The Abdus Salam International Centre for Theoretical Physics

United Nations Educational, Scientific and Cultural Organization

IAEA International Atomic Energy Agency

1856-48

2007 Summer College on Plasma Physics

30 July - 24 August, 2007

Fluctuations in Non Uniform Tokamak Plasmas

P.K. Shukla

Ruhr-Universitaet Bochum
Bochum, Germany

Fluctuations in nonuniform tokamak plasmas

P. K. Shukla
Ruhr University Bochum, Germany

August 23, 2007

Contents

1 Introduction

Tokamak plasmas are inhomogeneous and magnetized. In tokamak plasmas, we can have multi-scale sized fluctuations.

What are the scale sizes in plasmas?

Electron Debye radius

$$\lambda_{De} = \left(\frac{T_e}{4\pi n_e e^2} \right)^{1/2}$$

Electron gyroradius

$$\rho_e = \frac{V_{te}}{\omega_{ce}}, \text{ where } V_{te} = \left(\frac{T_e}{m_e} \right)^{1/2}, \ \omega_{ce} = \frac{eB_0}{m_e c}.$$

Ion gyroradius

$$\rho_i = \frac{V_{ti}}{\omega_{ci}}, \text{ where } V_{ti} = \left(\frac{T_i}{m_i} \right)^{1/2}, \ \omega_{ci} = \frac{eB_0}{m_i c}.$$

Ion sound gyroradius

$$\rho_s = \frac{C_s}{\omega_{ci}}.$$

Ion sound speed

$$C_s = \frac{T_e}{m_i}.$$

Electron skin depth

$$\lambda_e = \frac{c}{\omega_{pe}}.$$

Ion skin depth

$$\lambda_i = \frac{c}{\omega_{pi}}.$$

Electron plasma frequency

$$\omega_{pe} = \left(\frac{4\pi n_0 e^2}{m_e} \right)^{1/2}.$$

Ion plasma frequency

$$\omega_{pi} = \left(\frac{4\pi n_0 e^2}{m_i} \right)^{1/2}.$$

What are the characteristic time scales?

Inverse gyro periods

$$\omega_{ce}^{-1}, \qquad \omega_{ci}^{-1}.$$

Inverse plasma periods

$$\omega_{pe}^{-1}, \qquad \omega_{pi}^{-1}.$$

What are the physical quantities to be measured?

Density fluctuations:

$$\frac{n_1}{n_0}.$$

Potential fluctuations:

$$\frac{e\varphi}{T_e}.$$

Electric field fluctuations:

$$\mathbf{E} = -\boldsymbol{\nabla}\varphi - \frac{1}{c}\frac{\partial \mathbf{A}}{\partial t}.$$

Velocity fluctuations:

$$\mathbf{V}/C_s, \quad \mathbf{V}/V_A.$$

Magnetic fluctuations:

$$\mathbf{B}/\mathbf{B}_0, \text{ where } \mathbf{B} = \mathbf{B}_\perp + \hat{\mathbf{z}}\ B_z.$$

$$\mathbf{B} = \boldsymbol{\nabla} \times \mathbf{A}, \text{ and } V_A = \frac{B_0}{(4\pi n_0 m_i)^{1/2}}.$$

Plasma beta

$$\beta = \frac{4\pi n_0 (T_e + T_i)}{B_0^2} \equiv \frac{4\pi n_0 T}{B_0^2} = \frac{\text{Plasma kinetic energy}}{\text{Magnetic energy density}}$$

2 Electrostatic fluctuations

EQUILIBRIUM:

Steady state $(\partial/\partial t = 0)$.

MHD equilibrium

$$\rho(\mathbf{U}_0 \cdot \boldsymbol{\nabla})\mathbf{U}_0 = \frac{1}{c}\mathbf{J}_0 \times \mathbf{B}_0 - \boldsymbol{\nabla}\mathbf{P}_0,$$

where $\rho = m_i n_i$ is the ion mass density.

$$\boldsymbol{\nabla} \times \mathbf{B}_0 = \frac{4\pi}{c}\mathbf{J}_0,$$

$$P_0 = n_0 T_0; \quad T_0 = T_{e0} + T_{i0}.$$

Modified Burnoulli equation for plasmas

$$\frac{\rho}{2}\nabla U_0^2 - \rho \mathbf{U}_0 \times (\boldsymbol{\nabla} \times \mathbf{U}_0) = \frac{\mathbf{B}_0 \cdot \boldsymbol{\nabla}\mathbf{B}_0}{4\pi} - \boldsymbol{\nabla}\left(P_0 + \frac{B_0^2}{8\pi}\right).$$

Choose profiles of U_0, B_0, and P_0, which satisfy the above equation. We thus have a self consistent equilibrium.

ELECTROSTATIC FLUCTUATIONS:

$$\mathbf{E} = -\boldsymbol{\nabla}\varphi\left(\mathbf{r}, t\right).$$

Low-frequency $\omega \ll \omega_{ci}$, Long wavelength $\lambda \gg \rho_i$

We have two possibilities (1) flute modes $k_{\|} = 0$ and (2) non-flute modes $k_{\|} \neq 0$

(1) flute modes $k_{\|} = 0$, B_0 =constant.

Convective cells/zonal flows

$$\mathbf{U}_{e\perp} \simeq \frac{c}{B_0}\hat{\mathbf{z}} \times \boldsymbol{\nabla}\varphi + \mathbf{U}_{De} + \frac{c}{B_0\omega_{ce}}\left(\frac{\partial}{\partial t} + \mu_e\nabla_\perp^2\right)\boldsymbol{\nabla}_\perp\varphi,$$

$$\mathbf{U}_{i\perp} \simeq \frac{c}{B_0}\hat{\mathbf{z}} \times \boldsymbol{\nabla}\varphi + \mathbf{U}_{Di} - \frac{c}{B_0\omega_{ci}}\left(\frac{\partial}{\partial t} + \mu_i\nabla_\perp^2\right)\boldsymbol{\nabla}_\perp\varphi.$$

Electron and ion gyroviscosities are

$$\mu_e = 0.51\ \nu_{ee}\rho_e^2$$

$$\mu_i = 0.3\ \nu_{ii}\rho_i^2$$

Electron and ion diamagnetic drifts

$$\mathbf{U}_{De} = -\frac{c}{eB_0n_0}\hat{\mathbf{z}} \times \boldsymbol{\nabla}\tilde{P}_e; \qquad \tilde{P}_e = n_0T_{e1} + \ n_1T_{e0}$$

$$\mathbf{U}_{Di} = \frac{c}{eB_0n_0}\hat{\mathbf{z}} \times \boldsymbol{\nabla}\tilde{P}_i; \qquad \tilde{P}_i = n_0T_{i1} + \ n_1T_{i0}.$$

Linearized electron and ion continuity equations are

$$\frac{\partial n_{e1}}{\partial t} - \frac{c}{B_0}\frac{\partial n_0}{\partial x}\frac{\partial\varphi}{\partial y} + \frac{cn_0}{B_0\omega_{ce}}\left(\frac{\partial}{\partial t} + \mu_e\nabla_\perp^2\right)\nabla_\perp^2\varphi = 0,$$

$$\frac{\partial n_{i1}}{\partial t} - \frac{c}{B_0}\frac{\partial n_0}{\partial x}\frac{\partial\varphi}{\partial y} - \frac{cn_0}{B_0\omega_{ci}}\left(\frac{\partial}{\partial t} + \mu_i\nabla_\perp^2\right)\nabla_\perp^2\varphi = 0.$$

Note that $\boldsymbol{\nabla}\cdot(n_0\mathbf{U}_{De}) = 0$ and $\boldsymbol{\nabla}\cdot(n_0\mathbf{U}_{Di}) = 0$.

The Poisson equation is

$$\nabla_\perp^2\varphi = 4\pi e(n_{e1} - n_{i1}).$$

From the above equation, we obtain for a dense plasma $(\omega_{pi}^2 \gg \omega_{ci}^2)$

$$\left(\frac{\partial}{\partial t} + \mu_i\nabla_\perp^2\right)\nabla_\perp^2\varphi = 0.$$

This equation describes the decay of the ion vorticity, defined as

$$\bar{U}=\frac{c}{B_0}\boldsymbol{\nabla}\times(\hat{\mathbf{z}}\times\boldsymbol{\nabla}\varphi)\equiv\frac{c}{B_0}\hat{\mathbf{z}}\nabla_\perp^2\varphi,$$

$$\varphi\propto\exp\left(-i\omega t+i\mathbf{k}_\perp\cdot\mathbf{r}\right),$$

$\omega(\mathbf{k})$ is the wave frequency (wave vector).

$$\omega=-i\mu_i k_\perp^2\equiv -i\Gamma.$$

Decay rate (damping rate)

$$\Gamma=0.3\ \nu_{ii}k_\perp^2\rho_i^2.$$

Life-time of convective cells

$$\Gamma^{-1}\propto\frac{1}{k_\perp^2\rho_i^2}.$$

Thus long wavelength $(k_\perp\rho_i\ll 1)$ would have longer life-time —Okuda-Dawson (Phys. Fluids 1973).

In thermal equilibrium, convective cells (c c) have finite energy density. Thermal c c can cause cross-field (across $B_0\hat{\mathbf{z}}$) transport, since the charged particles random walk in the fluctuating electric fields of c c

$$D_\perp=\int\langle\mathbf{U}(t)\cdot\mathbf{U}(t+\tau)\rangle\,d\tau,$$

$$\mathbf{U}(t)=\frac{c}{B_0}\mathbf{E}_\perp(t)\times\hat{\mathbf{z}},$$

$D_\perp\propto 1/B_0$, Bohm-like scaling.

2.1 Zonal flows

- radial symmetric perturbations having $k_r\equiv k_x$
  - n_0 k_y or poloidal wave number $k_y=0$, $k_z=0$. c c is then

$$\omega=-i\ \nu_{ii}k_x^2\rho_i^2 \tag{2.1.1}$$

c c/Zonal flows are nonlinearly excited by drift waves due to the Reynolds stresses.

2.2 Drift waves in collisionless plasmas

$\omega\ll\omega_{ci}$, $k_\parallel\neq 0$

$$\mathbf{U}_{e\perp}\simeq\frac{c}{B_0}\hat{\mathbf{z}}\times\boldsymbol{\nabla}\varphi-\frac{cT_e}{eB_0n_0}\hat{\mathbf{z}}\times\boldsymbol{\nabla}n_{e1}+\hat{\mathbf{z}}U_{ez}\ , \tag{2.2.1}$$

$$\mathbf{U}_{i\perp} \simeq \frac{c}{B_0}\hat{\mathbf{z}} \times \boldsymbol{\nabla}\varphi + \frac{cT_i}{eB_0 n_0}\hat{\mathbf{z}} \times \boldsymbol{\nabla} n_{i1} - \frac{c}{B_0\omega_{ci}} \times \left(\frac{\partial}{\partial t} + V_{i*}\frac{\partial}{\partial y}\right)\boldsymbol{\nabla}_\perp\varphi + \hat{\mathbf{z}}U_{iz} \ . \tag{2.2.2}$$

Insert the electron fluid velocity above into the linearized electron continuity equation to obtain

$$\frac{\partial n_{e1}}{\partial t} - \frac{c}{B_0}\frac{\partial n_0}{\partial x}\frac{\partial \varphi}{\partial y} + n_0\frac{\partial}{\partial z}U_{ez} = 0, \tag{2.2.3}$$

where the parallel component of the electron momentum equation is

$$\frac{\partial U_{ez}}{\partial t} = \frac{e}{m_e}\frac{\partial}{\partial x}\left(\varphi - \frac{T_e}{e}\frac{n_{e1}}{n_0}\right), \tag{2.2.4}$$

used $\mathbf{U}_{De} \cdot \boldsymbol{\nabla} U_{ez} = -\left(\boldsymbol{\nabla} \cdot \boldsymbol{\Pi}_e\right)_z / m_e n_0$; $\boldsymbol{\Pi}_e$ is the collisionless electron stress tensor. From the above two equations, we can deduce the Boltzmann law

$$\frac{n_{e1}}{n_0} \approx \frac{e\varphi}{T_e}, \tag{2.2.5}$$

for $\partial^2 n_{e1}/\partial t^2 \ll V_{te}^2 \partial^2 n_{e1}/\partial z^2$ and $\chi_n = \partial^2\varphi/\partial t\partial y \ll \omega_{ce}\partial^2\varphi/\partial z^2$, where $\chi_n = d\ln n_0/dx$ is the inverse density gradient scale length. The Boltzmann law dictates that the electrons rapidly thermalize along the external magnetic field $B_0\hat{\mathbf{z}}$ in the ES perturbations where parallel (to $B_0\hat{\mathbf{z}}$) phase speed is much smaller than the speed of light.

