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# On Shear Flow Instabilities and its Applications.

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# ON SHEAR FLOW INSTABILITIES AND ITS APPLICATIONS

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# Plan Of The Talk

1. Classical Shear Flow Instability (1965)
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4. Drift Wave Instability
5. Comparison With Previous Works
6. SF Instabilities In EPI Plasmas
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# 1. Classical SF-Instability

- N.D' Angelo [1], pointed out an important low frequency shear flow instability in 1965, in plasmas

i). Without density gradient

ii). With density gradient

Using two fluid equations with $T_e = T_i$

Similar instabilities were studied by many authors including drift waves, Alfven waves etc. both with fluid and kinetic approaches.

- The basic dispersion relation of D' Angelo has been analyzed in the limit $\partial_t \sim \Omega_i$ as well in 2002 [2].
- Two important points are to be discussed here. Using fluid model

I. The contribution of the ion diamagnetic drift in the convective derivative of the polarization drift by the relevant collision-less Stress Tensor part is cancelled.

{ This was unknown at early times when SF instability was discussed (1965) and followed by other authors as in 1972 [3] }

II. A minor algebraic error in the calculations in Ref. [1] produced a physically incorrect result , in the case $\nabla n_{jo} \neq 0$.Our aim is to follow an algebraically correct way to obtain the linear dispersion relation for SF-Instabilities in the plasma with hot ions $(T_i \neq 0)$ using two fluid equations in both cases

i). $\nabla n_{0j} = 0$ ; ii). $\nabla n_{j0} \neq 0$

and to look at physics accordingly. The focus will be on the case

$$\left|\partial_t\right| << \Omega_i = \frac{eB_0}{m_i}$$

# 2. Basic Equations And The Model

Assumptions

i). $\mathrm{B}_0 = \mathrm{B}_0\mathbf{z}=$ Constant and $\mathbf{j}_0 = 0$

ii). $\mathbf{v}_{0j} = \upsilon_{j0}(\mathrm{x})\mathbf{z},$ $(\mathrm{j} = \mathrm{e}, \mathrm{i})$

