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Short wavelength ballooning mode in tokamaks

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A skin size ($k_\perp \lesssim \omega_{pe}/c$) plasma mode characterized by a dispersion relation $\omega \simeq ck_\perp k_\parallel/k_{De}$ (k_{De} the electron Debye wavenumber), adiabatic ions, and $\omega \ll k_\parallel v_{Te}$, in a uniform plasma is destabilized in the tokamak geometry by a modest electron temperature gradient η_e and ballooning parameter α_e. When unstable, a large electron thermal diffusivity emerges because of the cross-filed wavelength much longer than that of the conventional electron temperature gradient (ETG) mode.

I. INTRODUCTION

The role played by plasma turbulence having cross-field wavelengths of the order of the electron skin depth c/ω_{pe} in anomalous transport has long been noticed in the past. Mikhailovskii [1] made a detailed study of the so-called universal drift mode due to the intrinsic electron Landau damping and found that the maximum growth rate

$$\gamma_{\max} \simeq \frac{c_s}{4\sqrt{\pi}L_n},$$

is reached in the wavenumber regime

$$k_\perp \gtrsim \frac{\omega_{pe}}{c},$$

where $c_s = \sqrt{T_e/M_i}$ is the ion acoustic speed, L_n the density gradient scale length. Simple mixing length estimate for particle diffusivity yields

$$D \simeq \frac{c_s}{4\sqrt{\pi}L_n}\left(\frac{c}{\omega_{pe}}\right)^2. \tag{1}$$

As is well known, the universal drift mode in slab geometry with magnetic shear is absolutely stable [2]. Subsequent studies revealed that in tokamaks, the toroidicity induced drift mode may be unstable [3] and the diffusivity may prevail with some toroidicity corrections. Ohkawa [4] proposed an electron thermal diffusivity in tokamaks,

$$\chi_e = \frac{v_{Te}}{qR}\left(\frac{c}{\omega_{pe}}\right)^2, \tag{2}$$

where v_{Te}/qR is the electron transit frequency with v_{Te} the electron thermal velocity and qR the connection length of the helical magnetic structure. In the formula, the skin depth plays the role of decorrelation length in the radial direction and the transit frequency plays the role of decorrelation rate. It is noted that the derivation of the Ohkawa diffusivity was heuristic. The presence of skin size instability (or turbulence) was assumed but the origin of the turbulence was left unexplained. One feature of the thermal diffusivity which has relevance to experimental observation is that it gives a somewhat natural explanation for the empirical Alcator type scaling law for the energy confinement time [5],

$$\tau_E \propto n, \tag{3}$$

where n is the plasma density. More recently, it is becoming clear that the lower edge of the unstable $k_\perp$ spectrum of the electromagnetic electron temperature gradient (ETG) mode is in the regime $k_\perp \simeq \omega_{pe}/c$ and the presence of skin size plasma turbulence appears to be responsible for the anomalous electron thermal diffusivity.

In the past, electromagnetic dispersion relations involving the skin depth c/ω_{pe} have been considered largely in the limit of $\omega > k_\parallel v_{Te}$ (electron transit frequency). For example, in the slab geometry, the well known dispersion relation for the electromagnetic drift mode

$$\omega = \frac{(1+\eta_e)\,\omega_{*e}}{1+(ck_\perp/\omega_{pe})^2}, \tag{4}$$

emerges in the limit $\omega > k_\parallel v_{Te}$. Here $\eta_e = d\,(\ln T_e)\,/d\,(\ln n_0)$ is the electron temperature gradient relative to the density gradient. In tokamaks, this is modified as

$$\frac{\omega-\omega_{*e}\,(1+\eta_e)}{\omega-\omega_{De}} - \frac{\eta_e\omega_{*e}\omega_{De}}{(\omega-\omega_{De})^2} + \left(\frac{ck_\perp}{\omega_{pe}}\right)^2 = 0,\ \ \omega > k_\parallel v_{Te}, \tag{5}$$

