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Magnetic Stochasticity: origin, consequences, control

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Magnetic Stochasticity

origin, consequences, control

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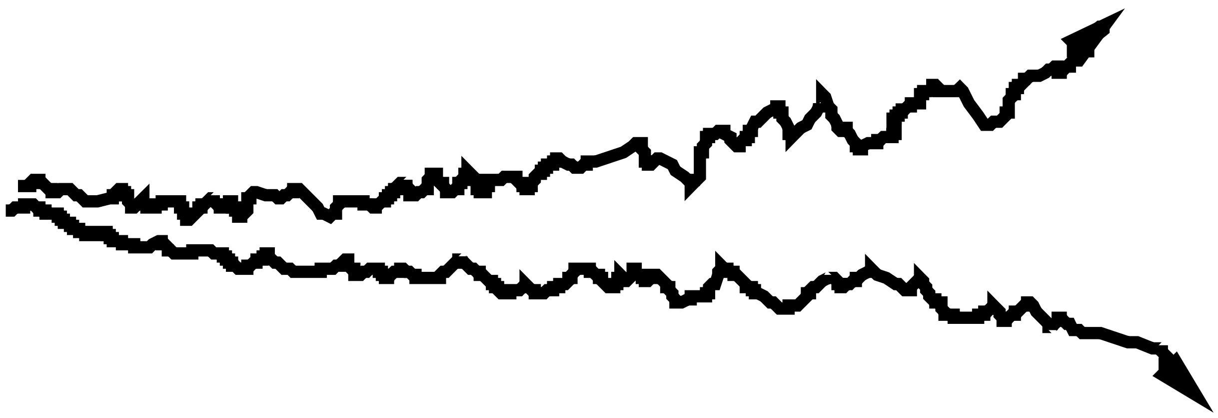
Ordered field

Stochastic field



affects transport, since particles follow field lines

Applications

- Fusion:

  can defeat confinement,

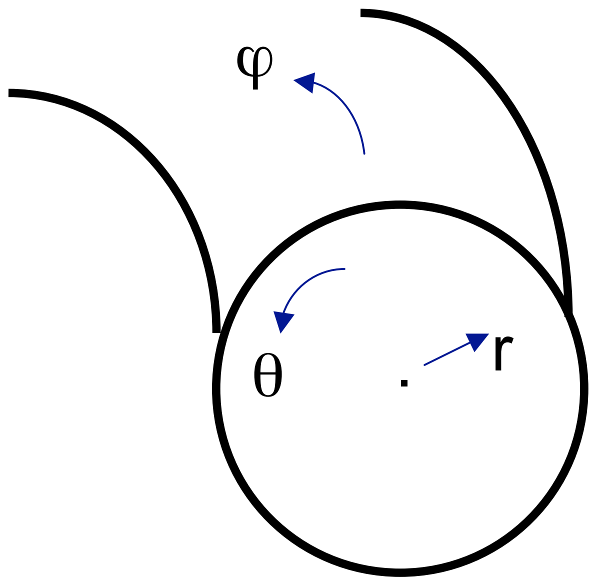
  can be used to control heat flow

- Astrophysics

  cooling flows in galaxy clusters

  (stochastic field affects heat conduction)

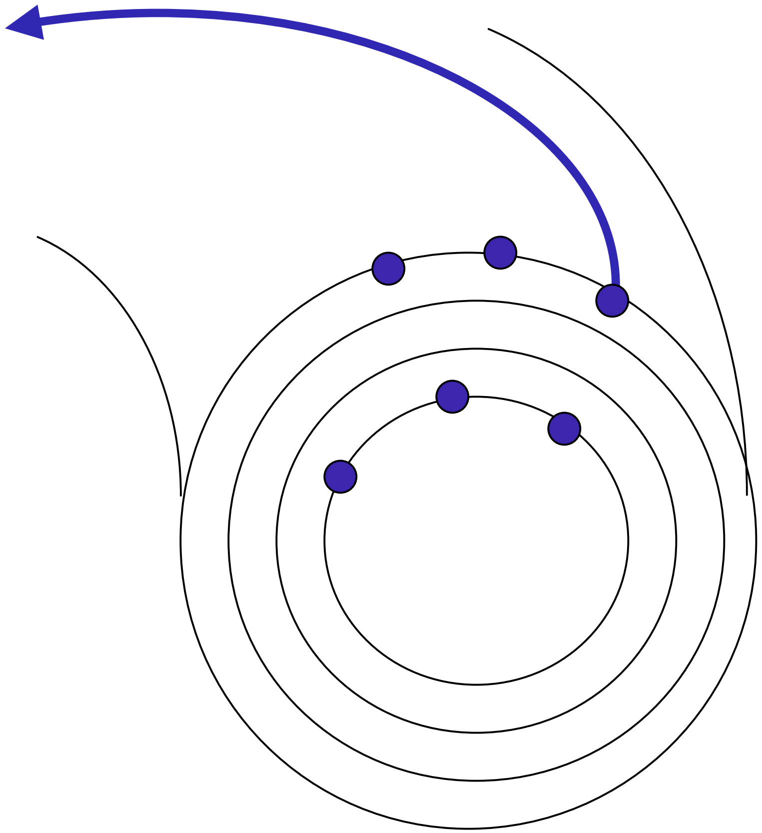
Consider torus with helical field



$$\vec{B}(r) = \underbrace{\vec{B}_\vartheta(r)}_{\text{poloidal}} + \underbrace{\vec{B}_\varphi(r)}_{\text{toroidal}}$$

Field lines nearly lie on circles

magnetic surfaces



surfaces upon which B lines reside

Ideally surfaces are concentric tori,

Add a small perturbation in radial magnetic field $\tilde{B}_r$

with $$\frac{\tilde{B}_r}{B} << 1$$

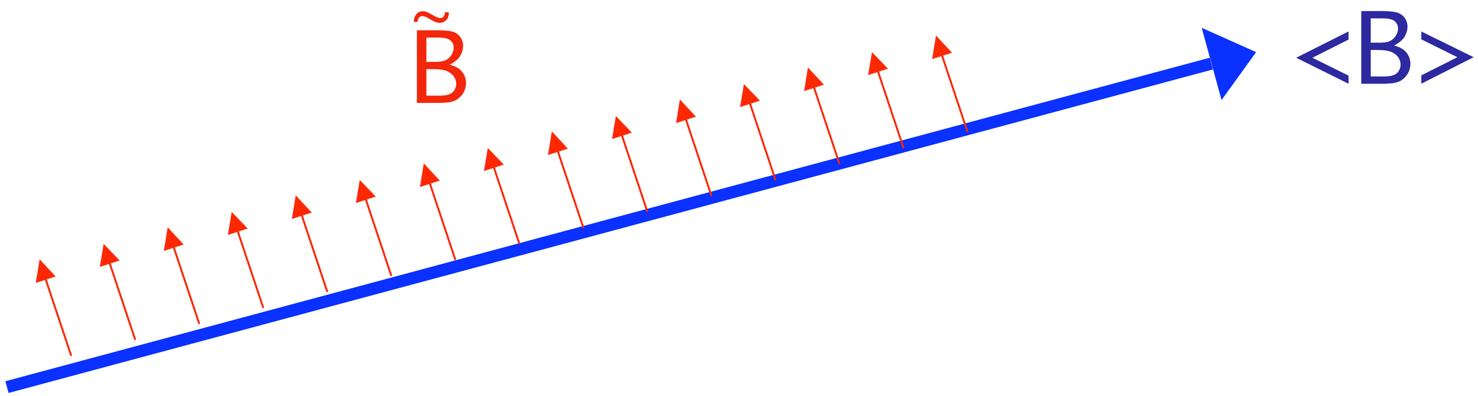
Possible sources of perturbations:

- instability
- magnetic field error
- deliberate additional field

How can a small perturbation have a large effect on the field structure?

If $k_{||} \sim 0$ (resonance), then

small magnetic fluctuation --> large field line excursion



add a perturbation (e.g., an instability)

$$\vec{B} = \vec{B}(r) + \hat{r}\tilde{B}_r(r)\sin(m\vartheta - n\varphi)$$

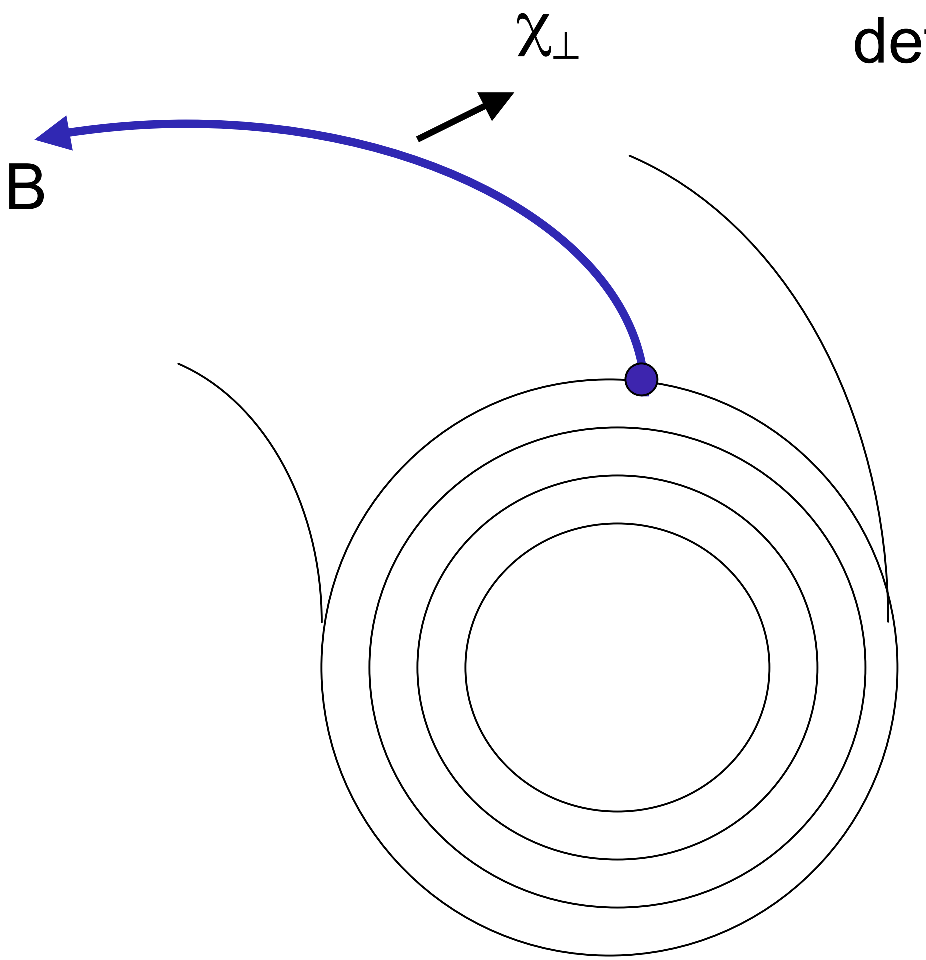
thus, $\vec{k} = \frac{m}{r}\hat{\vartheta} - \frac{n}{R}\hat{\varphi}$

consider region near $k_{\parallel} = 0$

or $\vec{k} \bullet \vec{B} = \frac{m}{r}B_\vartheta - \frac{n}{R}B_\varphi = 0$

$$q = \frac{m}{n}$$

where $q = \frac{rB_\varphi}{RB_\vartheta}$



define coordinate $\perp \ \vec{B}, \hat{r}$

$$\chi_\perp = m\vartheta - n\varphi$$

$$\tilde{B}_r = \tilde{B}_r(r)\sin\chi_\perp$$

Perturbation is constant
along B at one radius

Field line equation

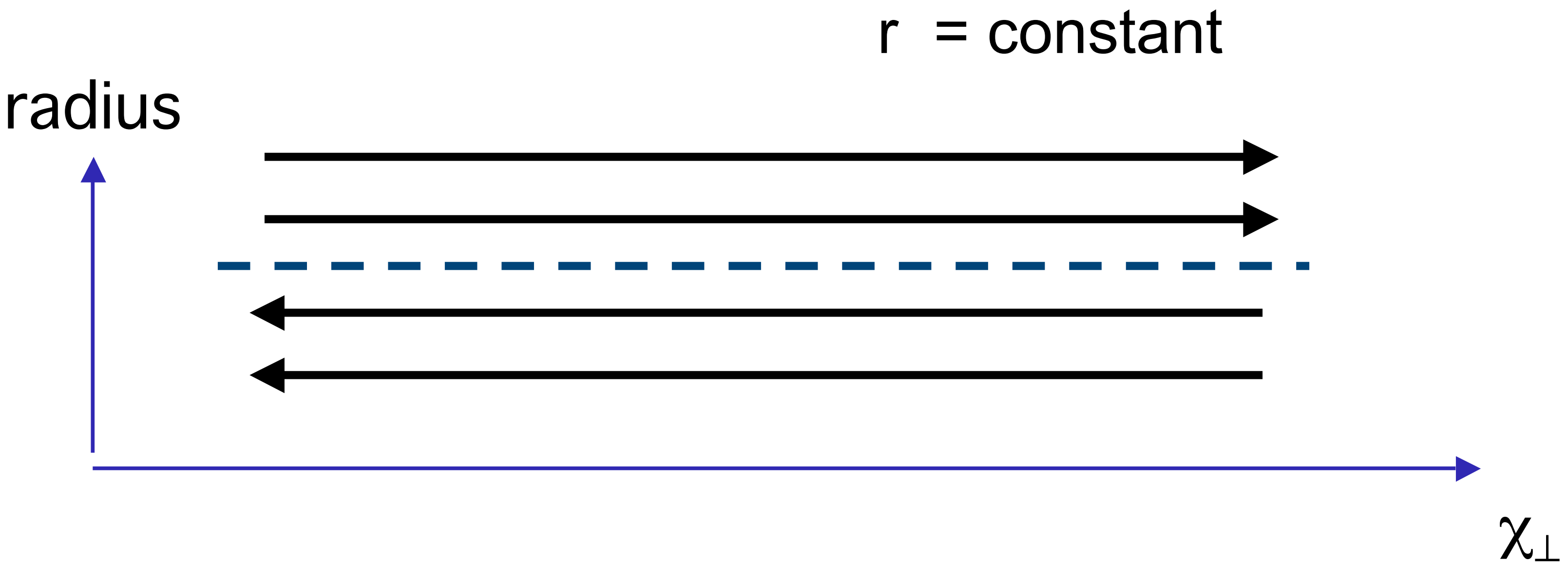
$$\frac{dr}{B_r} = \frac{h_\chi d\chi_\perp}{B_\perp}$$

$$\frac{dr}{\tilde{B}_r \sin\chi_\perp} = \frac{h_\chi d\chi_\perp}{B'_\perp r}$$

$$\frac{rdr}{d\chi_\perp} = h_\chi \frac{\tilde{B}_r}{B'_\perp} \sin\chi_\perp$$

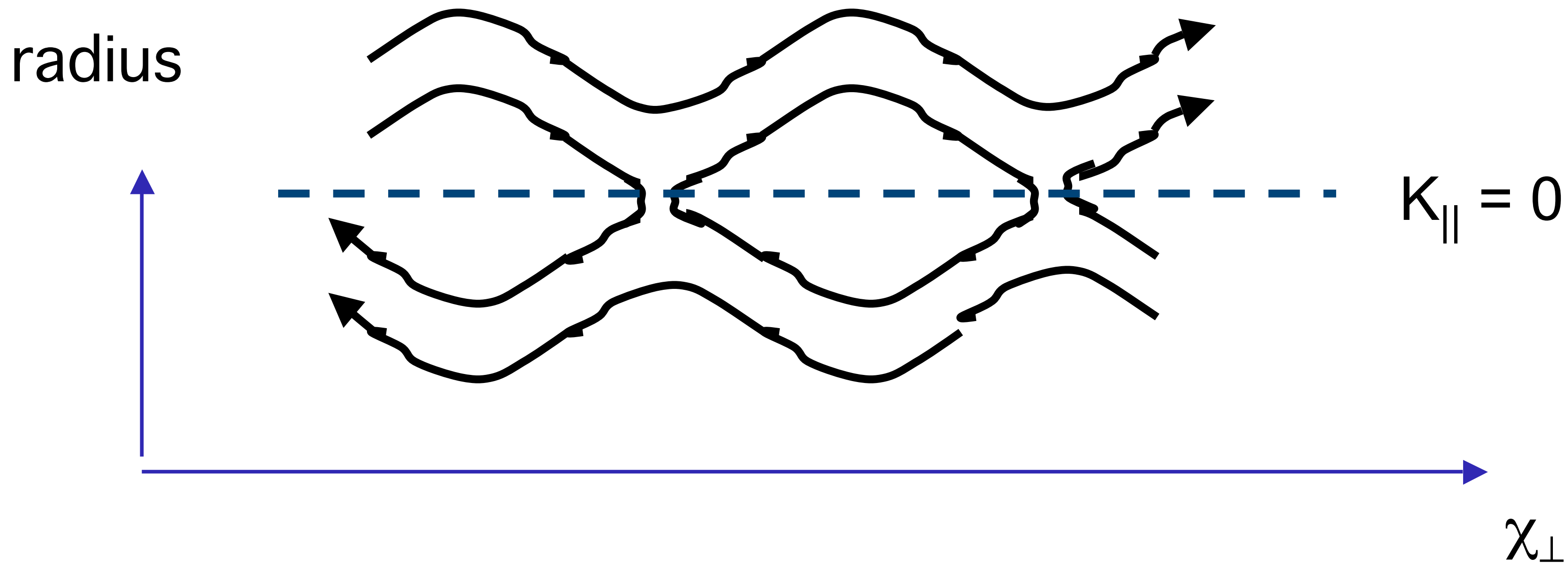
$$r = \pm\sqrt{\left(\frac{h_\chi}{4}\frac{\tilde{B}_r}{B'_\perp}\cos\chi_\perp + C\right)}$$

