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Turbulence in Interstellar Medium - Part 3

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Turbulence in Interstellar Medium

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International Center of Theoretical Physics
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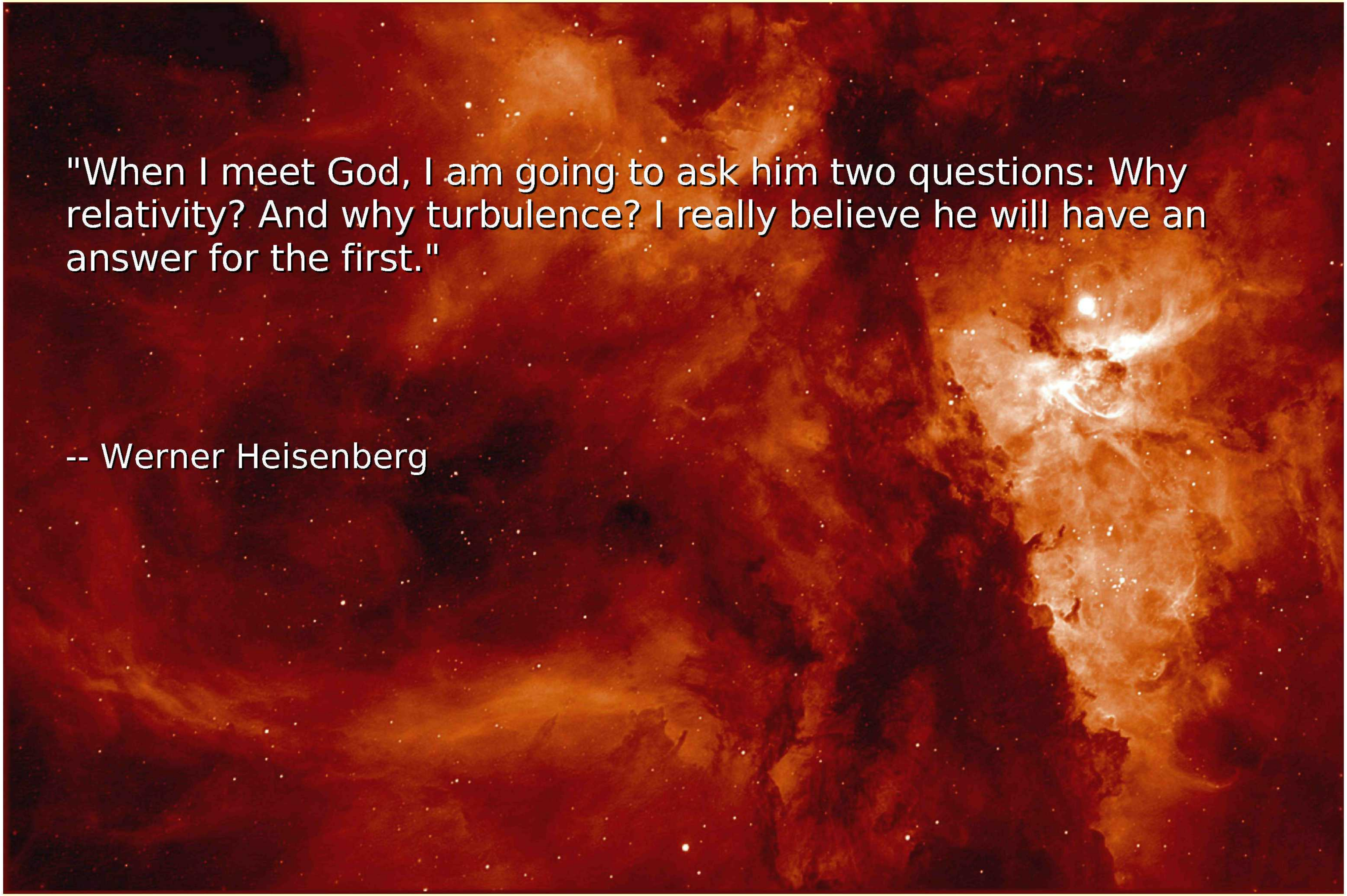
"When I meet God, I am going to ask him two questions: Why relativity? And why turbulence? I really believe he will have an answer for the first."

-- Werner Heisenberg



Src: Nebula in our galaxy

Turbulence – General Definition

In fluid dynamics, turbulence or turbulent flow is a flow regime characterized by chaotic, stochastic property changes.

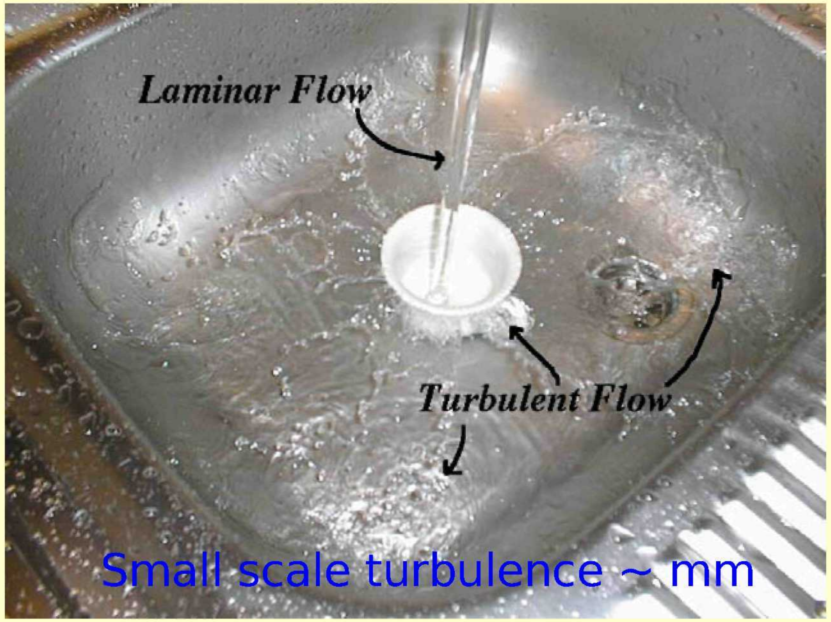
Random in space, time and upredictable.

This includes low momentum diffusion, high momentum convection, and rapid variation of pressure and velocity in space and time.

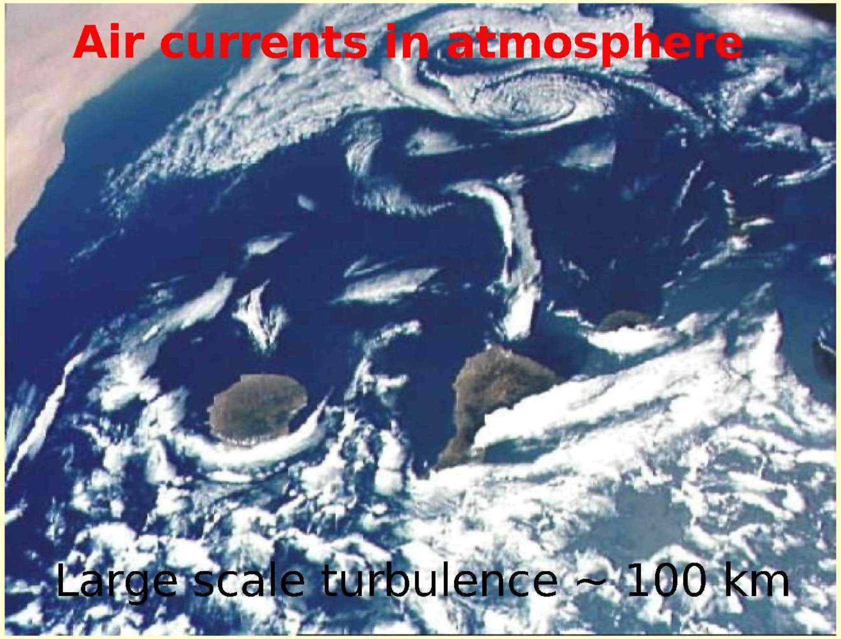
Flow that is not turbulent is called laminar flow.

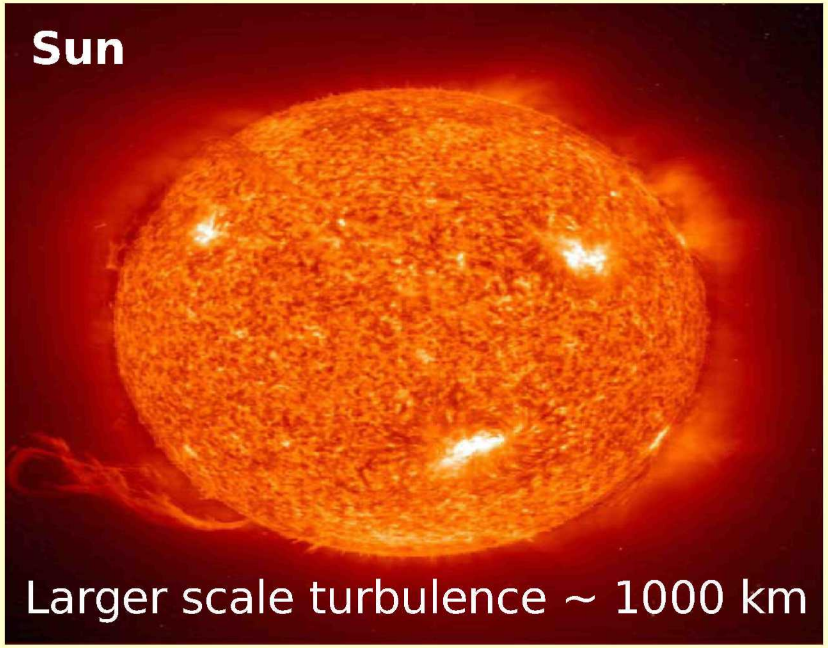
Source - wikipedia

Turbulence – Examples

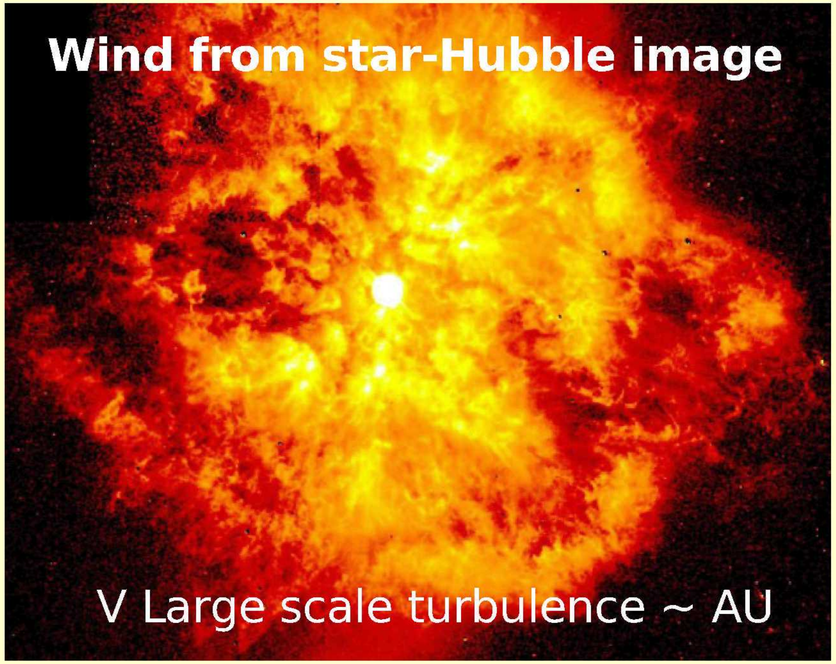


HD

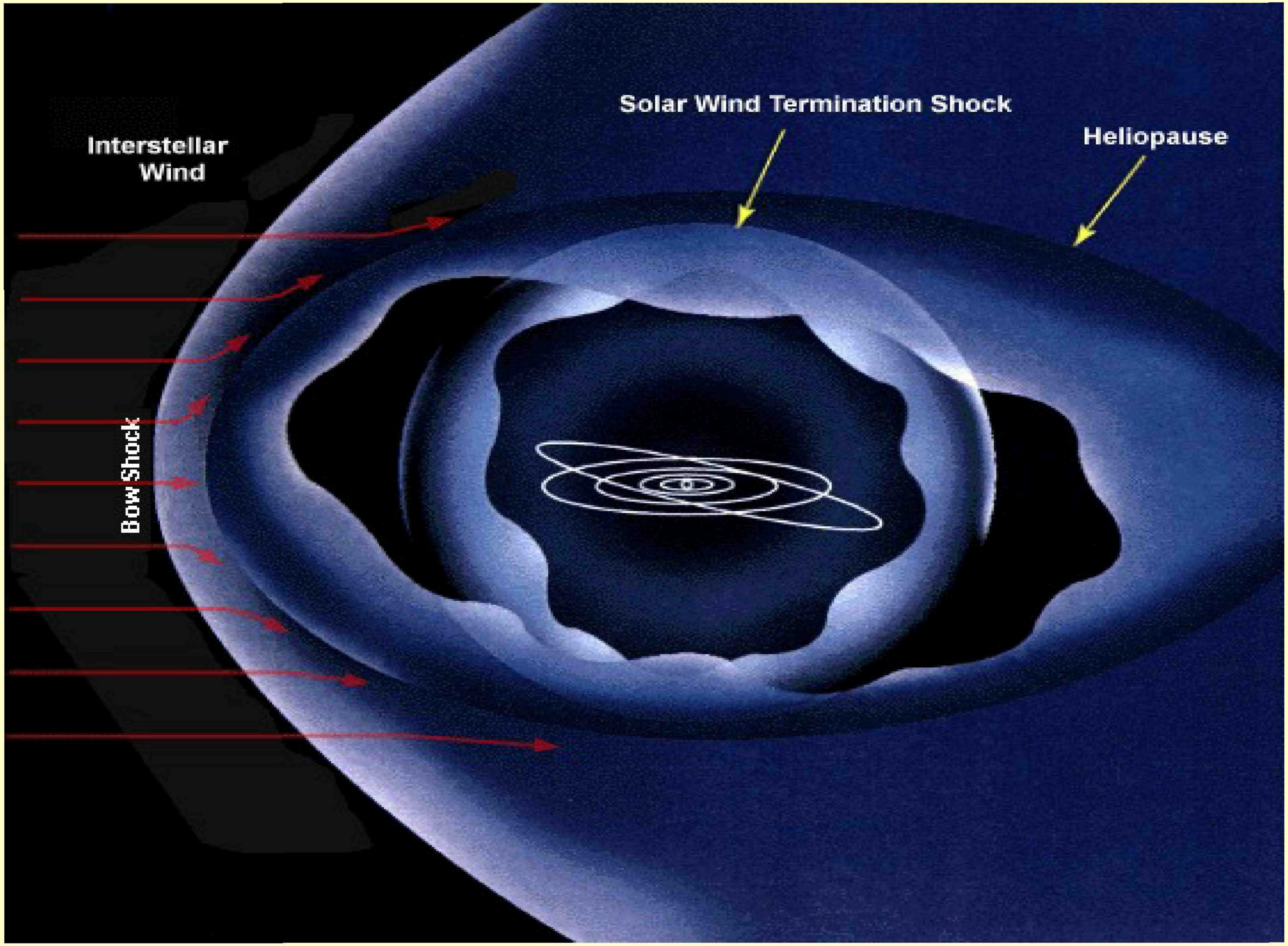




MHD

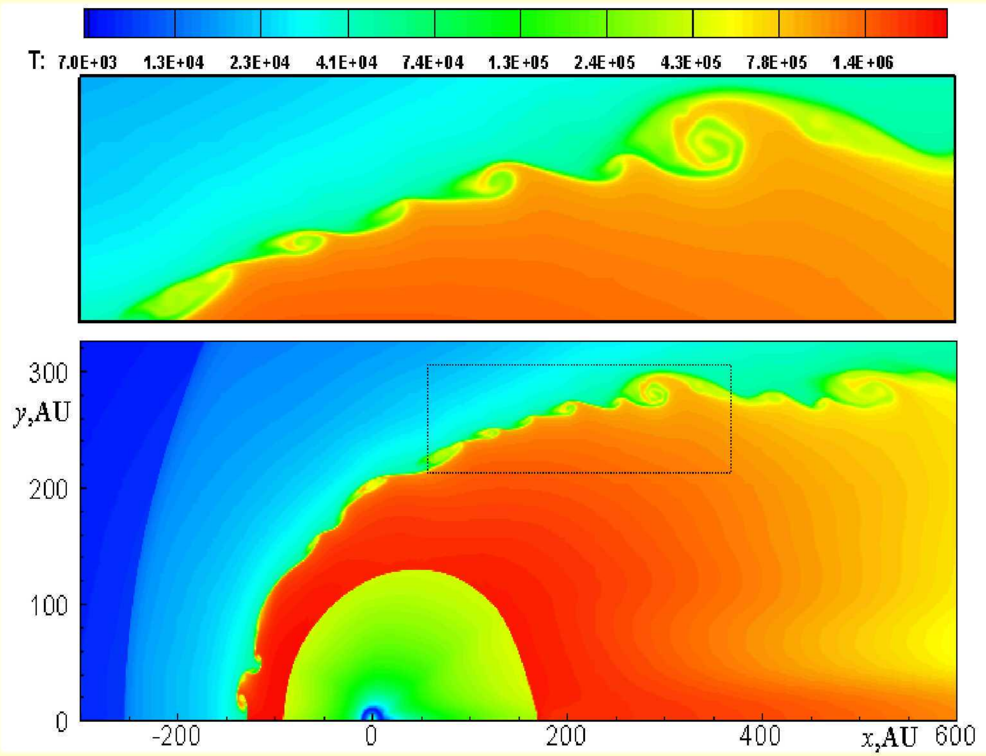


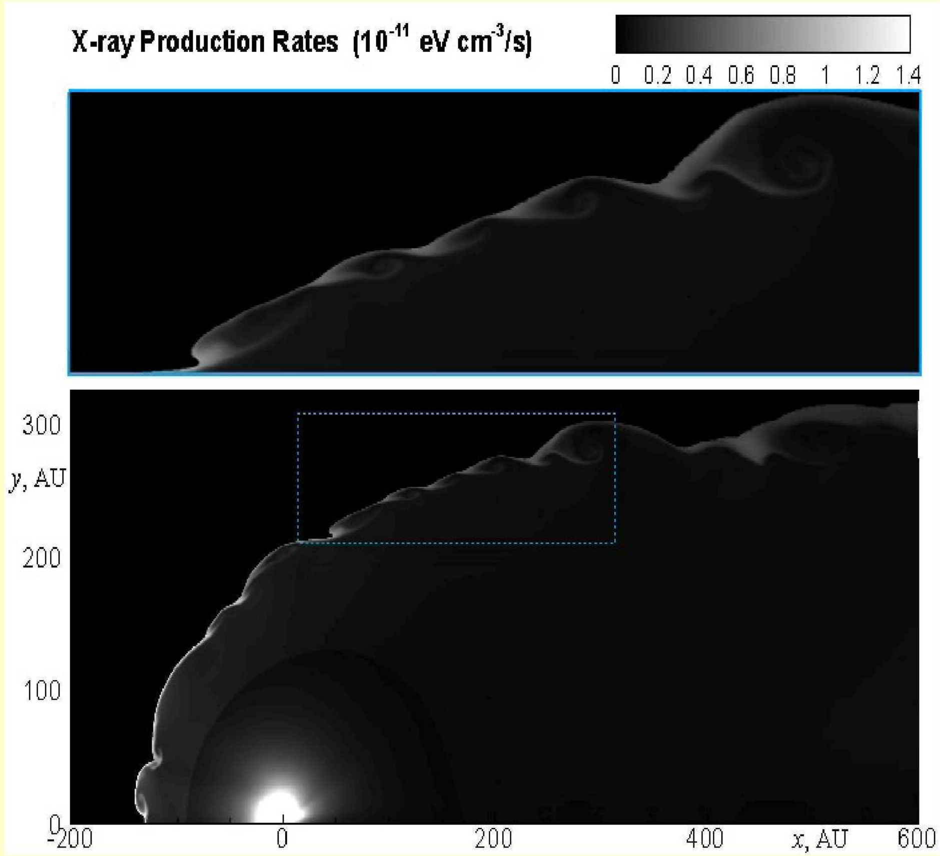
Solar Wind Interaction with Interstellar Medium – A Turbulence Perspective



src:NASA, GSFC

Observation and Modeling of KH Instability





Nonlinear development of this instability can lead to small scale heliospheric turbulence. (Pogorelov et al 2006, 07).