Furthermore, inserting the ion fluid velocity into the linearized ion continuity equation, we have

$$\frac{\partial n_{i1}}{\partial t} - \frac{c}{B_0}\frac{\partial n_0}{\partial x}\frac{\partial \varphi}{\partial y} - \frac{cn_0}{B_0\omega_{ci}}\left(\frac{\partial}{\partial t} + V_{i*}\frac{\partial}{\partial y}\right)\nabla_\perp^2\varphi = -n_0 U_{iz}, \tag{2.2.6}$$

$$\frac{\partial U_{iz}}{\partial t} = -\frac{e}{m_i}\frac{\partial}{\partial z}\left(\varphi + \frac{T_i}{e}\frac{n_{i1}}{n_0}\right). \tag{2.2.7}$$

In the parallel ion momentum equation, we use

$$\mathbf{U}_{Di} \cdot \boldsymbol{\nabla} U_{iz} = -\frac{\boldsymbol{\nabla} \cdot \boldsymbol{\Pi}_i}{m_i n_0}.$$

$\boldsymbol{\Pi}_i$ is the collisionless ion stress tensor. Using $n_{e1} = n_{i1} \equiv n_0 e\varphi/T_e$, we have for $\omega_{pi}^2 \gg \omega_{ci}^2$ (assumption for the quasi-neutrality-no need to use Poisson's equation) and $k_\perp^2\rho_i^2 \ll 1$

$$\frac{\partial}{\partial t}(1 - \rho_s^2\nabla_\perp^2)\varphi - V_{e*}\frac{\partial}{\partial y}\varphi + \frac{T_e}{e}\frac{\partial}{\partial z}U_{iz} = 0, \tag{2.2.8}$$

$$\frac{\partial U_{iz}}{\partial t} = -\frac{e}{m_i}\left(1+\frac{T_i}{T_e}\right)\frac{\partial}{\partial z}\varphi, \tag{2.2.9}$$

$V_{e*} = (cT_e/eB_0)\ \chi_n$; $\rho_s = C_s/\omega_{ci}$. Assuming φ and U_{iz} are proportional to $\exp\left(-i\omega t + i\mathbf{k}_\perp\cdot\mathbf{r}+ik_z z\right)$, we obtain from the above the dispersion relation

$$\omega^2 - \frac{\omega\omega_{e*}}{1+b} - \frac{k_z^2 V_s^2}{1+b} = 0, \tag{2.2.10}$$

for coupled drift and ion-sound waves in nonuniform magnetoplasmas. Here $\omega_{e*} = -k_y V_{e*}$, $b = k_\perp^2\rho_s^2$, $V_s^2 = C_s^2\left(1+T_i/T_e\right)$.

Solutions of the quadratic equation above are

$$\omega = \frac{1}{2}\omega_D \pm \frac{1}{2}\left(\omega_D^2 + 4\omega_{IA}^2\right)^{1/2}, \tag{2.2.11}$$

respectively accelerated (+) and retarded (-) drift waves.

$$\omega_D = \frac{\omega_{e*}}{1+b},\ \omega_{IA}^2 = \frac{k_z^2 V_s^2}{1+b}. \tag{2.2.12}$$

No density gradient

$$\omega = \omega_{IA} = \frac{k_z V_s}{(1+b)^{1/2}}, \tag{2.2.13}$$

dispersive ion-acoustic waves. For $\omega_{IA}^2 \ll \omega_D^2$, we have

$$\omega = \omega_D = \frac{\omega_{e*}}{1+b} \tag{2.2.14}$$

- dispersive drift waves
    - dispersion causes from the ion polarization drift or the perpendicular ion inertia.

2.3 Drift wave instability in a collisional plasma

$|\partial/\partial t| \ll \nu_{ei}$ = electron ion collision frequency, $\omega_{ci} = eB_0/m_i c$.

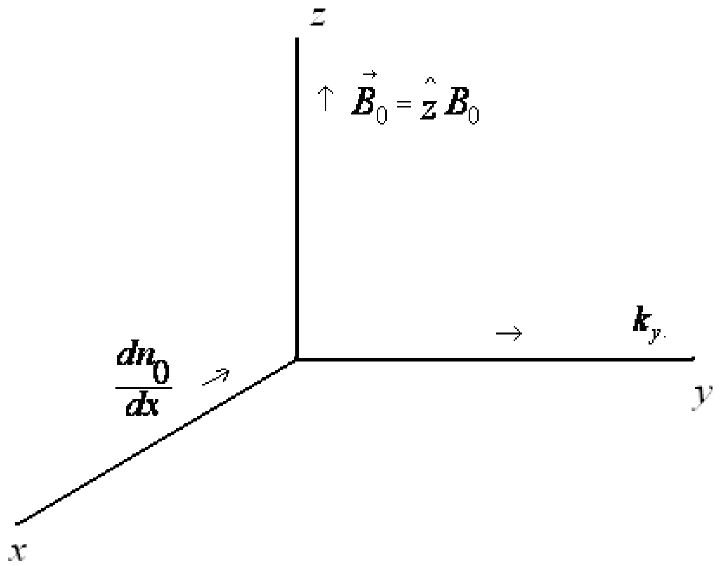
$$\mathbf{U}_{e\perp} \simeq \frac{c}{B_0}\hat{\mathbf{z}}\times\nabla\varphi - \frac{cT_e}{eB_0 n_0}\hat{\mathbf{z}}\times\nabla n_{e1} + \hat{\mathbf{z}}U_{ez}\ , \tag{2.3.1}$$

2D ions

$$\mathbf{U}_{i\perp} \simeq \frac{c}{B_0}\hat{\mathbf{z}}\times\nabla\varphi + \frac{cT_i}{eB_0 n_0}\hat{\mathbf{z}}\times\nabla n_{i1} - \frac{c}{B_0\omega_{ci}}\left(\frac{\partial}{\partial t} + V_{i*}\frac{\partial}{\partial y}\right)\nabla_\perp\varphi, \tag{2.3.2}$$

where $V_{i*} = (cT_i/eB_0)\ \partial\ln n_0(x)/\partial x$.

$$m_e\nu_{ei}U_{ez} = e\frac{\partial\varphi}{\partial z} - \frac{T_e}{en_0}\frac{\partial}{\partial z}n_{e1}. \tag{2.3.3}$$



Insert (2.3.1) and (2.3.3) in the linearized continuity equation and obtain

$$\left(\frac{\partial}{\partial t} - D\frac{\partial^2}{\partial z^2}\right) n_{e1} - \frac{c}{B_0}\frac{dn_0}{dx}\frac{\partial \varphi}{\partial y} + \frac{n_0 e}{m_e \nu_{ei}}\frac{\partial^2 \varphi}{\partial z^2} = 0, \tag{2.3.4}$$

where $D = V_{te}^2/\nu_{ei}$, $V_{te} = \sqrt{T_e/m_e}$ is the electron thermal speed. Substitute (2.3.2) in the linearized continuity equation and obtain

$$\frac{\partial n_{i1}}{\partial t} - \frac{c}{B_0}\frac{dn_0}{dx}\frac{\partial \varphi}{\partial y} - \frac{cn_0}{B_0 \omega_{ci}}\left(\frac{\partial}{\partial t} + V_{i*}\frac{\partial}{\partial y}\right)\frac{\partial^2 \varphi}{\partial y^2} = 0. \tag{2.3.5}$$

Assuming that n_{e1}, n_{i1}, and φ are proportional to $\exp\left(-i\omega t + ik_y y + ik_z z\right)$, we have from (2.3.4) and (2.3.5) after Fourier transformation

$$\left(Dk_z^2 - i\omega\right)\frac{n_{e1}}{n_0} = i\left(\frac{k_y c}{B_0}\frac{d\ln n_0}{dx} + \frac{ek_z^2}{m_e \nu_{ei}}\right)\varphi, \tag{2.3.6}$$

and

$$\frac{n_{i1}}{n_0} = -\left[\frac{k_y c}{\omega B_0}\frac{d\ln n_0}{dx} + \frac{k_y^2 c(\omega - \omega_{i*})}{\omega B_0 \omega_{ci}}\right]\varphi, \tag{2.3.7}$$

where $\omega_{i*} = k_y V_{i*}$.

Invoking the quasi-neutrality condition, i.e., $n_{e1} = n_{i1}$(holding for a dense magnetized plasma with $\omega_{pi}^2 \gg \omega_{ci}^2$, where ω_{pi} is the ion plasma frequency), we obtain from (2.3.6) and (2.3.7)

$$Dk_z^2\left(\frac{\omega_{ci}\chi_n}{\omega k_y} + 1\right) - i\omega + \frac{\omega_{LH}^2 k_z^2}{\nu_{ei}k_y^2} = 0, \tag{2.3.8}$$

which is the desired dispersion relation. Here $\chi_n = d\ln n_0/dx$ and $\omega_{LH} = \sqrt{\omega_{ce}\omega_{ci}}$ is the lower-hybrid resonance frequency. In deriving (2.3.8), we have noted that in our quasi-neutral plasma the divergence of the electron and ion fluxes involving the $-c\hat{\mathbf{z}} \times \boldsymbol{\nabla}\varphi/B_0$ particle drift are equal, and therefore they cancel each other.

Two comments are in order. First, in the absence of the density inhomogeneity, we obtain from (2.3.8)

$$\omega = -i\frac{k_z^2\omega_{LH}^2}{k_y^2\nu_{ei}}\left(1 + k_y^2\rho_s^2\right), \tag{2.3.9}$$

which is a purely damped mode. Here $\rho_s = C_s/\omega_{ci}$ is the ion sound gyroradius. Second, letting $\omega = \omega_k + i\gamma_k$ in (2.3.8), where $\gamma_k \ll \omega_k$, we obtain for the real and imaginary parts of the frequency, respectively,

$$\omega_k = -\frac{\omega_{ci}\chi_n k_y \rho_s^2}{1 + k_y^2\rho_s^2}, \tag{2.3.10}$$

and

$$\gamma_k \approx \frac{\omega_k^2\nu_{en}}{k_z^2 V_{te}^2}\frac{k_y^2\rho_s^2}{\left(1 + k_y^2\rho_s^2\right)}. \tag{2.3.11}$$

It turns out that ω_k is positive for $\chi_n < 0$. The growth rate γ_k is proportional to $k_y^2\rho_s^2$, indicating that the finite ion polarization drift is essential for the dissipative drift wave instability. Physically, instability arises because the plasma density fluctuation cannot keep in phase with the drift wave potential due to a non-Boltzmann electron density distribution arising from electron-neutral collisions and the ion polarization drift that separates the charges. Thus, the energy stored in the equilibrium density gradient is channelled to drift waves via collisions.

The cross-field ion-flux in the presence of enhanced drift-like waves fluctuations reported above can be obtained as

$$\Gamma_x = \langle n_{i1} U_{ix}^\star + c.\ c.\rangle\,, \tag{2.3.12}$$

where the angular bracket denotes the ensemble average and *c.c.* stands for the complex conjugate. We use

$$n_{i1} \approx i\frac{\gamma_k k_y c}{\omega_k^2 B_0}\frac{dn_0}{dx}\varphi_k, \tag{2.3.13}$$

from (2.3.7), and $U_{ix} = -(c/B_0)\ i\ k_y\varphi_k$ to obtain from (2.3.12)

$$\Gamma_x = -D_i\left|\frac{dn_0}{dx}\right|, \tag{2.3.14}$$

where the ion diffusion coefficient is

$$D_i = 2\frac{c^2}{B_0^2}\sum_k \frac{\gamma_k}{\omega_k^2} k_y^2 \left|\varphi_k\right|^2. \tag{2.3.15}$$

By invoking the mixing length hypothesis, one obtains

$$|\varphi_k|^2 \sim \left(\frac{\pi^2 B_0^2}{2c^2}\right)\frac{\omega_k^2}{k_y^4}. \tag{2.3.16}$$

Hence

$$D_i \sim \frac{\pi^2}{2}\sum_k \frac{\gamma_k}{k_y^2}, \tag{2.3.17}$$

which is the Kadomtsev scaling for the diffusion coefficient.