Field lines without perturbations

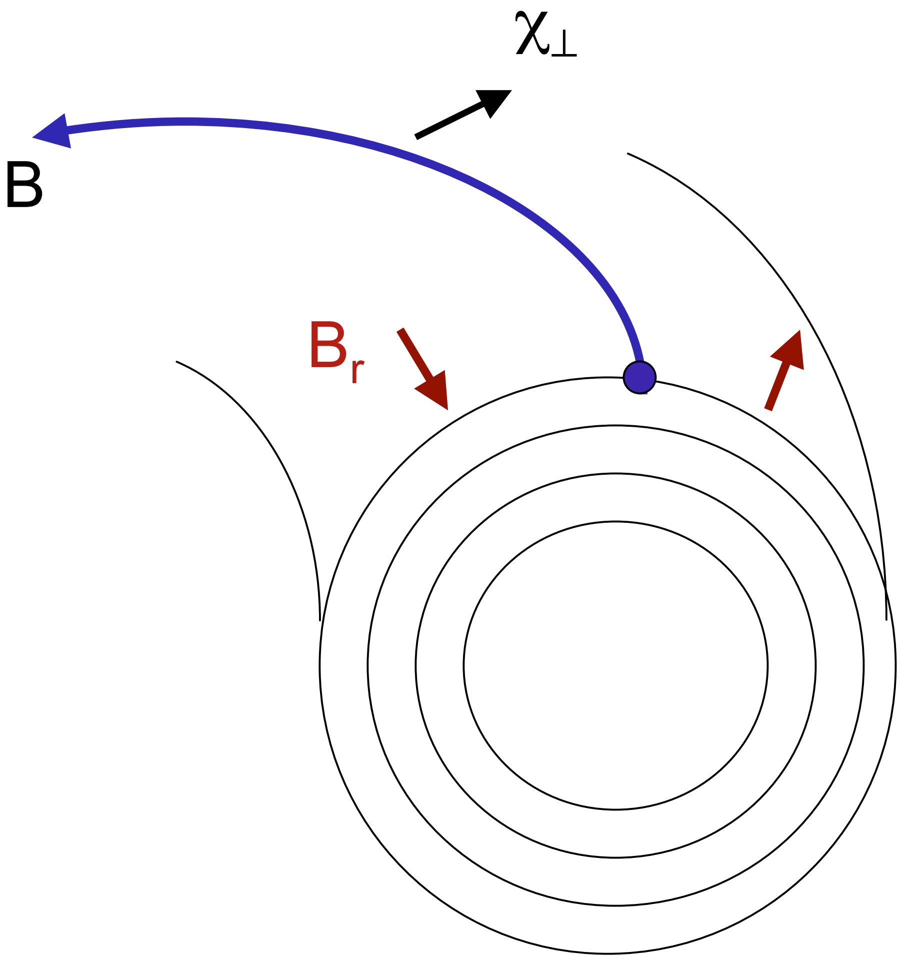


Field lines with perturbations

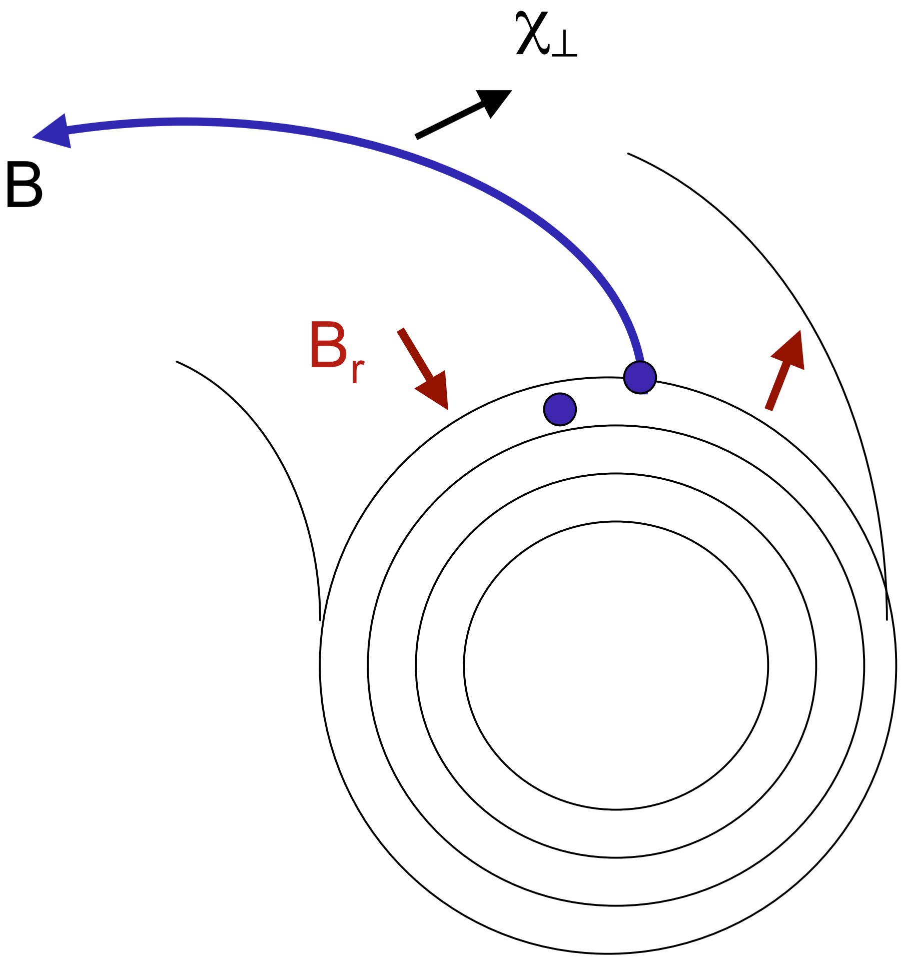
$$r = \pm\sqrt{\left(h_\chi \frac{\tilde{B}_r}{B_\perp}\cos\chi_\perp + C\right)}$$

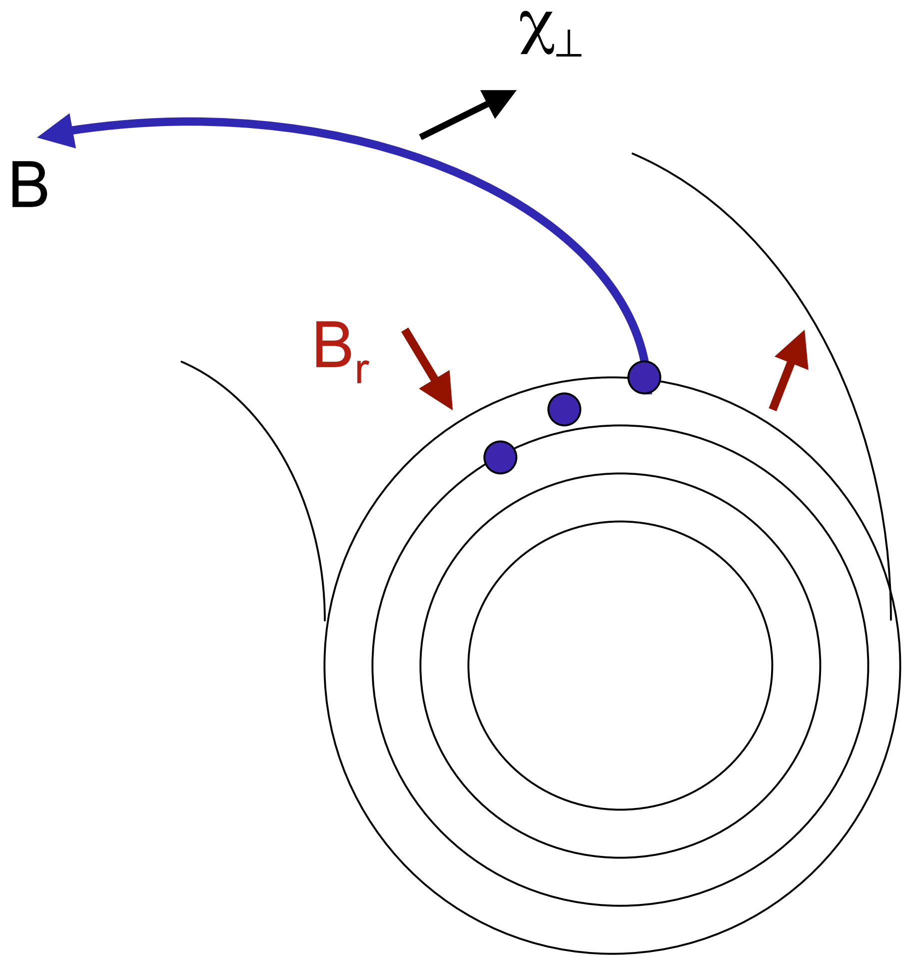


reconnection has occurred

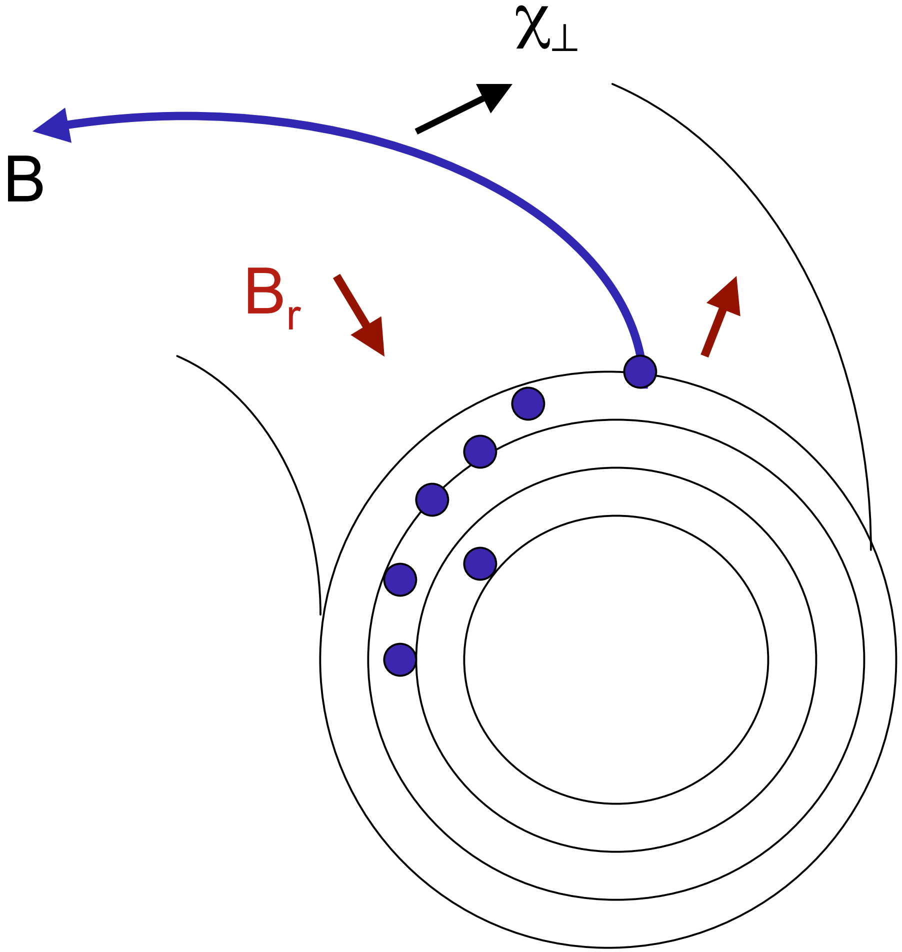


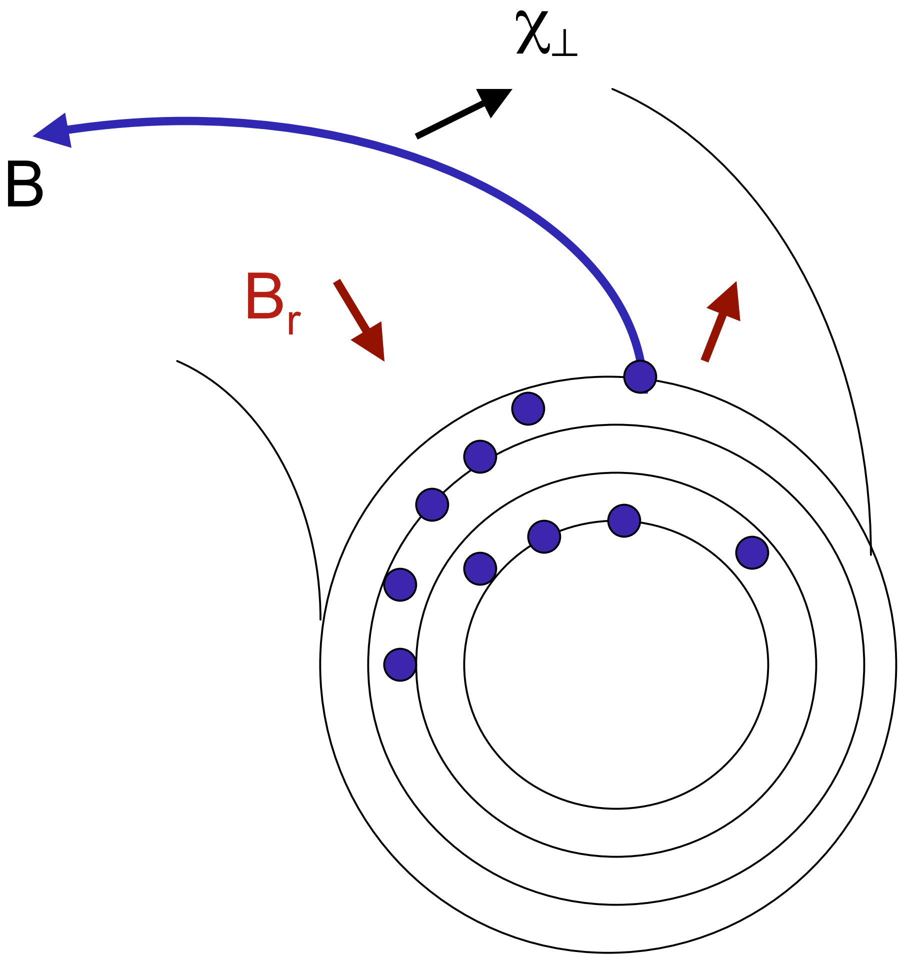
$$\tilde{B}_r \sim \sin(m\vartheta - n\varphi)$$


