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**Electrostatic Wave Modes in a 4 Component Electron Positron Plasma**

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# ICTP Autumn College on Plasma Physics

# Electrostatic Wave Modes in a 4-Component Electron-Positron Plasma

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## Outline of Talk

- Introduction
- Model Equations
- The Linear Dispersion Relation for the electrostatic Modes
- Numerical Results for electrostatic mode
- Conclusions

- Linear wave modes in a two-temperature electron-positron plasma are studied.

- Electron-positron plasmas are known to occur naturally in astrophysical situations.

- Also occur in laboratory based laser-plasma experiments. Driven by an increased interest to test astrophysical plasma theories under laboratory conditions.

- Theoretical studies have largely focussed on the relativistic regime (Sturrock, 1971; Lakhina and Buti, 1981, Shukla, 1985; Shukla and Stenflo, 1993; Verheest and Lakhina, 1996)

- Studies have also been largely on single temperature electron-positron plasmas. plasmas.

- However, nonrelativistic electron-positron plasmas could also relevant in astrophysical plasmas - electron-positron plasmas are known to radiate very effectively by cyclotron emission, and must therefore eventually cool.

- Theoretical studies of electron-positron plasmas in the nonrelativistic regime have been largely neglected.

- Tsytovich and Wharton, 1978 - theoretical results as well as an idea for a magnetic mirror device to perform electron-positron plasma experiments.

- Iwamoto, 1993 - kinetic treatment of linear waves in an electron-positron plasma examining both longitudinal and transverse collective modes, including the Bernstein modes.

- Stewart and Laing , 1993 - used a multi-fluid approach to study a few aspects of wave propagation.

- Zank and Greaves, 1995 - comprehensive two-fluid model to describe collective modes in a single temperature $e^{-} - e^{+}$ plasma.

- Need for studies of fundamental plasma behaviour of such nonrelativistic plasmas both to add to current understandings of the theory as well as to inform several proposed laboratory based experiments involving such plasmas.

## Nonlinear Studies

- Bharuthram (1992) - investigated the existence and properties of arbitrary amplitude double layers in an unmagnetized $e^{-} - e^{+}$ plasma.

- Pillay and Bharuthram (1992) then investigated the possibility of large amplitude solitons in an unmagnetized $e^{-} - e^{+}$ plasma.

- Verheest *et al.*(1996), investigated acoustic solitons in an unmagnetized $e^{-} - e^{+}$ plasma, containing equal hot and cool components of both species.

**Model:**

- We extend the model of Zank and Greaves to a homogeneous magnetized, four-component $e^- - e^+$ plasma.

- Cool electrons and positrons with equal temperatures and equilibrium densities denoted by $T_c$ and $n_{oc}$, respectively

- Hot electrons and positrons with equal temperatures and equilibrium densities denoted by $T_h$ and $n_{oh}$, respectively

- Wave propagation is at an angle $\theta$ to the ambient magnetic field $\mathbf{B}_o = (B_0, 0, 0)$

**Basic Equations:**

- The hot isothermal species have a Boltzmann distribution.
  Their densities are, respectively,

$$n_{eh} = n_{oh} \exp\left(\frac{e\phi}{T_h}\right) \tag{1}$$

$$n_{ph} = n_{oh} \exp\left(\frac{-e\phi}{T_h}\right) \tag{2}$$

- The cooler adiabatic species are governed by the fluid equations.