iii). $\nabla \mathrm{n}_{j0} = -\mathbf{x}\left(\frac{d n_{j0}}{d x}\right)$

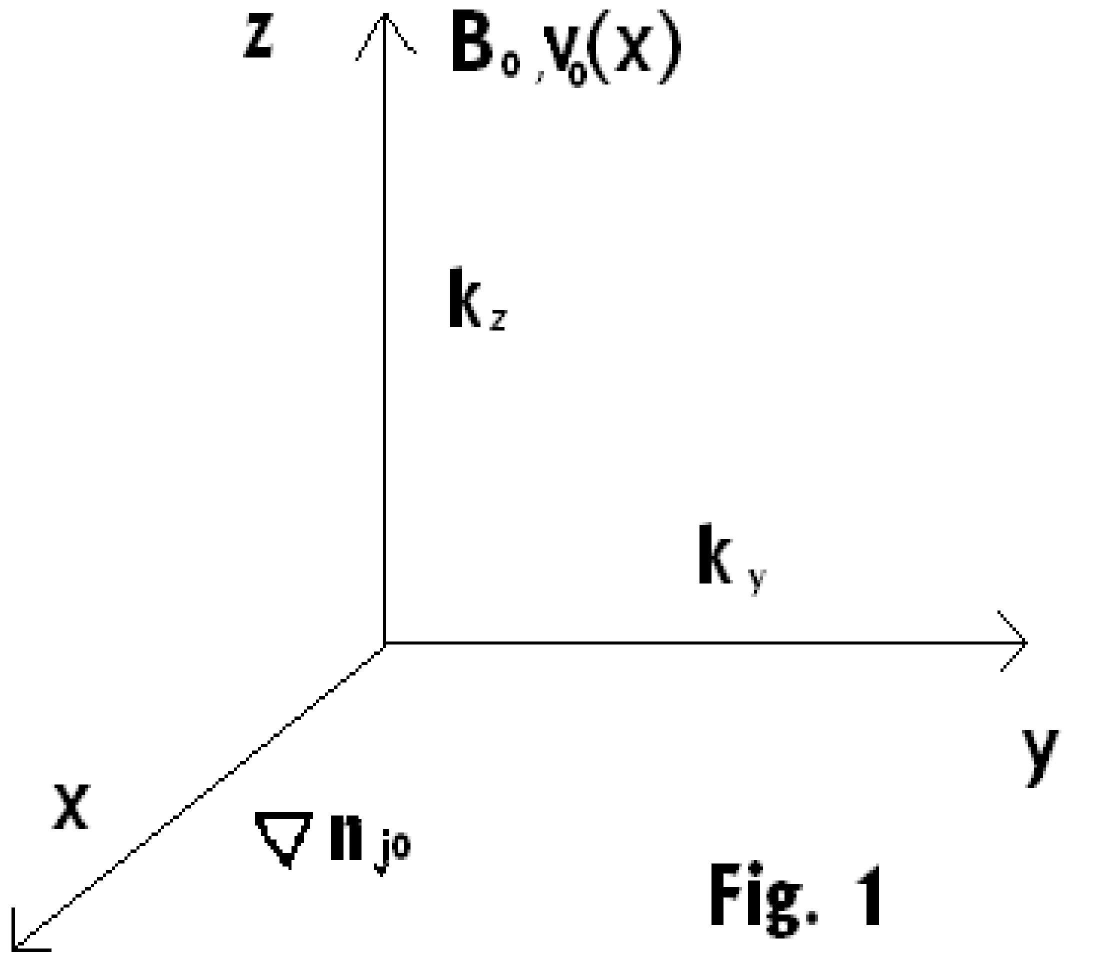


Fig. 1

iv). $T_j \neq 0$

v). $\mathbf{E} = -\nabla \varphi$

vi). $\mathbf{k} = \left(0, \mathbf{k}_y, \mathbf{k}_z\right); \dfrac{\mathbf{k}_z}{\mathbf{k}_y} << 1.$

Steady state demands,

$$\mathbf{v}_{0\perp j} = \mathbf{v}_{jD0} = -\left(\frac{T_j}{q_j B_0}\right)\nabla \ln n_{j0} \times \mathbf{z} \qquad (2.0)$$

Perturbed Perpendicular Velocities for

$\partial_t << \Omega_i = \dfrac{e\mathbf{B}_i}{m_i},$

$$\mathbf{V}_{\perp j} = \frac{1}{B_0}\left(\mathbf{E}_\perp \times \mathbf{z}\right) - \frac{T_j}{q_j B_0}\left(\nabla \ln n_j \times \mathbf{z}\right) - \frac{1}{q_j B_0}\left(\frac{\nabla . \overleftrightarrow{\Pi}_j}{n_j}\right)$$

$$- \frac{1}{\Omega_j}\left(\partial_t . \mathbf{V}_j . \nabla\right)\mathbf{V}_j \times \mathbf{z}$$

$$= \mathbf{v}_E + \mathbf{v}_{Dj} + \mathbf{v}_{\Pi j} + \mathbf{v}_{pj} \qquad (2.1)$$

Since $\nabla.\mathbf{v}_E = 0$, the continuity equation becomes

$$\partial_t n_j + \nabla n_{j0}.\mathbf{v}_E + \nabla_\perp.\left[n_j\left(\mathbf{v}_{pj} + \mathbf{v}_{\Pi j}\right)\right] + \partial_z\left(n_j \mathbf{v}_{jz}\right) = 0 \qquad (2.2)$$

In the linear limit,

$$\nabla.\left[n_j(\mathbf{v}_{pj} + \mathbf{v}_{\Pi j})\right]_1 = \nabla.\left[\frac{n_{j0}}{\Omega_j}\partial_t\left(\mathbf{z}\times\mathbf{v}_{j1}\right)\right] \qquad (2.3)$$

