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The Rayleigh-Taylor instability of a plasma foil accelerated by the radiation pressure of an ultra intense laser pulse.

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The Rayleigh-Taylor instability of a plasma foil accelerated by the radiation pressure of an ultra intense laser pulse.

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Second lecture (relativistc physics)

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Radiation pressure can be a very effective mechanism of momentum transfer to charged particles.

This mechanism was introduced long ago[1].

Physical conditions of interest range from stellar structures and radiation generated winds[2], to high accuracy optical experiments[3] and optical traps, to the formation of "photon bubbles" in very hot stars and accretion disks[4].

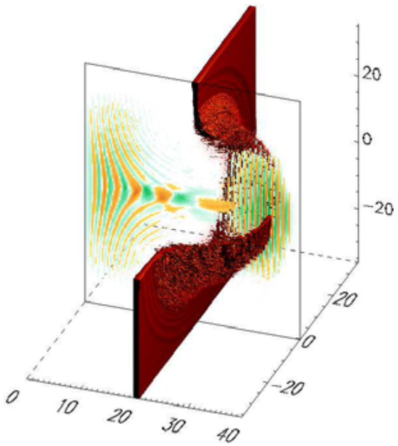
[1] A.S. Eddington, *MNRAS* **85**, 408 (1925).

[2] E.A. Milne, *MNRAS* **86**, 459 (1926); S. Chandrasekhar, *MNRAS* **94**, 522 (1934), N.J. Shaviv, *ApJ* **532**, L137 (2000).

[3] P.F. Cohadon, *et al.*, *Phys. Rev. Lett.* **83**, 3174 (1999), A. Ashkin, *Phys. Rev. Lett.* **24**, 156 (1970).

[4] J. Arons, *ApJ* **388**, 561 (1992); C.F. Gammie, *MNRAS* **297**, 929 (1998); M.C. Begelman, *ApJ* **551**, 897 (2001).

Particle acceleration by radiation pressure has been considered in the laboratory[5] and in high energy astrophysical environments[6].



[5]T. Esirkepov, *et al.*, *Phys. Rev. Lett.* **92**, 175003 (2004); S.V. Bulanov, *et al.*, *Plasma Phys. Rep.* **30**, 196 (2004); F. Pegoraro, *et al.*, *Phys. Lett. A* **347**, 133 (2005); W. Yu, *et al.*, *Phys. Rev.* E **72**, 046401 (2005), A. Macchi, *et al.*, *Phys. Rev. Lett.* **94**, 165003, (2005); J. Badziak, *et al.*, *Appl. Phys. Lett.* **89**, 061504, (2006).

[6]P. Goldreich, *Phys. Scripta* **17**, 225 (1978); T. Piran, *ApJ* **257**, L23 (1982); V. S. Berezinskii, *et al.*, *Astrophysics of Cosmic Rays*, (Elsevier, Amsterdam, 1990).

Radiation pressure arises from the "coherent" interaction of the radiation with the particles in the medium.

In an electron-ion plasma radiation pressure acts mostly on the lighter particles, the electrons, with a force that is quadratic in the wave field amplitude.

Ions on the contrary are accelerated by the charge separation field caused by the electrons pushed by the radiation pressure.
This collective acceleration mechanism is very efficient[7] when the number of ions inside the electron cloud is much smaller than that of the electrons.

[7] V.I. Veksler, in *Proc. CERN Symposium on High Energy Accelerators and Pion Physics*, Geneva, **1**, 80 (1956).

The electric fields produced by the interaction of ultra-short and ultra-intense laser pulses with a thin target make it possible to obtain multi-MeV, high density, highly collimated proton and ion beams[8] of extremely short duration, in the sub-picosecond range.

Such laser pulses may also open up the possibility of exploring high energy astrophysical phenomena, such as in particular the formation of photon bubbles in the laboratory.

[8] M. Borghesi, *et al.*, *Fus. Sc. & Techn.* **49**, 412 (2006); B.M. Hegelich, *et al.*, *Nature* **439**, 441 (2006); H. Schwoerer, *et al.*, *Nature* **439**, 445 (2006); L. Willingale, *et al.*, *Phys. Rev. Lett.* **96**, 245002 (2006), + references therein.

The RPDA Regime

Different regimes of plasma ion acceleration have been discussed in the literature[9]

A critical factor for a number of applications is the efficiency of the energy conversion.