2.4 Generation of drift waves by sheared flows

Let us now suppose that a magnetized plasma contains an equilibrium magnetic field-aligned ion velocity gradient $d\mathbf{U}_0/dx$. The latter is created by accelerating a group of ions along the external magnetic field direction $B_0\hat{\mathbf{z}}$ by the electric field. We focus on the low-frequency ($\ll \omega_{ci}$) electrostatic waves $\mathbf{E} = -\boldsymbol{\nabla}\varphi$. The electrons obey the Boltzmann electron density distribution, given by (2.2.5). The ion density perturbation in the presence of the equilibrium parallel ion drift $U_0(x)\ \hat{\mathbf{z}}$ is obtained from (c.f. Eq. (2.2.6))

$$\begin{aligned}&\left(\frac{\partial}{\partial t}+U_0\frac{\partial}{\partial z}\right)n_{i1}-\frac{c}{B_0}\frac{dn_0}{dx}\frac{\partial\varphi}{\partial y}\\ &=\frac{cn_0}{B_0\omega_{ci}}\left(\frac{\partial}{\partial t}+U_0\frac{\partial}{\partial z}+V_{i*}\frac{\partial}{\partial y}\right)\nabla_\perp^2\varphi-n_0\frac{\partial}{\partial z}U_{iz},\end{aligned} \tag{2.4.1}$$

where the parallel component of the ion fluid velocity perturbation is now obtained from (c.f. Eq. (2.2.7))

$$\left(\frac{\partial}{\partial t}+U_0\frac{\partial}{\partial z}\right)U_{iz}-\frac{c}{B_0}\frac{dU_0}{dx}\frac{\partial\varphi}{\partial y}=-\frac{e}{m_i}\frac{\partial}{\partial z}\left(\varphi+\frac{T_i}{e}\frac{n_{i1}}{n_0}\right). \tag{2.4.2}$$

We replace n_{i1} by $n_{e1} = n_0 e\varphi/T_e$ in (2.4.1) and (2.4.2) to obtain

$$\left(\frac{\partial}{\partial t}+U_0\frac{\partial}{\partial z}\right)\varphi+V_{e*}\frac{\partial}{\partial y}\varphi-\rho_s^2\omega_{ci}\left(\frac{\partial}{\partial t}+U_0\frac{\partial}{\partial z}\right)\nabla_\perp^2\varphi+\frac{T_e}{e}\frac{\partial}{\partial z}U_{iz}=0, \tag{2.4.3}$$

and

$$\left(\frac{\partial}{\partial t}+U_0\frac{\partial}{\partial z}\right)U_{iz}=-\frac{e}{m_i}\left[(1+\tau)\frac{\partial}{\partial z}-S\frac{\partial}{\partial y}\right]\varphi, \tag{2.4.4}$$

where $V_{e*} = -(cT_e/eB_0)\ \chi_n$, $\tau = T_i/T_e$, $S = \acute{U}_0/\omega_{ci}$, $\acute{U}_0 = dU_0/dx$, and $\rho_i^2\nabla_\perp^2 \ll 1$ has been assumed.

Supposing that φ and U_{iz} are proportional to $\exp(-i\omega t + ik_y y + ik_z z)$, we Fourier transform (2.4.3) and (2.4.4) to obtain

$$\left[\Omega(1+k_y^2\rho_s^2)+\omega_{e*}\right]\varphi-\frac{T_e}{e}k_z U_{iz}=0, \tag{2.4.5}$$

and

$$\Omega U_{iz} = \frac{e}{m_i}\left[(1+\tau)k_z - Sk_y\right]\varphi = 0, \tag{2.4.6}$$

where $\Omega = \omega - ik_zU_0$ and $\omega_{e*} = -k_yV_{e*}$. Eliminating U_{iz} from (2.4.5) by using (2.4.6), we obtain the dispersion relation

$$\Omega^2 + \Omega\frac{\omega_{e*}}{1+bs} - \frac{k_z^2C_s^2}{1+bs}\left(1+\tau - S\frac{k_y}{k_z}\right) = 0, \tag{2.4.7}$$

which has the solutions

$$\Omega = -\frac{\Omega_*}{2} \pm \frac{1}{2}\left[\Omega_*^2 + 4\Omega_{IA}^2\left(1+\tau - S\frac{k_y}{k_z}\right)\right]^{1/2}, \tag{2.4.8}$$

where $\Omega_* = \omega_{e*}/\left(1+bs\right)^{1/2}$, $bs = k_y^2\rho_s^2$, $\Omega_{IA} = k_zC_s/\left(1+bs\right)^{1/2}$. Equation (2.4.8) reveals an oscillatory instability if

$$S\frac{k_y}{k_z} > \frac{\Omega_*^2 + 4\Omega_{IA}^2\left(1+\tau\right)}{4\Omega_{IA}^2}. \tag{2.4.9}$$

In the absence of the density gradient, we have from (2.4.7)

$$\omega = k_zU_0 \pm \Omega_{IA}\left(1+\tau - S\frac{k_y}{k_z}\right)^{1/2}, \tag{2.4.10}$$

which also exhibits an oscillatory instability if

$$S\frac{k_y}{k_z} > 1+\tau. \tag{2.4.11}$$

The present instability is known as the parallel ion velocity shear (PIVS) instability.

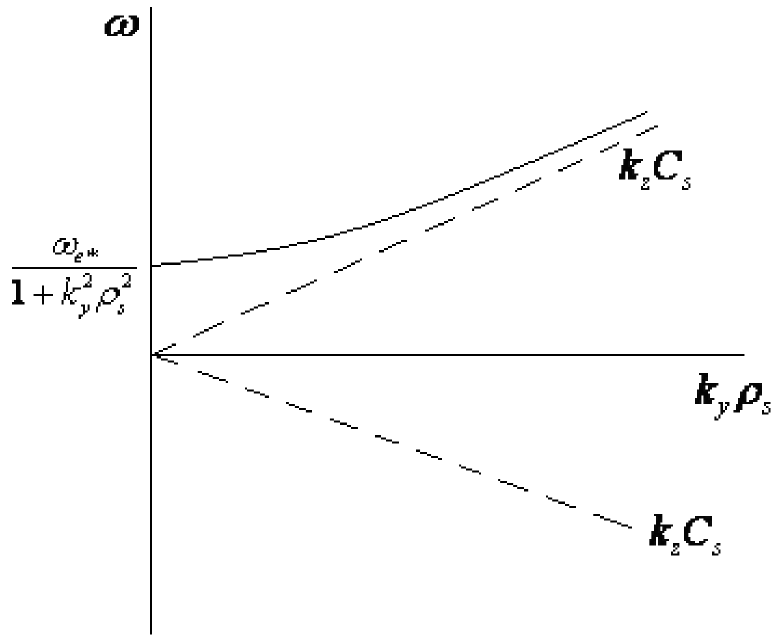
From the figure, we observe that for large k_z the drift wave turns into the ion acoustic wave. Clearly, ion parallel motion is neglected when $k_zC_s = \omega_{e*} = -k_yV_{e*}$ (if $k_y\rho_s \ll 1$). For typical JET parameters, we have $C_s \approx 10^8$ cm/s and $|V_{e*}| \approx 10^5$ cm/s, i.e., we have to require $k_z \ll k_y \times 10^{-3}$ in order to neglect the parallel ion dynamics. The Boltzmann electron distribution is valid if $k_zV_{te} \gg k_yV_{e*}$, which means $k_z \gg 0.25\times10^{-4}k_y$ for JET.

Quasilinear diffusion

Let us consider the particle transport across the magnetic field direction in the presence of non-thermal low frequency fluctuations. We compare the continuity and diffusion equations, which read, respectively,

$$\frac{\partial n}{\partial t} + \boldsymbol{\nabla}.(n\boldsymbol{U}) = 0,$$

and



$$\frac{\partial n}{\partial t} = \mathbf{\nabla .}(D\mathbf{\nabla} n),$$

where the flux $\mathbf{\Gamma} = n\boldsymbol{U}$, according to Fick's low, is

$$\mathbf{\Gamma} = -D\mathbf{\nabla} n.$$

Here D is diffusion coefficient. In a nonuniform plasma a harmonic wave will obtain a superimposed slow space variation of the amplitude due to the plasma inhomogeneity, i.e.

$$\varphi = \varphi(x)\exp(-i\omega t + i\mathbf{k}\cdot\mathbf{r}) + c.c.$$

The flux in the x-direction over the harmonic variation is

$$\langle \Gamma_x \rangle = \sum_k n_{ek} U_k^* + c.c.$$

According for a departure from the Boltzmann low, say due to collisions, or wave-electron interactions, we write

$$n_{e1k} = n_0(1 - i\delta_k)\frac{e\varphi_k}{T_e}.$$

Since the leading order particle drift velocity is $U_k = -i\,(c/B_0)\,k_y\varphi_k$, we have

$$\langle \Gamma_{ex} \rangle = \frac{2n_0 c T_e}{eB_0}\sum_k \left|\frac{e\varphi_k}{T_e}\right|^2 k_y\delta_k.$$

Hence, the diffusion coefficient is

$$D_{ex} = \frac{2cT_e}{eB_0\chi} \sum_k \left| \frac{e\varphi_k}{T_e} \right|^2 k_y \delta_k,$$

where $\chi = -d \ln n_0/dx$. We observe that the diffusion is due to the imaginary part of the electron density fluctuations. For drift waves, we can write

$$n_{i1k} = n_0 \frac{\omega_{e*}}{\omega_k} \frac{e\varphi_k}{T_e},$$

in the region for $\omega_k \gg k_\parallel C_s$ and $k_\perp^2 \rho_s^2 \ll 1$. The ion flux is

$$\langle \Gamma_{ix} \rangle = \frac{n_0 c T_e}{eB_0} \sum_k \left| \frac{e\varphi_k}{T_e} \right|^2 i k_y \frac{\omega_{e*}}{\omega_k} + c.c.$$

Letting $\omega_k = \omega_r + i\gamma_k$, we have

$$\langle \Gamma_{ix} \rangle = \frac{2n_0 c T_e}{eB_0} \sum_k k_y \rho_s \frac{k_y \gamma_k}{\omega_r^2 + \gamma_k^2} \left| \frac{e\varphi_k}{T_e} \right|^2 .$$

The cross field ion diffusion coefficient is then

$$D_i = \frac{2cT_e}{eB_0} \sum_k k_y \rho_s \frac{k_y \gamma_k C_s}{\omega_k^2 + \gamma_k^2} \left| \frac{e\varphi_k}{T_e} \right|^2 .$$

2.5 The influence of magnetic shear on drift waves

Let us suppose that the external magnetic field is of the form

$$\mathbf{B}(x) = B_0 \left(\hat{\mathbf{z}} + \frac{x}{L_s} \hat{\mathbf{y}} \right), \tag{2.5.1}$$

where L_s is the characteristic scale length of the magnetic field variation. This is a kind of magnetic shear. We see that the effect of the transverse variation of the magnetic field is regarded to as magnetic shear is to twist the magnetic field. A toroidal eigenmode will also be twisted according to its poloidal and toroidal mode numbers. At a certain value of r it has the same degree of twisting as the magnetic field and $k_z = 0$ at the mode rational surface defined by $\mathbf{k} \cdot \mathbf{B} = 0$. At large r the poloidal field will have a projection on z. In Cartesian system of coordinate, its variation is accounted for by writing

$$k_z = \frac{x}{L_s} k_y. \tag{2.5.2}$$

In order to study the properties of drift waves in plasmas with magnetic shear, we have to solve a differential equation for the field variation in x, and the solution for the mode frequency becomes an eigenvalue problem. Thus, perturbation are of the form

$$\varphi = \varphi(x)\exp(ik_y y + ik_z z - i\omega t) + c.c. \tag{2.5.3}$$

Accordingly, we have from Eqs. () and ()

$$(\omega - \omega_{e*})\varphi - \omega\rho_s^2\left(\frac{\partial^2}{\partial x^2} - k_y^2\right)\varphi - \frac{T_e}{e}k_z U_{iz} = 0, \tag{2.5.4}$$

and

$$\omega U_{iz} = \frac{e}{m_i}(1+\tau)k_z\varphi. \tag{2.5.5}$$

From the above two equations, we obtain

$$\rho_s^2\frac{\partial^2\varphi}{\partial x^2} - \left(1 + k_y^2\rho_s^2 - \frac{\omega_{e*}}{\omega}\right)\varphi + \frac{k_z^2 V_s^2}{\omega^2}\varphi = 0. \tag{2.5.6}$$

On using (2.5.2), we can write (2.5.6) as

$$\frac{\partial^2\varphi}{\partial x^2} - \frac{k_y^2\omega_{ci}^2(1+\tau)}{\omega_{e*}^2 L_s^2}x^2\varphi + k_s^2\left(\frac{\omega_{e*}}{\omega} - 1 - k_y^2\rho_s^2\right)\varphi = 0, \tag{2.5.7}$$

where $k_s = \rho_s^{-1}$. We have approximated ω by ω_{e*} in the term proportional to x^2, since this term is assumed to be small. Possible solutions of (2.5.7) are

$$\varphi = H_n(i\xi)\exp(\pm i\xi^2/2), \tag{2.5.8}$$

where H_n is a Hermit polynomial of order n and $\xi = (k_y\omega_{ci}(1+\tau)/\omega_{e*}L_s)^{1/2}$.