Eq. (2.3) is the same as that of Eq. (2.41) of Ref. [2]. In Eq. (2.3), substitute,

$$\mathbf{v}_{j1} = \mathbf{v}_{E1} + \mathbf{v}_{Dj1}$$

Then Eq. (2.3) becomes,

$$\nabla.\left[n_j\left(\mathbf{v}_{pj}+\mathbf{v}_{\Pi j}\right)\right]_1=\frac{n_{j0}}{B_0\Omega_j}\partial_t\left(\nabla_\perp.\mathbf{E}_{\perp i}\right)$$

(2.4)

$$-\frac{T_j}{q_jB_0\Omega_j}\partial_t\nabla_\perp^2 n_{ji}+\frac{n_{j0}}{\Omega_j}v_{jz0}(x)\partial_z$$

$$\left[\frac{1}{B_0}\nabla_\perp.\mathbf{E}_{\perp\mathbf{1}}-\frac{T_j}{q_jB_0}\frac{\nabla_\perp^2 n_{j1}}{n_{j0}}\right]$$

Using (2.4) in Continuity Eq. (2.2), we obtain

$$\left(\partial_t+v_{j0z}\partial_z\right)n_{j1}+\nabla n_{j0}.\mathbf{v}_{E1}+\frac{n_{j0}}{B_0\Omega_j}\left(\partial_t+v_{j0z}\partial_z\right)$$

$$\left(\nabla_\perp.\mathbf{E}_{\perp 1}\right)-\frac{T_j}{q_jB_0\Omega_j}\left(\partial_t+v_{j0z}\partial_z\right)\nabla_\perp^2 n_{j1}+n_{j0}\partial_z v_{jz1}=0$$

(2.5)

- Using the relation for hot ions,

$$-ik_z c_i^2 \rho_i^2 k_y^2 \left( \frac{e\varphi_1}{T_i} + \frac{n_{i1}}{n_{i0}} \right) + \left( \mathbf{v}_{iD0} . \nabla \right) v_{iz1} = -\frac{1}{m_i} \left( \frac{\nabla . \mathbf{\Pi}_i}{n_i} \right)_1 \qquad (2.6)$$

The parallel ion equation of motion becomes,

$$\left( \partial_t + v_{ioz} \partial_z \right) v_{iz1} + v_{ix1} \partial_x \upsilon_{i0z}(x) = \frac{q_i}{m_i} E_{z1} - \frac{T_{i0}}{m_i n_{i0}} \partial_z n_{i1} - ik_z c_i^2 \rho_i^2 k_y^2 \left( \frac{e\varphi_1}{T_i} + \frac{n_{i1}}{n_{i0}} \right)$$

where $c_i^2 = \frac{\gamma_i T_i}{m_i}; \rho_i^2 = \frac{c_i^2}{\Omega_i^2}.$ (2.7)

- Note: Here $\gamma_i$ is ratio of specific heats.

$$\left(\nabla.\Pi_i\right).\mathbf{z}=\frac{1}{\Omega_i}\Big\{\left(\nabla_\perp p_i\times\mathbf{z}\right).\nabla\upsilon_{iz}+$$

$$\nabla_\perp p_i.\partial_z\left(\mathbf{z}\times\mathbf{v}_{i\perp}\right)+p_i\mathbf{z}\partial_z.\left(-\nabla_\perp.\mathbf{v}_{i\perp}\right)\Big\}$$

- The linear equations (2.5) & (2.7) for ions can be written as,

$$\Omega_\omega^2\frac{n_{i1}}{n_{i0}}-\omega_e^*\Omega_\omega\Phi_1+ \qquad (2.8)$$

$$\left(\rho_s^2k_y^2\Phi_1+\rho_i^2k_y^2\frac{n_{i1}}{n_{i0}}\right)\Omega_\omega-k_zv_{iz1}=0$$