In the Radiation Pressure Dominant Acceleration (RPDA) the ion acceleration in a plasma is directly due to the radiation pressure of the e.m. pulse[10].

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[9] S.V. Bulanov, *et al.*, in *Reviews of Plasma Physics*, ed. V.D. Shafranov, **22**, 227 (Kluwer Acad., N.Y., 2001); G.A. Mourou, *et al.*, *Rev. Mod. Phys.* **78**, 309 (2006) + references therein.

[10] T. Esirkepov, *et al.*, *Phys. Rev. Lett.* **92**, 175003 (2004); S.V. Bulanov, *et al.*, *Plasma Phys. Rep.* **30**, 196 (2004).

In this regime, the ions move forward with almost the same velocity as the electrons and thus have a kinetic energy well above that of the electrons.

In contrast to the other regimes, this acceleration process is highly efficient and the ion energy per nucleon is proportional to the e.m. pulse energy.

This acceleration mechanism can be illustrated by considering a thin, dense plasma foil, made of electrons and protons, pushed by an ultra intense laser pulse in conditions where the radiation cannot propagate through the foil, while the electron and the proton layers move together and can be regarded as forming a (perfectly reflecting) relativistic plasma mirror co-propagating with the laser pulse.

The frequency of the reflected e.m. wave is reduced by

$$\frac{(1 - v/c)}{(1 + v/c)} \approx \frac{1}{4}\gamma^2, \qquad \text{with} \quad \gamma = \left(1 - v^2/c^2\right)^{-1/2}$$

and v the mirror velocity.
Thus the plasma mirror is accelerated and acquires from the laser the energy

$$(1 - 1/4\gamma^2)\mathcal{E},$$

where $\mathcal{E}$ is the incident laser pulse energy in the laboratory frame (LF).

For large values of γ practically all the e.m. pulse energy is transferred to the mirror, essentially in the form of proton kinetic energy.

This high efficiency of the e.m. energy conversion into the fast protons opens up a wide range of applications.

For example it can be exploited in the design of proton dump facilities for spallation sources or for the production of large fluxes of neutrinos[11].

[11] S.V. Bulanov, *et al.*, *Nucl. Instr. Meth. A* **540**, 25 (2005).

Rayleigh-Taylor instabilities

Both in the astrophysical and in the laser plasma contexts, the onset of Rayleigh-Taylor-like instabilities[12] *may affect the interaction of the plasma with the radiation pressure.*

In this case the e.m. radiation may eventually dig through the plasma and make it porous to the radiation (and allow e.g., for super-Eddington luminosities) or, in the case of a plasma foil accelerated by a laser pulse, may tear it into clumps[13] and broaden the energy spectrum of the fast ions.

[12] Heavier medium accelerated by a lighter medium.
[13] K. Kifonidis, *et al.*, *Astron. Astrophys.* **408**, 621 (2003).

In this lecture I will discuss the stability of a plasma foil in the ultra relativistic conditions that are of interest for the RPDA regime[14].

We find that

• in the relativistic regime the growth of the instability is slower than in the nonrelativistic regime

• proper tailoring of the pulse amplitude can allow for stable foil acceleration.

• With the help of two-dimensional (2D) Particle in cell (PIC) simulations, we find that the nonlinear development of the instability leads to the formation of *high-density, high-energy plasma clumps*.

[14] Photon bubbles and ion acceleration in a plasma dominated by the radiation pressure of an electromagnetic pulse, F. Pegoraro, S.V. Bulanov, (2007) Phys. Rev. Lett. in press

Radiation Pressure Acceleration of a Thin foil Mirror

The equation of motion of an element of area $|d\Sigma|$ of a perfectly reflecting mirror can be written in the LF as

$$d\mathbf{p}/dt = P d\mathbf{\Sigma},$$

where $\mathbf{p}$ is the momentum of the mirror element, $d\mathbf{\Sigma}$ is normal to the mirror surface and P is the Lorentz invariant radiation pressure.

For the sake of geometrical simplicity, we refer to a 2D configuration where the mirror velocity and the mirror normal vector remain coplanar (in the x-y plane, x being the direction of propagation of the e.m. pulse), i.e., we assume that the mirror does not move or bend along z.