but this dispersion relation predicts stable modes unless $\eta_e < 0$ (electron temperature profile opposite to that of the density). Here, ω_{De} is the electron magnetic drift frequency due to the magnetic curvature. In the same limit $\omega > k_\parallel v_{Te}$, the dispersion relation of the predominantly electrostatic ETG mode is subject to a small electromagnetic correction involving the electron skin depth [6 Kim-Horton]

$$\tau + \frac{\omega_{*e}}{\omega} + \left((k_\perp \rho_e)^2 - \frac{\omega_{De}}{\omega}\right)\left(1 - \frac{\omega_{*ep}}{\omega}\right) = \frac{\left(k_\parallel v_{Te}\right)^2}{\omega^2}\left(1 - \frac{\omega_{*ep}}{\omega}\right)\frac{k_\perp^2}{k_\perp^2 + \left(\omega_{pe}/c\right)^2}. \tag{6}$$

To circumvent this problem, it has been suggested [7] that the nonlinear Doppler shift $\mathbf{k}\cdot\mathbf{v}_E$ due to the $E \times B$ drift $\mathbf{v}_E$ may exceed the electron transit frequency in strong turbulence, and skin size electromagnetic turbulence may exist *nonlinearly* being driven by shorter wavelength electrostatic ETG mode.

Finally, in a recent fully kinetic analysis of electromagnetic ETG mode in tokamaks [8], the following electron thermal diffusivity,

$$\chi_e = \frac{q v_{Te}}{\sqrt{RL_{Te}}}\left(\frac{c}{\omega_{pe}}\right)^2 \sqrt{\beta_e}, \tag{7}$$

has been found where q is the safety factor, β_e is the electron beta factor and L_{Te} is the electron temperature gradient scale length. The ratio between Eq. (2) and (6) is

$$q^2\sqrt{\frac{R}{L_{Te}}}\sqrt{\beta_e},$$

which is of order unity in tokamaks.

In this lecture, stability of toroidal drift modes in the wavelength regime $k_\perp \simeq \omega_{pe}/c$ is reviewed. In particular, it is shown that tokamak discharges are linearly unstable against an electromagnetic mode (ballooning mode) characterized by adiabatic ions $\left(k_\perp \rho_i\right)^2 \gg 1$ (ρ_i the ion Larmor radius) and adiabatic electrons with a phase velocity parallel to the magnetic field smaller than the electron thermal speed $\omega \lesssim k_\parallel v_{Te}$. In these limits, a simple hydrodynamic ballooning mode emerges which is symbolically described by the following dispersion relation,

$$\left(\omega - \omega_{*e}\right)\left(\omega - \omega_{De}\right) + \eta_e \omega_{*e}\omega_{De} - \frac{\left(\omega - \omega_{*e}\right)^2}{1+\tau} = \frac{c^2 k_\parallel k_\perp^2 k_\parallel}{k_{De}^2}, \tag{8}$$

where $\tau = T_e/T_i$, $k_{De} = \omega_{pe}/v_{Te}$ is the electron Debye wavenumber, and $k_\perp(k_\parallel)$ is the wavenumber perpendicular (parallel) to the ambient magnetic field. The mode is destabilized by a modest electron temperature gradient η_e and electron ballooning parameter defined by

$$\alpha_e = q^2 \frac{R}{L_n} (1 + \eta_e) \beta_e. \tag{9}$$

The mode is intrinsically electromagnetic (because it is a ballooning mode, although there is no resemblance to the ideal MHD ballooning mode), and is not a result of correction to electrostatic modes such as the familiar electron temperature gradient (ETG) mode. The right hand side of Eq. (8) may be approximated by

$$\frac{c^2 k_\| k_\perp^2 k_\|}{k_{De}^2} \simeq \left(\frac{ck_\theta}{\omega_{pe}}\right)^2 v_{Te}^2 k_\|^2, \tag{10}$$

which indicates the electron transit mode and skin depth are intimately related. It is noted that the Debye screening factor $(k_\perp / k_{De})^2 \ll 1$ manifests itself in the dispersion relation even though charge neutrality holds.