χ⊥
B
Br

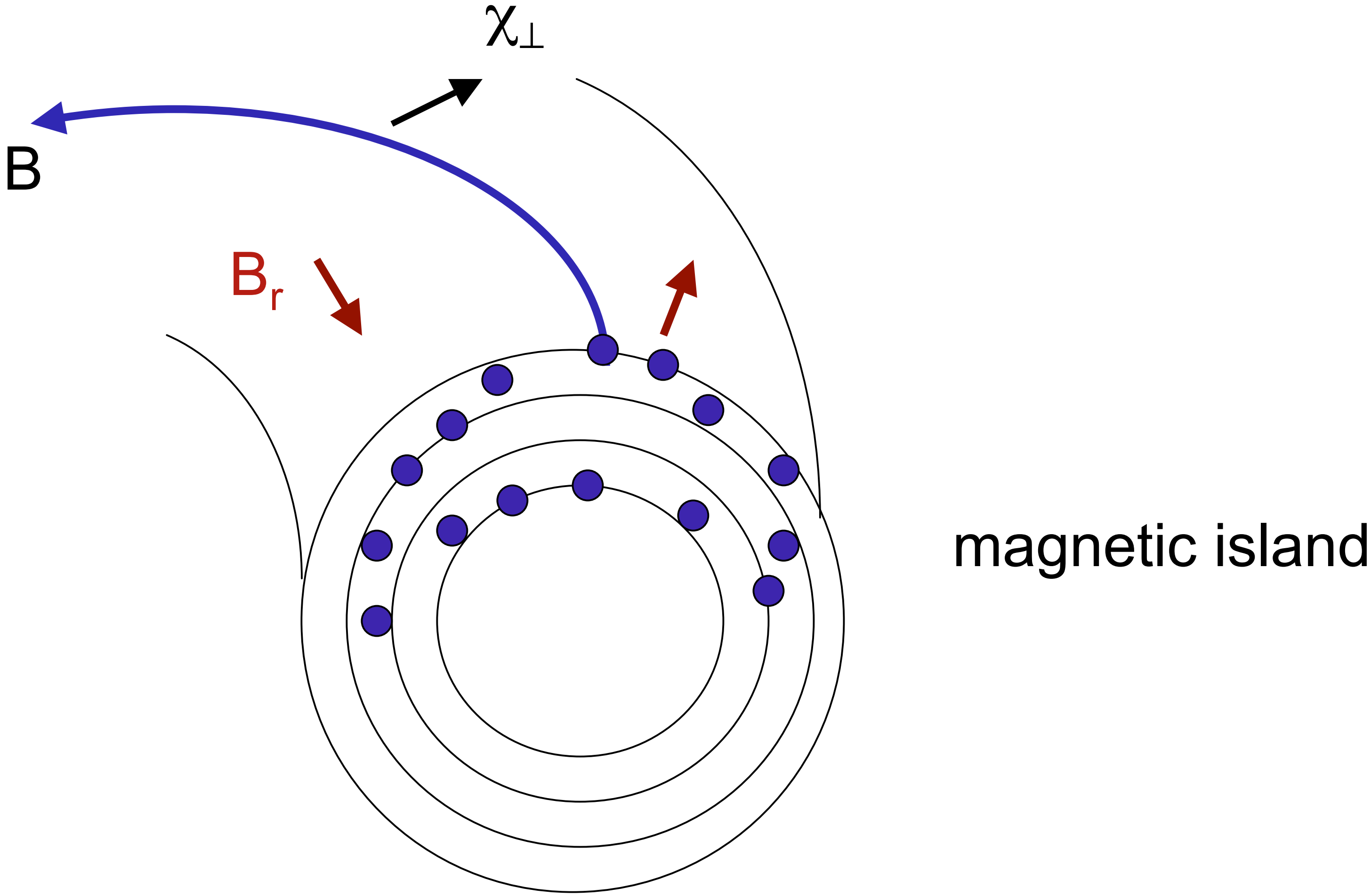


$\chi_\perp$
B
B_r


χ⊥
B
B r

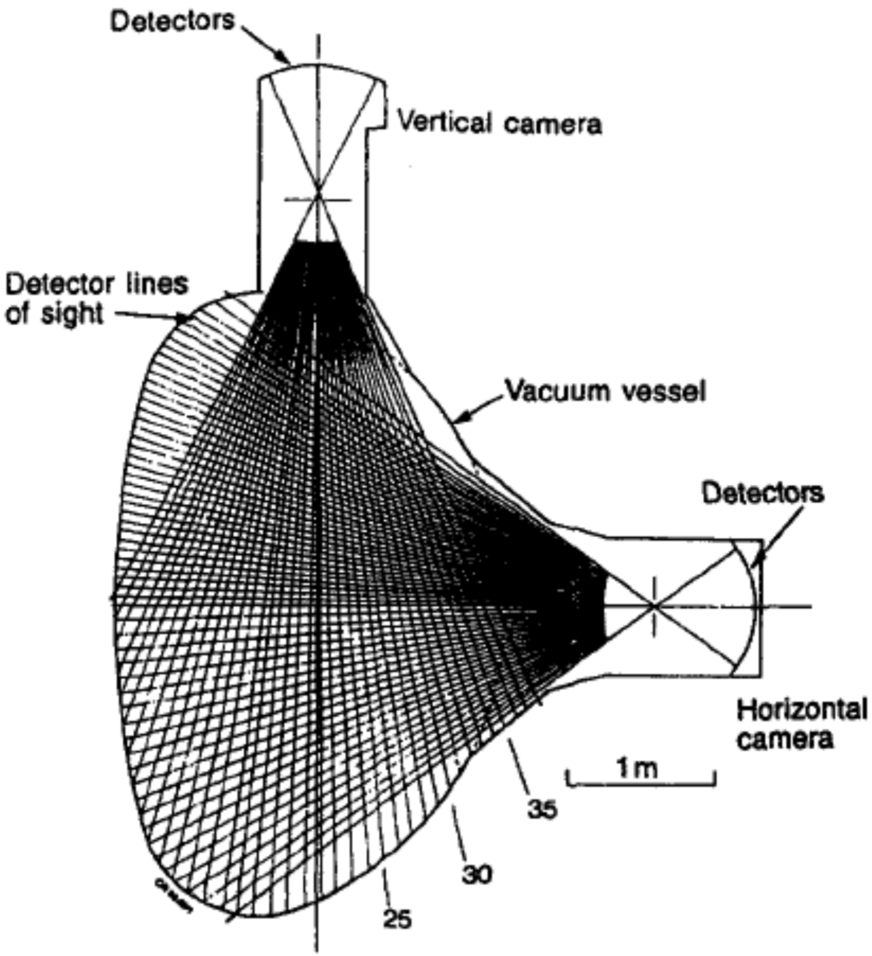


χ⊥
B
B r

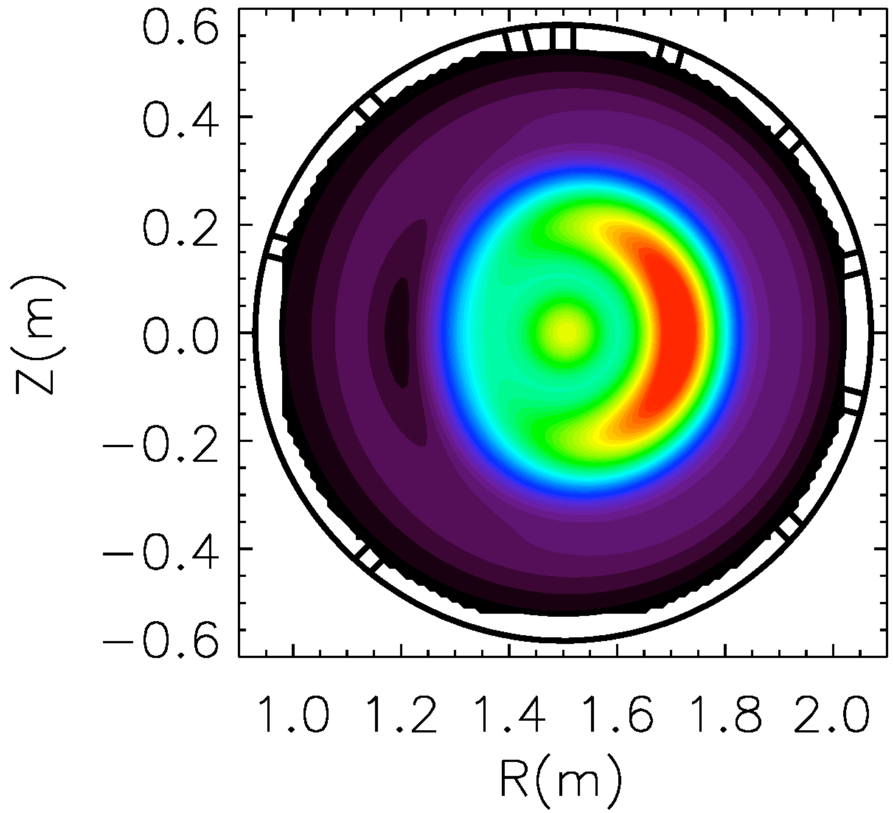


χ⊥
B
B_r
magnetic island

Soft xray tomography maps magnetic surfaces



SXR images of magnetic surfaces



With one dominant tearing instability

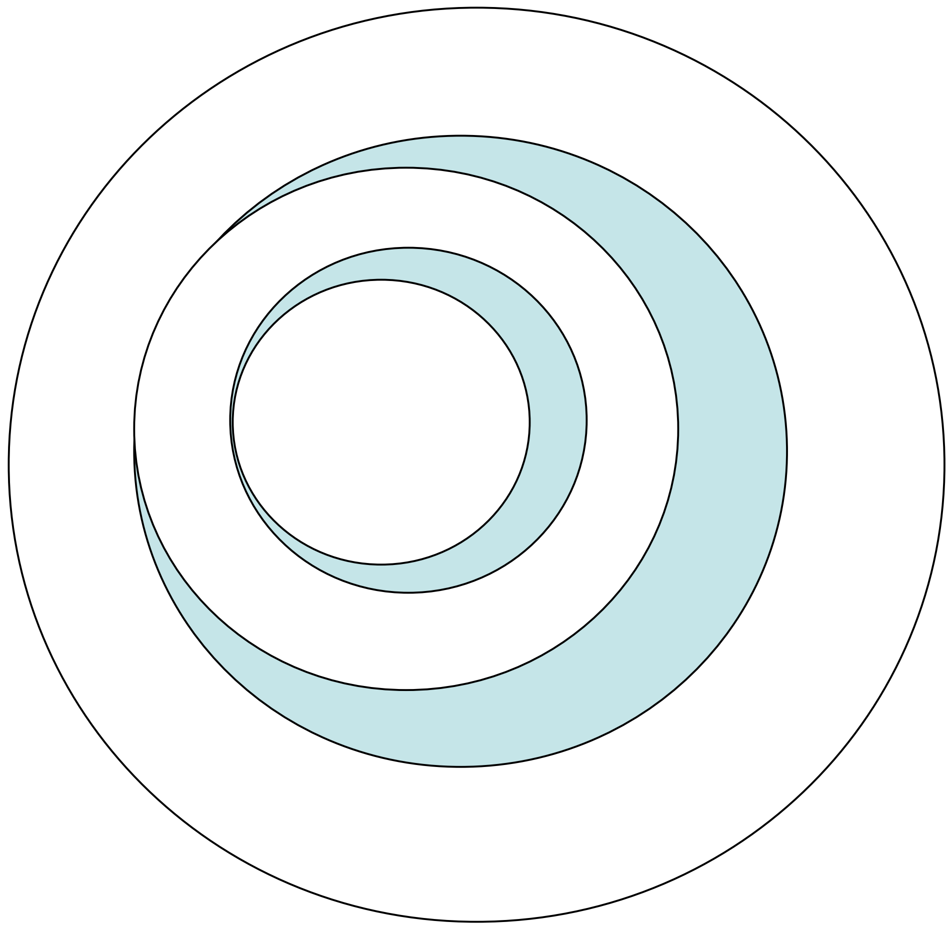
From a reversed field pinch experimet

(similar to a tokamak)