  The continuity equations,

$$\frac{\partial n_c}{\partial t} + \nabla.(n_c \mathbf{v}_c) = 0 \tag{3}$$

- the equations of motion,

$$\frac{\partial \mathbf{v}_c}{\partial t} + \mathbf{v}_c.\nabla \mathbf{v}_c = \frac{e}{m_e}\nabla\phi - \frac{e}{m_e}\frac{(\mathbf{v}_c \times \mathbf{B}_o)}{c} - \frac{T_c}{n_c m_e}\nabla n_c \tag{4}$$

- The system is closed by the Poisson equation.

$$\nabla^2\phi = -4\pi e(n_{ph} - n_{eh} + n_{pc} - n_{ec}) \tag{5}$$

- The dispersion law for linear modes:

$$\omega^4 - \omega^2(k^2 v_{tc}^2 + \Omega_e^2) + k^2 v_{tc}^2 \Omega_e^2 \cos^2\theta \quad (6)$$
$$= \frac{k(n_{oc}/n_{oh}) v_{th}^2 (\omega^2 - \Omega_e^2 \cos^2\theta)}{1 + \frac{1}{2}k^2\lambda_{dh}^2}$$

- where:

$$v_{tc} = \sqrt{\frac{T_c}{m_e}}$$

$$v_{th} = \sqrt{\frac{T_h}{m_e}}$$

$$\Omega_e = \frac{eB_o}{m_e c}$$

- $$\lambda_{dh}^2 = \frac{T_h}{4\pi n_{oh} e^2}$$

- Defining the electron-acoustic speed as:

$$\begin{aligned} v_{ea}^2 &= \omega_{pc}^2 \lambda_{dh}^2 \\ &= \left(\frac{n_{oc}}{n_{oh}}\right) v_{th}^2 \end{aligned} \tag{7}$$

the dispersion relation may be expressed as:

$$\begin{aligned} \omega^4 - \omega^2(k^2 v_{tc}^2 + \Omega_e^2) + k^2 v_{tc}^2 \Omega_e^2 \cos^2\theta \\ = \frac{k v_{ea}^2 (\omega^2 - \Omega_e^2 \cos^2\theta)}{1 + \frac{1}{2} k^2 \lambda_{dh}^2} \end{aligned} \tag{8}$$

**The electron-acoustic limit:**

- Consider the limit:

$$kv_{tc} \ll \omega \ll \Omega_e \cos\theta$$

- We obtain from the general dispersion relation:

$$-\omega^2\Omega_e^2 = -\frac{k^2 v_{ea}^2 \Omega_e^2 \cos^2\theta}{1+\frac{1}{2}k^2\lambda_{dh}^2}$$

- which is the dispersion relation for the electron-acoustic mode:

$$\omega = \frac{k_{\|} v_{ea}}{\sqrt{1+\frac{1}{2}k^2\lambda_{dh}^2}}$$

**The upper hybrid limit:**

- If $k\lambda_{dh}^2 >> 1$, then the dispersion relation may be written as:

$$\begin{aligned} \omega^2(\omega^2 - k^2 v_{tc}^2 - \Omega_e^2 - 2\omega_{pc}^2) + k^2 v_{tc}^2 \omega_e^2 & \\ + 2\omega_{pc}^2 \omega_e^2 \cos^2\theta &= 0 \end{aligned} \tag{9}$$

- Further, if we impose $T_c \to 0$, we obtain:

$$\omega^4 - \omega^2 \omega_{UH}^2 + 2\omega_{pc}^2 \Omega_e^2 \cos^2\theta = 0$$

- where, $\omega_{UH}$ is the upper hybrid frequency, defined by:

$$\omega_{UH}^2 = \Omega_e^2 + 2\omega_{pc}^2$$

- and $\omega_{pc}^2 = (n_{oc}/n_{oh})\omega_{ph}^2$

The full dispersion relation is analysed numerically using the following normalisations:

