The Abdus Salam
International Centre for Theoretical Physics

United Nations
Educational, Scientific and
Cultural Organization

IAEA
International Atomic Energy Agency

**SMR/1856-5**

**2007 Summer College on Plasma Physics**

***30 July - 24 August, 2007***

# Nonlinear Dynamics of Incoherent Superstrong Radiation in a Plasma.

N. Tsintsadze

*Georgian Academy of Sciences*
*Andronikashvili Institute of Physics*
*Tbilisi, Georgia*

Strada Costiera 11, 34014 Trieste, Italy - Tel. +39 040 2240 111; Fax +39 040 224 163 - sci_info@ictp.it, www.ictp.it

# Nonlinear Dynamics of Incoherent Superstrong Radiation in a Plasma


## N. L. Tsintsadze

### Co-author: L. N. Tsintsadze

Department of plasma physics, institute of physics,
G.A.S. Georgia

## Analogy between a photon in a plasma and a free material particle

$$\varepsilon = c\sqrt{p^2 + m_0^2 c^2}$$

or

$$\varepsilon = \frac{m_0 c^2}{\sqrt{1 - \frac{v^2}{c^2}}}$$

## Photon in a Vacuum

$$v \leqslant c$$

$$\omega = kc, \qquad v_\varphi = \frac{\omega}{k} = \frac{\partial \omega}{\partial k} = c$$

$$\varepsilon_\gamma = p.c$$

**then** $p = 0,\ \varepsilon_\gamma = 0\ (\omega = 0)$

we can say in the vacuum photon exists only in motion.
However the light can be stopped in different mediums

$$(\overrightarrow{p} = \overrightarrow{k} = 0, \text{ but } \omega \neq 0, \varepsilon_\gamma \neq 0\ )$$

**For a plasma**

$$\omega = \sqrt{\omega_p^2 + k^2c^2}$$

**or**

$$\varepsilon_\gamma = c\sqrt{p^2 + m_\gamma^2 c^2}$$

or

$$\varepsilon_\gamma = \frac{m_\gamma c^2}{\sqrt{1 - \frac{u_\gamma^2}{c^2}}}$$

$$u_\gamma = c\left(1 - \frac{\omega_p^2}{\omega^2}\right)^{1/2} \leq c \qquad m_\gamma = \frac{\hbar\omega_p}{c^2}$$

**Momentum of the photon** $u_\gamma = \frac{\partial\omega}{\partial k}$ is the group velocity.

$$\overrightarrow{p_\gamma} = \hbar\,\overrightarrow{k} = \frac{m_\gamma\,\overrightarrow{u_\gamma}}{\sqrt{1 - \frac{u_\gamma^2}{c^2}}}$$

**Thus the wave packets of light propagate with a group velocity $(u_\gamma < c)$ in accordance with the theory of relativity**

# Skin depth

$$\lambda_c = \frac{2\pi c}{\omega_p} = \frac{2\pi\hbar}{m_\gamma c}$$

takes the simple meaning of the Compton wavelength of a photon in a plasma

In the relativistic theory a coordinate uncertainty in a frame of reference in which the particle is moving with energy $\varepsilon$ is

$$\Delta q \sim \frac{c\hbar}{\varepsilon}$$

# For Photons

$$\Delta q = \frac{c}{\omega} = \begin{cases} \lambda & \omega_p^2 << k^2c^2 \\ \lambda_c & \omega_p^2 >> k^2c^2 \end{cases}$$

**or the characteristic dimensions of the problem should be large in comparison with the wavelength or the Compton length**

**In the quantum field theory the eigenvalues of the Hamiltonian are**

$$E = \sum_{\overrightarrow{k},\sigma} \left( n_{\overrightarrow{k},\sigma} + \frac{1}{2} \right) \hbar\omega(\overline{k})$$

## Wigner-Moyal Equation in quantum theory

$$\left[\frac{\partial}{\partial t}+\frac{\overrightarrow{p}}{m}\frac{\partial}{\partial \overline{r}}-\frac{2}{\hbar}\sin\left(\frac{\hbar}{2}\frac{\partial}{\partial \overline{p}}\cdot\frac{\partial}{\partial \overline{r}}\right)\right]V(\overline{r})F(\overline{r},\overline{p},t)=0$$