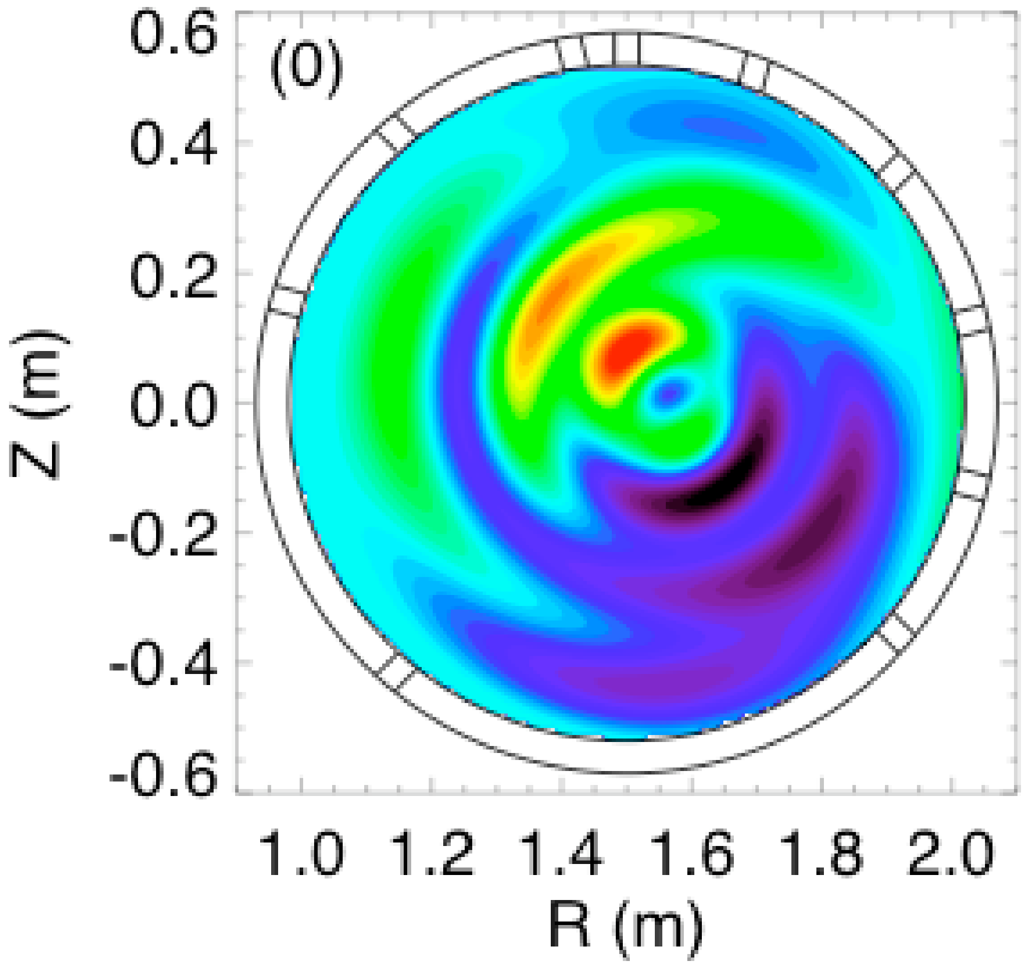
Can have multiple islands

Two modes (m_1, n_1) and (m_2, n_2) produce 2 islands,

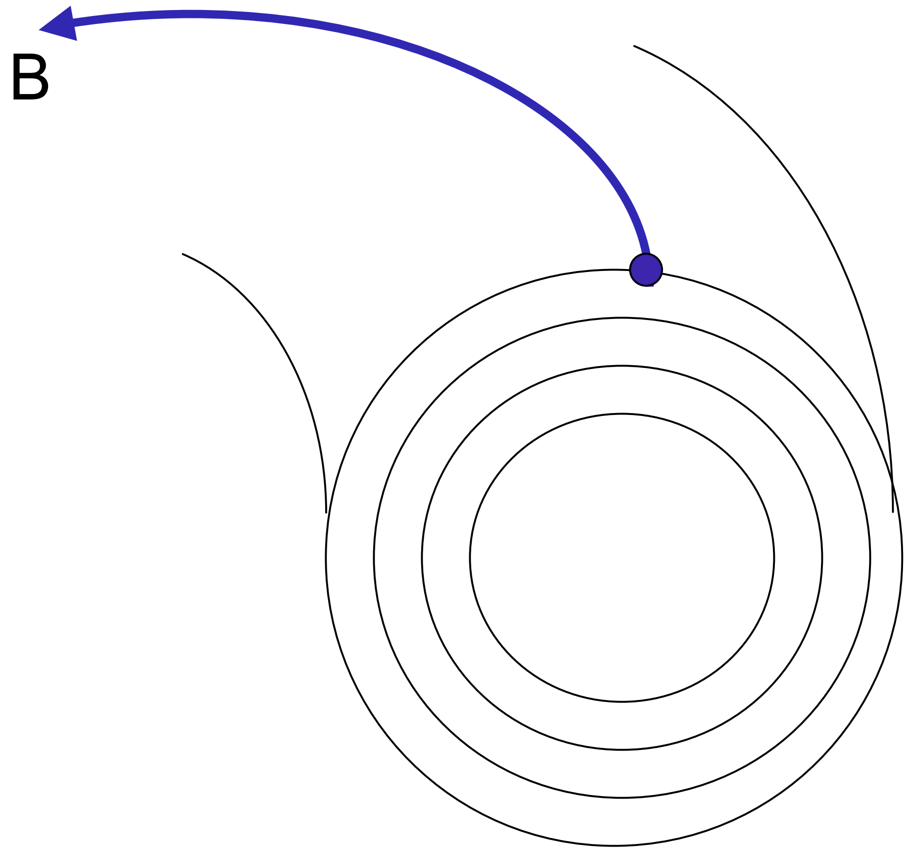
At radii where $q(r) = \frac{m_1}{n_1}$ and $q(r) = \frac{m_2}{n_2}$

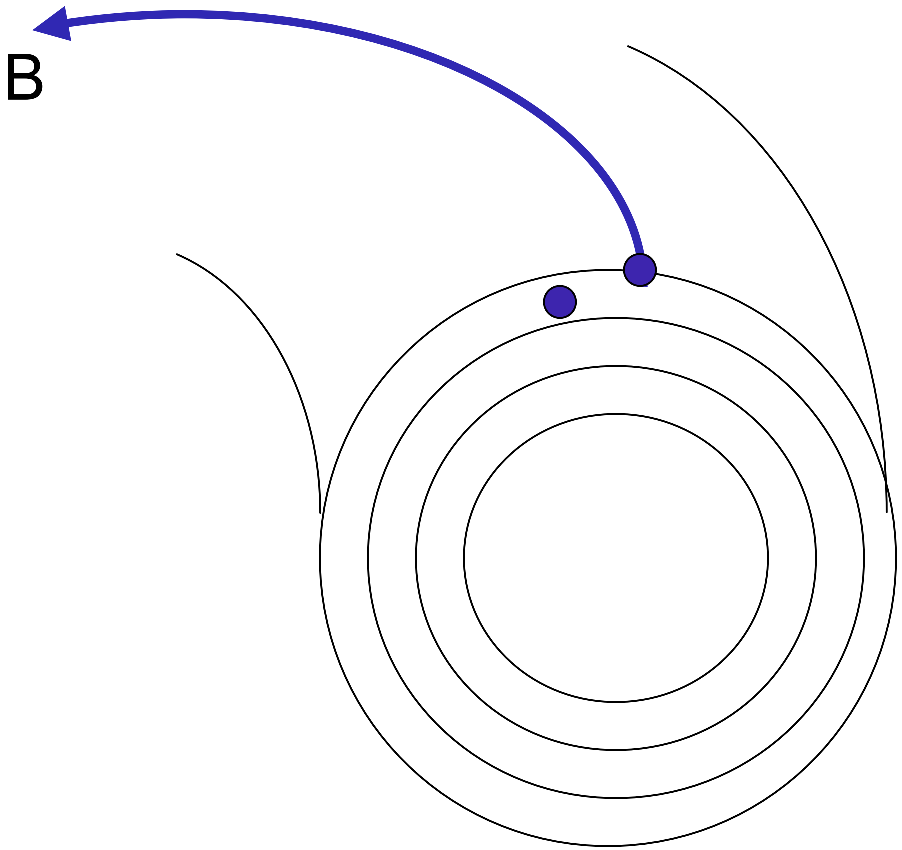


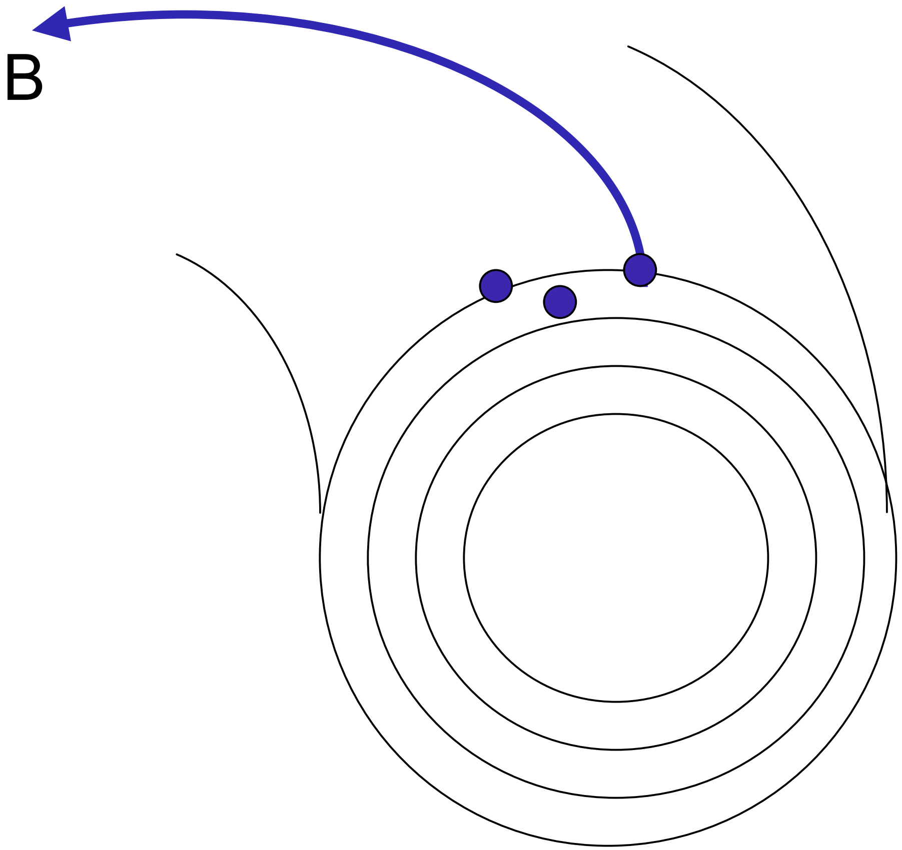
SXR tomography with 2 tearing modes



If magnetic islands overlap,
field lines wander stochastically




B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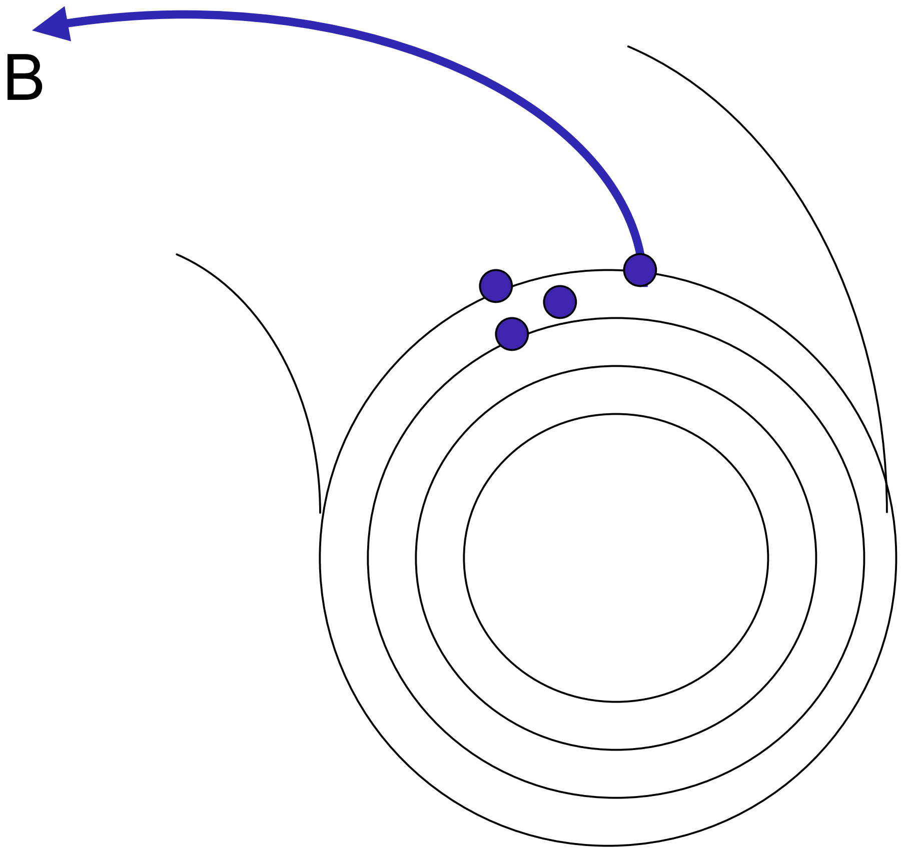
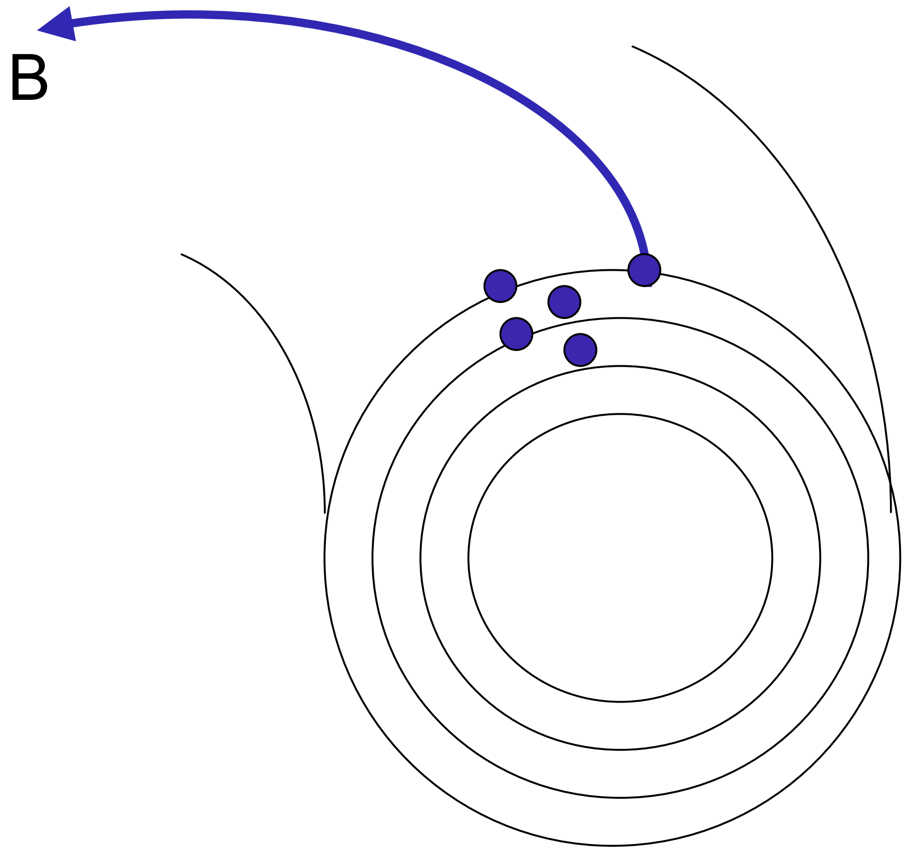
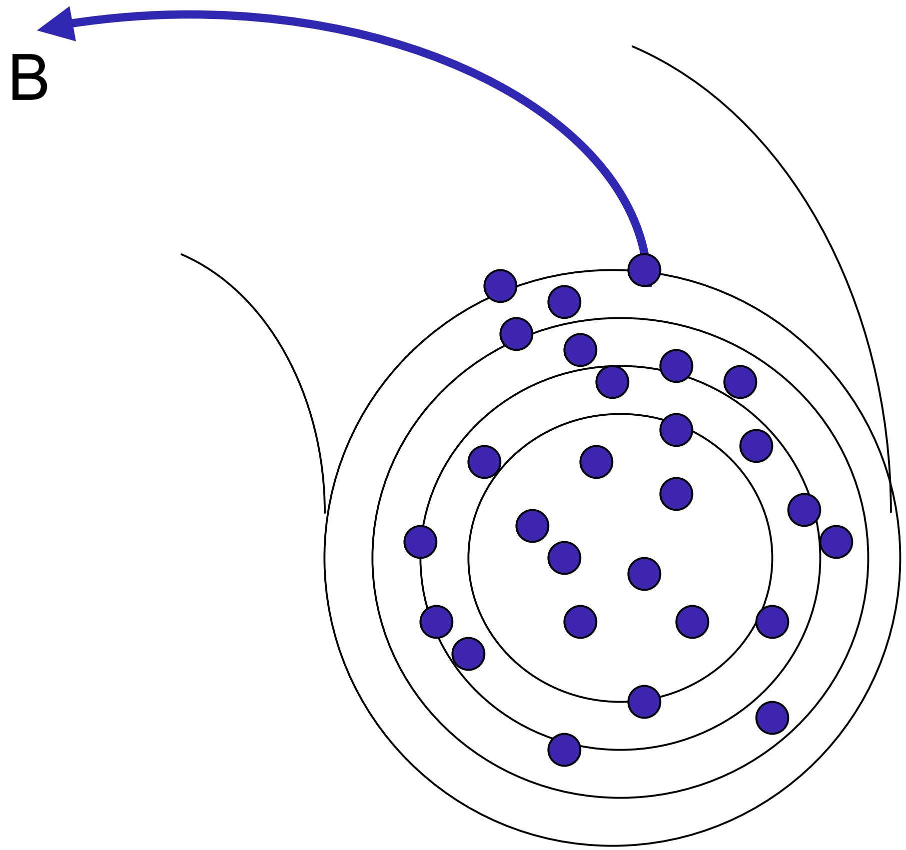
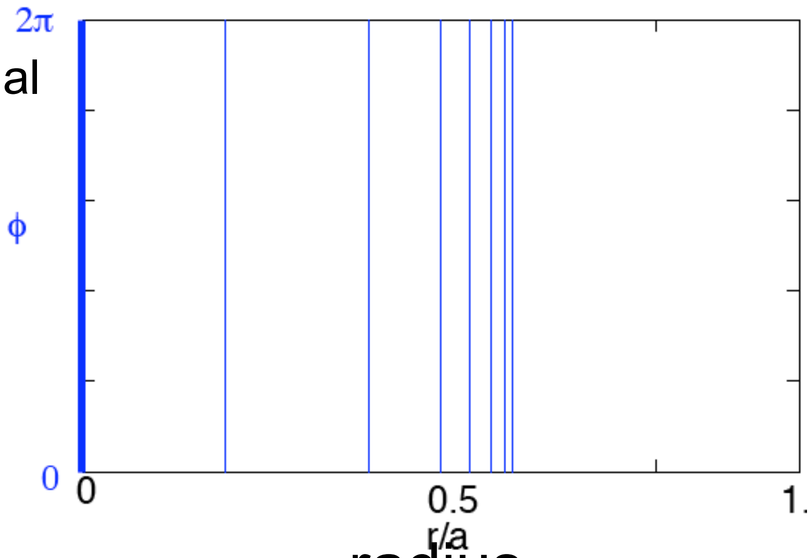