- $$\omega \to \omega_{pe} = \sqrt{\frac{4\pi n_o e^2}{m_e}}$$

  where

  $$n_o = n_{oc} + n_{oh}$$

- $$\text{temperatures} \to T_h$$

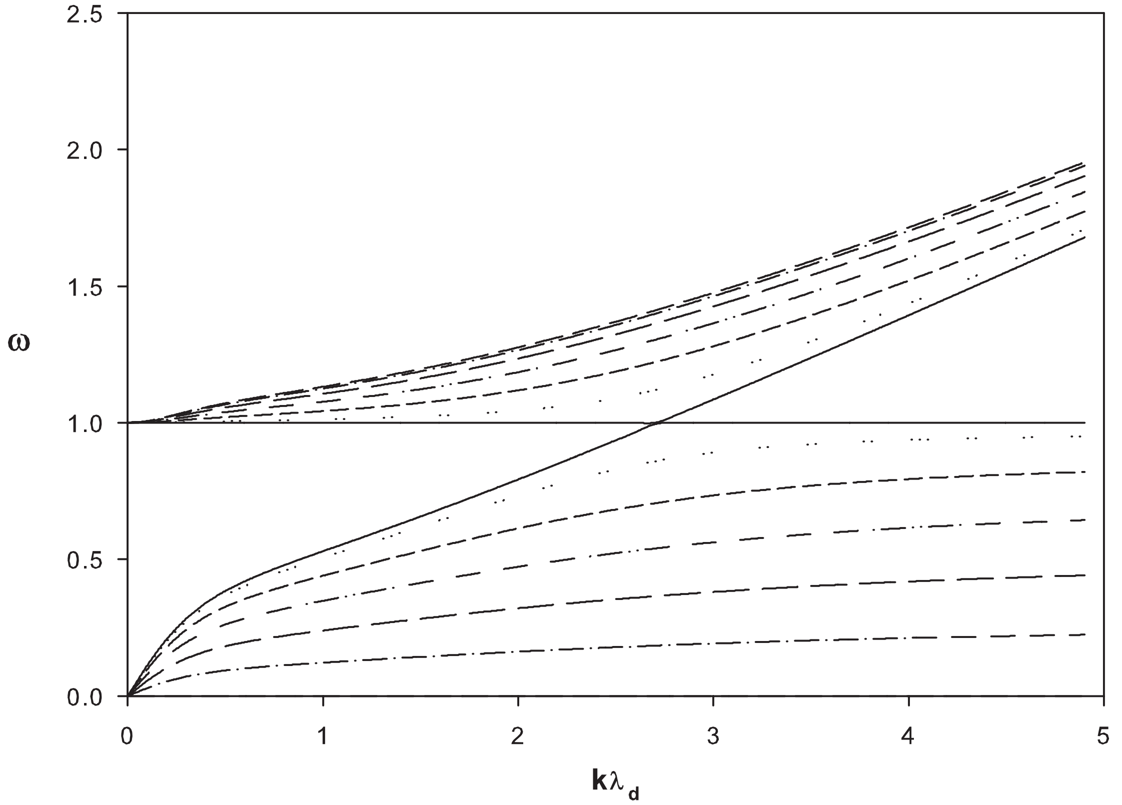


Figure 1: D-R for electrostatic waves for various angles of propagation showing the acoustic and cyclotron branches. $T_h/T_c = 100$; $n_{oc}/n_{oh} = 0.1$ and $\theta = 0^o$,$15^o$, $30^o$, $45^o$; $60^o$; $75^o$ and $90^o$.