Substituting (2.5.8) into (2.5.7), we obtain the condition

$$\frac{V_{e*}\omega_{ci}(1+\tau)L_s}{C_s^2}\left(\frac{\omega_{e*}}{\omega} - 1 - k_y^2\rho_s^2\right) = \pm i(2n+1), \tag{2.5.9}$$

which determines the eigenvalue ω. The $\pm$ sign in (2.5.8) and (2.5.9) is related to the direction of the wave propagation. Assuming the presence of absorbing boundaries the group velocity must be outward. Since this corresponds to an inward phase velocity we have to choose the minus sign in Eq. (2.5.9). This leads to convective damping for drift waves with outgoing group velocity. For $n = 0$ mode, we have for $\tau \ll 1$

$$\omega \approx \omega_{e*}(1 - k_y^2\rho_s^2)\left(1 - i\frac{L_n}{L_s}\right), \tag{2.5.10}$$

which corresponds to

$$\varphi = \Phi\exp(-i\xi^2/2), \tag{2.5.11}$$

where Φ is a constant.

2.6 Ion temperature gradient instability

The electrostatic ion-temperature gradient (ITG) mode in the slab geometry occur due to the coupling between the ion sound waves and an ion drift mode that depends on the ion temperature gradient. The ion temperature perturbation thus plays a crucial role. In the ITG mode, the electron density distribution is the Boltzmann, viz.

$$n_{e1} = n_0 \frac{e\varphi}{T_e}, \tag{2.6.1}$$

valid for $\omega/k_z \ll V_{te}$. In the absence of the density gradient, the ion motion is governed by

$$\frac{\partial}{\partial t} n_{i1} + n_0 \frac{\partial}{\partial z} U_{iz} = 0, \tag{2.6.2}$$

$$\frac{\partial}{\partial t} U_{iz} = -\frac{e}{m_i}\frac{\partial}{\partial z}\left(\varphi + \frac{T_i}{e}\frac{n_{i1}}{n_0} + \frac{T_{i1}}{e}\right), \tag{2.6.3}$$

$$\frac{3}{2}\frac{\partial}{\partial t} T_{i1} - \frac{3}{2}\frac{c}{B_0}\frac{dT_{i0}}{dx}\frac{\partial \varphi}{\partial y} + T_{i0}\frac{\partial U_{iz}}{\partial z} = 0. \tag{2.6.4}$$

Assuming $n_{i1} = n_{e1}$, we have from (2.6.1)-(2.6.4)

$$\frac{\partial}{\partial t}\varphi + \frac{T_e}{e}\frac{\partial}{\partial z} U_{iz} = 0, \tag{2.6.5}$$

and

$$\frac{\partial^2}{\partial t^2} U_{iz} = -\frac{e}{m_i}\left(1+\sigma\right)\frac{\partial^2}{\partial z \partial t}\varphi - \frac{c}{m_i B_0}\frac{\partial T_{i0}}{\partial x}\frac{\partial^2 \varphi}{\partial z \partial y}, \tag{2.6.6}$$

where $\left|\partial^2/\partial_t^2\right| \gg V_{ti}^2 \partial^2/\partial_z^2$ has been assumed is deriving (2.6.6). We have denoted $\sigma = T_{i0}/T_e$. Eliminating U_{iz} from (2.6.5) by using (2.6.6), we obtain

$$\frac{\partial^3}{\partial t^3}\varphi - C_s^2\left(1+\sigma\right)\frac{\partial^2}{\partial z^2}\frac{\partial}{\partial t}\varphi - \frac{cT_e}{eB_0 m_i}\frac{dT_{i0}}{dx}\frac{\partial^2 \varphi}{\partial z^2 \partial y} = 0. \tag{2.6.7}$$

Supposing that φ is proportional to $exp(-i\omega t + ik_y y + ik_z z)$, we Fourier transform (2.6.7) to obtain the dispersion relation

$$\omega^3 - \omega k_z^2 C_s^2 (1+\sigma) + k_z^2 C_s^2 \Omega_* = 0, \tag{2.6.8}$$

where $\Omega_* = k_y V_{t*}$ and $V_{t*} = (cT_{i0}/eB_0)\,(d\ln T_{i0}/dx)$.

For $\omega > k_z C_s (1+\sigma)^{1/2}$, we obtain from (2.6.8)

$$\omega^3 = -k_z^2 C_s^2 (1+\sigma)\Omega_* = 0, \tag{2.6.9}$$

which has an unstable solution,

$$\omega = \left(1 + i\frac{\sqrt{3}}{2}\right)\left(k_z^2 V_s^2 \Omega_*\right)^{1/3}, \tag{2.6.10}$$

where $V_s = C_s(1+\sigma)^{1/2}$.

The growth rate of the mode is

$$\gamma = \frac{\sqrt{3}}{2}\left(k_z^2 V_s^2 \Omega_*\right)^{1/2}, \tag{2.6.11}$$

for $\Omega_* > 0$. Physically, instability occurs due to the linear coupling between the ion sound waves the mode $\omega = \Omega_*$. The energy to drive instability comes from the ion temperature gradient, which creates an adverse phase lag between the ion temperature perturbation and the potential/density perturbation.

2.7 Nonlinear generation of convective cells/zonal flows by drift waves

Large scale (long wavelength) convective cells/zonal flows (also regarded to as sheared flows) can be generated by drift large amplitude electrostatic waves due to parametric instabilities. The perpendicular components of the electron and ion fluid velocities in the presence of nonlinearly coupled low-frequency ($\ll \omega_{ci}$) drift waves and zonal flows (ZFs) are, respectively,

$$\mathbf{U}_{e\perp}^d \approx \frac{c}{B_0}\hat{\mathbf{z}} \times \boldsymbol{\nabla}\varphi - \frac{cT_e}{eB_0 n_0}\hat{\mathbf{z}} \times \boldsymbol{\nabla} n_{e1}, \tag{2.7.1}$$

$$\begin{aligned}\mathbf{U}_{i\perp}^d \approx \frac{c}{B_0}\hat{\mathbf{z}} \times \boldsymbol{\nabla}\varphi + \frac{cT_i}{eB_0 n_0}\hat{\mathbf{z}} \times \boldsymbol{\nabla} n_{i1} - \frac{c}{B_0\omega_{ci}}\left(\frac{\partial}{\partial t} + \mathbf{U}_{i\perp}^d - \mu_i \nabla_\perp^2\right)\boldsymbol{\nabla}_\perp\varphi \\ - \frac{c^2}{B_0^2\omega_{ci}}\left[\left(\hat{\mathbf{z}} \times \boldsymbol{\nabla}\varphi \cdot \boldsymbol{\nabla}\right)\boldsymbol{\nabla}_\perp\psi + \left(\hat{\mathbf{z}} \times \boldsymbol{\nabla}\psi \cdot \boldsymbol{\nabla}\right)\boldsymbol{\nabla}_\perp\varphi\right],\end{aligned} \tag{2.7.2}$$

for the drift waves (denoted by the superscript d), and

$$\mathbf{U}_{e\perp}^z \approx \frac{c}{B_0}\hat{\mathbf{z}} \times \boldsymbol{\nabla}\psi, \tag{2.7.3}$$

and

$$\mathbf{U}_{i\perp}^z \approx \frac{c}{B_0}\hat{\mathbf{z}} \times \boldsymbol{\nabla}\psi - \frac{c}{B_0\omega_{ci}}\left[\left(\frac{\partial}{\partial t} - \mu_i \nabla_\perp^2\right)\boldsymbol{\nabla}_\perp\psi + \frac{c}{B_0}\left\langle\left(\hat{\mathbf{z}} \times \boldsymbol{\nabla}\varphi \cdot \boldsymbol{\nabla}\right)\right\rangle\boldsymbol{\nabla}_\perp\varphi\right]. \tag{2.7.4}$$

For the zonal flows (denoted by the superscript z). Here φ and ψ are the electrostatic potentials of the drift waves and zonal flows, respectively. The angular brackets denote averaging over one period of the drift waves. Inserting (2.7.2) into the linearized continuity equation and using the quasi-neutrality condition $n_{i1} = n_0 e\varphi/T_e$, we obtain the equation for the drift waves

$$\frac{\partial}{\partial t}(1-\rho_s^2\nabla_\perp^2)\varphi + V_{e*}\frac{\partial}{\partial y}\varphi + \frac{c}{B_0}\hat{\mathbf{z}} \times \boldsymbol{\nabla}\psi \cdot \boldsymbol{\nabla}\left(1-\rho_s^2\nabla_\perp^2\right)\varphi = 0, \tag{2.7.5}$$

where the drift wave frequency is assumed to be much larger than $\nu_{ii}k_\perp^2\rho_i^2$.

The equation for azimuthally symmetric zonal flows is obtained by inserting (2.7.3) and (2.7.4) into the linearized electron and ion continuity equations and combine them with the Poisson equation. The result is

$$\left(\frac{\partial}{\partial t} - \mu_i \nabla_\perp^2\right) \nabla_\perp^2 \psi + \frac{c}{B_0} \left\langle \hat{\mathbf{z}} \times \boldsymbol{\nabla}\varphi \cdot \boldsymbol{\nabla}\nabla_\perp^2 \varphi \right\rangle = 0. \qquad (2.7.6)$$

The last term in the left-hand side of (2.7.6) represents the Reynold stress (or the ponderomotive force) since its origin is rooted in the nonlinear ion polarization drift or the ion advection in the presence of the drift wave potential. Equations (2.7.5) and (2.7.6) are the desired equations for studying the excitation of zonal flows by large amplitude drift waves.

The nonlinear interactions between a finite amplitude drift pump wave $(\omega_0, \mathbf{k}_0)$ and zonal flows $(\omega, \mathbf{k})$ excite upper and lower drift sidebands $(\omega_\pm, \mathbf{k}_\pm)$. Thus, we decompose the drift wave potential as

$$\varphi = \varphi_{0+} exp\left(-i\omega_0 t + i\mathbf{k}_0 \cdot \mathbf{r}\right) + \varphi_{0-} exp\left(i\omega_0 t - i\mathbf{k}_0 \cdot \mathbf{r}\right) + \sum_{+,-} \varphi_\pm exp\left(-i\omega_\pm t + i\mathbf{k}_\pm \cdot \mathbf{r}\right), \qquad (2.7.7)$$

where $\omega_\pm = \omega \pm \omega_0$ and $\mathbf{k}_\pm = \mathbf{k} \pm \mathbf{k}_0$ are the frequency and wave vectors of the sidebands and the subscript 0 ($\pm$) stands for the pump (sideband). Inserting (2.7.7) into (2.7.5) and Fourier transforming, we obtain

$$D_\pm \varphi_\pm = \pm i \frac{c}{B_0 a_\pm} \left(\hat{\mathbf{z}} \times \mathbf{k}\right) \cdot \mathbf{k}_0 \left(1 + k_{\perp 0}^2 \rho_s^2\right) \varphi_{0\pm} \hat{\psi}, \qquad (2.7.8)$$

where $D_\pm = \omega_\pm - \omega_{e*\pm}$, $\omega_{e*\pm} = -k_{y\pm} V_{e*}$, and $a_\pm = 1 + k_{\perp\pm}^2 \rho_s^2$. In deriving (2.7.8), we have introduced

$$\psi = \hat{\psi} \exp(-i\omega t + i\mathbf{k} \cdot \mathbf{r}), \qquad (2.7.9)$$

and matched the phasors in order to satisfy the frequency and wave vector selection rules.