B


B

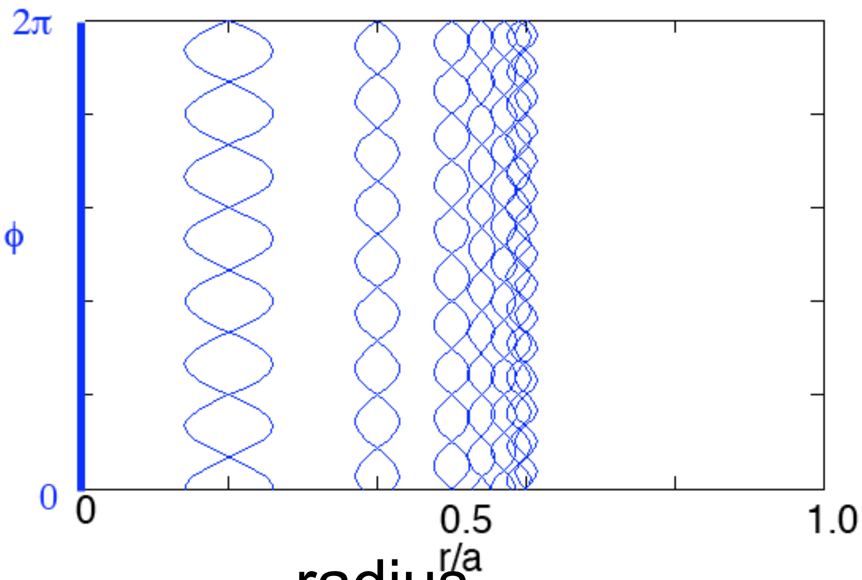

B

Can also see island structure in a toroidal cut

toroidal angle



radius



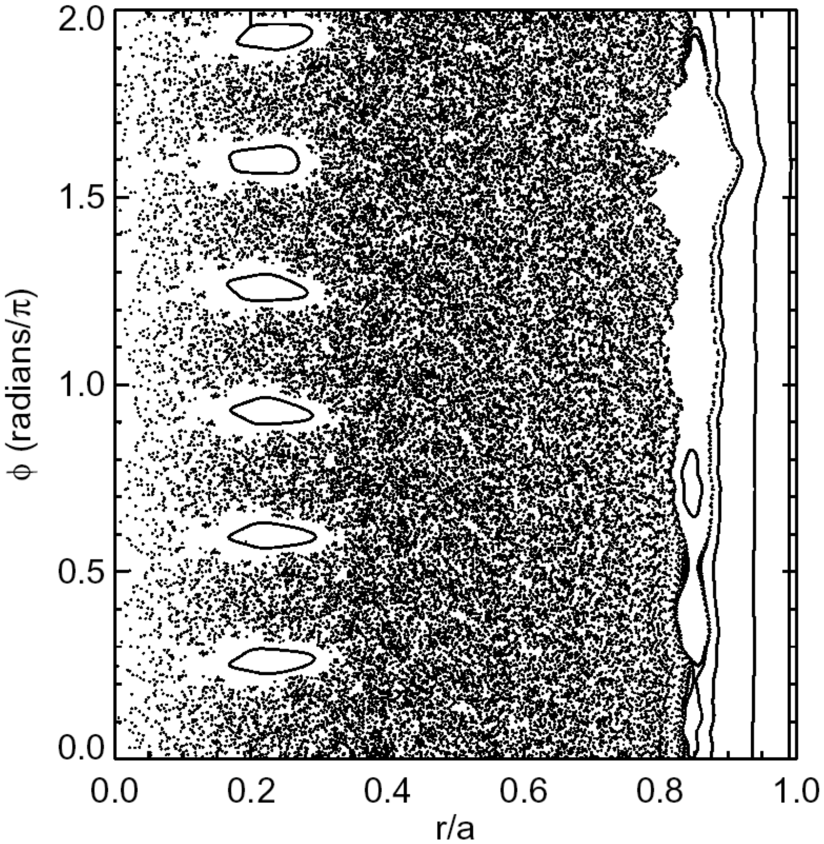
radius

If islands overlap → stochasticity

From MHD computation for multiple tearing modes

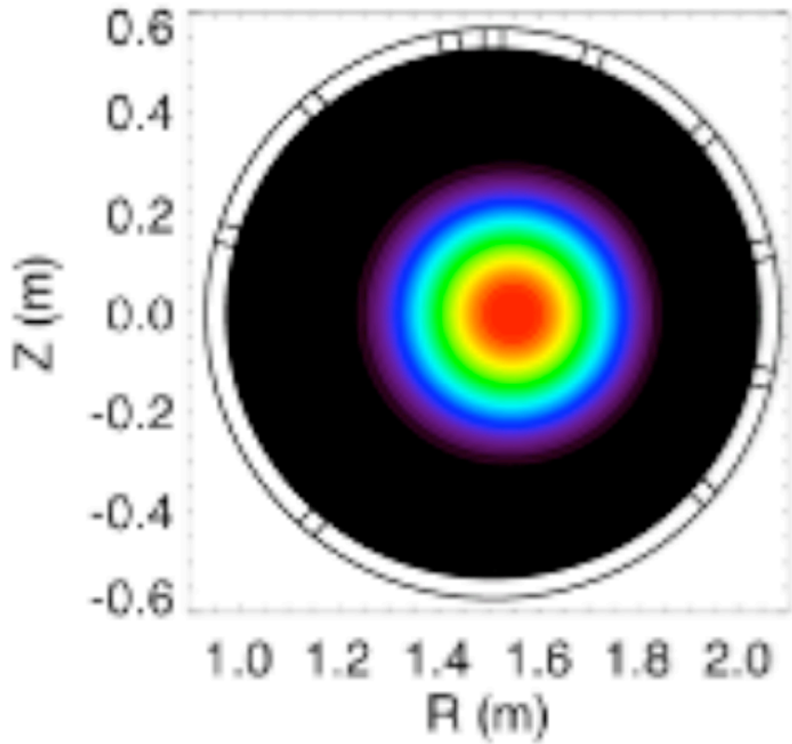
(with experimental input)

toroidal
angle



radius

SXR tomography in plasma with multiple tearing modes

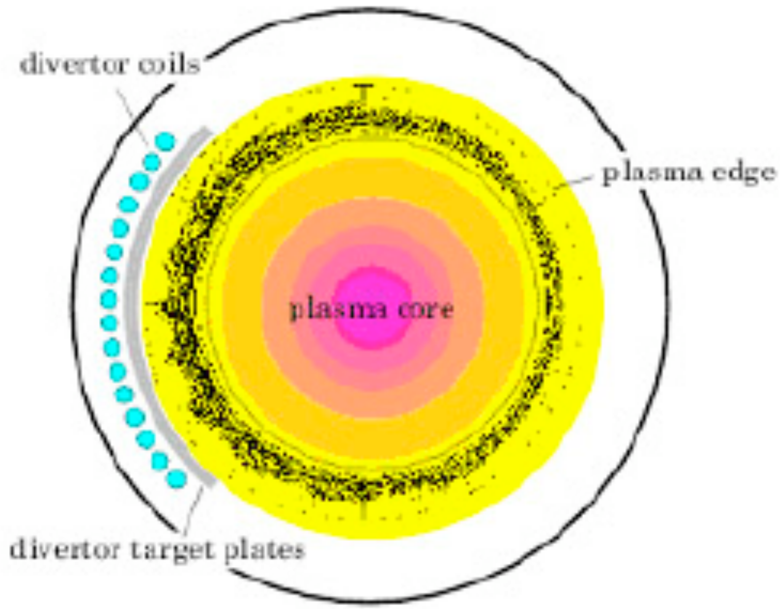


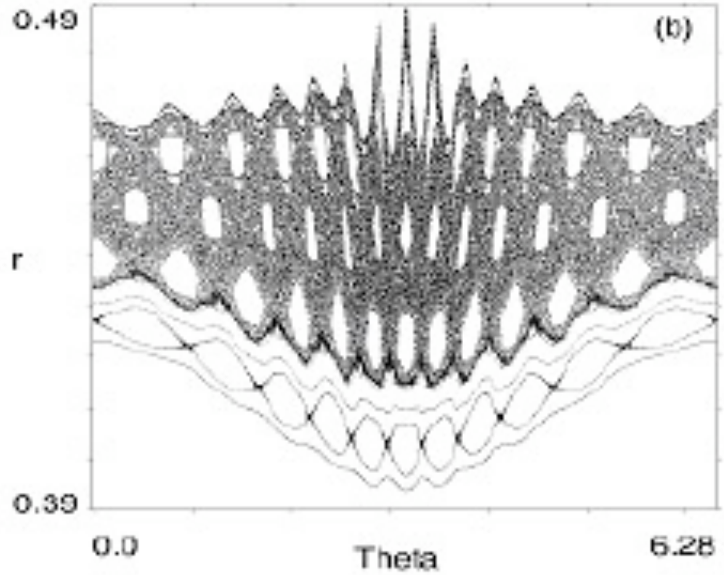
no islands remaining

An example of intentional stochasticity in a tokamak

Perturbations added by coils

Purpose: to make the edge stochastic to spread the heat flux to the walls



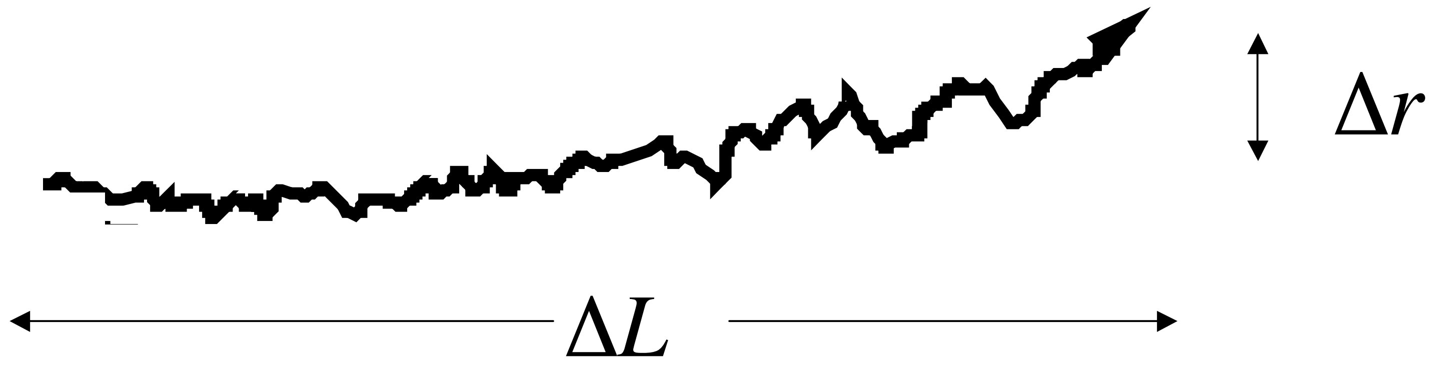


A consequence of magnetic stochasticisty:
enhanced transport

Simple estimate of transport

Particles follow stochastic, diffusing field lines

Define diffusion coefficient for the field lines



$$D_M = \frac{(\Delta r)^2}{\Delta L}$$

magnetic diffusion coefficient

let $\Delta r \sim \frac{\tilde{B}_r}{B} L_c$ where L_c is a correlation length

$$\Delta L \sim L_c$$

then,

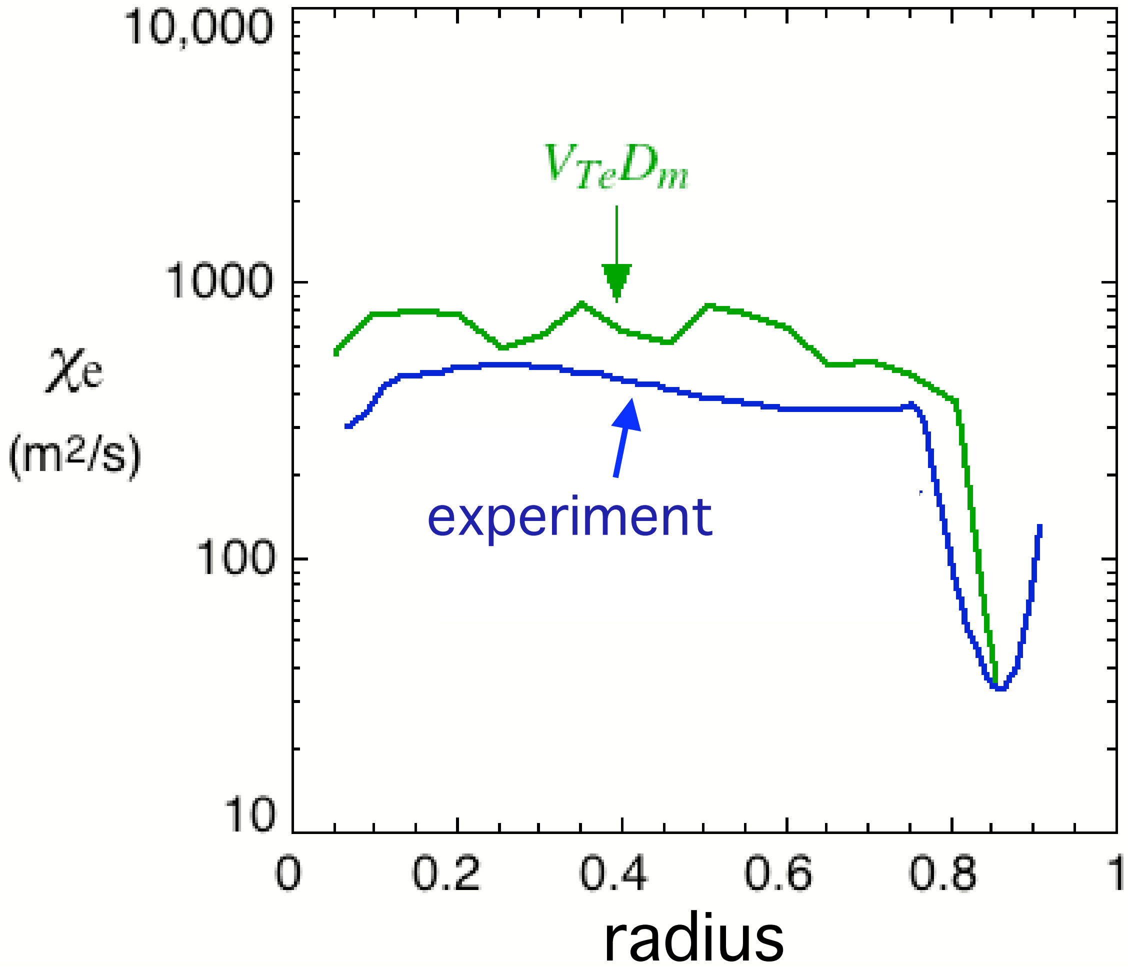
$$D_M \approx L_c \left(\frac{\tilde{B}_r}{B} \right)^2$$