and

$$\Omega_{\omega} v_{iz1} + \left( \frac{1}{\Omega_i} \partial_x v_{i0z} \right) c_s^2 k_y \Phi_1 = \left(1 + \rho_i^2 k_y^2\right) \left( c_s^2 k_z \Phi_1 + c_i^2 k_z \frac{n_{i1}}{n_{i0}} \right) \quad (2.9)$$

where $\Omega_{\omega} = \left(\omega - \omega_{0z}\right); \omega_{0z} = \upsilon_{i0z} k_z;$

$$\omega_e^* = \mathbf{k.V}_{eD0} = \frac{T_c}{eB_0} \kappa_{en} k_y; \kappa_{en} = \left| \frac{1}{n_{e0}} \frac{dn_{e0}}{dx} \right|;$$

$$\rho_s^2 = c_s^2 / \Omega_i^2 ; c_s^2 = T_e / m_i ; \Phi_i = e\varphi_1 / T_e$$

. If $\mathbf{v}_{\mathbf{j0}} = 0$, we approximate (in drift wave regime)

$$\frac{n_{i1}}{n_{i0}} \simeq \frac{n_{e1}}{n_{e0}} \simeq \frac{\omega_e^*}{\omega} \Phi_1 \qquad \text{(A)}$$

when the hot ion case is considered. Then,

$$\nabla . \left[ n_i \left( v_{pi} + v_{\Pi i} \right) \right]_1$$

$$\simeq -i n_{i0} \rho_s^2 k_y^2 \left( \omega - \omega_i^* \right) \Phi_1 \qquad (2.10)$$

which is equation (2.43) of Ref. [2]. Then the electrostatic dispersion relation becomes,

$$\omega^2 - \omega_e^*\omega - c_s^2 k_z^2 = -\rho_s^2 k_y^2 \left(\omega - \omega_i^*\right)\omega \qquad (2.11)$$

It is important to stress that in the absence of the shear flow $\left(v_{j0} = 0\right)$, the electrostatic dispersion relations obtained in several previous studies including Ref. [1] do not yield Eq. (2.11) as a limiting case.

- The reason is that the cancellation related to the diamagnetic / polarization and stress tensor drifts has not been taken into account in those investigations

Let,

$$n_{e1} \simeq n_{e0} e^{e\varphi / T_e} \simeq n_{e0} \frac{e\varphi_1}{T_e} \qquad (2.12)$$

Eqs. (2.8), (2.9), (2.12) along with $n_{i1} \simeq n_{e1}$ yield,

$$\left(1+\rho_s^2 k_z^2+\rho_i^2 k_y^2\right)\Omega_\omega^2-\omega_e^*\Omega_\omega+A_i c_s^2 k_y k_z \qquad (2.13)$$

$$-\left(1+\rho_i^2 k_y^2\right)\left(c_s^2+c_i^2\right)k_z^2=0$$

where $$A_i = \left(\frac{dv_0}{dx}\right)\frac{1}{\Omega_i}$$

- Note : For $v_0 = 0$, Eq. (2.13) reduces to Eq. (2.11), when we use $\omega \frac{n_{i1}}{n_{i0}} \simeq \omega_e^* \Phi_1$ during the derivation of (2.10) used in (2.13). Note that in deriving (2.13) we replace

$$\frac{n_{i1}}{n_0} = \frac{n_{e1}}{n_0}$$

and use Boltzmann density distribution for $n_{e1}$ Eqs. (A) has not been used. So the Eqs. (II) is not clearly seen in Eqs. (2.13) as a limit. If Eqs. (A) is used as an approximation for $n_{i1}$ in (2.6) assuming that we are in drift wave regime, then hot ion contribution in the form of $\omega_i^*$ appears as on the rhs of (2.11).