The radiation pressure P is given in terms of the amplitude of the electric field E_M of the incident e.m. pulse and of the pulse incidence angle θ_M in the CMF by

$$P = (E_M^2/2\pi)\cos^2\theta_M,$$

where

$$E_M^2 = (\omega_M^2/\omega_0^2)\ E_0^2$$

with

$$\omega_M^2/\omega_0^2 = (1-\beta\cos\phi)^2/(1-\beta^2),$$

the subscript 0 denotes quantities in the LF and ϕ the angle the mirror velocity β makes with the x-axis in the laboratory frame. The angle θ_M vanishes when the incidence angle θ_0 in the LF vanishes and $\phi = 0, \pi$, but is a fast increasing function of γ for $\theta_0 \neq 0$, or $\phi \neq 0, \pi$[15].

[15]Kinematic considerations show that the laser pulse can no longer reach the receding mirror when $|\sin\phi| > 1/\gamma$. This inequality constrains the maximum value of γ that can be obtained with a non perfectly collimated beam, see also E.S. Phinney, *MNRAS* **198**, 1109 (1982).

Then, the equation of motion of a mirror element of unit length along z and uniform density n_0 in the LF is

$$\bullet\bullet \quad \frac{\partial p_x}{\partial t} = \frac{P}{n_0 l_0}\,\frac{\partial y}{\partial s}, \qquad \frac{\partial p_y}{\partial t} = -\frac{P}{n_0 l_0}\,\frac{\partial x}{\partial s} \tag{1}$$

with

$$\frac{\partial x}{\partial t} = \beta_x c \quad \text{and} \quad \frac{\partial y}{\partial t} = \beta_y c.$$

Here

$$p_{x,y} = m_i c \frac{\beta_{x,y}}{(1-\beta^2)^{1/2}}$$

are the spatial components of the momentum 4-vector of the mirror element, l_0 is the mirror thickness and m_i is the ion mass.
Lagrangian coordinates, x_0 and y_0, have been adopted such that $x, y = x, y(x_0, y_0, t)$ and $ds = (dx_0^2 + dy_0^2)^{1/2}$.

In the non relativistic limit and for constant P, Eqs.(1) coincide with Ott's equations [16] for the motion of a thin foil.

Assuming that the unperturbed mirror moves along the x-axis, i.e. that the initial conditions correspond to a flat mirror along y_0, so that $dx_0 = 0,\ dy_0 = ds$ and $\theta_M = 0$, we write Eqs.(1) as[17]

$$\frac{dp_x^0}{dt} = \frac{E_0^2}{2\pi n_0 l_0}\,\frac{m_i c\gamma_0 - p_x^0}{m_i c\gamma_0 + p_x^0}, \tag{2}$$

where p_x^0 is the unperturbed x component of momentum and depends on the variable t only and $m_i^2 c^2 \gamma_0^2 = m_i^2 c^2 + \left(p_x^0\right)^2$.

[16] E. Ott, *Phys. Rev. Lett.* **29**, 1429 (1972).

[17] Eq.(2) has been analyzed in T. Esirkepov, *et al.*, *Phys. Rev. Lett.* **92**, 175003 (2004) and S.V. Bulanov, *et al.*, *Plasma Phys. Rep.* **30**, 196 (2004)

The electric field of the e.m. pulse at the mirror position $x(t)$ depends on time as $E_0 = E_0[t - x(t)/c]$.

Let us introduce the phase of the wave $\psi = \omega_0[t - x^0(t)/c]$, at the unperturbed mirror position $x^0(t)$ as a new independent variable.
Differentiating with respect to time, we obtain

$$\frac{d\psi}{dt} = \omega_0 \frac{m_i c \gamma_0 - p_x^0}{m_i c \gamma_0}. \tag{3}$$

Using the variable ψ and the normalized fluence of the e.m. pulse

$$w(\psi) = \int_0^{\psi} (R(\psi')/\lambda_0)\, d\psi',$$

with

$$R(\psi) = E_0^2(\psi)/(m_i n_0 l_0 \omega_0^2),$$

and $\lambda_0 = 2\pi c/\omega_0$, the solution of Eq.(2) with the initial condition $p_x^0(0) = 0$ is

$$\bullet\bullet \quad p_x^0(\psi) = m_i c \frac{w(\psi)[w(\psi)+2]}{2[w(\psi)+1]}, \tag{4}$$

while from Eq.(3) we obtain that t and ψ are related by

$$\bullet\bullet \quad \psi + \int_0^{\psi} \left[w(\psi') + w^2(\psi')/2\right] d\psi' = \omega_0 t. \tag{5}$$