II. REVIEW OF THE KINETIC BALLOONING MODE (KBM)

Since the electron ballooning mode to be discussed in this lecture is closely related to the kinetic ballooning mode that has been revealed in [9], we briefly review this mode first. In the analysis, ideal MHD approximation is avoided and the ion density perturbation is evaluated numerically according to the gyro-kinetic formula,

$$\begin{aligned} n_i &= -\frac{e\phi}{T_i} n_0 + \int \frac{\omega + \hat{\omega}_{*i}(v^2)}{\omega + \hat{\omega}_{Di}(v)} J_0^2(\Lambda_i) f_{Mi} d\mathbf{v} \frac{e\phi}{T_i} n_0 \\ &= -\frac{e\phi}{T_i} (1 - I_i) n_0, \end{aligned} \tag{11}$$

where

$$I_i = \int \frac{\omega + \hat{\omega}_{*i}(v^2)}{\omega + \hat{\omega}_{Di}(v)} J_0^2(\Lambda_i) f_{Mi} d\mathbf{v},$$

is the non-adiabatic part of the ion density perturbation with

$$\hat{\omega}_{*i}(v^2) = \omega_{*i}\left[1 + \eta_i\left(\frac{v^2}{v_{Ti}^2} - \frac{3}{2}\right)\right], \quad \omega_{*i} = \frac{cT_i}{eB^2} (\nabla \ln n_0 \times \mathbf{B}) \cdot \mathbf{k}, \tag{12}$$

$$\hat{\omega}_{Di}(\mathbf{v}) = \frac{Mc}{eB^3}\left(\frac{1}{2} v_\perp^2 + v_\|^2\right) (\nabla B \times \mathbf{B}) \cdot \mathbf{k}, \tag{13}$$

being the energy dependent ion diamagnetic drift frequency and velocity dependent ion magnetic drift frequency, respectively. Eq. (11) is subject to $\omega \gg k_\parallel v_{Ti}$ as appropriate to the ballooning mode which is essentially a destabilized Alfven mode.

For electrons, we assume $\omega \ll k_\parallel v_{Te}$. In this limit, perturbed electron density and parallel current are

$$\begin{aligned} n_e &= \int f_e d\mathbf{v} \\ &= \int \left(\phi - \frac{\omega - \hat{\omega}_{*e}\left(v^2\right)}{\omega - \hat{\omega}_{De}\left(\mathbf{v}\right) - k_\parallel v_\parallel}\left(\phi - \frac{v_\parallel}{c}A_\parallel\right)\right) f_{Me} d\mathbf{v}\frac{e}{T_e}n_0 \\ &\simeq \left(\phi - \frac{\omega - \omega_{*e}}{ck_\parallel}A_\parallel\right)\frac{e}{T_e}n_0, \end{aligned} \tag{14}$$

and

$$\begin{aligned} J_{\parallel e} &= -e\int v_\parallel \left[\phi - \frac{\omega - \hat{\omega}_{*e}\left(v^2\right)}{\omega - \hat{\omega}_{De}\left(\mathbf{v}\right) - k_\parallel v_\parallel}\left(\phi - \frac{v_\parallel}{c}A_\parallel\right)\right] f_{Me} d\mathbf{v}\frac{e}{T_e}n_0 \\ &\simeq \frac{n_0 e^2}{k_\parallel T_e}\left[\left(\omega_{*e} - \omega\right)\phi + \frac{\left(\omega - \omega_{*e}\right)\left(\omega - \omega_{De}\right) + \eta_e \omega_{*e}\omega_{De}}{ck_\parallel}A_\parallel\right]. \end{aligned} \tag{15}$$

Then from charge neutrality condition $n_i = n_e$ and parallel Ampere's law

$$\nabla_\perp^2 A_\parallel = -\frac{4\pi}{c}J_{\parallel e}, \tag{16}$$

we obtain the following mode equation,

$$c^2\frac{k_\parallel k_\perp^2 k_\parallel}{k_{De}^2} = -\frac{\left(\omega - \omega_{*e}\right)^2}{1 + \tau\left(1 - I_i\right)} + \left(\omega - \omega_{*e}\right)\left(\omega - \omega_{De}\right) + \eta_e \omega_{*e}\omega_{De}, \tag{17}$$

where

$$k_\parallel = -\frac{i}{qR}\frac{\partial}{\partial\theta}, \tag{18}$$

$$k_\perp^2 = k_\theta^2\left[1 + \left(s\theta - \alpha\sin\theta\right)^2\right], \tag{19}$$

$$\omega_{Dj}\left(\theta\right) = \frac{cT_j}{eBR}k_\theta\left[\cos\theta + \sin\theta\left(s\theta - \alpha\sin\theta\right)\right]. \tag{20}$$

The mode equation has been analyzed in [9] and the existence of the kinetic ballooning mode outside the stability boundary of the ideal ballooning mode demonstrated, including the regimes of the so-called second stability and negative shear.