- Let

$$g = \left(1 + \rho_s^2 k_y^2 + \rho_i^2 k_y^2\right) = h + \rho_s^2 k_y^2$$

$$h = \left(1 + \rho_i^2 k_y^2\right)$$

- Eq. (2.13) becomes,

$$g\,\Omega_\omega^2 - \omega_e^* \Omega_\omega + A_i c_s^2 k_y k_z - h\left(c_s^2 + c_i^2\right) k_z^2 = 0 \qquad (2.14)$$

with roots,

$$\Omega_{\omega} = \frac{1}{2g}\left\{\omega_e^{*} \pm \left[\omega_e^{*2} - 4g\left(A_i c_s^2 k_y k_z - h\left(c_s^2 + c_i^2\right)k_z^2\right)\right]^{\frac{1}{2}}\right\} \qquad (2.15)$$

The instability criterion becomes,

$$\omega_e^{*2} < 4g\left(A_i c_s^2 k_y k_z - h(c_s^2 + c_i^2)k_z^2\right) \qquad (2.16)$$

In the homogeneous limit, the condition for purely growing instability becomes,

$$h\left(1 + \frac{T_i}{T_e}\right)\frac{k_z}{k_y} < A_i \qquad (2.17)$$

The instability condition is modified mainly due to $\rho_i^2 k_y^2$- term in h.

# 3. Role Of The Collision-Less ion Stress Tensor

- For $T_i \neq 0$,
- We note that $\left(\mathbf{v}_{D0i}.\nabla\right)\mathbf{v}_i -$ term cancels with a part of collision-less ion stress tensor.
- Therefore, ion drift frequency in the linear dispersion relation does not appear because of $\left(\partial_t + \mathbf{v}_{D0i}.\nabla\right)\mathbf{v}_i -$ term. In the basic paper on shear flow instability (D. Angelo mode 1965), the mathematical treatment does not take into account this fact. The reason is that, this cancellation (probability) was not known.

# 4. Drift Wave Instability

- For $T_i << T_e$, Eq. (2.13) becomes ,

$$\left(1 + \rho_s^2 k_y^2\right)\Omega_\omega^2 - \omega_e^* \Omega_\omega + A_i c_s^2 k_y k_z - c_s^2 k_z^2 = 0 \qquad (4.1)$$

$$\Omega_\omega = \frac{1}{2a}\left\{\omega_e^* \pm \left[\omega_e^{*2} - 4a \left(A_i c_s^2 k_y k_z - c_s^2 k_z^2\right)\right]^{1/2}\right\} \qquad (4.2)$$

$a = \left(1 + \rho_s^2 k_y^2\right)$. Here $\Omega_\omega$ is Doppler shifted frequency. Eq. 4.2 gives,

$$\omega = \omega_{0z} + \frac{1}{2a}\{\omega_e^* \pm [\omega_e^{*2} - 4a(A_i c_s^2 k_y k_z - c_s^2 k_z^2)]^{1/2}\} \quad (4.3)$$

Shear flow drift wave (SFDW) instability condition becomes

$$\omega_e^{*2} < 4a\left(A_i c_s^2 k_y k_z - c_s^2 k_z^2\right) \quad (4.4)$$

With real frequency

$$\omega_\gamma = \left(\omega_{0z} + \frac{\omega_e^*}{2a}\right) \quad (4.5)$$

In the homogeneous case the instability condition for purely growing mode is $\frac{k_y}{k_z} < A_i$

# 5. Comparison with Previous Works

For $T_e = T_i$ the dispersion relation (2.13) becomes,

$$\left(1 + 2\rho_i^2 k_y^2\right)\Omega_\omega^2 - \omega_e^* \Omega_\omega + A_i c_i^2 k_z k_y - \left(1 + \rho_i^2 k_y^2\right) 2 c_i^2 k_z^2 = 0 \qquad (5.1)$$

where,

$$\Omega_\omega = \left(\omega - \omega_{0z}\right).$$

The equivalent dispersion relation in Ref. [1] is

$$\Omega^2 - 2\left(\kappa_{ne}\rho_i\right)\left(c_i k_y\right)\Omega + 2A_i c_i^2 k_z k_y - 2c_i^2 k_z^2 = 0 \qquad (5.2)$$

- where, $\Omega = \left(\omega - \upsilon_{D0i} k_y - \omega_{0z}\right)$.