For a constant amplitude e.m. pulse, when $R = R_0$, these expressions reduce to

$$w(\psi) = (R_0/\lambda_0)\,\psi,$$

and to

$$\psi + (R_0/\lambda_0)\,\psi^2/2 + (R_0/\lambda_0)^2\,\psi^3/6 = \omega_0 t, \qquad (6)$$

with

$$p_x^0 \approx m_i c\,(R_0/\lambda_0)\,\omega_0 t$$

for $t \ll \omega_0^{-1}\,(\lambda_0/R_0)$ and, for $t \gg \omega_0^{-1}\,(\lambda_0/R_0)$,

$$\bullet\bullet \quad p_x^0 \approx m_i c\,(3R_0\omega_0 t/2\lambda_0)^{1/3} \ .$$

These relationships are obtained under the assumption that the charge separation electric field $E_{||}$ which accelerates the ions is sufficiently large, so that

$$E_{||} = 2\pi n_0 e l_0 > \left(E_0^2/2\pi n_0 l_0\right)\left[\left(m_i c\gamma_0 - p_x^0\right)/\left(m_i c\gamma_0 + p_x^0\right)\right].$$

The latter condition can be rewritten in terms of the dimensionless laser pulse amplitude

$$a_0 = eE_0/m_e\omega_0$$

and of the dimensionless parameter[18]

$$\epsilon_0 = 2\pi n_0 e^2 l_0/m_e\omega_0 c \quad \text{as}$$

$$a_0 < \epsilon_0[(m_i c\gamma_0 + p_x^0/m_i c\gamma_0 - p_x^0]^{1/2}$$

and is equivalent to the full opacity condition for a thin overdense plasma foil in the CMF. In the opposite limit the ions are accelerated by the electric field

$$E_{||} = 2\pi n_0 e l_0 \quad \text{until} \quad p_x^0 m_i c(2a_0/\epsilon_0)^{1/2}$$

when they enter the RPDA regime.

[18] V.A. Vshivkov, *et al.*, *Phys. Plasmas* **5**, 2727 (1998).

The Instability of the accelerated foil

Let as now investigate the linear stability of the accelerated mirror with respect to perturbations $x^1(y_0,\psi),\ y^1(y_0,\psi)$ that bend the plasma foil.
Linearizing Eqs.(1) around the solution given by Eq.(4) we obtain

- $$\frac{\partial}{\partial\psi}\left[\frac{p_x^0(\psi)}{m_i c}\frac{\partial x^1}{\partial\psi}\right] = \frac{R(\psi)}{2\pi}\frac{\partial y^1}{\partial y_0}, \tag{7}$$

- $$\frac{\partial}{\partial\psi}\left[\frac{m_i c}{p_x^0(\psi)}\frac{\partial y^1}{\partial\psi}\right] = -\frac{R(\psi)}{2\pi}\frac{\partial x^1}{\partial y_0}. \tag{8}$$

Here we retain only the leading terms in the ultrarelativistic limit $p_x^0/m_i c \gg 1$ for the foil motion and neglect a term proportional to $(\partial R/\partial\psi)\ x^1/\lambda_0$.

We look for WKB solutions of the form

$$y^1(y_0,\psi) \propto \exp\left[\int_0^{\psi}\Gamma(\psi')d\psi' - iky_0\right], \tag{9}$$

with growth rate $\Gamma \gg 1$. We find

$$\Gamma(\psi) = \left[kR(\psi)/2\pi\right]^{1/2}, \quad \text{with} \quad x^1 \sim -iy^1(m_i c/p_x^0).$$

For a constant amplitude pulse, using Eq.(6), we obtain

$$y^1 \propto \ \exp\left[(t/\tau_r)^{1/3} - iky_0\right], \tag{10}$$

where

$$\bullet\bullet \quad \tau_r = \omega_0^{-1}(2\pi)^{3/2}R_0^{1/2}/(6k^{3/2}\lambda_0^2)$$

is the characteristic time of the instability in the ultrarelativistic limit.

The time scale of the instability is proportional to the square root of the ratio between the the radiation pressure and the ion mass.
Thus, the larger the ion mass, the faster the perturbation grows while the larger the radiation pressure the slower the perturbation.

In the non-relativistic limit the perturbation grows as

$$x^1, y^1 \;\propto\; \exp\left[t/\tau - iky_0\right], \tag{11}$$

where

$$\tau = \omega_0^{-1}(2\pi/kR_0)^{1/2},$$

In the ultrarelativistic limit the instability develops more slowly with time than in the non-relativistic case: $t^{1/3}$ instead of t. In addition the nonrelativistic time τ is inversely proportional to the square root of the radiation pressure.

