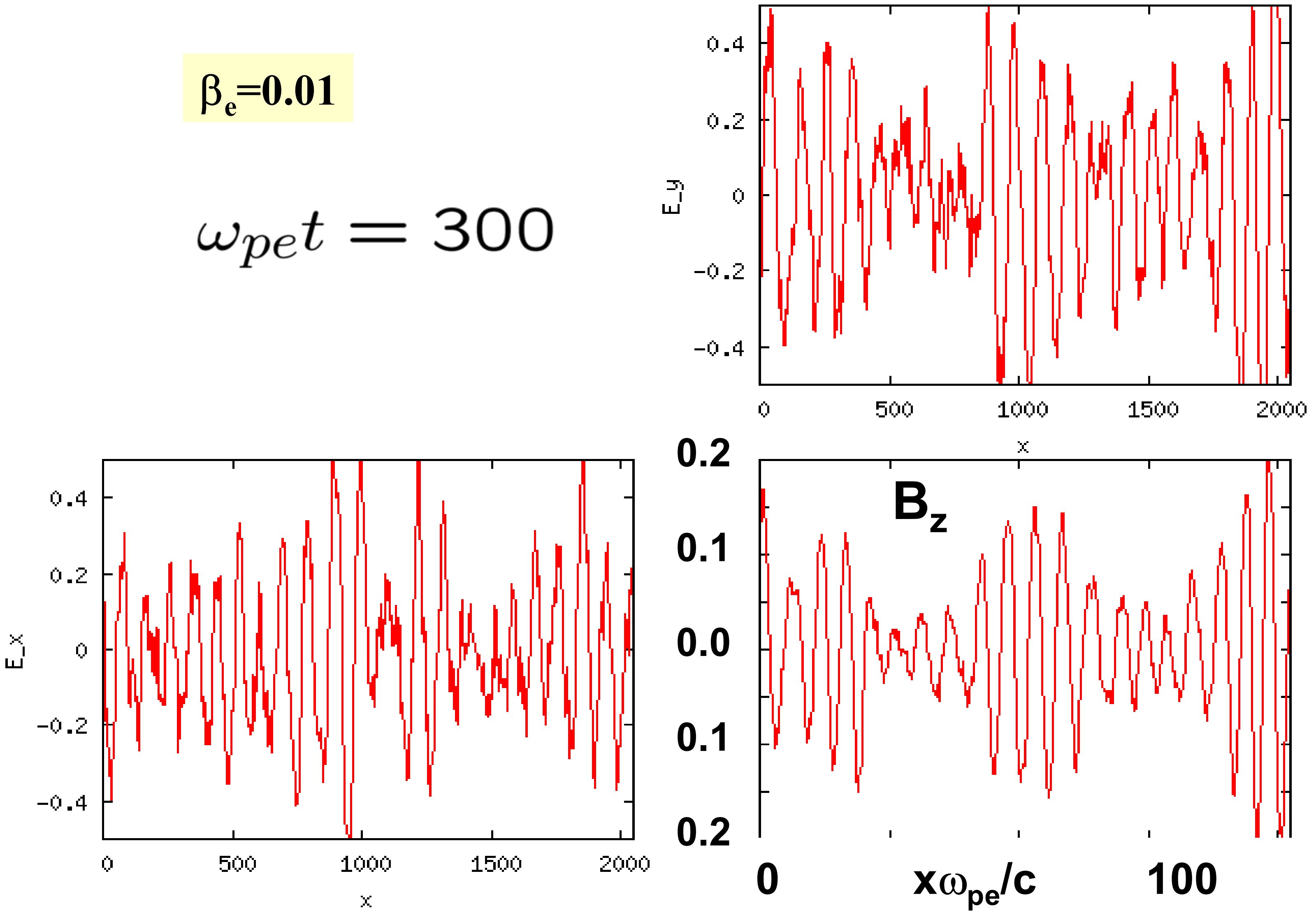
$T_{e\perp}/T_{e\parallel} = 3$ (t=0)