Why fluid becomes turbulent?

Hydrodynamic fluid momentum equation (Navier-Stokes eqn)

$$\rho\left(\frac{\partial}{\partial t}+\boldsymbol{V}\cdot\nabla\right)\boldsymbol{V}=-\nabla p+\mu\nabla^{2}\boldsymbol{V}$$

Nonlinear term

Viscous term

$$R_e=\frac{\boldsymbol{V}\cdot\nabla\boldsymbol{V}}{\mu\nabla^{2}\boldsymbol{V}}=\frac{v^{2}/L}{\mu v/L^{2}}=\frac{vL}{\mu}\gg 1$$

$$R_e=\frac{Inertial\ Forces}{Viscous\ Forces}$$

The (dimensionless) Reynolds number characterizes whether flow conditions lead to laminar or turbulent flow;

e.g. for pipe flow, a Reynolds number above about 2300 will be turbulent.

For Magnetohydrodynamic (MHD) flows, additional nonlinearity JXB arises.
It leads to a magnetic Reynolds number.

Turbulent Fluid

Turbulence causes the formation of eddies of many different length scales.

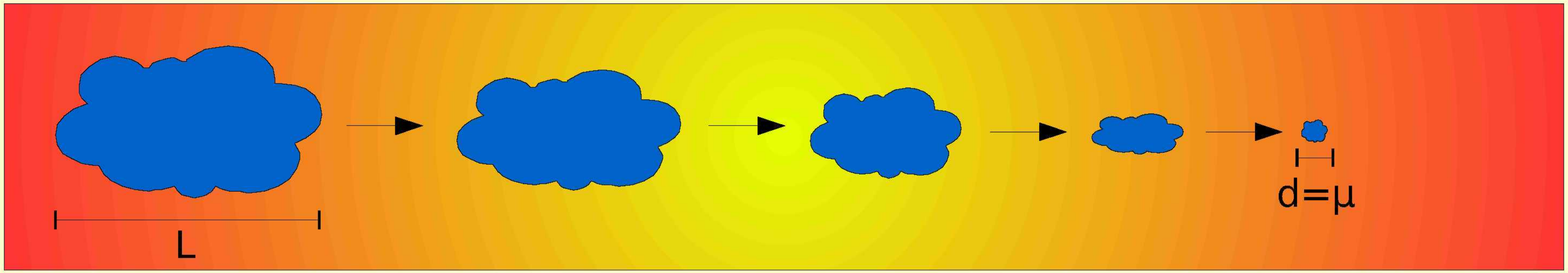
Most of the kinetic energy of the turbulent motion is contained in the large scale structures.

The energy "cascades" from these large scale structures to smaller scale structures by an inertial and essentially inviscid mechanism.

This process continues, creating smaller and smaller structures which produces a hierarchy of eddies.

Eventually this process creates structures that are small enough that molecular diffusion becomes important and viscous dissipation of energy finally takes place.

Great Orion Nabula



Turbulent Fluid

It is possible to find some particular solutions of the Navier-Stokes equations governing fluid motion.

All such solutions are unstable at large Reynolds numbers.

Sensitive dependence on the initial and boundary conditions makes fluid flow irregular both in time and in space

So that a statistical description is needed.

Self-similarity is broken so the statistical description is modified.

Still, the complete description of turbulence remains one of the unsolved problems in physics.

"When I meet God, I am going to ask him two questions: Why relativity? And why turbulence? I really believe he will have an answer for the first." -- Heisenberg

Properties of an Ideal Turbulent Fluid

Since turbulence processes deal with random, unsteady nonlinear fluctuations, one can study their statistical properties

Turbulence can be driven by instabilities (KH, RT, Shear, Drift etc)

Isotropic

the products and squares of the velocity components and their derivatives are independent of direction, or, more precisely, invariant with respect to rotation and reflection of the coordinate axes in a coordinate system moving with the mean motion of the fluid.

E.g;

$$k_x \simeq k_y \simeq k_z ; ik_x = \partial / \partial x$$

Homogeneous

Turbulent flows whose statistical properties are invariant under translation in space and/or time,

correlations are invariant, e.g

$$\langle U(x,t) \rangle = \langle U(x+L, t+\tau) \rangle$$

Random

Mean of fluctuations is zero

$$\langle U(x,t) \rangle = 0$$

$$\langle U(x,t) \cdot U(x+L, t+\tau) \rangle = 0$$

Cross correlation is zero.

Prob. Distr. Is Gaussian.

Note: These properties can be changed in the presence of mean or large scale flows.

Statistical Properties leads one to study many features of a hydro/MHD turbulent fluid

Fluid equations for Hydro fluid

$$\frac{\partial \rho_n}{\partial t} + \nabla\cdot(\rho_n \boldsymbol{U}_n) = 0$$

$$\rho_n \left(\frac{\partial}{\partial t} + \boldsymbol{U}_n \cdot \nabla \right) \boldsymbol{U}_n = -\nabla P_n$$

$$\frac{\partial}{\partial t} \left(\frac{1}{2} \rho_n \boldsymbol{U}_n^2 + \frac{P_n}{\gamma - 1} \right) + \nabla\cdot \left(\frac{1}{2} \rho U_n^2 \boldsymbol{U}_n + \frac{\gamma}{\gamma - 1} \frac{P_n}{\rho_n} \rho_n \boldsymbol{U}_n \right) = 0$$

The MHD model of plasma

$$\frac{\partial \rho_p}{\partial t} + \nabla\cdot(\rho_p \boldsymbol{U}_p) = 0$$

$$\rho_p \left(\frac{\partial}{\partial t} + \boldsymbol{U}_p \cdot \nabla \right) \boldsymbol{U}_p = -\nabla P_p + \frac{1}{4\pi} \boldsymbol{J} \times \boldsymbol{B}$$

$$\frac{\partial \boldsymbol{B}}{\partial t} = \nabla \times (\boldsymbol{U}_p \times \boldsymbol{B})$$

$$\frac{\partial}{\partial t} \left(\frac{1}{2} \rho_p \boldsymbol{U}_p^2 + \frac{P_p}{\gamma - 1} + \frac{B^2}{8\pi} \right) + \nabla\cdot \left(\frac{1}{2} \rho_p U_p^2 \boldsymbol{U}_p + \frac{\gamma}{\gamma - 1} \frac{P_p}{\rho_p} \rho_p \boldsymbol{U}_p - (\boldsymbol{U}_p \times \boldsymbol{B}) \times \boldsymbol{B} \right) = 0$$

$$\nabla\cdot \boldsymbol{B} = 0$$

Above set of equations are fully compressible. For simplicity, let us look at the incompressible equations. $\rho_p, \rho_n = const, \quad \nabla\cdot \boldsymbol{U}_n = 0, \nabla\cdot \boldsymbol{U}_p = 0$

$$\left(\frac{\partial}{\partial t} + \boldsymbol{U}_n \cdot \nabla \right) \boldsymbol{U}_n = -\frac{1}{\rho_0} \nabla P_n$$

$$\nabla^2 P = \nabla\cdot(\boldsymbol{U}_n \cdot \nabla \boldsymbol{U}_n)$$

$$\rho_0 \left(\frac{\partial}{\partial t} + \boldsymbol{U}_p \cdot \nabla \right) \boldsymbol{U}_p = -\nabla P_p + \frac{1}{4\pi} \boldsymbol{J} \times \boldsymbol{B}$$

$$\frac{\partial \boldsymbol{B}}{\partial t} = \nabla \times (\boldsymbol{U}_p \times \boldsymbol{B}), \quad \nabla\cdot \boldsymbol{B} = 0$$

$$\nabla^2 P = \nabla\cdot[\boldsymbol{U}_n \cdot \nabla \boldsymbol{U}_n + (\boldsymbol{B}\cdot\nabla)\boldsymbol{B}]$$

In the absence of sources, sinks the ideal invariants of the hydrodynamic system are

Kinetic Energy $$E=\frac{1}{2}\int V^2\, d^3\, v$$

Fluid vorticity $$\Omega=\frac{1}{2}\int (\nabla\times V)^2\, d^3\, v$$

Invariants of magnetohydrodynamic (MHD) system are

		Conserved?
Kinetic Energy	$E_k = \frac{1}{2} \int V^2 d^3 v$	No
Magnetic Energy	$E_m = \frac{1}{2} \int B^2 d^3 v$	No
Total Energy	$E = \frac{1}{2} \int (V^2 + B^2) d^3 v$	Yes (2D, 3D)
Cross Helicity	$H_c = \frac{1}{2} \int (V \cdot B) d^3 v$	Yes (2D, 3D)
Magnetic Helicity	$H_m = \frac{1}{2} \int (A \cdot B) d^3 v$	yes (3D)
Kinetic Helicity	$H_k = \frac{1}{2} \int (V \cdot \omega) d^3 v$	No
Mean-square mag potential	$A = \frac{1}{2} \int A^2 d^3 v$	yes(2D)
Enstrophy	$\Omega = \frac{1}{2} \int \omega^2 d^3 v$	No

Conserved quantities play important role in determining the flow of energy in MHD turbulence.

e.g. If there exist 2 invariants, the system will exhibit dual cascades (forward+inverse).

Forward cascades – Energy flows from larger scales to smaller scales

Inverse cascades – Energy migrates from smaller scales to larger scales

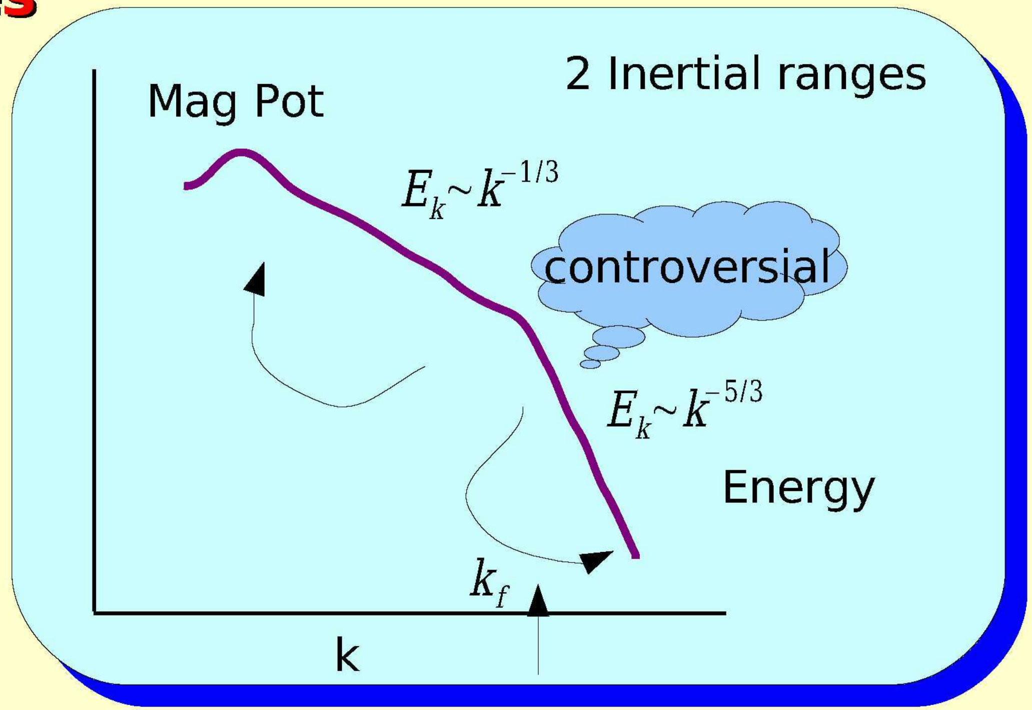


Fig : Cascades in 2D MHD

Energy cascades follow certain phenomenology, from Kolmogorov or kraichnan theory.

Kolmogorov theory : Energy transfer is governed by nonlinear eddy interactions.

Typically, nonlinear energy transfer time is given by $\tau_{nl} \sim l / v_l \sim (k v_k)^{-1}$

Energy dissipation rates in ISM turbulence $\varepsilon \sim \frac{E_k}{\tau_{nl}} \sim \frac{v_k^2}{(k v_k)^{-1}} \sim k v_k^3$

Energy spectrum depends upon (1) energy dissipation rates, (2) modes

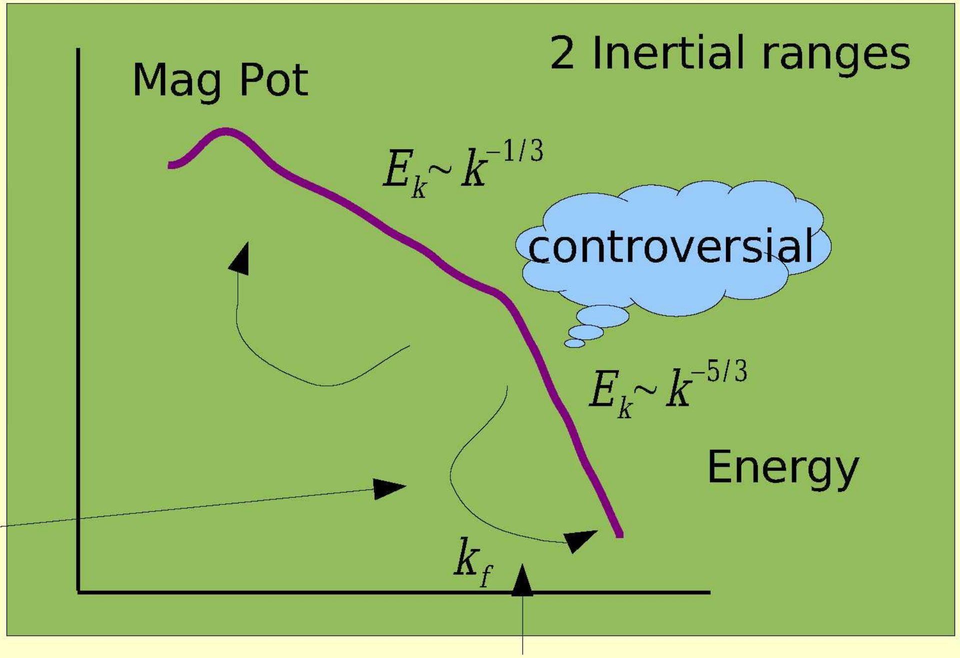
$$E_k \sim \varepsilon^{\alpha} k^{\beta}$$

$$k^{-1} v_k^2 \simeq \left(k v_k^3\right)^{\alpha} k^{\beta} \sim v_k^{3\alpha} k^{\alpha+\beta}$$

$$3\alpha = 2, \quad \alpha + \beta = -1$$
$$\alpha = 2/3, \quad \beta = 5/3$$

Energy Spectrum (2D and 3D MHD)

$$E_k \sim \varepsilon^{2/3} k^{-5/3}$$

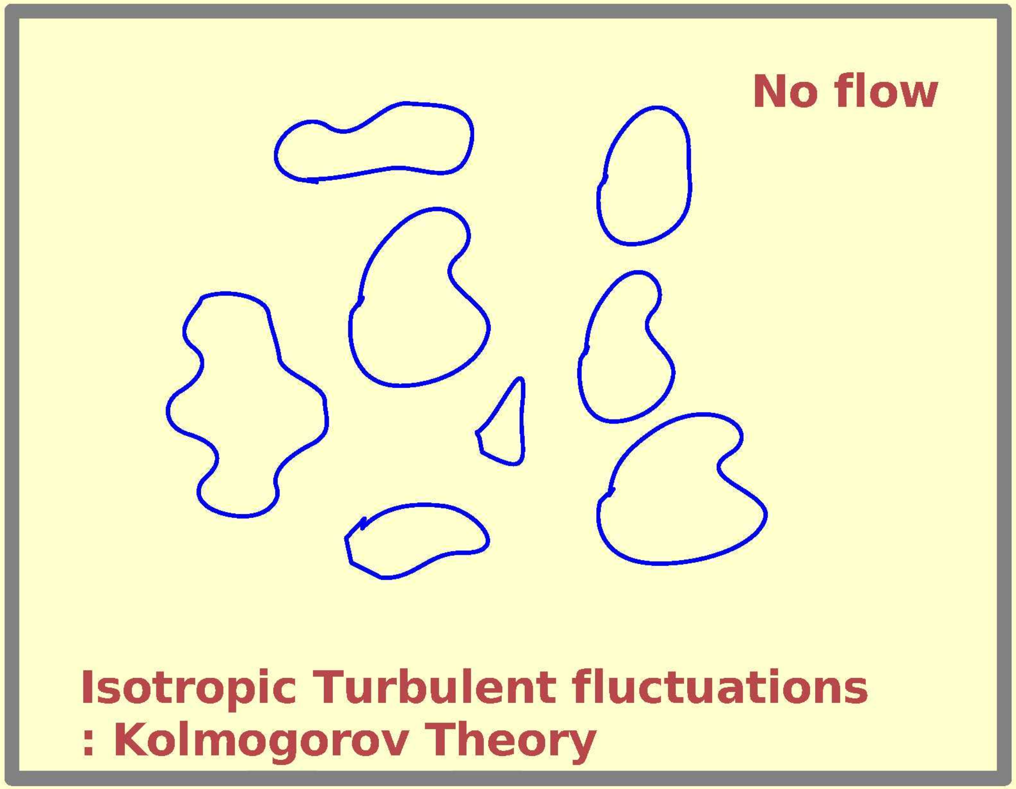


Similarly, energy spectrum for *magnetic potential* can be obtained as follows

$$\varepsilon \sim \frac{A_k}{\tau_{nl}} \sim \frac{k^{-2} v_k^2}{(k v_k)^{-1}} \sim \frac{v_k^3}{k}$$

Energy Spectrum for the mean mag potential regime (2D MHD)

$$E_k \sim \varepsilon^{2/3} k^{-1/3}$$

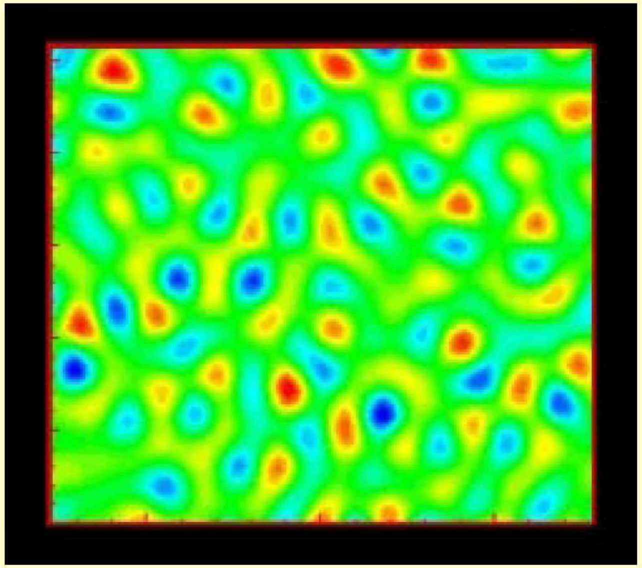


A turbulent fluid involves interactions amongst a wide range of spatial and temporal length-scales.