If we express Eqs.(10,11) in terms of the unperturbed momentum p_x^0, in both limits we find an exponential growth of the form

$$\bullet\bullet \quad y^1(y_0, p_x^0) \propto \exp\left[\kappa p_x^0/(m_i c) - iky_0\right],$$

where

$$\kappa = (k\lambda_0)^{1/2}/(2\pi R_0/\lambda_0)^{1/2}$$

i.e.,

$$\kappa \propto (k/l_0)^{1/2}(m_i/m_e)^{1/2}(\omega_{pe,0}/\omega_0)\ a_0^{-1},$$

with

$$\omega_{pe,0}^2 = 4\pi n_0 e^2/m_e.$$

Stabilization with tailored EM pulses

This exponential growth of the perturbation with the unperturbed momentum for a constant amplitude pulse can be stopped by tailoring the shape of the e.m. pulse. We refer to the ultrarelativistic limit and define $\Phi(\psi) = \int_0^\psi \Gamma(\psi')d\psi'$. From Eq.(4), which in this limit takes the form

$$p_x^0(\psi) = m_i c \int_0^\psi \left(R(\psi')/2\lambda_0\right) d\psi',$$

we see that the stability condition can be formulated as follows: *it is possible to choose the dependence of the e.m. pressure $R(\psi)$ on the phase ψ such that, for $\psi \to \psi_m$, the ion momentum $p_x^0(\psi)$ grows, formally to infinity, while $\Phi(\psi)$ remains finite.* Here ψ_m is either finite or equal to infinity.

As an example we can take $R(\psi)$ of the form

$$R(\psi) = R_0(1 - \psi/\psi_m)^{-\alpha}\ \chi(\psi_1 - \psi),$$

with $1 < \alpha < 2$, $\chi(x) = 1$ for $x > 0$, and $\chi(x) = 0$ for $x < 0$ and $\psi_m > \psi_1$ so as to keep the pulse fluence finite.

In this case the maximum value of the ion momentum

$$p_x^0/(m_i c) \approx R_0\psi_m(1 - \psi_1/\psi_m)^{1-\alpha}/[2\lambda_0(\alpha - 1)],$$

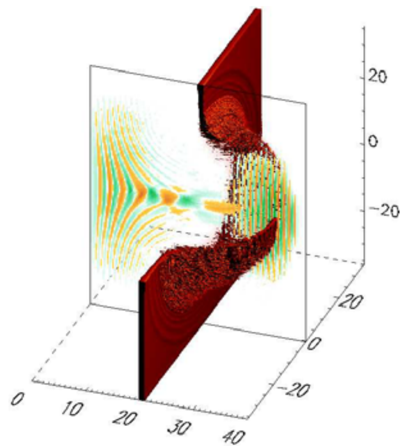
tends to infinity for $\psi_1 \to \psi_m$, while

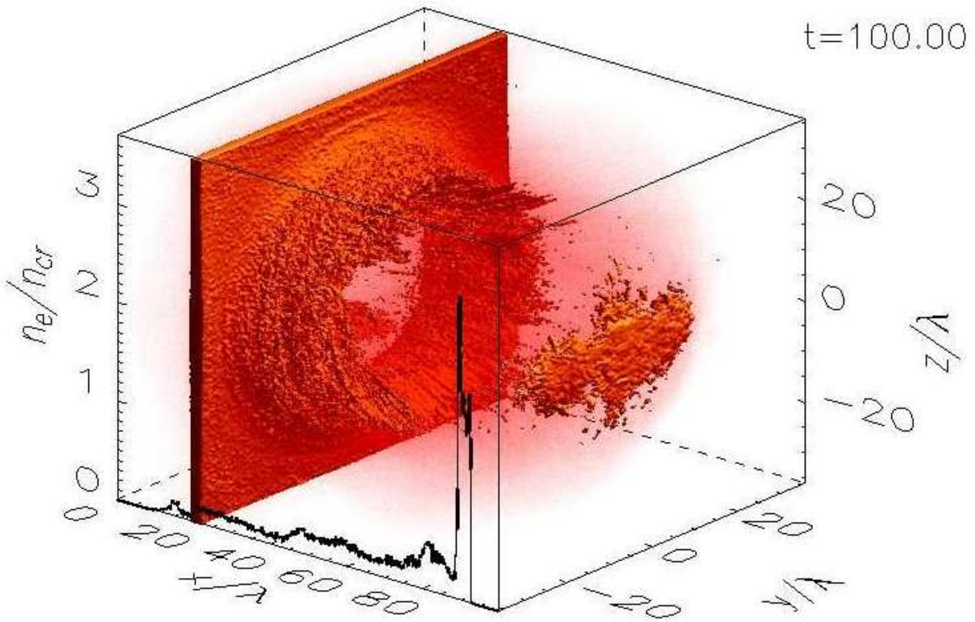
$$\Phi(\psi_m) = (2kR_0/\pi)^{1/2}\psi_m[1 - (1 - \psi_1/\psi_m)^{1-\alpha/2}]/(2 - \alpha)$$