In short wavelength limit $(k_\perp \rho_i)^2 \gg 1$, ions become adiabatic,

$$n_i = -\frac{e\phi}{T_i} n_0, \;\; I_i \to 0,$$

and the dispersion relation reduces to [8]

$$c^2 \frac{k_\parallel k_\perp^2 k_\parallel}{k_{De}^2} = -\frac{(\omega - \omega_{*e})^2}{1+\tau} + (\omega - \omega_{*e})(\omega - \omega_{De}) + \eta_e \omega_{*e} \omega_{De}. \tag{21}$$

Ions constitute neutralizing background in this case and instability, if any, is due to electron diamagnetic current coupled to the magnetic drift, *i.e.*, ballooning effect.

III. LOCAL ANALYSIS

In a uniform plasma $\omega_{*e} = \omega_{De} = 0$, the dispersion relation

$$(\omega - \omega_{*e})(\omega - \omega_{De}) + \eta_e \omega_{*e} \omega_{De} - \frac{(\omega - \omega_{*e})^2}{1+\tau} = \frac{c^2 k_\parallel k_\perp^2 k_\parallel}{k_{De}^2}, \tag{22}$$

reduces to the known form [10],

$$\omega^2 = \frac{1+\tau}{\tau} \left(\frac{c k_\parallel k_\perp}{k_{De}} \right)^2 = \left(\frac{c k_\perp}{\omega_{pe}} \right)^2 k_\parallel^2 \frac{T_e + T_i}{m_e}. \tag{23}$$

The conditions of adiabatic ions $(k_\perp \rho_i)^2 \gg 1$ and adiabatic electrons $\omega < k_\parallel v_{Te}$ impose the range in the cross field wavenumber $k_\perp$ such that

$$\frac{1}{\rho_i} < k_\perp \lesssim \frac{\omega_{pe}}{c}, \tag{24}$$

where c/ω_{pe} is the collisionless electron skin depth. This is possible if the plasma β factor exceeds the electron/ion mass ratio, $\beta \gg m_e/m_i \simeq 3 \times 10^{-4}$, which is well satisfied in tokamaks. However, the dispersion relation in Eq. (22) pertinent to tokamaks is not subject to $ck_\perp < \omega_{pe}$. The electron acoustic mode $\omega = k_\parallel \sqrt{(T_e + T_i)/m_e}$ and the skin depth c/ω_{pe} are intimately related in this mode and the skin depth naturally appears as a characteristic scale length. The mode can be destabilized in toroidal geometry through the ballooning effect or through electron Landau damping when α_e is subcritical.

The quadratic dispersion relation in Eq. (22) may be solved if the norm of the parallel gradient $k_\parallel$ is specified. As a rough estimate, we assume $k_\parallel \simeq 1/(2qR)$. Then the root is given by

$$\frac{\omega}{\omega_{*e}} = 1 - \frac{1+\tau}{2\tau}\left(1-2\varepsilon_n\right) + i\frac{\gamma}{\omega_{*e}}, \tag{25}$$

where γ/ω_{*e} is the normalized growth rate

$$\frac{\gamma}{\omega_{*e}} = \frac{1+\tau}{2\tau}\sqrt{\frac{4\tau}{1+\tau}\left(2\varepsilon_n\eta_e - \frac{\varepsilon_n\left(1+\eta_e\right)}{2\alpha_e}\right) - \left(1-2\varepsilon_n\right)^2}. \tag{26}$$