Random eddies, isotropic, no preferred direction etc.

Isotropic Turbulence

Dastgeer et al 2004



Two dimensional MHD Simulations of Dastgeer (2004).

Constant contours of magnetic field potential.

Fluctuations are isotropic, and random.

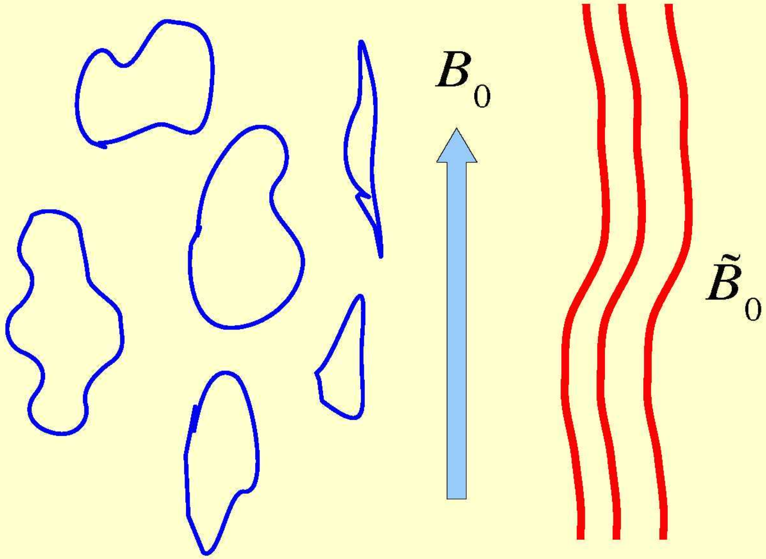
No directional preference.

Enormous body of work has been done in isotropic, homogeneous, fluid and MHD turbulence.

But, isotropy can be violated in solar wind or ISM by mean flow/magnetic field.

Iroshnikov-Kraichnan theory : Energy transfer is governed by Alfvenic wave packets.

In the presence of a mean or background flow field, viz., magnetic field, turbulent fluctuations can preferentially lie along the field lines.

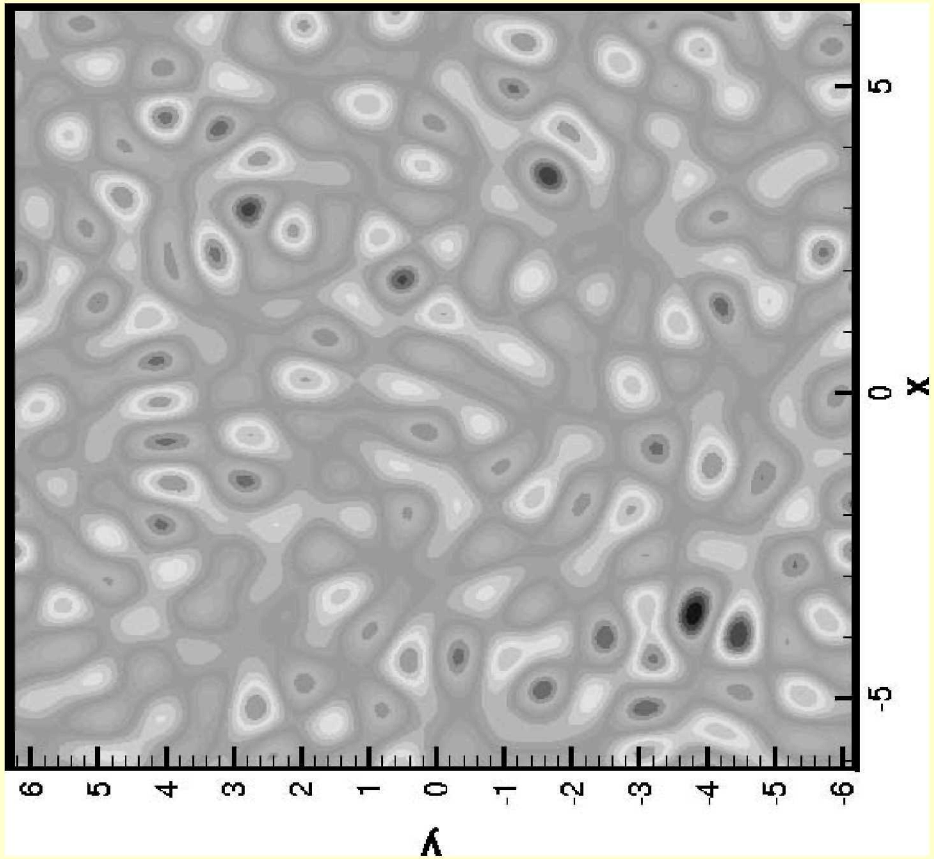


Turbulent fluctuations : Anisotropic

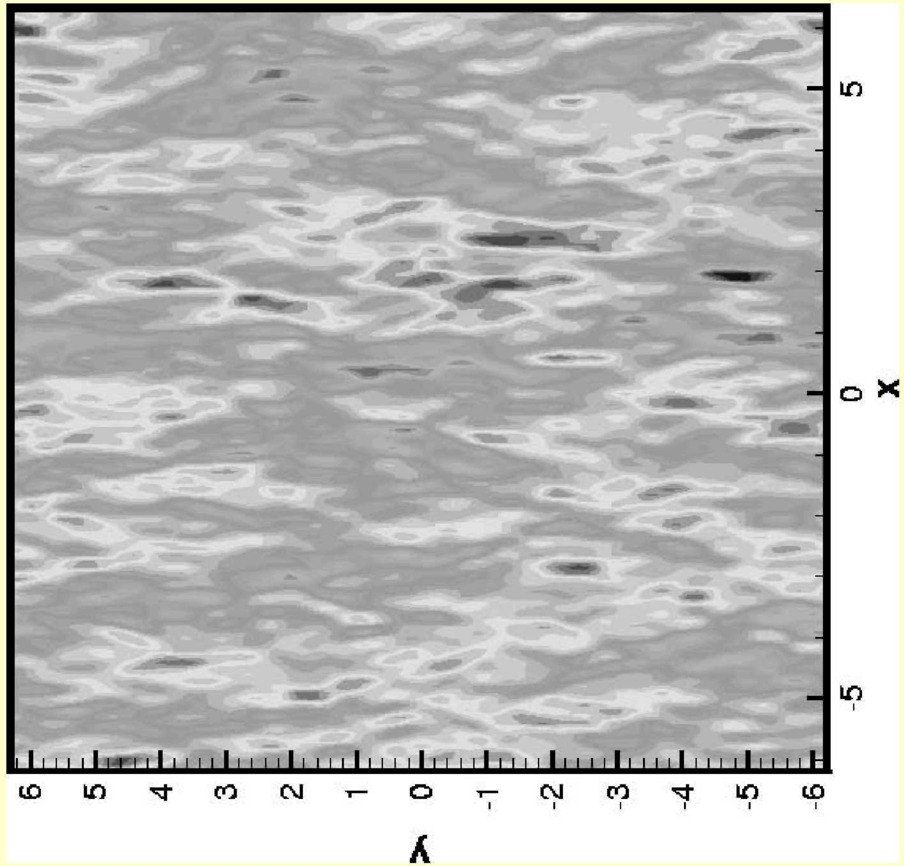
Mean Magnetic field is present

Anisotropic MHD Turbulence

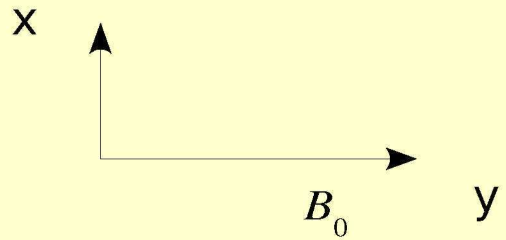
t=0, magnetic potential



t=50, magnetic potential



Dastgeer & Zank, ApJ (2003)



Anisotropy is observed in many other simulations.

SW, ISM observations

IK Theory – Dimensional Phenomenology

When mean or background magnetic field is much stronger than turbulent fluctuations (oppositely moving wave packets) interact weakly.

$$B_0 \gg U_k$$

Alfvenic time is effective time for energy transfer (compared to the eddy time)

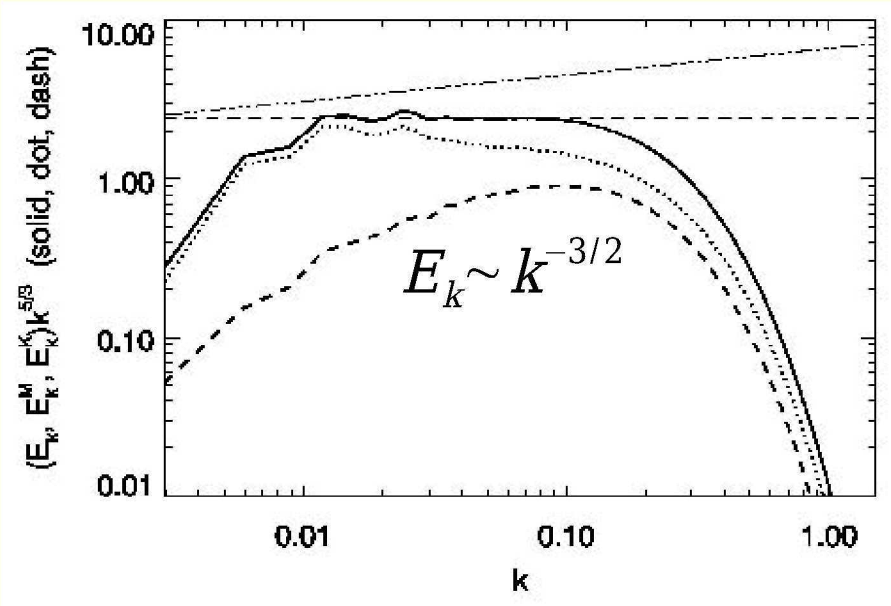
$$\tau_A = (B_0 k)^{-1}$$

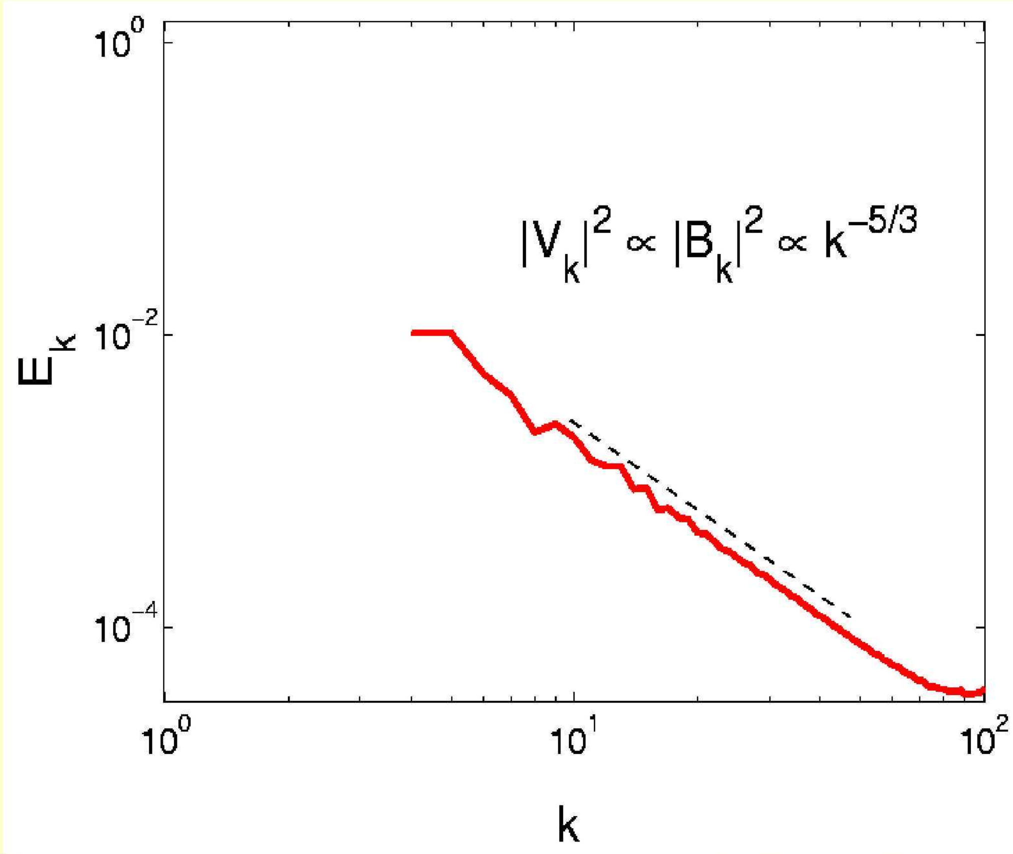
Energy dissipation rate $\varepsilon = \dfrac{E_k}{\tau_{nl}} \left(\dfrac{\tau_A}{\tau_{nl}} \right)$

$$\boxed{E_k \sim A (\varepsilon B_0)^{1/2} k^{-3/2}}$$

5/3 or 3/2 : Controversial still

MHD Spectra : Controversy





Muller et al show 3/2 spectra in 3D MHD turbulence.

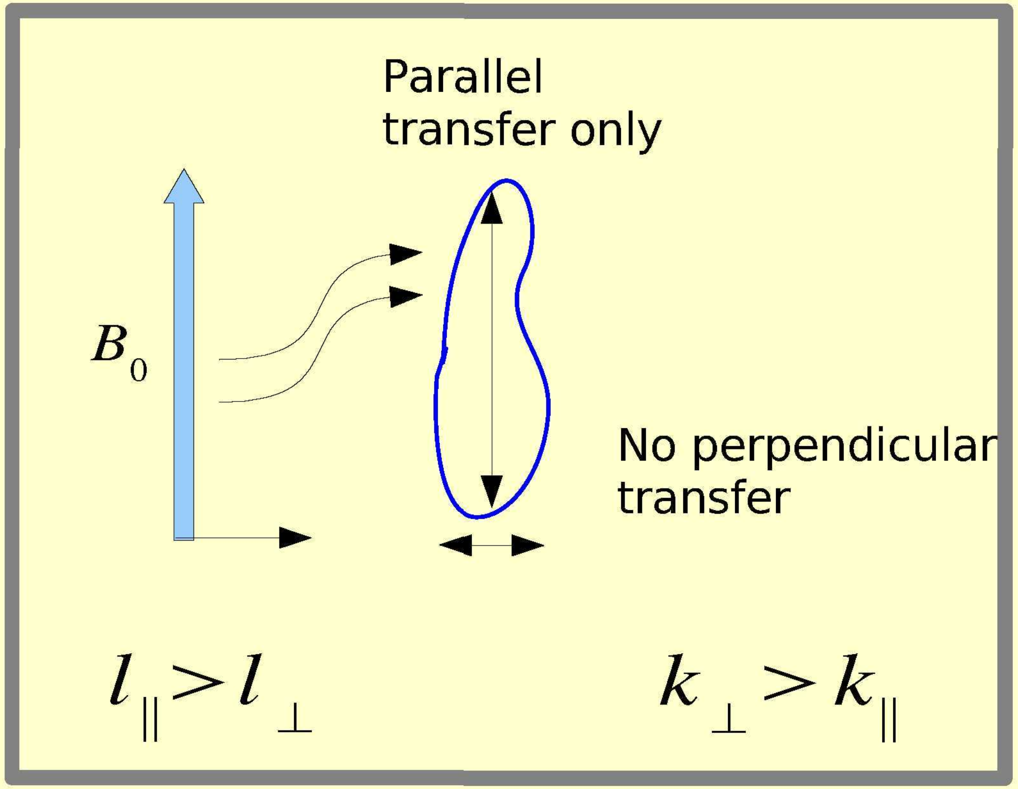
Dastgeer & Zank (2007)

Other work: Maron & Goldreich 2003 show 3/2 spectra.

3D MHD simulations in SW/ISM turbulence.