Evolution of unstable whistlers - PIC simulations: β_e=0.01, 1.0

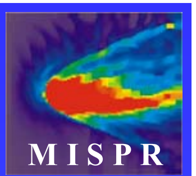


β_e=0.01

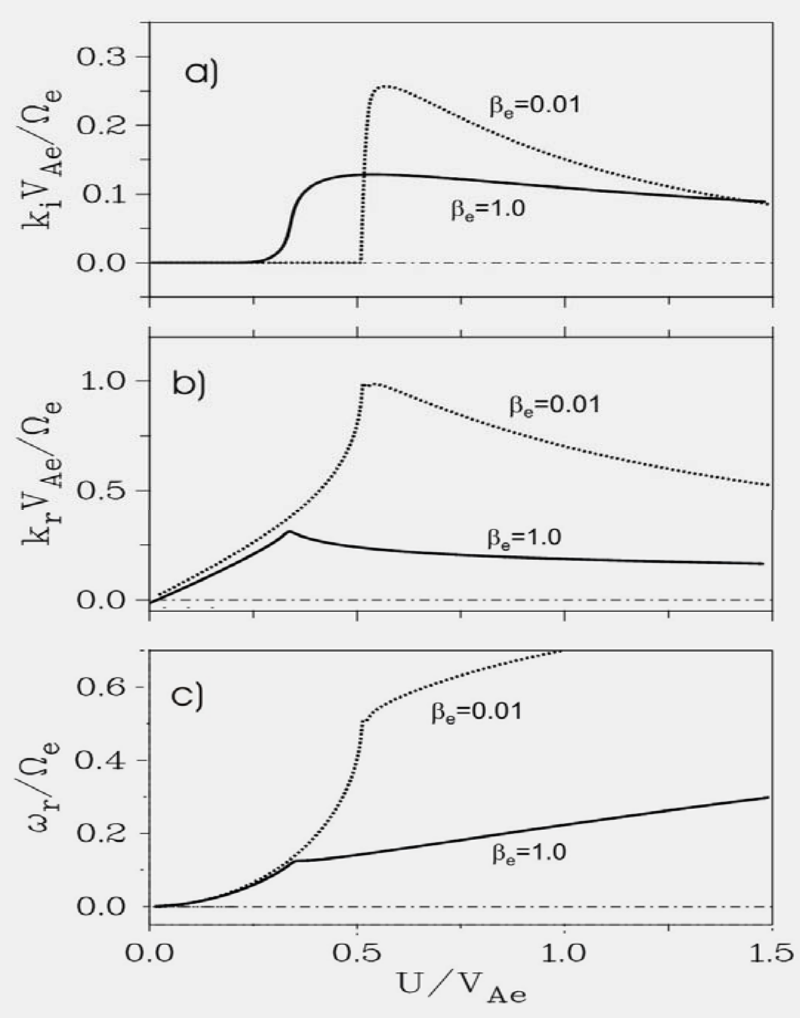
$$\omega_{pe} t = 300$$



$\lambda\omega_{pe}/c \approx 100/16 \approx 2\pi$ → $kc/\omega_{pe} \approx 1$



Kinetic dispersion theory of stationary whistlers: $\omega \rightarrow kU$



Kinetic dispersion relation: D(ω,k)=0

$$x^2 - \frac{y}{x\sqrt{\beta_e (T_{e\perp}/T_{e||})}} W(z) - 0.5\left(\frac{T_{e\perp}}{T_{e||}} - 1\right) W'(z) = 0$$

$x = kc/\omega_e$, $y = \omega/\Omega_e$, $z = (y-1)/x⌧\beta_e$

Transition to <u>stationary</u> waves, $\omega \rightarrow kU$:

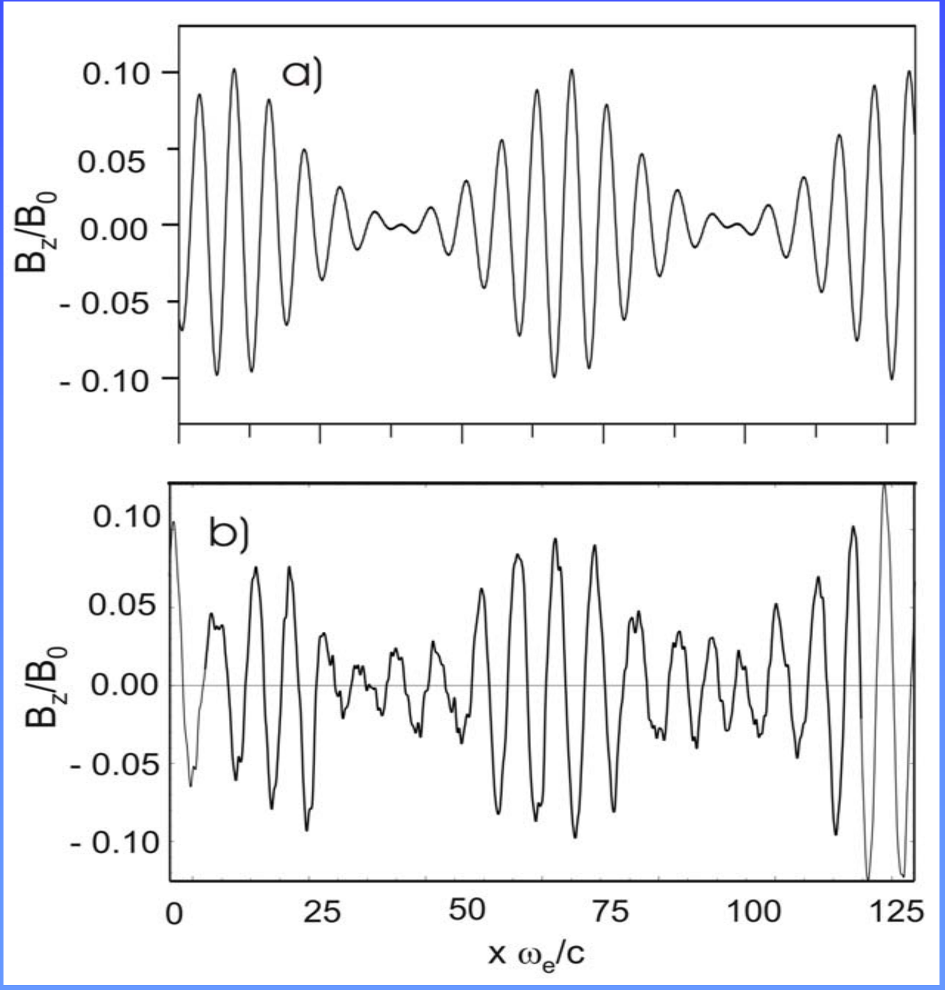
↓

D(kU,k)=0

↓

k=k(U)

Wave form of Whistler Oscilliton and PIC simulation compared



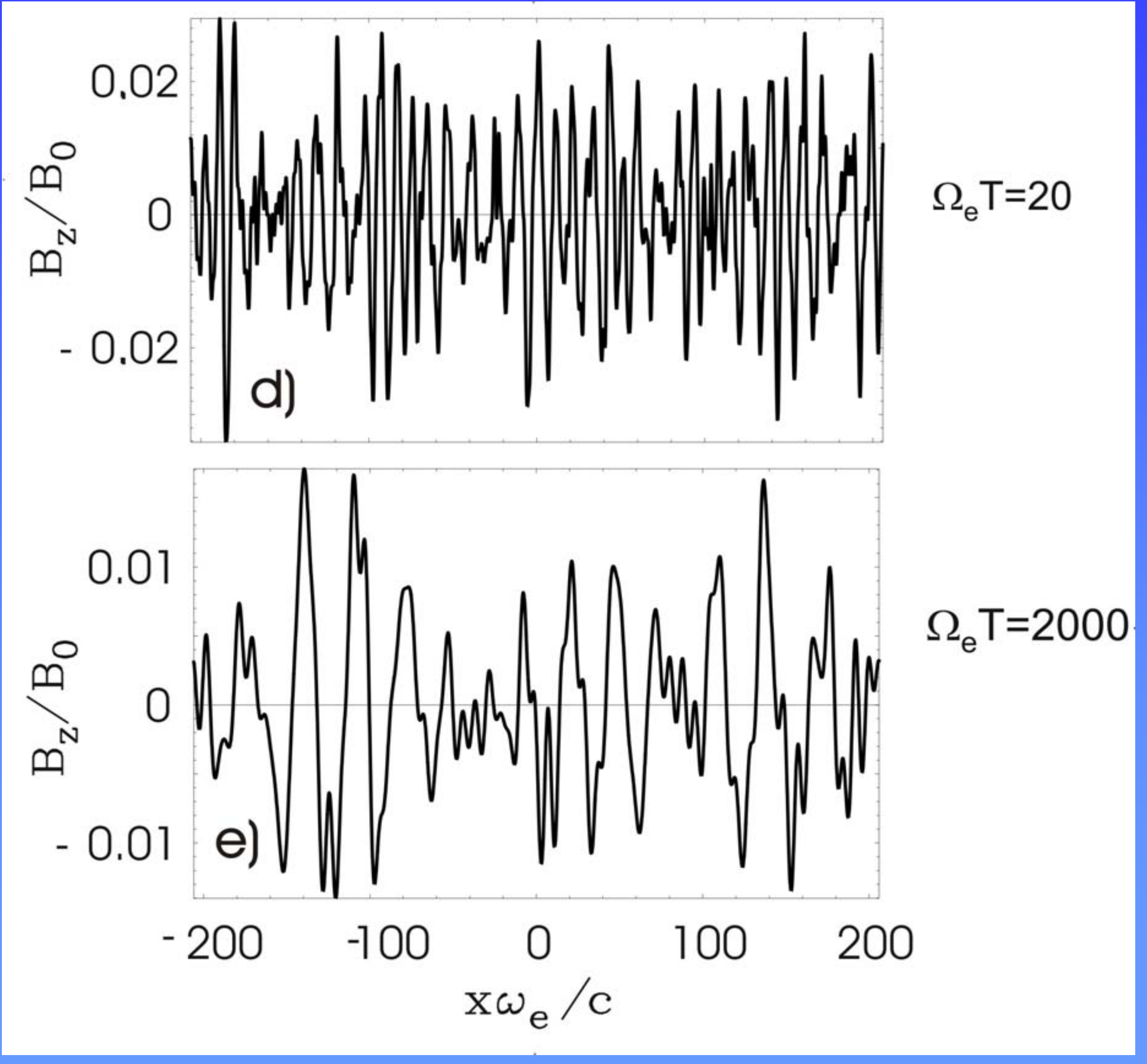
Whistler oscilliton obtained for M=0.5025. To get wave packets, a initial amplitude at infinity of $B_{y,\infty}$=0.002 was taken.