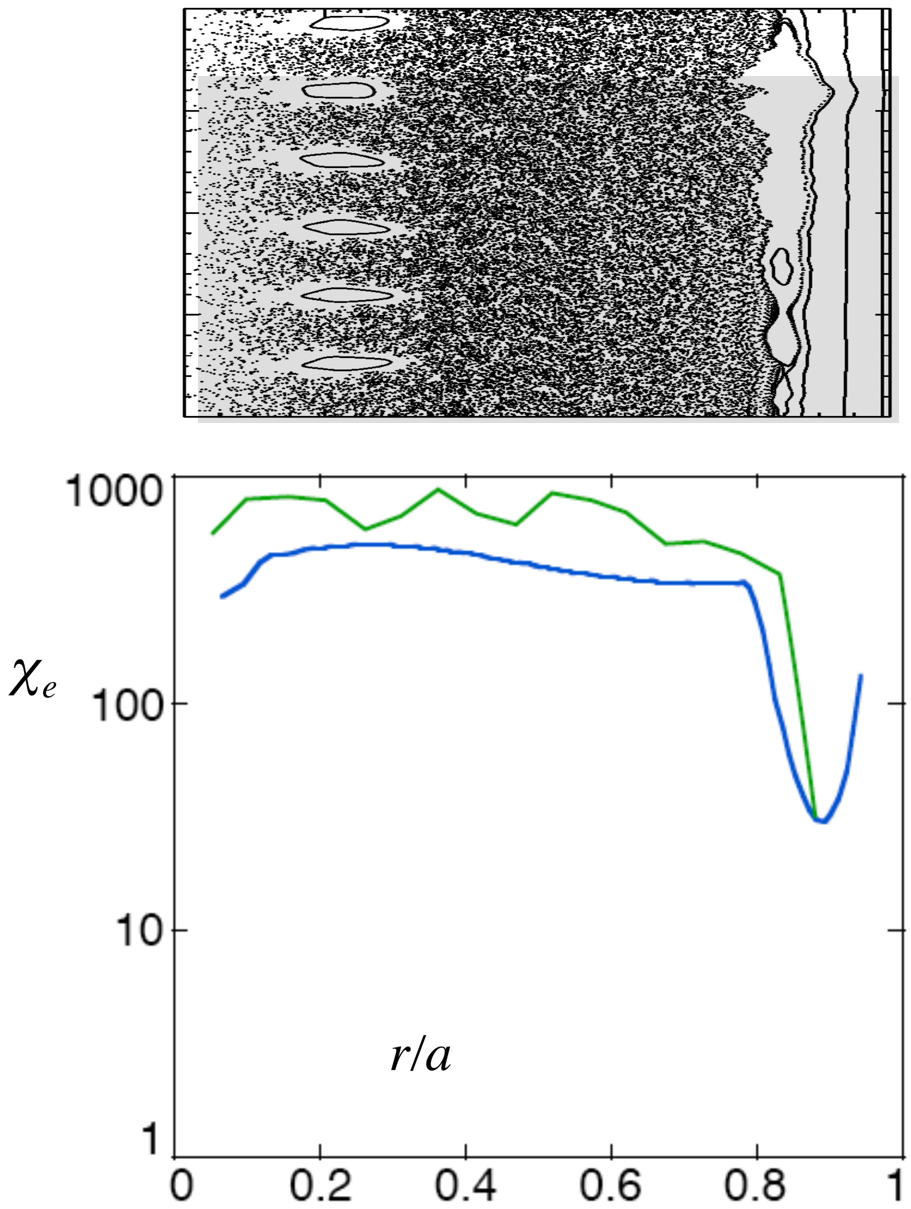
Rechester-Rosenbluth

Particle diffusion coefficient

$$D \approx v_{th} D_M$$

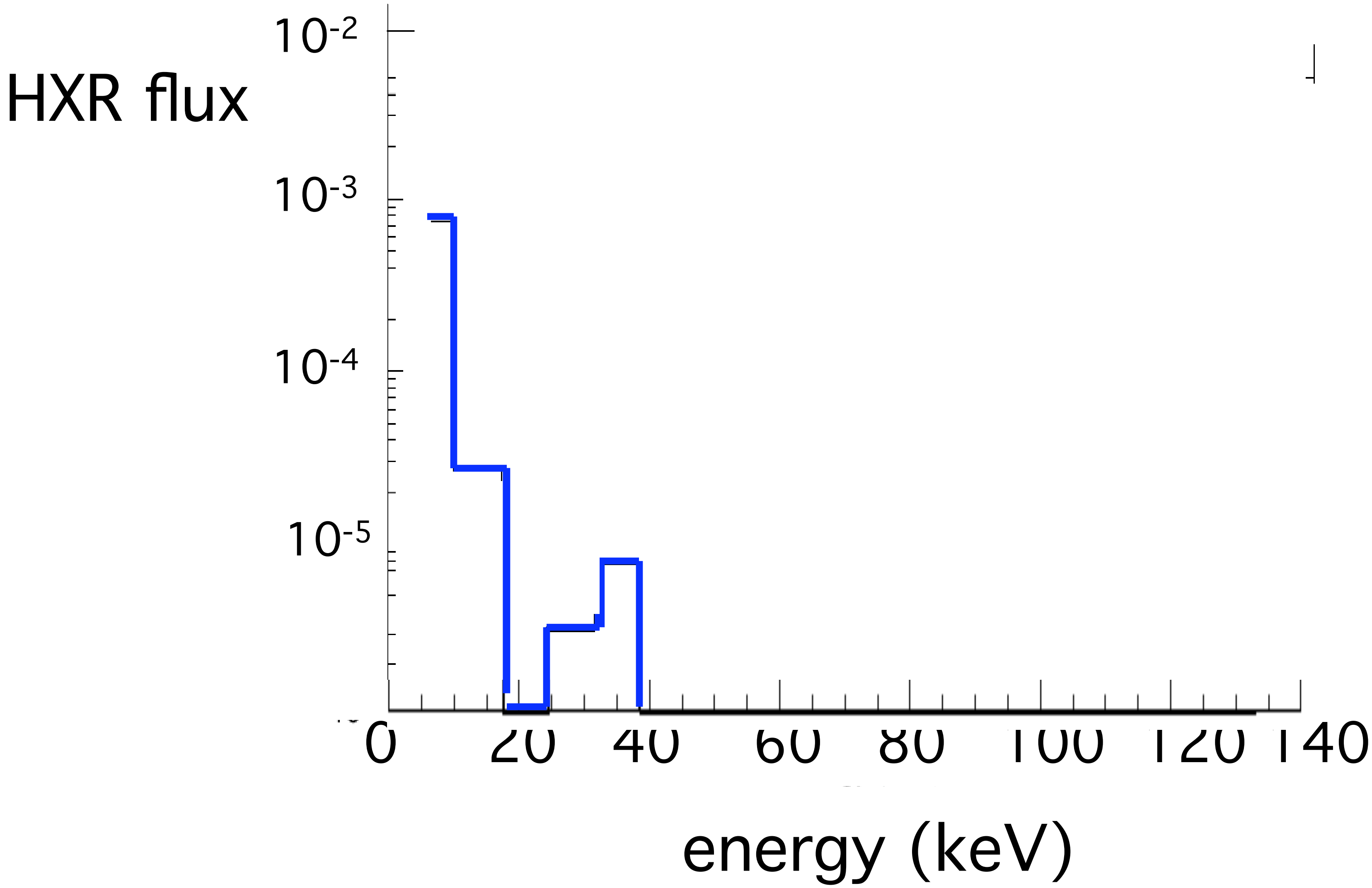
Measured energy diffusion coefficient consistent with simple stochastic diffusion





Consistent with
inferred puncture plot

High energy electrons poorly confined



Roughly consistent with D ~ v

How to measure particle transport from stochastic fields?

for electrons, the radial particle flux, due to streaming parallel to B is

$$\Gamma_r = \left\langle \vec{\Gamma}_{\parallel e} \bullet \hat{r} \right\rangle = \left\langle \Gamma_{\parallel e} \frac{\vec{B}}{B} \bullet \hat{r} \right\rangle = \frac{\left\langle \Gamma_{\parallel e} B_r \right\rangle}{\left\langle B \right\rangle}$$

or

$$\Gamma_r = \frac{\left\langle \tilde{\Gamma}_{\parallel e} \tilde{B}_r \right\rangle}{\left\langle B \right\rangle}$$

How to measure particle transport from stochastic fields?

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$$\Gamma_r = \left\langle \vec{\Gamma}_{\parallel e} \bullet \hat{r} \right\rangle = \left\langle \Gamma_{\parallel e} \frac{\vec{B}}{B} \bullet \hat{r} \right\rangle = \frac{\left\langle \Gamma_{\parallel e} B_r \right\rangle}{\left\langle B \right\rangle}$$

or

$$\Gamma_r = \frac{\left\langle \tilde{\Gamma}_{\parallel e} \tilde{B}_r \right\rangle}{\left\langle B \right\rangle}$$

writing $\Gamma_e = n v_e = \dfrac{j_e}{e}$

$$\Gamma_r = \frac{\left\langle \tilde{j}_{\parallel e} \tilde{B}_r \right\rangle}{e\left\langle B \right\rangle}$$

Adding momentum and energy transport

$$\text{particle flux} = \frac{\left\langle \tilde{j}_{\|e} \tilde{B}_r \right\rangle}{e\left\langle B \right\rangle}$$

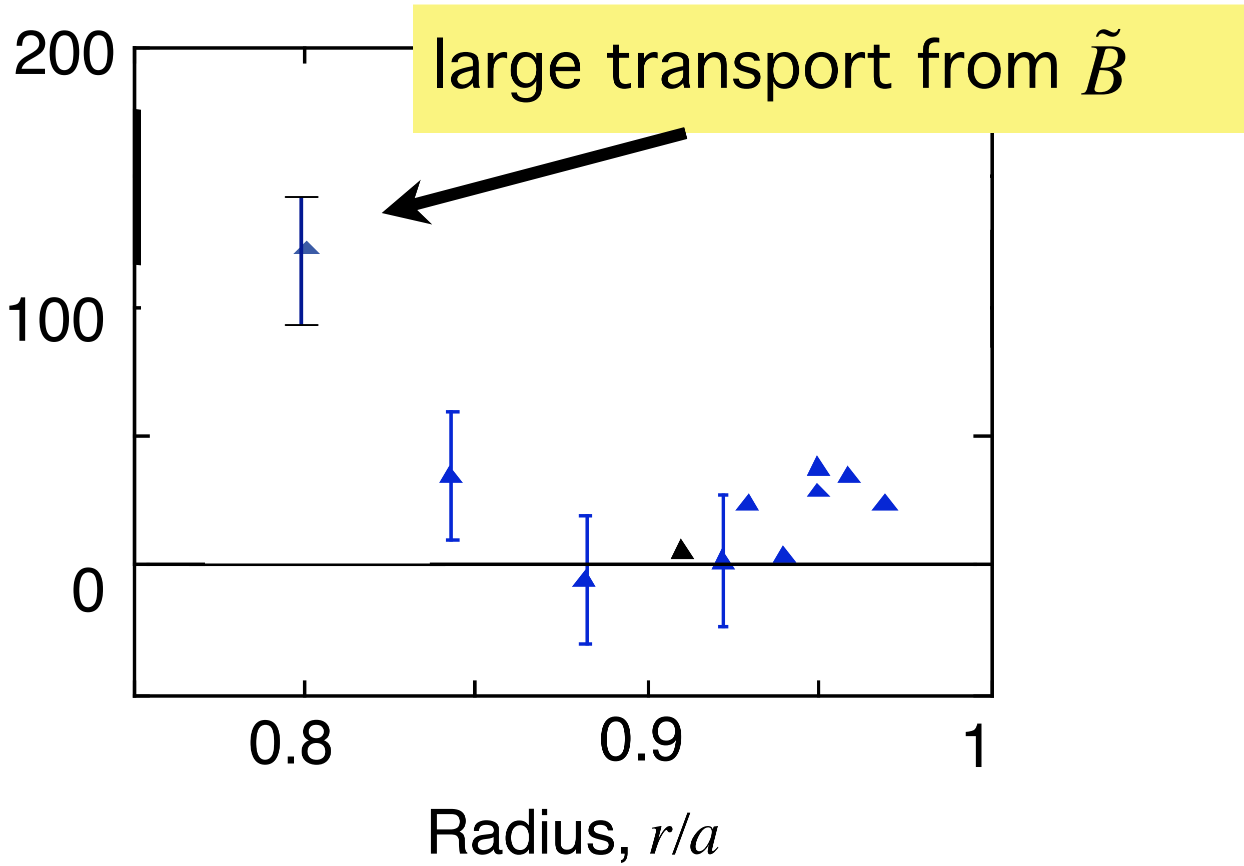
$$\text{momentum flux} = \frac{\left\langle \tilde{p}_{\|e} \tilde{B}_r \right\rangle}{e\left\langle B \right\rangle}$$

$$\text{energy flux} = \frac{\left\langle \tilde{Q}_{\|e} \tilde{B}_r \right\rangle}{e\left\langle B \right\rangle}$$

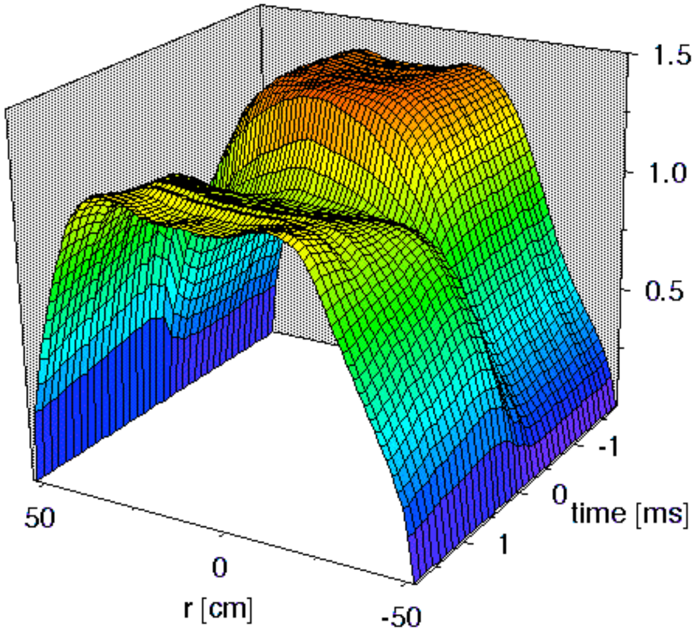
Edge measurements of energy flux

$$Q_r = \frac{\langle \tilde{Q}_{\parallel} \tilde{B}_r \rangle}{\langle B \rangle}$$

(kW/m²)



$$n_e(r)\ [10^{13} cm^{-3}]$$

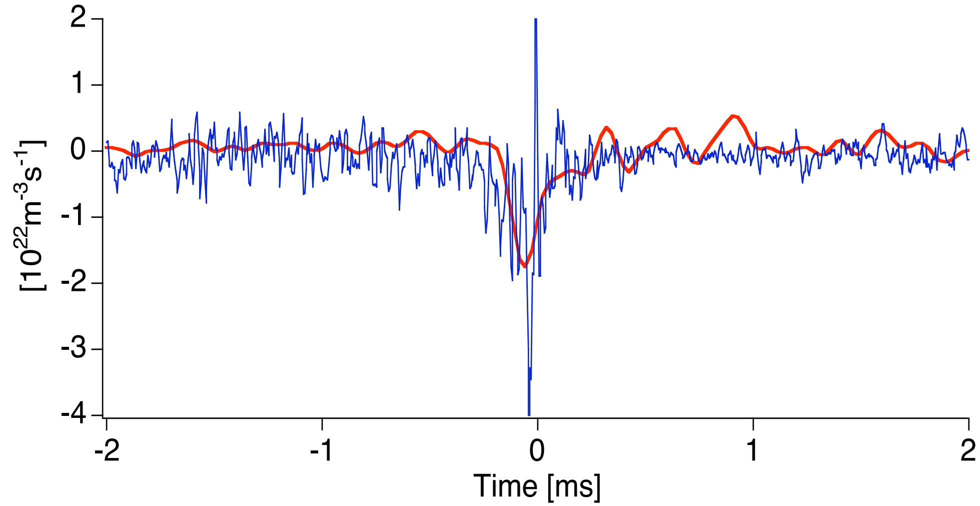


$$\text{particle flux} = \frac{\left\langle \tilde{j}_{\|e} \tilde{B}_r \right\rangle}{e\left\langle B \right\rangle}$$