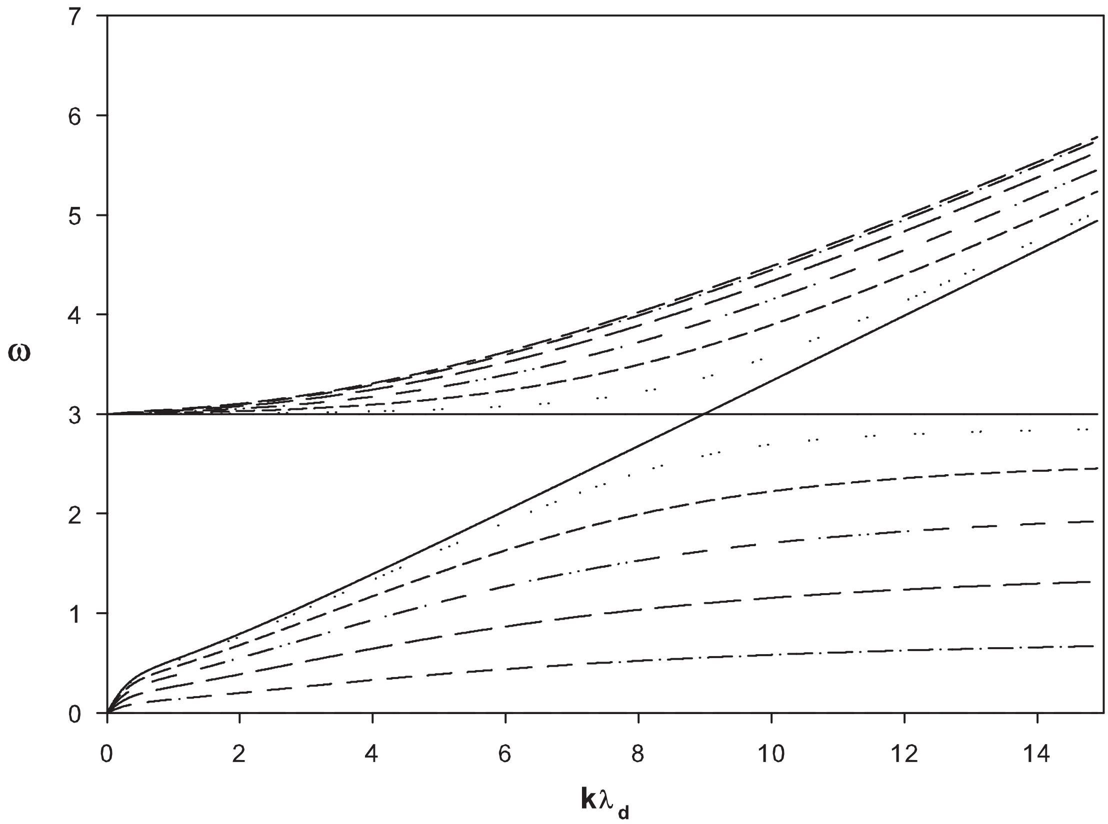


Figure 2: D-R for electrostatic waves for various angles of propagation showing the acoustic and cyclotron branches. $T_h/T_c = 100$; $n_{oc}/n_{oh} = 0.1$ and $\theta = 0^o, 15^o, 30^o, 45^o; 60^o; 75^o$ and $90^o$.