## Relativistic Kinetic Equation for the Photon Gas

$$\frac{\partial}{\partial t}N(\overline{k},\omega,\overline{r},t)+\frac{c^2}{\omega}(\overline{k}\cdot\overline{\nabla})N(\overline{k},\omega,\overline{r},t)$$

$$\omega_p^2\sin\frac{1}{2}(\nabla_r\cdot\nabla_{\overline{k}}-\frac{\partial}{\partial t}\frac{\partial}{\omega})\rho\frac{N}{\omega}=0$$

where $\rho=\frac{n_e}{n_{0e}}\frac{1}{\gamma}$, $\gamma$ **is the relativistic gamma factor of the electrons**

## gamma can be expressed as

$$\gamma(\bar{r},t) = \sqrt{1+Q} = \sqrt{1+\beta\int\frac{d\bar{k}}{(2\pi)^3}\int\frac{d\omega}{2\pi}\frac{N(\bar{k},\omega,\bar{r},t)}{\omega}}$$

$$\beta = \frac{2\hbar\omega_p^2}{m_0 n_0 c^2}$$

**The total number of Photons**

$$N = 2\int d\bar{r}\int\frac{d\bar{k}}{(2\pi)^3}\int\frac{d\omega}{2\pi}N(\bar{k},\omega,\bar{r},t) = const$$

**Hence the chemical potential of the photon gas is not zero**

**In the geometric optics approximation the one-particle Liouville-Vlasov equation with an additional term**

$$\frac{\partial N}{\partial t} + \frac{c^2}{\omega}(\overline{k}\cdot\overline{\nabla})N - \frac{\omega_p^2}{2}(\nabla\rho\cdot\nabla_k - \frac{\partial\rho}{\partial t}\frac{\partial}{\omega})\frac{N}{\omega} = 0$$

**here there are two forces of distinct nature which can change the occupation number of photons**

$$\nabla\rho = \nabla\left(\frac{n_e}{n_0}\right) = \frac{1}{\gamma}\left(\nabla\frac{n_e}{n_0} - \frac{1}{\gamma}\frac{n_e}{n_0}\nabla\gamma\right)$$

**1st one is just Compton scattering process, 2nd is new type of Compton scattering-photon scatters on the wave packet**

# Existence of the longitudinal photons

**We have shown that** $\frac{\delta n_e}{n_{0e}} << \frac{\delta n_\gamma}{n_{0\gamma}}$

$$1+\frac{\omega_p^2}{2\gamma^3}\beta\int\frac{d^3k}{\omega(k)}\left\{\frac{N_0^+(\overline{k}+\overline{q}/2)}{\omega(\overline{k}+\overline{q}/2)}-\frac{N_0^-(\overline{k}-\overline{q}/2)}{\omega(\overline{k}-\overline{q}/2)}\right\}\frac{1}{\frac{\overline{q}kc^2}{\omega(k)}-\Omega}=0$$

$\Omega$ and $\overline{q}$ **are the frequency and wave vector of the longitudinal photons.**

$$\varepsilon(p)=\sqrt{v^2\cdot p^2+\left(\frac{p^2}{2m_{eff}}\right)^2}$$

**well known Bogoliubov energy spectrum**
**(microscopic theory of the super fluidity)**

## Pauli Equation for the Photon Gas

$$\frac{\partial N(\overline{k},t)}{\partial t} = \sum_{\pm} \int \frac{d^3q}{(2\pi)^3} \left[W \pm (\overline{k}+\overline{q},\overline{k}) N(\overline{k}+\overline{q},t) - W \pm (\overline{k},\overline{k}+\overline{q}) N(\overline{k},t)\right]$$

where $W \pm (\overline{k},\overline{q})$ is the scattering rate

## Bose-Einstein Condensation in Photon Gas

where $\varepsilon_k = c\sqrt{p^2 + m_\gamma^2 c^2} - m_\gamma c^2$

# The Photon density

**Because** $n_\gamma > 0$,

$$n_\gamma = 2\int \frac{d\overline{p}}{(2\pi\hbar)^3} N(\overline{p}, r)$$

**in any point of space**

$$m_\gamma(\overline{r})c^2 > \mu_\gamma$$

The critical temperature of the B.E condensation is determined for the fixed points

$$m_\gamma(r_f)c^2 = \frac{\hbar\omega_p}{\gamma^{1/2}(r_f)} = \mu_\gamma$$

For non-relativistic temperature

$$T_c \sim n_\gamma^{2/3}$$

For ultra relativistic case

$$T_c \sim n_\gamma^{1/3}$$

**When the temperature is below critical $T_c$, the occupation number reads**

$$N(\overline{p},\overline{r}) = \frac{1}{\exp\left(\frac{\varepsilon_k}{T_\gamma}\right) - 1} + 4\pi^3 n_{0\gamma}\delta(\overline{p})$$