The condition for instability is given

$$\alpha_e\left[8\varepsilon_n\frac{\tau\eta_e}{1+\tau} - \left(1-2\varepsilon_n\right)^2\right] > 2\varepsilon_n\frac{\tau}{1+\tau}\left(1+\eta_e\right). \tag{27}$$

The source of instability is in the interchange drive term $\left(1+\eta_e\right)\omega_{*e}\omega_{De}$ due to the combination of unfavorable magnetic curvature and electron pressure gradient. The mode described by Eq. (22) may thus be called an electron ballooning mode. When compared with the ideal MHD ballooning mode symbolically described by

$$\omega\left(\omega+\omega_{*i}\right)\left(k_\perp\rho_i\right)^2 = \left(k_\perp\rho_i\right)^2\left(k_\parallel V_A\right)^2 - \left(1+\eta_e\right)\omega_{De}\omega_{*i} - \left(1+\eta_i\right)\omega_{Di}\omega_{*i}, \tag{28}$$

where V_A is the Alfven speed, the role of stabilizing Alfven frequency term $k_\perp\rho_i k_\parallel V_A$ in MHD ballooning mode is played by the modified electron transit frequency $\left(ck_\perp/\omega_{pe}\right)k_\parallel v_{Te}$. As is well known, the growth rate of the ideal MHD ballooning mode is practically independent of the ion finite Larmor radius parameter $k_\perp\rho_i$ since $\omega_{De}\omega_{*i} \propto \left(k_\perp\rho_i\right)^2$, while the growth rate of the electron ballooning mode is proportional to $k_\perp$.

The condition for the instability given in Eq. (27) is for hydrodynamic ballooning mode and may be relaxed if kinetic effects (electron kinetic resonance) are implemented. Fully kinetic analysis has revealed that the instability persists even in electrostatic limit although the growth rate is small [8]. The compressive magnetic perturbation $B_\parallel$ has little influence on the instability [11].

In the mode described by Eq. (23) for a uniform plasma, energy equipartition holds between the magnetic energy and thermal potential energy. They are out of phase and the sum of the two energy forms is constant, consistent with the general constraint on energy relationship in plasma waves [12]. The magnetic energy density associated with the wave is

$$\begin{aligned} U_m &= \frac{1}{8\pi}k_\perp^2 A_\parallel^2 = \frac{1}{8\pi}\frac{(1+\tau)^2 c^2 k_\parallel^2 k_\perp^2}{\omega^2}\phi^2 \\ &= \frac{1}{8\pi}\tau(1+\tau)k_{De}^2\phi^2 = \frac{1}{8\pi}(1+\tau)k_{Di}^2\phi^2, \end{aligned} \tag{29}$$

while the potential energy density is

$$\begin{aligned} U_p &= \frac{1}{2}n_0 T_i\left(\frac{n_i}{n_0}\right)^2 + \frac{1}{2}n_0 T_e\left(\frac{n_e}{n_0}\right)^2 \\ &= \frac{1}{8\pi}k_{Di}^2\phi^2 + \frac{1}{2}\frac{n_0 e^2}{T_e}\left(\phi - \frac{\omega}{ck_\parallel}A_\parallel\right)^2 \\ &= \frac{1}{8\pi}(1+\tau)k_{Di}^2\phi^2, \end{aligned} \tag{30}$$

in agreement with the magnetic energy density. Here the charge neutrality relationship $(1+\tau)ck_\parallel\phi = \omega A_\parallel$ has been substituted. It is noted that the dispersion relation is independent of electron and ion masses and thus no kinetic energy is involved in the wave described by Eq. (22).

IV. NONLOCAL ANALYSIS

In order to confirm destabilization of the mode by the ballooning effect in a more rigorous manner, a fully kinetic, electromagnetic integral equation code [13] has been employed to find the mode frequency and growth rate. We consider a high temperature, low β tokamak discharge with eccentric circular magnetic surfaces. Trapped electrons are ignored for simplicity. Also, the magnetosonic perturbation ($\mathbf{A}_\perp$) is ignored in light of the low β assumption and we employ the two-potential (ϕ and $A_\parallel$) approximation to describe electromagnetic modes. As in the preceding section, the basic field equations are the charge neutrality condition (subject to $k^2 \ll k_{De}^2$)