Measure convective part

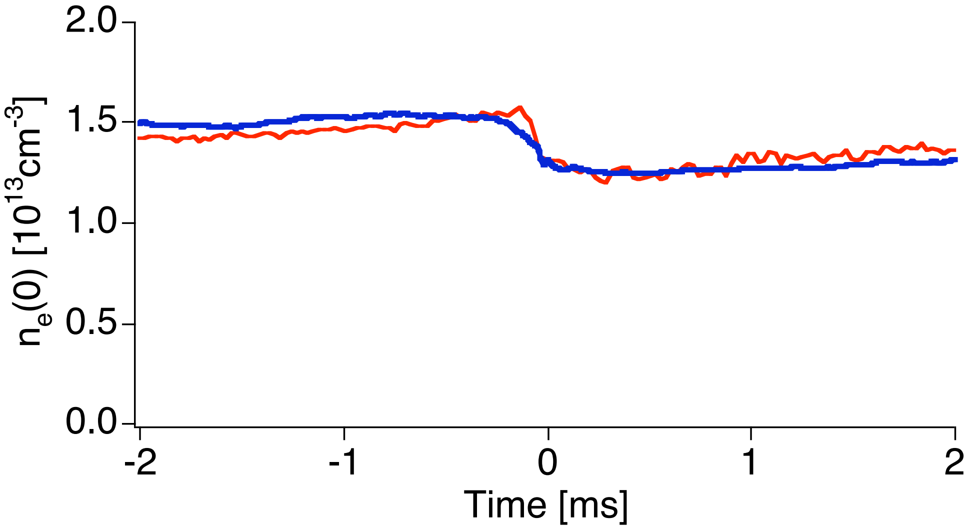
$$\Gamma_r = V_{//,e} \frac{< \tilde{n}_e \tilde{b}_r >}{B}$$

Density change is balanced by particle transport



$$\frac{\partial \langle n_e \rangle}{\partial t}$$

$$-\nabla \bullet \Gamma_r$$



$$\langle n_e \rangle$$

$$-\int \nabla \bullet \Gamma_r \, dt$$

$$\frac{\partial \langle n_e \rangle}{\partial t} + \nabla \bullet \Gamma_r \approx 0$$

Control of magnetic stochasticity

Technique:

reduce energy source for tearing instability

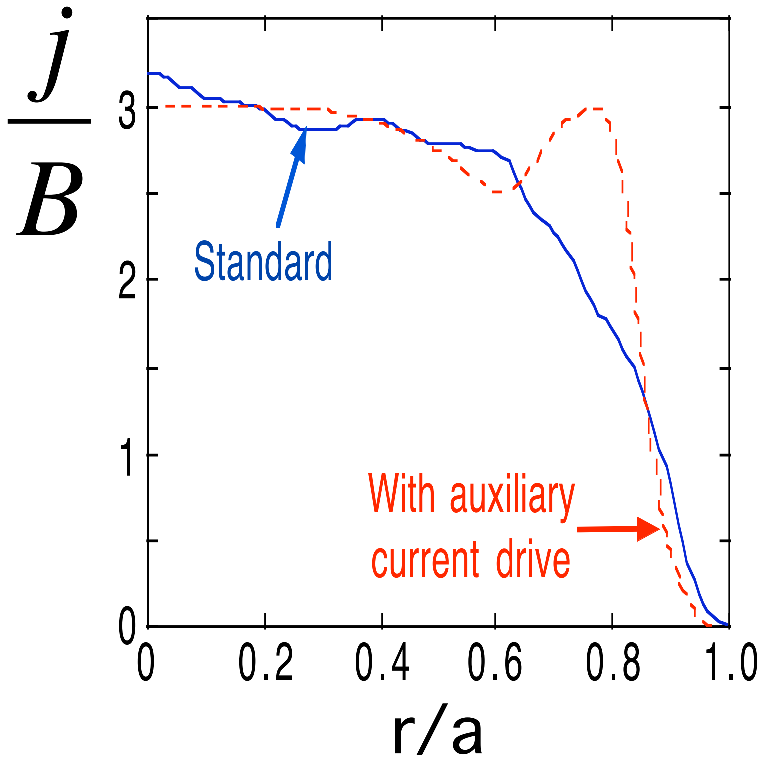
$$\nabla \frac{j_{\|}}{B}$$

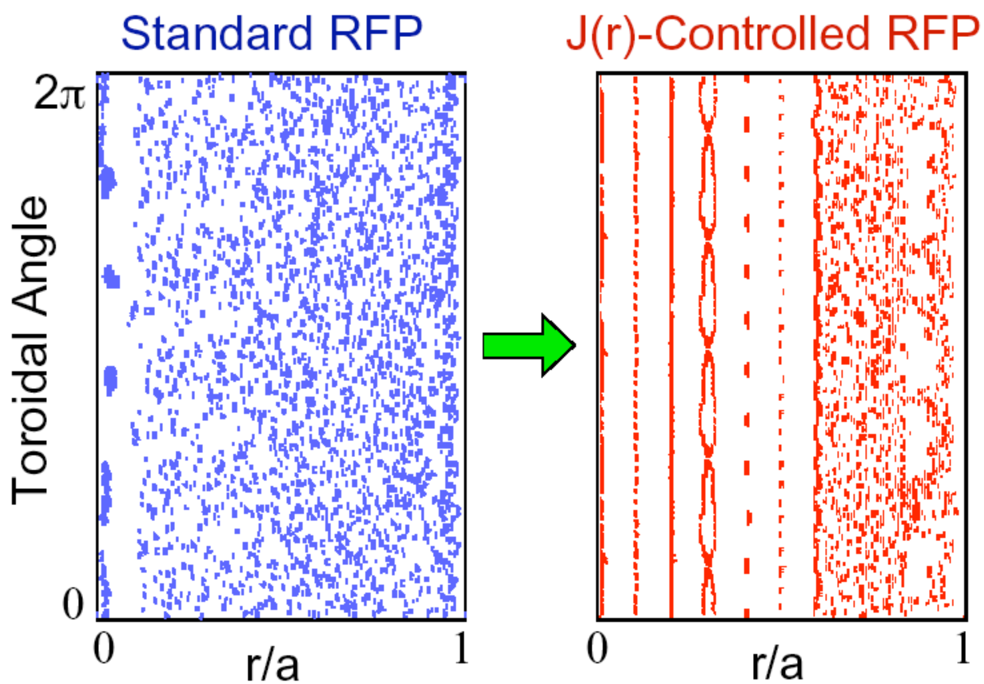
by controlling the current density profile

Transport reduction by current profile control

From nonlinear MHD computation:

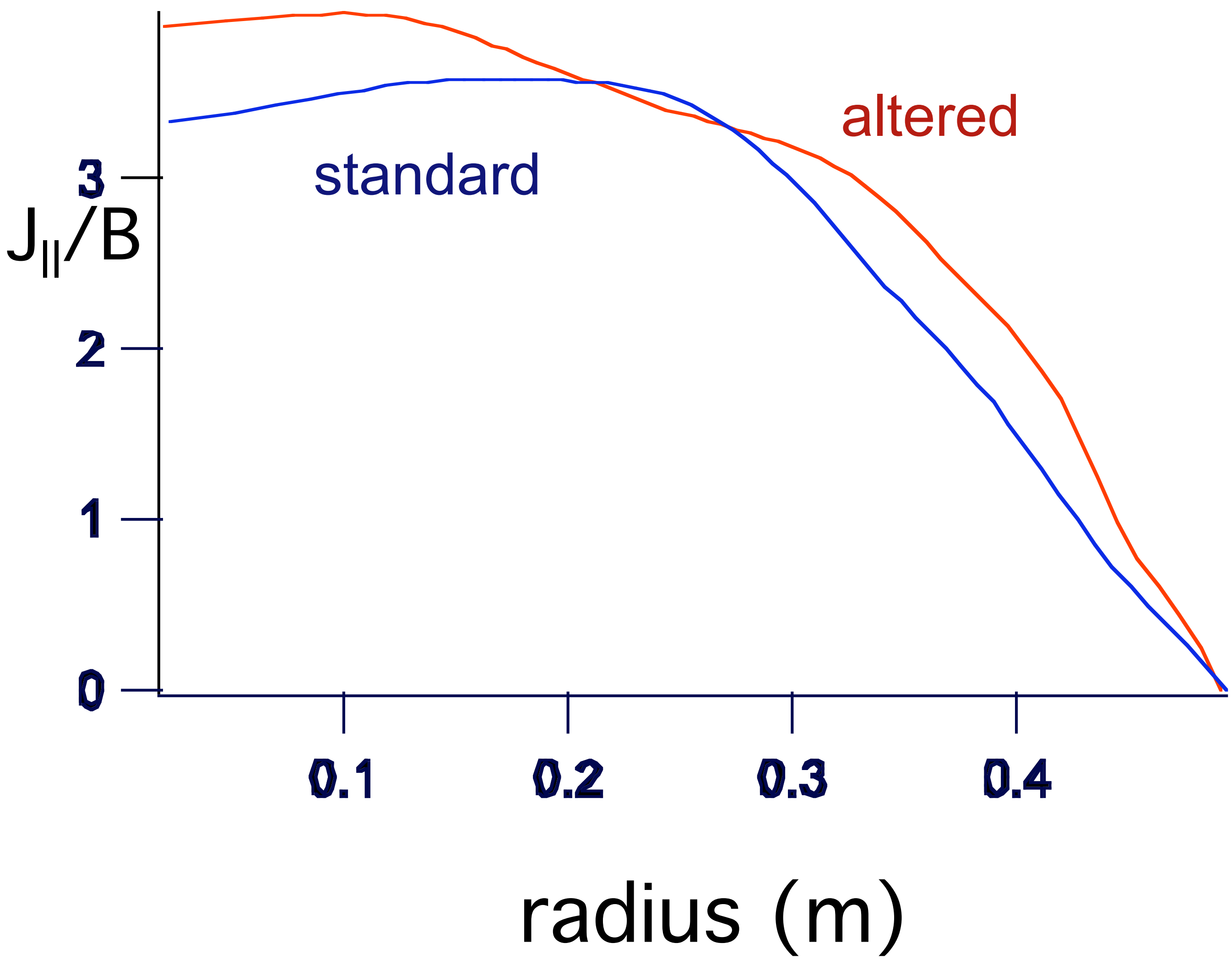
Adding edge current ⇒ reduces fluctuations and chaos



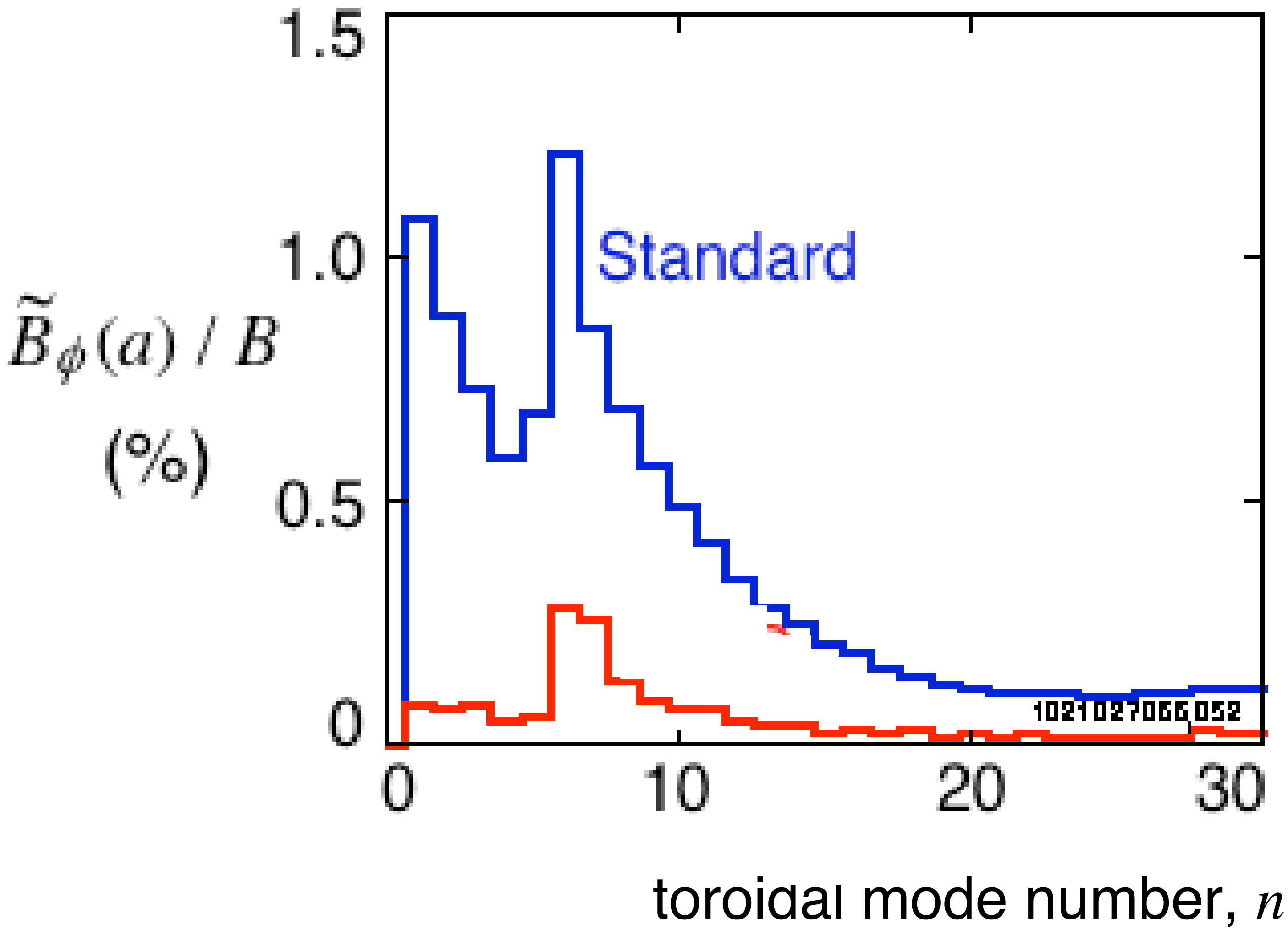


Sovinec

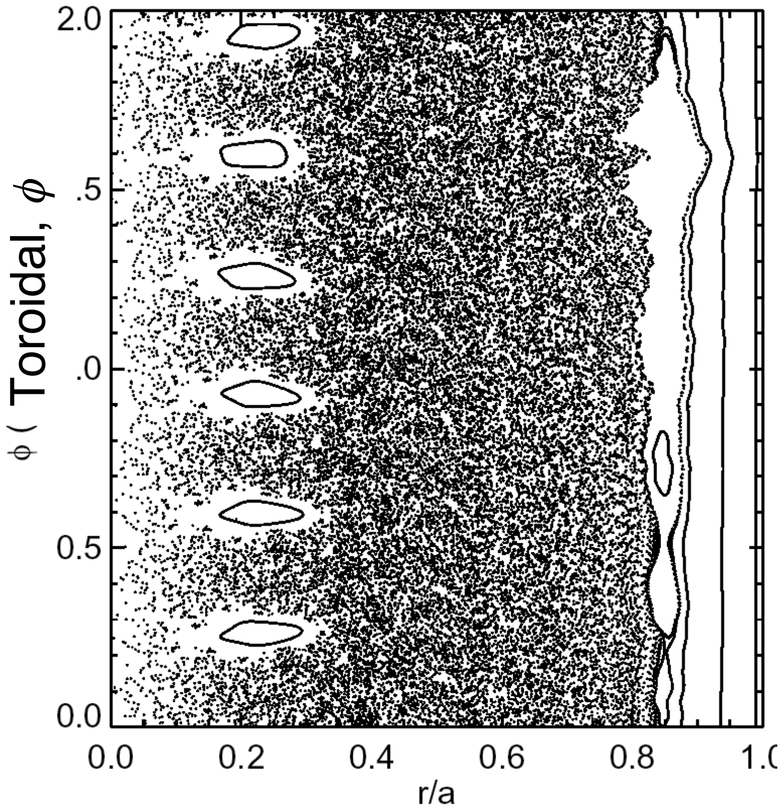
Experimental alteration of current density profile