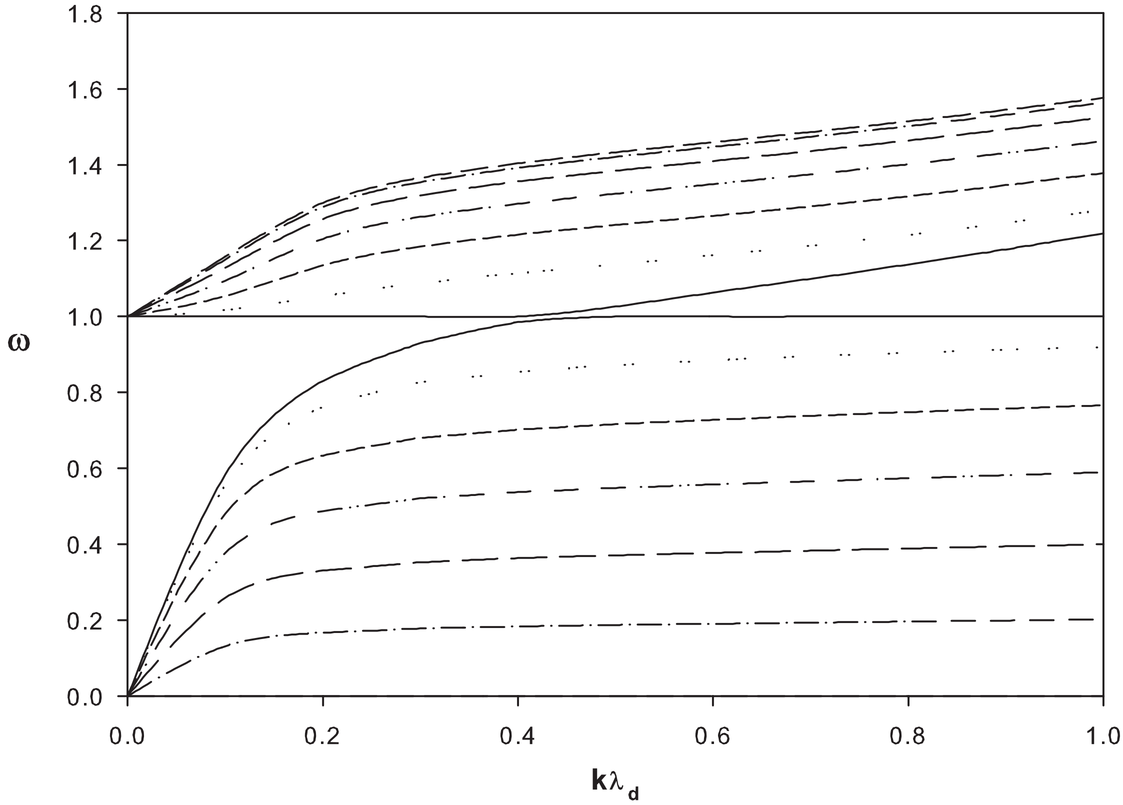


Figure 3: D-R for electrostatic waves for various angles of propagation showing the acoustic and cyclotron branches. $T_h/T_c = 100$; $n_{oc}/n_{oh} = 0.5$ and $\theta = 0^o$,$15^o$, $30^o$, $45^o$; $60^o$; $75^o$ and $90^o$.