**The problem of BEC and evaporation of the Bose-Einstein condensate can be investigated by Fokker-Planck equation, which we shall derive using Pauli equation. We suppose that**

$$|\overline{q}| << |\overline{k}|, \quad \text{and } \Omega << \omega$$

$$W(\overline{k}+\overline{q},\overline{k})N(\overline{k}+\overline{q}) \approx W(k)N(k) + \overline{q}\frac{\partial}{\partial \overline{k}}(WN)_{q=0} + \frac{q_i q_j}{2}\frac{\partial^2 WN}{\partial k_i \partial k_j}$$

$$\frac{\partial N}{\partial t} = a\frac{\partial}{\partial \bar{k}}(\bar{k}N) + \frac{D_0}{2}\nabla_k^2 N$$

where $a = \frac{D_0}{2\sigma_k^2}$ , $D_0$ **are the dynamic friction and diffusion coefficients, respectively**

**First we neglect the diffusion term and consider 1D case, the solution of which is**

$$N = \frac{f}{k} = \text{const}\ e^{at}$$

**Second**

$$\frac{\partial N}{\partial t} = \frac{D_0}{2}\nabla_k^2 N$$

Assuming that initially all the photons are in ground sate with

$$k = 0, \text{ or } N_0 = 4\pi^3 n_0 \delta(\bar{k})$$

The solution is

$$N(k,t) = \frac{n_0 e^{-\frac{k^2}{2D_0 t}}}{(2\pi D_0 t)^{1/2}}$$

From here

$$< k^2 >= D_0 t$$

We have derived a relation between the diffusion time, $t_p$ and the time of condensation

$$t_p / t_c = k^2 r_0^2, \text{ which is always } >> 1$$

# First Law of Relativistic Thermodynamics

$$\left(e - i - \gamma \quad or \quad e - p - \gamma\right).$$

$$dE_t = dQ - P_t dV ,$$

where

$$P_t = \sum_{\alpha} \left( P_{\perp\alpha} + P_{\parallel\alpha} \right) + P_{\gamma}$$

In the case of relativistically intence circular polarized EM field, we use the distribution function

$$f_{\alpha} = n_{0\alpha} \frac{\delta\left( \vec{P}_{\perp\alpha} + \frac{e_{\alpha}}{c} \vec{A}_{\perp} \right)}{2m_{0\alpha} c \sqrt{1+a_{\alpha}^2} \cdot K_1\left(\beta_{\alpha} \sqrt{1+a_{\alpha}^2}\right)}$$

$$\exp\left\{ - \frac{c\sqrt{m_{0\alpha}^2 c^2 + P_{\perp\alpha}^2 + P_{\parallel\alpha}^2}}{T_{\alpha}} \right\}$$

**where**

$$a_\alpha^2 = \left( \frac{e_\alpha \vec{A}_\perp}{m_{0\alpha} c^2} \right)^2, \quad \beta_\alpha = \frac{m_{0\alpha} c^2}{T_\alpha},$$

$$P_{\perp\alpha} = \frac{2}{3} \cdot \frac{n_\alpha m_{0\alpha} c^2 a_\alpha^2}{\sqrt{1+a_\alpha^2}} \cdot \frac{K_0\left(\beta_\alpha \sqrt{1+a_\alpha^2}\right)}{K_1\left(\beta_\alpha \sqrt{1+a_\alpha^2}\right)},$$

$$P_{\parallel\alpha} = \frac{1}{3} \cdot n_\alpha \cdot T_\alpha,$$

$$dQ_t = T_e dS_e + T_{i(P)} dS_{(i.P)} + dS_\gamma,$$

Where $S_\alpha = -V \int dp_\parallel \int dp_\perp f_\alpha \ln f_\alpha$

The entropy per particle

$$s_\alpha = \frac{S_\alpha}{N} = -\left[ \ln \frac{n_\alpha}{2 m_{0\alpha} c K_1(\beta_\alpha)} + 1 - \beta_\alpha \sqrt{1 + a_\alpha^2} \frac{K_2\left(\beta_\alpha \sqrt{1 + a_\alpha^2}\right)}{K_1\left(\beta_\alpha \sqrt{1 + a_\alpha^2}\right)} \right]$$