Furthermore, inserting (2.7.7) and (2.7.9) into (2.7.6) and Fourier transforming, we have

$$\left(\omega + i\Gamma_z\right) \hat{\psi} = i \frac{c}{B_0} \frac{\left(\hat{\mathbf{z}} \times \mathbf{k}_0\right) \cdot \mathbf{k}}{k_\perp^2} \left(K_-^2 \varphi_{0+} \varphi_- - K_+^2 \varphi_{0-} \varphi_+\right), \qquad (2.7.10)$$

where $\Gamma_z = \mu_i k_\perp^2$ and $K_\pm^2 = k_{\perp\pm}^2 - k_0^2$.

Combining Eqs. (2.7.8) and (2.7.10), we readily obtain the nonlinear dispersion relation

$$\omega + i\Gamma_z = -\frac{c^2 \left|\varphi_0\right|^2}{B_0^2} \frac{\left|\hat{\mathbf{z}} \times \mathbf{k}_0 \cdot \mathbf{k}\right|^2}{k_\perp^2} \left(1 + k_{\perp 0}^2 \rho_s^2\right) \sum_{+,-} \frac{K_\pm^2}{a_\pm D_\pm}, \qquad (2.7.11)$$

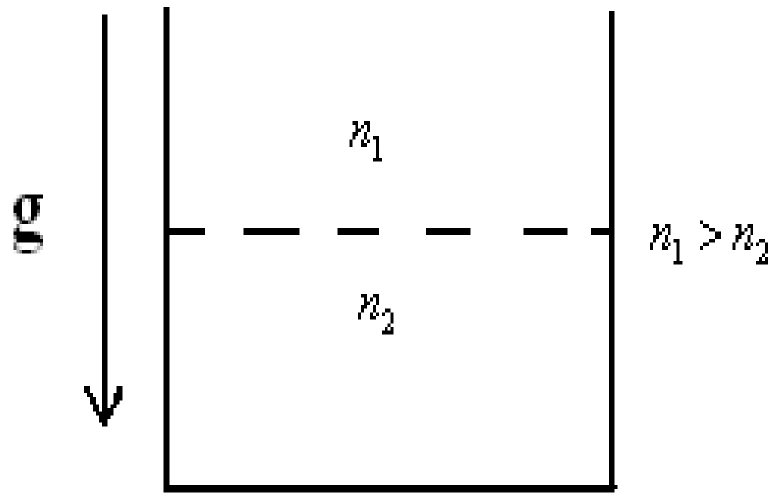


Figure 1: RT instability

where $|\varphi_0|^2 = \varphi_{0+}\,\varphi_{0-}$. For $|\omega| \gg \Gamma_z$ and $\mathbf{k}_{\perp 0} \gg \mathbf{k}_{\perp}$, we obtain from (2.7.11)

$$\omega^2 \approx -2\frac{c^2\,|\varphi_0|^2}{B_0^2}\frac{|\hat{\mathbf{z}}\times\mathbf{k}_0\cdot\mathbf{k}|^2}{k_\perp^2}\mathbf{k}_0\cdot\mathbf{k}_\perp, \tag{2.7.12}$$

which depicts a purely growing ($\omega = i\gamma_z$) instability. For $\mathbf{k}_0\cdot\mathbf{k}_\perp > 0$, the growth rate obtained from (2.7.12), for the azimuthally symmetric zonal flow excitation is

$$\gamma_z = \sqrt{2}\frac{c\,|\varphi_0|}{B_0}\frac{|\hat{\mathbf{z}}\times\mathbf{k}_0\cdot\mathbf{k}|}{k_\perp}\,|\mathbf{k}_0\cdot\mathbf{k}_\perp|^{1/2} \tag{2.7.13}$$

The expression (2.7.13) predicts that the growth rate is proportional to the pump wave electric field $k_0\,|\varphi_0|$.

2.8 Interchange instability

The interchange instability also referred to as the Rayleigh-Taylor (RT) instability, arises in plasmas containing nonuniform density and magnetic field inhomogeneities. The interchange instability has similarity with the RT instability in which a heavy fluid resting on light fluid, falls vertically downward and intermixes, generating flute-like $k_z = 0$ unstable perturbations.

Thus, there is a density gradient which opposes the force due to gravity. The latter represents a center-field force caused by the magnetic field curvature. In plasmas, inhomogeneous magnetic fields having curvature gradients a centrifugal force, due to the thermal motion of charged particles along the field lines, which is directed in a direction opposite to the density gradient. The interchange modes cause a convective transport because they interchange magnetic flux tubes of different pressure.

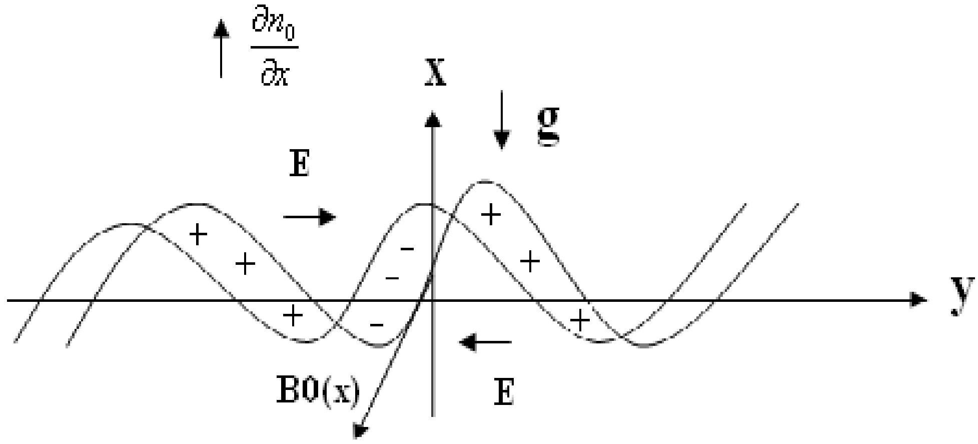


Figure 2: The mechanism of instability

The source of instability is the difference in gravity (curvature) drifts of electrons and ions, which in a combination with a density perturbation leads to a charge separation and associated electric field **E**, as shown in the figure.

In a nonuniform magnetoplasma, at equilibrium, we have

$$\frac{\partial}{\partial x}\left(n_0(x)T_0 + \frac{B_0^2(x)}{8\pi}\right) = 0, \tag{2.8.1}$$

where $T_0 = T_{e0} + T_{i0}$ is the sum of the electron and ion temperatures. In the presence of the low-frequency ($\ll \omega_{ci}$) electrostatic perturbations the appropriate electron and ion fluid velocities are

$$\mathbf{U}_{e\perp} \approx \frac{c}{B_0}\hat{\mathbf{e}}_\parallel \times \boldsymbol{\nabla}\varphi - \frac{cT_e}{eB_0n_0}\hat{\mathbf{e}}_\parallel \times \boldsymbol{\nabla} n_{e1}, \tag{2.8.2}$$

$$\mathbf{U}_{i\perp} \approx \frac{c}{B_0}\hat{\mathbf{e}}_\parallel \times \boldsymbol{\nabla}\varphi + \frac{cT_i}{eB_0n_0}\hat{\mathbf{e}}_\parallel \times \boldsymbol{\nabla} n_{i1} - \frac{c}{B_0\omega_{ci}}\left(\frac{\partial}{\partial t} + V_{i*}\frac{\partial}{\partial y}\right)\boldsymbol{\nabla}_\perp\varphi, \tag{2.8.3}$$

where $\hat{\mathbf{e}}_\parallel = \mathbf{B}_0/B_0$ is the unit vector representing the curvature effect.

Inserting (2.8.2) and (2.8.3) into the linearized electron and ion continuity equations, we obtain respectively

$$\left(\frac{\partial}{\partial t} + \mathbf{U}_{Be}\cdot\boldsymbol{\nabla}\right)n_{e1} + c\boldsymbol{\nabla}\cdot\left(\frac{n_0}{B_0}\hat{\mathbf{e}}_\parallel \times \boldsymbol{\nabla}\varphi\right) = 0, \tag{2.8.4}$$

and

$$\left(\frac{\partial}{\partial t} - \mathbf{U}_{Bi}\cdot\boldsymbol{\nabla}\right)n_{i1} + c\boldsymbol{\nabla}\cdot\left(\frac{n_0}{B_0}\hat{\mathbf{e}}_\parallel \times \boldsymbol{\nabla}\varphi\right) - \frac{cn_0}{B_0\omega_{ci}}\left(\frac{\partial}{\partial t} + V_{i*}\frac{\partial}{\partial y}\right)\nabla_\perp^2\varphi = 0, \tag{2.8.5}$$

where $\mathbf{U}_{Be,Bi} = (cT_{e,i}/eB_0)\,\hat{\mathbf{e}}_\parallel \times \left[\boldsymbol{\nabla}\ln B_0 + \left(\hat{\mathbf{e}}_\parallel \cdot \boldsymbol{\nabla}\right)\hat{\mathbf{e}}_\parallel\right]$ is the magnetic drift velocity the ∇B_0 and the convective effects.

Subtracting (2.8.5) from (2.8.4) and using the quasi-neutrality condition $n_{i1} = n_{e1} = n_1$, we obtain

$$\left(\frac{\partial}{\partial t} + V_{i*}\frac{\partial}{\partial y}\right)\nabla_\perp^2\varphi + \frac{B_0\omega_{ci}}{cn_0}\mathbf{U}_B \cdot \boldsymbol{\nabla} n_1 = 0, \tag{2.8.6}$$

where $\mathbf{U}_B = \mathbf{U}_{Be} + \mathbf{U}_{Bi} = \left[c\left(T_e + T_i\right)/eB_0\right]\hat{\mathbf{e}}_\parallel \times \left[\boldsymbol{\nabla}\ln B_0 + \hat{\mathbf{e}}_\parallel \cdot \boldsymbol{\nabla}\hat{\mathbf{e}}_\parallel\right].$

Equation (2.8.4) gives

$$\left(\frac{\partial}{\partial t} + \mathbf{U}_{Be}\cdot\boldsymbol{\nabla}\right)n_1 - \frac{n_0 c}{B_0}\left(\mathbf{k}_{ne} - \mathbf{k}_{Be}\right)\cdot\boldsymbol{\nabla}\varphi = 0, \tag{2.8.7}$$

where $\mathbf{k}_{ne} = \hat{\mathbf{e}}_\parallel \times \boldsymbol{\nabla}\ln n_0(x)$ and $\mathbf{k}_{Be} = \hat{\mathbf{e}}_\parallel \times \left[\boldsymbol{\nabla}\ln B_0 + \hat{\mathbf{e}}_\parallel \cdot \boldsymbol{\nabla}\hat{\mathbf{e}}_\parallel\right].$

Supposing that φ and n_1 are proportional to $\exp(-i\omega t + i\mathbf{k}_\perp\cdot\mathbf{r}_\perp)$ and Fourier transform (2.8.6) and (2.8.7) and combine the resultant equations to obtain the dispersion relation

$$\left(\omega - \Omega_{Be}\right)\left(\omega - \omega_{i*}\right) - \omega_{ci}\Omega_B\frac{\mathbf{k}\cdot\left(\mathbf{k}_{ne} - \mathbf{k}_{Be}\right)}{k_\perp^2} = 0, \tag{2.8.8}$$

where $\Omega_{Be} = \mathbf{k}\cdot\mathbf{U}_B \equiv 2k_y cT/eB_0R_c$, R_c is the radius of curvature, $\omega_{i*} = k_y V_{i*}$, and $\Omega_B = \mathbf{k}\cdot\mathbf{U}_{Be} \equiv 2k_y cT_e/eB_0R_c$.