Spectral Anisotropy (Not Included in IK Theory)

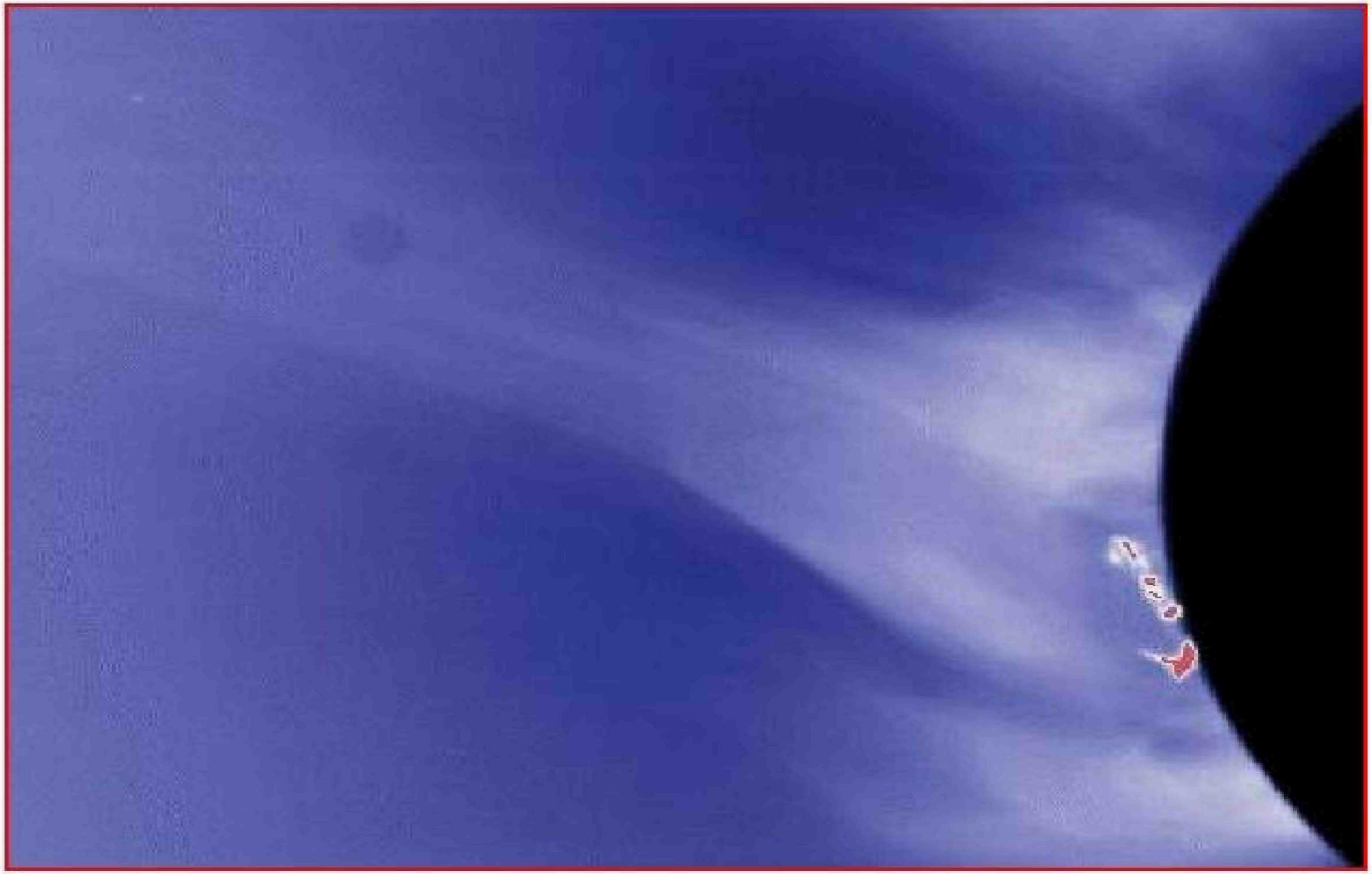
Transfer of energy may proceed amongst neighboring scales in a disparate manner.



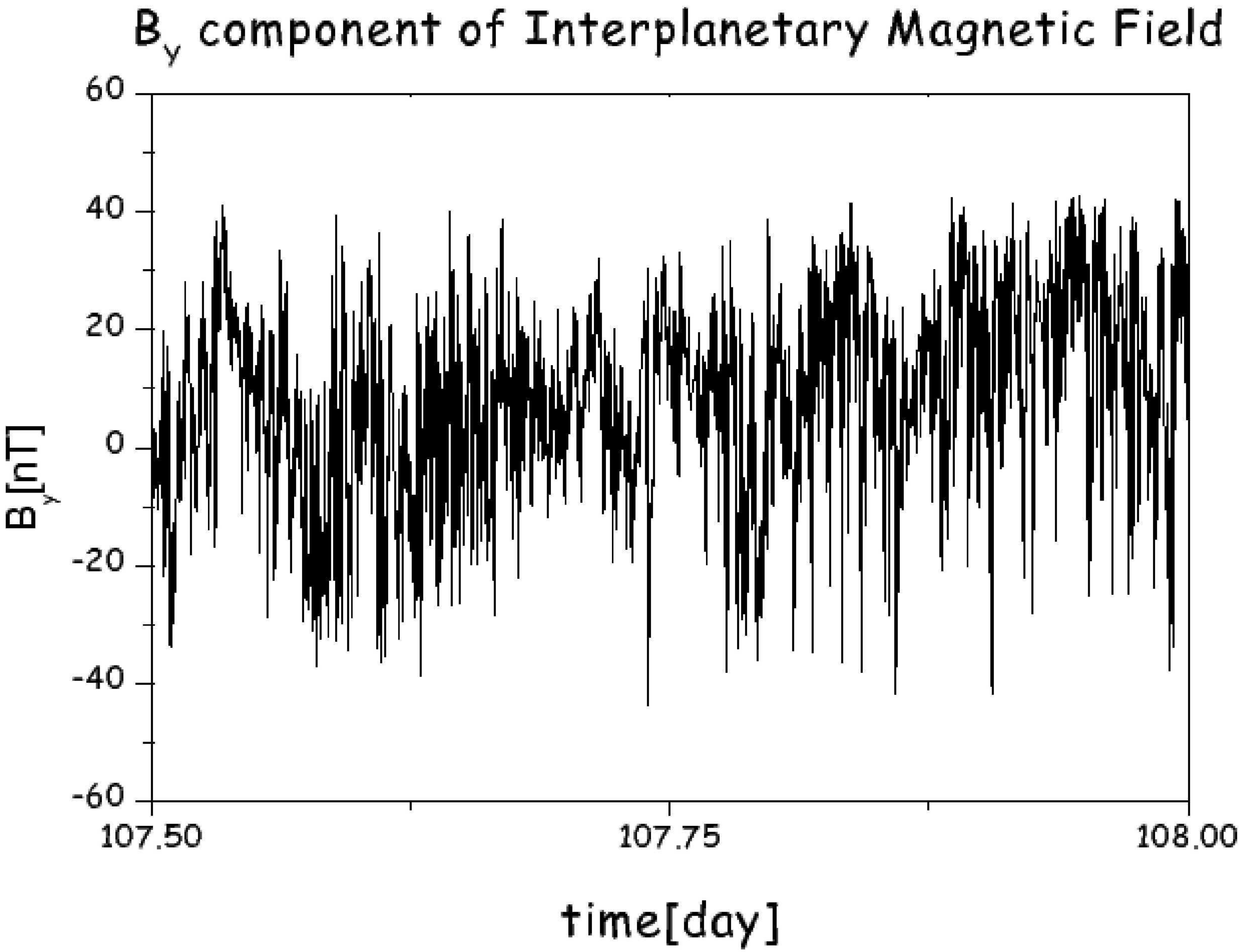
Isotropic $k_{\perp} \simeq k_{\|}$

Anisotropic $k_{\perp} \neq k_{\|}$

In-situ measurements of Solar Wind turbulence

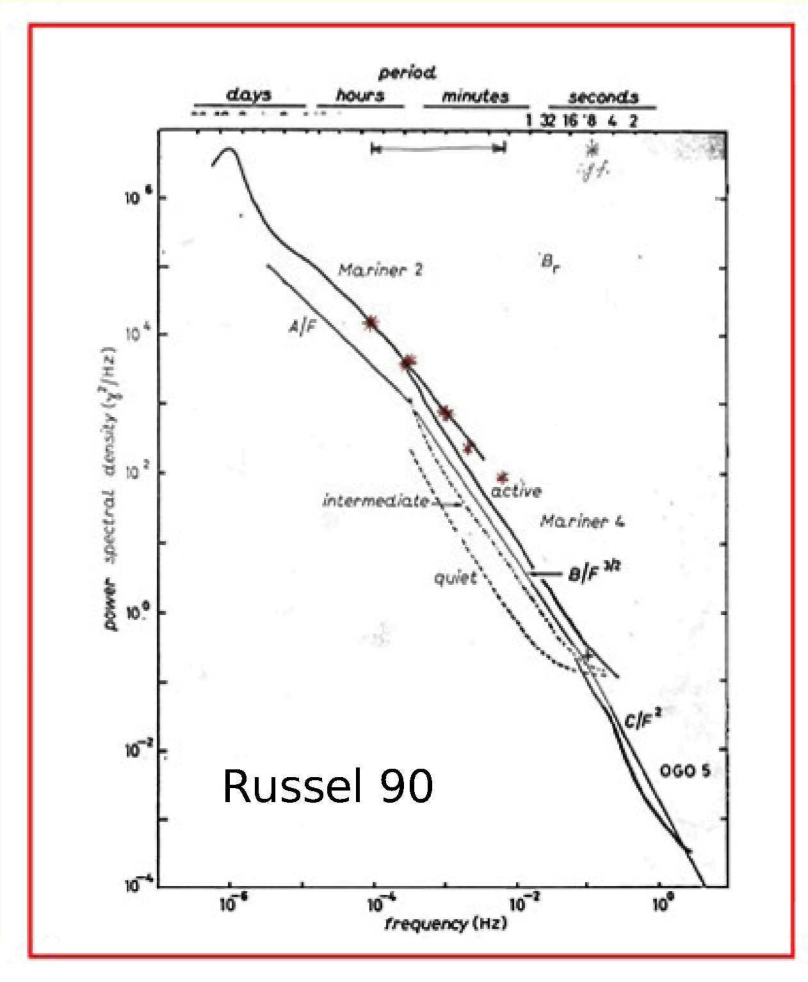


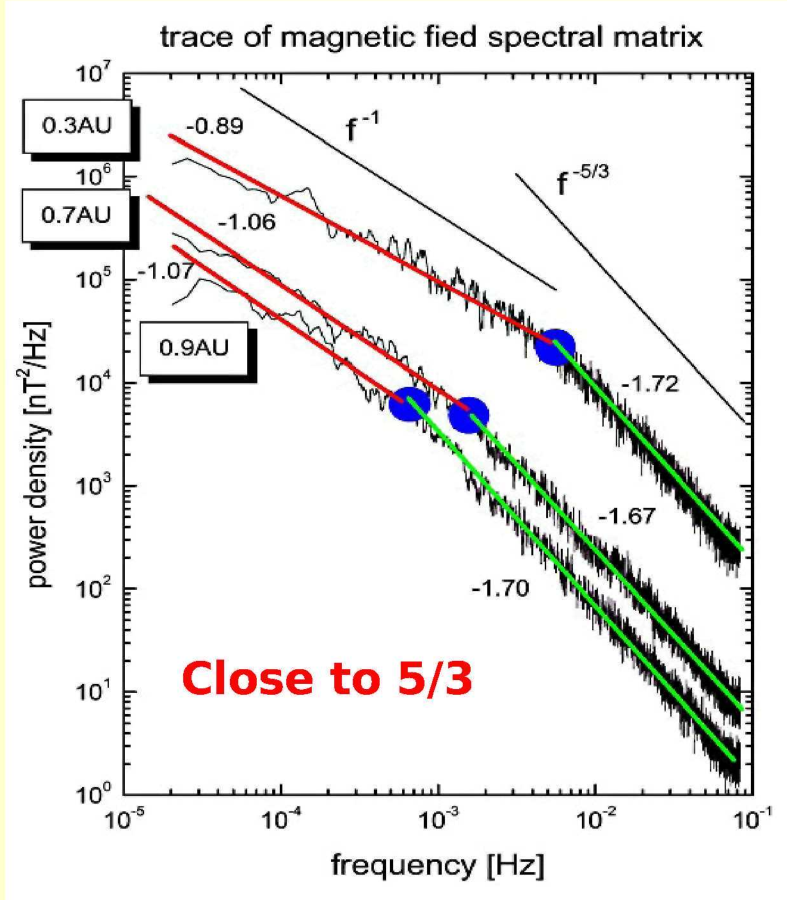
Helmet streamer during a solar eclipse. Slow wind leaks into the interplanetary space along the flanks of this coronal structure. (Figure taken from High Altitude Observatory, 1991).



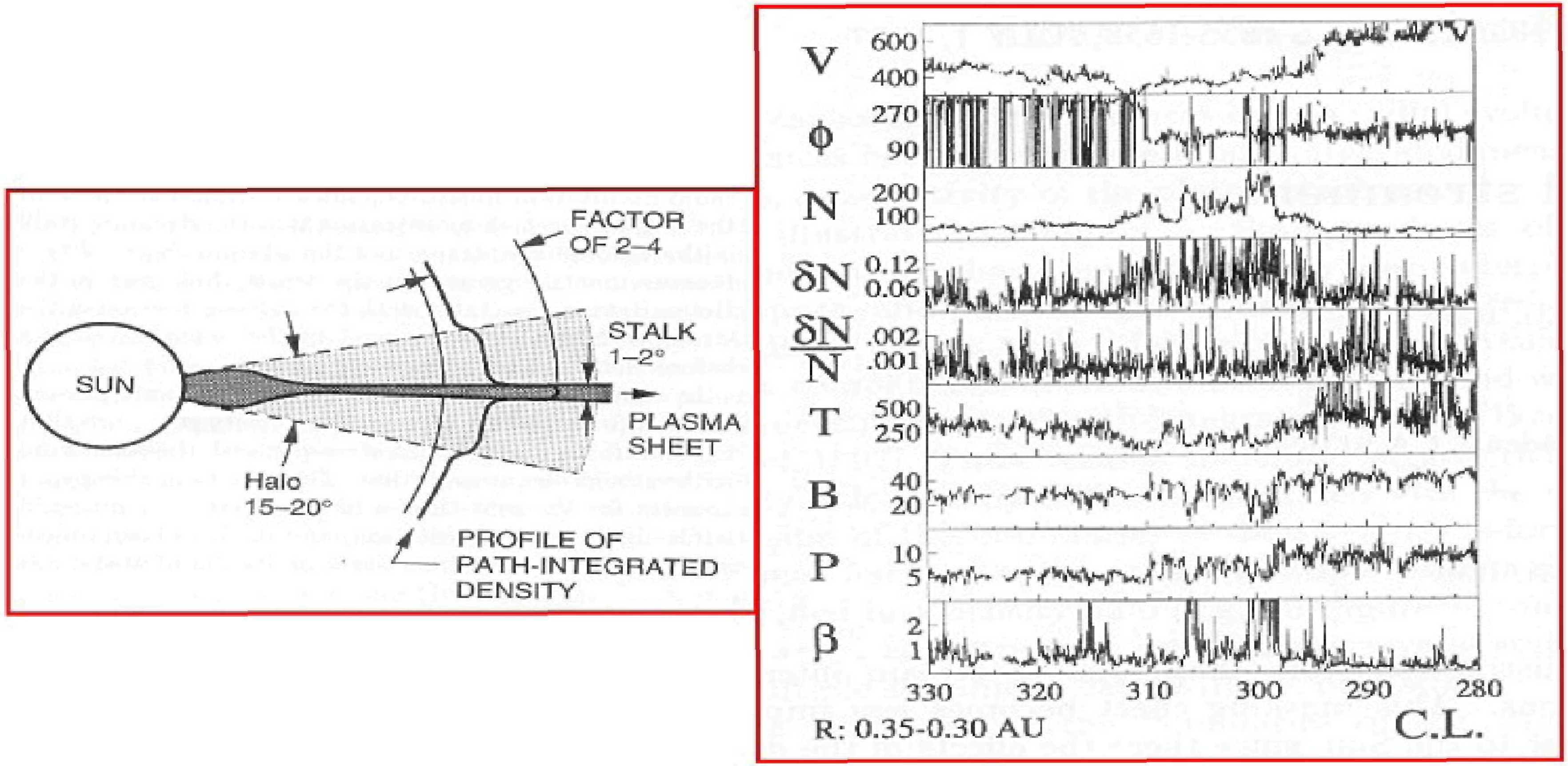
B_y component of Interplanetary Magnetic Field
60
40
20
0
-20
-40
-60
B_y[nT]
107.50
107.75
108.00
time[day]