# For photons

$$P_\gamma = \frac{T_\gamma^4 \beta_\gamma^2}{\pi^2 (\hbar c)^3} \sum_{l=1}^{\infty} \frac{e^{\beta_\gamma \cdot l}}{l^2} K_2(\beta_\gamma \cdot l),$$

$$S_\gamma = \frac{V T_\gamma^3 \beta_\gamma^2}{\pi^2 (\hbar c)^3} \sum_{l=1}^{\infty} \frac{e^{\beta_\gamma \cdot l}}{l^2}$$

$$\left[ l\beta_\gamma \left( 1 - \frac{l\beta_\gamma}{2} \right) K_3(l\beta_\gamma) + \frac{l^2 \beta_\gamma^2}{2} K_1(l\beta_\gamma) \right].$$

For an adiabatic process the energy is conserved in each subsystem, i.e.

$ds_\alpha = 0$ and

$$\frac{n_\alpha e^{-\beta_\alpha \sqrt{1+a_\alpha^2}\, G}}{K_1(\beta_\alpha)} = const$$

$$G = \frac{K_2\left(\beta_\alpha \sqrt{1+a_\alpha^2}\right)}{K_1\left(\beta_\alpha \sqrt{1+a_\alpha^2}\right)}$$

$$\beta_\alpha \sqrt{1+a_\alpha^2} << 1, \quad m_\alpha c^2 \sqrt{1+a_\alpha^2} << T_\alpha,$$

$$\frac{n_\alpha \left(1+\frac{e_\alpha^2 A^2}{T_\alpha^2}\right)}{T_\alpha} = const$$

In the opposite limit, that is for nonrelativistic temperatures, $\beta_\alpha \sqrt{1+a_\alpha^2} >> 1,$ we obtain

$$n_\alpha \frac{e^{-\beta_\alpha\left(\sqrt{1+a_\alpha^2}-1\right)}}{T_\alpha^{1/2}} = const$$

**For the photons the asymptotic behavior of the entropy, in the case**

$$\beta_\gamma = \frac{m_\gamma c^2}{T_\gamma} << 1, \quad S_\gamma = S_{0\gamma}\left(1 + 0{,}83 \cdot \beta_\gamma\right),$$

**where** $$S_{0\gamma} = \frac{4\pi^2}{45}\left(\frac{T_\gamma}{\hbar c}\right)^3 \cdot V$$

**is the entropy in vacuum.**

**For the case** $\beta_\gamma >> 1,$

$$S_\gamma = S_{0\gamma} \cdot 0{,}48\, \beta_\gamma^{3/2}.$$

In this case the entropy depends on the temperasture and the volume as follows

$$S_\gamma \approx T_\gamma^{3/2} V^{1/4}$$

Thus, for the adiabatic process, we obtain

$$T_\gamma V^{1/6} = T_\gamma V^{\Gamma - 1} = const\ .$$

So, the specific heat for the photon gas is

$$\Gamma = \frac{c_p}{c_V} = \frac{7}{6}.$$

# Fluctuation of the number of photons

$$\left\langle \left(\Delta N_\gamma\right)^2 \right\rangle = \frac{T_\gamma N_\gamma^2}{V^2}\left(\frac{\partial V}{\partial P_\gamma}\right)_{T_\gamma} .$$

In the vacuum $\left(\frac{\partial V}{\partial P_\gamma}\right)_{T_\gamma} = \infty,\ P_\gamma \sim T^4$

In the plasma $P_\gamma = P_\gamma(T, V)$

and $\left(\frac{\partial V}{\partial P_\gamma}\right)_{T_\gamma}$ is finite.

We now examine fluctuations in the distribution of photons over the various "quantum" states. The mean values of the occupation numbers $n_k$ in the $k$ *th* quantum state is

$$\langle n_k \rangle = n_\gamma = \frac{1}{e^{\frac{\varepsilon(k)-\mu_\gamma}{T_\gamma}} - 1}$$

The mean square fluctuation of the occupation number of photons is

$$\left\langle (\Delta n_k)^2 \right\rangle = T_\gamma \frac{\partial n_\gamma}{\partial \mu} \quad or \quad \left\langle (\Delta n_k)^2 \right\rangle = n_\gamma (1 + n_\gamma).$$

**The first term reflacts the corpuscular behavior of the photons, wheareas the second term is of the wave origin. In the case, when** $|\varepsilon(\kappa)-\mu| >> T_\gamma$ **the first term is larger than the second one.**