Possible solutions of (2.8.8) are

$$\omega = \frac{1}{2}\left(\Omega_{Be} + \omega_{i*}\right) \pm \frac{1}{2}\left[\left(\Omega_{Be} + \omega_{i*}\right)^2 + 4\left(\Omega_B\omega_{ci}\mathbf{k}\cdot\left(\mathbf{k}_{ne} - \mathbf{k}_{Be}\right)/k_\perp^2\right)\right]^{1/2} \tag{2.8.9}$$

Equation (2.8.9) depicts an oscillatory instability for $\mathbf{k}_{ne} < 0$, $\mathbf{k}_{Be} > 0$ and

$$\omega_{ci}\Omega_B\left|\mathbf{k}_{ne} + \mathbf{k}_{Be}\right| > \left(\Omega_{Be} + \omega_{i*}\right)^2/4 \tag{2.8.10}$$

The growth rate of the interchange instability, obtained from (2.8.9) above threshold is

$$\gamma = \sqrt{\omega_{ci}\Omega_B}\left|\mathbf{k}\cdot\left(\mathbf{k}_{ne} + \mathbf{k}_{Be}\right)\right|^{1/2} \tag{2.8.11}$$

3 Electromagnetic waves

3.1 Hall-MHD waves

We discuss the properties of electromagnetic waves in a uniform collisionless plasma within the framework of the Hall-MHD equations. The latter consist of the ion continuity equation

$$\frac{\partial n_i}{\partial t} + \boldsymbol{\nabla}.(n_i\mathbf{U}_i) = 0, \tag{3.1.1}$$

the ion momentum equation

$$\rho_i \left(\frac{\partial}{\partial t} + \mathbf{U}_i . \boldsymbol{\nabla} \right) \mathbf{U}_i = \frac{\mathbf{J} \times \mathbf{B}}{c} - \boldsymbol{\nabla} P. \tag{3.1.2}$$

Ampere's law

$$\boldsymbol{\nabla} \times \mathbf{B} = \frac{4\pi}{c} \mathbf{J}, \tag{3.1.3}$$

and Faraday's law

$$\begin{aligned} \frac{\partial \mathbf{B}}{\partial t} &= -c \boldsymbol{\nabla} \times \mathbf{E} \\ &\equiv \boldsymbol{\nabla} \times \left[\left(\mathbf{U}_i - \frac{c \boldsymbol{\nabla} \times \mathbf{B}}{4\pi e n_0} \right) \times \mathbf{B} \right], \end{aligned} \tag{3.1.4}$$

where $P = (T_e + T_i)n$, $n_e = n_i = n$, and $\rho_i = n_i m_i$. The $\boldsymbol{\nabla} \times \mathbf{B}$ term in the right-hand side of (3.1.4) represents the Hall velocity.

Letting $n_i = n_0 + n_i$, and $\mathbf{B} = \mathbf{B}_0 \hat{\mathbf{z}} + \mathbf{B}_1$, where $n_1 \ll n_0$ and $\mathbf{B}_1 \ll \mathbf{B}_0$, we obtain from (3.1.1)-(3.1.4), the linear dispersion relation in the usual manner

$$\left(\omega^2 - k_z^2 V_A^2 \right) D_{ms} = \frac{\omega^2}{\omega_{ci}^2} (\omega^2 - k^2 V_s^2) k_z^2 k_\perp^2 V_A^4, \tag{3.1.5}$$

where $D_{ms} = \omega^4 - \omega^2 k^2 (V_s^2 + V_A^2) - k_z^2 k^2 V_s^2 V_A^2$ with $V_A = B_0 / \sqrt{4\pi n_0 m_i}$, $V_s = \sqrt{(T_e + T_i)/m_e}$. Equation (3.1.5) shows that the Alfven and magnetoacoustic waves are coupled due to the effect of finite ω/ω_{ci}. Equation (3.1.5) also contains the low-frequency ($\omega \ll \omega_{ci}$) kinetic Alfven wave (KAW)

$$\omega = k_z V_A (1 + k_y^2 \rho_s^2)^{1/2}, \tag{3.1.6}$$

in the limit $\omega \ll k V_s$, $V_s \ll V_A$, as well as long wavelength (in comparison with the electron skin depth $\lambda_e = c/\omega_{pe}$) electron whistlers. The latter usually have $\sqrt{\omega_{ce} \omega_{ci}} \ll \omega \ll kc \ll \omega_{pe}$.

The dynamics of nonlinear whistler is governed by the electron-MHD equation

$$\frac{DB}{Dt} - B . \boldsymbol{\nabla} \mathbf{U} = 0, \tag{3.1.7}$$

where $B = \mathbf{B} - \lambda_e^2 \nabla^2 \mathbf{B}$, $D/Dt = (\partial/\partial t) + \mathbf{U} . \boldsymbol{\nabla}$ and $\mathbf{U} = -c \boldsymbol{\nabla} \times \mathbf{B} / 4\pi e n$ is the electron fluid velocity. In the whistler dynamics, the ions do not participate since $|D/Dt| \gg \omega_{pi}, \omega_{ci}$. In the linear limit Eq. (3.1.7) admits obliquely propagating whistlers whose frequency is

$$\omega = \frac{k^2 c^2 \omega_{ce} \cos\theta}{\omega_{pe}^2 + k^2 c^2}, \tag{3.1.8}$$

where $\cos\theta = k_z/k$. In the long wavelength limit, namely $kc \ll \omega_{pe}$, we have from (3.1.8)

$$\omega = \frac{k_z k c^2 \omega_{ce}}{\omega_{pe}^2}, \tag{3.1.9}$$

which reduces to

$$\omega = \frac{k_z^2 c^2 \omega_{ce}}{\omega_{pe}^2}, \tag{3.1.10}$$

when the perpendicular component of the wavevector k_x is zero (N.B. $k = \sqrt{k_x^2 + k_z^2}$).

Introduction of the ion dynamics in the description of whistlers gives rise to the modification of the index of refraction $N = k_z^2 c^2/\omega^2$. We have for $\omega \ll \omega_{ce} \cos\theta$

$$N \approx 1 \pm \frac{\omega_{pe}^2}{\omega\omega_{ce}\cos\theta} - \frac{\omega_{pi}^2}{\omega(\omega \mp \omega_{ci})\cos\theta}. \tag{3.1.11}$$

For $\theta = 0$, we have from (3.1.11)

$$\frac{k_z^2 c^2}{\omega^2} \approx \pm\frac{\omega_{pe}^2}{\omega\omega_{ce}} - \frac{\omega_{pi}^2}{\omega(\omega \mp \omega_{ci})}, \tag{3.1.12}$$

where $\omega_{pe}^2/\omega\omega_{ce} \gg 1$. Here + (-) refers to the right-(left)-hand circular polarized electromagnetic waves represented by

$$\mathbf{E} = \mathbf{E}_\perp(\hat{\mathbf{x}} \pm i\hat{\mathbf{y}}) \exp(-i\omega t + ik_z z), \tag{3.1.13}$$

where $\hat{\mathbf{x}}$ and $\hat{\mathbf{y}}$ are the unit vectors along the x and y axes, respectively. In the low-frequency limit (i.e. $\omega \ll \omega_{ci}$), we have from (3.1.12)

$$\omega \approx k_z V_A \left(1 \pm \frac{k_z V_A}{2\omega_{ci}}\right), \tag{3.1.14}$$

which is the dispersive Alfven wave. The dispersion comes from the finite ω/ω_{ci} effect.

3.2 Drift Alfven waves

Drift Alfven waves (DAWs) arise in a nonuniform plasma containing the equilibrium density gradient dn_0/dx . The properties of the DAWs are determined either from the dispersion relation that is deduced by using the two-fluid or a kinetic description. In the two-fluid model of the low-frequency ($\ll \omega_{ci}$) electromagnetic waves we express the electromagnetic fields as,

$$\mathbf{E} = -\boldsymbol{\nabla}\varphi - \frac{1}{c}\frac{\partial A_z}{\partial t}, \tag{3.2.1}$$

and

$$\mathbf{B} = \boldsymbol{\nabla} A_z \times \hat{\mathbf{z}} \equiv \mathbf{B}_\perp \tag{3.2.2}$$

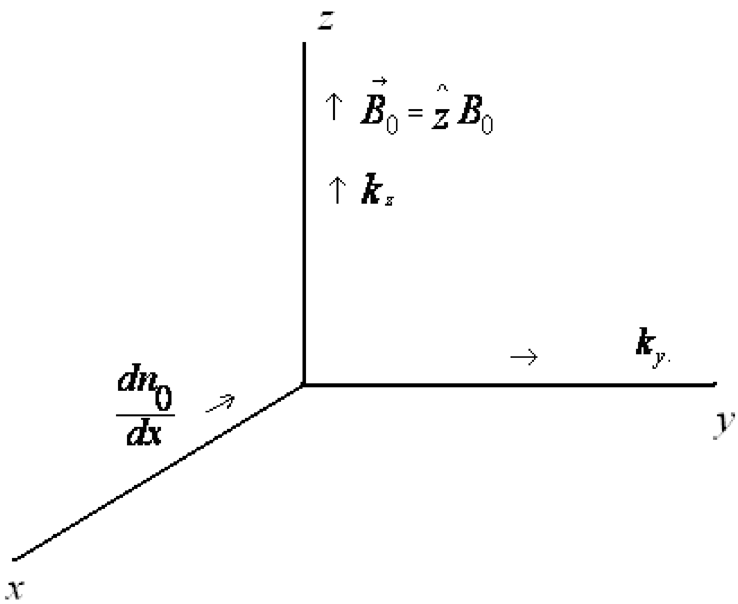


Figure 3: Drift wave geometry

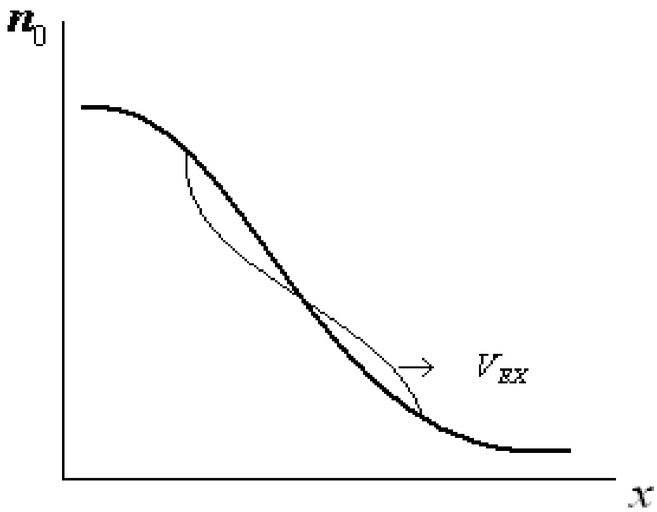


Figure 4: Convective density perturbation caused by the $\mathbf{E} \times \mathbf{B}_0$ drift

where A_z is the z-component of the vector potential. In the low-$\beta(\ll 1)$ approximation, we neglect the compressional magnetic field perturbation (viz. $\mathbf{A}_\perp = 0$). The perpendicular components of the electron and ion fluid velocities will be here.

$$\mathbf{U}_{e\perp} \approx \frac{c}{B_0}\hat{\mathbf{z}} \times \boldsymbol{\nabla}\varphi - \frac{cT_e}{eB_0 n_0}\hat{\mathbf{z}} \times \boldsymbol{\nabla} n_{e1}, \tag{3.2.3}$$

$$\mathbf{U}_{i\perp} \approx \frac{c}{B_0}\hat{\mathbf{z}} \times \boldsymbol{\nabla}\varphi + \frac{cT_i}{eB_0 n_0}\hat{\mathbf{z}} \times \boldsymbol{\nabla} n_{i1} - \frac{c}{B_0\omega_{ci}}\left(\frac{\partial}{\partial t} + V_{i*}\frac{\partial}{\partial y}\right)\boldsymbol{\nabla}_\perp\varphi. \tag{3.2.4}$$

The parallel component of the Ampere's law gives

$$U_{ez} \approx U_{iz} + \frac{c}{4\pi e n_0}\nabla_\perp^2 A_z, \tag{3.2.5}$$

where the parallel component of the ion fluid velocity U_{iz} is determined from

$$\frac{\partial U_{iz}}{\partial t} = -\frac{e}{m_i}\frac{\partial}{\partial z}\left(\varphi + \frac{T_i}{e}\frac{n_{i1}}{n_0}\right) - \frac{e}{m_i c}\frac{\partial A_z}{\partial t}. \tag{3.2.6}$$

The parallel component of the inertialess electron momentum equation gives

$$\left(\frac{\partial}{\partial t} + V_{e*}\frac{\partial}{\partial y}\right)A_z + c\frac{\partial}{\partial z}\left(\varphi - \frac{T_e}{e}\frac{n_{e1}}{n_0}\right) = 0. \tag{3.2.7}$$

Now inserting (3.2.3) and (3.2.4) into the linearized electron and ion continuity equations, we have

$$\frac{\partial n_{e1}}{\partial t} - \frac{c}{B_0}\frac{dn_0}{dx}\frac{\partial\varphi}{\partial y} + n_0\frac{\partial}{\partial z}\left(U_{iz} + \frac{c}{4\pi e n_0}\nabla_\perp^2 A_z\right) = 0. \tag{3.2.8}$$

$$\frac{\partial n_{i1}}{\partial t} - \frac{c}{B_0}\frac{dn_0}{dx}\frac{\partial\varphi}{\partial y} - \frac{cn_0}{B_0\omega_{ci}}\left(\frac{\partial}{\partial t} + V_{i*}\frac{\partial}{\partial y}\right)\nabla_\perp^2\varphi + n_0\frac{\partial}{\partial z}U_{iz} = 0. \tag{3.2.9}$$