The wave length is given by

$kc/\omega_e \approx 1$

Snapshot of **PIC simulation** for a cold plasma, β_e=0.01 at $\Omega_e t$=300

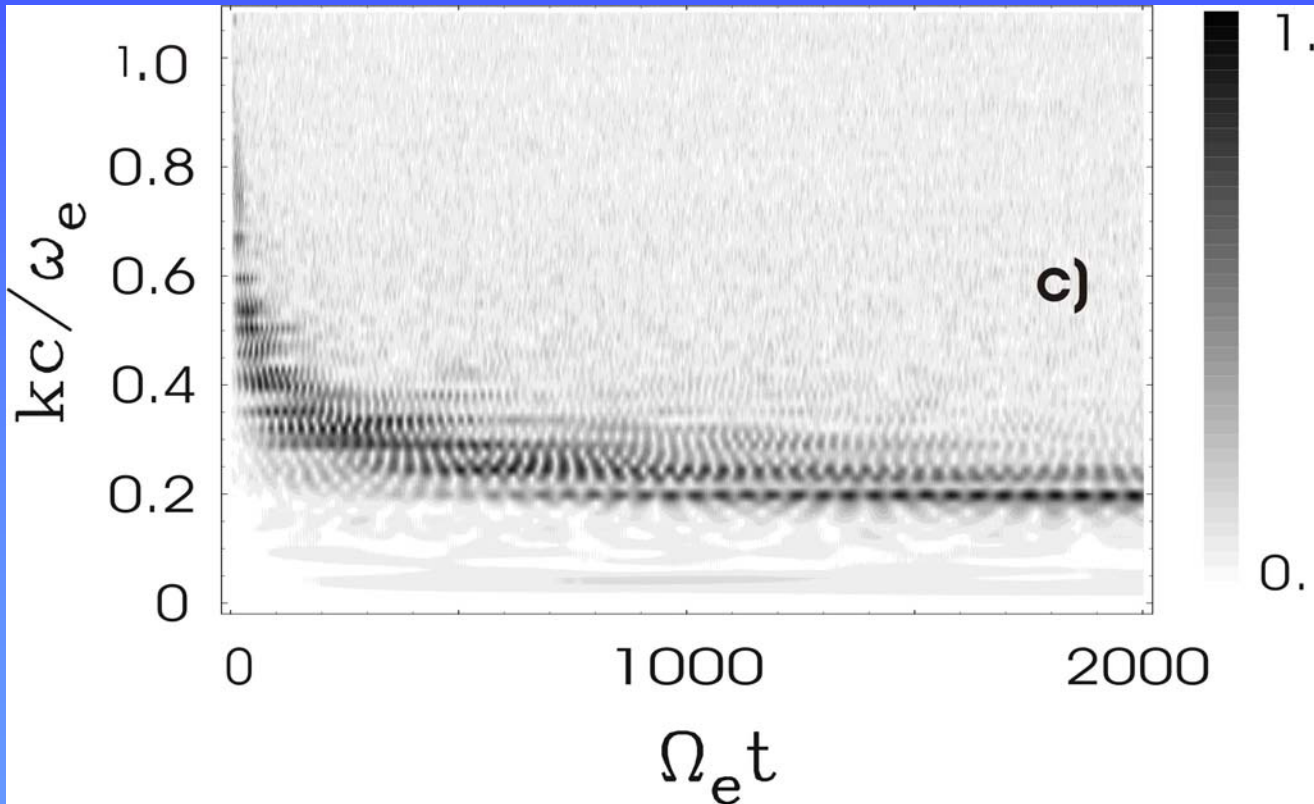
Time evolution of magnetic field profiles