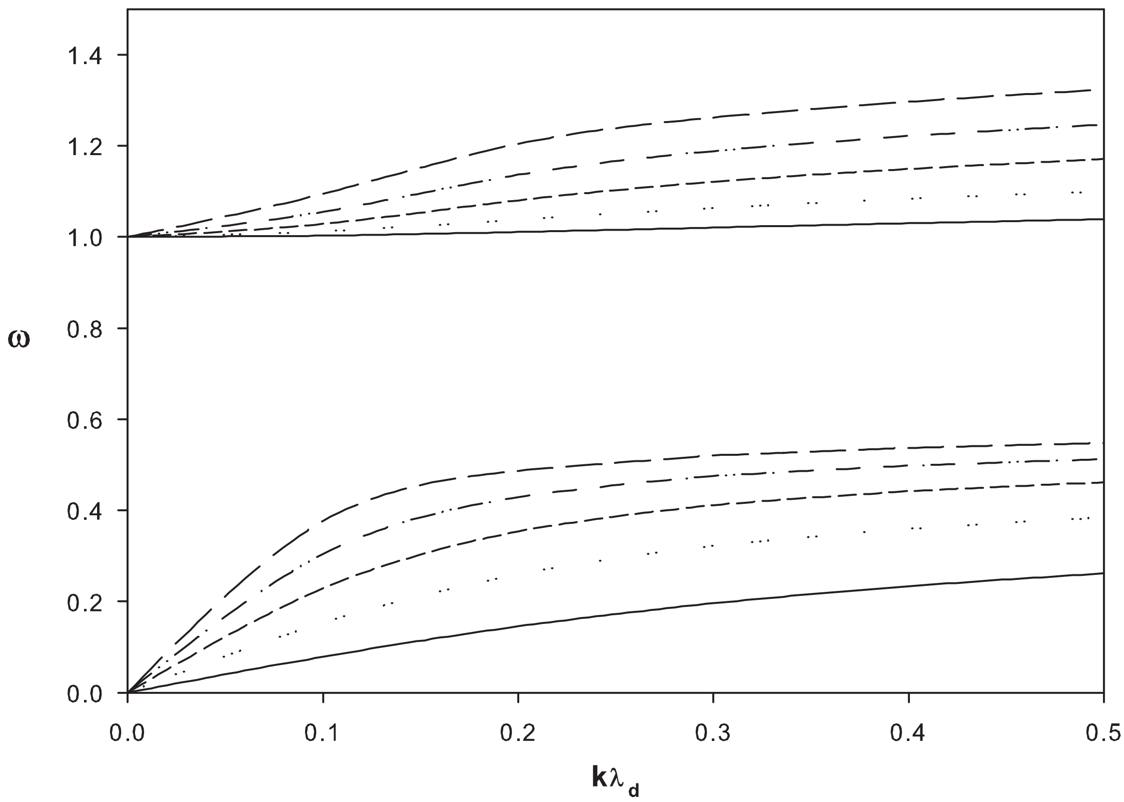


Figure 4: D-R for electrostatic waves for various density ratios and fixed angle of propagation. $T_h/T_c$ = 100; $\theta$ = 45$^o$. $n_{oc}/n_{oh}$ = 0.1; 0.2; 0.3; 0.4; 0.5.

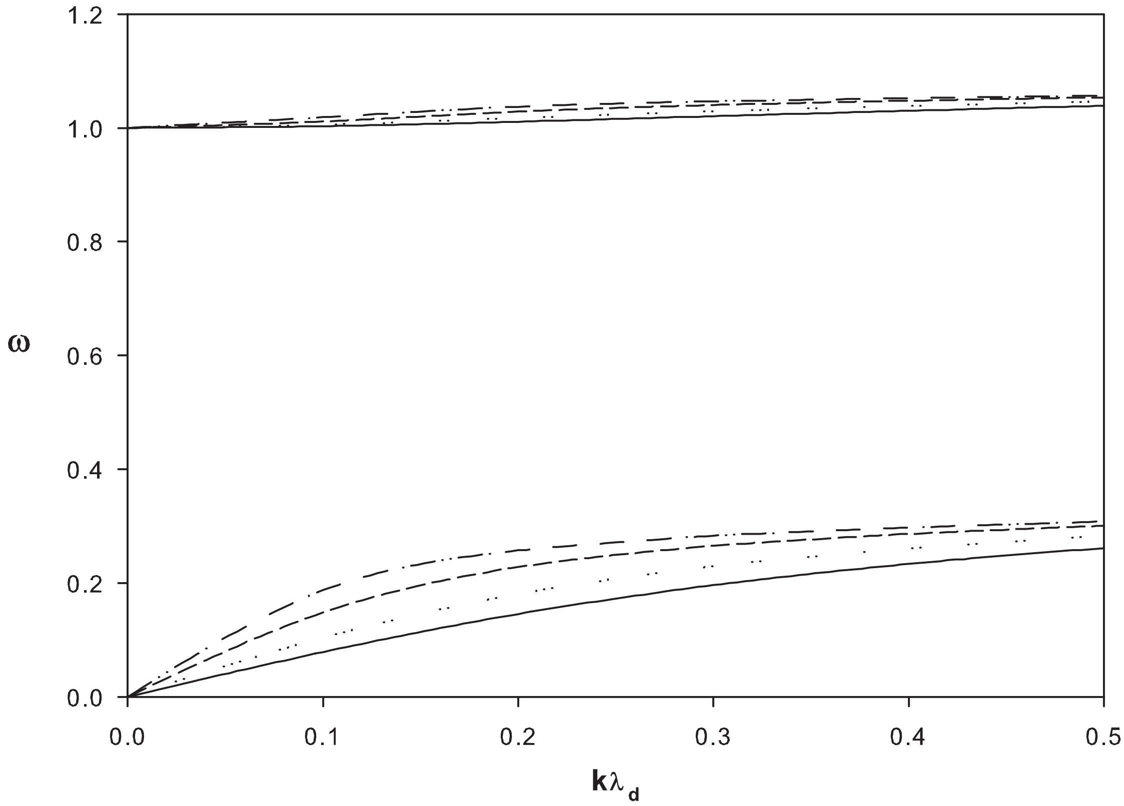


Figure 5: D-R for electrostatic waves for various temperatures ratios and fixed angle of propagation. $n_{oc}/n_{oh}$ = 0.1; $\theta$ = 45$^o$. $T_h/T_c$ = 100; 200; 500; 1000