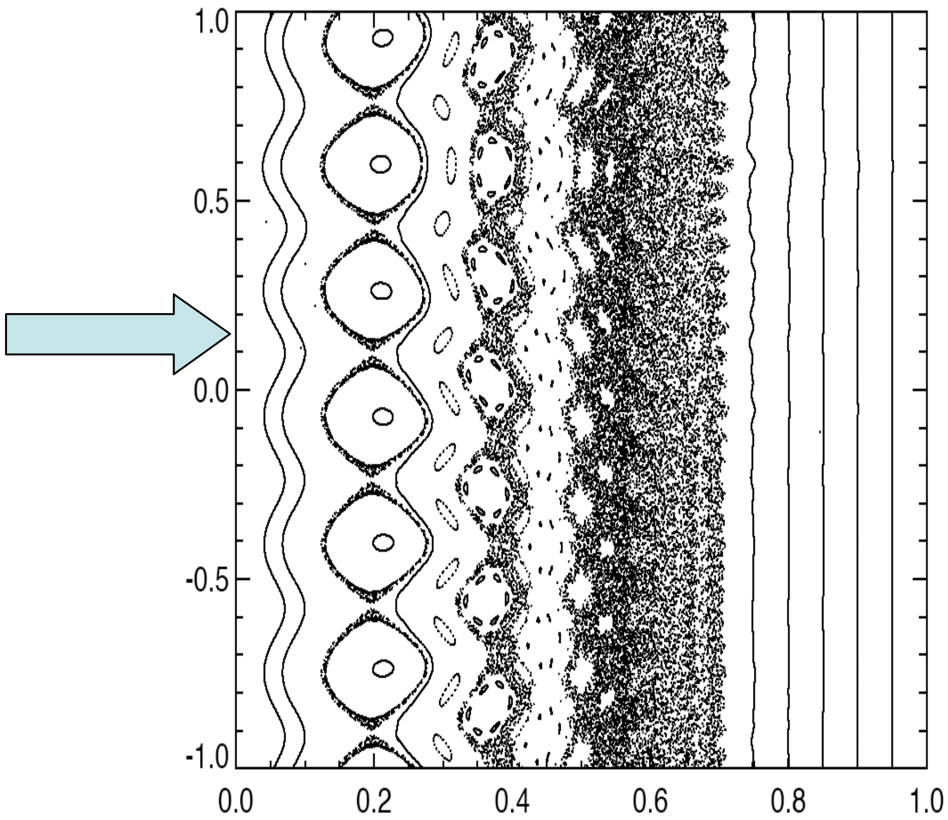
Magnetic fluctuations reduced



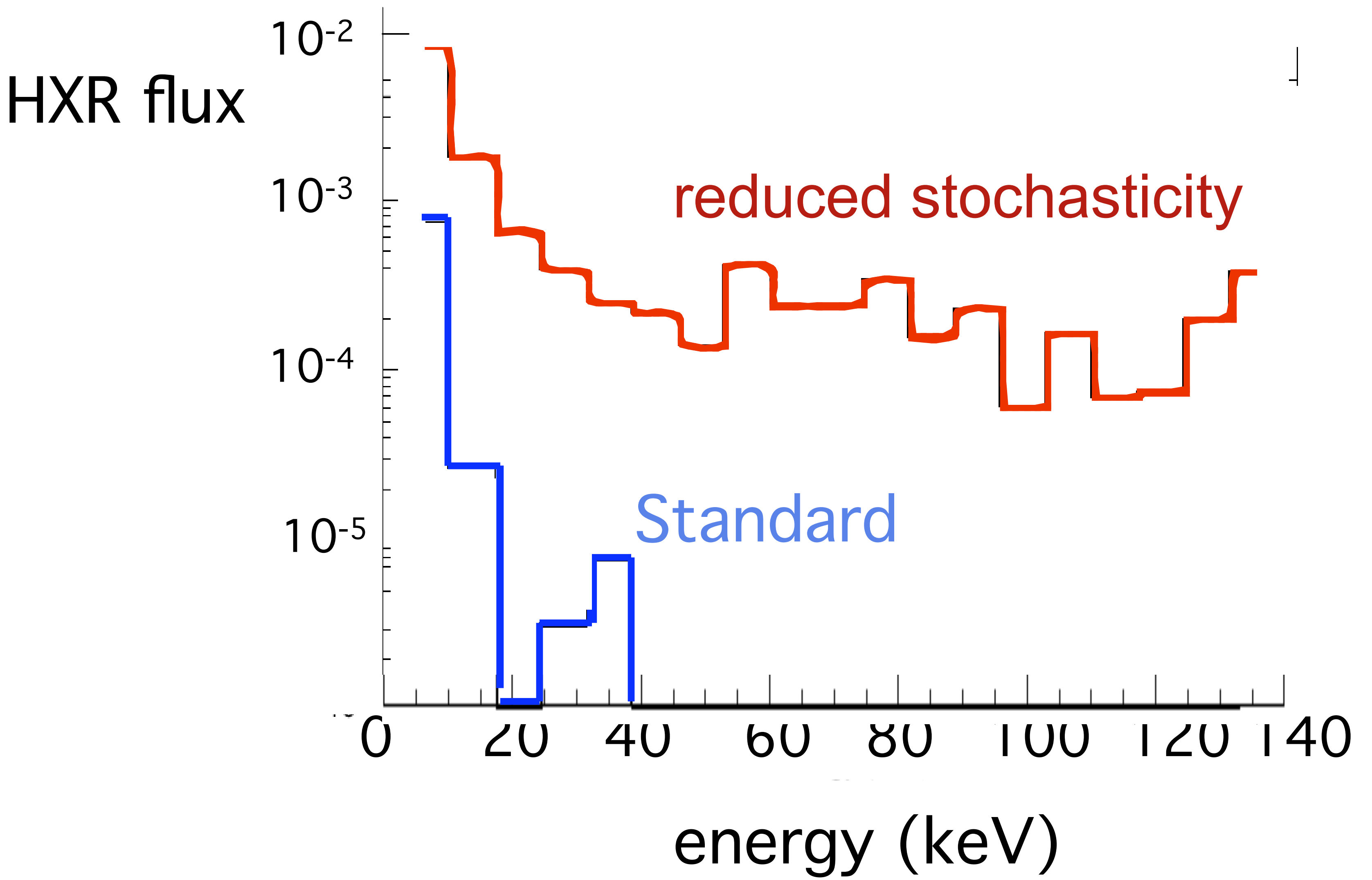
Magnetic stochasticity reduced in experiment



radius



Electrons confined to energy > 100 keV

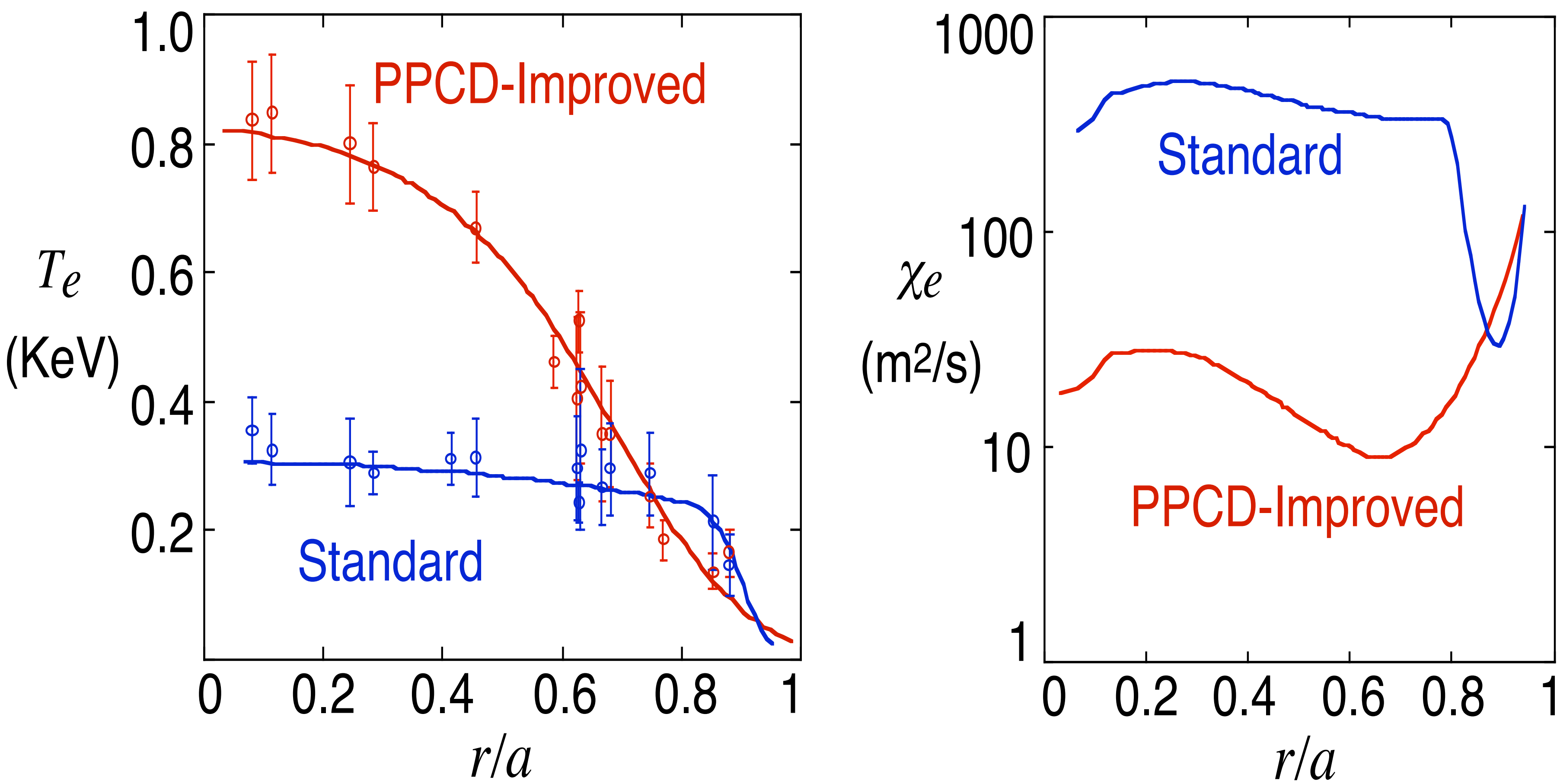


HXR spectrum with reduced stochasticity

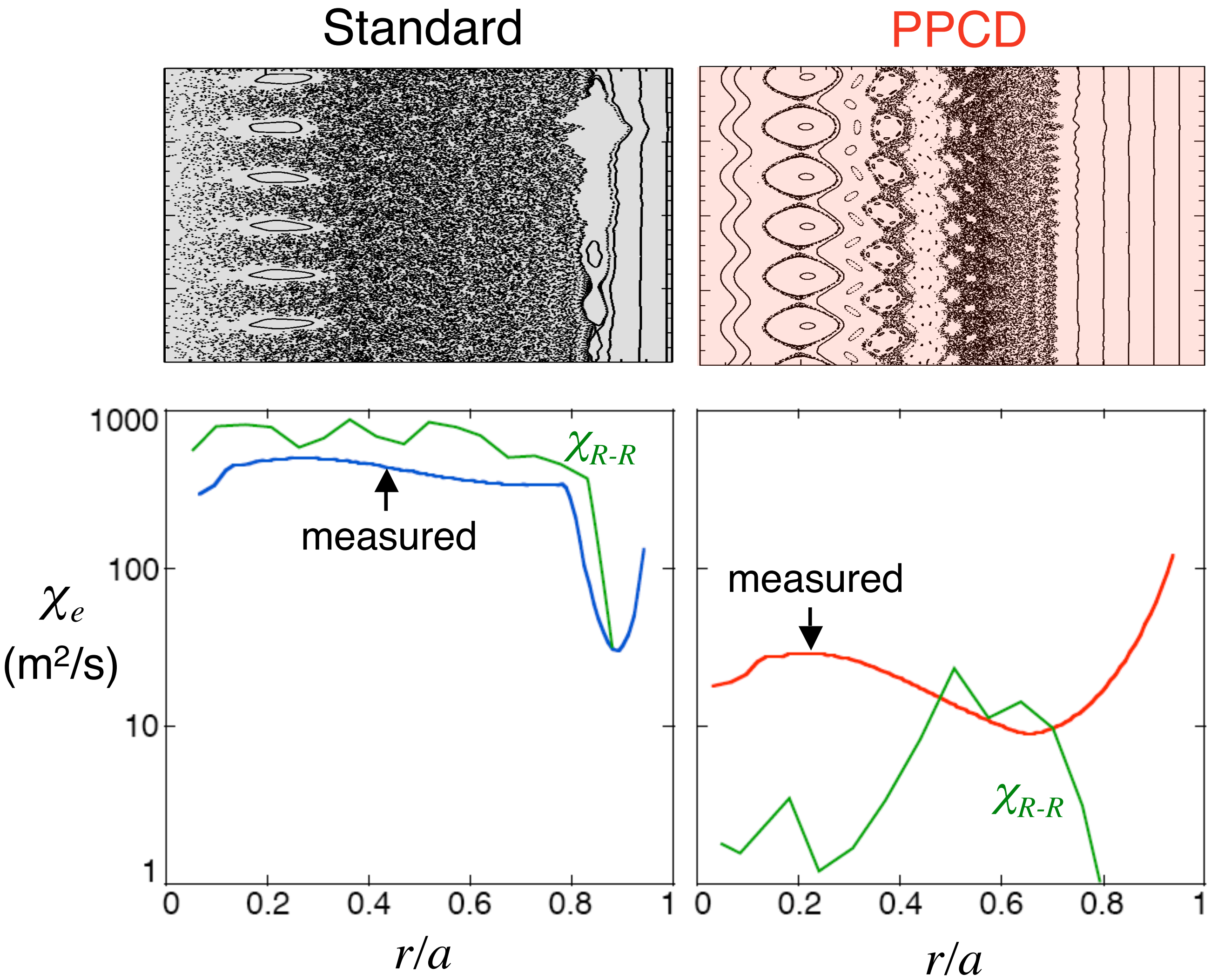
D approximately independent of v

Transport not dominated by magnetic fluctuations,

Possibly dominated by electrostatic fluctuations, through $\tilde{E} \times B$ drifts temperature increases   energy diffusion decreases



Compare with theoretical expression for diffusion coefficient



Summary

- Stochasticity has large effect on transport
- Stochasticity is controllable
- A theory for transport in stochastic field is not yet available