**In the opposite case** $|\varepsilon(\kappa)-\mu| << T_\gamma$ **, the distribution function** $n_\gamma >> 1$.

Thus the relative fluctuations of the number of photons does not decrease, when the mean number of photons increases, so that

$$\frac{\left\langle \left(\Delta n_k\right)^2 \right\rangle}{n_\gamma^2} \sim 1 .$$

## Boltzmann H-theorem for a photon gas.

In the limit of the spatial homogeneity for the distribution function of photons the Pauli equation was derived by L. Tsintsadze (2003)

$$\frac{\partial N\left(\vec{k},t\right)}{\partial t}=\sum_{\pm}\int\frac{d^3k'}{(2\pi)^3}W_{\pm}\left(\vec{k}',\vec{k}\right)$$

$$\left[\frac{\omega\left(\vec{k}\right)}{\omega\left(\vec{k}'\right)}N\left(\vec{k}',t\right)-N\left(\vec{k},t\right)\right],$$

**where**

$$W_{\pm}\left(\vec{k}',\vec{k}\right)=\frac{\pi}{4}\frac{\omega_{P}^{4}\,|\,\delta\rho\left(q\right)|^{2}}{\omega\left(k\right)\omega\left(\vec{k}\pm\vec{q}/2\right)}\delta\left(\Omega-\vec{q}\cdot\vec{u}_{\pm}\right)$$

$$\vec{k}'=\vec{k}+\vec{q},\quad \vec{u}_{\pm}=\frac{\left(\vec{k}\pm\vec{q}/2\right)c^{2}}{\omega\left(\vec{k}\pm\vec{q}/2\right)},\ \rho=\frac{n}{n_{0}}\frac{1}{\gamma}$$

$$S=-k_{B}V\int\frac{d^{3}k}{4\pi^{3}}\left[N\left(\vec{k},t\right)\ln N\left(\vec{k},t\right)-\left(N\left(\vec{k},t\right)+1\right)\right]$$
$$\ln\left(1+N\left(\vec{k},t\right)\right)$$

where $k_B$ is the Bolizmann's constant.

$$\frac{dS}{dt} = k_B V \int \frac{d^3k}{4\pi^3} \ln\left( \frac{1 + N(\vec{k}, t)}{N(\vec{k}, t)} \right) \frac{\partial N(\vec{k}, t)}{\partial t}$$

Using the Pauli equation we obtain

$$\frac{dS}{dt} = \frac{k_B V}{2} \sum_{\pm} \int \frac{d^3k}{(2\pi)^3} \int \frac{d^3k'}{(2\pi)^3} W_{\pm}(\vec{k}, \vec{k}') F(\vec{k}', \vec{k}),$$

where $F(\vec{k}', \vec{k}) =$

$$\ln\left(\frac{1+N(\vec{k},t)}{1+N(\vec{k}',t)} \cdot \frac{N(\vec{k}',t)}{N(\vec{k},t)}\right)\left(N(\vec{k}',t) - N(\vec{k},t)\right).$$

Which is non-negative in any cases, i.e.

$$N(\vec{k}',t) > N(\vec{k},t)$$

or reverse. Thus,

$$\frac{dS}{dt} \geq 0.$$

# *Adiabatic Photon Self-Capture*

In the geometric optics approximation the Wigner Moyal equation reduces to the one particle Liouville-Vlasov equation

$$\frac{\partial}{\partial t}N(\vec{k},\vec{r},t)+\left(\vec{u},\vec{\nabla}\right)N(\vec{k},\vec{r},t)-\nabla U\cdot\nabla N(\vec{k},\vec{r},t)=0,$$

where

$$U=k_P^2\delta\rho.$$

Let $l$ and $\tau$ be the charasteristic length and time of variation of the potential. Supposing that

$$\tau >> \frac{l}{u}.$$

With this condition the solution of the Liouville- Vlasov equation is

$$N(r,k) = n_{0\gamma} \cdot \frac{1}{\left(2\pi\,\sigma_0^2\right)^{3/2}} \exp\left(-\frac{k^2 + k_P^2 \delta\rho}{2\sigma_0^2}\right),$$

where $\sigma_0$ is the spectral-width and $\delta\rho$ can be ***$<0$, or $>0$.***

$$\delta\rho = \frac{\delta n}{n_0 \gamma} + \frac{1}{\gamma} - \frac{1}{\gamma_0}.$$