remains finite. In addition, for $\alpha < 3/2$ the acceleration time is finite.

Results of PIC Simulations

The PIC simulations presented in T. Esirkepov, *et al.*, *Phys. Rev. Lett.* **92**, 175003 (2004); S.V. Bulanov, *et al.*, *Plasma Phys. Rep.* **30**, 196 (2004); show a stable phase of the RPDA of protons where a portion of the foil, with the size of the pulse focal spot, is pushed forward by a super-intense e.m. pulse.




t=100.00
n_e/n_{cr}
0
1
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80
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0
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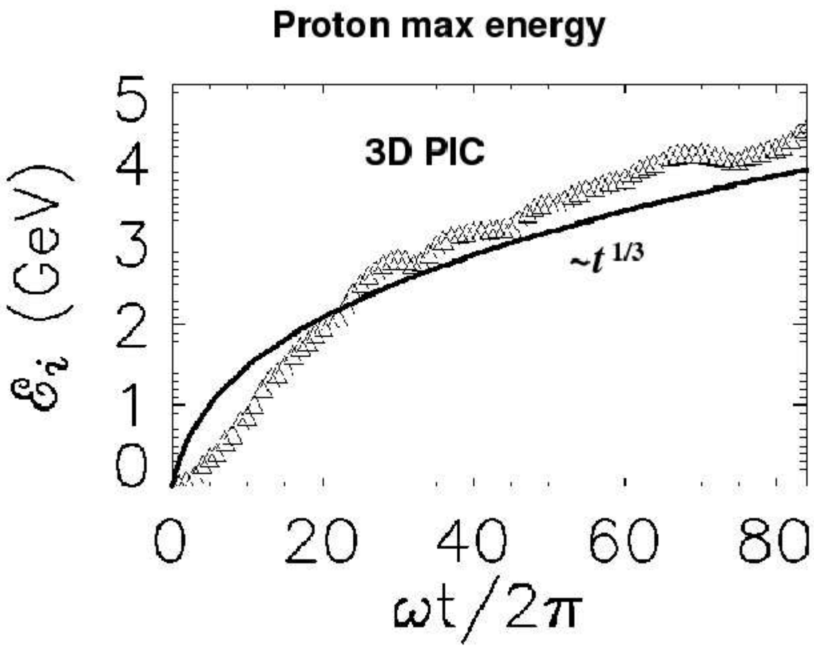
y/λ


Proton max energy
3D PIC
~$t^{1/3}$
$\mathcal{E}_i$ (GeV)
0
1
2
3
4
5
0
20
40
60
80
$\omega t/2\pi$

The wavelength of the reflected radiation is substantially larger than that of the incident pulse, as consistent with the light reflection from the co-moving relativistic mirror with the e.m. energy transformation into the kinetic energy of the plasma foil.

At this stage, the initially planar plasma slab is transformed into a “cocoon” inside which the laser pulse is almost confined. The protons at the front of the “cocoon” are accelerated up to energies in the multi-GeV range at a rate that agrees with the $t^{1/3}$ scaling predicted by the analytical model. The proton energy spectrum has a narrow feature corresponding to a quasi-monoenergetic beam, but part of it extends over a larger energy interval.

In order to investigate the onset and the nonlinear evolution of the instability of the foil we have performed a series of numerical simulations with heavier ions (aiming at a faster growth rate), using the 2D version of the PIC e.m. relativistic code REMP, which is based on the “density decomposition” scheme[19].