In-situ measurements of Solar Wind turbulence

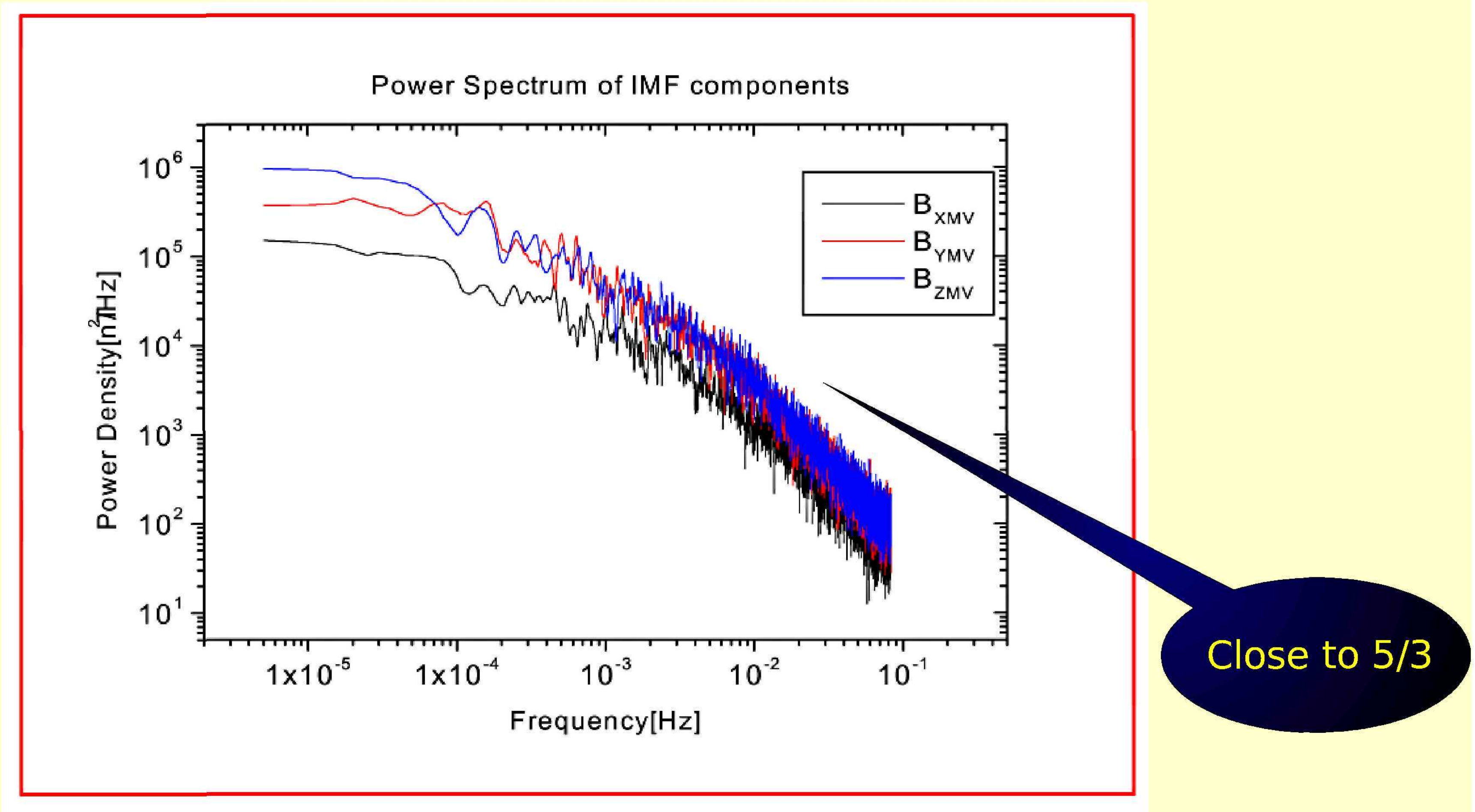




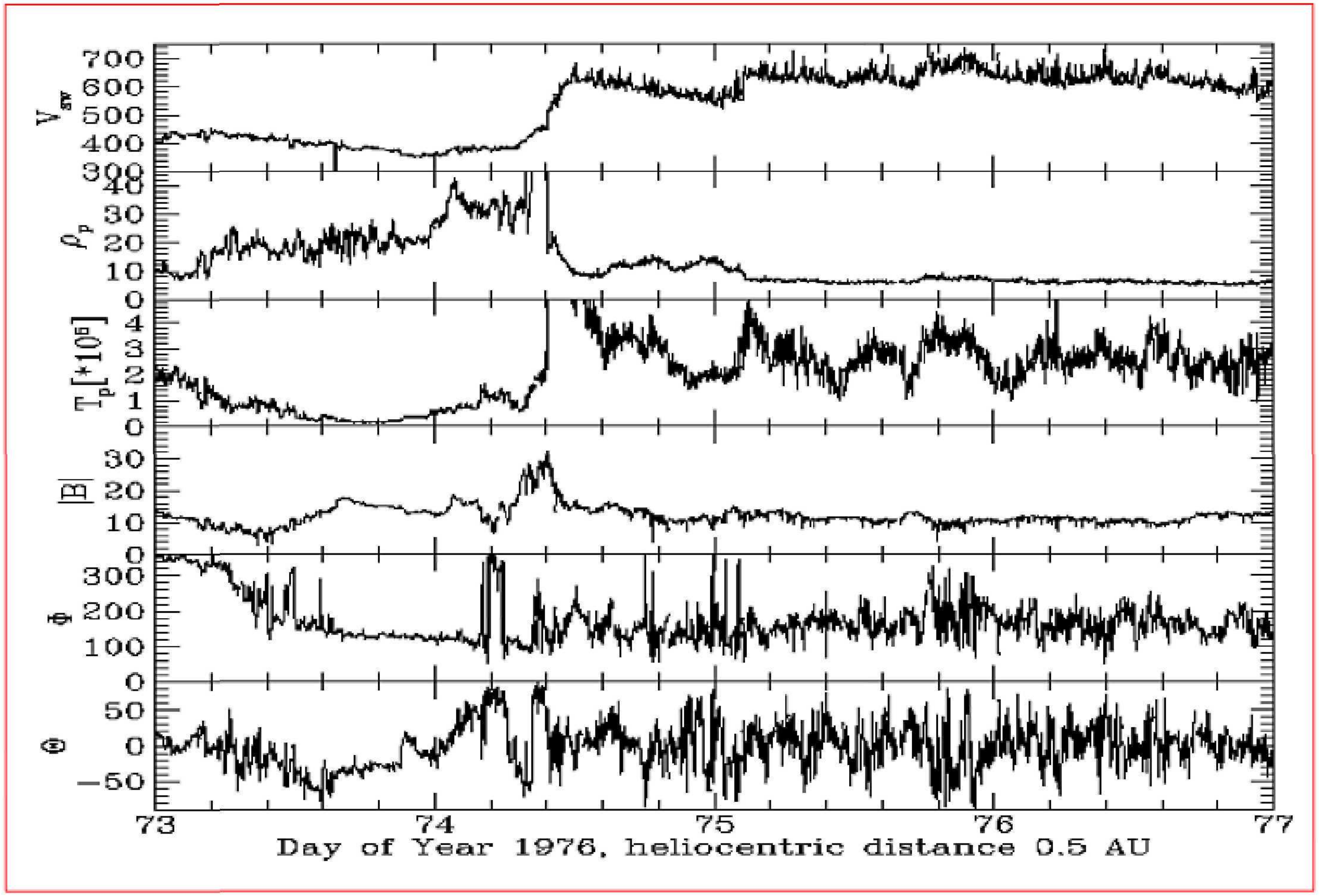
Power density spectra of magnetic field fluctuations observed by Helios 2 between 0.3 and 1 AU. The spectral break (blue dot) shown by each spectrum, moves to lower and lower frequency as the heliocentric distance increases. (Bavassano 82)



Left panel: a simple sketch showing the configuration of a helmet streamer and the density profile across this structure. Right panel: Helios 2 observations of magnetic field and plasma parameters across the heliospheric current sheet. From top to bottom: wind speed, magnetic field azimuthal angle, proton number density, density fluctuations and normalized density fluctuations, proton temperature, magnetic field magnitude, total pressure, and plasma beta, respectively (Bavassano 1970).



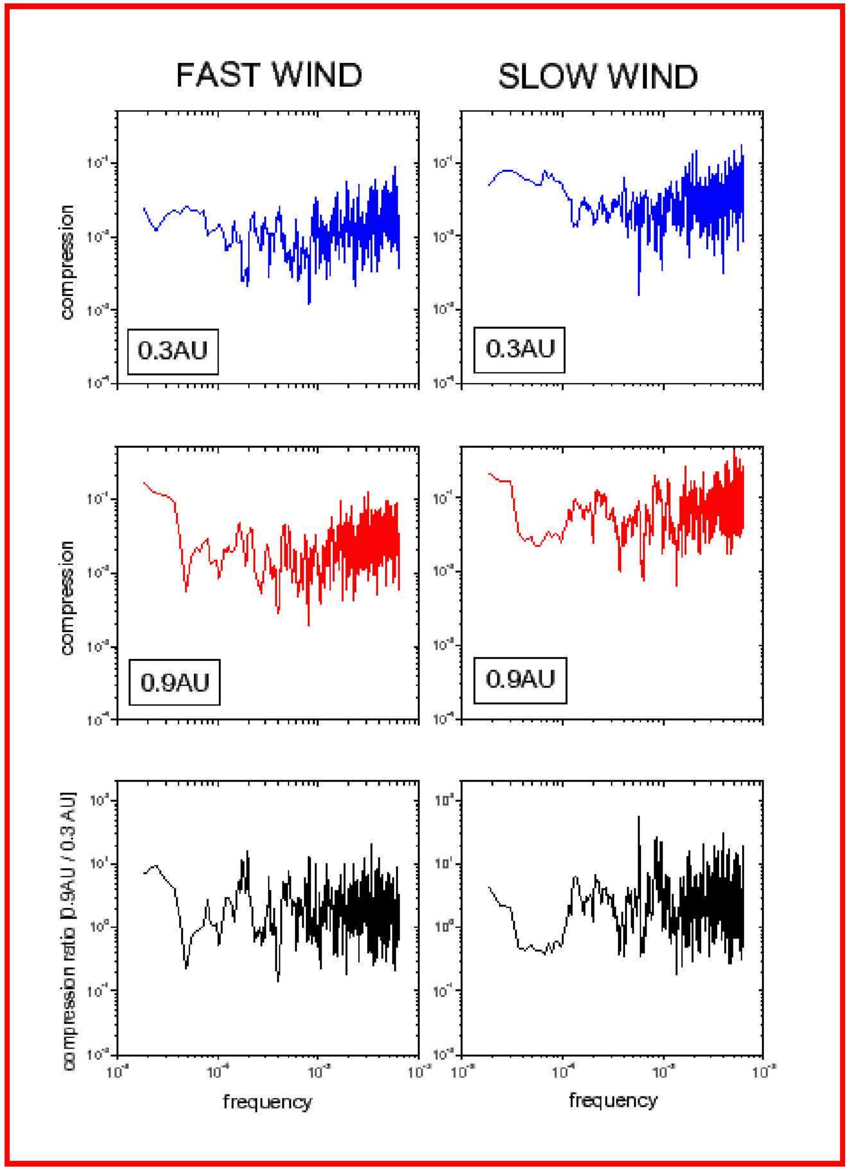
Power density spectra of the three components of IMF after rotation into the minimum variance reference system. The black curve corresponds to the minimum variance component, the blue curve to the maximum variance, and the red one to the intermediate component. (Marsch & Tu 1990)



High velocity streams and slow wind as seen in the ecliptic during solar minimum

Compressive Solar Wind turbulence

Compressive Solar Wind turbulence

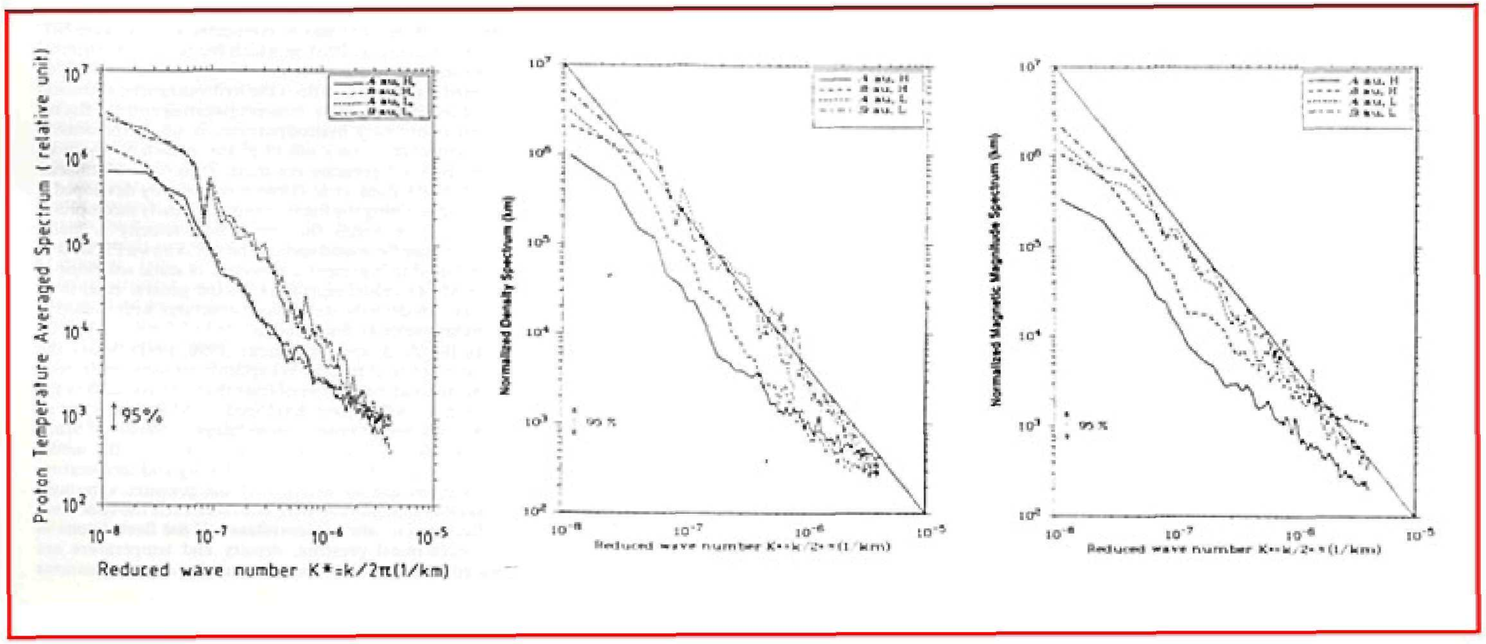


Bruno & Bavassano 1990, 92

Interplanetary medium is slightly compressive, magnetic field intensity and proton number density experience fluctuations over all scales and the compression depends on both the scale and the nature of the wind. As a matter of fact, slow wind is generally more compressive than fast wind.

The first two rows show magnetic field compression (see text for definition) for fast (left column) and slow (right column) wind at 0.3 AU (upper row) and 0.9 AU (middle row). The bottom panels show the ratio between compression at 0.9 AU and compression at 0.3 AU. This ratio is generally greater than 1 for both fast and slow wind.

Temperature density magnetic field



Marsch & Tu 1990

Observations of ISM turbulence

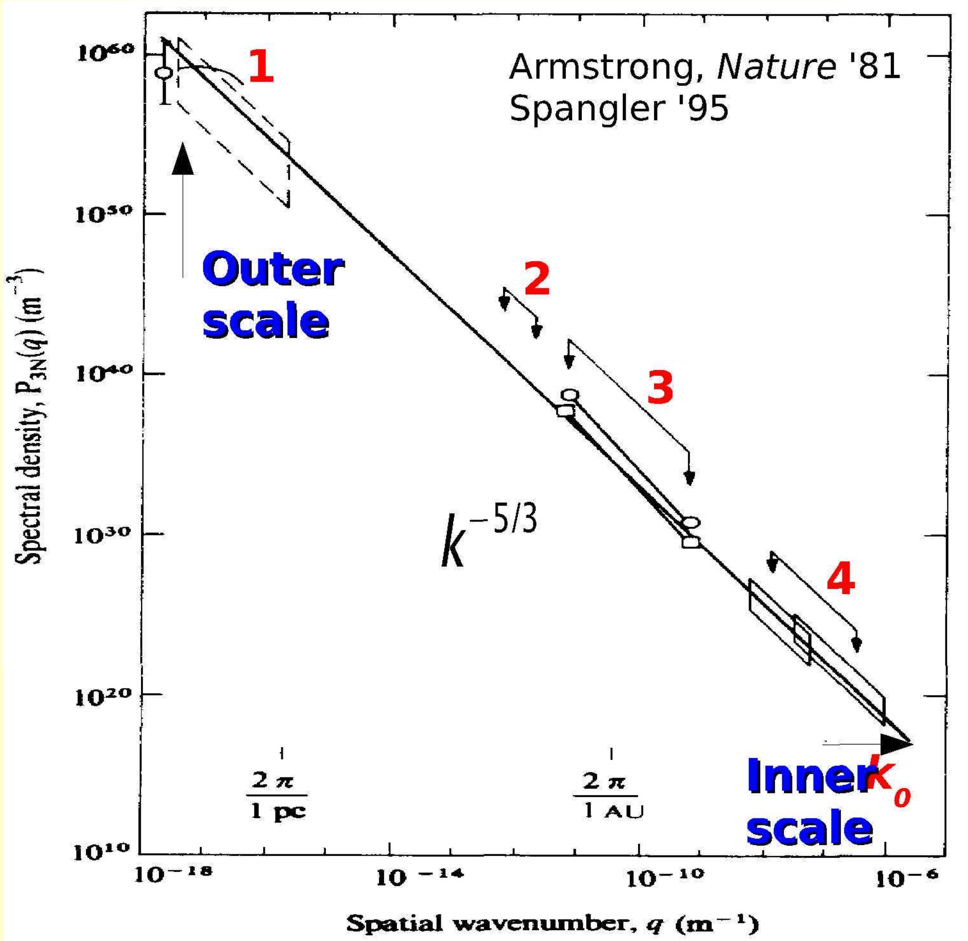
Radio Scintillations measure power density spectrum in local ISM.

Power law spans 12 decades in wavenumber space.

Exhibits a $k^{-5/3}$ Kolmogorov-like spectrum.

Shocks, struc. (scales smaller than turb) do not modify spectrum.