- The Eq. 48 of Ref. [3] , similar to (5.2) is

$$\Omega^2 - \left(\omega_e^* + \omega_i^*\right)\Omega + A_i c_s^2 k_z k_y - 2c_i^2 k_z^2 = 0 \qquad (5.3)$$

where $c_s^2 = \dfrac{\left(\gamma_i T_i + \gamma_e T_e\right)}{m_i} = \dfrac{2T_i}{m_i} = c_i^2$ has been used for the isothermal case .

Note:

I. Both (5.2) & (5.3) are different from (5.1), and both can not reduce to (2.11) in the limit

$$v_{0z} = 0$$

II. Even in the homogeneous limit $\left(\nabla n_{0j} = 0\right)$;

our result is different from Ref. [1].

The roots of (5.1) are

$$\Omega_\omega = \frac{1}{2p}\{\omega_e^* \pm [\omega_e^{*2} - 4p(A_i c_i^2 k_z k_y - (1+\rho_i^2 k_y^2) 2 c_i^2 k_z^2]^{1/2}\} \tag{5.4}$$

The instability condition for inhomogeneous case is

$$\omega_e^{*2} < 4p[A_i c_i^2 k_z k_y - (1+\rho_i^2 k_y^2) 2 c_i^2 k_z^2] \tag{5.5}$$

where,

$$p = (1 + 2\rho_i^2 k_y^2).$$

This is basically the drift wave instability due to shear flow and it was not clear in Ref. [1]. In case of homogeneous plasma, there exists a purely growing shear flow instability in the limit,

$$2\left(1+\rho_i^2 k_y^2\right)\frac{k_z}{k_y} < A_i \qquad (5.6)$$

Note that (5.6) is slightly different from the original work of Ref. [1]. Now a minor error is pointed out. in the derivations after the set of equations (11) of Ref. [1], the term

$$d(N_1/n_0)/dx \equiv [dN_1(x)/dx]/n_0 + \lambda N_1/n_0$$

has been omitted. Here, $N_1$ is the x-dependent mode amplitude and

$$n_0(x) = N_0 \exp(-\lambda x).$$

In the local approximation, the first term may be omitted but the second one should be kept as in the other steps in the derivations. Using the notation from Ref. [1], this will give in the dispersion equation

$$\varsigma^2 - 3\Lambda\beta\varsigma - 2\gamma(\gamma - a\beta) = 0,$$

- which has a factor 3 in the second term instead of the factor 2 in Ref. [1], Here

$$\varsigma = (\omega - k_y v_{0y} - k_z v_{0z}) / \Omega_i .$$

$$\gamma = k_z \rho_i$$

$$\Lambda = \lambda \rho_i = \kappa_n \rho_i$$

$$\beta = k_y \rho_i$$

- Although the instability condition is modified to read

$$A > (k_z / k_y)[1 + 9k_y^2 \rho_i^2 \lambda^2 / \left(8k_z^2\right)]$$

instead of

$$A > (k_z / k_y)[1 + 9k_y^2 \rho_i^2 \lambda^2 / \left(2k_z^2\right)]$$

as given in Ref. [1], the correction is not only the matter of numbers. More importantly, using the equilibrium conditions the dispersion equation in Ref. [1], should yield the real part of the frequency

$$\omega_r = -k_y \upsilon_{y0} / 2 + k_z \upsilon_{z0}$$

$$+[3e/(2m_i \Omega_i)](d\varphi_0 / dx).$$

- The first term here, which survives even in the absence of equilibrium potential $\varphi_0$ indicates the presence of drift mode, and it is missing in Eq. (18) of Ref. [1]. But the expression given above is still different compared to our result, which is due to the cancellation of the terms, explained in the previous text. Note that in the later work [3] $\omega_e^*$ and $\omega_i^*$ appear in dispersion relation [Eq. (48) of this reference], but this expression does not reduce to Eq. (11) either.

# 6. SF Instability in EPI Plasmas

- The ion equations remain the same as in case of the EI Plasma.