The size of the computation box is $95\lambda \times 40\lambda$ with a mesh of 80 cells per λ. Such a high spatial resolution is required because the interaction of a super intense e.m. pulse with an overdense plasma slab is accompanied by strong plasma compression.
The total number of quasi-particles in the plasma region is equal to 2×10^6.

A thin plasma slab, of width 20λ and thickness 0.5λ, is localized at $x = 20\lambda$. The plasma is made of fully ionized aluminum ions with $Z = 13$, the ion to electron mass ratio is 26.98×1836. The electron density is equal to $64 n_{cr}$, with $n_{cr} = \omega_0^2 m_e / 4\pi e^2$ the critical density.

[19] T. Esirkepov, *Comput. Phys. Commun.* **135**, 144 (2001).

An s-polarized laser pulse with electric field along the z-axis is initialized in vacuum at the left-hand side of the plasma slab.
The pulse has a "gaussian" envelope given by $a_0 \exp(-x^2/2l_x^2 - y^2/2l_y^2)$, with $l_x = 40\lambda$, $l_y = 20\lambda$.
The dimensionless pulse amplitude, a_0, corresponds for $\lambda = 1\mu m$ to the intensity $I = 1.37 \times 10^{23}\ W/cm^2$, close to the value that is expected for the recently proposed super-power lasers such as $HiPER$ and ELI [20].

The boundary condition are periodic along the y-axis and absorbing along the x-axis for both the e.m. radiation and the quasi-particles.

For our choice of the laser-target parameters the opacity condition is fulfilled.

The results of these simulations are shown in the following figures at $t = 75,\ 87.5$.

[20] M. Dunne, *Nature Physics* **2**, 2 (2006); M. Schirber, *Science* **310**, 1610 (2005); http://www.hiper-laser.org/; http://loa.ensta.fr/ .

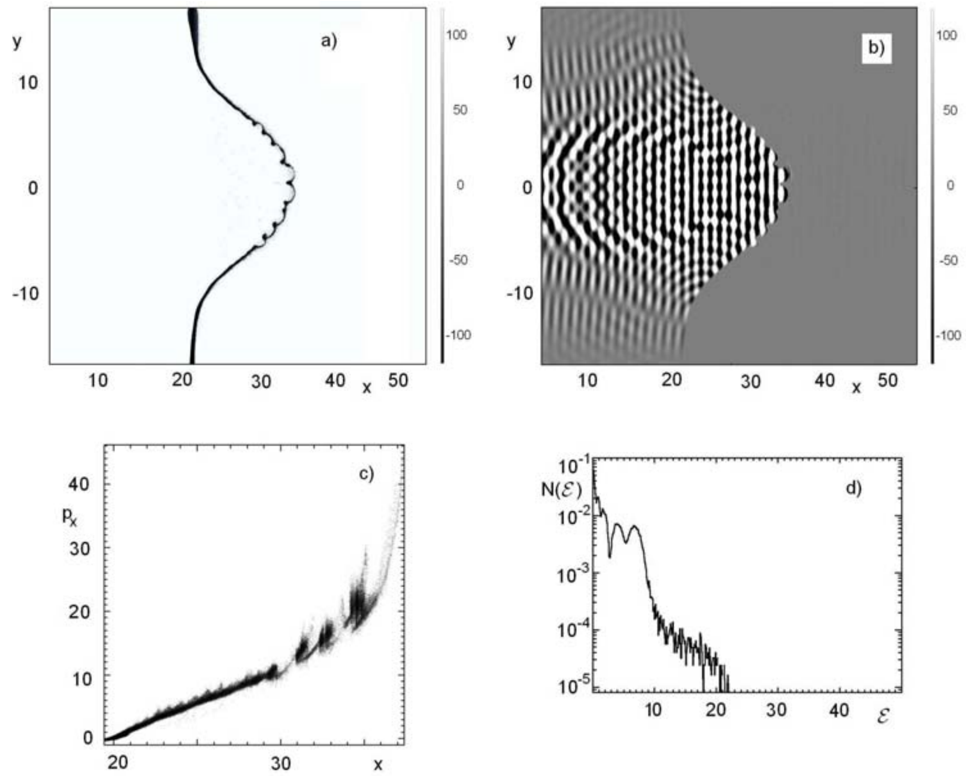


Figure 1: a): ion density distribution in the x-y plane; b) distribution of the electric field E_z; c) ion phase plane (p_x, x); d) ion energy spectrum at $t = 75$.