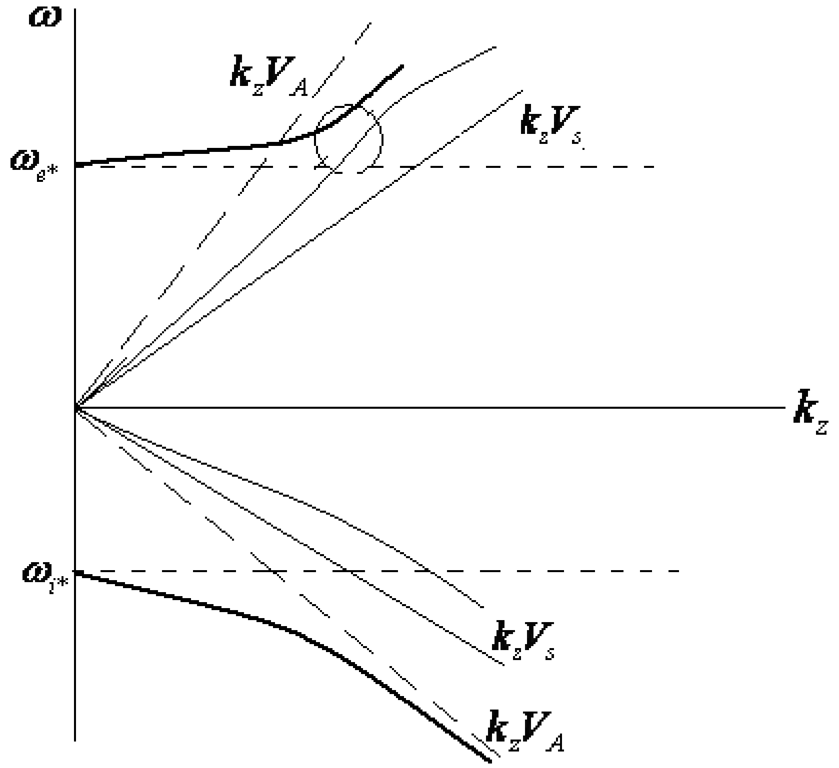
Subtracting (3.2.9) from (3.2.8) and using $n_{i1} = n_{e1} \equiv n_1$, we obtain

$$\left(\frac{\partial}{\partial t} + V_{i*}\frac{\partial}{\partial y}\right)\nabla_\perp^2\varphi + \frac{V_A^2}{c}\frac{\partial}{\partial z}\nabla_\perp^2 A_z = 0. \tag{3.2.10}$$

Equations (3.2.6), (3.2.7), (3.2.8) and (3.2.10) are the desired governing equations for the deriving the linear dispersion relation for the DAWs in the local approximation. Assuming that n_1, φ, U_{iz} and A_z are proportional to $exp\,(-i\omega t + i\,\mathbf{k}_\perp \cdot \mathbf{r}_\perp + ik_z z)$, we Fourier transform the governing equations above, and combine the resultant equations to obtain

$$[\omega\,(\omega - \omega_{e*}) - k_z^2 V_s^2][\omega\,(\omega - \omega_{i*}) - k_z^2 V_A^2] = k_\perp^2\rho_s^2 k_z^2 V_A^2\,\,(\omega - \omega_{i*})\,\omega, \tag{3.2.11}$$

where $V_s = (C_s^2 + V_{ti}^2)^{1/2}$.



The dispersion relation (3.2.11) has four branches, as shown in above figure. From the above figure we notice that for $V_A > V_s$, the two branches with $k_z \neq 0$ do not intersect, instead they obtain a region of strong coupling (shown by a circle) in the figure. In the latter, the drift and Alfven waves thus coupled. Such a coupling occurs in a plasma with $\beta > m_e/m_i$, since $\omega/k_z < V_{te}$ requires that $V_A < V_{te}$. Equation (3.2.11) admits unstable solutions due to the linear coupling between the drift and Alfven waves.

3.3 The magnetic drift mode

The two-dimensional magnetic drift mode (MDM) occurs in an electron-ion plasma with fixed ion background. The MDM involves the electron motion along $B_0\hat{\mathbf{z}}$ and perturbs the magnetic field to produce $\mathbf{B}_\perp = \boldsymbol{\nabla}_\perp A_z \times \hat{\mathbf{z}}$. The dynamics of inertial electrons is governed by the parallel electron momentum equation

$$\frac{\partial U_{ez}}{\partial t} = \frac{e}{mc}\left(\frac{\partial}{\partial t} + V_{e*}\frac{\partial}{\partial y}\right) A_z, \tag{3.3.1}$$

where U_{ez} is given by

$$U_{ez} \approx \frac{c}{4\pi e n_0}\nabla_\perp^2 A_z, \tag{3.3.2}$$

we note that the electric field of the 2D MDM is

$$E_z = -\frac{1}{c}\frac{\partial A_z}{\partial t}, \tag{3.3.3}$$

and there are no density and potential fluctuation with the mode .

From (3.3.1) and (3.3.2), we obtain

$$\frac{\partial}{\partial t}\left(1 - \lambda_e^2 \nabla_\perp^2\right) A_z + V_{e*}\frac{\partial}{\partial y} A_z = 0 \tag{3.3.4}$$

Supposing that A_z is proportional $exp(-i\omega t + i\ \mathbf{k}_\perp \cdot \mathbf{r})$, we Fourier transform (3.3.4) to obtain

$$\omega = \frac{\omega_{e*}}{1 + k_\perp^2 \lambda_e^2}, \tag{3.3.5}$$

which is the frequency of the 2D dispersive magnetic drift mode. The dispersion arises from the parallel electron inertial force .

3.4 Pseudo-three-dimensional convective cells

For low frequency (in comparison with the ion gyrofrequency) electrostatic pseudo 3D convective cells, we have the Boltzmann law

$$\frac{n_{e1}}{n_0} \approx \frac{e\varphi}{T_e} \tag{3.4.1}$$

which is valid for low-parallel (to $B_0\hat{\mathbf{z}}$)phase speed (in comparison with ion electron thermal speed) disturbances. The ion density perturbation n_{i1} is determined from

$$\frac{\partial n_{i1}}{\partial t} - \frac{c n_0}{B_0 \omega_{ci}}\left(\frac{\partial}{\partial t} + 0.3\nu_{ii}\rho_i^2\nabla_\perp^2\right)\nabla_\perp^2\varphi = 0, \tag{3.4.2}$$

where the ion motion is assumed two-dimensional in a plane perpendicular to $\hat{\mathbf{z}}$.

Replacing n_{i1} by $n_0 e\varphi/T_e$ in the quasi-neutrality approximation, we have

$$\frac{\partial}{\partial t}(1 - \rho_s^2\nabla_\perp^2)\varphi - 0.3\nu_{ii}\rho_i^2\rho_s^2\nabla_\perp^4\varphi = 0. \tag{3.4.3}$$

Assuming that φ is proportional to $exp(-i\omega t + i\mathbf{k}_\perp \cdot \mathbf{r})$, we obtain from (3.4.3) after Fourier transformation

$$\omega = -i\frac{0.3\nu_{ii}k_\perp^2\rho_i^2\rho_s^4}{1 + k_\perp^2\rho_s^2} \equiv -i\gamma_D, \tag{3.4.4}$$

which yields the damping rate γ_D of the pseudo-3D convective cells/ zonal flows.

3.5 Nonlinear excitation of zonal flows by kinetic Alfven waves

The dispersive kinetic Alfven waves (DKAWs) are the low-frequency ($\ll \omega_{ci}$) electromagnetic waves, which have the parallel electric field as well as the perpendicular (to $B_0\hat{\mathbf{z}}$) components of the electric and magnetic fields. Thus, the electromagnetic fields are denoted by

$$\mathbf{E} = -\boldsymbol{\nabla}\phi - \frac{1}{c}\frac{\partial A_z}{\partial t}, \tag{3.5.1}$$

and

$$\mathbf{B} = -\boldsymbol{\nabla}A_z \times \hat{\mathbf{z}} \equiv \mathbf{B}_\perp, \tag{3.5.2}$$

where φ is the scalar potential and A_z is the parallel (to $\hat{\mathbf{z}}$) component of the vector potential. The DKAWs are accompanied with finite density perturbation

$$n_1 = n_0\frac{c}{B_0\omega_{ci}}\nabla_\perp^2\varphi, \tag{3.5.3}$$

associated with the ion polarization drift. Since the parallel phase speed of the DKAWs is much smaller than the electron thermal speed $V_{te} = (T_e/m_e)^{1/2}$, they appear in a plasma with an intermediate plasma β, viz.

$$\frac{m_e}{m_i} \ll \beta \ll \frac{4\pi n_0(T_e+T_i)}{B_0^2} \ll 1, \tag{3.5.4}$$

values. In view of the low-β approximation, the compressional magnetic field perturbation can be neglected in the DKAW dynamics. The DKAW frequency is

$$\omega = k_z V_A(1+k^2\rho^2), \tag{3.5.5}$$

where $\rho = (\rho_s^2 + 3\rho_i^2)^{1/2}$ is the effective ion gyroradius and ρ_i is the ion thermal gyroradius. In the following, we consider the dynamics of the DKAW in the presence of the zonal flows in plasma. The electron and ion fluid velocities for our purposes are

$$\mathbf{U}_{e\perp}^{kA} \approx \frac{c}{B_0}\hat{\mathbf{z}}\times\boldsymbol{\nabla}\varphi - \frac{cT_e}{eB_0}\hat{\mathbf{z}}\times\boldsymbol{\nabla}n_1, \tag{3.5.6}$$

and

$$\begin{aligned}\mathbf{U}_{i\perp}^{kA} \approx\ & \frac{c}{B_0}\hat{\mathbf{z}}\times\boldsymbol{\nabla}\varphi - \frac{c}{B_0\omega_{ci}}\left[\left(\frac{\partial}{\partial t} + \nu_{in} + \mu_i\nabla_\perp^2\right)\boldsymbol{\nabla}_\perp\varphi + \frac{c}{B_0}(\hat{\mathbf{z}}\times\boldsymbol{\nabla}\psi\cdot\boldsymbol{\nabla})\boldsymbol{\nabla}_\perp\varphi\right.\\ & \left. + \frac{c}{B_0}(\hat{\mathbf{z}}\times\boldsymbol{\nabla}\varphi\cdot\boldsymbol{\nabla})\boldsymbol{\nabla}_\perp\psi\right],\end{aligned} \tag{3.5.7}$$

where φ and ψ are the potentials of the KAWs (denoted by the superscript KA), and zonal flows, respectively, and $T_e \gg T_i$ has been assumed.

The approximate electron and ion fluid velocities in zonal flows in the presence of the DKAWs are, respectively,

$$\mathbf{U}_{e\perp}^{zF} \approx \frac{c}{B_0}\hat{\mathbf{z}} \times \boldsymbol{\nabla}\psi + \frac{\langle U_{ez}\mathbf{B}_\perp\rangle}{B_0}, \tag{3.5.8}$$

and

$$\mathbf{U}_{i\perp}^{zF} \approx \frac{c}{B_0}\hat{\mathbf{z}}\times\boldsymbol{\nabla}\psi - \frac{c}{B_0\omega_{ci}}\left[\left(\frac{\partial}{\partial t} + \nu_{in} + \mu_i\nabla_\perp^2\right)\boldsymbol{\nabla}_\perp\psi + \frac{c}{B_0}\left\langle(\hat{\mathbf{z}} \times \boldsymbol{\nabla}\varphi \cdot \boldsymbol{\nabla})\boldsymbol{\nabla}_\perp\varphi\right\rangle\right], \tag{3.5.9}$$

where the parallel component of the electron fluid velocity in the DKAWs is

$$U_{ez}^{kA} \approx \frac{c}{4\pi e n_0}\nabla_\perp^2 A_z, \tag{3.5.10}$$

which is obtained from the parallel component of Ampere's law with $\mathbf{B}_\perp = \boldsymbol{\nabla}A_z \times \hat{\mathbf{z}}$. In Eq. (3.5.10), we have neglected the parallel ion motion, as we are isolating the ion-acoustic waves in our intermediate β plasma. The last term in the right-hand side of Eqs. (3.5.8) and (3.5.9) are the nonlinear Lorentz force and the Reynolds stresses of the DKAWs, which reinforce 2D zonal flows.