  Electrons and positrons follow Boltzmann distributions

$$n_{e1} \simeq n_{e0} e^{e\varphi 1 / T_e} \simeq n_{e0} \frac{e\varphi_1}{T_e} \qquad (6.1)$$

and

$$n_{p1} = n_{p0} e^{-e\varphi 1 / T_{ip}} \simeq -n_{p0} \frac{e\varphi_1}{T_p} \qquad (6.2)$$

Then poisson eq. yields

$$\frac{n_{i1}}{n_{i0}} \simeq \eta \Phi_1 \qquad (6.3)$$

where $\Phi_1 = \dfrac{e\varphi_1}{T_e}, \eta = (1 + \sigma + \lambda_{De}^2 k^2)\dfrac{n_{0e}}{n_{i0}},$

$$\sigma = {n_{p0}T_e}\big/{n_{e0}T_p}, \lambda_{De}^2 = \frac{\varepsilon_0 T_e}{n_{e0}e^2}$$

Then using (6.3) in the ion continuity equation (2.8) along with ion parallel eq. (2.9), we obtain

$$\lambda\Omega_\omega^2 - \omega_e^*\Omega_\omega + A_i c_s^2 k_y k_z \qquad (6.4)$$

$$-(1 + \rho_i^2 k_y^2)(c_s^2 + \eta c_i^2)k_z^2 = 0$$

where $\lambda = (1+\rho_i^2 k_y^2)\eta + \rho_s^2 k_y^2$

Note in Eq. (6.4),

$$\omega_e^* = \frac{T_e}{eB_0}\frac{1}{n_{i0}}\frac{dn_{i0}}{dx}$$

and

$$n_{i0} = (n_{e0} - n_{p0}) \qquad (6.5)$$

For $n_{p0} = 0$, and $\lambda_{D0}^2 k^2 << 1$, we have $\sigma = 0$ and hence $\eta = 1$ which is the EI case.

The instability criteria for homogeneous and inhomogeneous plasmas are similar to EI-case

Note: Our is different from of Ref. [1] as estimated above in section 5.

- We find

$$\omega = \omega_{0z} + \frac{1}{2\lambda}\{\omega_e^* \pm [\omega_e^{*2} - 4\lambda(A_i c_s^2 k_y k_z - (1+\rho_i^2 k_y^2)(c_s^2 + \eta c_i^2)k_z^2]^{\frac{1}{2}}\}. \quad (6.6)$$

The instability condition in this case reads

$$A_i > \frac{\omega_e^{*2}}{4\lambda c_s^2 k_y k_z} + (1+\rho_i^2 k_y^2)(1+\eta\frac{\gamma_i T_i}{T_e})\frac{k_z}{k_y}, \quad (6.7)$$

For homogeneous plasmas it becomes

$$A_i > (1+\rho_i^2 k_y^2)(1+\eta\frac{\gamma_i T_i}{T_e})\frac{k_z}{k_y} \quad (6.8)$$

we conclude that the presence of positrons has a stabilizing effect.

# 7. SF Instability in Dusty Plasmas

- Let us assume now that apart from electrons and ions, the plasma contains negatively charged dust grains as the third component, which take the role of ions from the previous case. Then the equilibrium demands

$$n_{i0} = n_{e0} + z_d n_{d0} \qquad (7.1)$$

where $j = e, i, d$ and $q_d = -ez_d$ . Since $m_i << m_d$ therefore we assume,

$$n_{i1} \simeq n_{i0} e^{-e\varphi_1 / T_i} \simeq -n_{0i} \frac{e\varphi_1}{T_i} \qquad (7.2)$$

- Electrons also follow Boltzmann density distribution then Poisson Eq. Yields

$$\frac{n_{d1}}{n_{d0}} = -\mu\Phi_1 \qquad (7.3)$$

where $\mu = 1 + \lambda_{De}^2 k^2, \Phi_1 = ez_d\varphi_1 / T_{eff}$,

$T_{eff} = z_d n_{0d} T_i T_e / (n_{0e} T_i + n_{0i} T_e)$,

and $\lambda_{De}^2 = \varepsilon_0 T_{eff} / (n_{d0} e^2 z_d^2)$.