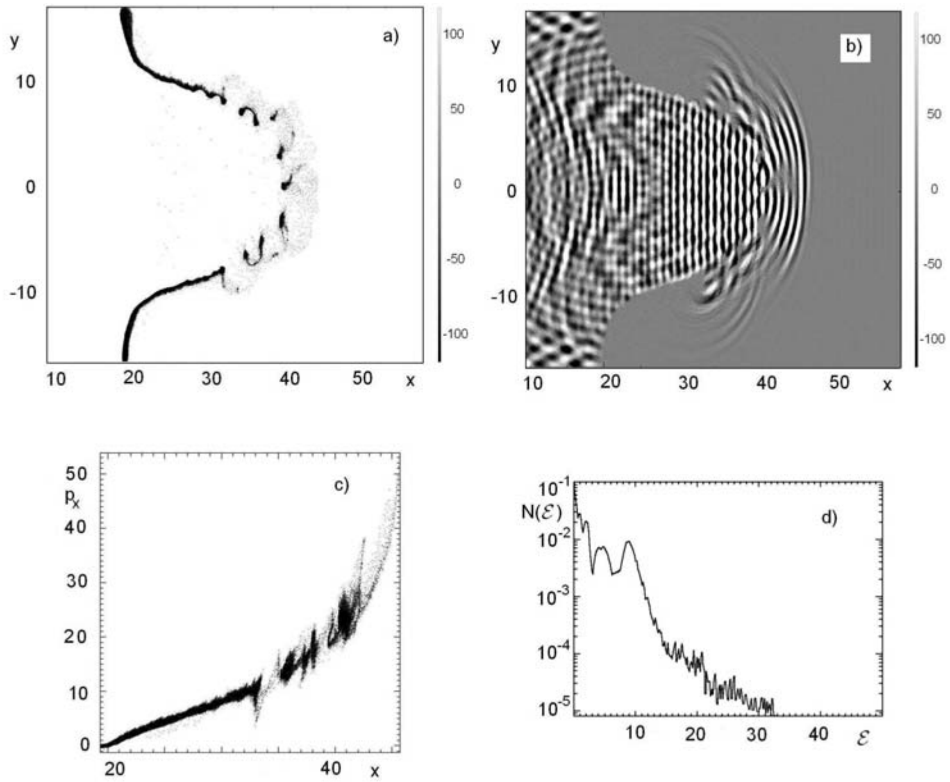


Figure 2: Same as in Fig.1 but at $t = 87.5$.

The distribution in the x-y plane of the ion density and of the z-component of the electric field E_z are shown in frames a) and b), frames c) show the ion phase plane, (p_x, x) and frames d) the energy spectrum of the aluminum ions. The ion momentum and kinetic energy are given in GeV/c and in GeV, respectively. The wavelength λ of the incident radiation and its period $2\pi/\omega_0$ are chosen as units of length and time.

We see the typical initial stage of the Rayleigh-Taylor instability with the formation of cusps and of multiple-bubbles in the plasma density distribution. These are accompanied by a modulation of the e.m. pulse at its front,

At this time the ions are accelerated forward, as seen in their phase plane. Their energy spectrum is made up of quasi-monoenergetic beamlets which correspond to the cusp regions, and of a relatively high energy tail which is formed by the ions at the front of the bubbles.

At $t = 87.5$ we see that the fully nonlinear stage of the instability results in the formation of several clumps in the ion density distribution with more diffuse, lower density plasma clouds between them.

The e.m. wave partially penetrates through, and partially is scattered by, the clump-plasma layer.

The high energy tail in the ion spectrum grows much faster than in the stable case.

At later times, because of the mass reduction of the diffuse clouds at the front of the pulse, the maximum ion energy scales linearly with time.

The local maxima at relatively lower energy correspond to the plasma clumps.

Conclusions

In the relativistic regime the Rayleigh-Taylor instability of a plasma foil accelerated by the radiation pressure of the reflected e.m. pulse develops much more slowly than in the non-relativistic regime.

In the former limit its timescale is inversely proportional to the square root of the ratio between the radiation pressure and the ion mass while in the latter this dependence is reversed.

The use of a properly tailored e.m. pulse with a steep intensity rise can stabilize the shell acceleration.

Numerical simulations show that the nonlinear development of the instability leads to the formation of high-density, high-energy plasma clumps and to a relatively higher rate of ion acceleration in the regions between the clumps.

For the HiPER and ELI laser systems the ion energy can easily reach several tens of GeV.