$$n_i(\phi, A_\parallel) = n_e(\phi, A_\parallel), \tag{31}$$

and the parallel Ampere's law,

$$\nabla_\perp^2 A_\parallel = -\frac{4\pi}{c}J_\parallel(\phi, A_\parallel), \tag{32}$$

where the density perturbations are given in terms of the perturbed velocity distribution functions f_i and f_e by

$$n_i = \int f_i d\mathbf{v}, \quad n_e = \int f_e d\mathbf{v}, \tag{33}$$

and the parallel current by

$$J_\parallel = e \int v_\parallel (f_i - f_e) d\mathbf{v}. \tag{34}$$

The perturbed distribution functions f_i and f_e can be found from the gyro-kinetic equation in the form

$$f_i = -\frac{e\phi}{T_i} f_{Mi} + g_i(v,\theta) J_0(\Lambda_i), \tag{35}$$

$$f_e = \frac{e\phi}{T_e} f_{Me} + g_e(v,\theta) J_0(\Lambda_e), \tag{36}$$

where $g_{i,e}$ are the nonadiabatic parts that satisfy

$$\left(i\frac{v_\parallel(\theta)}{qR}\frac{\partial}{\partial\theta} + \omega + \widehat{\omega}_{Di} \right) g_i = (\omega + \widehat{\omega}_{*i}) J_0(\Lambda_i) \left(\phi - \frac{v_\parallel}{c} A_\parallel \right) \frac{e}{T_i} f_{Mi}, \tag{37}$$

$$\left(i\frac{v_\parallel(\theta)}{qR}\frac{\partial}{\partial\theta} + \omega - \widehat{\omega}_{De} \right) g_e = -(\omega - \widehat{\omega}_{*e}) J_0(\Lambda_e) \left(\phi - \frac{v_\parallel}{c} A_\parallel \right) \frac{e}{T_e} f_{Me}. \tag{38}$$

Here, θ is the extended poloidal angle in the ballooning space, J_0 is the Bessel function with argument $\Lambda_{i,e} = k_\perp v_\perp / \omega_{ci,e}$,

$$k_\perp^2 = k_\theta^2 \left[1 + (s\theta - \alpha\sin\theta)^2 \right],$$

$$\widehat{\omega}_{Dj} = 2\varepsilon_n \omega_{*j} \left[\cos\theta + (s\theta - \alpha\sin\theta)\sin\theta \right] \left(\frac{1}{2}\hat{v}_\perp^2 + \hat{v}_\parallel^2 \right),$$

and qR is the connection length. For circulating particles, g_j $(j = i, e)$ can be integrated as

$$v_\parallel > 0, \qquad g_j^+ = -i\frac{e_j f_{Mj}}{T_j} \int_{-\infty}^{\theta} d\theta' \frac{qR}{|v_\parallel|} e^{i\beta_j} (\omega - \widehat{\omega}_{*j}) J_0(\Lambda_j') \left(\phi(\theta') - \frac{|v_\parallel|}{c} A_\parallel(\theta') \right), \tag{39}$$

$$v_\parallel < 0, \qquad g_j^- = -i\frac{e_j f_{Mj}}{T_j} \int_{\theta}^{\infty} d\theta' \frac{qR}{|v_\parallel|} e^{-i\beta_j} (\omega - \widehat{\omega}_{*j}) J_0(\Lambda_j') \left(\phi(\theta') + \frac{|v_\parallel|}{c} A_\parallel(\theta') \right), \tag{40}$$

where

$$\beta_j(\theta,\theta') = \int_{\theta'}^{\theta} \frac{qR}{|v_\parallel|} [\omega - \widehat{\omega}_{Dj}(\theta'')] d\theta''.$$