The dust continuity equation can be written as

$$\Omega_\omega n_{d1} + n_{d0}\omega_d^* \Phi_1 - \frac{n_{d0}}{B_0\Omega_d}\Omega_\omega k_y^2 \frac{T_{eff}}{ez_d}\Phi_1 + \frac{T_d k_y^2}{ez_d B_0 \Omega_d}\Omega_\omega n_{d1} - n_{d0} k_z \upsilon_{dz1} = 0 \qquad (7.4)$$

where $\Omega_{\omega} = \omega - \upsilon_{0dz} k_z , \omega_d^* = T_{eff} k_{nd} k_y /(e z_d B_0) > 0$
and $\kappa_{nd} = dn_{d0} /(n_{d0} dx)$.

The parallel component of the equation of motion gives

$$\Omega_{\omega} \upsilon_{dz1} + \frac{T_{eff}}{e z_d B_0} k_y (d_x \upsilon_{dx0}) \Phi_1 = \qquad (7.5)$$

$$(1 + \rho_d^2 k_y^2)(-\frac{T_{eff}}{m_d} k_z \Phi_1 + \frac{T_{d0}}{m_d} k_z \frac{n_{d1}}{n_{d0}})$$

The linear dispersion relation can be written as

$$\chi \Omega_{\omega}^2 - \omega_d^* \Omega_{\omega} - c_d^2 k_z k_y A_d - (1 + \rho_d^2 k_y^2) \qquad (7.6)$$

$$(1 + \mu \frac{T_{d0}}{T_{eff}}) c_d^2 k_z^2 = 0$$

where

$$\chi = \mu + \rho_d^2 k_y^2 (1 + \mu \frac{T_{d0}}{T_{eff}}), \ A_d = \frac{1}{\Omega_d} \frac{dv_{d0}}{dx},$$

$$c_d^2 = \frac{T_{eff}}{m_d}, \quad \text{and} \quad \rho_d = \frac{c_d}{\Omega_d}.$$

Eq. (7.6) has the roots

$$\omega = \omega_{0z} + \frac{1}{2\chi}[\omega_d^* \pm \{\omega_d^{*2} + 4\chi(c_d^2 k_z k_y A_d \qquad (7.7)$$

$$+(1 + \rho_d^2 k_y^2)(1 + \mu \frac{T_{d0}}{T_{eff}}) c_d^2 k_z^2)\}^{1/2}]$$

The instability sets in provided that $A_d < 0$, and

$$\left| A_d \right| > (1 + \rho_d^2 k_y^2)(1 + \mu \frac{T_{d0}}{T_{eff}}) c_d^2 k_z^2 \qquad (7.8)$$

Thus the dust drift wave can become unstable if the shear flow gradient is negative.

Note that SF instability for EI , and EPI plasmas appear in the case of positive shear flow $0 < A_i$.

# 8. Conclusions [4]

1. The shear flow instability has been revisited taking into account the effects of the cancellation of the zeroth-order diamagnetic drift term in the linear polarization drift against a part of the collision-less stress tensor. This cancellation perhaps was unknown at the time when the shear flow instability was discussed for the first time in Ref. [1]. It was a real pioneering work and it was followed by many authors in subsequent years.

2. The dispersion relation Eq. (18) of Ref. [1] needs to be corrected to understand the physical mechanism in detail.

3. It is also necessary to note that in the limit $T_i \neq 0$ the contribution of Eq. (2.10) can not be neglected even in the case of homogeneous plasma. The factor $\rho_i^2 k_y^2$ appears in the dispersion relations and in the instability conditions as well.

4. Furthermore, the shear flow case has been discussed for the electron-positron-ion and dusty plasmas.

5. In case of the dust drift wave coupled with dust acoustic wave the instability can appear if the shear flow is negative i.e. $A_d < 0$.

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