If ***δρ<0*** in some region, and in the rest of the space ***δρ>0,*** then we have two sorts of photons. For the case ***δρ>0,*** $k^2+k^2_\rho\delta\rho>0,$ and for the density of photons

$$n_\gamma = n_{0\gamma}\exp\left(-\frac{k_P^2\delta\rho}{\sigma_0^2}\right)$$

But in the case, when there are some photons in the cavity, then the motion of photons takes place in a finite region of space, i. e. they are trapped in the potentiall well

$$U = -k_P^2\left|\delta\rho\right|.$$

Therefore $n_\gamma$ we can now represent as

$$n_\gamma = n_\gamma^{trap} + n_\gamma^{untrap},$$

**where** $$\frac{n_\gamma^{trap}}{n_{0\gamma}} = \frac{4}{3\sqrt{\pi}}\left(\frac{k_P}{\sqrt{2}\cdot\sigma_0}\left|\delta\rho\right|^{1/2}\right)^3$$

**and** $$\frac{n_\gamma^{untrap}}{n_{0\gamma}} = \left(1-\frac{4}{\sqrt{\pi}}\int_0^{\eta_0} d\eta\cdot\eta^2 e^{-\eta^2}\right)\cdot e^{-\eta_0^2},$$

**where** $$\eta = \frac{k}{\sqrt{2}\cdot\sigma_0} \quad and \quad \eta_0 = \frac{k_P\left|\delta\rho\right|^{1/2}}{\sqrt{2}\cdot\sigma_0},$$

**When** $\eta_0 >> 1$, $(k_\rho\,|\delta\rho|^{1/2} >> \sqrt{2}\,\sigma_0)$, **then** $n_\gamma^{untrap} \to 0$. **In the opposite limit,** $\eta_0 << 1$, **for the density of photons we obtain**

$$n_\gamma = n_{0\gamma}\left\{1+\eta_0^2 - \frac{8}{15\cdot\sqrt{\pi}}\eta_0^{5/3}\right\}.$$

# Uniform Expansion of the Photon Gas

For the ultrarelativistic photon gas, i. e.

$$T(t) = T_0 \left( \frac{V_0}{V(t)} \right)^{1/3} \frac{1}{1 + \zeta \left( \frac{V(t)}{V_0} \right)^{1/3}} ,$$

where

$$\zeta = 0,29 \frac{m_\gamma (V_0) c^2}{T_0}$$

and the suffix 0 denotes the constant initial value.

In order to determine the explicit dependence ***T(t)*** and ***V(t)***, we study the spherically symmetric case. Using the continaity equation with assumption

$$n_\gamma(t) = n_{0\gamma}\left(\frac{R_0}{R(t)}\right)^3 \quad and \quad u_r = u_0\frac{r}{R(t)}$$

we obtain for the radius

$$R(t) = R_0 + u_0 t.$$

**Thus**

$$T(t) = T_0 \frac{R_0}{R(t)} \cdot \frac{1}{1 + \zeta \frac{R(t)}{R_0}}.$$

**Nonrelativistic photon gas, i. e.**

$$\varepsilon_\gamma \approx m_\gamma c^2 + P_\gamma^2 / 2m_\gamma$$

$$T(t) = T_0 \left( \frac{V_0}{V(t)} \right)^{1/6} \quad or \quad T(t) = T_0 \left( \frac{R_0}{R(t)} \right)^{1/2}.$$

# References


- L.N.Tsintsadze et al., Proc. Int. Conf. Superstrong Fields in Plasmas (Varenna, 1997), (AIP,1998), 170
- N.L.Tsintsadze et al., Phys. Plasmas 5, 3609 (1998)
- N.L.Tsintsadze et al., Phys. Plasmas 7, 2348 (2000)
- J.T.Mendonca et al., PRE 62, 4276 (2000)
- L.N.Tsintsadze et al., J.Plasma Phys.55,407,1996
- L.N.Tsintsadze, Phys. Plasmas 11, 855 (2004); (physics/0207074); Hiroshima Univ., Annual Report of VBL 7, 68 (2002); VBL 8, 90 (2003)
- L.N.Tsintsadze, Focus on Astrophysics Research, ed. by Louis V.Ross (Nova Science Publishers, NY, 2003), 147; (astro-ph/0212124)
- N.L.Tsintsadze et al., Physica Scripta T113, 24 (2004); (astro-ph/0611702)