Substituting (3.5.8) and (3.5.7) into

$$\boldsymbol{\nabla} \cdot \mathbf{J} = en_0\boldsymbol{\nabla} \cdot (\mathbf{U}_{i\perp}^{kA} - \mathbf{U}_{e\perp}^{kA} + U_{ez}^{kA}\hat{\mathbf{z}}) = 0, \tag{3.5.11}$$

we have

$$\frac{\partial}{\partial t}\nabla_\perp^2\varphi + \frac{V_A^2}{c}\frac{\partial}{\partial z}\nabla_\perp^2 A_z + \frac{c}{B_0}(\hat{\mathbf{z}} \times \boldsymbol{\nabla}\psi \cdot \boldsymbol{\nabla})\nabla_\perp^2\varphi + \frac{c}{B_0}(\hat{\mathbf{z}} \times \boldsymbol{\nabla}\varphi \cdot \boldsymbol{\nabla})\nabla_\perp^2\psi = 0, \tag{3.5.12}$$

where we have assumed that $|\partial\varphi/\partial t| \gg (\nu_{in} + \mu_i\nabla_\perp^2)\varphi$. Equation (3.5.11) assumes the quasineutrality condition $n_{e1} = n_{i1} = n_1$, which holds in a dense plasma with $\omega_{pi} \gg \omega_{ci}$.

From the parallel component of the inertialess electron equation of motion, we obtain

$$\frac{\partial A_z}{\partial t} + c\frac{\partial}{\partial z}\left(\varphi - \frac{T_e n_1}{en_0}\right) + \frac{c}{B_0}\hat{\mathbf{z}} \times \boldsymbol{\nabla}\psi \cdot \boldsymbol{\nabla}A_z = 0. \tag{3.5.13}$$

On the other hand, the ion continuity equation, together with (3.5.7), yields

$$\left(\frac{\partial}{\partial t} + \frac{c}{B_0}\hat{\mathbf{z}} \times \boldsymbol{\nabla}\psi \cdot \boldsymbol{\nabla}\right)\left(n_1 - \frac{c}{B_0\omega_{ci}}\nabla_\perp^2\varphi\right) = 0. \tag{3.5.14}$$

Equations (3.5.12)-(3.5.14) are the desired equations for the DKAWs in the presence of zonal flows. The equation for 2D zonal flows is obtained by inserting (3.5.8) and (3.5.9) into the electron and ion continuity equations, respectively, substituting the resultant equations into the Poisson equation

$$\nabla_\perp^2\frac{\partial\varphi}{\partial t} = 4\pi\left(\frac{\partial n_{e1}}{\partial t} - \frac{\partial n_{i1}}{\partial t}\right), \tag{3.5.15}$$

we obtain

$$\left(\frac{\partial}{\partial t} + \nu_{in} + 0.3\nu_{ii}\rho_i^2\nabla_\perp^2\right)\nabla_\perp^2\psi + \frac{c}{B_0}\left\langle(\hat{\mathbf{z}}\times\boldsymbol{\nabla}\varphi\cdot\boldsymbol{\nabla})\nabla_\perp^2\varphi\right\rangle$$

$$= \frac{V_A^2}{c}\left\langle(\hat{\mathbf{z}}\times\boldsymbol{\nabla}A_z\cdot\boldsymbol{\nabla})\nabla_\perp^2 A_z\right\rangle, \tag{3.5.16}$$

where to lowest order, we must use

$$\frac{\partial A_z}{\partial t} + \frac{c}{V_A^2}\frac{\partial\varphi}{\partial t} = 0, \tag{3.5.17}$$

into the last term of Eq. (3.5.16) to eliminate A_z in terms of φ.

Let us now derive a general dispersion relation for the modulational instability of a constant amplitude DAW pump against zonal flow perturbation. For this purpose, we decompose the high frequency potentials into those of the pump and two sidebands, viz.

$$\varphi = \varphi_{0+}\exp(-i\omega_0 t + i\mathbf{k}_0\cdot\mathbf{r}) + \varphi_{0-}\exp(i\omega_0 t - i\mathbf{k}_0\cdot\mathbf{r}) + \sum_{+,-}\varphi_\pm\exp(-i\omega_\pm t + i\mathbf{k}_\pm\cdot\mathbf{r}), \tag{3.5.18}$$

$$A_z = A_{z0+}\exp(-i\omega_0 t + i\mathbf{k}_0\cdot\mathbf{r}) + A_{z0-}\exp(i\omega_0 t - i\mathbf{k}_0\cdot\mathbf{r}) + \sum_{+,-}A_{z\pm}\exp(-i\omega_\pm t + i\mathbf{k}_\pm\cdot\mathbf{r}), \tag{3.5.19}$$

$\omega_\pm = \Omega \pm \omega_0$ and $\mathbf{k}_\pm = \mathbf{K} \pm \mathbf{k}_0$ are the frequency and wave vector of the upper and lower DAW sidebands. The subscripts $0_\pm$ and $\pm$ represent the pump and sidebands, respectively. Furthermore, we assume that

$$\Psi = \varphi\exp(-i\Omega t + i\mathbf{K}\cdot\mathbf{r}), \tag{3.5.20}$$

where Ω and $\mathbf{K}$ are the frequency and wave vector of zonal flows. We now inserts Eqs. (3.5.18)-(3.5.20) into Eq. (3.5.12)-(3.5.14) and Fourier transform them and combine the resultant equations to obtain

$$D_\pm\varphi_\pm = \pm\frac{ic}{B_0}(\hat{\mathbf{z}}\times\mathbf{k}_0).\mathbf{K}\left(\omega_0 + \omega_\pm\frac{k_{0\perp}^2 - \mathbf{K}_\perp^2}{k_{\perp\pm}^2}\right)\varphi_{0\pm} \tag{3.5.21}$$

where

$$D_\pm = \omega_\pm^2 - k_{z0}^2 V_A^2(1 + k_{\perp\pm}^2\rho_s^2) = \pm 2\omega_0(\Omega - \mathbf{K}_\perp\cdot\mathbf{V}_{g\perp} \mp \delta),$$

with

$$\omega_0 = k_{z0}V_A(1 + k_{0\perp}^2\rho_s^2)^{1/2},\ \mathbf{V}_{g\perp} = k_{0\perp}\rho_s^2 k_{z0}^2 V_A^2/\omega_0,\ \text{and}\ \delta = k_{z0}^2 V_A^2 K_\perp^2\rho_s^2/2\omega_0.$$

On the other hand, inserting Eqs. (3.5.18)-(3.5.20) into Eq. (3.5.16) and Fourier transforming the resultant equation, we have

$$(\Omega + i\Gamma_z)\varphi = i\frac{2c}{B_0}\frac{(\hat{\mathbf{z}}\times\mathbf{k}_0)\cdot\mathbf{K}}{K_\perp^2}\left(1 - \frac{\omega_0^2}{k_{z0}^2 V_A^2}\right)$$

$$\times\left(K_-^2\varphi_{0+}\varphi_- - K_+^2\varphi_{0-}\varphi_+\right), \tag{3.5.22}$$

where

$$\Gamma_z = \nu_{in} + 0.3\nu_{ii}K^2\rho_i^2 \text{ and } K_\pm^2 = k_{\perp\pm}^2 - k_0^2 \equiv K_\perp^2 \pm 2\mathbf{k}_0\cdot\mathbf{K}_\perp.$$

Equation (3.5.22) reveals that the coupling constant on the right hand side remains finite only if $\omega_0 \neq k_{0z}V_A$. Thus, dispersion to Alfven waves is required in order for the parametric coupling between the DKAWs and Zonal flows to remain intact. By using Eq. (3.5.21), we can eliminate $\varphi_\pm$ from (3.5.22), obtaining the nonlinear dispersion relation

$$\Omega + i\Gamma_z = \frac{2c^2\omega_o|\varphi_0|^2}{B_0^2}\frac{|\,\hat{\mathbf{z}}\times\mathbf{k}_0\cdot\mathbf{K}\,|^2}{K_\perp^2}k_{0\perp}^2\rho_s^2\sum_{+,-}\frac{K_\pm^2 x_\pm^2}{k_{\perp\pm}^2 D_\pm} \qquad (3.5.23)$$

where $x_\pm^2 = K_\perp^2 \pm \mathbf{k}_0\cdot\mathbf{K}_\perp$. We see that the coupling constant in the right hand of (3.5.23) proportional to $k_{0\perp}^2\rho_s^2$, which is a feature of the DKAWs. For long wavelength zonal flows with $|K_\perp| \ll |k_{0\perp}|$. Eq. (3.5.23) reduces to

$$(\Omega+i\Gamma_z)\left[(\Omega - \mathbf{K}_\perp\cdot\mathbf{V}_{g\perp})^2 - \delta^2\right] = -2K_\perp^2c^2\delta\frac{|E_{0\perp}|^2}{B_0^2}k_{0\perp}^2\rho_s^2|(\hat{\mathbf{K}_\perp}\cdot\hat{\mathbf{k}_{0\perp}})(\hat{\mathbf{z}}\times\hat{\mathbf{k}_0}\cdot\hat{\mathbf{K}})|^2 \qquad (3.5.24)$$

where $|E_{0\perp}|^2 = k_{0\perp}^2|\varphi_0|^2$ and $\hat{K}_\perp$ and $\hat{k}_{0\perp}$ are the unit vectors.

We analyze Eq. (3.5.24) in two limiting cases. First, we let $\Omega = \mathbf{K}\cdot\mathbf{V}_g + i\gamma_m$ in Eq. (3.5.24) and obtain for $\gamma_m, \Gamma_z \ll |\mathbf{K}_\perp\cdot\mathbf{V}_g|$, the growth rate

$$\gamma_m = \left[\frac{2K_\perp^2c^2k_{0\perp}^2\rho_s^2\delta}{|\mathbf{K}_\perp\cdot\mathbf{V}_{g\perp}|}\frac{|E_{0\perp}|^2}{B_0^2}|(\hat{\mathbf{K}}_\perp\cdot\hat{\mathbf{k}}_{0\perp})(\hat{\mathbf{z}}\times\hat{\mathbf{k}_0}\cdot\hat{\mathbf{K}})|^2 - \delta^2\right]^{1/2} \qquad (3.5.25)$$

The expression (3.5.25) shows that the modulation instability sets in if

$$|E_{0\perp}|^2 > \frac{B_0^2\delta|\mathbf{K}_\perp\cdot\mathbf{V}_{g\perp}|}{2K_\perp^2c^2k_{0\perp}^2\rho_s^2|(\hat{\mathbf{K}}_\perp\cdot\hat{\mathbf{k}_{0\perp}})(\hat{\mathbf{z}}\times\hat{\mathbf{k}}_0\cdot\hat{\mathbf{K}})|^2}. \qquad (3.5.26)$$

Second for $\Omega \gg \Gamma_z$, $\mathbf{K}_\perp\cdot\mathbf{V}_{g\perp}$, δ, we obtain from (3.5.24)

$$\Omega^3 \approx -2K_\perp^2c^2\delta\frac{|E_{0\perp}|^2}{B_0^2}k_{0\perp}^2\rho_s^2, \qquad (3.5.27)$$

which admits a reactive instability whose growth rate is

$$\gamma_r \approx (K_\perp c)^{2/3}\delta^{1/3}(k_{0\perp}\rho_s)^{2/3}\left(\frac{|E_{0\perp}|}{B_0}\right)^{2/3}, \qquad (3.5.28)$$

we observe from Eq. (3.5.28) that the growth rate is proportional to two-third power of $k_{0\perp}\rho_s$ and the DKAW pump electric field strength $|E_{0\perp}|$.

4 Discussion

In this talk, we have presented properties of thermal and nonthermal fluctuations in magnetized plasmas existing tokamaks and space environments. We focused on the linear properties of electrostatic 2D convective cells (two-dimensional zonal flows), pseudo 3D drift waves, as well as the Hall MHD waves and Alfven waves. We also presented several mechanisms that are responsible for generating long and short scale fluctuations. The mechanisms include the linear excitations by the free energy sources contained in equilibrium plasma inhomogeneities (e.g. velocity shear, density, temperature and magnetic field gradients), and the nonlinear excitation involving the parametric pressures. Nonthermal fluctuations are shown to produce cross-field transports of the plasma particles and heat. The results are relevant for understanding the properties of multiscale fluctuations and associated anomalous particle and heat transports in fusion plasmas.

Acknowledgement This research was partially supported by the Deutsche Forschungsgemeinschaft.