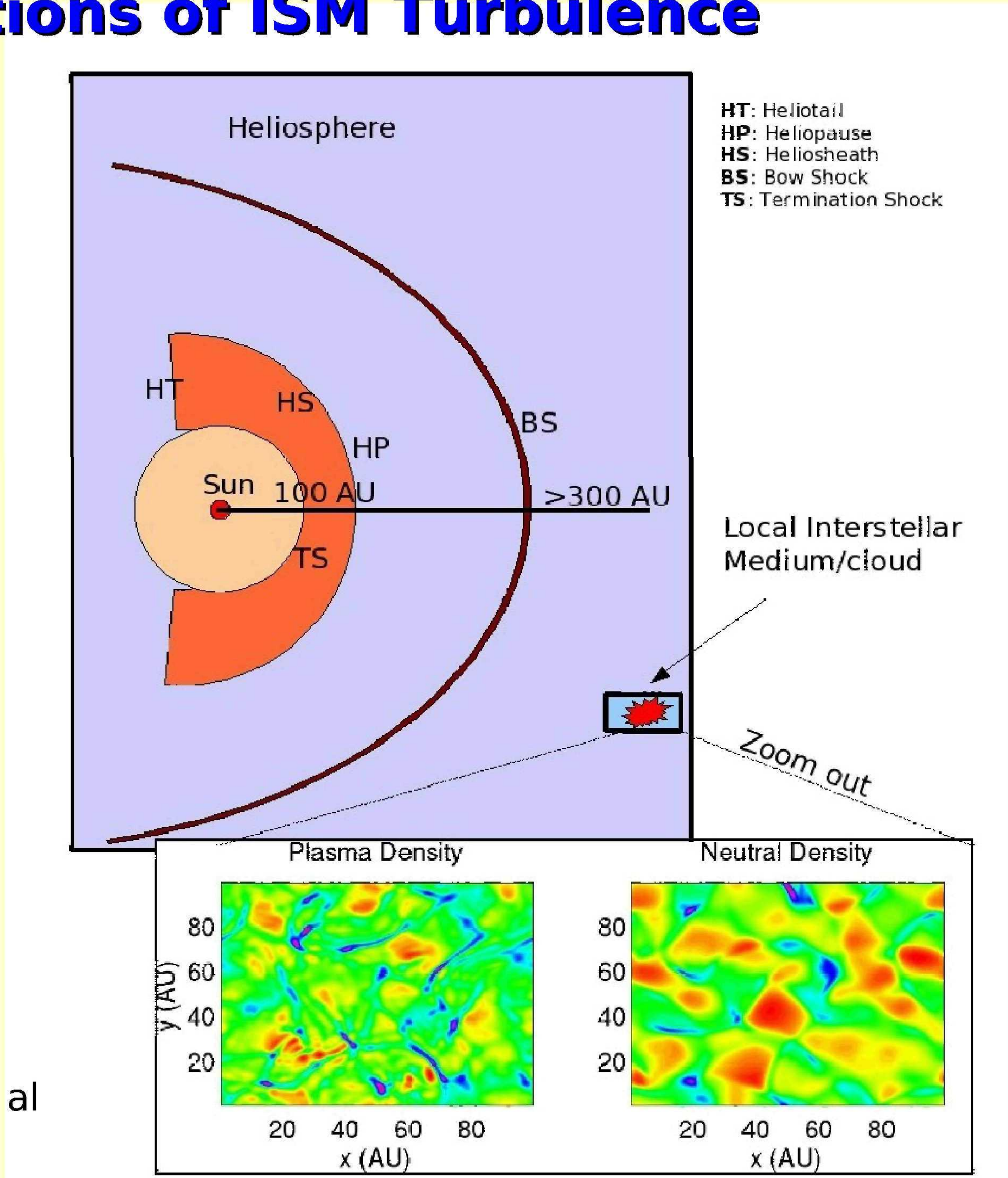
Homogeneous, isotropic, *composite* **1)** ISM velocity fluc., **2)** Disp measu flucs, **3)** Refractive scint. **4)** strong ISS, decorrelation & ang. broad



K_0 is cutoff--inner scales-- dissipation by radiative cooling and ion-neutral collision.

Hydrodynamic incompressible turb--$k^{-5/3}$

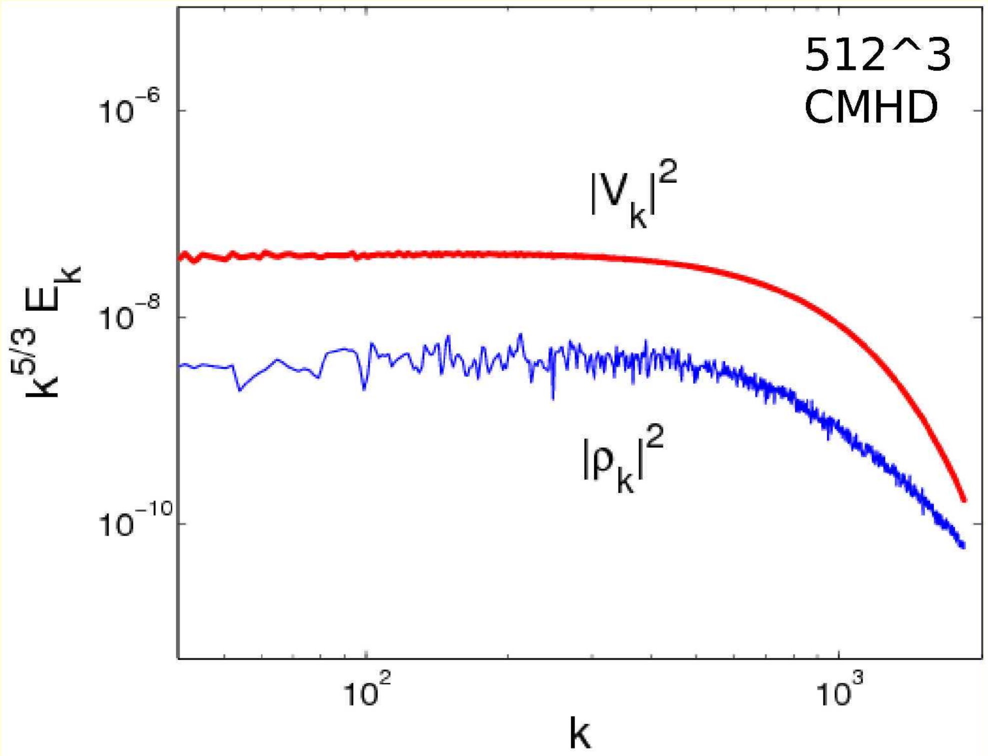
3D Simulations of ISM Turbulence



From: Dastgeer et al 2006, 07

Turbulent Density Spectrum

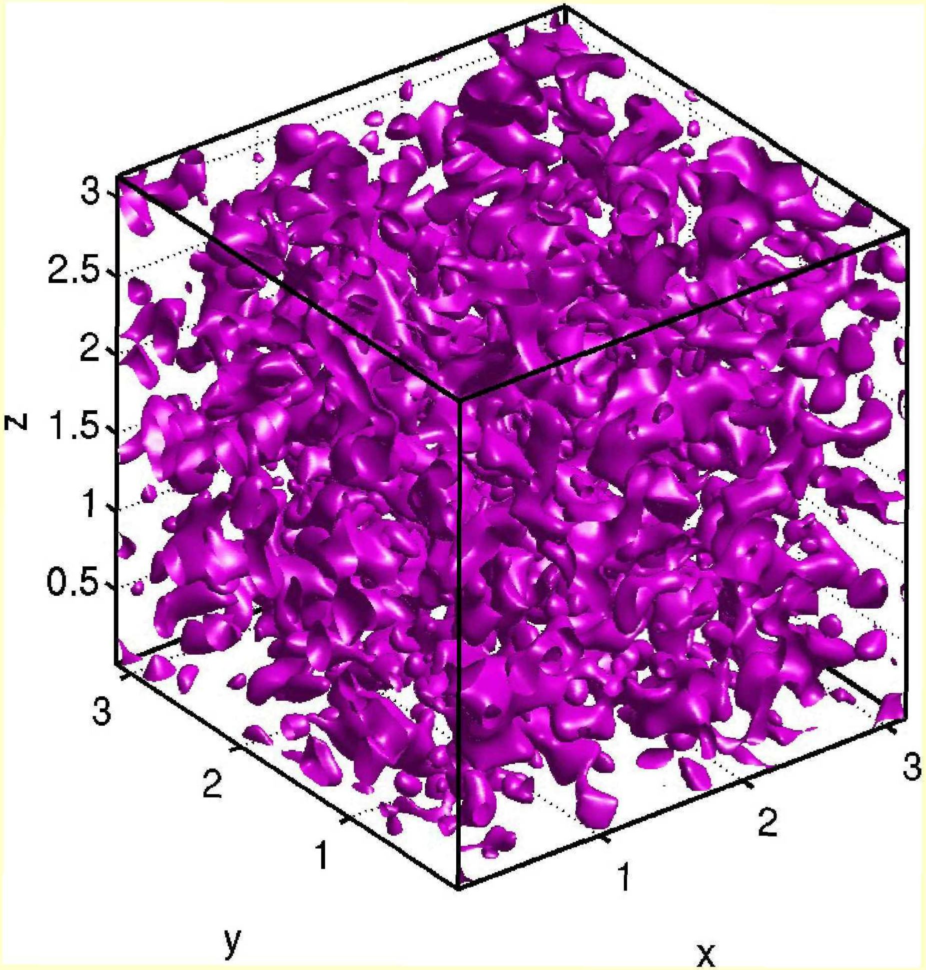
Dastgeer (2007)



Thanks SDSC

- Initially, spectra are steep (high Mach regime); density do not follow vel
- Steady state density flucs show passive scalar spectrum (observed in the ISM/SW)
- Explanation; $\partial_t \rho_k + i \sum_k \delta(k+k') \boldsymbol{U}_k \cdot \boldsymbol{k}' \rho_k \simeq -i \boldsymbol{k} \cdot \boldsymbol{U}_k;$ $\rho_k = \ln \rho(k,t) / \sqrt{\sum_k |\boldsymbol{U}_{\boldsymbol{k}}|^2}$
- Simulations indicate $i \boldsymbol{k} \cdot \boldsymbol{U} \rightarrow O(\varepsilon^2) \ll 1;$ **Passive spectrum**

Mode Structures - B_x



Iso-surface of Bx

From: Dastgeer et al
2006, 07

End of Lecture

Thank You

Properties of a Turbulent Fluid

Interstellar medium (ISM) turbulence can be driven by a number of ways.

- The large scale turbulence is excited by

- Viscous dissipation rates are much smaller than energy transfer rates
- Instabilities such as Kelvin-Helmohltz and other fluid instabilities
- Expansion of protostellar wind in stars, supernovae, and combinations (Elmegreen 04)

- Small scale turbulence may be caused by

- Sonic reflection of shock waves hitting clouds
- Cosmic ray streaming, instabilities
- Energy cascades from larger scales

Steeper Spectra : Scaling Arguments

In ISM turbulence, charge exchange interactions introduce multiple length, time scales that alter nonlinear energy transfer rates.

Typically, nonlinear energy transfer time is given by $\tau_{nl} \sim l/v_l \sim (k v_k)^{-1}$

Charge exchange interactions can change the nonlinear time scale by a factor $k_{ce}/k \leq 1$

New nonlinear time-scale $\tau_{NL} \sim \frac{k_{ce}}{k} \frac{1}{k v_k}$

$$\tau_{NL} \sim \frac{k_{ce} v_k}{k v_k} \frac{1}{k v_k} \sim \frac{\tau_{nl}^2}{\tau_{ce}}; \quad \tau_{ce} \sim (k_{ce} v_k)^{-1}$$

CEX time-scale

Energy dissipation rates in ISM turbulence

$$\varepsilon \sim \frac{E_k}{\tau_{NL}} \sim \frac{v_k^2}{k_{ce}/k^2 v_k} \sim \frac{k^2 v_k^3}{k_{ce}}$$

Energy spectrum depends upon (1) energy dissipation rates, (2) modes

$$E_k \sim \varepsilon^\alpha k^\beta$$

$$k^{-1} v_k^2 \sim \left(\frac{k^2 v_k^3}{k_{ce}} \right)^\alpha k^\beta \sim v_k^{3\alpha} k^{2\alpha+\beta} k_{ce}^{-\alpha}$$

$$\alpha = 2/3, \quad \beta = -7/3$$

ISM is Partially ionized: Plasma + Neutrals

- Interstellar medium (ISM) is composed of partially ionized plasma, hence neutral component must be taken into account in understanding ISM turbulent interactions.
- Neutrals are coupled to ISM plasma through charge exchange (CEX) (PZW 1995).
- Neutral plays a crucial role: e.g. damp plasma waves, influences energy cascades, etc (Spangler 1991).
- Neutrals play a critical role in global heliosphere, shock location, heating etc.

It is *important* to develop a self-consistent understanding of coupled plasma-neutral dynamics in ISM turbulence

- We develop ISM turbulence model that couples plasma and neutral through CEX (by treating them two different fluids).
- We currently restrict to two-dimensional interactions. Plasma and neutral fluids are described by MHD and hydrodynamic equations.

Outline of this talk

- The model [two-fluid, 2D model, 2D offers higher resolution/long inertial range]
- Usual 2D turbulence (MHD & HD) [spectral properties, for comparison with ISM]
- Simulations of dissipative ISM turbulence [study spectra, compare with theory]
- Spectral features, scaling arguments, effect of CEX in energy transfer rates in ISM
- Mode structures [evolution of physical variables]
- Charge exchange spectra [coupled plasma-neutral spectra]
- Conclusions

Plasma-Neutral ISM Turbulence Model

The MHD model of plasma

$$\frac{\partial \rho_p}{\partial t}+\nabla\cdot(\rho_p \boldsymbol{U}_p)=0$$

$$\rho_p\left(\frac{\partial}{\partial t}+\boldsymbol{U}_p\cdot\nabla\right)\boldsymbol{U}_p=-\nabla P_p+\frac{1}{4\pi}\boldsymbol{J}\times\boldsymbol{B}+\boldsymbol{Q}_m(\boldsymbol{U}_p,\boldsymbol{U}_n)$$

$$\frac{\partial \boldsymbol{B}}{\partial t}=\nabla\times(\boldsymbol{U_p}\times\boldsymbol{B})$$

$$\frac{\partial}{\partial t}\left(\frac{1}{2}\rho_p \boldsymbol{U}_p^2+\frac{P_p}{\gamma-1}+\frac{B^2}{8\pi}\right)+\nabla\cdot\left(\frac{1}{2}\rho_p U_p^2\boldsymbol{U}_p+\frac{\gamma}{\gamma-1}\frac{P_p}{\rho_p}\rho_p\boldsymbol{U}_p-(\boldsymbol{U}_p\times\boldsymbol{B})\times\boldsymbol{B}\right)=Q_e(\boldsymbol{U}_p,\boldsymbol{U}_n)$$

Fluid equations for neutrals

$$\frac{\partial \rho_n}{\partial t}+\nabla\cdot(\rho_n \boldsymbol{U}_n)=0$$

$$\rho_n\left(\frac{\partial}{\partial t}+\boldsymbol{U}_n\cdot\nabla\right)\boldsymbol{U}_n=-\nabla P_n+\boldsymbol{Q}_m(\boldsymbol{U}_n,\boldsymbol{U}_p)$$

$$\frac{\partial}{\partial t}\left(\frac{1}{2}\rho_n \boldsymbol{U}_n^2+\frac{P_n}{\gamma-1}\right)+\nabla\cdot\left(\frac{1}{2}\rho\, U_n^2\boldsymbol{U}_n+\frac{\gamma}{\gamma-1}\frac{P_n}{\rho_n}\rho_n\boldsymbol{U}_n\right)=Q_e(\boldsymbol{U}_n,\boldsymbol{U}_p)$$

Charge exchange terms

$$\sigma=[(2.1-0.092\ln V_{rel})10^{-7}\,cm]^2;\quad v_{rel}=|\boldsymbol{U}_p-\boldsymbol{U}_n|$$

$$\boldsymbol{Q}_m(\boldsymbol{U}_n,\boldsymbol{U}_p)=m\sigma\, n_p n_n(\boldsymbol{U}_p-\boldsymbol{U}_n)\left[U^A+\frac{v_{th_p}^2}{\delta\, v_{u_p,u_n}}-\frac{v_{th_n}^2}{\delta\, v_{u_n,u_p}}\right]$$

$$Q_e(\boldsymbol{U}_n,\boldsymbol{U}_p)=\frac{1}{2}m\sigma\, n_p n_n U^A(U_p^2-U_n^2)+\frac{3}{4}m\sigma\, n_p n_n(v_{th_p}^2\Delta v_{u_p,u_n}-v_{th_n}^2\Delta v_{u_n,u_p})-m\sigma\, n_p n_n\left[\boldsymbol{U}_p\cdot(\boldsymbol{U}_p-\boldsymbol{U}_n)\frac{v_{th_p}^2}{\delta\, v_{u_p,u_n}}-\boldsymbol{U}_n\cdot(\boldsymbol{U}_n-\boldsymbol{U}_p)\frac{v_{th_n}^2}{\delta\, v_{u_n,u_p}}\right]$$

Normalization

We normalize plasma and neutral fluids by typical parameters

$$(\hat{\rho}_p, \hat{\rho}_n) = (\rho_p, \rho_n)/\rho_0,$$
$$(\hat{\boldsymbol{U}}_p, \hat{\boldsymbol{U}}_n) = (\boldsymbol{U}_p, \boldsymbol{U}_n)/U_0,$$
$$(\hat{P}_p, \hat{P}_n) = (P_p, P_n)/\rho_0 U_0^2$$
$$\hat{\boldsymbol{B}} = \boldsymbol{B}/U_0\sqrt{\rho_0}$$

Normalization of charge exchange terms

$$\hat{\boldsymbol{Q}}_m = \frac{l_0}{U_0^2 \rho_0} \boldsymbol{Q}_m$$
$$\hat{Q}_e = \frac{l_0}{U_0^3 \rho_0} Q_e$$

CEX interaction length-scale: intrinsic scale

$$\hat{\sigma} = n_0 l_0 \sigma = \frac{\sigma}{\sigma_{ce}}; \quad \sigma_{ce} = \frac{1}{n_0 l_0} = k_{ce}^2$$