Substitution of perturbed distribution functions into charge neutrality and parallel Ampere's law yields

$$\nabla^2\phi = -4\pi \sum_{j=i,e} e_j \left(-\frac{e_j}{T_j}\phi + \int \left[g_j^+(\theta) + g_j^-(\theta)\right] J_0(\Lambda_j) d\mathbf{v} \right), \tag{41}$$

$$\nabla_\perp^2 A_\parallel(\theta) = -\frac{4\pi}{c} \sum_{j=i,e} e_j \int v_\parallel \left[g_j^+(\theta) - g_j^-(\theta)\right] J_0(\Lambda_j) d\mathbf{v}, \tag{42}$$

where $\int d\mathbf{v} = 2\pi \int_0^\infty v_\perp dv_\perp \int_0^\infty dv_\parallel$. This system of inhomogeneous integral equations can be solved by employing the method of Fredholm in which the integral equations are viewed as a system of linear algebraic equations [14]. In the numerical code, the velocity space integration is executed using Gauss-Hermite approximation.

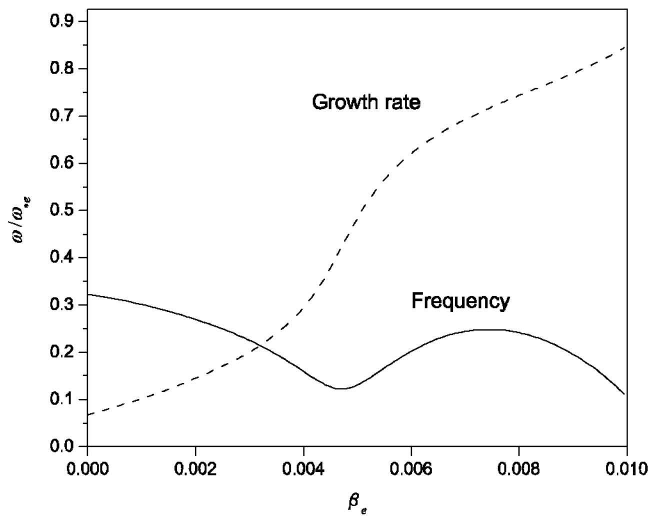

FIG. 1: The normalized mode frequency ω_r/ω_{*e} and growth rate γ/ω_{*e} as functions of the electron beta β_e when $ck_\theta/\omega_{pe} = 0.3$, $L_n/R = 0.2$, $\eta_e = 2$, $q = 2$, $s = 1$.

Figure 1 shows the β_e dependence of the normalized eigenvalue ω/ω_{*e} (frequency ω_r/ω_{*e} and growth rate γ/ω_{*e}) when $ck_\theta/\omega_{pe} = 0.3$, $T_e = T_i$, $L_n/R = 0.3$, $\eta_e = 2$, $s = 1$, $q = 2$, $m_i/m_e = 1836$ (hydrogen). The growth rate increases rapidly with the electron pressure (β_e) indicating that the instability is indeed driven by the ballooning effect. The growth rate found in the numerical analysis qualitatively agrees with the analytic expression presented in Eq. (26).

The mixing length electron thermal diffusivity $\chi_e = \gamma/k_\perp^2$ normalized by the Ohkawa diffusivity

$$\chi_{Ohkawa} = \frac{v_{Te}}{qR}\left(\frac{c}{\omega_{pe}}\right)^2, \tag{43}$$

is shown in Fig. 2 as a function of $k^2 = (ck_\theta/\omega_{pe})^2$ when $L_n/R = 0.2$, $\eta_e = 2$, $s = 1$, $q = 2$, and $\beta_e = 0.4$ %. The maximum diffusivity occurs at $ck_\perp/\omega_{pe} \simeq 0.3$.

In light of the analytic dispersion relation found in this study, it may be concluded that a tokamak discharge can be strongly unstable in the wavelength regime $k_\perp \simeq \omega_{pe}/c$ which is the lower end of unstable $k_\perp$ spectrum. The growth rate is of the order of $\gamma \simeq \sqrt{\eta_e\omega_{*e}\omega_{De}}$. Therefore, for the electron thermal diffusivity, the following estimate emerges,

$$\chi_e \simeq \frac{\sqrt{\eta_e\omega_{*e}\omega_{De}}}{k_\perp^2} = \frac{qv_{Te}}{\sqrt{RL_T}}\left(\frac{c}{\omega_{pe}}\right)^2\sqrt{\beta_e}, \tag{44}$$

where L_T is the temperature gradient scale length. The proportionality of the diffusivity to the safety factor, $\chi_e \propto q$, stems from the condition of most active thermal transport, $\gamma \simeq k_\parallel v_{Te}$, which yields $k_\perp \propto 1/q$ [15].