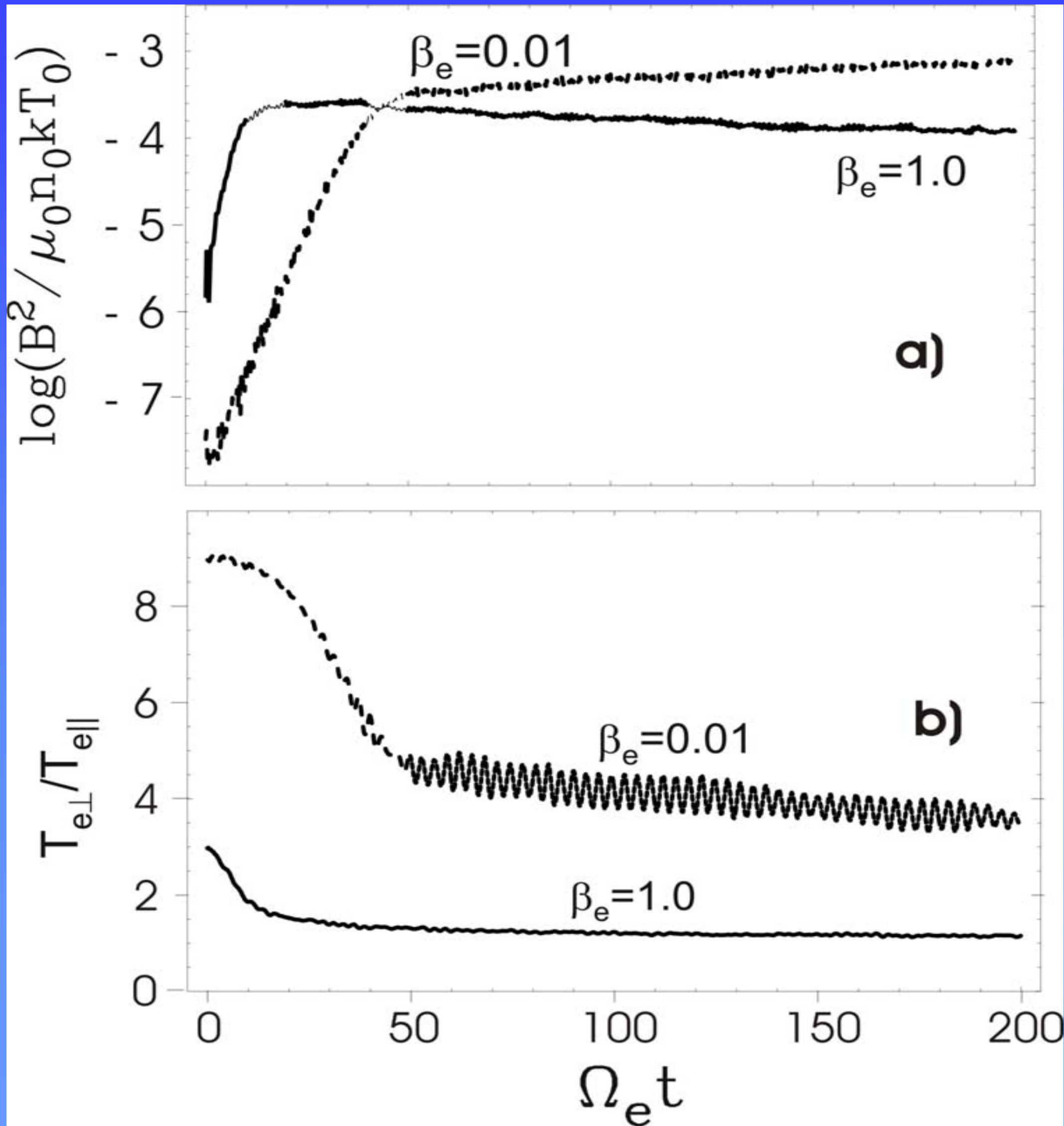
β_e=1.0

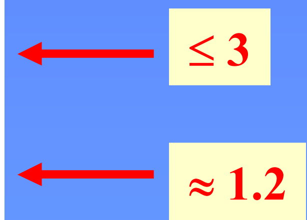
Magnetic field energy versus wave number and time:

PIC simulations: $\beta_e = 1.0$

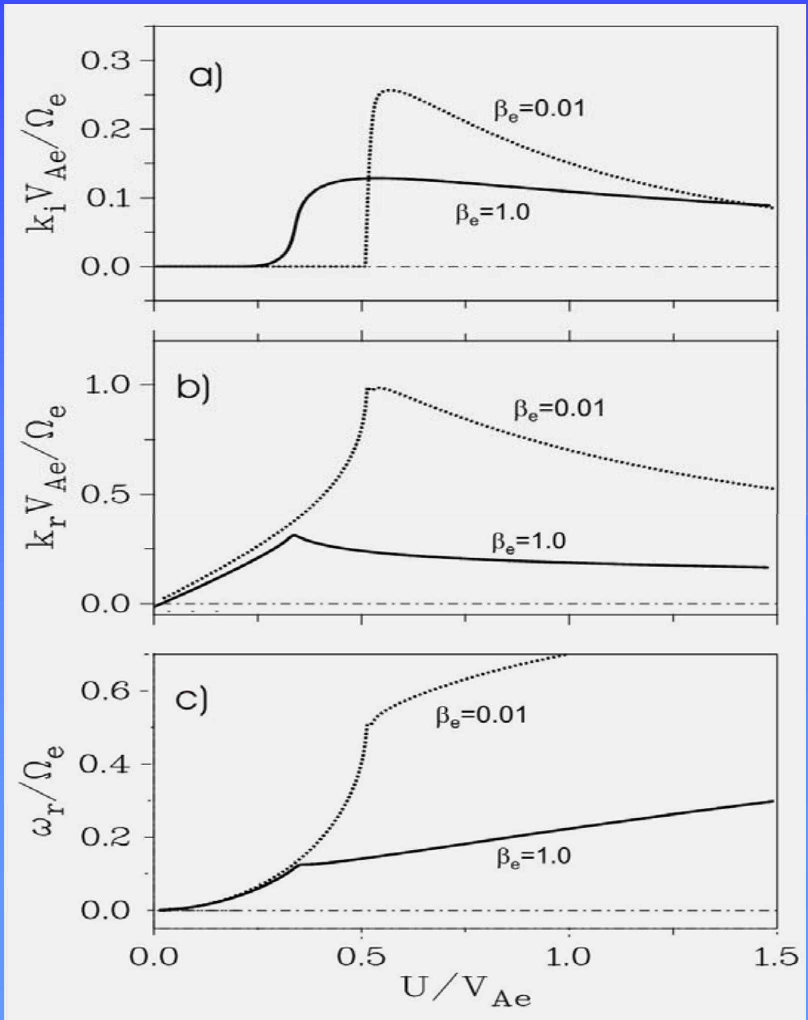


Evolution of unstable whistlers - PIC simulations: β_e=0.01, 1.0





Kinetic dispersion theory of stationary whistlers: $\omega \to kU$



Kinetic dispersion relation: $D(\omega,k)=0$

$$x^2 - \frac{y}{x\sqrt{\beta_e (T_{e\perp}/T_{e||})}} W(z) - 0.5\left(\frac{T_{e\perp}}{T_{e||}} - 1\right) W'(z) = 0$$

$x = kc/\omega_e$, $y = \omega/\Omega_e$, $z = (y-1)/x$⌦β_e

Transition to stationary waves, $\omega \to kU$:

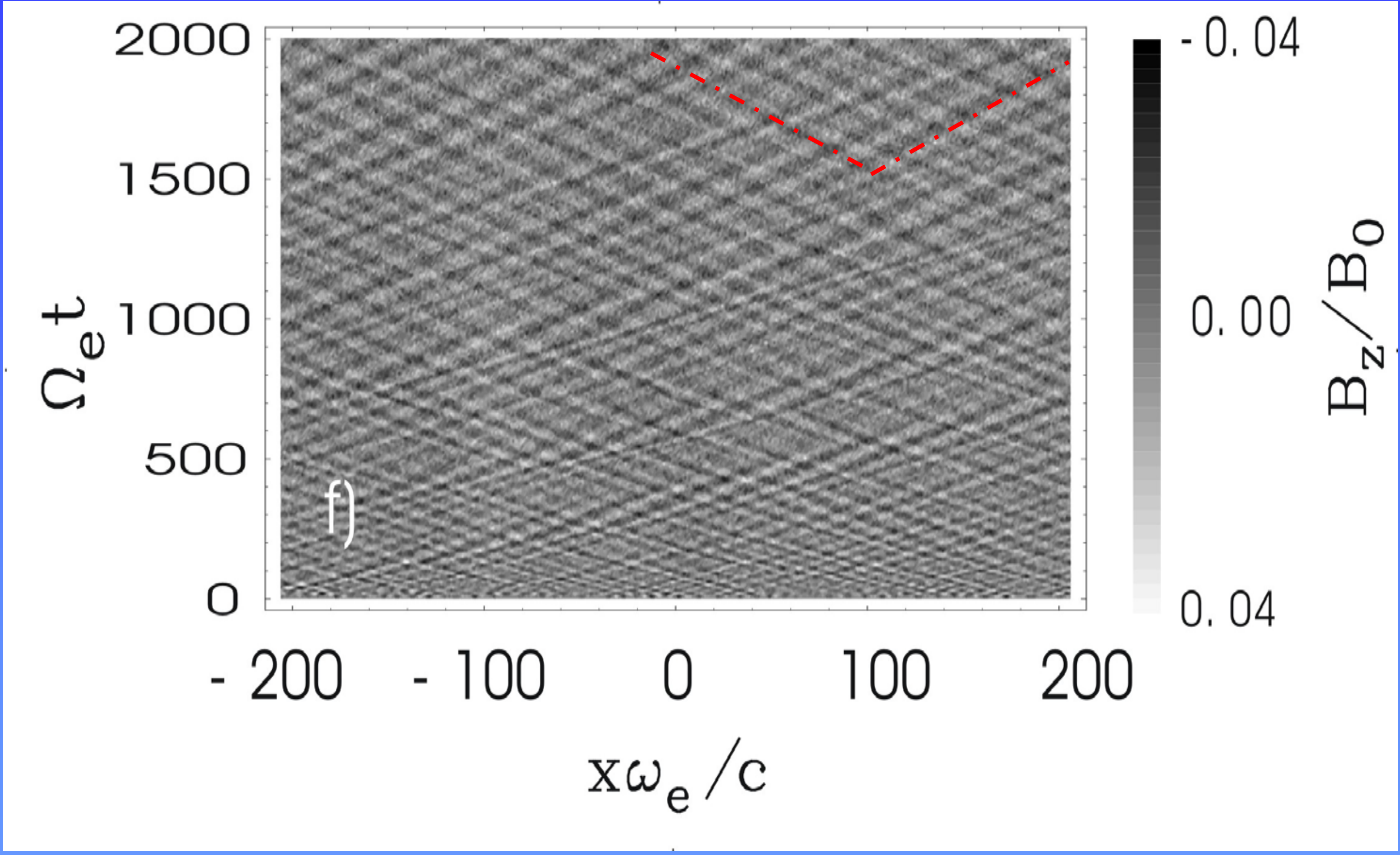
↓

$D(kU,k)=0$

↓

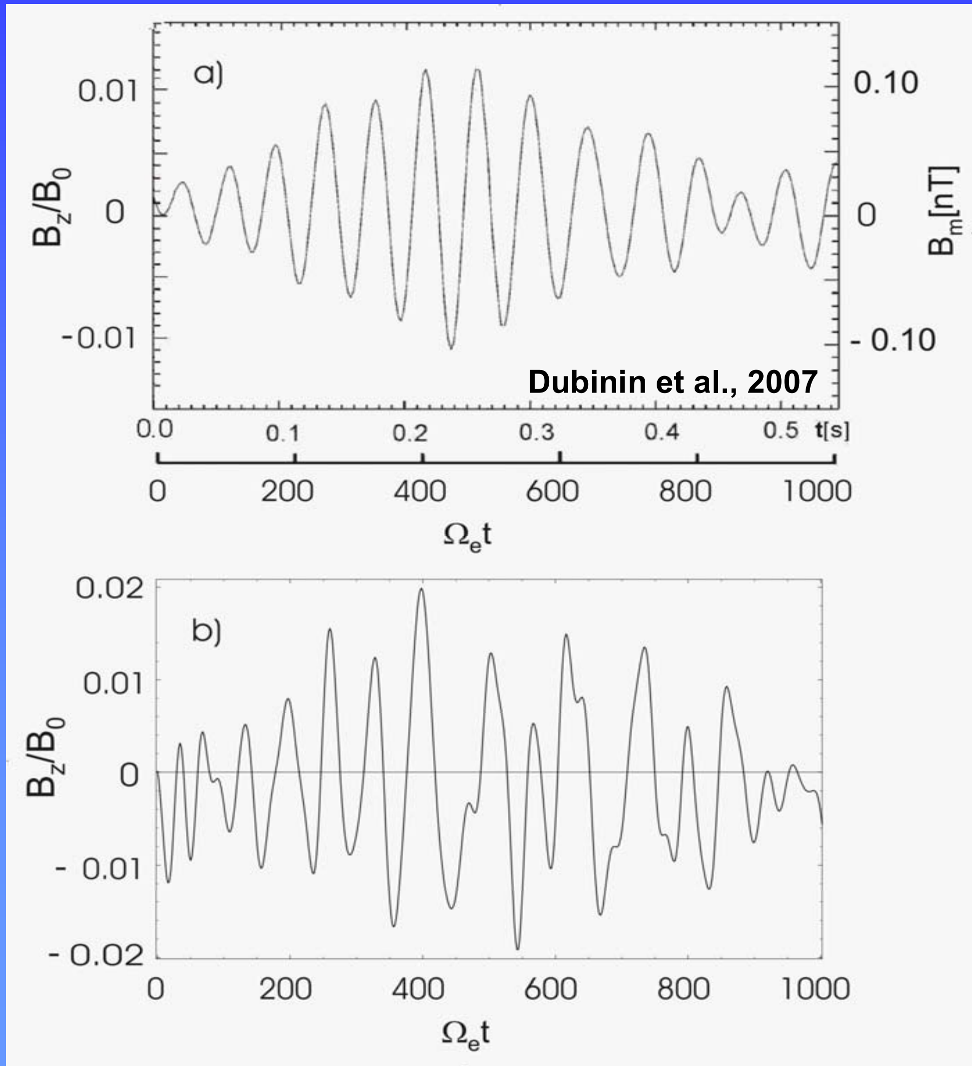
$k=k(U)$

x,t evolution of the magnetic field component B_z/B_0



$V_{ph}/V_{Ae} \approx 100/400 = 0.25$

Single whistler wave packet from Cluster data and PIC simulation results



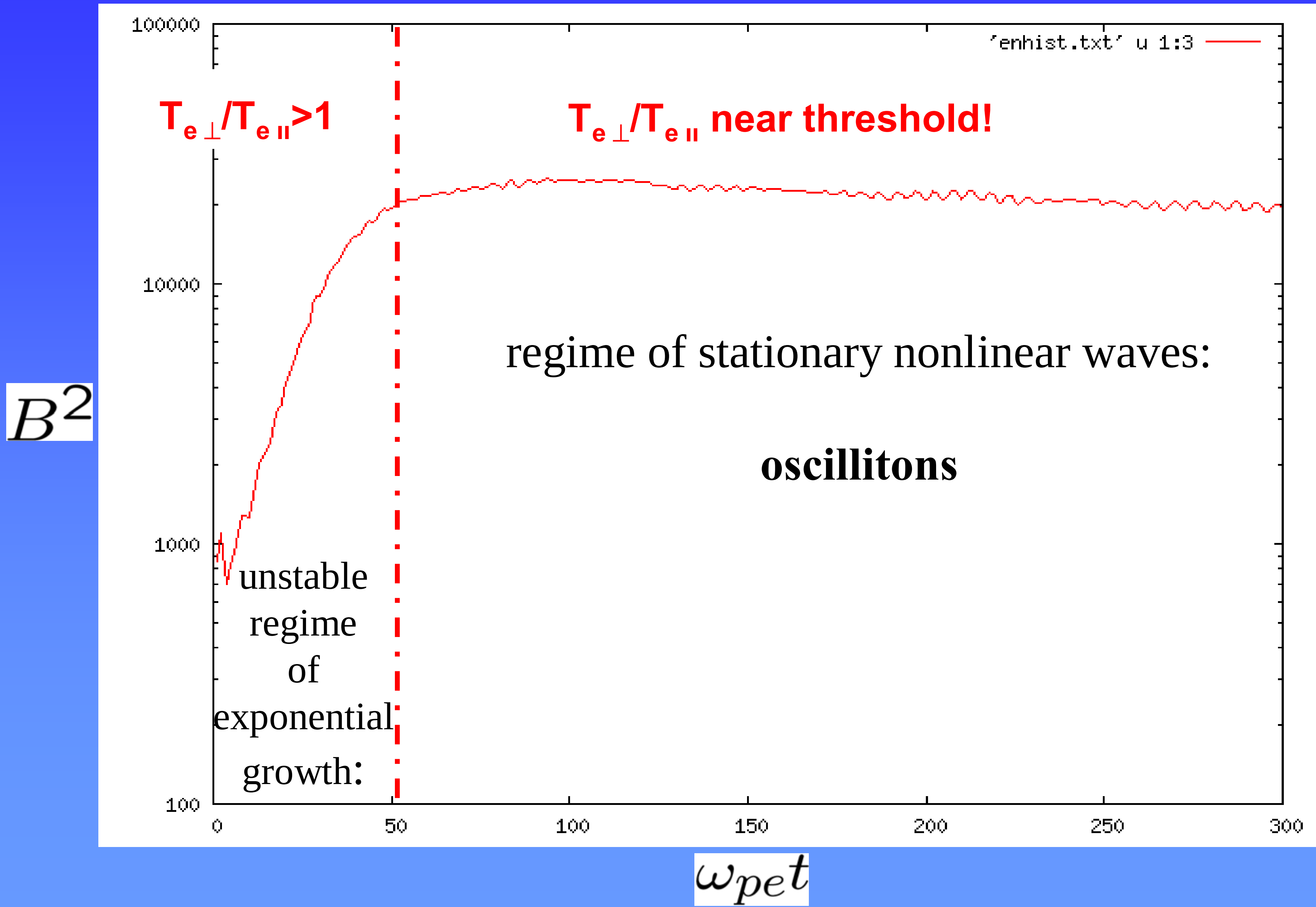
Wave form of magnetic field variations in a single wave packet

Temperal evolution of a magnetic field component from PIC simulations:

βe=1.0,

$(Te\perp/Te\|)_0$=3

Formation of Whistler Oscillitons



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Summary of simulation results:

The *initial* phase of whistler wave excitation is characterized by the parameters of the temperature anisotropy instabilty: frequency, wave number and maximum growth rate are in agreement with the linear Vlasov theory.

Reaching saturation the wave pattern changes character and tends to develop into stationary structures according to the predictions of the theory of *whistler oscillitons*. In particular, frequency and wavelength of the structures differ significantly from that during the unstable regime.