Time and length scales normalizations

$$\hat{t} = \frac{t}{l_0/U_0}, \quad \hat{x} = \frac{x}{l_0}$$

Charge exchange modes $k_{ce}^{-1} \sim \sqrt{n_0 l_0} \simeq \lambda_{ce}/2\pi$

(1) $k > k_{ce} \; : \; l < \lambda_{ce}$

(2) $k \sim k_{ce} \; : \; l \sim \lambda_{ce}$

(3) $k < k_{ce} \; : \; l > \lambda_{ce}$

Fewer CEX

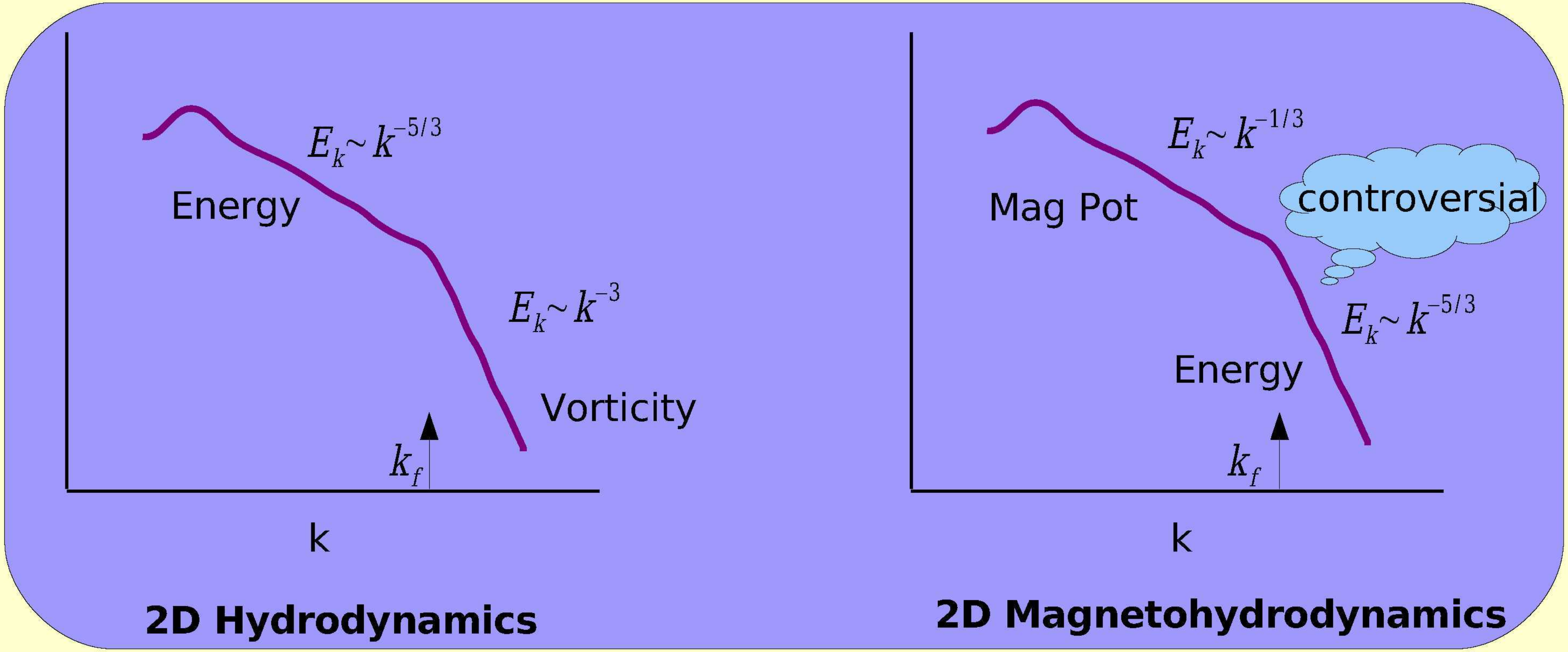
lot more CEX

Charge exchange coupling introduces multiple length-scales in ISM turbulence

2D turbulence - Kolmogorov theory

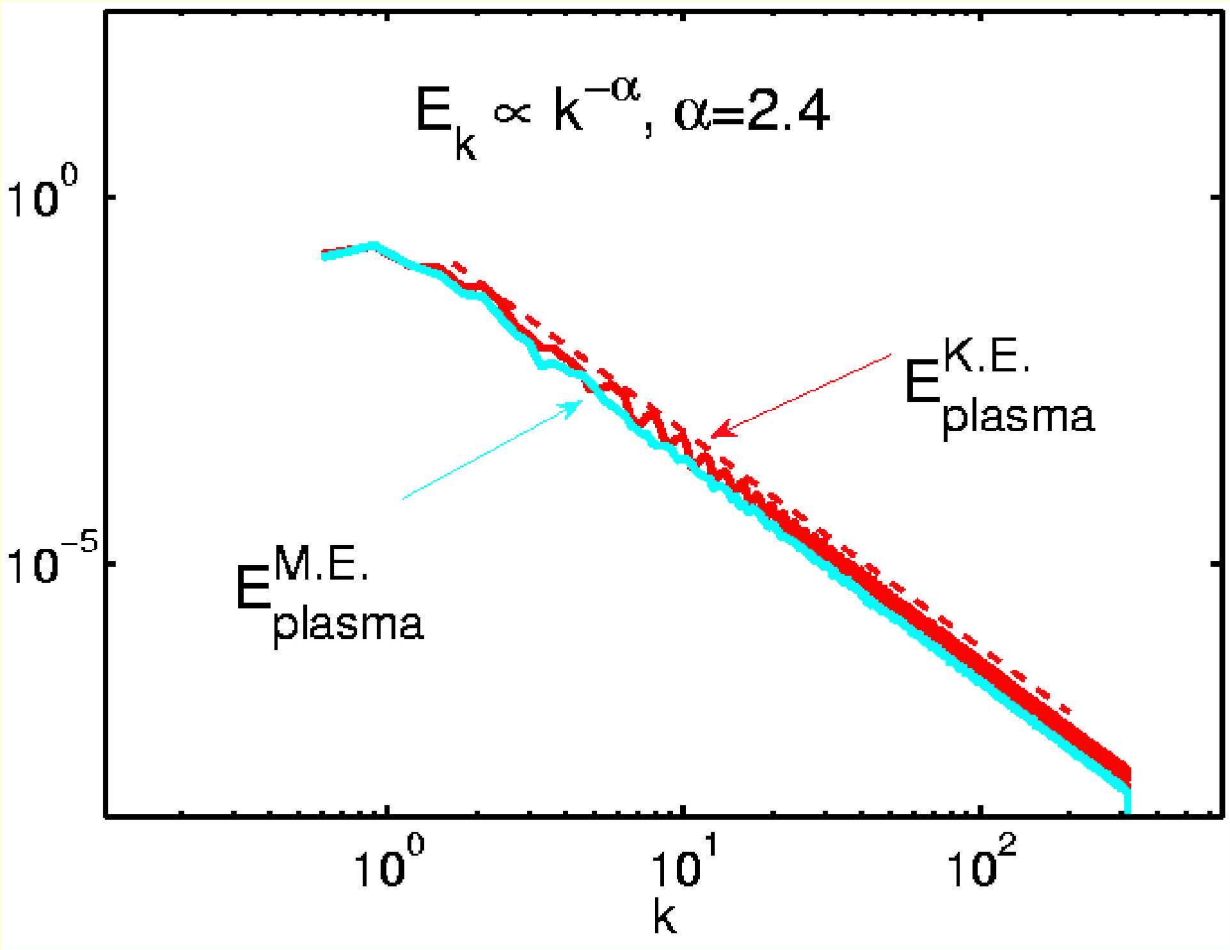
•In 2D hydrodynamic (neutral fluid) turbulence, enstrophy follows a forward cascade while energy follows inverse cascade.

•In 2D MHD (plasma fluid) turbulence, forward cascade of energy co-exists with an inverse cascade of magnetic vector potential.



What happens to Kolmogorov's spectrum in a coupled ISM plasma-neutral system?

Coupled ISM Simulations- Plasma Spectra

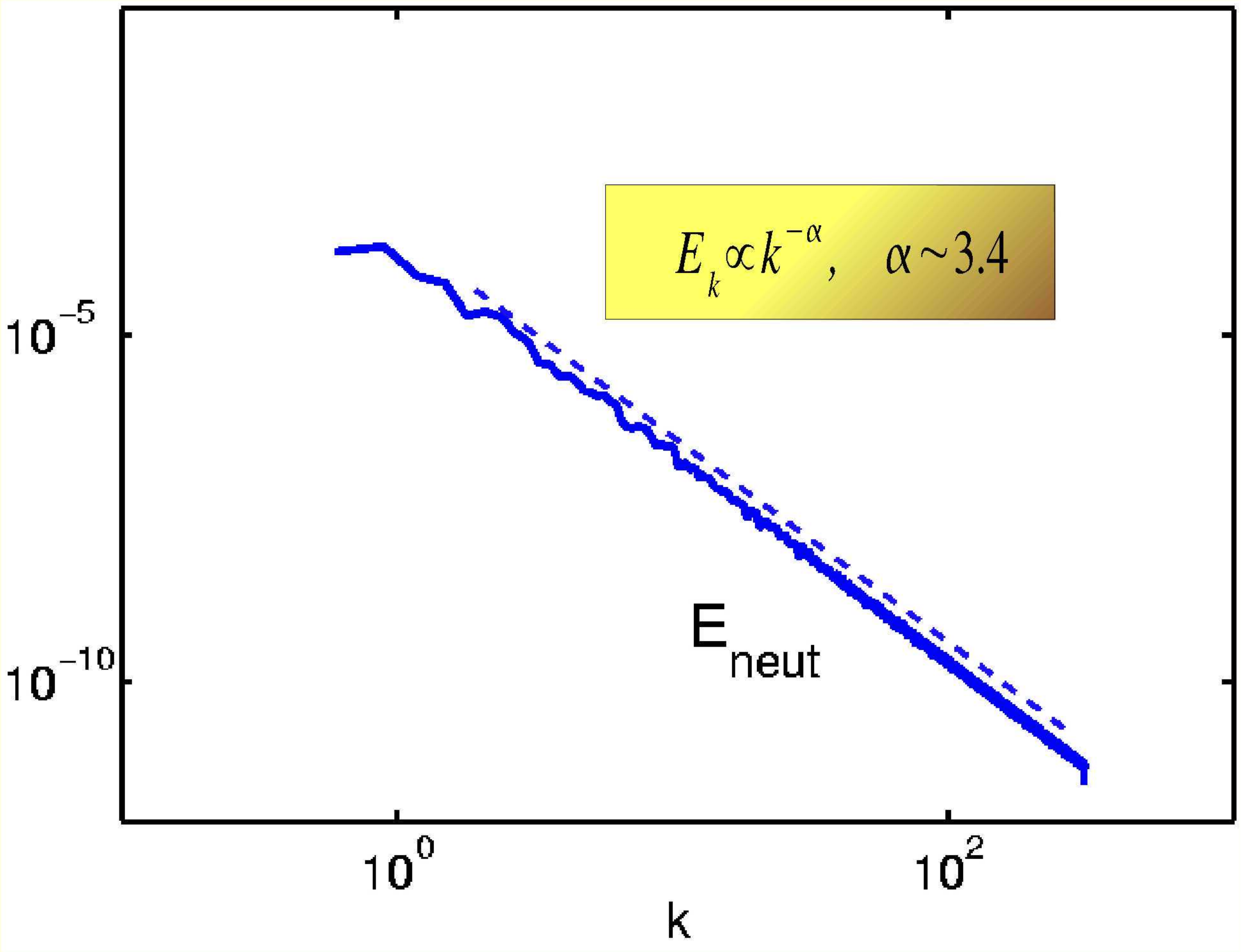


Close to $E_k \sim k^{-7/3}$

$E_{ke} \sim E_{me}$

Spectra Steeper than forward cascade energy spectrum in MHD

Coupled ISM Simulations- Neutral Spectra



Spectra Steeper than forward cascade Hydro spectrum

Close to

$$E_k \sim k^{-11/3}$$

Why the coupled ISM spectra are steeper?

Scaling Arguments - Continues

$$E_k \sim \varepsilon^{2/3} k^{-7/3}$$

Similarly, energy spectrum for *magnetic potential* can be obtained as follows

$$\varepsilon \sim \frac{A_k}{\tau_{NL}} \sim \frac{k^{-2} v_k^2}{k_{ce}/k^2 v_k} \sim \frac{v_k^3}{k_{ce}}$$

$$\boxed{E_k \sim \varepsilon^{2/3} k^{-1}}$$

Consistent with our simulations

Similar calculations for neutral fluid yield

$$E_k \sim \varepsilon^{2/3} k^{-11/3}$$

Forward cascade of enstrophy on energy spectrum

$$E_k \sim \varepsilon^{2/3} k^{-7/3}$$

Inverse cascade of energy

Coupled ISM versus ordinary turbulence

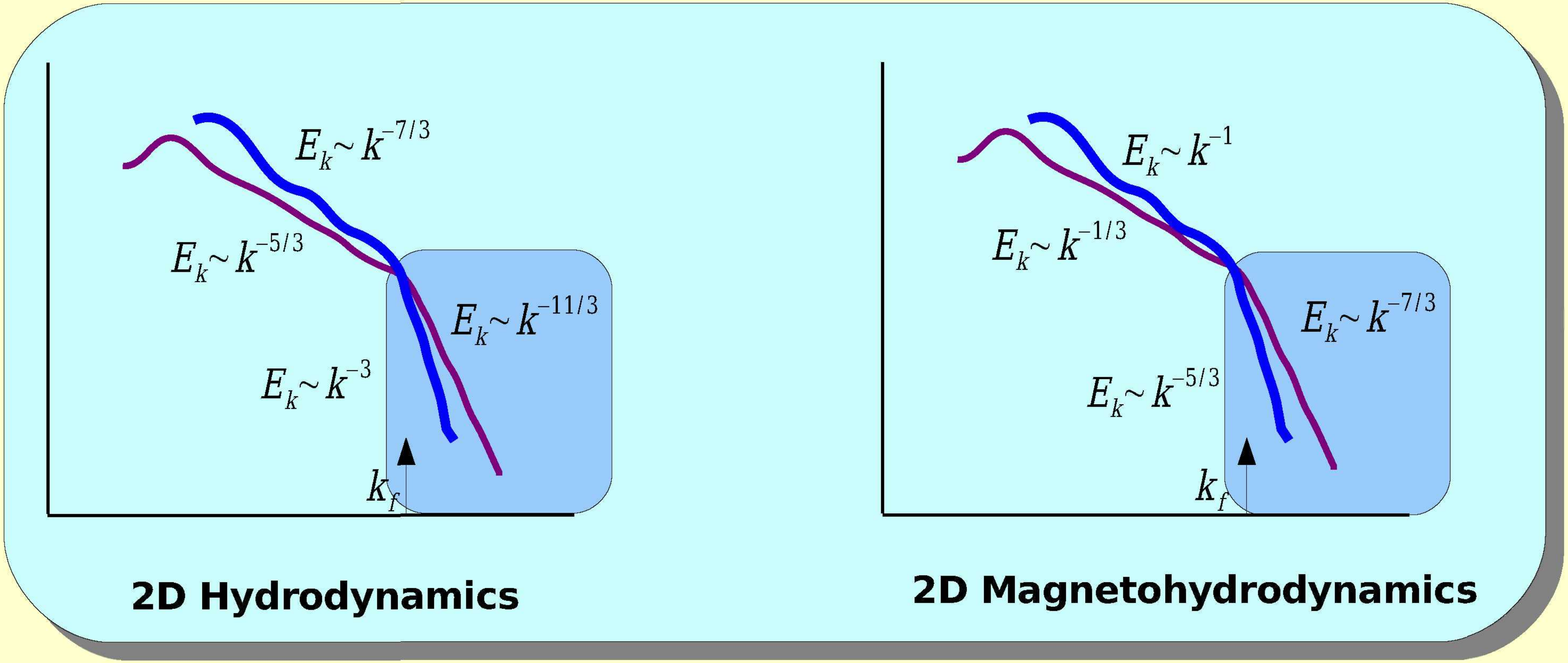
In ordinary MHD turbulence $k_{ce} \sim k \rightarrow \tau_{NL} \sim \tau_{nl}$

$E_k \sim \varepsilon^{2/3} k^{-5/3}$ Forward cascade of energy

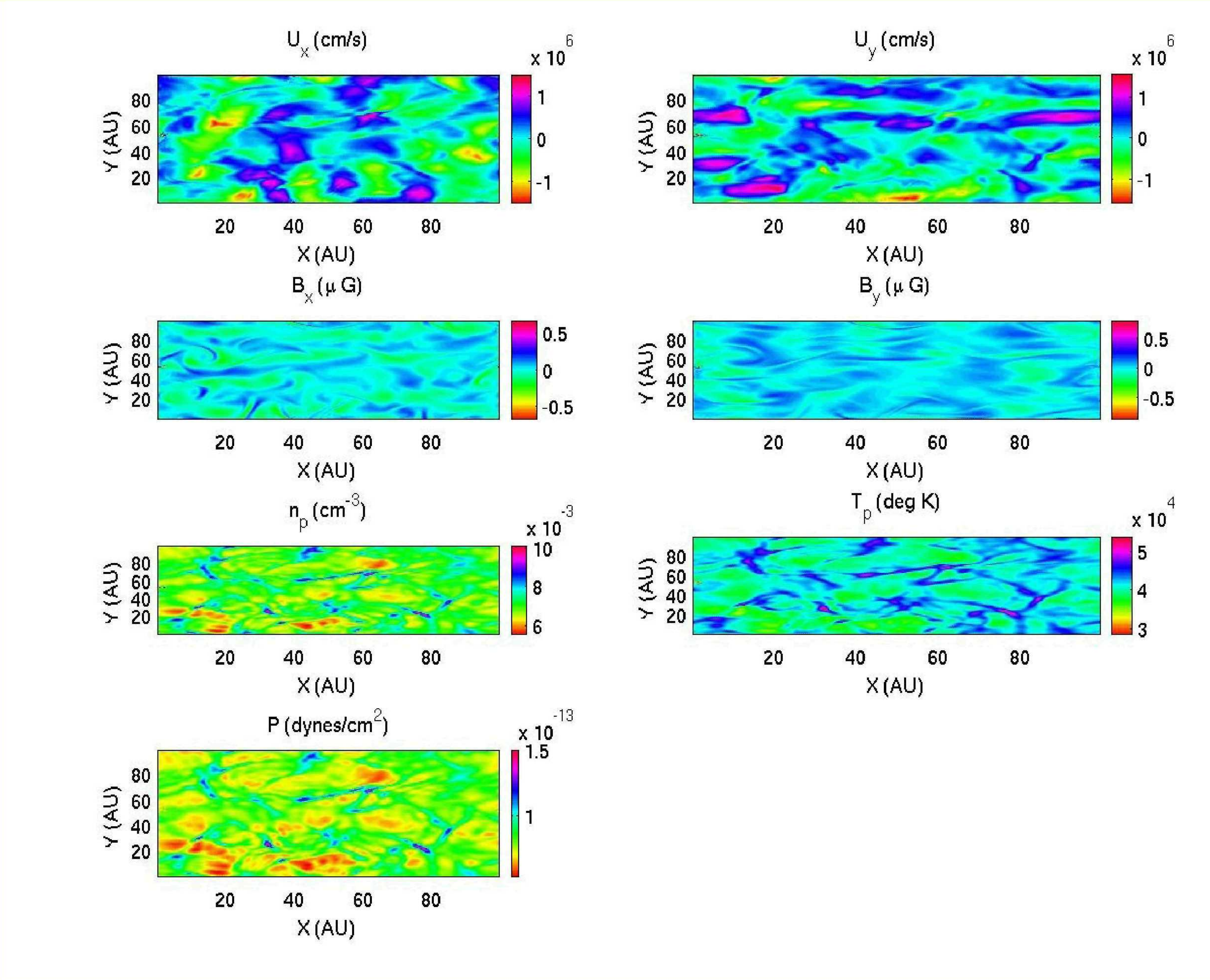
$E_k \sim \varepsilon^{2/3} k^{-1/3}$ Inverse cascade of energy