V. CONCLUSIONS

The local dispersion relation

$$(\omega - \omega_{*e})(\omega - \omega_{De}) + \eta_e\omega_{*e}\omega_{De} - \frac{(\omega - \omega_{*e})^2}{1+\tau} = \frac{c^2 k_\parallel k_\perp^2 k_\parallel}{k_{De}^2},$$

describes the short wavelength electron ballooning mode subject to the adiabatic electron response $\omega \lesssim k_\parallel v_{Te}$. The mode is intrinsically electromagnetic while the conventional ETG mode is subject to $\omega > k_\parallel v_{Te}$ for which electromagnetic effects appear only as a small correction. Because of the long wavelength nature of the instability ($c/\omega_{pe} \gg \rho_e$), large electron thermal transport emerges even in simple mixing length estimate. The following formula for the electron thermal diffusivity has been found,

$$\chi_e = \frac{qv_{Te}}{\sqrt{RL_T}}\left(\frac{c}{\omega_{pe}}\right)^2\sqrt{\beta_e}.$$

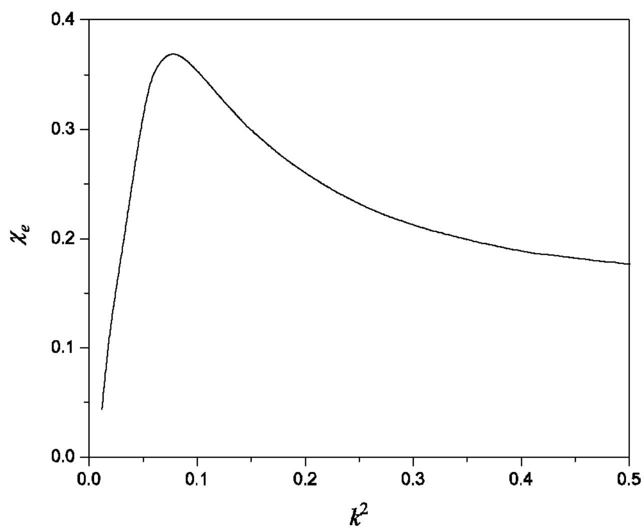

FIG. 2: χ_e/χ_{Ohkawa} vs. $k^2 = (ck_\theta/\omega_{pe})^2$ when $L_n/R = 0.2$, $\eta_e = 2$, $s = 1$, $q = 2$, and $\beta_e = 0.004$.

In the previous investigations [3], it was proposed skin size plasma turbulence may exist if the nonlinear Doppler shift $\mathbf{k} \cdot \mathbf{v}_{E\times B}$ ($\mathbf{v}_{E\times B}$ being the $E \times B$ drift) exceeds the electron transit frequency $\mathbf{k} \cdot \mathbf{v}_{E\times B} > k_\| v_{Te}$. The main finding in the present investigation is that the skin depth manifests itself even in the adiabatic limit and governs the lower end of the k spectrum of the ETG mode. It is noted that in the limit of large ballooning parameter α_e, the dispersion relation reduces to

$$(\omega - \omega_{*e})(\omega - \omega_{De}) + \eta_e \omega_{*e} \omega_{De} - \frac{(\omega - \omega_{*e})^2}{1+\tau} = 0,$$

which resembles that of the electrostatic ETG mode in the limit $\omega > k_\| v_{Te}$,

$$\tau - \frac{\omega_{*e} - \omega_{De}}{\omega - \omega_{De}} + \frac{\eta_e \omega_{*e} \omega_{De}}{(\omega - \omega_{De})^2} = 0.$$

In summary, an electromagnetic ballooning instability having cross-field wavelengths of the order of electron skin depth has been identified analytically and confirmed with an integral equation code which is fully kinetic and electromagnetic. In tokamaks, the mode is destabilized by a modest electron ballooning parameter α_e. A large electron thermal diffusivity emerges because of the long wavelength nature of the instability.

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