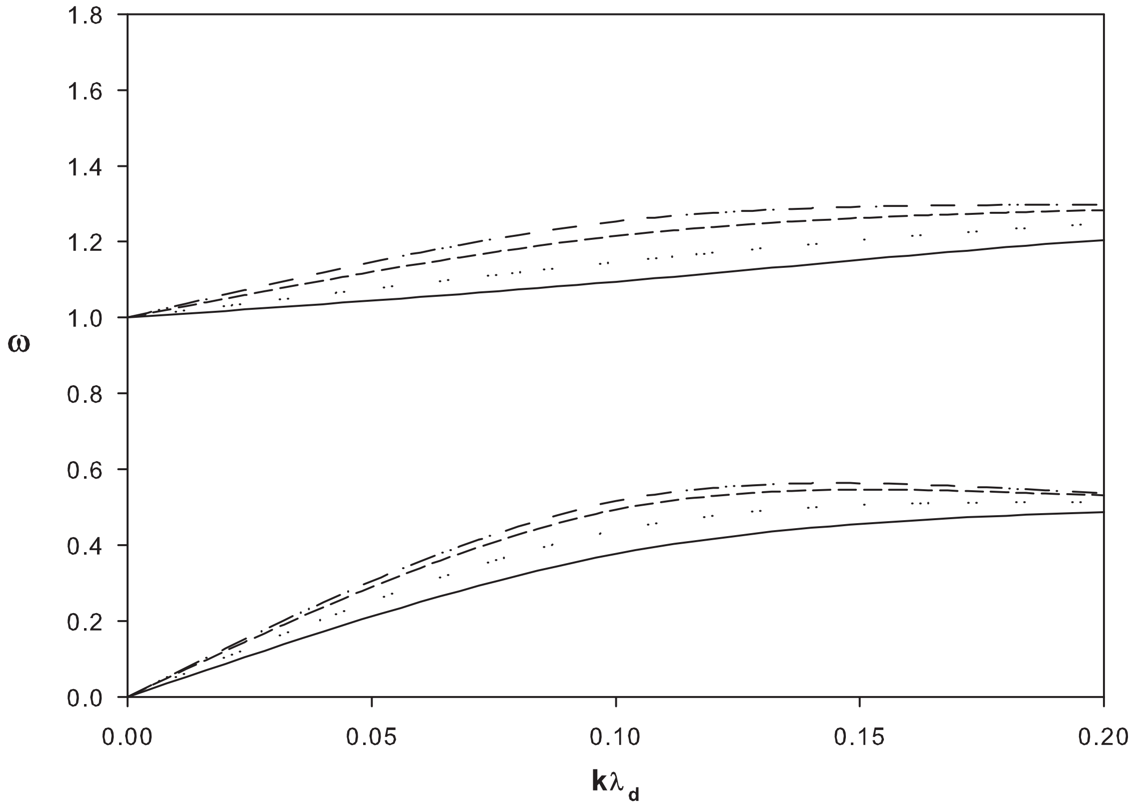


Figure 6: D-R for electrostatic waves for various temperatures ratios and fixed angle of propagation. $n_{oc}/n_{oh}$ = 0.5; $\theta$ = 45$^o$. $T_h/T_c$ = 100; 200; 500; 1000

## Conclusions

- In a two temperature electron-positron plasma, the acoustic mode is of the electron-acoustic type, rather than the Langmuir-type obtained by Zank et. al.

- The plasma frequency for the upper-hybrid mode is fundamentally that of the hot component, factored down/up by the ratio of cold to hot number densities.

- Presence of a cold population reduces the order of magnitude of the frequencies obtained for the typical acoustic- and cyclotron branches.

- Increasing the cold population forces the existence of the purely acoustic-like modes to lower $k\lambda d$-domains (i.e. long wavelength limits.

THE END