In ordinary HD turbulence $E_k \sim \varepsilon^{2/3} k^{-3}$ Forward

$E_k \sim \varepsilon^{2/3} k^{-5/3}$ Inverse



Evolution of plasma fluid



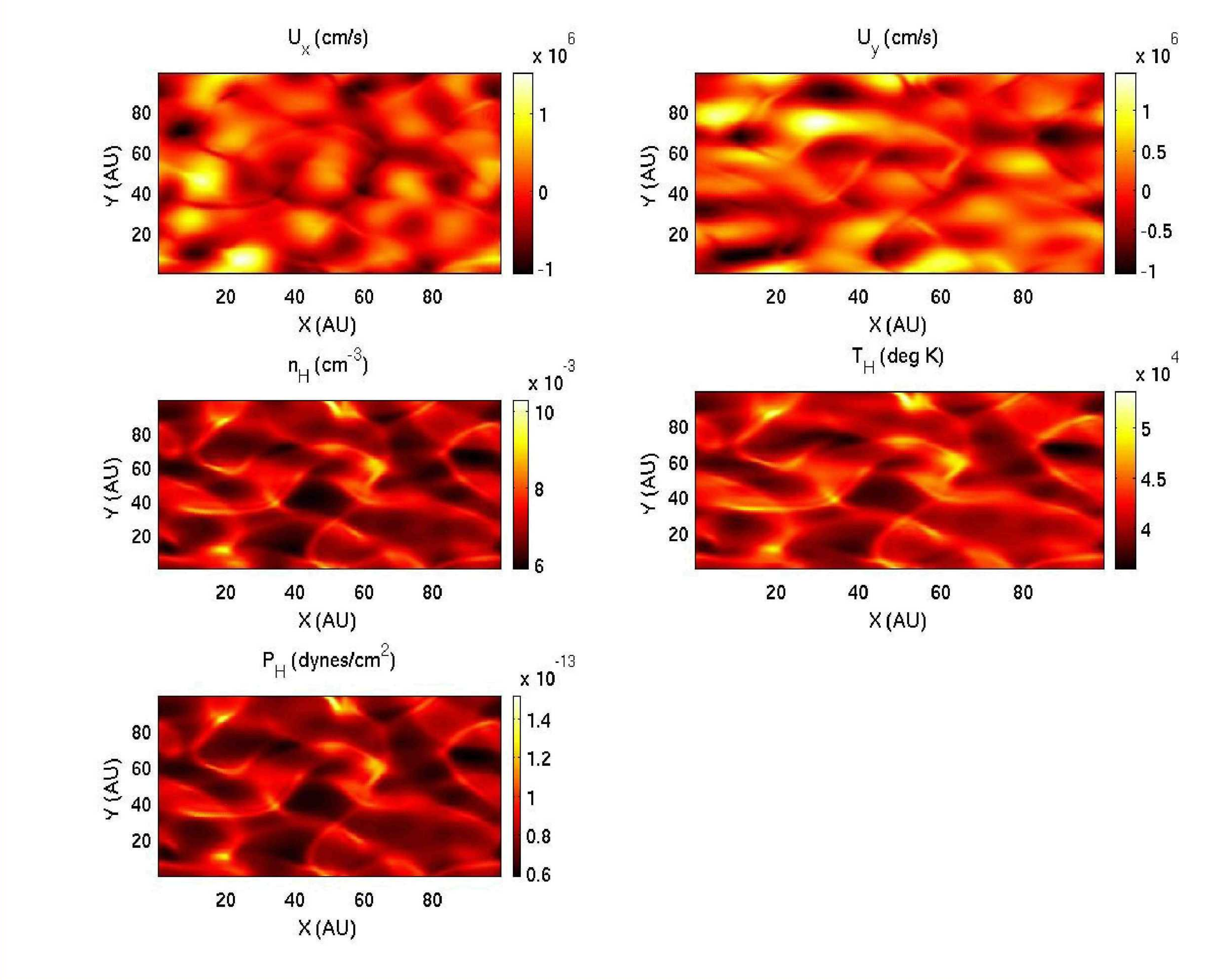
For typical ISM parameters

Momentum transfer rates are enhanced

Formation of Curret-sheets is slowed down

Density forms smaller scales

Evolution of neutral fluid



Momentum transfer rates are decreased

Steeper Spectra Enhance Energy Cascade

ISM turbulent spectra are steeper than ordinary turbulent spectra. CEX interactions introduce multiple time & space scales and alter nonlinear cascades.

Characteristic length scales are typically smaller or equal to CEX length-scales.

$$l \le \lambda_{ce}$$
$$k \ge k_{ce}$$

$$\frac{k_{ce}}{k} \le 1$$

$$\tau_{NL} \sim \frac{k_{ce}}{k} \frac{1}{k v_k} < \tau_{nl}$$

The new nonlinear interaction time, due to CEX, is smaller than ordinary nonlinear time.

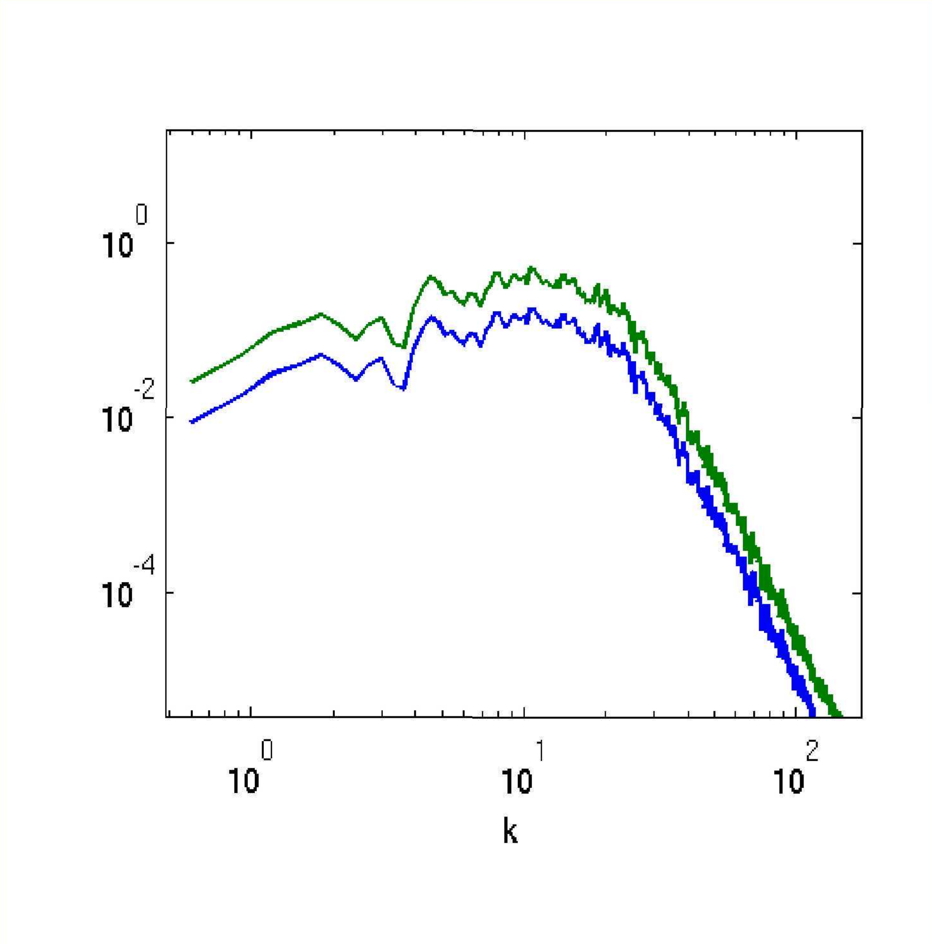
This means energy transfer rates $(\varepsilon_{cex} \sim E_k / \tau_{NL})$ **are enhanced due to CEX**

Rapid transfer of energy takes place. This leads to steepening of power spectra.

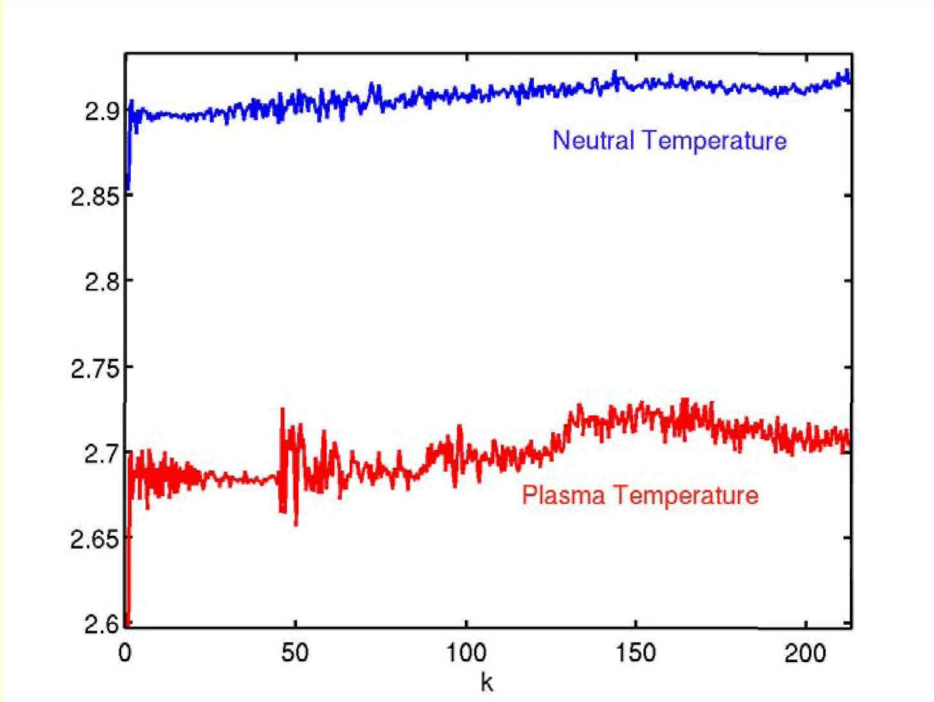
This is unlike Alfven effect, where waves along magnetic field direction inhibit spectral transfer and flatten the energy spectra.

$$\frac{\varepsilon_{cex}}{\varepsilon} \sim \frac{E_k / \tau_{NL}}{E_k / \tau_{nl}} > 1$$

Density and temperature Spectra



Density and pressure spectra



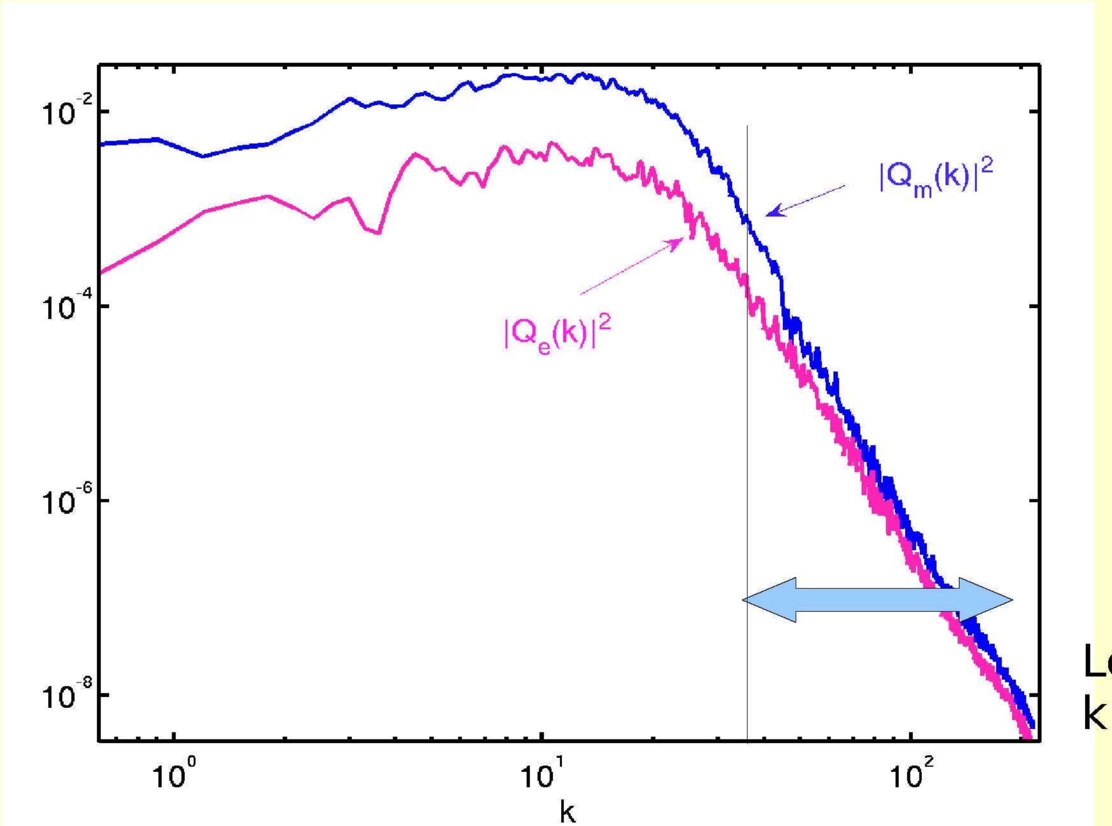
Temperature spectra

Equipartition among turbulent modes

Charge exchange Spectra

CEX are dominant on larger-scales, i.e low k modes effeciently transfer energy and momentum amongst the modes.

Higher k modes are less efficient in transferring energy and momentum via charge exchange mechanism.



Less efficient
k 's

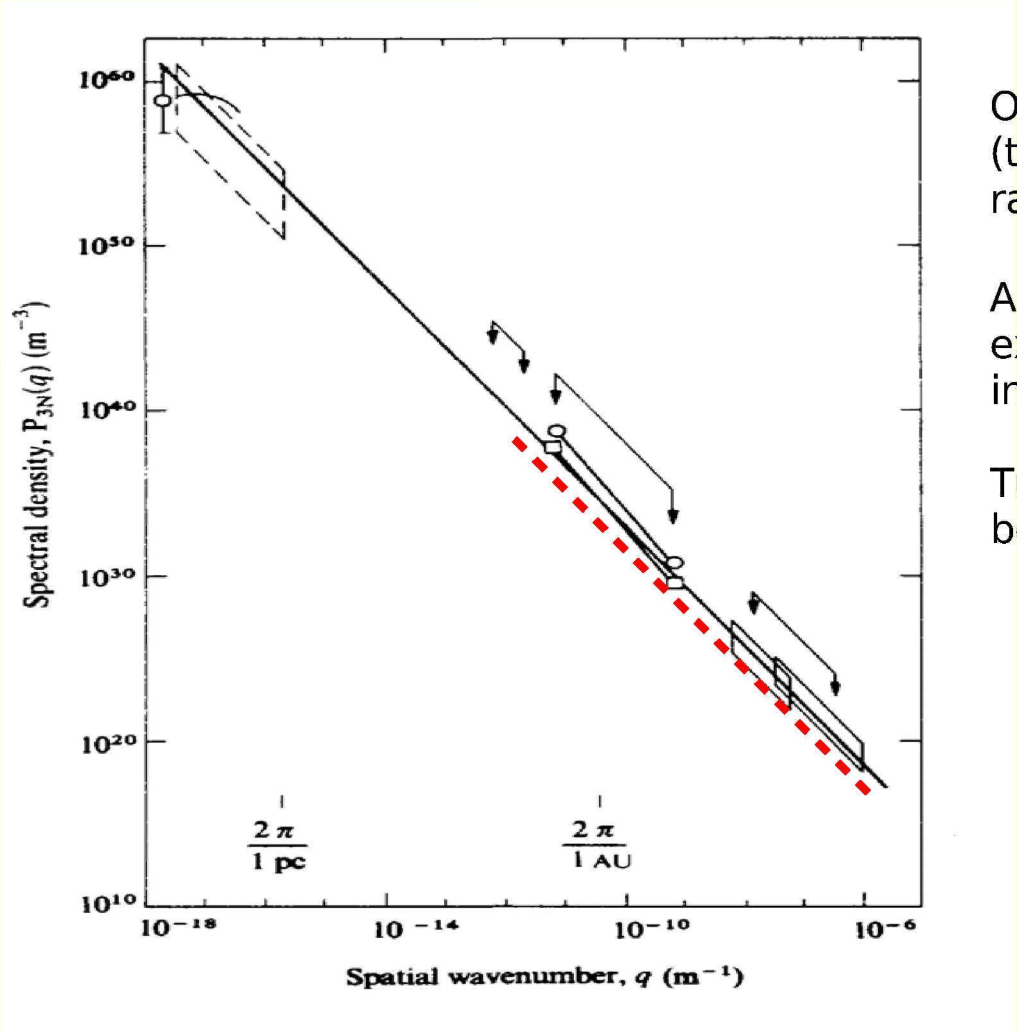
Simulations are consistent with theory and scaling arguments

Conclusions

- A coupled plasma-neutral fluid turbulence model of Interstellar medium (ISM) is developed. The two fluids are coupled through charge exchange.
- Charge exchange introduces multiple length, time-scales in ISM turbulence.
- Energy cascade rates are enhanced.
- Steepening of turbulent spectra. Different from usual Kolmogorov turbulence.
- CEX are dominant at relatively large-scales.
- Smaller scales cannot be coupled by CEX.

Where are we?

Armstrong et al '81



Our simulations cover a region upto 30-50 AU (turbulent correlation length) in the inertial range ISM density spectrum.

A broad range of parameters needs to be explored to understand a variety of turbulent interactions bewteen neutral and plasma.

Turbulent length-scales $l_0 \gg \lambda_{ce}\ :\ k \ll k_{ce}$ are to be explored to understand broader spectrum.

Simulations

We have developed two different codes to simulate coupled plasma-neutral fluid ISM turbulence model.

Configuration space code : Mode structures, in configuration space, are studied using 2D finite difference code.

- This code uses MacCormack two-step predictor-corrector finite difference method.
- It is 2^{nd} order accurate in space.
- Periodic boundary condition is used.

Spectral code: Spectral investigations are done using Spectral-code.

- It uses descritized Fourier transform of turbulent fluctuations.
- Periodic Bcs.
- Time integration is performed by RK4 method.
- Spectral resolution up to 1024^2 is achieved.
- Parallelized.
- Yields a longer inertial range, that cannot be achieved from finite difference code.
- Preserves $\nabla \cdot \boldsymbol{B} = 0$ condition at all time.
- Initial spectrum is close to k^-2.
- Phases are random.